Bulk Higgs and gauge fields in a multiply warped braneworld model

Ashmita Das,* R. S. Hundi,† and Soumitra SenGupta‡

Department of Theoretical Physics, Indian Association for the Cultivation of Science, 2A & 2B Raja Subodh Chandra Mullick Road, Kolkata 700 032, India

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We readdress the problems associated with bulk Higgs and the gauge fields in a five-dimensional Randall-Sundrum model by extending the model to six dimensions with double warping along the two extra spatial dimensions. In this six-dimensional model, we have a freedom of two moduli scales as against one modulus in the five-dimensional model. With a little hierarchy between these moduli, we can obtain the right magnitude for W and Z boson masses from the Kaluza-Klein modes of massive bulk gauge fields where the spontaneous symmetry breaking is triggered by bulk Higgs. We also have determined the gauge couplings of the standard model fermions with Kaluza-Klein modes of the gauge fields. Unlike the case of the five-dimensional model with a massless bulk gauge field, here, we have shown that the gauge couplings and the masses of the Kaluza-Klein gauge fields satisfy the precision electroweak constraints and also obey the Tevatron bounds.

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I. INTRODUCTION

The hierarchy between electroweak and Planck scales can be addressed in extra-dimensional models. Among these, the model proposed by Randall and Sundrum (RS) assumes warp geometry of the space-time in five dimensions [1]. The fifth dimension has Planck scale length r_c and is compactified on the space S^1/Z_2. Two three-branes are supported on either side of this fifth dimension. The exponential suppression along the fifth dimension naturally suppresses the Planck scale quantities of one three-brane into the electroweak scale on the second three-brane, which is identified as a TeV-brane and can be interpreted as our Universe. In the original RS model, the standard model fields are assumed to lie on the TeV-brane, while only gravity propagates in the bulk. Later works have explored the phenomenology of bulk standard model fields in the warped geometry model [2–6]. It was, however, shown that the models with bulk gauge and Higgs fields where the spontaneous symmetry breaking takes place in the bulk encounter serious problems. The two main problems are

(i) The non-Abelian gauge fields acquire masses through the Higgs vacuum expectation value (vev) generated through spontaneous symmetry breaking in the bulk. This vev, being a bulk parameter, has a magnitude of the order of the Planck scale and, therefore, lends a very large bulk mass $\sim$ Planck mass to the gauge boson in the bulk. As a result, the lowest-lying masses in the Kaluza-Klein (KK) tower of the gauge boson on the visible brane become $\sim$TeV, which fails to comply with the W and Z boson masses ≤ 100 GeV. If one tries to reduce it by adjusting the bulk parameters, then that would jeopardize the unique feature of the Planck-scale-to-TeV-scale warping, i.e., the resolution of the gauge hierarchy problem which was the original motivation of such a warped geometry model.

(ii) For an Abelian gauge boson with zero bulk mass, the massless KK mode on the TeV-brane corresponds to a photon. However, the first excited state in the KK tower has an unacceptably large coupling with fermions. This puts a stringent bound on the mass of this state, such that the model may survive the direct search bound at the Fermilab Tevatron, as well as precision electroweak constraints. However, the mass of this first KK mode turns out to be much lower than the above bound. Once again, it is impossible either to reduce the coupling or to increase the mass by adjusting the bulk parameters without disturbing the resolution of the gauge hierarchy/fine tuning problem.

We refer our readers to [3,6–10], where both these problems have been discussed in detail. Recently, in a generalized five-dimensional RS model with a nonflat visible brane, by adjusting the brane cosmological constant, the problem coming from the precision electroweak tests has been averted, and the bulk Higgs problem has also been resolved [7]. The brane cosmological constant, however, was found to be negative, implying that the visible brane in such a case is an anti-de Sitter three-brane.

In the present work, we address these problems from a different viewpoint, i.e., in the backdrop of a six-dimensional doubly warped model with flat three-branes, which is an extension of the original RS model to more than one extra dimension [11]. In this six-dimensional model, two extra spatial coordinates are compactified such that the space-time manifold is $[M^{(1,3)} \times S^1/Z_2] \times S^1/Z_2$. Compared to the RS model, here, the two extra

*tpad@iacs.res.in
†tprsh@iacs.res.in
‡tpssg@iacs.res.in

dimensions, denoted by angular coordinates y and z, are doubly warped. Four four-branes are located at the orbifolded points: $y = 0, \pi$; $z = 0, \pi$. The intersection of any two four-branes gives a three-brane. The three-brane located at $(y, z) = (\pi, 0)$ is identified with our Universe. Analogous to the RS setup, the mass scale suppression can be felt along both the coordinates y and z. We can choose the moduli of these coordinates, say, R_y and r_z, such that TeV-scale masses can be generated on the visible brane located at $(y, z) = (\pi, 0)$. Since there is an extra freedom through an additional modulus in this model, compared to the five-dimensional RS model, we explore whether the above problems relating to bulk Higgs can be solved in the six-dimensional doubly warped model by adjusting the moduli R_y and r_z suitably.

We organize our paper as follows. In the following section, we explain some essential features of the six-dimensional doubly warped model. In Sec. III, we describe the KK mode analysis of gauge bosons and fermions in six-dimensional bulk and the corresponding modes on the visible three-brane. In Sec. IV, we present our results and argue that the precision electroweak tests put no additional constraints on this model. We conclude in Sec. V.

II. THE SIX-DIMENSIONAL DOUBLY WARPED MODEL

As explained previously, the six-dimensional doubly warped model has a space-time of six dimensions, and the extra two spatial dimensions are orbifolded by Z_2 symmetry [11]. The manifold under consideration is $[M^{(1,3)} \times S^1/Z_2] \times S^1/Z_2$, with four noncompact dimensions denoted by x^μ, $\mu = 0, \cdots, 3$. Since we are interested in the doubly warped model, the metric in this model can be chosen as

$$ds^2 = b^2(z)[a^2(y)\eta_{\mu\nu}dx^\mu dx^\nu + R_y^2 dy^2] + r_z^2 dz^2. \quad (1)$$

As explained before, the angular coordinates y and z represent the extra spatial dimensions with moduli R_y and r_z, respectively. The Minkowski matrix in the usual four dimensions has the form $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$. The functions $a(y)$, $b(z)$ give warp factors in the y and z directions, respectively. The total bulk-brane action of this model has a form [11]

$$S = S_6 + S_5, \qquad S_6 = \int d^4x dy dz \sqrt{-g_6}(R_6 - \Lambda),$$
$$S_5 = \int d^4x dy dz [V_1\delta(y) + V_2\delta(y - \pi)] + \int d^4x dy dz [V_3\delta(z) + V_4\delta(z - \pi)]. \quad (2)$$

Here, $V_{1,2}$ and $V_{3,4}$ are brane tensions of the branes located at $y = 0, \pi$ and $z = 0, \pi$, respectively. Λ is the cosmological constant in six dimensions. The three-branes are located at the intersection points of the four four-branes.

After solving Einstein's equations, the solutions to the warp functions of the metric, as given in Eq. (1), have a form [11]

$$a(y) = \exp(-c|y|), \qquad b(z) = \frac{\cosh(kz)}{\cosh(k\pi)},$$
$$c \equiv \frac{R_y k}{r_z \cosh(k\pi)}, \qquad k \equiv r_z\sqrt{\frac{-\Lambda}{10M_P^4}}. \quad (3)$$

Here, M_P is the Planck scale. Just as in the case of the five-dimensional RS model, the second derivative of the above solutions at the boundaries yields the following brane tensions for the visible (at $y = \pi$, $z = 0$) and Planck three-branes (at $y = 0$, $z = \pi$) [11],

$$V_{\mathrm{vis}} = -8M_P^2\sqrt{\frac{-\Lambda}{10}},$$
$$V_{\mathrm{Planck}} = 8M_P^2\sqrt{\frac{-\Lambda}{10}}(\mathrm{sech}(k\pi) - \tanh(k\pi)), \quad (4)$$

where k depends on the bulk cosmological constant Λ. The warp factors $a(y)$ and $b(z)$, which give the largest suppression from the $y = 0$, $z = \pi$ brane to the $y = \pi$, $z = 0$ brane. For this reason, we can interpret the three-brane formed out of the intersection of four-branes at $y = \pi$ and $z = 0$ as our standard model brane. The suppression factor f on the standard model brane can be written as

$$f = \frac{\exp(-c\pi)}{\cosh(k\pi)}. \quad (5)$$

The desired suppression of 10^{-16} on the standard model brane can be obtained for different combinations of the parameters c and k. However, from the relation for c in Eq. (3), it can be noticed that, in order not to have a large hierarchy in the moduli R_y and r_z, either c or k must be large, whereas the other must be small, e.g., $c \sim 11.4$ and $k \sim 0.1$. It should be noted that such a small hierarchy also exists in the original RS model, where there is an order-one hierarchy between the modulus r and k due to the Planck-to-Tev-scale warping condition $kr \sim 11.5$. In this case, as c and k differ by just one order, therefore, the hierarchy between the two moduli is also only by order one. However, due to the exponential character of the warp factor, such a small hierarchy between the moduli yields large warping along y and a small warping along the z direction. Such a condition, thus, naturally emerges from the requirements of

(i) getting the desired hierarchy of 10^{-16} between the Planck three-brane and the visible three-brane
(ii) and having both the moduli R_y and r_z close to the Planck length.

One question that may arise is whether the two moduli can be stabilized to these desired values, which are very close to the Planck length. For the five-dimensional RS model, it was shown by Goldberger and Wise [2,12] that, by introducing a bulk scalar field and tuning the vev of the scalar field at the boundary, one can stabilize the only modulus of the theory near the Planck length. In this case, a similar procedure can be employed by introducing two independent scalar fields in the bulk, each with either y or z dependence. Once again, by choosing an appropriate vev at the boundaries, the two moduli R_y and r_z can be stabilized to the desired values. These choices of the moduli essentially stabilize the choices of c and k through Eq. (3). In the context of string theory, such moduli stabilization has been discussed using fluxes [13].

It has been argued that this feature may offer an explanation of the small mass hierarchy among the standard model fermions [11].

The six-dimensional model which has been described here is thus viable in explaining the hierarchy between the Planck scale and the electroweak scale without introducing a large hierarchy between the moduli R_y and r_z. In this model, KK modes of bulk scalar fields have been studied [14]. Bulk fermion fields have also been studied in this model, with a possibility of localizing them on a four-brane [15]. However, the possibility of bulk gauge and Higgs fields in this model has not been explored yet. In the following section, we derive the KK modes of the gauge field, fermion fields, and the corresponding couplings to estimate the viability of this model in respect to the problems discussed earlier. We reiterate that our aim is to explore whether we can put the Higgs in the bulk of such a six-dimensional multiply warped model without invoking any contradiction with the precision electroweak test [3,6], as was encountered in the five-dimensional RS model.

III. GAUGE BOSONS AND FERMIONS IN THE BULK

In this section, we explain the KK decomposition and eigenvalue equations of KK gauge bosons and KK fermions which arise from the respective bulk fields after integrating over the two extra dimensions of the model.

A. KK modes of the gauge bosons

For simplicity, we consider a U(1) gauge theory, but our derivation given below is applicable to non-Abelian theory, as well. In a realistic model, the gauge fields can acquire nonzero masses due to spontaneous symmetry breaking. In our model, the Higgs mechanism can take place in the bulk of the six dimensions, and the vev of the Higgs field will be of the order of the Planck scale. The vev of the Higgs field contributes to generate the bulk mass for the gauge field. Hence, after spontaneous symmetry breaking, the invariant action can be written as

$$S_G = \int d^4x dy dz \sqrt{-G}\Big(-\frac{1}{4}G^{MK}G^{NL}F_{KL}F_{MN} - \frac{1}{2}M^2 G^{MK}A_M A_K\Big), \tag{6}$$

where M is the bulk mass $\sim M_P$, and $G = \det(G_{AB})$ is the determinant of the metric G_{AB}, which is given in Eq. (1). $F_{KL} = \partial_K A_L - \partial_L A_K$ is the gauge field strength. Exploiting the gauge symmetry, we can choose the gauge where $A_4 = A_5 = 0$. The KK decomposition of the gauge field can be taken as

$$A_\mu = \sum_{n,p} A_\mu^{(n,p)}(x)\xi_n(y)\chi_p(z)/\sqrt{R_y r_z}. \tag{7}$$

The KK fields in the four dimensions $A_\mu^{(n,p)}$ carry two indices n and p, due to the two additional dimensions of the model. The functions $\xi_n(y)$ and $\chi_p(z)$ give KK wave functions in the y and z directions, respectively. Substituting the above KK decomposition into Eq. (6) and integrating over the y and z coordinates, we demand that the resulting action in the four dimensions must have a form

$$\sum_{n,p} -\frac{1}{4}F_{\mu\nu}^{(n,p)}F^{(n,p)\mu\nu} - \frac{1}{2}m_{n,p}^2 A_\mu^{(n,p)}A^{(n,p)\mu}, \tag{8}$$

where $m_{n,p}$ is the mass of the KK field $A_\mu^{(n,p)}$. This can be achieved, provided the KK wave functions satisfy the following orthonormality condition:

$$\int dy \xi_n(y)\xi_{n'}(y) = \delta_{nn'}, \qquad \int dz b(z)\chi_p(z)\chi_{p'}(z) = \delta_{pp'}. \tag{9}$$

Moreover, in addition to the above normalization conditions, the following eigenvalue equations for the ξ_n and χ_p must also be satisfied:

$$\frac{1}{R_y^2}\partial_y(a^2\partial_y\xi_n) - m_p^2 a^2\xi_n = -m_{n,p}^2\xi_n, \qquad \frac{1}{r_z^2}\partial_z(b^3\partial_z\chi_p) - M^2 b^3\chi_p = -m_p^2 b\chi_p. \tag{10}$$

Here, m_p is a mass parameter which is determined by solving the equation for $\chi_p(z)$, and the value of m_p determines the KK mass $m_{n,p}$ through the eigenvalue equation for ξ_n, as given above.

The second part of Eq. (10) can be solved by approximating $b(z) \sim \exp[-k(\pi - z)] = \exp[-k\tilde{z}]$. By writing $\tilde{\chi}_p(z) = \exp(-3k\tilde{z}/2)\chi_p(z)$, the eigenvalue equation for $\tilde{\chi}_p$ takes the form

$$z_p^2\frac{d^2\tilde{\chi}_p}{dz_p^2} + z_p\frac{d\tilde{\chi}_p}{dz_p} + (z_p^2 - \nu_p^2)\tilde{\chi}_p = 0, \tag{11}$$

where $z_p = \frac{m_p}{k'}\exp(k\tilde{z})$ and $\nu_p^2 = \frac{9}{4} + (\frac{M}{k'})^2$. Here, $k' = k/r_z$. The solutions to the above equation are Bessel functions of order ν_p, and we can write

$$\chi_p(z) = \frac{1}{N_p}\exp\left(\frac{3}{2}k\tilde{z}\right)[J_{\nu_p}(z_p) + b_p Y_{\nu_p}(z_p)], \tag{12}$$

where N_P and b_p are some constants. By demanding that the function $\chi_p(z)$ be continuous at the orbifold fixed points $z = 0$, π, we get the following approximate solution, which determines the spectrum for m_p:

$$3J_{\nu_p}(x_{\nu_p}) + x_{\nu_p}(J_{\nu_p-1}(x_{\nu_p}) - J_{\nu_p+1}(x_{\nu_p})) = 0, \tag{13}$$

where $x_{\nu_p} = \frac{m_p}{k'}\exp(k\pi)$. After solving for m_p using the above equation, we can compute the KK mass $m_{n,p}$ by solving the first part of Eq. (10). By writing $\tilde{\xi}_n = \exp(-c|y|)\xi_n$, the eigenvalue equation for $\tilde{\xi}_n(y)$ becomes

$$y_n^2\frac{d^2\tilde{\xi}_n}{dy_n^2} + y_n\frac{d\tilde{\xi}_n}{dy_n} + (y_n^2 - \nu_n^2)\tilde{\xi}_n = 0, \tag{14}$$

where $y_n = \frac{m_{n,p}}{k'}\exp(c|y|)\cosh(k\pi)$ and $\nu_n^2 = 1 + (\frac{m_p}{k'})^2\cosh^2(k\pi)$. The solution for $\xi_n(y)$ can be written in terms of a Bessel function of order ν_n, multiplied by a growing exponential factor as

$$\xi_n(y) = \frac{1}{N_n}\exp(c|y|)[J_{\nu_n}(y_n) + b_n Y_{\nu_n}(z_n)], \tag{15}$$

where N_n and b_n are some constants. Again, by demanding that the function $\xi_n(y)$ be continuous at the orbifold fixed points $y = 0$, π, the following equation determines the KK mass $m_{n,p}$:

$$J_{\nu_n}(x_{\nu_n}) + x_{\nu_n}(J_{\nu_n-1}(x_{\nu_n}) - J_{\nu_n+1}(x_{\nu_n}))/2 = 0, \tag{16}$$

where

$$x_{\nu_n} = \frac{m_{n,p}}{k'}\exp(c\pi)\cosh(k\pi). \tag{17}$$

The actual KK mass of a gauge field is found by first solving Eq. (13) for m_p and then solving Eq. (16), which is described in the previous paragraph. The wave function of these KK gauge fields is product of wave functions given in Eqs. (12) and (15). In the above analysis, if we put the bulk gauge boson mass $M = 0$, we easily obtain the various KK mode solutions and the corresponding masses. In this case, the lowest-lying KK mode is massless, which corresponds to the standard model photon.

A nice feature of the KK gauge fields in the six-dimensional doubly warped model is that their wave functions can be decomposed into product of functions in the two extra dimensions, a feature which may not be evident for the bulk fermion fields, which is the subject of the next subsection.

B. KK modes of the fermions

The invariant action for a bulk fermion field Ψ in six dimensions is [3,5,6]

$$S_f = \int d^4x dy dz\sqrt{-G}\left\{E_a^A\left[\frac{i}{2}(\bar{\Psi}\Gamma^a\partial_A\Psi - \partial_A\bar{\Psi}\Gamma^a\Psi) + \frac{\omega_{bcA}}{8}\bar{\Psi}\{\Gamma^a, \sigma^{bc}\}\Psi\right] - M_f\bar{\Psi}\Psi\right\}. \tag{18}$$

Here, the capital letter A denotes an index in the curved space, and the lowercase letters a, b, and c denote indices in the tangent space. ω_{bcA} is a spin connection, and E_a^A is an inverse vielbein. M_f is the bulk mass, and $\sigma^{bc} = \frac{i}{2}[\Gamma^b, \Gamma^c]$. The Dirac matrices Γ^a in six dimensions would be 8×8, and they can be taken as [16]

$$\Gamma^\mu = \gamma^\mu \otimes \sigma^0, \quad \Gamma^4 = i\gamma_5 \otimes \sigma^1, \quad \Gamma^5 = i\gamma_5 \otimes \sigma^2. \tag{19}$$

Here, γ^μ are the Dirac matrices in four dimensions, and $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. σ^i, $i = 1, 2, 3$, are the Pauli matrices, and σ^0 is the 2×2 unit matrix. The chirality in six dimensions is defined by the matrix $\bar{\Gamma} = \Gamma^0\Gamma^1\Gamma^2\Gamma^3\Gamma^4\Gamma^5$, as $\bar{\Gamma}\Psi_\pm = \pm\Psi_\pm$. The chiral fermions in six dimensions have both a left- and right-handed chirality of four dimensions, which can be projected by the operators $P_{L,R} = (1 \mp i\Gamma^0\Gamma^1\Gamma^2\Gamma^3)/2$.

As explained in Sec. I, we are interested in estimating the gauge coupling of standard model fermions to the KK gauge bosons. We take the bulk mass of the fermions M_f to be zero, since the masses of standard model fermions are much below the Planck scale. The term that is associated with the spin connection in Eq. (18) would give no contribution, since the metric in Eq. (1) is diagonal. Hence, in our particular case of interest, we expand the first term of Eq. (18), which has the following form:

$$\begin{aligned}S_f = \int d^4x dy dz\{&b^4a^3R_yr_zi(\bar{\Psi}_{+L}\Gamma^\mu\partial_\mu\Psi_{+L} \\ &+ \bar{\Psi}_{+R}\Gamma^\mu\partial_\mu\Psi_{+R} + \bar{\Psi}_{-L}\Gamma^\mu\partial_\mu\Psi_{-L} \\ &+ \bar{\Psi}_{-R}\Gamma^\mu\partial_\mu\Psi_{-R}) + [\bar{\Psi}_{+L}(\Gamma^4D_y + \Gamma^5D_z)\Psi_{+R} \\ &+ \bar{\Psi}_{+R}(\Gamma^4D_y + \Gamma^5D_z)\Psi_{+L} + \bar{\Psi}_{-L}(\Gamma^4D_y + \Gamma^5D_z)\Psi_{-R} \\ &+ \bar{\Psi}_{-R}(\Gamma^4D_y + \Gamma^5D_z)\Psi_{-L}]\},\end{aligned} \tag{20}$$

where the differential operators are defined as $D_y = \frac{i}{2}b^4r_z(a^4\partial_y + \partial_y a^4)$ and $D_z = \frac{i}{2}a^4R_y(b^5\partial_z + \partial_z b^5)$. In the subscript of the fields Ψ, the $\pm$ indicates the chirality in six dimensions, and the L and R stand for the left- and right-handed chirality of the four dimensions. The terms within the square brackets of the above equation give effective masses for the KK modes in the four dimensions. These terms indicate that, in general, we cannot decompose the wave functions into y and z parts separately, like what we have done for the KK wave functions of the gauge bosons as described previously in Eq. (7). Hence, for the bulk fermions, the KK decomposition can be taken as

$$\Psi_{+L,-R}(x^\mu,y,z)=\frac{1}{\sqrt{R_y r_z}}\sum_{j,k}\psi^{(j,k)}_{+L,-R}(x^\mu)f^{(j,k)}_{+L,-R}(y,z)\otimes\begin{pmatrix}1\\0\end{pmatrix},$$
$$\Psi_{-L,+R}(x^\mu,y,z)=\frac{1}{\sqrt{R_y r_z}}\sum_{j,k}\psi^{(j,k)}_{-L,+R}(x^\mu)f^{(j,k)}_{-L,+R}(y,z)\otimes\begin{pmatrix}0\\1\end{pmatrix}. \quad (21)$$

In the above equation, various fields of the form $\psi^{(j,k)}(x^\mu)$ are the KK fields living in the four dimensions, and the f's are the KK wave functions, depending on both y and z coordinates. Substituting the above KK decomposition into Eq. (20) and integrating over the y and z, we can get the action of the form

$$\begin{aligned}S_f = \int d^4x\sum_{j,k}&\bar\psi^{(j,k)}_{+L}i\gamma^\mu\partial\psi^{(j,k)}_{+L}+\bar\psi^{(j,k)}_{+R}i\gamma^\mu\partial\psi^{(j,k)}_{+R}\\&+\bar\psi^{(j,k)}_{-L}i\gamma^\mu\partial\psi^{(j,k)}_{-L}+\bar\psi^{(j,k)}_{-R}i\gamma^\mu\partial\psi^{(j,k)}_{-R}\\&-M_{j,k}(\bar\psi^{(j,k)}_{+L}\psi^{(j,k)}_{+R}+\bar\psi^{(j,k)}_{+R}\psi^{(j,k)}_{+L}\\&+\bar\psi^{(j,k)}_{-L}\psi^{(j,k)}_{-R}+\bar\psi^{(j,k)}_{-R}\psi^{(j,k)}_{-L}),\end{aligned} \quad (22)$$

provided the following normalization and eigenvalue equations for the KK wave functions are satisfied:

$$\int dydzb^4(z)a^3(y)(f^{(j,k)}_{+R,+L,-R,-L}(y,z))^* f^{(j',k')}_{+R,+L,-R,-L}(y,z) = \delta^{j,j'}\delta^{k,k'}, \quad (23)$$

$$\begin{aligned}(i\mathcal{D}_y+\mathcal{D}_z)f^{(j,k)}_{+R}(y,z)&=-M_{j,k}f^{(j,k)}_{+L}(y,z),\\(-i\mathcal{D}_y+\mathcal{D}_z)f^{(j,k)}_{+L}(y,z)&=-M_{j,k}f^{(j,k)}_{+R}(y,z),\\(i\mathcal{D}_y+\mathcal{D}_z)f^{(j,k)}_{-L}(y,z)&=M_{j,k}f^{(j,k)}_{-R}(y,z),\\(-i\mathcal{D}_y+\mathcal{D}_z)f^{(j,k)}_{-R}(y,z)&=M_{j,k}f^{(j,k)}_{-L}(y,z),\end{aligned} \quad (24)$$

where the differential operators are $\mathcal{D}_y=\frac{i}{2R_y}\times(4\partial_y a+2a\partial_y)$ and $\mathcal{D}_z=\frac{i}{2r_z}a(5\partial_z b+2b\partial_z)$. Here, $M_{j,k}$ is the mass of the KK fermion $\psi^{(j,k)}$.

As explained previously, we are interested in standard model fermion coupling with the KK modes of the gauge field. The zero mode of the KK fermions is identified with the standard model fermions. The wave function for these fields can be solved from Eq. (24) by putting $M_{j,k}=0$. Here, we show the solution for the wave function $f^{(0,0)}_{+R}(y,z)$, and the solutions for other chiral fermions can be analogously worked out. The eigenvalue equation we are interested in is

$$(i\mathcal{D}_y+\mathcal{D}_z)f^{(0,0)}_{+R}(y,z)=0, \quad (25)$$

where the differential operators $\mathcal{D}_y$ and $\mathcal{D}_z$ are defined below Eq. (24). For the zero-mode case, we can write the function $f^{(0,0)}_{+R}$ as a product of y and z parts, say, $f^{(0,0)}_{+R}(y,z)=f_y(y)f_z(z)$. This simplification happens only for the zero-mode case, since the factor $a(y)$ in the operators $\mathcal{D}_{y,z}$ can be taken out, and the right-hand side of the above equation is zero. Substituting this form of $f^{(0,0)}_{+R}(y,z)$ into the above equation, we get

$$-\frac{1}{R_y}\frac{(-4c+2\partial_y)f_y}{f_y}+i\frac{1}{r_z}\frac{(5\partial_z b+2b\partial_z)f_z}{f_z}=0. \quad (26)$$

Since the functional dependence on y and z is completely separated out, we can solve for f_y and f_z by taking $\frac{(-4c+2\partial_y)f_y}{f_y}=c_1$, where c_1 is a separation constant. In terms of c_1, the functional dependences of f_y and f_z are given below:

$$f_y(y)=\exp\left(\frac{1}{2}(c_1+4c)y\right),$$
$$f_z(z)=\frac{\exp(\frac{-ic_1r_z}{kR_y}\tan^{-1}(\tanh(kz/2))\cosh(k\pi))}{\cosh^{5/2}(kz)}. \quad (27)$$

The value of c_1 can be worked out in terms of c and k by normalizing the wave function $f^{(0,0)}_{+R}(y,z)$ using Eq. (23).

IV. BULK PHENOMENOLOGY

In the previous section, we have given a description of the KK modes of the gauge bosons and fermions in the bulk of a six-dimensional doubly warped model. Now, using the mode expansion for the bulk fields, we can calculate the gauge couplings of standard model fermions with the KK modes of the gauge bosons.

We now address the two problems mentioned in the beginning. Recall that, in the five-dimensional RS model, it is found that, for the nonzero bulk mass for the non-Abelian gauge field, the lowest-lying mode has mass much higher than 100 GeV, i.e., the masses for W and Z bosons. Also, for the massless gauge boson, the gauge coupling with the first exited KK gauge boson is larger than 1, and, hence, the standard model fermions are strongly coupled [3,6]. Because of this, a stringent lower bound of ~10 TeV on the mass of the first exited KK boson arose because of the precision electroweak tests.

In this section, we repeat this exercise in the six-dimensional doubly warped model and will show that, due to the presence of an additional modulus, we can tune the lowest KK mode mass for the non-Abelian gauge field near 100 GeV, although the spontaneous symmetry breaking takes place in the bulk with a bulk Higgs field with vev ~ Planck scale. Furthermore, for a gauge boson with a zero bulk mass, the coupling-to-mass ratio of the first excited KK mode can survive the precision electroweak test without putting any additional restriction on the model.

The action between the bulk fermions and gauge bosons can be written as [3,6]

$$S_{\rm int} = \int d^4x dy dz \sqrt{-G} g_{6d} \bar{\Psi}(x^\mu, y, z) i \Gamma^a E_a^A A_A(x^\mu, y, z), \Psi(x^\mu, y, z), \tag{28}$$

where g_{6d} is the gauge coupling in the six dimensions, as has been discussed in Sec. III. Substituting the KK decomposition for the gauge and fermion fields in the above equation and also reminding that we are working in the gauge choice where $A_4 = A_5 = 0$, we get the gauge coupling in the four dimensions as

$$g_{+R}^{(j,k)(n,p)} = \int dy dz g_0 \pi b^4 a^3 (f_{+R}^{(j,k)}(y,z))^* f_{+R}^{(j,k)}(y,z) \xi_n(y) \chi_p(z), \tag{29}$$

where $g_0 = g_{6d}/\sqrt{\pi R_y \pi r_z}$ is the effective four-dimensional gauge coupling. In the above equation, we have given a gauge coupling for the KK fermion $\psi_{+R}^{(j,k)}$ with the KK gauge field $A_\mu^{(n,p)}$. Similarly, the gauge couplings with the other KK fermions can be easily obtained by replacing the $+$ with $-$ and R with L accordingly in the above equation. The KK wave functions, f's, ξ, and χ, in the above equation, should be the normalized wave functions, as given by Eqs. (9) and (23). However, in our particular case of interest, where we are interested in precision electroweak tests, we compute gauge couplings of the standard model fermions with the KK gauge fields. Hence, the wave functions for the fermions are of the form in Eq. (27), and the corresponding functions for the KK gauge fields are given in Eqs. (12) and (15).

Now, in order to compute the gauge couplings, the unknown parameters that need to be fixed are k, c, r_z, and the bulk mass of the gauge fields M. The nonzero value for the bulk gauge mass M is around the Planck scale. We can determine the remaining parameters by making the following demands: (a) the lowest nonzero mass of the KK tower of the bulk gauge boson should be identified with either a W or Z boson mass, (b) the suppression f of Eq. (5) should be $\sim 10^{-16}$, and (c) the hierarchy between the moduli R_y and r_z should not be too large. The expression for the KK gauge boson mass is given in Eq. (17). For the lowest nonzero KK gauge boson mass, which is identified as $m_{1,1}$, the root x_{ν_n} would be $\mathcal{O}(1)$. The factor $\exp(c\pi)\cosh(k\pi)$ in this equation, which is the inverse of f, should be $\sim 10^{16}$. By demanding that the lowest KK mode $m_{1,1}$ have a mass of ~ 100 GeV, from Eq. (17), we can naively estimate that $k' \sim 10^{17}$ GeV. Since we would argue that $k \sim 0.1$, a consistent value for the scale r_z is $\frac{1}{r_z} = 7 \times 10^{17}$ GeV, which is about 14 times smaller than the Planck scale. The parameters k and c can be determined from the fact that we should not get a large hierarchy between the moduli R_y and r_z, and we should also get the desired suppression of $f \sim 10^{-16}$ on the standard model brane. We have estimated that, for $k = 0.25$ and $c = 11.52$, the ratio between the moduli is $\frac{R_y}{r_z} = 61$, which is not unacceptably large, and the suppression f also came out to be 1.45×10^{-16}. For these particular values of k, c, and $1/r_z$ we have given the gauge couplings and the corresponding masses of the excited KK gauge fields in Table I. In this table, the gauge couplings $g^{(i,j)}$, where i and j are integers, of the standard model fermion are given as a fraction of the four-dimensional coupling g_0. In the case of $\frac{M}{k'} = 0.5$ or $= 1.0$, we have found that the lowest nonzero mode has a mass of about 95 GeV. Hence, this mode can be identified with the W or Z gauge boson. Since we have got the right amount of W and Z boson masses for the above described values of k, c, and $\frac{1}{r_z}$, we use the same set of values to get the gauge couplings and KK gauge boson masses in the case where the bulk mass M is zero. In this case, we have found that the lowest mode $m_{0,0}$ has zero mass, which can be identified with the photon state. The nonzero KK masses of the photon field and their corresponding gauge coupling values are given in Table II.

As stated in Sec. I, the five-dimensional RS model suffers from the precision electroweak tests, due to the fact that the first excited KK gague boson has a coupling larger than 1 with the standard model field. To parametrize the precision electroweak constraints in extra-dimensional models, the following quantity has been defined [3,17]:

TABLE I. The gauge couplings $g^{(0,0)(i,j)}$ of the standard model fermions are given in the form $\tilde{g}^{i,j} = \frac{g^{(0,0)(i,j)}}{g_0}$, where g_0 is the effective four-dimensional gauge coupling. The masses of the KK gauge bosons $m_{i,j}$ are given in GeV units. M is the bulk gauge boson mass, and $k' = k/r_z$. The nonzero values for $\frac{M}{k'}$ are indicated in the table. $\frac{1}{r_z} = 7 \times 10^{17}$ GeV, $k = 0.25$, and $c = 11.52$. The lowest-lying mode $m_{1,1}$, which corresponds to a W or Z boson, is not included in the table.

$\frac{M}{k'} = 0.5$			$\frac{M}{k'} = 1.0$		
$m_{1,2} = 143.78$	$m_{1,3} = 194.15$	$m_{1,4} = 244.52$	$m_{1,2} = 148.33$	$m_{1,3} = 199.21$	$m_{1,4} = 249.58$
$\tilde{g}^{1,2} = 0.0015$	$\tilde{g}^{1,3} = 0.0028$	$\tilde{g}^{1,4} = 0.0009$	$\tilde{g}^{1,2} = 0.0006$	$\tilde{g}^{1,3} = 0.0027$	$\tilde{g}^{1,4} = 0.0011$
$m_{2,1} = 180.23$	$m_{2,2} = 237.94$	$m_{2,3} = 295.40$	$m_{2,1} = 184.53$	$m_{2,2} = 243.25$	$m_{2,3} = 300.97$
$\tilde{g}^{2,1} = 0.0127$	$\tilde{g}^{2,2} = 0.0012$	$\tilde{g}^{2,3} = 0.0023$	$\tilde{g}^{2,1} = 0.0124$	$\tilde{g}^{2,2} = 0.0005$	$\tilde{g}^{2,3} = 0.0022$
$m_{3,1} = 261.73$	$m_{3,2} = 322.73$	$m_{3,3} = 383.48$	$m_{3,1} = 266.29$	$m_{3,2} = 328.30$	$m_{3,3} = 389.56$
$\tilde{g}^{3,1} = 0.0106$	$\tilde{g}^{3,2} = 0.0010$	$\tilde{g}^{3,3} = 0.0019$	$\tilde{g}^{3,1} = 0.0104$	$\tilde{g}^{3,2} = 0.0004$	$\tilde{g}^{3,3} = 0.0019$

TABLE II. The gauge couplings $g^{(0,0)(i,j)}$ of the standard model fermions are given in the form $\tilde{g}^{i,j} = \frac{g^{(0,0)(i,j)}}{g_0}$, where g_0 is the effective four-dimensional gauge coupling. The masses of the KK gauge bosons $m_{i,j}$ are given in GeV units. M is the bulk gauge boson mass, which is taken to be zero, and $k' = k/r_z$. The values of k, c, and r_z, in this case, are the same as those in Table I. The lowest-lying mode $m_{0,0}$, which corresponds to the photon, is not included in the table.

$\frac{M}{k'} = 0$		
$m_{1,1} = 93.15$	$m_{1,2} = 142.0$	$m_{1,3} = 192.38$
$\tilde{g}^{1,1} = 0.0168$	$\tilde{g}^{1,2} = 0.0019$	$\tilde{g}^{1,3} = 0.0028$
$m_{2,1} = 178.45$	$m_{2,2} = 235.91$	$m_{2,3} = 293.12$
$\tilde{g}^{2,1} = 0.0128$	$\tilde{g}^{2,2} = 0.0015$	$\tilde{g}^{2,3} = 0.0023$
$m_{3,1} = 259.96$	$m_{3,2} = 320.71$	$m_{3,3} = 381.21$
$\tilde{g}^{3,1} = 0.0107$	$\tilde{g}^{3,2} = 0.0012$	$\tilde{g}^{3,3} = 0.0020$

$$V = \sum_n \left(\frac{g_n}{g_0} \frac{m_W}{M_n} \right)^2. \quad (30)$$

Here, m_W is the mass of the W gauge boson, and M_n is the higher KK gauge boson mass. The summation on n in the above equation is over all the higher KK gauge masses M_n with corresponding gauge couplings g_n. In our six-dimensional model, the index n would be replaced by a pair of integers, and we should sum over all nonzero higher-order modes. It has been shown that, by fitting to the precision electroweak observables, the quantity V should satisfy the condition $V < 0.0013$ at 95% confidence level [3]. It can be easily checked that this bound can be respected by the gauge couplings and the KK masses of Tables I and II. From both these tables, we can notice that the gauge couplings are decreasing with the increase of the KK gauge masses for a particular value of $\frac{M}{k'}$. Hence, in the summation of Eq. (30), only the first few higher KK modes are relevant. We have checked that, for $\frac{M}{k'} = 0.5$, V has come out to be about 5×10^{-5}. In the case of the photon where the bulk mass is zero, the value of V is found out to be about 0.000 257. From these results, we can conclude that precision electroweak tests can be satisfied in the six-dimensional doubly warped model without introducing too much hierarchy in the moduli R_y and r_z.

At the Tevatron, the higher KK gauge bosons have been searched in the channel $P\bar{P} \to X \to e^+e^-$, and a limit of $M_T > 700$ GeV on the heavy vector gauge boson (X) mass has been put in [18]. In our case, the gauge couplings of the higher KK modes have been reduced from the four-dimensional coupling g_0 by some factors, which are given in Tables I and II. Hence, in our case, the Tevatron bounds for the nonzero KK mode masses should be greater than $700 \times \tilde{g}^{i,j}$ GeV. As an example, the KK mode of 93.15 GeV mass of Table II has the gauge coupling ratio of 0.0168. Hence, the lower bound from the Tevatron on this KK mode mass would be about 12 GeV, which is much lower than our calculated value of 93.15 GeV. Likewise, from each column of Tables I and II, it can be easily seen that the abovementioned Tevatron bounds can be satisfied. So, the six-dimensional warped model is not only free from the precision electroweak constraints, but also from the Tevatron limits.

V. CONCLUSIONS

The extra-dimensional phenomenological models in a warped geometry encounter problems in putting the Higgs and the gauge fields in the bulk. It was shown that it is impossible to construct proper W and Z boson masses on the brane from the KK modes of a non-Abelian bulk gauge field through spontaneous symmetry breaking in the bulk. Also, proper coupling and masses for the first KK excitation of a massless bulk gauge field, consistent with an electroweak precision test, as well as a Fermilab Tevatron mass bound, is hard to obtain without changing the bulk parameter of the theory from their desired values. In this work, we have shown that it is possible to resolve both these problems in a multiply warped geometry model where there is more than one modulus. Considering a six-dimensional model, we have shown that, by setting one of the moduli approximately two orders smaller than the Planck scale, we can have the mass for the lowest-lying mode of the bulk gauge field (with bulk mass $\sim M_P$, acquired through a spontaneous symmetry breaking in the bulk) on the TeV-brane to be of the order of 100 GeV, which, therefore, may be identified with the W or Z boson mass. Moreover, such a choice for the moduli, which does not contradict the main spirit of the RS model, lowers the coupling of the first KK mode excitation of a massless bulk gauge field so that it can escape the electroweak precision test. We have determined the KK mode masses, as well as their couplings, for different choices of the parameter of the theory, namely, the ratio of the bulk mass and the bulk cosmological constant. In the entire analysis, the value of the warp factor is maintained at 10^{-16}, so that the resolution of the gauge hierarchy problem, the main objective of these models, can be achieved. These findings can be easily extended to models with an even larger number of warped extra dimensions [11]. One would then arrive at similar conclusions with a lesser hierarchy among different moduli. We can, therefore, conclude that a consistent description of a bulk Higgs and gauge field with spontaneous symmetry breaking in the bulk can be obtained in a warped geometry model if the RS model in five dimensions is generalized to six or higher dimensions with more than one modulus. The phenomenology of these models, therefore, becomes an interesting area of study for the forthcoming collider experiments.

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