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Dark matter models with uniquely spin-dependent detection possibilities

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With much higher sensitivities due to coherence effects, it is often assumed that the first evidence for direct dark matter detection will come from experiments probing spin-independent interactions. We explore models that would be invisible in such experiments, but detectable via spin-dependent interactions. The existence of much larger (or even only) spin-dependent tree-level interactions is not sufficient, due to potential spin-independent subdominant or loop-induced interactions. We find that, in such a way, most models with detectable spin-dependent interactions would also generate detectable spin-independent interactions. Models in which a light pseudoscalar acts as the mediator seem to uniquely evade this conclusion. We present a particular viable dark matter model generating such an interaction.

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I. INTRODUCTION

The sensitivity of dark matter (DM) direct detection experiments is undergoing rapid progress and is expected to continue in the next decade. There are a number of proposed experiments which will probe complementary aspects of dark matter properties with much better sensitivities than the existing ones: DM mass, spin-independent (SI) and spin-dependent (SD) cross sections, the dependence of the cross sections on the target nuclei, directional information, etc.

The focus, rightly, is often on the detection of spin-independent DM interactions, because, due to a coherence effect, the SI interaction cross section with heavy nuclei is enhanced by A^2, the number of nucleons in a nucleus, and is, therefore, expected in many models to be the dominant interaction in DM detectors.

There is a good chance that, in the not-too-distant future, direct detection experiments will be able to extend their sensitivity to cover the full detectable parameter space for SI cross sections, down to the 10^{-48} cm^2 level, below which atmospheric neutrinos constitute an irreducible background.

Prior studies [1–4] have considered the relationship between SI and SD cross sections, concluding that the two are typically correlated when a viable dark matter candidate is present. Most of the discussions have been in the context of the minimal supersymmetric standard model. (Similar statements have been made about DM candidates in universal extra dimensions [1] and little Higgs models [3], as well.) In general, the common wisdom is that SI experiments have a much better chance of first direct detection discovery.

The generality of this conclusion cannot be addressed by merely considering operators; one must explore the underlying models which determine relationships between operator coefficients. For example, the conclusions stated above ultimately stemmed from the assumption of DM with electroweak charges, which generically implies both mediators with at least weak-scale masses to justify null results thus far and couplings to the Higgs leading to SI signals. Once this condition is relaxed, the relationship between SI and SD cross sections becomes weaker, and models in which SD interactions are more easily accessible, or even the only interaction accessible in direct detection experiments, become feasible.

Here, we point out that, in order to impose the last condition, i.e., uniquely SD detection, the consideration of subleading effects is crucial. Since, due to coherence effects, SI experiments are more sensitive than SD ones (currently by 5 orders of magnitude), a loop-induced SI process might be only marginally more difficult or possibly even as easy to detect than a tree-level SD one. Upon considering these additional operators, we find that models with light pseudoscalars are uniquely capable of generically evading such detection modes.

Although several ingredients of our analysis appear in the literature [5,6], the impact of light mediators on a general analysis of operators has not been heretofore discussed, and the effect of loop corrections on DM scattering has not been considered in this context. In Sec. II, we review current bounds on SI and SD cross sections and the expected improvements. Sec. III then constitutes the bulk of the paper. We discuss operators relevant for the detection of DM particles, including operators which become important in the case of light mediators. We then consider which models could generate exclusively SD interactions and calculate the loop-induced interactions that would simultaneously be present. In Sec. IV, we construct a viable model achieving our goals, in which the SI interaction is out of reach, but the SD interaction may be detected in future experiments. Sec. V concludes.

II. PROSPECTS OF DIRECT DETECTION

The best SI bounds come, at present, from XENON10 [7], CDMS [8], and XENON100 [9], with the highest sensitivity from XENON100 near 3×10^{-44} cm^2 at 50 GeV. In general, optimal sensitivity is for DM masses of the order of the mass of the recoiling nucleus. At higher masses, the sensitivity decreases roughly as $1/m_{\rm DM}$. Within the coming years, XENON100, LUX, and SuperCDMS can improve these bounds down to the 10^{-45} cm^2 or possibly near the 10^{-46} cm^2 level. Ultimately, multiton xenon or germanium experiments can achieve sensitivities to 10^{-47} cm^2 or maybe even 10^{-48} cm^2, at which point atmospheric neutrinos form an irreducible background [10–12] and achieving sensitivity to lower SI DM-nucleon interactions seems unfeasible.

For SD detection, the best current limit for DM-proton interaction is near 2×10^{-38} cm^2 from SIMPLE [13], with slightly weaker bounds from COUPP [14], KIMS [15], and PICASSO [16], at similar optimal masses as above. For DM-neutron cross sections, the best bound is from XENON10 [17] at 5×10^{-39} cm^2 at optimal sensitivity near 30 GeV. Within the next few years, COUPP [18], PICASSO [19], and XENON100 should improve these to a few $\times\, 10^{-40}$ cm^2, for both protons and neutrons. These limits could then be extended to near 5×10^{-41} cm^2 with experiments such as DMTPC, or to 5×10^{-43} cm^2 for a 500 kg extension of COUPP [20].

Bounds on direct detection cross sections can also come indirectly from other experiments. One source is from DM annihilation signals from the Sun. The annihilation at equilibrium is proportional to the rate of DM capture, which is driven by the same interactions as direct detection. In this case, the SI terms are not so strongly enhanced over the SD ones, since this capture is mostly due to light nuclei, almost entirely hydrogen and helium. (Some enhancement does occur due to small amounts of Fe and O, but bounding the SD interaction, neglecting the SI contribution, is conservative.) Super-Kamiokande [21] and IceCube [22] used this to place limits on SD proton interactions at around 10^{-38} cm^2, assuming annihilations primarily to $b\bar{b}$. Above $m_{\rm DM} \sim 250$ GeV, IceCube could even place a bound at 2×10^{-40} cm^2 if the DM annihilated to W^+W^-. However, these indirect bounds do not apply in the case of light mediators, which will be discussed below, since, if the annihilations proceed through a light on-shell particle, decays to neither heavy quarks nor W bosons occur.

Other bounds can be placed from constraints on operators from collider searches [23–25]. In cases where the mediator can be integrated out, these searches place bounds on interactions of very light dark matter better than those of direct detection, while remaining competitive with them for SD interactions of DM that can be directly produced at the Tevatron. The expected LHC reach is expected to also remain competitive with direct detection sensitivites of upcoming expierments [24]. However, for mediators light enough to be produced on-shell, the bound deteriorates rapidly [25] and is also not applicable for the class of models we discuss below.

III. GENERAL CONSIDERATIONS

A. Operator analysis

In order to survey possible models, we first identify all operators through which dark matter may interact with detectors. In doing so, we will see which interactions give us the signals we are looking for and which operators need to be suppressed by small coefficients or forbidden by symmetries. Similar operator analyses have been considered before in Refs. [2,6,26]. We present it here as a guide to possible types of underlying structure.

We assume that the mediator is heavy enough so that, for the purposes of direct detection, describing the interaction of dark matter via a contact term is a reasonable approximation. Beyond this, we want to consider interactions with dark matter of arbitrary spin, without making additional assumptions, such as parity conservation. At the structural level of the operators, this encompasses both elastic and inelastic scattering. Having two (or more) fields of different mass in the DM sector only leads to differences in kinematics and the presence of operators that are otherwise zero for Majorana fermions and real bosons for symmetry reasons (discussed below).

The smallest number of operators, as expected, is furnished by scalar dark matter candidates. These are listed in Table I. Note that $\mathcal{O}_3^s$ and $\mathcal{O}_4^s$ are nonvanishing only if the dark matter candidate is complex.

For fermionic dark matter, the operators are listed in Table II. If the dark matter candidate is a Majorana fermion, the operators $\mathcal{O}_5^f$, $\mathcal{O}_7^f$, $\mathcal{O}_9^f$, and $\mathcal{O}_{10}^f$ are absent, as they are odd under charge conjugation. There are only two operators with tensor couplings. Since $\sigma^{\mu\nu}\gamma^5 = i\epsilon^{\mu\nu\rho\sigma}\sigma_{\rho\sigma}/2$, not all (pseudo)tensor-(pseudo)tensor combinations are linearly independent. In addition, $\mathcal{O}_7^f$ has separate SD terms suppressed independently by v^2 and q^2, while $\mathcal{O}_6^f$, commonly referred to as the anapole moment coupling, has contributions to both SI and SD cross sections with different suppression factors. (Here, as elsewhere in the paper, v is the velocity of DM in the halo, approximately 10^{-3}, while q is the momentum transfer in the interaction.)

Finally, in Table III, we give the possible operators for vector dark matter candidates. Similar to the case of scalar dark matter, the operators $\mathcal{O}_3^v$ and $\mathcal{O}_4^v$ are only present if the vector is complex.

TABLE I. Operators relevant for scalar dark matter detection. The suppression factor given is for the relevant cross section. Operators $\mathcal{O}_3^s$ and $\mathcal{O}_4^s$ are only allowed for complex scalars.

Operator	SI/SD	Suppression
$\mathcal{O}_1^s = \phi^2\bar{q}q$	SI	—
$\mathcal{O}_2^s = \phi^2\bar{q}\gamma^5 q$	SD	q^2
$\mathcal{O}_3^s = \phi^\dagger\partial^\mu\phi\bar{q}\gamma_\mu q$	SI	—
$\mathcal{O}_4^s = \phi^\dagger\partial^\mu\phi\bar{q}\gamma_\mu\gamma^5 q$	SD	v^2

TABLE II. Operators relevant for fermionic dark matter detection. Operators $\mathcal{O}_5^f$, $\mathcal{O}_7^f$, $\mathcal{O}_9^f$, and $\mathcal{O}_{10}^f$ only exist if the dark matter is Dirac. Notations as in Table I.

Operator	SI/SD	Suppression
$\mathcal{O}_1^f = \bar{\chi}\chi\bar{q}q$	SI	—
$\mathcal{O}_2^f = \bar{\chi}i\gamma^5\chi\bar{q}q$	SI	q^2
$\mathcal{O}_3^f = \bar{\chi}\chi\bar{q}i\gamma^5 q$	SD	q^2
$\mathcal{O}_4^f = \bar{\chi}\gamma^5\chi\bar{q}\gamma^5 q$	SD	q^4
$\mathcal{O}_5^f = \bar{\chi}\gamma^\mu\chi\bar{q}\gamma_\mu q$	SI	—
$\mathcal{O}_6^f = \bar{\chi}\gamma^\mu\gamma^5\chi\bar{q}\gamma_\mu q$	SI SD	v^2 q^2
$\mathcal{O}_7^f = \bar{\chi}\gamma^\mu\chi\bar{q}\gamma_\mu\gamma^5 q$	SD	v^2 or q^2
$\mathcal{O}_8^f = \bar{\chi}\gamma^\mu\gamma^5\chi\bar{q}\gamma_\mu\gamma^5 q$	SD	—
$\mathcal{O}_9^f = \bar{\chi}\sigma^{\mu\nu}\chi\bar{q}\sigma_{\mu\nu}q$	SD	—
$\mathcal{O}_{10}^f = \bar{\chi}i\sigma^{\mu\nu}\gamma^5\chi\bar{q}\sigma_{\mu\nu}q$	SI	q^2

TABLE III. Operators relevant for vector dark matter detection. Operators $\mathcal{O}_3^v$ and $\mathcal{O}_4^v$ only exit for complex vector fields. Notations as in Table I.

Operator	SI/SD	Suppression
$\mathcal{O}_1^v = B^\mu B_\mu\bar{q}q$	SI	—
$\mathcal{O}_2^v = B^\mu B_\mu\bar{q}\gamma^5 q$	SD	q^2
$\mathcal{O}_3^v = B_\mu^\dagger\partial^\nu B^\mu\bar{q}\gamma_\nu q$	SI	—
$\mathcal{O}_4^v = B_\mu^\dagger\partial^\nu B^\mu\bar{q}\gamma_\nu\gamma^5 q$	SD	v^2
$\mathcal{O}_5^v = B^\mu\partial_\mu B^\nu\bar{q}\gamma_\nu q$	SI	v^2q^2
$\mathcal{O}_6^v = B^\mu\partial_\mu B^\nu\bar{q}\gamma_\nu\gamma^5 q$	SD	q^2
$\mathcal{O}_7^v = \epsilon_{\mu\nu\rho\sigma}B^\mu\partial^\nu B^\rho\bar{q}\gamma^\sigma q$	SI SD	v^2 q^2
$\mathcal{O}_8^v = \epsilon_{\mu\nu\rho\sigma}B^\mu\partial^\nu B^\rho\bar{q}\gamma^\sigma\gamma^5 q$	SD	—

There are a large number of operators that could mediate SD interactions. However, for our purposes, some of these may be ignored right away. For example, $\mathcal{O}_6^f$ and $\mathcal{O}_7^v$ lead to both SD and SI interactions of comparable magnitudes. It may naively seem that all operators that come with kinematic suppression factors can be dismissed just as easily. After all, with DM in the galactic halo at such low velocities, the nonrelativistic limit is appropriate for detection, and, traditionally, such operators have indeed been neglected. Let us examine this assumption more carefully.

Within the dominant weakly interacting massive particle paradigm, the mediator has typically been assumed to be at the weak scale, with direct detection occurring with $\mathcal{O}(100\ \text{MeV})$ momentum transfers and $\mathcal{O}(100\ \text{keV})$ recoil energies. In that case, the integrated-out mediator sets the scale of the operators through a factor of $1/m_W^2$. In the nonrelativistic limit, terms like $\bar{\psi}\gamma^5\psi$ are suppressed by factors of $|\vec{q}|/2m_N$ or $|\vec{q}|/2m_{\text{DM}}$, while others, like $\bar{\psi}\gamma^\mu\gamma^5\psi$, have some components scale as v. Operators with any of these factors can typically be dismissed, because they are suppressed by $O(10^3)$. This means that, even if present, such interactions can be ignored. For example, in the case of Majorana fermion dark matter, such as the neutralino in supersymmetric models, the only two operators that need to be considered are scalar-scalar and axial-vector–axial-vector [11,27,28]; all others are highly suppressed.

However, recent interest in explaining various possibly DM-related anomalies have introduced models with $O(\text{GeV})$ mediator particles. In this case, if the leading operators were suppressed or forbidden for some symmetry reason, the traditionally subleading operators could lead to contributions of the correct magnitude to be accessible to current or future direct detection experiments. As pointed out in Ref. [29], these two statements may, in fact, be connected, since the spontaneous breaking of a symmetry forbidding the appearance of certain operators can provide for a natural explanation for the presence of light (pseudo) Nambu-Goldstone scalars.

This opens up new possibilities. If SI operators without kinematic suppression factors are forbidden or highly suppressed for other reasons, the set of operators which may lead to a detectable SD signal becomes much larger.

B. Renormalizable models

If we wish to remain agnostic about the nature of the DM-nucleon interactions, we can say no more. However, if a further step is to be taken, it seems most conservative to assume that the DM comes from some theory with renormalizable interactions in which the operators leading to direct detection come from heavy states that have been integrated out. One can then ask what sort of renormalizable interactions could lead to the operators given above. Such a procedure was followed in Ref. [5]. Here, we quote their results, along with the additional possibilities afforded by interactions yielding kinematically suppressed operators.

For scalar DM, the only option for generating solely SD operators seems to be a t-channel exchange of a light pseudoscalar, which yields $\mathcal{O}_2^s$. While such an interaction breaks parity, given that parity is badly broken already in the standard model (SM), this is not a serious concern.

For fermionic DM, several possibilities present themselves. Once again, a t-channel light pseudoscalar exchange produces solely SD interactions via $\mathcal{O}_4^f$. Additionally, for Majorana fermions, the t-channel exchange of a vector with axial couplings, either the SM Z or a new Z', will generate only a single kinematically unsuppressed operator, $\mathcal{O}_8^f$. Other options are an s- or u-channel coupling through either a scalar or vector, provided the couplings are chiral, in which case $\mathcal{O}_8^f$ is generated again. If the couplings are not chiral, $\mathcal{O}_1^f$ is produced, as well.

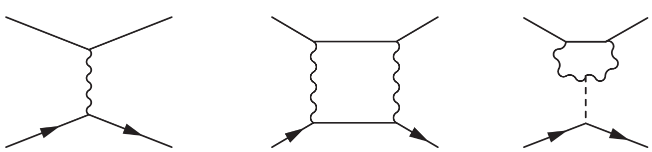
Finally, for vector DM, a light pseudoscalar in the t channel produces only $\mathcal{O}_2^v$, which breaks parity as in the scalar case. Alternatively, an s- or u-channel coupling through a fermion makes $\mathcal{O}_8^v$ the leading operator, if the coupling is chiral while the vector boson is real.

C. Loops and subleading interactions

Suppose that one is presented with a model in which one of the above SD interactions is the only one present or dominant over others by many orders of magnitude. Does that mean that only an experiment sensitive to SD interactions would see a signal? Not necessarily.

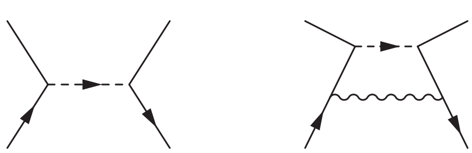
The bounds on SI cross sections are currently 5–7 orders of magnitude higher than the SD ones, and this looks to continue to be the case in the future. Therefore, if any of the SD interactions discussed above induce subleading SI couplings, such an effect could potentially be visible in a SI experiment. There are two sources for such effects. First, there are kinematically suppressed contributions of tree-level scattering that were ignored above. These are easily estimated from Tables I, II, and III given earlier. Second, the tree-level SD interactions can induce SI couplings at loop level. These are not as simple to estimate and should be calculated to confirm their effect.

Let us consider a Z (or Z' exchange) with a Majorana fermion, as in Fig. 1(a). While the dominant contribution comes from $\mathcal{O}_8^f$, also present is $\mathcal{O}_6^f$, the anapole coupling. We see that this gives rise to a SI interaction suppressed by v^2. Similarly, both the scalar exchange of Fig. 2(a) and the equivalent diagram for vector exchange give an anapole coupling after using Fiertz identities. A fermion exchange of the same form in the case of vector DM produces $\mathcal{O}_7^v$, as well as $\mathcal{O}_8^v$, in the chiral limit, which again mediates a v^2-suppressed SI coupling. In all of these cases, there is a SI scattering cross section no more than $O(10^6)$ smaller than the SD one, independent of any other field content of a model. This means that such interactions would be seen in SI experiments simultaneously or in the next generation of experiments after they appear in SD ones. Only the pseudoscalar exchanges evade this, as they lead to no v^2-suppressed subleading contributions to DM-nucleon scattering at all.



(a) Tree level (b) Loop processes

FIG. 1. The tree- and loop-level contributions to the scattering of Majorana fermions through a Z boson. For all box diagrams, the crossed box diagram is included in calculations but not depicted. In the last diagram, a Higgs mediates the scattering through a Z loop.



(a) Tree level (b) Loop processes

FIG. 2. The tree- and loop-level contributions to the scattering of Majorana fermions through an s-channel scalar.

All the aforementioned interactions should also be computed at the one-loop level. While these will be suppressed by loop factors and extra couplings, they may also generate SI interactions. For large enough couplings, these loops might even give rise to interactions larger than the kinematically suppressed ones discussed above and so might be even more readily detectable.

Without making any further assumptions about the underlying model, we can already identify diagrams which will produce SI interactions at loop level. For SD interactions involving a t-channel exchange, at a minimum, exchanging two mediators in a box diagram will give rise to a SI interaction. For an s or u-channel process, a SI loop-level contribution can come from a loop with W or Z bosons exchanged between the quarks.

Consider the exchange of a Z with axial couplings to quarks. (We will discuss the case of a Z' shortly.) In that case, the quark-level operator for tree-level scattering [Fig. 1(a)] is

$$\frac{g_2^2}{2\cos^2\theta_W} T_3^q \frac{Q}{2} \frac{1}{m_Z^2} \bar{\chi}\gamma^\mu\gamma^5\chi\bar{q}\gamma_\mu\gamma^5 q, \tag{1}$$

where Q is the coupling of the DM to the Z. Then, the DM-proton SD cross section generated is (see Appendixes A and B for details)

$$\sigma_{\rm SD}^{\chi p} \approx (1.5\times 10^{-39}\ {\rm cm}^2)\left(\frac{Q}{0.1}\right)^2, \tag{2}$$

with the DM-neutron cross sections about 20% smaller. In this case, two one-loop processes lead to SI effective interactions: one with two Z exchanges and a Higgs coupling through a Z loop to the DM [Fig. 1(b)]. We work in the limit $m_q \ll m_Z \ll m_{\rm DM}$. (This limit is generally the one in which the DM has the correct relic abundance in models where the only coupling of the DM to the quarks is through electroweak bosons, while foregoing the last inequality only yields $O(1)$ changes; see Ref. [30].) The SI contribution to the effective coupling is then [30,31][1]

[1] In deriving this result, along with those following, we have set several quark operators, such as

$$m_q\bar{\chi}\chi\bar{q}q, \qquad \bar{\chi}\chi\bar{q}i\not{\partial}q,$$
$$\frac{4}{3m_{\rm DM}}\bar{\chi}i\partial_\mu\gamma_\nu\chi\bar{q}i\Big(i\partial^\mu\gamma^\nu + \partial^\nu\gamma^\mu - \frac{1}{2}g^{\mu\nu}\not{\partial}\Big)q,$$

which all simplify to $m_q\bar{\chi}\chi\bar{q}q$ on shell, but can have different nuclear matrix elements, to their on-shell value. In fact, this seems to yield a conservative estimate, as, out of the nuclear matrix elements known, the first one has the smallest value (for a detailed discussion of these issues, see Ref. [32]).

$$\frac{1}{4\pi}\frac{g_2^4 Q^2}{\cos^4\theta_W m_Z}\left[\frac{(T_3^q)^2}{2m_Z^2}+\frac{1}{4m_h^2}\right]m_q\bar{\chi}\chi\bar{q}q. \tag{3}$$

Taking a reference value of $m_h = 120$ GeV, these interactions will induce a SI cross section of

$$\sigma_{\rm SI}^{\chi N} = (4\times 10^{-47}\ {\rm cm}^2)\left(\frac{Q^2}{0.1}\right)^2. \tag{4}$$

Asking that the SD signal be just beyond current SD experimental bounds implies $Q \sim 0.3$, giving a SI cross section of 4×10^{-47} cm^2. This, while not detectable in experiments underway, is feasible with ones in preparation.

This result would make the v^2-suppressed contribution to SI scattering dominant. However, it is worth mentioning that this cross section acts as a lower bound—it could be that the DM particle is part of a larger representation of $SU(2)$, in which case, additional loops involving W's would also contribute. Generally, the size of the cross section grows as n^2, with n the dimension of the representation [31], making it possible for the loop contribution to be dominant and not merely competitive with the kinetically suppressed contribution, and even being large enough to be discovered simultaneously with the SD signal.

If one wishes to consider models with a new Z', then the existence of a Z' with Higgs coupling becomes model-dependent. To talk about a lower bound, we can then ignore the contribution of the second term in the effective coupling. The heavier mediator mass that such a model would entail would have to be offset with a larger coupling in order to be detectable. Thus, at loop level, one would generally expect the effective interaction to be of at least similar size, or possibly larger, due to the higher power of the coupling appearing in the loop-induced term.

If one considers the possibility of a light Z', which is not ruled out by collider constraints down to the GeV range for gauge couplings smaller than the SM by 10^{-2}, the situation discussed above would be reversed, and one would expect a smaller loop-induced contribution. However, the SI contribution, due to kinematically suppressed operators, is insensitive to changes in the mediator mass and would still be present. Constructing a model without such operators and without significant fine-tuning seems extremely difficult. It is difficult to say more in generality, due to the large freedom in assigning masses and charges under a new gauge group.

Now, let us consider DM with chiral couplings to the SM via an s or u channel. The most model-independent loop-level processes here come from box diagrams with the quarks exchanging a W or Z boson, an example of which is given in Fig. 2(b). The contributions of the loops have completely different forms, depending on whether the coupling of the DM is left- or right-handed. However, in all cases, the loop-level processes only give rise to suppressed SD contributions. In addition to DM of the form in Fig. 2, this is also true for the cases of fermionic DM with a vector mediator and vector DM with a fermion mediator of similar topologies. In this case, we find that the most reliable lower bound on a SI cross section in this case comes from the v^2-suppressed contribution to the tree-level interaction discussed earlier.

Finally, let us turn to the box diagrams induced in the cases of light pseudoscalar exchange, Fig. 3(b). First, we consider the case of scalar DM. At tree level, the operator obtained after integrating out the pseudoscalar is

$$\frac{1}{m_a^2}\xi y_q m_\phi \phi^\dagger\phi\bar{q}i\gamma^5 q, \tag{5}$$

where y_q is the Yukawa coupling of the quark, so ξ absorbs both the coupling of the DM and mediator and any scaling to Yukawas of the mediator-quark coupling. This leads to a tree-level cross section of

$$\sigma_{\rm SD}^{\phi p} \approx (8\times 10^{-37}\ {\rm cm}^2)\left(\frac{\xi}{0.1}\right)^2\left(\frac{1\ {\rm GeV}}{m_a}\right)^4. \tag{6}$$

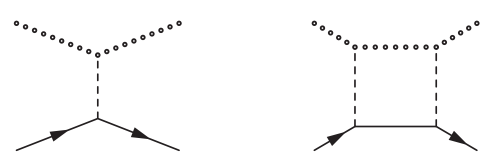
(See Appendix B for the definition of the cross section in cases of kinematically suppressed operators.) For a mediator with mass of a few GeV and $\xi = 0.01$, this would be accessible to currently running searches.

The calculation of the loop diagram in the same limits as the previous Z-mediated case does not give as compact of an answer, but can be expressed in closed form in terms of Passarino-Veltman scalar integrals [33], computed with the use of FeynCalc [34] as

$$\begin{aligned}&\frac{1}{(4\pi)^2}\xi^2 y_q^2[C_0(m_\phi^2,0,m_\phi^2;m_\phi^2,m_a^2,0)\\&-C_0(m_\phi^2,m_\phi^2,0;m_a^2,m_\phi^2,m_a^2)\\&+m_a^2 D_0(m_\phi^2,m_\phi^2,0,0,0,m_\phi^2;m_a^2,m_\phi^2,m_a^2,0)]\phi^\dagger\partial^\mu\phi\bar{q}\gamma_\mu q.\end{aligned} \tag{7}$$

A numerical evaluation of the coefficients shows the C_0 and D_0 functions with these parameters to scale as $\ln(m_a/m_\phi)$ and $\ln^2(m_a/m_\phi)$, respectively, beyond their overall $1/m_\phi^2$ dependence. Using a fiducial value of $m_a/m_\phi = 0.01$ gives

$$\frac{1}{(4\pi)^2}\frac{\xi^2 y_q^2}{m_\phi^2}C_S\phi^\dagger\partial^\mu\phi\bar{q}\gamma_\mu q, \tag{8}$$



(a) Tree level (b) Loop processes

FIG. 3. The tree- and loop-level contributions to scattering DM mediated by a light pseudoscalar. The dotted line can represent either a scalar, fermion, or vector boson.

where $C_S \approx 80$. Note that, if the DM were real, this operator vanishes identically, and there is no loop-induced coupling at one-loop order at all. If present, the cross section induced is

$$\sigma_{\rm SI}^{\phi N} \approx (4\times 10^{-54}\ {\rm cm}^2)\left(\frac{\xi}{0.1}\right)^4\left(\frac{100\ {\rm GeV}}{m_\phi}\right)^4, \quad (9)$$

undetectable for any choice of parameters that would make the SD cross section detectable.

The case of vector DM is very similar. For

$$\frac{1}{m_a^2}\xi y_q m_B B_\mu^\dagger B^\mu \bar{q} i\gamma^5 q, \quad (10)$$

the tree-level cross section takes the same value as Eq. (6). Meanwhile, the loop-induced coupling is

$$\begin{aligned}\frac{1}{(4\pi)^2}\xi^2 y_q^2\Big\{&C_0(m_B^2, 0, m_B^2; m_B^2, m_a^2, 0)\\ &- C_0(m_B^2, m_B^2, 0; m_a^2, m_B^2, m_a^2)\\ &+ m_a^2 D_0(m_B^2, m_B^2, 0, 0, 0, m_B^2; m_a^2, m_B^2, m_a^2, 0)\\ &+\frac{1}{4m_B^2}[B_0(m_B^2; m_a^2, m_B^2)\\ &- B_0(m_B^2; 0, m_B^2)]\Big\}B_\nu^\dagger\partial^\mu B^\nu\bar{q}\gamma_\mu q, \end{aligned}\quad (11)$$

which numerically evaluates to

$$\frac{1}{(4\pi)^2}\frac{\xi^2 y_q^2}{m_B^2}C_V B_\nu^\dagger \partial^\mu B^\nu \bar{q}\gamma_\mu q, \quad (12)$$

with $C_V \approx 80$ very close to the scalar case, giving a loop-induced SI cross section, as in Eq. (9), and similarly giving no contribution if the DM were real.

The case of fermionic DM is slightly different. This is because the tree-level operator responsible for scattering is

$$\frac{1}{m_a^2}\xi y_q \bar{\chi} i\gamma^5\chi\bar{q}i\gamma^5 q \quad (13)$$

and, therefore, is parametrically suppressed by q^4, instead of the previous cases' q^2. The tree-level cross section then becomes

$$\sigma_{\rm SD}^{\chi p} \approx (3\times 10^{-43}\ {\rm cm}^2)\left(\frac{\xi}{0.1}\right)^2\left(\frac{1\ {\rm GeV}}{m_a}\right)^4. \quad (14)$$

We see that, due to the greater momentum suppression, we require a lighter mediator mass and cannot afford the coupling of the DM to be as small as in the bosonic case above. In this case, a cross section detectable in current experiments would require, for example, a mediator with $m_a = 100$ MeV and $\xi = 0.1$.

Meanwhile, the effective coupling from computing the loop diagram in the same limits as the other cases is

$$\begin{aligned}\frac{1}{(4\pi)^2}\frac{\xi^2 y_q^2}{m_\chi^2}\Big\{&\Big[\frac{1}{2}+\frac{m_\chi^2}{2}C_0(m_\chi^2, m_\chi^2, 0; m_a^2, m_\chi^2, m_a^2) - m_\chi^2 C_0(0, m_\chi^2, m_\chi^2; 0, m_a^2, m_\chi^2)\Big]\bar{\chi}\gamma^\mu\chi\bar{q}\gamma_\mu q\\ &+\frac{3}{8}\Big[1 + B_0(m_\chi^2; 0, m_\chi^2) - B_0(0; m_a^2, m_a^2) + 4m_\chi^2 C_0(m_\chi^2, 0, m_\chi^2; m_\chi^2, m_a^2, 0) - m_\chi^2 C_0(m_\chi^2, m_\chi^2, 0; m_a^2, m_\chi^2, m_a^2)\\ &+ 3m_a^2 m_\chi^2 D_0(m_\chi^2, m_\chi^2, 0, 0, 0, m_\chi^2; m_a^2, m_\chi^2, m_a^2, 0)\Big]\frac{m_q}{m_\chi}\bar{\chi}\chi\bar{q}q\Big\}, \end{aligned}\quad (15)$$

which numerically yields

$$\frac{1}{(4\pi)^2}\frac{\xi^2 y_q^2}{m_\chi^2}\left(C_{F_1}\bar{\chi}\gamma^\mu\chi\bar{q}\gamma_\mu q + C_{F_2}\frac{m_q}{m_\chi}\bar{\chi}\chi\bar{q}q\right), \quad (16)$$

with $C_{F_1} \approx 4.8$ and $C_{F_2} \approx 170$. The magnitudes of these coefficients can be understood as arising from the $\ln(m_a/m_\phi)$ and $\ln^2(m_a/m_\phi)$ behavior of C_0 and D_0 mentioned above. The loop-level cross section is then

$$\sigma_{\rm SI}^{\chi N} \approx (3\times 10^{-56}\ {\rm cm}^2)\left(\frac{\xi}{0.1}\right)^4\left(\frac{100\ {\rm GeV}}{m_\chi}\right)^4. \quad (17)$$

We will confirm below in the explicit model of Sec. IV that the loop-induced coupling is indeed tiny, but it is simple to see here why this is generically so.

Unlike in the massive mediator cases, there are two mass scales in the dark sector, that of the DM itself and that of the mediator. At tree level, the lighter mediator mass is the one that appears in the denominator of the operator. However, at loop level, the value of the loop integral is parametrically controlled by the mass of the DM, the heaviest particle in the loop. Additionally, a pseudoscalar which is the Nambu-Goldstone boson of a broken symmetry would be expected to couple to quarks proportional to the masses of the quarks. Thus, at loop level, the effective operator would be expected to be suppressed by extra factors of quark Yukawa couplings. Together, both effects combine to make the loop-level coupling to be as many as 20 orders smaller than the tree-level one, with higher-order corrections to the nonrelativistic scattering approximation coming at similar orders as $q^2 v^4$, so that the SI-induced interaction is expected to be completely negligible.

IV. THE AXION PORTAL

We have just seen that, without tuning of couplings, models with light pseudoscalar mediators provide the unique method of avoiding any SI signal, while still producing a SD direct detection signature. Now, we turn to the question of whether a viable model producing DM with the correct abundance can have these features.

Coupling a light pseudoscalar to quarks is most efficiently achieved by adding a scalar field which spontaneously breaks a global symmetry and which, by mixing with the Higgs, gets a coupling to the SM. Allowing this scalar to have a new global charge, while adding new fermions charged under the same symmetry, ensures that the new scalar field is the only method for the new fermions to interact with the SM.

As a simple realization of such a mechanism, where the dominant interaction is $\mathcal{O}_4^f$ via a pseudoscalar interaction, we introduce, following Ref. [35], a scalar field charged under a new global $U(1)_X$ charge that is spontaneously broken to

$$S = \left(f_a + \frac{s}{\sqrt{2}}\right)\exp\left(\frac{ia}{\sqrt{2}f_a}\right). \quad (18)$$

This scalar field is coupled to a new fermion, which is vectorlike under the SM, through $\mathcal{L} = -\xi S \chi \chi^c + \mathrm{H.c.}$, so that, after the scalar field acquires a vacuum expectation value, the fermion receives a mass of $m_\chi = \xi f_a$, allowing it to act as dark matter, with stability ensured by the remnant of $U(1)_X$ after breaking.

In order for the pseudoscalar to interact with the SM, some known particles must also carry charges under the new $U(1)_X$. In a two-Higgs–doublet model, this can be accomplished by adding a term of the form

$$\mathcal{L} = \lambda S^n H_u H_d + \mathrm{H.c.}, \quad (19)$$

by assigning the appropriate charges to the Higgses and SM fermions and promoting the $U(1)_X$ to a Peccei-Quinn (PQ) symmetry. For $n = 2$, this coupling is of the same form as in the case of the Dine-Fischler-Srednicki-Zhitnitsky axion [36,37], while the $n = 1$ case functions like that of the PQ-symmetric limit of the next-to-minimal supersymmetric standard model [38]. We now have a dark matter candidate coupling to the SM through a massive scalar and an axionlike Nambu-Goldstone boson. The Nambu-Goldstone boson is assumed to get a small mass through an unspecified mechanism. Anticipating making the scalar heavy, by virtue of

$$\langle\sigma v\rangle_{\chi\chi^c\to sa} = \frac{m_\chi^2}{64\pi f_a^4}\left(1 - \frac{m_s^2}{4m_\chi^2}\right) + O(v^4), \quad (20)$$

a choice of, say, $m_s = f_a = 1$ TeV and $m_\chi = 1.1$ TeV (corresponding to $\xi = 1.1$) yields a cross section of 3×10^{-26} cm^3/s and so generates the correct order of magnitude for the relic abundance [35].

For direct detection, two channels present themselves. The scalar gives a SI cross section through the operator $\mathcal{O}_1^f$, due to mixing of the scalar with the two CP-even Higgses, while the light axionlike state yields a SD interaction, $\mathcal{O}_4^f$, by a similar mixing with the CP-odd Higgses. For our purposes, we need to check whether this tree-level SI cross section can be small enough to be completely negligible.

The mixing of the scalar with the two CP-even Higgses has a lot of arbitrariness to it, due to the 11 constants in the most general $U(1)_{\mathrm{PQ}}$-preserving two-Higgs–doublet and one-singlet potential. However, we can say that, barring accidental cancellations, this mixing will be $\epsilon = O(v_{\mathrm{ew}}/f_a)$, so that we may write the tree-level SI cross section as

$$\sigma_{\mathrm{SI}}^{\chi N} \approx (2\times 10^{-42}\ \mathrm{cm}^2)\xi^2\epsilon^2\left(\frac{100\ \mathrm{GeV}}{m_s}\right)^4. \quad (21)$$

(See Appendix A for a caveat on the values of the nuclear matrix elements in this calculation.) In the model considered in Ref. [35], m_s needed to be light, $O(10$ GeV), in order to provide a mechanism for Sommerfeld enhancement to explain astrophysical anomalies. In that case, the direct detection cross section was in tension with the SI bound and could only be slightly beyond current limits, at a few $\times 10^{-43}$ cm^2. However, if we impose no such condition, m_s could be larger. If it is at the electroweak scale, then the cross section is, at most, a few $\times 10^{-45}$ cm^2, smaller than the sensitivity of the next generation of direct detection experiments. If $m_s \sim O(1$ TeV), a reasonable choice given the scale of f_a in this setup, then the cross section becomes undetectably small, below the irreducible atmospheric neutrino limit.

Let us next consider the pseudoscalar channel. With the interaction kinetically suppressed by the momentum transfer as q^4, we cannot merely compute the cross section in the limit of $q^2 \to 0$ as we did in the scalar exchange case. Instead, we must define a cross section at a fixed momentum transfer. (See Appendix B for a more thorough discussion.) We choose to do so at $q_{\mathrm{ref}}^2 = (100\ \mathrm{MeV})^2$. Because the signal is different from that of unsuppressed interactions relative to the expected recoil energies, the sensitivities of experiments are modified. This was studied in Ref. [29], with the result that, at the same reference momentum transfer, optimal sensitivities of SD experiments to pseudoscalars remained at the same order of magnitude as in the unsuppressed case, but with $1/m_{\mathrm{DM}}$ scaling of the limits.

With this definition, we can compute the SD cross section for $q^2 = q_{\mathrm{ref}}^2$ as

$$\sigma_{\mathrm{SD}}^{\chi p} \approx (2\times 10^{-37}\ \mathrm{cm}^2)\xi^2\sin^2\theta\frac{q_{\mathrm{ref}}^2}{4m_\chi^2}\left(\frac{1\ \mathrm{GeV}}{m_a}\right)^4, \quad (22)$$

where $\tan\theta = n\sin 2\beta[v_{\mathrm{ew}}/(2f_a)]$ is the mixing of the s with the Higgses [39]. From this, we see that, given a DM mass $m_\chi = 1.1$ TeV, a pseudoscalar with a mass $m_a \approx 300$ MeV generates a cross section of 3×10^{-40} cm^2, within the range of the next generation of direct SD detection experiments. In fact, in a two-Higgs–doublet model like this, the nuclear matrix element also has a dependence on β, as up-type quarks couple with a coefficient proportional to $\cot\beta$, while down-type ones couple proportional to $\tan\beta$. We have evaluated the matrix elements for the above cross section at $\tan\beta = 1$. At large values of $\tan\beta$, the cross section can rise by almost 2 orders of magnitude.

Given the tiny size of the tree-level SI cross section, and in keeping with the discussion of the previous section, we should confirm that the loop-induced couplings fail to produce a detectable SI cross section. The calculation mostly mirrors that of Sec. III C. The only substantial difference is the aforementioned different coupling to up- and down-type quarks. As before, we evaluate the nuclear matrix elements at $\tan\beta = 1$, but this time, varying $\tan\beta$ cannot only modify the cross section by a factor of $O(1)$, as the suppression of $\sin^2\theta$ at high $\tan\beta$ is too strong, so we find

$$\sigma_{\rm SI}^{\chi N} \approx (3\times 10^{-56}\ {\rm cm}^2)\left(\frac{\xi \sin\theta}{0.1}\right)^4\left(\frac{100\ {\rm GeV}}{m_\chi}\right)^4, \tag{23}$$

with no additional implicit $\tan\beta$ dependence.

V. CONCLUSIONS

As the sensitivity of both SI and SD direct DM detection experiments increases, it is worth asking to what extent the discovery potential of the two methods is complementary. In this work, we have pointed out that, when one considers the full range of possible mediators, instead of being confined to new weak-scale particles, the range of possible viable interactions generating SD cross sections increases. At the same time, when one searches for interactions for which SD experiments are complimentary for discovery—ones which could not be seen in any SI experiments without the need for accidental cancellations or other tuning—it becomes necessary to take into account subleading contributions to scattering, such as suppressed operators and loop processes. The outcome is that the traditional models considered also generically produce SI interactions whose suppression is counterbalanced by the greater sensitivity of SI experiments. The list of viable candidates whose interaction with the SM can be described by tree-level mediators integrated out in a renormalizable model is then reduced to merely ones mediated by light pseudoscalars.

We have presented a realistic model of such interactions that generates the right DM abundance with a fermionic DM candidate without having other interactions generating detectable SI interactions.

Similar scenarios can also be considered with a scalar or vector dark matter candidate. Just as in the case of fermionic DM, $\mathcal{O}_1^{s,v}$ gives the leading interaction in the nonrelativistic limit, while $\mathcal{O}_2^{s,v}$ is kinematically suppressed. The necessary couplings between the pseudoscalar and the scalars or vectors cannot be generated in as simple a manner as those used above, so more model building will be required. However, the suppression is only by q^2, so the mass differences between the scalar and pseudoscalar do not have to be quite as large, and the couplings themselves can be smaller, so that the parameter space of couplings and the pseudoscalar mass are not as tightly limited by experiment, potentially making the exercise worthwhile.

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APPENDIX A: NUCLEAR MATRIX ELEMENTS

Here, we summarize how to compute the dark-matter–nucleon interaction cross sections from quark-level interactions. Much of this has been discussed in the DM literature, with the exception of the pseudoscalar matrix element, as it only plays a role in momentum-suppressed cross sections.

For a vector coupling, nuclear matrix elements are straightforward to compute, since a vector coupling to quarks is a conserved current, so the coupling to a nucleon is obtained from the sum of the currents of the valence quarks.

In the case of a scalar coupling to quarks, we are interested in the effective nucleon coupling induced by a quark-level coupling:

$$a_q m_q \bar{q} q \to f_N m_N \bar{N} N. \tag{A1}$$

We define the nuclear matrix elements conventionally by

$$\langle N | m_q \bar{q} q | N \rangle = m_N f_{Tq}^{(N)}. \tag{A2}$$

On including the coupling to gluons induced by integrating out heavy quark loops, f_N is given by

$$f_N = \sum_{q=u,d,s} f_{Tq}^{(N)} a_q + \frac{2}{27} f_{TG}^{(N)} \sum_{q=c,b,t} a_q, \tag{A3}$$

where $f_{TG}^{(N)} = 1 - \sum_{q=u,d,s} f_{Tq}^{(N)}$.

Unlike the u and d matrix elements, which can be extracted from πN scattering, the uncertainty associated with the strange quark matrix element $f_{Ts}^{(N)}$ is higher, which introduces a substantial uncertainty in the SI coupling to nucleons. Most studies use numerical values $f_{Ts}^{(N)} \gg f_{Tu,d}^{(N)}$ based on older calculations. A representative set of values is that used by the DarkSUSY package [40], wherein,

$$\begin{aligned} f_{Tu}^{(p)} &= 0.023, & f_{Td}^{(p)} &= 0.034, & f_{Ts}^{(p)} &= 0.14, \\ f_{Tu}^{(n)} &= 0.019, & f_{Td}^{(n)} &= 0.041, & f_{Ts}^{(n)} &= 0.14. \end{aligned} \tag{A4}$$

These are the values used for the numerical estimates given above and in most of the literature. However, recent lattice

QCD results give substantially smaller values, $f_{Ts}^{(N)} = 0.013 \pm 0.020$ [41] (see also [42,43]), and so the SI cross section from scalar exchange (if it couples proportionally to mass) may be smaller by a factor of 2–5 than numerical results quoted by many calculations.

For SD interaction, we need to consider the nuclear matrix elements induced by the quark-level axial-vector and pseudoscalar couplings,

$$d_q \bar{q}\gamma_\mu \gamma^5 q \to a_N \bar{N} s_\mu^{(N)} N, \tag{A5}$$

and

$$c_q m_q \bar{q} i\gamma^5 q \to g_N m_N \bar{N} i\gamma^5 N. \tag{A6}$$

For the axial-vector current, defining

$$\langle N|\bar{q}\gamma_\mu\gamma^5 q|N\rangle = s_\mu^{(N)}\Delta q^{(N)}, \tag{A7}$$

where $s_\mu^{(N)}$ is the spin of the nucleon, we have

$$a_N = \sum_{q=u,d,s} d_q \Delta q^{(N)}. \tag{A8}$$

The matrix elements coming from polarized deep inelastic scattering carry much smaller uncertainties than for the scalar SI interaction above. For our numerical results, we use again the DarkSUSY values,

$$\Delta u^{(p)} = \Delta d^{(n)} = 0.77, \qquad \Delta d^{(p)} = \Delta u^{(n)} = -0.40,$$
$$\Delta s^{(p)} = \Delta s^{(n)} = -0.12. \tag{A9}$$

More recent determinations favor slightly different values, and the Particle Data Group quotes $\Delta s^{(n)} = -0.09$, $\Delta d^{(n)} = 0.84$, and $\Delta u^{(n)} = -0.43$, with a 0.02 uncertainty for each [44]; the effect on our numerical results is negligible.

For the pseudoscalar current in Eq. (A6), the nucleon-level coupling is determined by the same axial-vector matrix elements above. The relationship is established through generalized Goldberger-Treiman relations. While not normally considered in dark matter detection, it has been well-studied in the axion literature [45,46]. Taking divergences of the axial currents and using the equations of motion for the quarks yields [47]

$$g_N = (c_u - \bar{c}_q\eta)\Delta u^{(N)} + (c_d - \bar{c}_q\eta z)\Delta d^{(N)} + (c_s - \bar{c}_q\eta w)\Delta s^{(N)}, \tag{A10}$$

where $\eta = (1 + z + w)^{-1}$, $z = m_u/m_d$, and $w = m_u/m_s$, while $\bar{c}_q$ is the mean of the quark coupling coefficients. Because of uncertainties in the value of z, the value of g_N can vary by as much as a factor of 2.

APPENDIX B: CROSS SECTIONS

In this Appendix, we provide a summary of cross sections for DM-nucleon interactions relevant for calculating the various cross sections discussed above in the nonrelativistic limit.

We first consider the unsuppressed operators in the limit of zero momentum transfer. SI cross sections can come from either scalar or vector quark couplings. Effective DM-nucleon scalar interactions for fermions of the form

$$f_N \bar{\chi}\chi\bar{N}N, \tag{B1}$$

which are derived from the quark-level couplings using nuclear matrix elements, as explained in Appendix A, lead to a DM-nucleus cross section

$$\hat{\sigma} = \frac{4}{\pi}\hat{\mu}^2[Zf_p + (A-Z)f_n]^2, \tag{B2}$$

for Majorana DM fermions. (For Dirac fermions, all results for Majorana fermions are divided by 4.) Here, $\hat{\mu}$ is the reduced mass of the DM-nucleus system. The per-nucleon cross section, which is usually quoted for comparisons, is

$$\sigma = \frac{4}{\pi}\mu^2\frac{1}{A^2}[Zf_p + (A-Z)f_n]^2, \tag{B3}$$

where μ is the reduced mass of the DM-nucleon system.

For scalar or vector dark matter, the relevant operators are (we include the DM mass to give all operators the same dimension)

$$f_N m_\phi \phi\phi\bar{N}N \quad \text{or} \quad f_N m_B B^\mu B_\mu \bar{N}N, \tag{B4}$$

and the nucleon cross section for either operator is

$$\sigma = \frac{1}{\pi}\mu^2\frac{1}{A^2}[Zf_p + (A-Z)f_n]^2. \tag{B5}$$

Vector interactions for fermions only exist in the case of Dirac DM:

$$b_N \bar{\chi}\gamma^\mu\chi\bar{N}\gamma_\mu N, \tag{B6}$$

where $b_p = 2b_u + b_d$ and $b_n = b_u + 2b_d$, due to vector current conservation, as discussed above in Appendix A. Then,

$$\sigma = \frac{1}{\pi}\mu^2\frac{1}{A^2}[Zb_p + (A-Z)b_n]^2. \tag{B7}$$

For the operators

$$b_N \phi^\dagger\partial_\mu\phi\bar{N}\gamma^\mu N \quad \text{or} \quad b_N B_\nu^\dagger\partial_\mu B^\nu\bar{N}\gamma^\mu N, \tag{B8}$$

which only exist for complex scalars or vectors, the cross section is

$$\sigma = \frac{1}{\pi}\mu^2\frac{1}{A^2}[Zb_p + (A-Z)b_n]^2. \tag{B9}$$

Unsuppressed SD interactions come solely from the quarks' axial currents. In the case of

$$a_N \bar{\chi}\gamma^\mu\gamma^5\chi\bar{N}\gamma_\mu\gamma^5 N, \tag{B10}$$

the DM-nucleus cross section is

$$\hat{\sigma} = \frac{16}{\pi}\hat{\mu}^2 a_N^2 J_N(J_N + 1), \tag{B11}$$

and, for a nucleon,

$$\sigma = \frac{12}{\pi}\mu^2 a_N^2. \tag{B12}$$

The only other unsuppressed SD interaction is for vector DM and comes from

$$a_N \epsilon^{\mu\nu\sigma\rho} B_\mu \partial_\nu B_\sigma \bar{N}\gamma_\rho\gamma^5 N. \tag{B13}$$

Here, the DM-nucleon cross section is

$$\sigma = \frac{2}{\pi}\mu^2 a_N^2. \tag{B14}$$

All of the above cross sections are quoted in the $q^2 \to 0$ limit. In this limit, interactions mediated by light pseudoscalars are all zero, so we need another way of expressing such cross sections. To do so, we will use the fact that, while in the nonrelativistic limit, $\bar{\psi}\psi \sim 2m$, $\bar{\psi}\gamma^5\psi \sim q^i\xi^\dagger\sigma^i\xi$, so that, using the results above, we can write (since $q^2 \approx |\vec{q}|^2$ in the nonrelativistic limit)

$$\bar{\chi}\gamma^5\chi\bar{N}\gamma^5 N \sim \frac{q^2}{4m_\chi^2}\frac{q^2}{4m_N^2}\bar{\chi}\gamma^\mu\gamma^5\chi\bar{N}\gamma_\mu\gamma^5 N. \tag{B15}$$

We then compute the cross section as above and quote a result at a reference value of q^2. We have chosen $q^2 = (100\text{ MeV})^2$, since, with $q^2 = 2m_N E_R$, where E_R is the recoil energy of the nucleus, this is a typical value for most SD detectors. Other momentum-suppressed operators can be handled the same way.

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