# Possible 2$S$ and 1$D$ charmed and charmed-strange mesons

Bing Chen,* Ling Yuan, and Ailin Zhang†

*Department of Physics, Shanghai University, Shanghai 200444, China*

(Received 22 February 2011; published 10 June 2011)

Possible 2$S$ and 1$D$ excited $D$ and $D_s$ states are studied, the charmed states $D(2550)^0$, $D^*(2600)$, $D(2750)^0$, and $D^*(2760)$ newly observed by the *BABAR* Collaboration are analyzed. The masses of these states are explored within the Regge trajectory phenomenology, and the strong decay widths are computed within the heavy-quark effective theory. Both the mass and the decay width indicate that $D(2550)^0$ is a good candidate for $2^1S_0$. The strong decay property of $D^*(2600)$ and $D^*_{s1}(2700)^\pm$ is described well by pure $2^3S_1$ states. If a mixing between $2^3S_1$ and $1^3D_1$ does exist, the mixing angle $\theta$ is not large and $2^3S_1$ is predominant. $D^*(2760)$ and $D^*_{sJ}(2860)^\pm$ are possibly the $1^3D_3$ $D$, and $D_s$, respectively. $D(2750)^0$ and $D^*(2760)$ seem two different states, and $D(2750)^0$ is very possibly the $1D(2^-, \frac{5}{2})$ though the possibility of $1D(2^-, \frac{3}{2})$ has not been excluded. There may exist an unobserved meson $D_{sJ}(2850)^\pm$ corresponding to $D^*_{sJ}(2860)^\pm$.

DOI: 10.1103/PhysRevD.83.114025 PACS numbers: 13.25.Ft, 11.30.Hv, 12.39.Hg

## I. INTRODUCTION

The properties of 2$S$ and 1$D$ $Q\bar{q}$ mesons have been studied for a long time. However, no such higher excited $Q\bar{q}$ state has been established for lack of experimental data. In the past years, some higher excited charmed or charmed-strange states were reported though most of them have not yet been pinned down [1]. It will be useful to study the possible 2$S$ and 1$D$ charmed and charmed-strange mesons systemically in time.

The first possible charmed radial excitation, $D^{*\prime}(2640)$, was reported by DELPHI [2]. This state is difficult to be understood as a charmed radially excited state for the observed decaying channel $D^{*+}\pi^+\pi^-$ and decay width <15 MeV [3,4]. Its existence has not yet been confirmed by other collaborations. $D_{sJ}(2632)^+$ is another puzzling state first observed by SELEX [5]. It has not been observed by other collaborations either. It seems impossible that $D_{sJ}(2632)^+$ is a conventional $c\bar{s}$ meson for its narrow decay width and anomalous branching ratio $\Gamma(D^0K^+)/\Gamma(D_s^+\eta) = 0.14 \pm 0.06$ [6] even if it does exist. In an early analysis of the spectrum within Regge trajectories phenomenology [7], it is pointed out that $D^{*\prime}(2640)$ and $D_{sJ}(2632)^+$ do not seem to be the orbital excited tensor states or the first radially excited states.

The observation of another three $D_s$ mesons: $D^*_{s1}(2700)^\pm$ [8–10], $D^*_{sJ}(2860)^\pm$ [9,10], and $D_{sJ}(3040)^+$ [10], has evoked much more study of highly excited $Q\bar{q}$ mesons. The masses and the decay widths of $D^*_{s1}(2700)^\pm$ and $D^*_{sJ}(2860)^\pm$ were reported by experiments. Furthermore, the ratios of branching fractions, $\frac{\mathcal{B}(D^*_{s1}(2700)^\pm\to D^*K)}{\mathcal{B}(D^*_{s1}(2700)^\pm\to DK)} = 0.91 \pm 0.13_{\rm stat} \pm 0.12_{\rm syst}$ and $\frac{\mathcal{B}(D^*_{sJ}(2860)^+\to D^*K)}{\mathcal{B}(D^*_{sJ}(2860)^+\to DK)} = 1.10 \pm 0.15_{\rm stat} \pm 0.19_{\rm syst}$, were measured. These states have been explored within some models. $D^*_{s1}(2700)^\pm$ was identified with the first radial excitation of $D^*_s(2112)^\pm$ [11,12], or the $D_s(1^3D_1)$ [13], or the mixture of them [14]. $D^*_{sJ}(2860)^\pm$ was interpreted as the $D_s(2^3P_0)$ [13,15] or the $D_s(1^3D_3)$ [12,13,16]. $D_{sJ}(3040)^+$ was identified with the radially excited $D_s[2P(1^+, \frac{1}{2})]$ [12,17]. However, theoretical predictions of these states are not completely consistent with experiments either on their spectrum or on their decay widths.

Four new charmed states, $D(2550)^0$, $D^*(2600)^0$, $D(2750)^0$, and $D^*(2760)^0$ [including two isospin partners $D^*(2600)^+$ and $D^*(2760)^+$] were recently observed by the *BABAR* collaboration [18]. Some ratios of branching fractions of $D^*(2600)^0$ and $D(2750)^0$ were also measured. In their report, an analysis of the masses and helicity-angle distributions indicates that $D(2550)^0$ and $D^*(2600)$ are possibly the first radially excited $S$-wave states $D(2^1S_0)$ and $D(2^3S_1)$, respectively, while the other two charmed candidates are possibly the 1$D$ orbitally excited states.

Theoretical analyses indicate that $D(2550)^0$ is a good candidate for $2^1S_0$ though the predicted narrow width of $2^1S_0$ is inconsistent with the observation [19–21]. $D^*(2600)^0$ is interpreted as a mixing state of $2^3S_1$ and $1^3D_1$ [19,20]. The calculation in Ref. [19] indicates that $D^*(2760)^0$ can be regarded as the orthogonal partner of $D^*(2600)^0$ (or $1^3D_3$), but this possibility [or $D^*(2760)$ is predominantly the $1^3D_1$] was excluded in Ref. [20], where $D^*(2760)$ is identified with the $1^3D_3$ state. In Ref. [20], the identification of $D(2750)^0$ and $D^*(2760)$ with the same resonance with $J^P = 3^-$ is not favored.

Obviously, these $D$ and $D_s$ candidates have not yet been pinned down. In addition to some theoretical deviations from experiments, some theoretical predictions of their strong decays are different in different models. Systematical study of these possible 2$S$ and 1$D$ states in

*chenbing@shu.edu.cn
†Corresponding author
zhangal@staff.shu.edu.cn

 

more models is required. In this paper, the method presented by Eichten *et al.* (EHQ's method) [22] is employed to study the strong decay of the heavy-light mesons. We will label them with the notation $nL(J^P, j_q)$ in most cases, where $n$ is the radial quantum number, $L$ is the orbital angular momentum, $J^P$ refers to the total angular momentum and parity, and $j_q$ is the total angular momentum of the light degrees of freedom.

The paper is organized as follows. In Sec. II, the spectrum of $2S$ and $1D$ $D_s$ and $D$ will be examined within the Regge trajectory phenomenology. In Sec. III, the two-body strong decay of these states will be explored with EHQ's method. Finally, we present our conclusions and discussions in Sec. IV.

## II. MASS SPECTRUM IN REGGE TRAJECTORIES

Linearity of Regge trajectories (RTs) is an important observation in particle physics [23]. In the relativized quark model [24], the RTs for normal mesons are linear. For $Q\bar{q}$ mesons, the approximately linear, parallel, and equidistant RTs were obtained both in $(J, M^2)$ and in $(n_r, M^2)$ planes in the framework of a QCD-motivated relativistic quark model [25].

However, when RTs are reconstructed with the experimental data, the linearity is always approximate. For orbitally excited states, Tang and Norbury plotted many RTs of mesons and indicated that the RTs are nonlinear and intersecting [26]:

$$M^2 = aJ^2 + bJ + c, \tag{1}$$

where the coefficients $a$, $b$, $c$ are fixed by the experimental data, and $|a| \ll |b|$ [26]. The coefficients are usually different for different RTs.

For radially excited light $q\bar{q}$ mesons, Anisovich *et al.* systematically studied the trajectories on the planes $(n, M^2)$ in the mass region up to $M < 2400$ MeV [27]. The RTs on $(n, M^2)$ plots behave as

$$M^2 = M_0^2 + (n-1)\mu^2, \tag{2}$$

where $M_0$ is the mass of the basic meson, $n$ is the radial quantum number, and $\mu^2$ is the slope parameter of the trajectory.

Possible $1S$ and $2S$ $D$ and $D_s$ states are listed in Table I, where $^\dagger D_s'(2635)$ is the predicted mass of $2S(1^-, \frac{1}{2})$ $D_s$ meson. It is easy to notice that these candidates for $1S$ and $2S$ meet well with the trajectories on the $(n, M^2)$ plot according to Eq. (2). The narrow charmed-strange state

TABLE I. $1S$ and $2S$ $D$ and $D_s$ mesons are shown.

| States | $(0^-, \frac{1}{2})$ | $(1^-, \frac{1}{2})$ | $(0^-, \frac{1}{2})$ | $(1^-, \frac{1}{2})$ |
|---|---|---|---|---|
| $2S$ | $D(2550)^0$ | $D_1^*(2600)$ | $^\dagger D_s'(2635)$ | $D_{s1}^*(2700)^\pm$ |
| $1S$ | $D(1869)^\pm$ | $D^*(2007)^0$ | $D_s(1968)^\pm$ | $D_s^*(2112)^\pm$ |
| $\mu^2$ (GeV$^2$) | 2.97 | 2.78 | 3.07 | 2.88 |

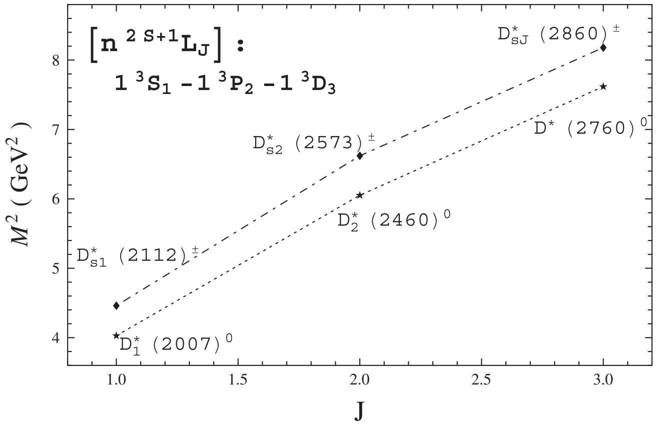


FIG. 1. Nonlinear RTs of the $D$ and $D_s$ triplet with $N$, $S = 1$. The polynomial fits are $M^2 = -0.23J^2 + 2.74J + 1.53$ (GeV$^2$) and $M^2 = -0.29J^2 + 3.03J + 1.72$ (GeV$^2$), respectively.

$D_{sJ}(2632)^+$ is located around the mass region of $2S$, $D_s$. However, the exotic relative branching ratio $\Gamma(D^0K^+)/\Gamma(D_s^+\eta) = 0.14 \pm 0.06$ excludes its $2S(1^-, \frac{1}{2})$ possibility. Therefore, we denote the $2S(1^-, \frac{1}{2})$ $D_s$ meson with $^\dagger D_s'(2635)$. As indicated in Ref. [12], the $2P$ candidate $D_{sJ}(3040)^+$ meets well with the trajectory on the $(n, M^2)$ plot.

The measured masses of $D(2750)^0$, $D^*(2760)$, and $D_{sJ}^*(2860)^\pm$ seem a little lower than most theoretical predictions of the $1D$ states [24,25,28]. In Fig. 1, nonlinear RTs of $D$ and $D_s$ states consisting of $1^3S_1(1^-)$, $1^3P_2(2^+)$, and $1^3D_3(3^-)$ were reconstructed, where the polynomial fits indicate $|a| \ll |b|$. In a relativistic flux tube model, a ratio $b_{hl}/b_{ll} = 2$ was obtained at the lowest order [29], where $b_{hl}$ is the coefficient for the heavy-light meson and $b_{ll}$ is the coefficient for the light-light meson in Eq. (1). The $b_{ll}$ (about 0.70–1.60) has been obtained in Ref. [26]. The fitted $b_{hl}$ of $D$ and $D_s$ in Fig. 1 is about 2.74 and 3.03, respectively. Obviously, the fitted ratio is consistent with the theoretical prediction.

Through the analysis of the spectrum only, $D(2550)^0$, $D^*(2600)$, and $D_{s1}^*(2700)^\pm$ are very likely the first radially excited $D$ and $D_s$ states, and $D^*(2760)$ and $D_{sJ}^*(2860)^\pm$ are likely the $1^3D_3$ states.

However, as is well known, the RTs can only give a preliminary analysis of the observed states, the investigation of the decay widths and the ratios of branching fractions will be more useful to shed light on the underlying properties of these states.

## III. DECAY WIDTH IN EHQ'S FORMULA

As is well known, in the heavy-quark symmetry theory, the heavy-light mesons degenerate in $j_q^P$, i.e., two orbital ground states form a spin doublet $1S(0^-, 1^-)$ with $j_q^P = \frac{1}{2}-$, and the decay amplitude satisfies certain symmetry relations due to the heavy-quark symmetry [30].

The decay properties of heavy-light mesons have been studied in detail in the heavy-quark effective theory. When $1/m_Q$ corrections to heavy-quark symmetry predictions for strong decay are ignored, the decays of the two mesons in one doublet are governed by the same transition strength [4,22,30,31]. As mentioned above, the concise method presented by Eichten *et al.* [22] is employed to study the decays of $D$ and $D_s$ mesons.

In the decay of an excited heavy-light meson $H$, characterized by $nL(J^P, j_q)$, to a heavy-light meson $H'$ $[n'L'(J'^{P'}, j'_q)]$ and a light hadron $h$ with spin $s_h$ and orbital angular momentum $l$ relative to $H'$, the two-body strong decay width is written as [4,22]

$$\Gamma^{H\to H'h} = \zeta(\mathcal{C}^{s_Q,j'_q,J'}_{j_h,j_q,J})^2 \mathcal{F}^{j_q,j'_q}_{j_h,l}(0)p^{2l+1}\exp\left(-\frac{p^2}{6\beta^2}\right); \quad (3)$$

where

$$\mathcal{C}^{s_Q,j'_q,J'}_{j_h,J,j_q} = \sqrt{(2J'+1)(2j_q+1)}\begin{Bmatrix} s_Q & j'_q & J' \\ j_h & J & j_q \end{Bmatrix}$$

and $\vec{j}_h = \vec{s}_h + \vec{l}$. $\mathcal{F}^{j_q,j'_q}_{j_h,l}(0)$ is the transition strength, and $p$ is the momentum of decay products in the rest frame of $H$. The coefficients $\mathcal{C}$ depend only upon the total angular momentum $j_h$ of the light hadron, and not separately on its spin $s_h$ and the orbital angular momentum $l$ of the decay. The $6-j$ symbols of the coefficients $\mathcal{C}$ exhibit the heavy-quark symmetry in the strong decays of heavy-light mesons [4,32]. The flavor factor $\zeta$ for different decay channels can be found in Ref. [28].

The value of parameter $\beta$ is important to the decay width. In Ref. [22], the momentum scale $\kappa$ was assumed universally $\approx 1$ GeV, which implies $\beta \approx 0.41$ GeV. In this work, the optimum value of $\beta$ is taken as 0.38 GeV. It is consistent with the harmonic oscillator parameter (0.35–0.50 GeV) which usually appears in the pseudoscalar-meson emission model [24], the chiral quark model [14,33,34], and the $^3P_0$ model [35–38].

Because of lack of measurements of partial widths in the charmed states, the decay width of $K$ mesons [i.e. $K_1(1270) \to \rho K$] was used to fix the transition strength in Ref. [22]. $c$ and $b$ quarks are much heavier than $u$, $d$, and $s$ quarks, so the open charm or bottom mesons provide better place to test EHQ's formula. Systematic studies of $S$- and $P$-wave heavy-light meons ($D$, $B$, $D_s$, and $B_s$ mesons) by EHQ's formula have been presented in Ref. [39].

The EHQ's formula is also obtained by the $^3P_0$ model where a unitary rotation between the bases of $Q\bar{q}$ mesons ($J^2$, $j_q^2$, $s_Q^2$, $J_z$) and $q\bar{q}$ mesons ($J^2$, $L^2$, $S^2$, $J_z$) has been performed [39]. In this way, the transition strength $\mathcal{F}^{j_q,j'_q}_{j_h,l}(0)$ obtained in the $^3P_0$ model includes only two parameters: the dimensionless parameter $\gamma$ and the harmonic oscillator parameter $\beta$ [39]. In fact, the nodal Gaussian form factor obtained by the $^3P_0$ model has been used for the transition strength $\mathcal{F}^{j_q,j'_q}_{j_h,l}(0)$ to interpret $D^{*\prime}(2640)$ in terms of EHQ's formula [4].

The relevant transition strengths $\mathcal{F}^{j_q,j'_q}_{j_h,l}(0)$ used in this paper are given in Table II. Some expressions in the table can be found in Refs. [36–38], and others are obtained in

TABLE II. The transition strength $\mathcal{F}^{j_q,j'_q}_{j_h,l}(0)$, where the sign "$\mathcal{P}$" denotes a light pseudoscalar-meson or a light vector meson is shown.

| $nL(j_q^P) \to nL(j_q^P) + \mathcal{P}$ | $\mathcal{F}^{j_q,j'_q}_{j_h,l}(0)$ | Polynomial of $p/\beta$ |
|---|---|---|
| $2S(\frac{1}{2}^-) \to 1S(\frac{1}{2}^-) + 0^-$ | $\mathcal{F}^{(1/2),(1/2)}_{1,1}(0)$ | $\frac{5^2}{3^4}\frac{1}{\beta^2}(1-\frac{2}{15}\frac{p^2}{\beta^2})^2$ |
| $2S(\frac{1}{2}^-) \to 1P(\frac{1}{2}^+) + 0^-$ | $\mathcal{F}^{(1/2),(1/2)}_{0,0}(0)$ | $\frac{1}{2\times 3^3}(1-\frac{7}{9}\frac{p^2}{\beta^2}+\frac{2}{27}\frac{p^4}{\beta^4})^2$ |
| $2S(\frac{1}{2}^-) \to 1P(\frac{3}{2}^+) + 0^-$ | $\mathcal{F}^{(1/2),(3/2)}_{2,2}(0)$ | $\frac{13^2}{3^7}\frac{1}{\beta^4}(1-\frac{2}{39}\frac{p^2}{\beta^2})^2$ |
| $1D(\frac{3}{2}^-) \to 1S(\frac{1}{2}^-) + 0^-$ | $\mathcal{F}^{(3/2),(1/2)}_{1,1}(0)$ | $\frac{5\times 2}{3^4}\frac{1}{\beta^2}(1-\frac{2}{15}\frac{p^2}{\beta^2})^2$ |
| $1D(\frac{3}{2}^-) \to 1S(\frac{1}{2}^-) + 1^-$ | $\mathcal{F}^{(3/2),(1/2)}_{1,1}(0)$ | $\frac{2^2}{3^4}\frac{1}{\beta^2}(1-\frac{2}{15}\frac{p^2}{\beta^2})^2$ |
| $1D(\frac{3}{2}^-) \to 1P(\frac{1}{2}^+) + 0^-$ | $\mathcal{F}^{(3/2),(1/2)}_{2,2}(0)$ | $\frac{5}{3^7}\frac{1}{\beta^4}(1+\frac{2}{15}\frac{p^2}{\beta^2})^2$ |
| $1D(\frac{3}{2}^-) \to 1P(\frac{3}{2}^+) + 0^-$ | $\mathcal{F}^{(3/2),(3/2)}_{0,0}(0)$ | $\frac{2^2\times 5}{3^3}(1-\frac{5}{18}\frac{p^2}{\beta^2}+\frac{1}{135}\frac{p^4}{\beta^4})^2$ |
| | $\mathcal{F}^{(3/2)),(3/2)}_{2,2}(0)$ | $\frac{13^2}{3^7\times 5}\frac{1}{\beta^4}(1-\frac{2}{39}\frac{p^2}{\beta^2})^2$ |
| $1D(\frac{5}{2}^-) \to 1S(\frac{1}{2}^-) + 0^-$ | $\mathcal{F}^{(5/2),(1/2)}_{3,3}(0)$ | $\frac{2^3}{3^6\times 5}\frac{1}{\beta^6}$ |
| $1D(\frac{5}{2}^-) \to 1S(\frac{1}{2}^-) + 1^-$ | $\mathcal{F}^{(5/2),(1/2)}_{3,3}(0)$ | $\frac{2^5}{3^7\times 5}\frac{1}{\beta^6}$ |
| | $\mathcal{F}^{(5/2),(1/2)}_{2,1}(0)$ | $\frac{2^4}{3^4}\frac{1}{\beta^2}(1-\frac{2}{15}\frac{p^2}{\beta^2})^2$ |
| $1D(\frac{5}{2}^-) \to 1P(\frac{1}{2}^+) + 0^-$ | $\mathcal{F}^{(5/2),(1/2)}_{2,2}(0)$ | $\frac{2^2\times 5}{3^7}\frac{1}{\beta^4}(1-\frac{1}{15}\frac{p^2}{\beta^2})^2$ |
| $1D(\frac{5}{2}^-) \to 1P(\frac{3}{2}^+) + 0^-$ | $\mathcal{F}^{(5/2),(3/2)}_{2,2}(0)$ | $\frac{2^5\times 7}{3^7\times 5}\frac{1}{\beta^4}(1-\frac{1}{42}\frac{p^2}{\beta^2})^2$ |
| | $\mathcal{F}^{(5/2),(3/2)}_{4,4}(0)$ | $\frac{2^4}{3^8\times 5\times 7}\frac{1}{\beta^8}$ |

the $^3P_0$ model in detail in Ref. [39]. For these transition strengths, a constant

$$\mathcal{G} = \pi^{1/2}\gamma^2 \frac{2^{10}}{3^4} \frac{\tilde{M}_B\tilde{M}_C}{\tilde{M}_A} \frac{1}{\beta} \tag{4}$$

was omitted. Here the phase space normalization of Kokoski and Isgur is employed [24,38]. $\tilde{M}_A$, $\tilde{M}_B$, $\tilde{M}_C$ are the "mock-meson" masses of $A$, $B$, $C$, respectively. The constant $\mathcal{G}$ absorbs the dimensionless parameter $\gamma$ in the $^3P_0$ model. The variation of the constant $\mathcal{G}$ with the mock-meson masses $\tilde{M}_i$ is slow.

In the analysis that follows, the decay widths of possible $2S$ and $1D$ $D$ and $D_s$ states are computed in terms of Eq. (3).

### A. $2^1S_0$ or $[2S(0^-, \frac{1}{2})]$

$D(2550)^0$ observed in the decay channel $D^{*+}\pi^-$ is a good candidate for a $2^1S_0$ charmed meson. Following the procedure in Ref. [39], we take the decay width of $D_2^*(2460)^0$ as an input and obtain the $d$-wave transition strength $\mathcal{F}_{2,2}^{(3/2),(1/2)}(0) = 0.964\ \text{GeV}^{-4}$, where

$$\mathcal{F}_{2,2}^{(3/2),(1/2)}(0) = \mathcal{G}\frac{2^2}{3^4}\frac{1}{\beta^4}.$$

All the other transition strengths $\mathcal{F}_{j_h,l}^{j_q,j_q'}(0)$ in Table II could be fixed easily once the mock-meson masses $\tilde{M}_i$ effect has been taken into account. According to our computation [39], the total decay width of $D(2550)^0$ is about 124.1 MeV. The dominating decay mode is the $D^*\pi$ channel with $\Gamma(D^*\pi) = 121.0$ MeV, and the decay width of another allowed $D_0^*(2400)\pi$ channel is 3.1 MeV [the mass of $D_0^*(2400)$ is taken as 2318 MeV [1]].

These results agree well with the experiments. It explains the fact that $D(2550)^0$ was first observed in $D^{*+}\pi^-$ [18]. In Fig. 2, the variation of the decay width with $\beta$ is plotted. Obviously, the observed decay width of $D(2550)^0$ is well obtained in the reasonable region of $\beta$ (0.35–0.42 GeV).

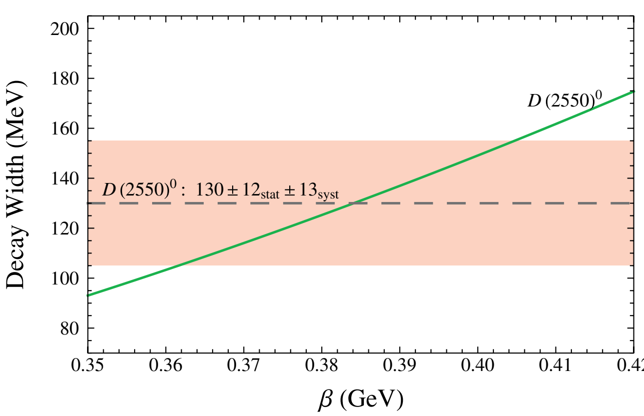


FIG. 2 (color online). The decay width versus $\beta$, where $D(2550)^0$ is taken as a pure $2^1S_0$ (green line) state. The dashed line refers to central values of the decay width given by experiment.

TABLE III. $D$, $D_s$ masses of the states $2^3S_1$ and $1^3D_1$ predicted in different models are shown (MeV).

| States | Ref. [11] | Ref. [24] | Ref. [25] | Ref. [40] | Ref. [41] |
|---|---|---|---|---|---|
| $D_1^*(2^3S_1)$ | ... | 2640 | 2632 | 2620 | 2636 |
| $D_1'^*(1^3D_1)$ | ... | 2820 | 2788 | 2710 | 2740 |
| $D_{s1}^*(2^3S_1)$ | 2711 | 2730 | 2731 | 2730 | 2714 |
| $D_{s1}'^*(1^3D_1)$ | 2784 | 2900 | 2913 | 2820 | 2804 |

In $D_s$ states, the mass of the $2^1S_0$ state is predicted around $2635 \pm 20$ MeV [a little smaller than the threshold of $D^*\eta$ and $D_0^*(2400)K$], and $D^*K$ is the only two-body strong decay channel. Our result for this decay channel is $\Gamma(D^*K) \approx 82.2 \pm 15.1$ MeV, so it is impossible that the observed $D_{sJ}(2632)^+$ is $2^1S_0$.

### B. Mixing states of $2^3S_1$ and $1^3D_1$

The predicted masses of $2^3S_1$, $D$ are almost about 2600–2640 MeV, and the masses of $2^3S_1$, $D_s$ are almost about 2710–2730 MeV (Table III) [24,25,40–42]. The spectrum and the helicity-angle distributions suggest that $D^*(2600)$ is the $2^3S_1$ [18]. In our analysis, it is possible to explain both $D^*(2600)$ and $D_{s1}^*(2700)^\pm$ as the pure $2^3S_1$ states. In this case, the variations of the branching fractions and decay widths with $\beta$ are given in Figs. 3 and 4, respectively. Obviously, theoretical decay widths and ratios in the reasonable region of $\beta$ are consistent with the experimental data.

In the charmonium system, $\psi(2S)$ and $\psi(3770)$ are two orthogonal partners of mixtures of $2^3S_1$ and $1^3D_1$ with $J^{PC} = 1^{--}$ [43]. This mixing scheme has also been employed to explain the decay width and the ratio of branching fractions of $D_{s1}^*(2700)^\pm$ and $D_{sJ}^*(2860)^\pm$ [14]. If this mixing does exist, there are two orthogonal partners ($J^P = 1^-$) of $D$ and $D_s$. They can be denoted as

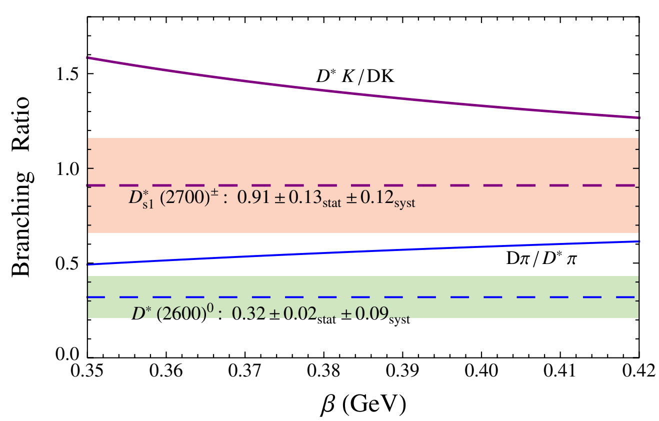


FIG. 3 (color online). Branching ratios of $D^*(2600)^0$ and $D_{s1}^*(2700)^\pm$ with $\beta$. The dashed lines refer to central values of decay width given by experiment.

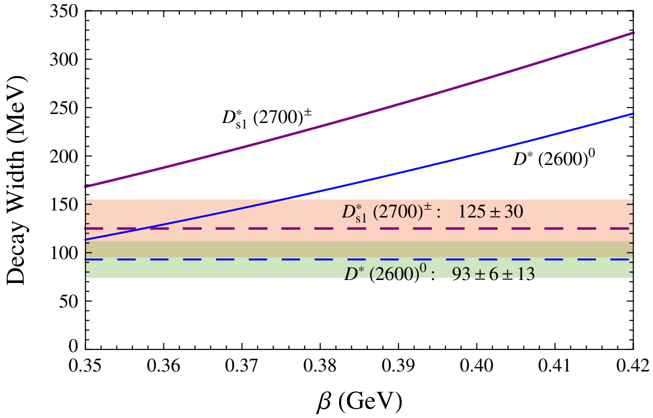


FIG. 4 (color online). Decay widths of $D^*(2600)^0$ and $D^*_{s1}(2700)^\pm$ with $\beta$. The dashed lines refer to central values of decay width given by experiment.

$$|(SD)_1\rangle_L = \cos\phi|2^3S_1\rangle - \sin\phi|1^3D_1\rangle,$$
$$|(SD)_1\rangle_R = \sin\phi|2^3S_1\rangle + \cos\phi|1^3D_1\rangle. \quad (5)$$

Details for the estimate of the decay width are given in the Appendix where $D^*(2600)$ is identified with the $|(SD)_1\rangle_L$ of $D$.

To proceed our analysis, the masses of pure $2^3S_1$ and $1^3D_1$ obtained in Refs. [24,41] are used. For $2^3S_1$, the masses from these two groups are almost the same. For $1^3D_1$, the mass given by Ref. [24] is much larger than that in Ref. [41] (Table III).

When mixing angles $\theta$ are treated as free variables, the decay widths and ratios of $D^*(2600)^0$ and $D^*_{s1}(2700)$ dependence on them are presented in Figs. 5 and 6, respectively. In the figures, the red lines and the blue lines

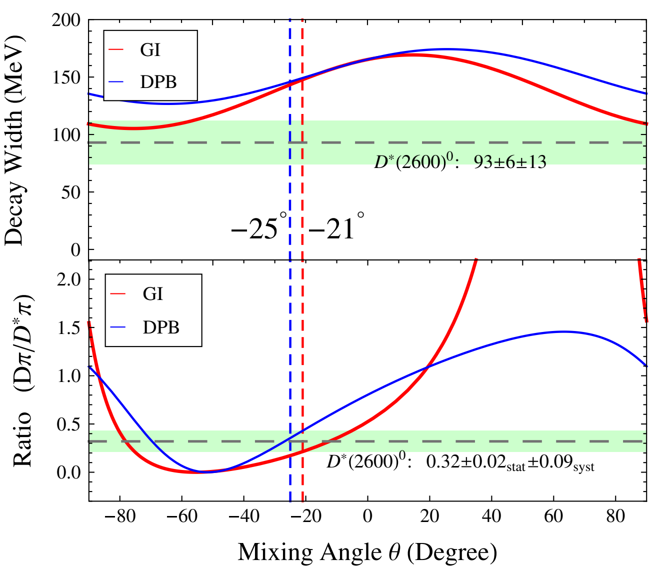


FIG. 5 (color online). Decay width and ratio $D\pi/D^*\pi$ of $D^*(2600)^0$ in the diagram method. The horizontal dashed lines refer to central values of the decay width given by experiment.

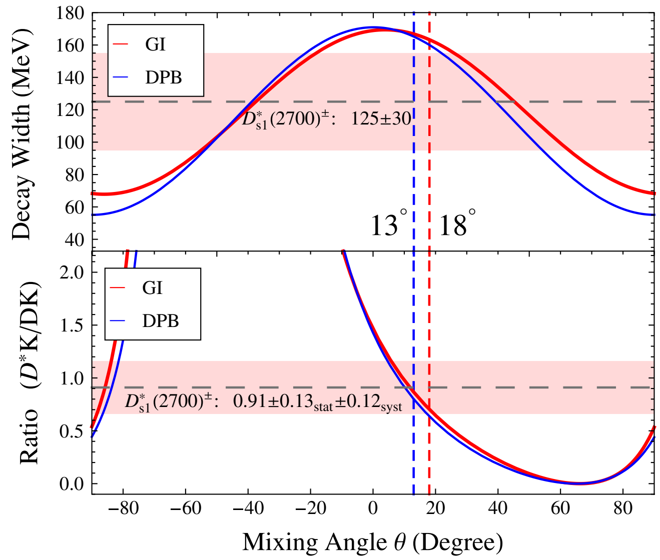


FIG. 6 (color online). Decay width and ratio $D^*K/DK$ of $D^*_{s1}(2700)$ in the diagram method. The horizontal dashed lines refer to central values of decay width given by experiment.

result from the predicted masses of the pure $2^3S_1$ and $1^3D_1$ in Ref. [24] and in Ref. [41], respectively. However, when $D^*(2600)^0$ and $D^*_{s1}(2700)$ are identified with the $|(SD)_1\rangle_L$ of $D$ and $D_s$, respectively, the mixing angles $\theta$ are fixed (Table IV). The mixing angle $\theta$ does not seem to be strongly dependent on the masses input of $1^3D_1$. The mixing angles determined from two different masses input are used as the reasonable boundaries of the variables. Obviously, the ratio $D\pi/D^*\pi$ of $D^*(2600)^0$ and $D^*K/DK$ of $D^*_{s1}(2700)$ in the reasonable region agree well with experiments. The decay widths are a little larger than the experimental data.

In summary, both $D^*(2600)^0$ and $D^*_{s1}(2700)^\pm$ can be explained as the pure $2^3S_1$ states. If the mixing between $2^3S_1$ and $1^3D_1$ exists, the mixing angle $\theta$ is not large and $2^3S_1$ is predominant.

The decay channels $D^*(2760)^0 \to D^+\pi^-$ and $D^*_{sJ}(2860)^+ \to D^0K^+$ have been observed. However, it is difficult to identify $D^*(2760)^0$ and $D^*_{sJ}(2860)^+$ with the

TABLE IV. Mixing angles $\theta$ determined by masses input in Refs. [24,41] are shown.

| | $D^*_1(2^3S_1 - 1^3D_1)$ | | $D^*_{s1}(2^3S_1 - 1^3D_1)$ | |
|---|---|---|---|---|
| | Theoretical prediction (Theo.) | Experimental data | Theoretical prediction | Experimental data |
| Ref. [24] | 2640 | 2608 | 2730 | 2709 |
| | 2820 | 2851 (Theo.) | 2900 | 2921 (Theo.) |
| | $\theta = -21°$ | | $\theta = 18°$ | |
| Ref. [41] | 2636 | 2608 | 2714 | 2709 |
| | 2740 | 2767 (Theo.) | 2804 | 2809 (Theo.) |
| | $\theta = -25°$ | | $\theta = 13°$ | |

$|(SD)_1\rangle_R$ of $D$ and $D_s$, respectively. In other words, if they are the orthogonal partners of $D^*(2600)^0$ and $D^*_{s1}(2700)^\pm$, respectively, the decay width of $D^*_{sJ}(2860)^+$ is broader than 200 MeV and the decay width of $D^*(2760)^0$) is broader than 110 MeV. These decay widths are much broader than the experimental results.

### C. $1^3D_3$ or $[1D(3^-, \frac{5}{2})]$

$D^*(2760)$ and $D^*_{sJ}(2860)^\pm$ are very possibly the $1^3D_3$ $D$ and $D_s$, respectively.

$D^*(2760)^0$ was observed in the decay channel $D^+\pi^-$ and was suggested to be a $D$-wave charmed meson [18]. If $D^*(2760)^0$ has the same $J^P$ with the $1^3D_1$, it would have a broad width through the mixing scheme mentioned above.

Under the assumption that both $D^*(2760)$ and $D^*_{sJ}(2860)^\pm$ are the $1^3D_3$ states, their partial widths and total decay widths are given in Table V. The predicted decay widths of them are in accord with experimental results.

$D(2750)^0$ has mass close to $D^*(2760)$, if these two states are the same state of $1^3D_3$, the predicted ratio $\Gamma(D^*(2760) \to D\pi)/\Gamma(D^*(2760) \to D^*\pi) = 1.78$ (see Table IV) is much larger than the observed $\mathcal{B}(D^*(2760)^0 \to D^+\pi^-)/\mathcal{B}(D(2750)^0 \to D^{*+}\pi^-) = 0.42 \pm 0.05 \pm 0.11$. This fact supports the suggestion that $D(2750)^0$ and $D^*(2760)$ are two different charmed states [18,20].

For $D^*_{sJ}(2860)^\pm$, the predicted $\Gamma(D^*_{sJ}(2860)^\pm \to D^*K)/\Gamma(D^*_{sJ}(2860)^\pm \to DK) = 0.43$ is much smaller than the experimental $\frac{\mathcal{B}(D^*_{sJ}(2860)^+\to D^*K)}{\mathcal{B}(D^*_{sJ}(2860)^+\to DK)} = 1.10 \pm 0.15_{\rm stat} \pm 0.19_{\rm syst}$.

It is apparent that the mass gaps of the corresponding ground state between $D$ and $D_s$ are about 100 MeV [1]. The mass gap between $D^*_{s1}(2700)^\pm$ and $D^*(2600)$, and the mass gap between $D^*_{sJ}(2860)^\pm$ and $D^*(2760)$ are also about 100 MeV. The mass gap supports the suggestion that $D^*_{s1}(2700)^\pm$ is a similar state as $D^*(2600)$ with the same $J^P$. Therefore, there should exist a charmed-strange $D_{sJ}(2850)^\pm$ which has the same $(J^P, j_q)$ of $D(2750)^0$ with mass close to $D^*_{sJ}(2860)^\pm$.

TABLE V. Two-body strong decays of the states $1^3D_3$ are shown.

| Modes [a] | $\Gamma_i$ (MeV) | Modes [b] | $\Gamma_i$ (MeV) | Modes [b] | $\Gamma_i$ (MeV) |
|---|---|---|---|---|---|
| $D^*\ K$ | 12.3 | $D^*\ \pi$ | 12.4 | $D_s\ K$ | 0.9 |
| $D\ K$ | 28.4 | $D\ \pi$ | 22.0 | $D_s^*\ K$ | 0.1 |
| $D_s^*\ \eta$ | 0.6 | $D^*\ \eta$ | 0.2 | $D_1'(2430)\pi$ | 1.1 |
| $D_s\ \eta$ | 3.0 | $D\ \eta$ | 0.8 | $D_1(2420)\pi$ | 0.4 |
| $D\ K^*$ | 0.5 | $D\ \rho$ | 0.1 | $D_2^*(2460)\pi$ | 1.3 |
| $D_s\ \omega$ | 0.2 | $D\ \omega$ | 0 | $\cdots$ | $\cdots$ |
| $\Gamma^{(a)}_{\rm total}$ | 44.9 | | | $\Gamma^{(b)}_{\rm total}$ | 39.3 |
| Experimental data | $48 \pm 7$ | | | Experimental data | $60.9 \pm 8.7$ |

[a]decay modes of $D^*_{sJ}(2860)$.
[b]those of $D^*(2760)$.

TABLE VI. Two-body strong decays of the states $(2^-, \frac{3}{2})$ and $(2^-, \frac{5}{2})$ are shown.

| Modes [a] | $(2^-, \frac{3}{2})$ | $(2^-, \frac{5}{2})$ | Modes [b] | $(2^-, \frac{3}{2})$ | $(2^-, \frac{5}{2})$ |
|---|---|---|---|---|---|
| $D^*\ \pi$ | 58.9 | 20.0 | $D^*\ K$ | 96.2 | 19.2 |
| $D^*\ \eta$ | 5.4 | 0.2 | $D_s^*\ \eta$ | 21.7 | 0.9 |
| $D_s^*\ K$ | 8.6 | 0.2 | $D\ K^*$ | 4.3 | 18.0 |
| $D\ \rho$ | 1.9 | 9.2 | $\cdots$ | $\cdots$ | $\cdots$ |
| $D\ \omega$ | 0.7 | 3.3 | $D_s\ \omega$ | 2.7 | 13.3 |
| $D_0^*(2400)\pi$ | 0.6 | 10.9 | $D_0^*(2400)K$ | 0.2 | 0.2 |
| $D_1'(2430)\pi$ | 0.2 | 1.4 | $\cdots$ | $\cdots$ | $\cdots$ |
| $D_1(2420)\pi$ | 0.5 | 1.4 | $\cdots$ | $\cdots$ | $\cdots$ |
| $D_2^*(2460)\pi$ | 1.2 | 0.3 | $\cdots$ | $\cdots$ | $\cdots$ |
| $\Gamma^{(\dagger)}_{\rm total}$ (MeV) | 77.9 | 47.9 | $\Gamma^{(\ddagger)}_{\rm total}$ (MeV) | 125.1 | 51.6 |
| Experimental data | $\cdots$ | $71 \pm 17$ | Experimental data | $\cdots$ | $\cdots$ |

[a]decay modes of $D(2750)^0$.
[b]$D_{sJ}(2850)$, respectively.

### D. $1D(2^-, \frac{3}{2})$ and $1D(2^-, \frac{5}{2})$

$D(2750)^0$ was observed in $D^{*+}\pi^-$ and is possibly a $1D(2^-, \frac{3}{2})$ or $1D(2^-, \frac{5}{2})$, there exists similar assignment for the suggested $D_{sJ}(2850)^\pm$. The partial widths of some two-body decay modes of $D(2750)^0$ and $D_{sJ}(2850)^\pm$ in the two possible assignments have been computed and presented in Table VI.

If $D_{sJ}(2850)^\pm$ is the $1D(2^-, \frac{3}{2})$, the predicted ratio of branching fraction $\mathcal{B}(D_{sJ}(2850) \to D^*K)/\mathcal{B}(D_{sJ}(2860) \to DK)$ is about 2.42. Theoretical predictions of the decay width and the ratio of branching fraction $\mathcal{B}(D^*(2760)^0 \to D^+\pi^-)/\mathcal{B}(D(2750)^0 \to D^{\star+}\pi^- = 0.52$ of $D(2750)^0$ are in accord with experiment.

If $D(2750)^0$ and $D_{sJ}(2850)^\pm$ are the $1D(2^-, \frac{5}{2})$, $D(2750)^0$, $D^*(2760)^0$ and $D_{sJ}(2850)^\pm$, $D^*_{sJ}(2860)^\pm$ form the $1D(2^-, 3^-)$ doublet of $D$ and $D_s$, respectively.

For charmed mesons $D(2750)^0$ and $D^*(2760)^0$, we obtained $\mathcal{B}(D^0[\frac{5}{2}^-] \to D^+\pi^-)/\mathcal{B}(D^0[\frac{5}{2}^-] \to D^{\star+}\pi^-) \approx 0.82$, which is a little larger than the observed $\mathcal{B}(D^*(2760)^0 \to D^+\pi^-)/\mathcal{B}(D(2750)^0 \to D^{*+}\pi^-) = 0.42 \pm 0.05 \pm 0.11$. We obtained $\mathcal{B}(D^+_{sJ}[\frac{5}{2}^-] \to D^*K)/\mathcal{B}(D^+_{sJ}[\frac{5}{2}^-] \to DK) \approx 0.92$ for the charmed-strange mesons $D_{sJ}(2850)^\pm$ and $D^*_{sJ}(2860)^\pm$, and the observed $\frac{\mathcal{B}(D^*_{sJ}(2860)^+\to D^*K)}{\mathcal{B}(D^*_{sJ}(2860)^+\to DK)} = 1.10 \pm 0.15_{\rm stat} \pm 0.19_{\rm syst}$. Theoretical predictions are in accord with experiments within the uncertainties of the $^3P_0$ model. In our computation, a spin counting has been used.

The two states in the doublet $1D(2^-, 3^-)$ $[1D(2^-, \frac{5}{2})$ and $1D(3^-, \frac{5}{2})]$ have masses close to each other while their mass splitting is comparable to the uncertainty of their masses, it will be difficult to distinguish these two states through the channel of $D\pi$ and $D^*\pi$. In this case, the partial width of $D^*\pi$ observed by experiment is the total one of $D(2750)^0$ and $D^*(2760)^0$. However, the state $1D(2^-, \frac{5}{2})$ decays through the $P$ wave and the $F$ wave while the state $1D(3^-, \frac{5}{2})$ can only decay through the $F$ wave. Therefore, the widths of decay channels $D\rho$ and $D\omega$ of $D(2750)^0$ would much broader than those of $D^*(2760)^0$. The observation of the channels $D\rho$ and $D\omega$ in forthcoming experiments will be useful to pin down these states.

## IV. CONCLUSIONS AND DISCUSSIONS

In this work, we study the possible $2S$ and $1D$ $D$ and $D_s$ states, especially the four new $D$ candidates observed by the *BABAR* Collaboration. Both the mass and the decay width indicate that $D(2550)^0$ is a good candidate of the $2^1S_0$ charmed state. The $2^1S_0$ $Ds$ meson is predicted to have mass about $2635 \pm 20$ MeV and decay width about $82.2 \pm 15.1$ MeV. The observed $D_{sJ}(2632)^+$ seems impossible the $2^1S_0$ $Ds$ meson if it exists.

$D^*(2600)$ and $D^*_{s1}(2700)^\pm$ can be explained as pure $2^3S_1$ states. If the mixing between $2^3S_1$ and $1^3D_1$ exists, the mixing angle $\theta$ is not large and $2^3S_1$ is predominant. The results indicate that the mixing angle $\theta$ is not strongly dependent on the input mass of $1^3D_1$. Our analysis does not support the possibility that $D^*(2760)$ and $D^*_{sJ}(2860)^\pm$ are the orthogonal partners of $D^*(2600)$ and $D^*_{s1}(2700)^\pm$, respectively.

If an unobserved meson, corresponding to $D(2750)^0$, $D_{sJ}(2850)^\pm$ exists, more measurement of $D^*_{sJ}(2860)^\pm$ is required. $D^*(2760)$ and $D^*_{sJ}(2860)^\pm$ could be identified with the $1^3D_3$ $D$ and $D_s$ states, respectively. $D(2750)^0$ and $D^*(2760)$ favor to form the doublet $1D(2^-, 3^-)$. The possibility that $D(2750)^0$ is the $1D(2^-, \frac{3}{2})$ state has not been excluded, so the observation of the channels $D\rho$ and $D\omega$ would be important for the identification of $D(2750)^0$ and $D^*(2760)^0$.

## ACKNOWLEDGMENTS

Bing Chen thanks Professor Tom Steele for useful discussions. This work is supported by the National Natural Science Foundation of China under Grants No. 10775093 and No. 11075102. Bing Chen is also supported by Shanghai University under the Graduates Innovation Fund SHUCX092016 of its contract: No. A. 16-0101-09-543.

## APPENDIX

When one considers the two-body strong decay of $D^*(2600)^0$ in the mixing scheme [Eq. (5)], the Eq. (3) should be written as

$$\Gamma^{H\to H'h} = \zeta p \mathcal{G} \sum_{\rm LS} |\cos\theta \mathcal{C}_1 \mathcal{P}^{\rm LS}_{2^3S_1\to H'h}(x_1) e^{-x_1^2/12} - \sin\theta \mathcal{C}_2 \mathcal{P}^{\rm LS}_{1^3D_1\to H'h}(x_2) e^{-x_2^2/12}|^2. \qquad \text{(A1)}$$

where

TABLE VII. The coefficients for different decay channels in heavy-quark effective theory. $S$, $P$, and $D$ refer to $S$, $P$, and $D$-wave decays.

| | $\mathcal{C}_1(2^3S_1)$ | | $\mathcal{C}_2(1^3D_1)$ | |
|---|---|---|---|---|
| ${}^1S_0 + {}^1S_0$ | $\sqrt{\frac{1}{3}}$ | $[P]$ | $-\sqrt{\frac{2}{3}}$ | $[P]$ |
| ${}^3S_1 + {}^1S_0$ | $-\sqrt{\frac{2}{3}}$ | $[P]$ | $-\sqrt{\frac{1}{3}}$ | $[P]$ |
| $1P(1^+, \frac{1}{2}) + {}^1S_0$ | 1 | $[S]$ | 1 | $[D]$ |
| $1P(1^+, \frac{3}{2}) + {}^1S_0$ | $\sqrt{\frac{1}{2}}$ | $[D]$ | $-1$ | $[S]$ |
| | $-$ | | $-\sqrt{\frac{1}{2}}$ | $[D]$ |
| ${}^3P_2 + {}^1S_0$ | $-\sqrt{\frac{1}{2}}$ | $[D]$ | $-\sqrt{\frac{1}{2}}$ | $[D]$ |

$$x_1 = \frac{p_1}{\beta}, \quad x_2 = \frac{p_2}{\beta},$$

$$p = \frac{\sqrt{[m^2_{D^*(2600)^0} - (m_{H'} + m_h)^2][m^2_{D^*(2600)^0} - (m_{H'} - m_h)^2]}}{2m_{D^*(2600)^0}},$$

$$p_1 = \frac{\sqrt{[m^2_{2^3S_1} - (m_{H'} + m_h)^2][m^2_{2^3S_1} - (m_{H'} - m_h)^2]}}{2m_{2^3S_1}},$$

$$p_2 = \frac{\sqrt{[m^2_{1^3D_1} - (m_{H'} + m_h)^2][m^2_{1^3D_1} - (m_{H'} - m_h)^2]}}{2m_{1^3D_1}};$$

$\mathcal{C}_1$ and $\mathcal{C}_2$ are coefficients for different decay channels (Table VII). $\mathcal{P}_{\rm LS}(x)$ are the channel-dependent polynomials.

For $2^3S_1$:

$$2^3S_1 \to \begin{cases} 1S_0 + 1S_0 \\ 3S_1 + 1S_0 \end{cases} : \frac{5}{3^4} x_1 \left(1 - \frac{2}{15} x_1^2\right),$$

$$2^3S_1 \to 1P\left(1^+, \frac{1}{2}\right) + 1S_0: \frac{1}{2^{1/2}3^{7/2}} \left(1 - \frac{7}{9} x_1^2 + \frac{2}{27} x_1^4\right),$$

$$2^3S_1 \to \begin{cases} 1P\left(1^+, \frac{3}{2}\right) + 1S_0 \\ {}^3P_2 + 1S_0 \end{cases} : \frac{13}{3^{11/2}} x_1^2 \left(1 - \frac{2}{39} x_1^2\right).$$

For $1^3D_1$:

$$1^3D_1 \to \begin{cases} 1S_0 + 1S_0 \\ 3S_1 + 1S_0 \end{cases} : \frac{5^{1/2}2^{1/2}}{3^4} x_2 \left(1 - \frac{2}{15} x_2^2\right),$$

$$1^3D_1 \to 1P\left(1^+, \frac{1}{2}\right) + 1S_0: -\frac{5^{1/2}}{3^{11/2}} x_2^2 \left(1 + \frac{2}{15} x_2^2\right),$$

$$1^3D_1 \to 1P\left(1^+, \frac{3}{2}\right) + 1S_0 \begin{cases} -\frac{2\cdot 5^{1/2}}{3^{7/2}}\left(1 - \frac{5}{18} x_2^2 + \frac{1}{135} x_2^4\right) \\ -\frac{13}{3^{11/2}} x_2^2 \left(1 - \frac{2}{39} x_2^2\right) \end{cases},$$

$$1^3D_1 \to {}^3P_2 + 1S_0: -\frac{13}{3^{11/2}5^{1/2}} x_2^2 \left(1 - \frac{2}{39} x_2^2\right).$$

[1] K. Nakamura *et al.* (Particle Data Group), J. Phys. G **37**, 075021 (2010).
[2] P. Abreu *et al.* (DELPHI Collaboration), Phys. Lett. B **426**, 231 (1998).
[3] D. Melikhov and O. Pene, Phys. Lett. B **446**, 336 (1999).
[4] P. R. Page, Phys. Rev. D **60**, 057501 (1999).
[5] A. V. Evdokimov *et al.* (SELEX Collaboration), Phys. Rev. Lett. **93**, 242001 (2004).
[6] T. Barnes, F. E. Close, J. J. Dudek, S. Godfrey, and E. S. Swanson, Phys. Lett. B **600**, 223 (2004).
[7] A. Zhang, Phys. Rev. D **72**, 017902 (2005).
[8] K. Abe *et al.* (Belle Collaboration), arXiv:hep-ex/0608031; J. Brodzicka *et al.* (Belle Collaboration), Phys. Rev. Lett. **100**, 092001 (2008).
[9] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **97**, 222001 (2006).
[10] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. D **80**, 092003 (2009).
[11] F. E. Close, C. E. Thomas, O. Lakhina, and E. S. Swanson, Phys. Lett. B **647**, 159 (2007).
[12] B. Chen, D.-X. Wang, and A. Zhang, Phys. Rev. D **80**, 071502(R) (2009).
[13] B. Zhang, X. Liu, W.-Z. Deng, and S.-L. Zhu, Eur. Phys. J. C **50**, 617 (2007).
[14] T. Abdullah and F. E. Close, Phys. Rev. D **5**, 2332 (1972).
[15] E. V. Beveren and G. Rupp, Phys. Rev. Lett. **97**, 202001 (2006).
[16] P. Colangelo, F. De. Fazio, and S. Nicotri, Phys. Lett. B **642**, 48 (2006).
[17] Zhi-Feng Sun and Xiang Liu, Phys. Rev. D **80**, 074037 (2009).
[18] J. Benitez *et al.*, in ICHEP2010, Paris, 2010 (unpublished); P. del Amo Sanchez *et al.* (*BABAR* Collaboration), Phys. Rev. D **82**, 111101 (2010).
[19] Z.-F. Sun, J.-S. Yu, X. Liu, and T. Matsuki, Phys. Rev. D **82**, 111501(R) (2010).
[20] X.-H. Zhong, Phys. Rev. D **82**, 114014 (2010).
[21] Z. G. Wang, Phys. Rev. D **83**, 014009 (2011).
[22] E. J. Eichten, C. T. Hill, and C. Quigg, Phys. Rev. Lett. **71**, 4116 (1993).
[23] G. Chew and S. Frautschi, Phys. Rev. Lett. **8**, 41 (1962).
[24] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985).
[25] D. Ebert, V. O. Galkin, and R. N. Faustov, Eur. Phys. J. C **66**, 197 (2010).
[26] A. Tang and J. W. Norbury, Phys. Rev. D **62**, 016006 (2000).
[27] A. V. Anisovich, V. V. Anisovich, and A. V. Sarantsev, Phys. Rev. D **62**, 051502(R) (2000).
[28] M. Di Pierro and E. J. Eichten, Phys. Rev. D **64**, 114004 (2001).
[29] M. G. Olsson and S. Veseli, Phys. Rev. D **51**, 3578 (1995).
[30] N. Isgur and M. Wise, Phys. Rev. Lett. **66**, 1130 (1991).
[31] A. F. Falk and M. Luke, Phys. Lett. B **292**, 119 (1992).
[32] A. V. Mannohar and M. B. Wise, *Heavy Quark Physics* (Cambridge University Press, Cambridge, 2000).
[33] A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984).
[34] F. E. Close and Z. P. Li, Phys. Rev. D **42**, 2194 (1990); Z. P. Li, Phys. Rev. D **48**, 3070 (1993); **50**, 5639 (1994); **52**, 4961 (1995); Phys. Rev. C **52**, 1648 (1995).
[35] L. Micu, Nucl. Phys. **B10**, 521 (1969); A. Le Yaouanc, L. Olivier, O. Pene, and J. C. Raynal, Phys. Rev. D **8**, 2223 (1973); A. Le Yaouanc, L. Oliver, O. Pene, and J. C. Raynal, *Hadron Transition in the Quark Model* (Gordon and Breach, New York, 1988), p. 311.
[36] E. S. Ackleh, T. Barnes, and E. S. Swanson, Phys. Rev. D **54**, 6811 (1996).
[37] T. Barnes, F. E. Close, P. R. Page, and E. S. Swanson, Phys. Rev. D **55**, 4157 (1997).
[38] R. Kokoski and N. Isgur, Phys. Rev. D **35**, 907 (1987).
[39] Bing Chen, Ling Yuan, and Ailin Zhang (unpublished).
[40] J. Zeng, J. W. Van Orden, and W. Roberts, Phys. Rev. D **52**, 5229 (1995).
[41] D.-M. Li, P.-F. Ji, and B. Ma, Eur. Phys. J. C **71**, 1582 (2011).
[42] T. A. Lahde, C. J. Nyfalt, and D. O. Riska, Nucl. Phys. **A674**, 141 (2000).
[43] J. L. Rosner, Phys. Rev. D **64**, 094002 (2001); J. L. Rosner, Ann. Phys. (N.Y.) **319**, 1 (2005); T. Barnes, S. Godfrey, and E. S. Swanson, Phys. Rev. D **72**, 054026 (2005); E. J. Eichten, S. Godfrey, H. Mahlke, and J. L. Rosner, Rev. Mod. Phys. **80**, 1161 (2008).