On $D \to X_u l^+ l^-$ within the Standard Model and frameworks like the littlest Higgs model with T Parity

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The $D \to X_u l^+ l^-$ transitions—branching ratios, forward-backward (FB) asymmetry A_{FB}^c, the CP asymmetry A_{CP}^c, and the CP asymmetry in the forward-backward asymmetry A_{FB}^{CP}—have two sources: for $D^{\pm}$ they represent a pure $\Delta C = 1$ & $\Delta Q = 0$ current interaction whereas neutral D mesons can also communicate via their antihadron. Standard model (SM) contributions to $\mathrm{BR}(D \to X_u l^+ l^-)$ come primarily from long distance dynamics, which overshadow short distance contributions by several orders of magnitude; still they fall much below the present upper experimental bounds. Even the SM contributions to A_{FB}^c, A_{CP}^c, and A_{FB}^{CP} are tiny, quite unlike in beauty hadrons. The branching ratios are hardly dented by contributions from the littlest Higgs models with T parity (LHT) even in the short distance regime, let alone in the SM long distances dynamics. Yet the asymmetries A_{FB}^c, A_{CP}^c, and A_{FB}^{CP} in these new physics models can be enhanced over SM predictions, as they arise purely from short distance dynamics; this can occur, in particular, for A_{FB}^c and A_{FB}^{CP}, which get enhanced by orders of magnitudes. Even such enhancements hardly reach absolute sizes for observable experimental effects for A_{FB}^c and A_{CP}^c. However, LHT contributions to A_{FB}^{CP} could be measured in experiments like the LHCb and the SuperB Collaboration. These results lead us to draw further conclusions on flavor-changing neutral-current interactions within LHT-like models through some simple scaling arguments that encapsulate the essence of flavor dynamics in and beyond the standard model.

I. INTRODUCTION

The discovery of charm quarks was and still is seen as a great success of the standard model (SM), since their existence was necessary for the observed suppression of strangeness changing neutral currents. Charm hadrons were also found in the expected mass range as was the predicted preference for decays to strange hadrons.

Yet, at last, some possible hint of new physics (NP) has appeared in charm physics. Compelling evidence for $D^0 - \bar{D}^0$ oscillations has been presented by the Belle, *BABAR*, and CDF Collaborations[1–3]. The HFAG Collaboration has combined the results on neutral D decays allowing CP violation [4,5]:

$$x_D = \frac{\Delta M_D}{\Gamma_D} = (0.63^{+0.19}_{-0.20})\%, \quad \left|\frac{q}{p}\right| = 0.91^{+0.18}_{-0.16}$$

$$y_D = \frac{\Delta \Gamma_D}{2\Gamma_D} = (0.75 \pm 0.12)\%, \quad \phi = -10.2^{+9.4}_{-8.9}(°). \tag{1}$$

The observation of $D^0 - \bar{D}^0$ oscillations is hardly disputed, while the relative size of x_D and y_D is not clear yet. Before these experimental results, most theorists argued that the SM predicts $x_D, y_D \lesssim 10^{-4}$—yet not all: in 1998, $x_D, y_D \lesssim 10^{-2}$ was called a SM *conservative* bound

[6]; in 2000 and 2003, a SM prediction obtained from a sophisticated operator product expansion yielded $x_D, y_D \sim \mathcal{O}(10^{-3})$ [7,8] and more recently in [9,10]; alternatively, in 2001 and 2004, a SM prediction on $D^0 - \bar{D}^0$ oscillations was based on $SU(3)$ breaking mostly in the phase space for y_D and then from a dispersion relation for x_D [11,12].

While the present experimental results on x_D and y_D can be accommodated within the available theoretical SM estimates, and no nonzero CP asymmetry has been seen yet, the observation of $D^0 - \bar{D}^0$ oscillation, however, has "wetted" the appetite on thinking of NP in charm decays. The authors of Ref. [13] consider a (approximately) $SU(2)_L$-invariant NP scenario; therefore NP contributions to $D^0 - \bar{D}^0$ and $K^0 - \bar{K}^0$ oscillations are not independent of each other.

There is a large variety of NP models, in which the Higgs boson appears as a pseudo-Nambu-Goldstone boson of a spontaneously broken global symmetry, namely, the "little Higgs" class of models [14–20]. To achieve this program, one needs at least heavy gauge bosons $W_H^{\pm}$, Z_H, and A_H, a heavy top partner T and a scalar triplet Φ as physical degrees of freedom as is implemented in the "littlest Higgs" model [22,23]. Studying electroweak precision observables shows that for such new states to arise below the 1 TeV scale, one needs an additional discrete symmetry [24,25], called T parity: the SM particles and the heavy top partner T are *even* and $W_H^{\pm}$, Z_H, A_H, and Φ are *odd*.

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A consistent implementation of T parity requires also the introduction of the so-called "mirror" fermions—one for each quark and lepton species—that are *odd* under T parity [25,26]. This creates the littlest Higgs model with T parity (LHT) [27]. While some theorists probably see it as intellectually economical, most experimentalists do not view it like that; at the same time they should understand that it can provide them with more work, but less so than supersymmetry. The most important point here is that the motivation for LHT models comes from *outside* flavor dynamics; at the same time they can create important nontrivial signals of NP in B, K, and D physics. The LHT models can—not necessarily, but possibly—affect $\Delta C = 2$ dynamics significantly. In particularly, it can generate sizable or even relatively large indirect CP violation in D^0 decays [29]. It can implement a *dynamical* realization of the symmetry approach described in Ref. [13].

Encouraged by the findings of Ref. [29] about the possible impacts of LHT models in $\Delta C = 2$ dynamics, we had looked at two $\Delta C = 1$ processes, $D^0 \to \gamma\gamma$ and $D^0 \to \mu^+\mu^-$ in a previous study [30]. While LHT failed to contribute significantly to the total decay rates in these channels, it had led us to some general conjectures on charm changing neutral currents (CCNC) within LHT-like scenarios. In this current work, we continue to analyze the impact of LHT on another $\Delta C = 1$ process to probe deeper into CCNCs in LHT-like scenarios, namely, to $D \to X_u l^+ l^-$. Similar to our previous work, we do not see any sizable enhancements in the global decay rates. However, in the presence of *large* weak phases in LHT-like models, it might seem surprising to find also very tiny contributions to the CP and forward-backward asymmetries, A^c_{CP} and A^c_{FB}. This is because the (nontrivial) SM asymmetries get produced by *short* distance dynamics, and LHT-like scenarios can create much larger A^c_{CP} and A^c_{FB} than the SM can; yet they are still small in their absolute size. However, the LHT contributions to the forward-backward CP asymmetry A^{CP}_{FB} are sizable—even large—such that they can be experimentally measured in the coming decade.

In this article, we discuss both short and long distance SM contributions to $D \to X_u l^+ l^-$ in Sec. II along with

A^c_{CP}, A^c_{FB}, and A^{CP}_{FB} in this channel. We will describe mostly $D^\pm \to X_u l^+ l^-$, since it is given only by $\Delta C = 1$ couplings, while one can also produce $D^0 \to X_u l^+ l^-$ by $\Delta C = 2$ couplings due to $D^0 - \bar{D}^0$ oscillations. We go on to briefly introduce LHT and its contributions to $D \to X_u l^+ l^-$ in Sec. III. Our quantitative findings are presented in Sec. IV. We take a more critical look at CCNCs in LHT-like models in Sec. V. In Sec. VI, we put forward some simple scaling arguments to explain why NP interventions such as in LHT-like models have effects of the size we see and the conclusion to this work follows in Sec. VII.

II. SM CONTRIBUTIONS TO $D \to X_u l^+ l^-$

The transition of $D \to X_u l^+ l^-$ must be produced by charm changing neutral currents, which are much weaker even than their strangeness and beauty analogues in the SM. These decay rates are tiny and dominated by long distances effects. Yet the forward-backward A^c_{FB}, CP asymmetries A^c_{CP} and CP asymmetry in the forward-backward asymmetry A^{CP}_{FB} could still be controlled by SM *short* distance dynamics. However, the dynamics scenery is very complex as shown below; conceptually very similar to $B \to X_s l^+ l^-$, but quantitatively at a smaller level. One can learn a lot by studying these *asymmetries* about short distance (SD) dynamics in general where different operators mix and an alternative perspective can be obtained from B studies. It turns out—not surprisingly—that a careful scrutiny needs huge statistics. This would be an important task for a SuperB factory like the recently approved SuperB project undertaken by the Istituto Nazionale di Fisica Nucleare [31–33]; LHCb might also be able to address it.

We will discuss first $D^\pm \to X_u l^+ l^-$, since it proceeds purely by a $\Delta C = 1$ interaction; then we will comment on lessons learned from neutral $D \to X_u l^+ l^-$ transitions, where $D^0 - \bar{D}^0$ oscillations can get involved.

A. $\Gamma_{SM}(D \to X_u l^+ l^-)$

The quark level process $c \to u l^+ l^-$ is described with an operator basis of the following ten operators:

$$O_1^{(q)} = (\bar{u}_L^\alpha \gamma_\mu q_L^\beta)(\bar{q}_L^\beta \gamma^\mu c_L^\alpha) \quad O_2^{(q)} = (\bar{u}_L^\alpha \gamma_\mu q_L^\alpha)(\bar{q}_L^\beta \gamma^\mu c_L^\beta) \quad O_3 = (\bar{u}_L^\alpha \gamma_\mu c_L^\alpha)\sum_q(\bar{q}_L^\beta \gamma^\mu q_L^\beta) \quad O_4 = (\bar{u}_L^\alpha \gamma_\mu c_L^\beta)\sum_q(\bar{q}_L^\beta \gamma^\mu q_L^\alpha)$$

$$O_5 = (\bar{u}_L^\alpha \gamma_\mu c_L^\alpha)\sum_q(\bar{q}_R^\beta \gamma^\mu q_R^\beta) \quad O_6 = (\bar{u}_L^\alpha \gamma_\mu c_L^\beta)\sum_q(\bar{q}_R^\beta \gamma^\mu q_R^\alpha) \quad O_7 = \frac{e}{16\pi^2}m_c(\bar{u}_L\sigma_{\mu\nu}c_R)F^{\mu\nu}$$

$$O_8 = \frac{g_s}{16\pi^2}m_c(\bar{u}_L\sigma_{\mu\nu}T^a c_R)G_a^{\mu\nu} \quad O_9 = \frac{e^2}{16\pi^2}(\bar{u}_L\gamma_\mu c_L)(\bar{l}\gamma^\mu l) \quad O_{10} = \frac{e^2}{16\pi^2}(\bar{u}_L\gamma_\mu c_L)(\bar{l}\gamma^\mu\gamma_5 l) \tag{2}$$

$$\mathcal{H}_{\text{eff}}(\mu) = -4\frac{G_F}{\sqrt{2}}\left[\sum_q C_1^{(q)}(\mu)O_1^{(q)}(\mu) + C_2^{(q)}(\mu)O_2^{(q)}(\mu) + \sum_{i=3}^{10} C_i(\mu)O_i(\mu)\right], \tag{3}$$

where $q = d, s, b$ and α, β are color indices. The charm operators O_i, $i = 1, \ldots, 10$ are analogous to those in b decays [34]. The effective weak Hamiltonian is expressed in Eq. (3) in terms of these operators taken at a scale μ. The coefficients describe renormalization of the operators at μ from the normalization scale. For the starting point, one can naturally chose $\mu = M_W$:

$$C_1^{(q)}(M_W) = 0 \qquad C_2^{(q)}(M_W) = 1 \qquad C_{3-6}(M_W) = 0$$

$$C_7(M_W) = -\frac{1}{2}\sum_{j=d,s} V_{uj}^* V_{cj}(D_0'(x_j) - D_0'(x_b))$$

$$C_8(M_W) = -\frac{1}{2}\sum_{j=d,s} V_{uj}^* V_{cj}(E_0'(x_j) - E_0'(x_b))$$

$$C_9(M_W) = \sum_{j=d,s} V_{uj}^* V_{cj}\left(\frac{Y_0(x_j)}{\sin^2(\theta_W)} - 4C_0(x_j)\right.$$
$$\left. - D_0(x_j) - (x_j \to x_b)\right)$$

$$C_{10}(M_W) = \sum_{j=d,s} V_{uj}^* V_{cj}\left(\frac{Y_0(x_j) - Y_0(x_b)}{\sin^2(\theta_W)}\right), \tag{4}$$

where $x_j = m_j^2/m_W^2$, with m_j being the masses of the internal down-type quarks $j = d, s, b$. Here we have used the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, $\sum_{j=d,s,b} V_{uj}^* V_{jc} = 0$ to eliminate the dependence of the Wilson coefficients on the third family. The limit $x_d \to 0$ cannot be taken as there is a logarithmic divergence in $D^0(x)$ in that limit. The form factors are defined in a modified way from Ref. [35]:

$$C_0(x) = \frac{1}{2}\left(\frac{x}{4} - \frac{3}{8}\frac{1}{(x-1)} + \frac{3}{8}\frac{2x^2 - x}{(x-1)^2}\log(x) + \gamma(x)\right)$$

$$D_0(x) = Q_d E_0(x) + \frac{x(9 - 43x + 28x^2)}{12(1-x)^3}$$
$$+ \frac{x(21 - 66x + 41x^2 - 2x^3)}{12(1-x)^4}\log(x) - 2\gamma(x)$$

$$E_0(x) = -\frac{2}{3}\log(x) + \frac{x^2(15 - 16x + 4x^2)}{6(1-x)^4}\log(x)$$
$$+ \frac{x(18 - 11x - x^2)}{12(1-x)^3}$$

$$D_0'(x) = Q_d E_0'(x) - \left(\frac{1}{3} + \frac{11x^2 - 7x + 2}{4(1-x)^3} + \frac{6x^3}{4(1-x)^4}\log(x)\right)$$

$$E_0'(x) = -\frac{5}{12} + \frac{1 - 5x - 2x^2}{4(x-1)^3} + \frac{3x^2}{2(x-1)^4}\log(x)$$

$$Y_0(x) = \frac{x}{8}\left(\frac{x-4}{x-1} + \frac{3x}{(x-1)^2}\log(x)\right). \tag{5}$$

Q_d is the charge of the internal down-type quarks. Here $\gamma(x)$ is the gauge dependent term which, for $\xi = 1$, is [36]

$$\gamma(x) = \frac{7}{8}\left(\frac{x}{(x-1)^2}\log(x) - \frac{1}{x-1}\right). \tag{6}$$

At scales $\mu < M_W$, one can express $\mathcal{H}_{\text{eff}}(\mu)$ using $C_i(\mu)$, $i = 1, \ldots, 10$ evolving through the two-loop QCD renormalization group equation. Actually, one has two regimes, M_W to m_b and m_b to m_c [37]; a matching condition as usual is applied through $\alpha_s(m_b, m_b; 4) = \alpha_s(m_b, m_b; 5)$. These operators mix via renormalization; in particular O_7 mixes with O_1, O_2 [34,38], O_{3-6}, and O_8. However, as pointed out in [38], O_7 is completely dominated by the two-loop QCD radiative correction, which was taken into account in [39,40]:

$$C_7(m_c) = \eta_c^{16/25}\eta_b^{16/23}C_7(m_W)$$
$$- \frac{16}{3}\left(\eta_c^{14/25}\eta_b^{14/23} - \eta_c^{16/25}\eta_b^{16/23}\right)C_8(m_W)$$
$$- V_{ub}^* V_{cb}\sum_{i=1}^{8}\sum_{j=1}^{6} C_j(m_b)X_{ji}\eta_c^{z_i} + \frac{\alpha_s(m_c)}{4\pi}C_2(m_c)$$
$$\times \left[V_{us}^* V_{cs}\left\{f^2\left(\frac{m_s}{m_c}\right) - f^2\left(\frac{m_d}{m_c}\right)\right\} + V_{ub}^* V_{cb}f^2\left(\frac{m_d}{m_c}\right)\right], \tag{7}$$

where

$$\eta_b = \frac{\alpha_s(m_W)}{\alpha_s(m_b)}, \qquad \eta_c = \frac{\alpha_s(m_b)}{\alpha_s(m_c)}. \tag{8}$$

The matrix $\mathbf{X}$, the vector $\mathbf{z}$, and the function f introduced above are given in Appendix A along with the Wilson coefficients for the operators O_{1-6} both at $\mu = m_b$ and $\mu = m_c$. We include the next to leading order correction to the running of the α_s with $\alpha_s(m_W) = 0.125$, $\alpha_s(m_b) = \alpha_s(m_b, m_W, 5)$, $\alpha_s(m_c) = \alpha_s(m_c, m_b, 4)$,

$$\alpha_s(\mu, \mu', n_f) = \frac{\alpha_s(\mu', \mu', n_f)}{v(\mu, \mu', n_f)}\left(1 - \frac{\beta_1(n_f)}{\beta_0(n_f)}\frac{\alpha_s(\mu', \mu', n_f)}{4\pi}\right.$$
$$\left. \times \frac{\log(v(\mu, \mu', n_f))}{v(\mu, \mu', n_f)}\right)$$

$$v(\mu, \mu', n_f) = 1 - \beta_0(n_f)\frac{\alpha_s(\mu', \mu', n_f)}{2\pi}\log\left(\frac{\mu'}{\mu}\right),$$

$$\beta_0(n_f) = 11 - \frac{2}{3}n_f, \quad \beta_1(n_f) = 102 - \frac{38}{3}n_f. \tag{9}$$

O_9 mixes with O_{1-6} beyond the leading order. However, only C_1 and C_2 are numerically significant and lead to an important cancellation amongst themselves. As we shall later see, the dominant contribution to the SM branching fraction comes from O_9; hence it is important to have a detailed look at it contrary to what was argued in [39,40]. The Wilson coefficient for O_9 after QCD corrections is given by [41]

$$C_9(m_c) = C_9(m_W)$$
$$+ \sum_{j=d,s} V_{uj}^* V_{cj}h(z_j, \hat{s})\left(\sum_{i=1,3,5} 3C_i(m_c) + C_{i+1}(m_c)\right) \tag{10}$$

$$h(z, \hat{s}) = Q_d \tilde{h}(z, \hat{s})$$

$$\tilde{h}(z, \hat{s}) = -\frac{4}{3} \log\frac{m_c}{\mu} - \frac{4}{3} \log(z) + \frac{4}{9} + \frac{8}{3}\frac{z^2}{s} - \frac{1}{3}\left(2 + \frac{4z^2}{s}\right)\sqrt{\left|1 - \frac{4z^2}{s}\right|} \times \begin{cases} 2\tan^{-1}\frac{1}{\sqrt{\frac{4z^2}{s}-1}} & \text{if } \hat{s} < 4z^2 \\ \log\left|\frac{\sqrt{1-\frac{4z^2}{s}}+1}{\sqrt{1-\frac{4z^2}{s}}-1}\right| - i\pi & \text{if } \hat{s} > 4z^2 \end{cases} \tag{11}$$

$$\frac{d}{d\hat{s}}\text{Br}_{\text{SM}}^{\text{SD}}(D \to X_u l^+ l^-) = \frac{1}{\Gamma_D}\frac{G_F^2 \alpha^2 m_c^5}{768\pi^5}(1-\hat{s})^2\left[(|C_9(\mu)|^2 + |C_{10}(\mu)|^2)(1+2\hat{s}) + 12\text{Re}(C_7(\mu)C_9^*(\mu)) + 4\left(1+\frac{2}{\hat{s}}\right)|C_7(\mu)|^2\right], \tag{12}$$

where $h(z_j, s)$ comes from the one-loop QCD correction to the four fermion operator and is given by [42] and

$$\hat{s} = \frac{(p_{l^+} + p_{l^-})^2}{m_c^2}, \qquad z_j = \frac{m_j}{m_c}.$$

As in the case of the analogous decay in B systems, the logarithmic term in $h(z, \hat{s})$ exactly cancels the logarithmic dependence in $C_9(m_W)$ that comes from the electromagnetic penguin and hence removes the logarithmic dependence on light quark masses at the scale $\mu = m_W$ as was pointed out in [39]. The importance of QCD correction was pointed out in [43], but we disagree with their argument that the purely electroweak "Inami-Lim" contribution to C_9 should be ignored as it is dependent on light quark mass and is reproduced as a limit of the QCD correction when $\hat{s} \to 0$. A careful look at the form of $h(z, \hat{s})$ shows that the logarithms have opposite signs in the Inami-Lim term and in the QCD correction as argued before [44]. A discussion of the logarithmic dependence of C_9 before including QCD corrections and its cancellation after including the same is discussed in [45] for the case of B mesons. A similar argument applies in the case of the D mesons too.

O_{10} does not suffer from any QCD corrections [39,46], which makes the assumption made in [40] unnecessary [47]. Finally, the differential decay branching fraction is given by Eq. (12) with $\mu = m_c = 1.2$ GeV. Integrating over $\hat{s}$ gives us the total decay rate. One has to be careful about not picking up the infrared divergence in the differential decay rate. We made an infrared cut on $\hat{s}$ at about an invariant dilepton momentum of 20 MeV. We get a branching fraction of

$$\text{BR}_{\text{SD}}^{\text{SM}}(D \to X_u e^+ e^-) \sim 3.7 \times 10^{-9}, \tag{13}$$

which is smaller than what is stated in [39,40,48], but larger than the number in [43] for reasons stated above. The same for muons in the final state is slightly smaller due to the finite mass of the muon. The SD-M contribution is domi-

nated by C_9, primarily with contributions from the purely electroweak part, coming almost entirely from the electromagnetic penguin, and an order of magnitude smaller contribution from the QCD correction coming from the four fermion operators O_1 and O_2. Contrary to what is stated in [43], C_7 provides only a subdominant contribution in spite of its huge enhancement from the two-loop $O(\alpha_s)$ contributions.

As we stated before, the SD contribution in SM is completely overshadowed by the long distance (LD) contribution that comes from intermediate vector meson states. These resonance contributions lead to a branching fraction estimated in [39]:

$$\text{BR}_{\text{LD}}^{\text{SM}}(D \to X_u e^+ e^-)$$
$$= \text{BR}^{\text{SM}}(D \to X_u e^+ e^-) \sim \mathcal{O}(10^{-6}). \tag{14}$$

Rough estimates can show that the SD contributions for the branching ratio (BR) are much smaller than LD contributions. Why did we (and other authors) undertake a time consuming operator-product expansion analysis? Finding incorrect statements in published literature maybe be intellectually acceptable, even if such statements are of only academic significance. However, we are driven by matters of much more practical interest: Some asymmetries on which distributions are based give a more direct access to SD dynamics, namely, A_{FB}^c and A_{CP}^c, as we shall discuss next.

B. A_{FB}^c, A_{CP}^c and A_{FB}^{CP}

Asymmetries between $D^\pm \to X_u l^+ l^-$ have a single source, namely, $\Delta C = 1$, $\Delta Q = 1$ currents. Later, we will comment on $D^0 \to X_u l^+ l^-$ vs $\bar{D}^0 \to X_u l^+ l^-$, where $D^0 - \bar{D}^0$ oscillations, in principle, could get involved.

The normalized forward-backward asymmetry is defined from the double differential decay rate as

$$A_{\text{FB}}^c(\hat{s}) = \frac{\int_{-1}^{1}\left[\frac{d^2}{d\hat{s}dz}\Gamma(D^+ \to X_u l^+ l^-) - \frac{d^2}{d\hat{s}dz}\Gamma(D^- \to X_u l^+ l^-)\right]\text{sgn}(z)dz}{\int_{-1}^{1}[d^2\Gamma(D^\pm \to X_u l^+ l^-)/d\hat{s}dz]dz} \tag{15}$$

$$= \frac{-3[\Re(C_{10}^*(\mu)C_9(\mu))\hat{s} + 2\Re(C_{10}^*(\mu)C_7(\mu)]}{(1+2s)(|C_9(\mu)|^2 + |C_{10}(\mu)|^2) + 4|C_7(\mu)|^2(1+\frac{2}{s}) + 12\Re(C_7(\mu)C_9^*(\mu))}. \tag{16}$$

After performing the integral over the angular distribution, we get the form in Eq. (16). Since C_{10} is real, $A_{\mathrm{FB}}^c(\hat{s})$ picks up the real part of C_9 and C_7, which are both in general complex. Integrating over $\hat{s}$ we get

$$A_{\mathrm{FB}}^c \sim 2 \times 10^{-6}. \qquad (17)$$

$A_{\mathrm{FB}}^c(\hat{s})$ is mostly proportional to C_{10} which is tiny in the SM as expected from the suppression of flavor-changing neutral-current interactions (FCNC) in charm physics. Unlike the integrated decay rate, A_{FB}^c is not very sensitive to infrared divergences. Since the angular distribution of the double differential decay rate is almost opaque to the LD contribution from SM [49], A_{FB}^c absorbs purely SM-SD contributions—yet is truly tiny.

The CP asymmetry parameter $A_{CP}^c(\hat{s})$ is defined as

$$A_{CP}^c(\hat{s}) = \frac{\frac{d}{d\hat{s}}\Gamma(D^+ \to X_u l^+ l^-) - \frac{d}{d\hat{s}}\Gamma(D^- \to X_{\bar{u}} l^+ l^-)}{\frac{d}{d\hat{s}}\Gamma(D^+ \to X_u l^+ l^-) + \frac{d}{d\hat{s}}\Gamma(D^- \to X_{\bar{u}} l^+ l^-)}. \qquad (18)$$

In general, any Wilson coefficient $C_i(\mu)$ in the differential decay rate can be written as

$$C_i(\mu) = \xi_i^0 + \lambda_i^j \xi^j,$$

where the summation over j is implied. For $\Gamma(\bar{D} \to X_{\bar{u}} l^+ l^-)$, $\lambda_j \to \lambda_j^*$. The numerator [Eq. (19)] and the denominator [Eq. (20)] will have contributions of the type

$$|C_i(\mu)|^2 \to \Im(\lambda^l \lambda^{m*})\Im(\xi_i^l \xi_i^{m*}) + 2\Im(\lambda^l)\Im(\xi_i^0 \xi_i^{l*})$$

$$\Re(C_i(\mu)C_j^*(\mu)) \to \Im(\lambda^l \lambda^{m*})\Im(\xi_i^l \xi_j^{m*}) + \Im(\lambda^l)\Im(\xi_i^l \xi_j^{0*})$$

$$+ \Im(\lambda^{m*})\Im(\xi_i^0 \xi_j^{m*}) \qquad (19)$$

$$|C_i(\mu)|^2 \to \Re(\lambda^l \lambda^{m*})\Re(\xi_i^l \xi_i^{m*})$$

$$+ 2\Re(\lambda^l)\Re(\xi_i^0 \xi_i^{l*}) + |\xi_i^0|^2$$

$$\Re(C_i(\mu)C_j^*(\mu)) \to \Re(\lambda^l \lambda^{m*})\Re(\xi_i^l \xi_j^{m*}) + \Re(\lambda^l)\Re(\xi_i^l \xi_j^{0*})$$

$$+ \Re(\lambda^{m*})\Re(\xi_i^0 \xi_j^{m*}) + \Re(\xi_i^0 \xi_j^{0*}). \qquad (20)$$

In the limit that only C_9 has an imaginary component and terms proportional to $V_{ub}^* V_{cb}$ are ignored, the results stated in [51] are realized. The numerator is sensitive only to the imaginary contributions from the Wilson coefficients to the decay rate and the denominator to the real contributions as should be the case; for the relative phases in the matrix element is needed for a CP asymmetry. Unlike in the B mesons, in the case of $D \to X_u l^+ l^-$ only C_{10} is purely real, and hence both C_7 and C_9 contribute to A_{CP}^c. The integrated asymmetry A_{CP}^c in SM turns out to be

$$A_{CP}^c = \frac{\Gamma(D^+ \to X_u l^+ l^-) - \Gamma(D^- \to X_{\bar{u}} l^+ l^-)}{\Gamma(D^+ \to X_u l^+ l^-) + \Gamma(D^- \to X_{\bar{u}} l^+ l^-)} \sim 3 \times 10^{-4},$$

$$(21)$$

i.e., still tiny. We also find that the bulk of A_{CP}^c comes from C_7 and is due to the presence of the two-loop $O(\alpha_s)$ contribution. Indeed, the C_9 contribution, which comes

from the mixing of O_{1-6} with O_9, serves only to suppress this contribution by an order of magnitude all of which stand in stark contrast to what happens in the analogous decay of the B mesons. Both of these contributions arise only after the inclusion of QCD radiative corrections and both of which are proportional to $V_{us}^* V_{cs}$; ignoring the term proportional to $V_{ub}^* V_{cb}$ as it is relatively much smaller.

We can also look at $A_{\mathrm{FB}}^{CP}(\hat{s})$, which is the normalized difference in the forward-backward asymmetry in $D \to X_u l^+ l^-$ and $\bar{D} \to X_{\bar{u}} l^+ l^-$ defined as [52]

$$A_{\mathrm{FB}}^{CP}(\hat{s}) = \frac{A_{\mathrm{FB}}^c(\hat{s}) + A_{\mathrm{FB}}^{\bar{c}}(\hat{s})}{A_{\mathrm{FB}}^c(\hat{s}) - A_{\mathrm{FB}}^{\bar{c}}(\hat{s})}. \qquad (22)$$

In the limit of CP symmetry, $A_{\mathrm{FB}}^c(\hat{s})$ and $A_{\mathrm{FB}}^{\bar{c}}(\hat{s})$, have to be exactly equal in magnitude but with an opposite sign [51,53]. As the forward-backward asymmetry is defined in terms of the positive antilepton, $A_{\mathrm{FB}}^c(\hat{s})$ and $A_{\mathrm{FB}}^{\bar{c}}(\hat{s})$ have opposite signs. $A_{\mathrm{FB}}^{CP}(\hat{s})$ is sensitive to the phase in C_7, C_9, and C_{10}. The SM offers phases only in C_7 and C_9 in $D \to X_u l^+ l^-$ and none in C_{10}. Hence the integrated asymmetry turns out to be tiny

$$\int A_{\mathrm{FB}}^{CP}(\hat{s})d\hat{s} = A_{\mathrm{FB}}^{CP} \sim 3 \times 10^{-5}. \qquad (23)$$

If NP brings about any new phases in either C_7, C_9, and C_{10}, A_{FB}^{CP} stands a chance of large enhancements.

Putting together A_{FB}^c, A_{CP}^c, and A_{FB}^{CP} gives us a good insight into the sizes of the phases in the Wilson coefficients. A_{FB}^c is sensitive to the size of C_{10} and the real parts of C_7 and C_9, while A_{CP}^c gives us an idea of the size of the phases in C_7 and C_9 and A_{FB}^{CP} is sensitive to phases in all C_7, C_9, and C_{10}. Within the SM, we can conclude from our numbers, the size of C_{10} is extremely small, as it should be, since it suffers from a very strong Glashow-Iliopoulos-Maiani mechanism suppression, and it also lacks a phase. Both C_7 and C_9 have phases because of the QCD corrections, which are more prominent in the case of D mesons than in B mesons as the purely electroweak contribution is truly tiny. Along with the decay rate, these provide us a very useful tool to probe into the flavor structure of any NP models and new sources of FCNC.

C. A Note on LD dynamics' impact on the asymmetries

The asymmetries discussed in the previous section do not incorporate any SM LD contributions, neither in their extraction from the difference in the partial decay rates nor in their normalization. This might be unreasonable considering that SM-SD contributions yield a branching ratio of only 1.5×10^{-9}, while SM LD yields something like $O(10^{-6})$, even considering that LD estimates come with very large uncertainties. Let us add a comment why LD physics have only a little impact on these asymmetries, either from the difference in the numerator or the normalization in the denominator.

For A_{CP}^c and A_{FB}^{CP}, it is obvious that SM LD physics cannot contribute a *CP* violating phase, and any *CP* asymmetry in this process has to come from SD physics whatever the origin is—SM or NP. A_{FB}^c is more sensitive to SM LD "pollution" in its definition as the difference in the hemispherical integral of the double differential decay rate. SM LD contributions to it will primarily come through final state interactions, and the dominance of light internal quarks in this process and quantitative statements on their sizes require separate analysis on each exclusive process. SM LD physics can also make their presence felt in all of these observables through the normalizations entering the definitions of these asymmetries. A natural way to remove such LD contributions is to cut off the dileptonic mass distribution around the ρ, ω, and ϕ widths.

Making such cuts, we find that it will *decrease* A_{FB}^c and *increase* A_{CP}^c and A_{FB}^{CP} by around 10% to 20%. Not surprisingly, these cuts affect A_{FB}^c more than A_{CP}^c and A_{FB}^{CP}. Similar effects arise, when we include LHT contributions as discussed below. More sophisticated cuts could yield even better results. As pointed out below, only A_{FB}^{CP} has a realistic chance to be measurable by LHCb and a super-flavor factory. It seems to us that such a theoretical uncertainty is fully acceptable for a search for NP as of now.

As a final point: including nonresonant LD effects and making a cut on the resonances seem to produce opposing effects and can well nullify each other; however only a detailed study can resolve this issue.

Throughout the rest of the article, we will stick to the definition of the asymmetries in terms of SD operators and infer on our results accordingly keeping in mind that a sufficiently motivated reader will already have had gone through this section by then. Any NP contribution to these asymmetries are strictly SD.

D. Comments on $D^0/\bar{D}^0 \to X_u l^+ l^-$

The branching ratios for neutral $D \to X_u l^+ l^-$ are again dominated by SM LD contributions and their size is comparable to what is stated in Eq. (14), namely, of order 10^{-6}. For A_{FB}^c and A_{CP}^c, one has a much more complex system in hand for neutral D mesons, because $D^0 - \bar{D}^0$ oscillations have been found on the level of 0.5%–1% for x_D and y_D, see Eq. (1), which might be still consistent with the SM. Since the SM asymmetries A_{FB}^c and A_{CP}^c are so tiny, the $D^0 - \bar{D}^0$ oscillation "background" is irrelevant.

III. ON LHT SCENARIOS

A. The flavor in LHT

The SM predictions presented above leave a large range in rates for these rare transitions, where NP could *a priori* make its presence felt. So-called little Higgs models mentioned in the Introduction have been studied extensively over the past decade as a possible NP scenario [14–19]. There the Higgs boson appears as a pseudo-Nambu-

Goldstone boson of a spontaneously broken global symmetry. Rather than attempting to solve the hierarchy problem, they "delay the day of reckoning" and address a maybe secondary, yet very relevant problem, namely, to reconcile the fact that the measured values of the electroweak parameters show no impact from NP, even on the level of quantum corrections with the expectation that NP quanta exist with masses around the 1 TeV scale so that they could be produced at the LHC. In some of these models, to achieve this program consistently, one needs an additional discrete symmetry [24–26] called *T* parity. One way of consistently implementing *T* parity also requires the introduction of the so-called mirror fermions—one for each quark (and lepton) species—that are *odd* under *T* parity and family-wise mass degenerate. This introduces two 3×3 mixing matrices, V_{Hd} and V_{Hu}, neither of which need to be close of the CKM matrix, but they are related to each other [54]:

$$V_{Hd}^\dagger V_{Hu} = V_{\text{CKM}}. \tag{24}$$

Since the CKM matrix does not differ too much from the identity matrix, one realizes that LHT contributions exhibit a clear correlation of the phases in the charm and strange sector.

In this note, we will analyze a subclass of little Higgs models, namely, littlest Higgs models with *T* parity (LHT) [22,28]. In our view, they possess several significant strong points:

(i) They contain several states with masses that can be below 1 TeV; i.e., those states should be produced and observed at the LHC.

(ii) Compared to supersymmetry models, they introduce many fewer new entities and observable parameters.

(iii) Their motivation as sketched above lies outside of flavor dynamics. Thus, they have not been "cooked up" to induce striking effects in the decays of hadrons with strangeness, charm, or beauty.

(iv) Nevertheless, they are *not* of the minimal flavor violating variety.

(v) The impact of LHT dynamics on K, B, and also D transitions has been explored in considerable detail, and potentially sizable effects have been identified [30,55–62].

(vi) Especially relevant for our study is the fact that they can have an observable impact on $D^0 - \bar{D}^0$ oscillations [29,63]. Also, sizable indirect *CP* violation can arise in D^0 decays [29] very close to the present experimental upper bounds. Having seen such large effects in $\Delta C = 2$ transitions coupled with the possibility of the existence of large *CP* violating phases, one would naturally ask whether it is possible to see the same in $\Delta C = 1$ transitions such as $D \to X_u l^+ l^-$.

B. LHT contributions to $D \to X_u l^+ l^-$

As T parity forbids tree level coupling of SM particles with the new T-odd particles, LHT makes its presence felt only through loop contributions from internal mirror fermions and heavy gauge bosons. Unlike the case of B and K, the new T-even heavy top quark does not contribute and hence any new contribution from LHT comes from the T-odd particles. The following are the modifications of the SM functions in Eq. (5). The auxiliary functions are defined explicitly in Appendix B.

$$C_1(x) = \frac{1}{64} \frac{v^2}{f^2} \left(xS(x) - 8xR_2(x) + \frac{3}{2}x + 2xF_2(x) \right)$$

$$D_1(x) = \frac{1}{4} \frac{v^2}{f^2} \left(D_0(x) + Q_u \frac{1}{2} E_0(x) + Q_u \frac{1}{10} E_0(x') \right)$$

$$E_1(x) = \frac{1}{4} \frac{v^2}{f^2} \left(E_0(x) + \frac{1}{2} E_0(x) + \frac{1}{10} E_0(x') \right)$$

$$D_1'(x) = \frac{1}{4} \frac{v^2}{f^2} \left(D_0'(x) + Q_u \frac{1}{2} E_0'(x) + Q_u \frac{1}{10} E_0'(x') \right)$$

$$E_1'(x) = \frac{1}{4} \frac{v^2}{f^2} \left(E_0'(x) + \frac{1}{2} E_0'(x) + \frac{1}{10} E_0'(x') \right)$$

$$Y_1(x, y) = \frac{1}{64} \frac{v^2}{f^2} [S(x) + F_W(x, y) - 4(G_Z(x, y)$$
$$+ G_A(x', y') + G_\eta(x, y))], \qquad (25)$$

where

$$x = \frac{m_{Hi}^2}{m_{W_H}^2} = \frac{m_{Hi}^2}{m_{Z_H}^2}, \qquad x' = ax, \qquad a = \frac{5}{\tan^2 \theta_W}$$

$$y = \frac{m_{l_H}^2}{m_{W_H}^2} = \frac{m_{l_H}^2}{m_{Z_H}^2}, \qquad y' = ay, \qquad \eta = \frac{1}{a}. \qquad (26)$$

$Q_u = 2/3$ is the charge of the up-type quarks, m_{H_i} is the mass of the mirror quark in the ith family, and m_{l_H} is the mass of the heavy internal neutrino. The functions $F_W(x, y)$, $G_Z(x, y)$, $G_A(x, y)$, and $G_\eta(x, y)$ are contributions from WW, ZZ, AA, and ZA box diagrams with heavy internal neutrinos, respectively. A complete list of Feynman diagrams can be found in [61]. Since the operator structure is the same in LHT as in SM, the expression for the decay rate and the asymmetries remain the same with the necessary modifications of the Wilson coefficients. QCD corrections to the LHT contributions have been ignored; after all, we do not know the model parameters, and these numerical exercises serve to show whether such models can be significant for such observables.

IV. NUMERICAL FINDINGS ON LHT CONTRIBUTIONS

The structure of the mirror fermion sector leaves us with a lot of liberty to choose the parameter space we wish to scan. However, constraints from B and K physics set very

stringent limits on the viable parameter space for probing D physics. In what follows, we define an operational parameter space and what effects LHT can bring about in $D \to X_u l^+ l^-$.

A. LHT parameter space

We continue to use the same parameter sets that we used to study the processes $D^0 \to \gamma\gamma$ and $D^0 \to \mu^+\mu^-$ in our previous work [30]. We vary the mirror fermion masses and the mixing angles and phases over the parameter sets, keeping the breaking scale of the nonlinear sigma model fixed at 1 TeV [64]. The LHT has 20 new parameters of which the ones which will be relevant to us are as follows:

(i) The LHT breaking scale $f = 1$ TeV is fixed by choice.

(ii) The masses of the three family-wise degenerate T-odd mirror quarks, m_{H1}, m_{H2}, m_{H3} range from 300 to 1000 GeV.

(iii) There are three independent mixing angles in V_{Hu}, θ_{12}^u, θ_{13}^u, θ_{23}^u.

(iv) There are three irreducible phases in V_{Hu}, δ_{12}^u, δ_{13}^u, δ_{23}^u.

The parameter space used for these analyses is a set that satisfies all experimental constrains from B and K physics. A small parameter set was also used, which did not follow such constraints to check whether constraints from B and K physics affects LHT contributions to D physics. However, even the parameter set that is not constrained does not have large mass hierarchies in the mirror fermion sector.

The mass spectrum for both the parameter sets is illustrated in Figs. 1. Using Eq. (24), the angles and phases of V_{Lu} were calculated from those of V_{Hd} and hence were constrained by B and K physics too for the first parameter set and not so for the second. Histograms of the parameter space of the angles and phases are shown in Figs. 2. The angles and phases are family-wise paired.

B. Impact on $\Gamma_{\text{SD}}(D \to X_u l^+ l^-)$

As we have seen in Sec. II A, the dominant SM contributions to $\Gamma_{\text{SD}}(D \to X_u l^+ l^-)$ are through the γ penguin in the O_9 operator and some from the mixing of O_{1-6} with O_9. The subdominant contributions come from the two-loop $O(\alpha_s)$ term in C_7. Hence, part of the dominant effect and the subdominant effect both come from QCD corrections. All other contributions are smaller by orders of magnitude. We saw in our previous work on $D \to \gamma\gamma$ and $D \to \mu^+\mu^-$ [30] that LHT is capable of producing large enhancements through box diagrams with internal heavy fermions and heavy gauge bosons just as in $D^0 - \bar{D}^0$ oscillation [29] and somewhat moderate enhancements to Z_L penguins. However, the enhancement to effective γ vertices are tiny compared to the SM contributions.

In $D \to X_u l^+ l^-$, we confirm our previous conclusions. The decay rate was calculated for three different internal
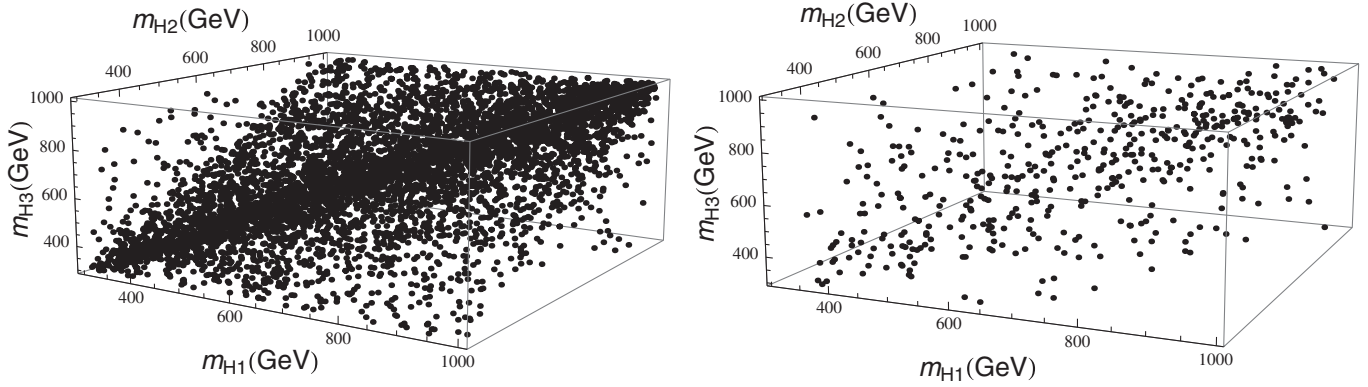
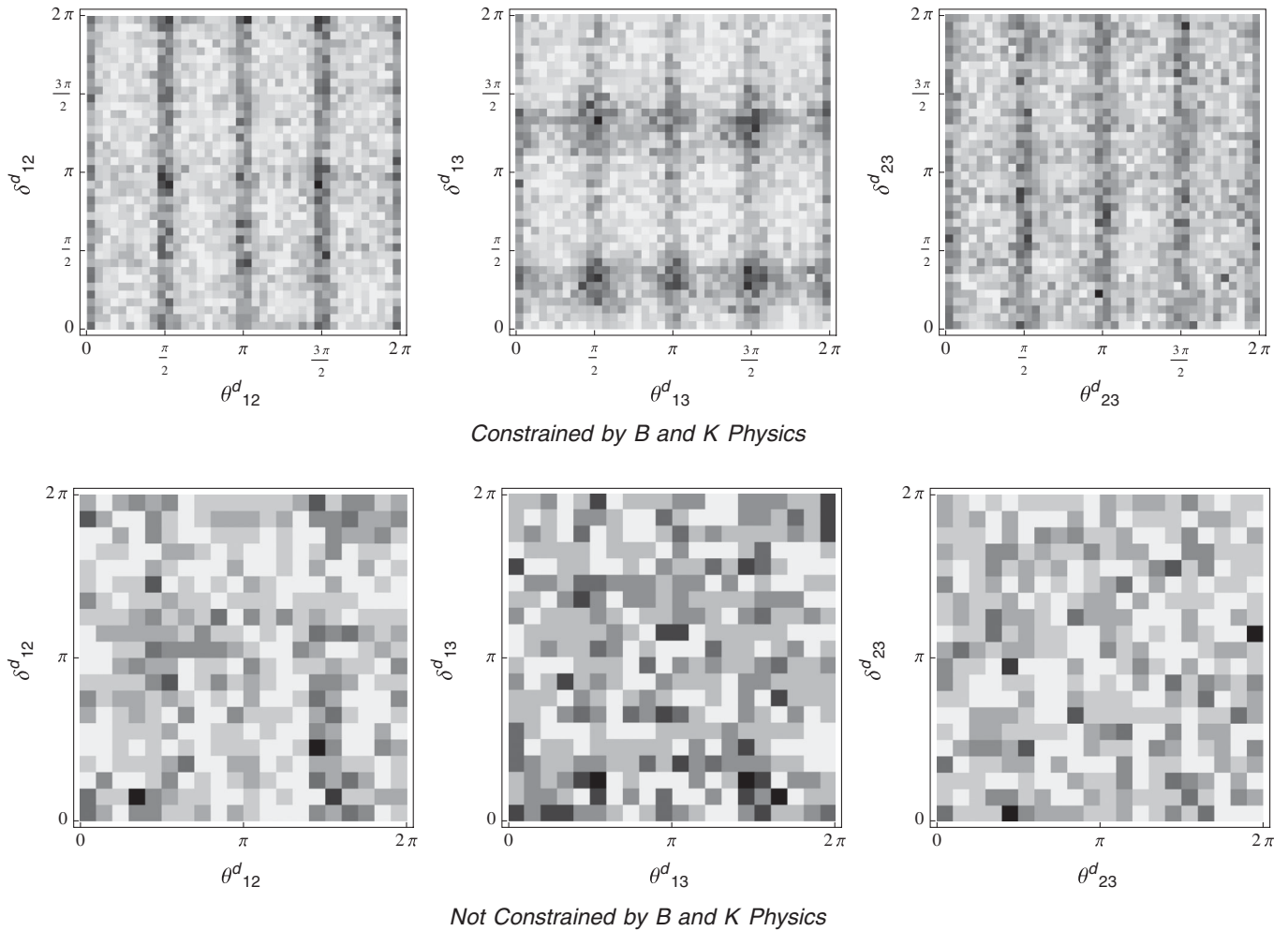FIG. 1.   Parameter space of the mass of the mirror quarks.

FIG. 2.   Histogram of the parameter space of the angles and phases in V_{Hd}. Counts in any bin is represented in gray scale, the darker representing higher density.

heavy neutrino masses of 400 GeV, 600 GeV, and 1100 GeV. We see very tiny change in both the differential decay rate and the integrated decay rate at $O(1\%)$, as can be seen from Fig. 3 in which the abscissa represents the percentage enhancement to the SD decay rate after

inclusion of LHT. Removing constraints from B and K physics does not make much difference either as can be understood from comparing Figs. 3(a) and 3(b). Our result is different from what was found in [66], which used the littlest Higgs model *without T* parity. Without T parity, the
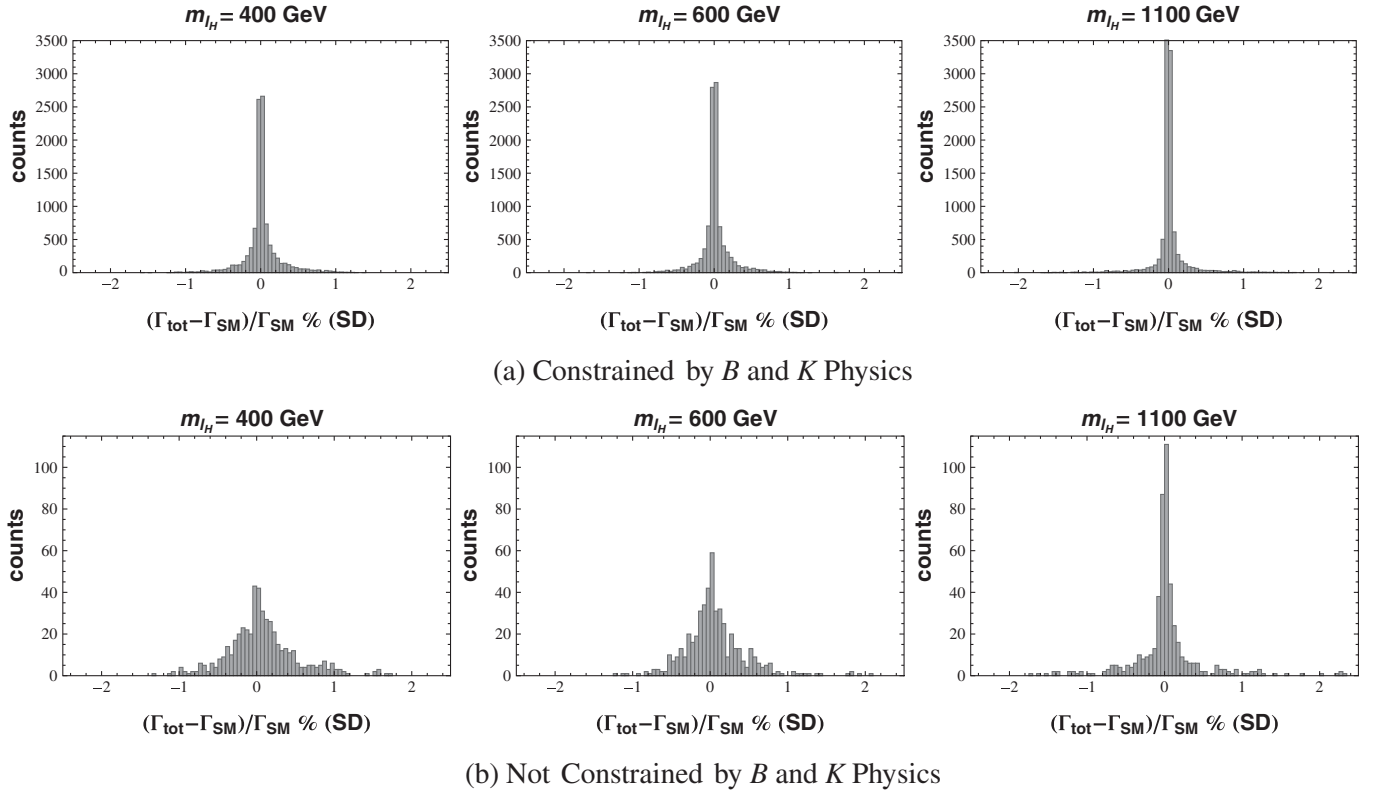
(a) Constrained by B and K Physics

(b) Not Constrained by B and K Physics

FIG. 3.   Percentage change of the SM-SD contribution to $\Gamma_{SD}(D \to X_u l^+ l^-)$ due to LHT effects.

$SU(2)$ custodial symmetry protecting the electroweak ρ parameter is explicitly broken at scales below 4 TeV [23], which is brought about by the $U(1)_H$ gauge boson. This model allows large tree level FCNC mediated by the coupling of the Z_H and A_H heavy gauge bosons with the SM quarks. The enhancements seen in [66] is because of the existence of these tree level FCNCs which are absent from LHT. Hence, LHT makes almost no changes to C_7 or C_9 above SM contributions and hence fails to enhance the decay rate. This essentially means that any LHT contribution to the decay rate fails to significantly dent SM-SD contributions and is completely swamped by SM LD effects.

C. Impact on A_{FB}^c, A_{CP}^c, and A_{FB}^{CP}

As we have seen earlier in Sec. II B, the SM contribution to A_{FB}^c is all but nonexistent due to tiny SM contributions to C_{10}. In LHT, C_{10} gets enhanced by orders of magnitude, which brings about a large enhancement in A_{FB}^c. This effect is similar to what was observed in [66], but comes from box diagrams involving T-odd heavy internal degrees of freedom rather than tree level FCNC. It is also commensurate with the enhancement we found in SD contribution to $\Gamma(D^0 \to \mu^+\mu^-)$ [30] from LHT. This is due to the fact that SD contribution to $\Gamma(D^0 \to \mu^+\mu^-)$ comes from O_{10}, and A_{FB}^c is highly sensitive to the same. However, even with such a large enhancement, the absolute value A_{FB}^c after including the LHT enhancement can at most be of $O(0.5\%)$ as can be seen from Fig. 4. Studying Figs. 4(a) and 4(b)

shows that removing constraints from B and K physics creates large *relative* enhancements—it can be as much as 1% (or rarely more)—but they do not enhance it to sizable absolute effects.

On the other hand, A_{CP}^c depends mostly on C_7 and C_9, which we have already seen suffers almost no enhancement from LHT. However, as A_{CP}^c is quite sensitive to the phases in these coefficients, it gets enhanced by a few factors over the SM value and can be as large as 4 times the SM value. This can be seen from Fig. 5, where we plot the ratio of the total A_{CP}^c including the LHT enhancements to the SM value of the same. However, this still keeps A_{CP}^c at $O(10^{-4}) - O(10^{-3})$ and hence the absolute measure of the CP asymmetry is still experimentally challenging. The unconstrained parameter set allows for slightly larger enhancements to A_{CP}^c, but is limited to almost the same order of magnitude.

The contributions from LHT models can enhance A_{FB}^{CP} so much as to bring it up to possibly measurable values. As pointed out in Sec. II B, A_{FB}^{CP} is sensitive to any phase in C_{10}. LHT cannot only enhance the magnitude of C_{10} by orders of magnitude, but also brings about the possibility of existence of a very large phase in it. For C_7 and C_9, the effect is dominated by phases from QCD radiative corrections. The existence of this large phase and the tangential dependence of A_{FB}^{CP} on it results in the huge enhancement that we see in A_{FB}^c, as illustrated in Fig. 6 where we plot the total A_{FB}^{CP} after the inclusion of LHT effects. This is
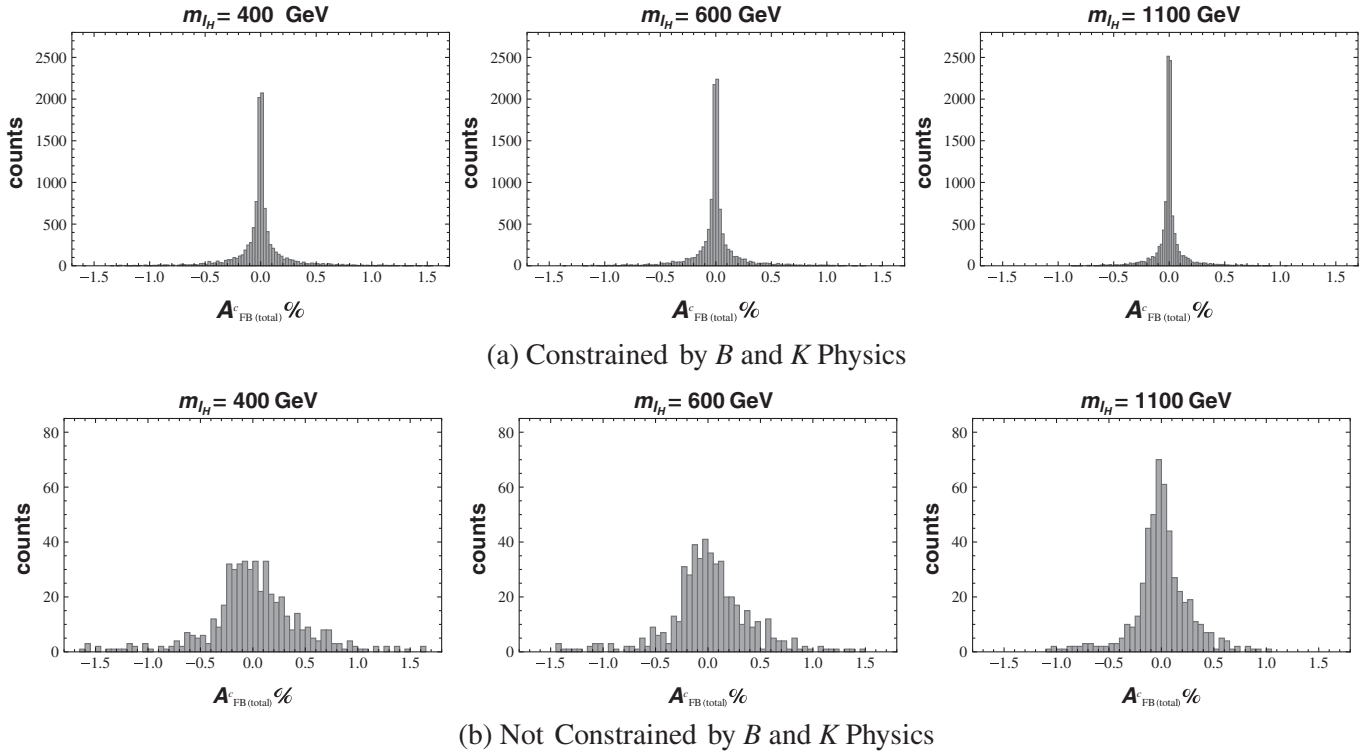
(a) Constrained by *B* and *K* Physics

(b) Not Constrained by *B* and *K* Physics

FIG. 4. A^c_{FB} after including LHT effects.

commensurate with what was observed in [29] for *CP* violation in $D^0 - \bar{D}^0$ oscillations. For both the constrained [Fig. 6(a)] and unconstrained [Fig. 6(b)] sets, more than 10% of the parameter set can produce asymmetries of

$O(10\%)$ or greater. As explained in Sec. II B, we relate A^{CP}_{FB} with SD contribution to $D \rightarrow l^+ l^- X$, which amounts to a branching ratio of 1.5×10^{-9}. With a sample of 10^{13} *D* mesons, NP intervention should be measurable
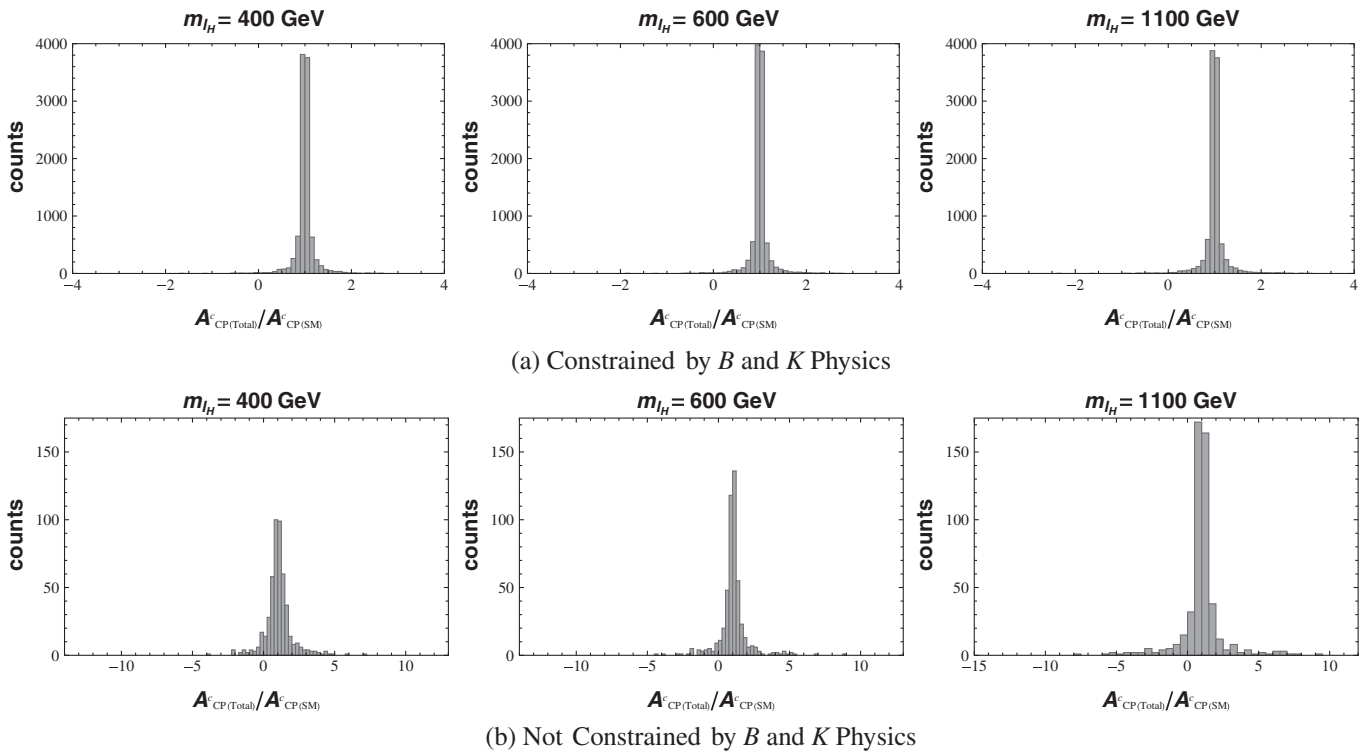

(a) Constrained by *B* and *K* Physics

(b) Not Constrained by *B* and *K* Physics

FIG. 5.   Enhancement to A^c_{CP} over SM, after including LHT effects.

(a) Constrained by B and K Physics

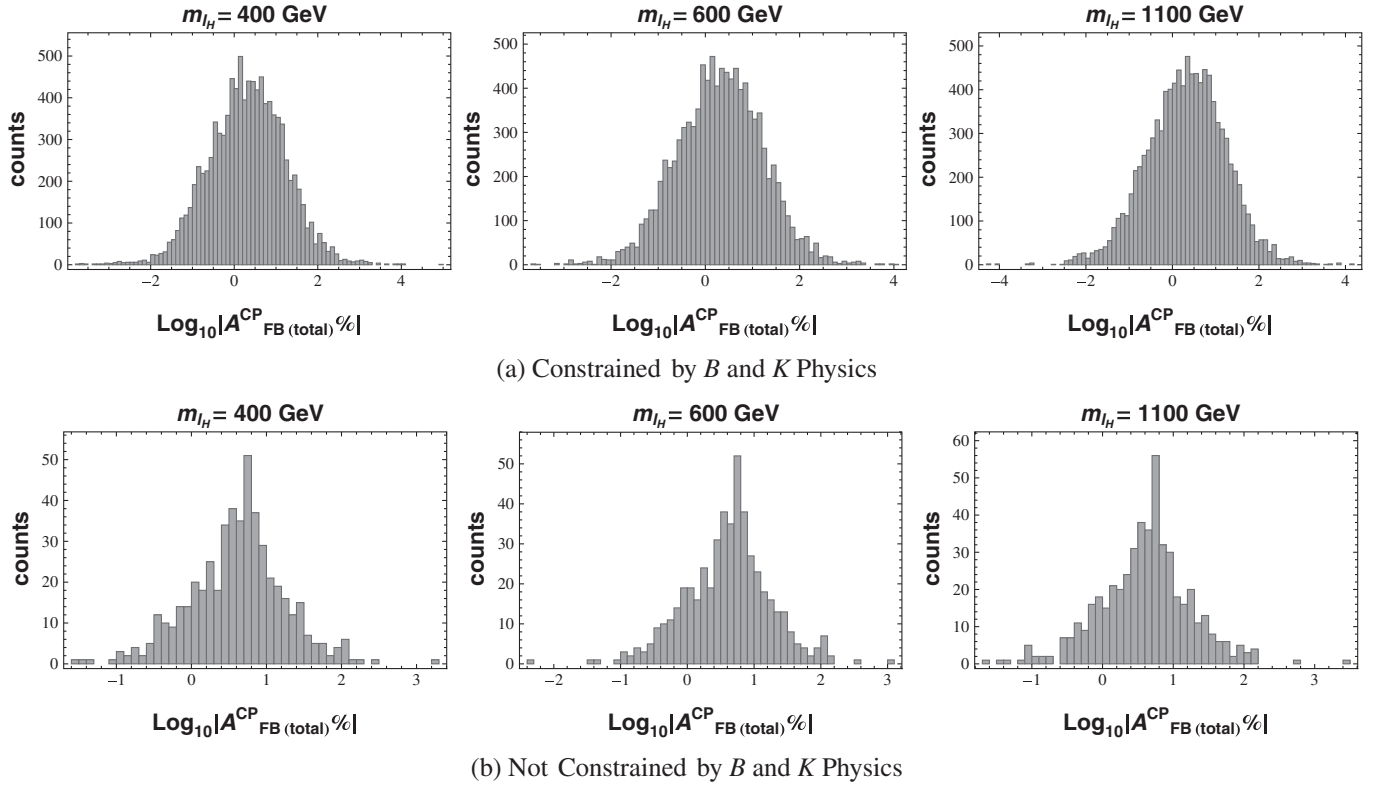(b) Not Constrained by B and K Physics

FIG. 6.   Large enhancements in A_{FB}^{CP}, after including LHT effects.

for $A_{FB}^{CP} > 5\%$. Such effects could be within the reach of to the currently running LHCb experiment and ones like the planned and approved SuperB Collaboration.

D. Correlation between A_{FB}^c and $\Gamma_{SD}(D \to X_u l^+ l^-)$

In Fig. 7, we plot the correlation between A_{FB}^c and $\Gamma_{SD}(D \to X_u l^+ l^-)$. These plots look at first chaotic, yet a careful (and time consuming) study reveals a pattern. For "low" heavy neutrino masses $m_{l_H} = 400$ GeV, 600 GeV, the LHT parameter sets that enhance $\Gamma_{SD}(D \to X_u l^+ l^-)$ can produce positive A_{FB}^c in some regions and negative ones in others; on the other hand, sets decreasing $\Gamma_{SD}(D \to X_u l^+ l^-)$ can also produce positive and negative A_{FB}^c, but in others regions. Those low heavy neutrino masses are

within the range of masses used for the mirror quarks in this study. However, for $m_{l_H} = 1100$ GeV, those LHT parameters that increase the SD branching ratio produce mostly a negative A_{FB}^c, while sets decreasing the SD branching ratio lead mostly a positive A_{FB}^c. At this mass, the mirror neutrino is heavier than any of the mirror quarks.

This behavior can be understood quite well. Dependence on the heavy neutrino mass exists only in the box diagrams. The sign of A_{FB}^c depends on the sign of C_{10}, which depends on the relative size of the mirror quarks and heavy neutrino masses, which comes from box diagrams. At less than 1 TeV, this can go either way, with the neutrino being either heavier or lighter than one or more of the mirror quarks, with slightly greater chances of being lighter than
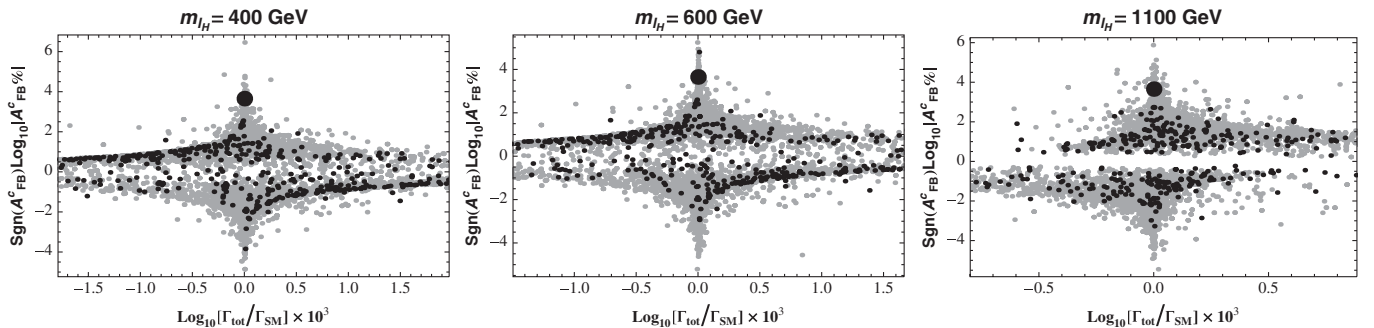

FIG. 7.   Correlation between A_{FB}^c and $\Gamma_{SD}(D \to X_u l^+ l^-)$. The gray points represent the constrained set and the black ones for the unconstrained one. The big black spot represents the SM values.

them. At above 1 TeV, the heavy neutrino is always heavier than the mirror quarks used. Also, enhancements to $\Gamma_{SD}(D \to X_u l^+ l^-)$ are quite sensitive to the box diagrams through both C_{10} and C_9. This leads to the sharp change in the correlation we see in Fig. 7.

We do not see any such correlation in A_{CP}^c vs $\Gamma_{SD}(D \to X_u l^+ l^-)$ or A_{FB}^c vs A_{CP}^c; after all, A_{CP}^c is blind to C_{10} and hence less sensitive to the box diagrams, as in this case they manifest themselves only through C_9. Furthermore, we see no correlation in A_{FB}^{CP} vs $\Gamma_{SD}(D \to X_u l^+ l^-)$ or A_{FB}^{CP} vs A_{FB}^c, since A_{FB}^{CP} is not affected by the magnitude of C_{10}, but rather by the CP violating phase in it; that information is lost in both $\Gamma_{SD}(D \to X_u l^+ l^-)$ and A_{FB}^c. Last, we see no correlation between A_{CP}^c and A_{FB}^{CP}, even though both are CP violating parameters. This reinforces our earlier statement that the sources of CP violation are distinct in these parameters, with the former coming from phases in C_7 and C_9 and the latter coming from a phase in C_{10}.

V. FURTHER INSIGHTS INTO FCNCS IN LHT-LIKE MODELS

Our work on the impact of LHT on $D^0 \to \gamma\gamma/\mu^+\mu^-$ [30] had lead us to some general conclusions on the structure of FCNCs within a LHT-like framework. As defined previously, this framework contains

(i) A second sector of fermions that are an exact copy of the SM ones.
(ii) Mass mixing matrices, which are unitary and loosely connected to V_{CKM} [Eq. (24)].
(iii) Possible large angles and phases in the mass mixing matrices.
(iv) Possible large hierarchies in the masses of the mirror quarks.
(v) A symmetry, like T parity, segregating the NP sector from the SM sector, hence forbidding tree level FCNC.

We shall, after further investigation, relax the second condition to:

(i) Mass mixing matrices that are constrained by a relationship between the one(s) connecting the new up-type quarks with the SM down-type quarks to the one(s) connecting the new down-type quarks with the SM up-type quarks.

In general, FCNCs are a very sensitive probe to the details of the flavor structure of both the SM and any NP models, as they highlight not only mass hierarchies within a theory but also are sensitive to phases within the same. Moreover, it is possible to disentangle the effect of phases and fermionic mass hierarchies on FCNCs in a model independent way if we have access to more observables.

$D \to X_u l^+ l^-$ wins over $D^0 \to \gamma\gamma$ and $D^0 \to \mu^+\mu^-$ by leaps and bounds in this respect. While $D^0 \to \gamma\gamma$ is sensitive mostly only to O_7 and $D^0 \to \mu^+\mu^-$ is sensitive only to O_{10}, $D \to X_u l^+ l^-$ is not only sensitive to all of that but

also to many more. Moreover, the final state being a three body final state, this channel can also be probed through forward-backward and CP asymmetries, hence opening the possibility of probing phases in any model, too.

As we noted above, the SD contribution to the branching fraction is dominated by the photonic penguin in O_9, while A_{FB}^c is highly sensitive to O_{10}, A_{CP}^c to the mixing between O_7 and O_9, and A_{FB}^{CP} to the phase in C_{10}. Studying $D \to X_u l^+ l^-$ in a sufficiently precise way, we can learn the impact of several operators and then comment on other rare decays. Logically, we should have started our analysis with $D \to X_u l^+ l^-$ and then applied our findings to the simpler cases of two-body rare decays. Instead, we started with our analysis of two-body rare decays, from which we extracted some conjectures; they happened to be correct in more general theoretical situations.

From our results, we see that, through LHT dynamics, A_{FB}^c gets orders of magnitude enhancement through the enhancement of C_{10}, which is commensurate with the orders of magnitude enhancement that we had seen in the SD contribution to $D^0 \to \mu^+\mu^-$, which was ultimately overshadowed by the LD contribution to the branching fraction. We see almost no enhancement to C_7, which agrees with the lack of enhancement that we noted in $D^0 \to \gamma\gamma$. It is not that C_7 gets absolutely no enhancement from LHT. The purely electroweak part of C_7 does get moderately enhanced. However, this enhancement is completely overshadowed by the SM two-loop $O(\alpha_s)$ QCD correction, something which is peculiar to the D meson system and not seen in the B system. In addition, we also see almost no enhancement to C_9 coming from the fact that photonic penguins are not enhanced by LHT, which results in a lack of enhancement to the branching fraction of $D \to X_u l^+ l^-$.

Intuitively, one might think that the situation for A_{CP}^c should be different as it is sensitive to phases in C_9 and C_7, and LHT allows for large phases in the mixing matrices. In a purely electroweak SM scenario, this would have been true. However, A_{CP}^c gets an unusual boost within the SM from the unusual two-loop $O(\alpha_s)$ QCD correction, something that LHT can barely overcome. Coupled to the fact that C_9 does not gain much from LHT, enhancements to A_{CP}^c fail to impress. The validity of the previous statement is further tested when we see orders of magnitude enhancement in A_{FB}^{CP} as it gets a boost from the introduction of a large phase from LHT while it is completely clean of phases from the SM.

So what does this tell us about LHT-like models and their effects in flavor physics? In principle, in any such model, large angles and phases in mass mixing matrices and large fermionic masses and sharp hierarchies amongst them are possible. However, one has to satisfy the experimental constraints that we already have in B and K physics. At this point, one will have to choose between large angles and phases or huge hierarchies in the fermionic masses as

experimental data already tell us that the extra fermions have to be heavy. Making such a choice automatically limits the size of NP intervention in yet unobserved FCNC processes, specially in $\Delta F = 1$ processes, even if $\Delta F = 2$ processes can escape these limits and absorb NP contributions. However, an exception to this rule occurs when these large phases from new physics are laid bare and have purely SM electroweak effects to compete with as in the case of A_{FB}^{CP}, where we see large effects even in a parameter, which is a measure of a $\Delta F = 1$ process. Hence, let us have a more thorough look at the diagrammatic details of NP intervention from LHT-like models.

VI. ON BOXES AND PENGUINS IN NEW PHYSICS

Because of CPT symmetry, CP violation can enter only through complex effective couplings. In the SM, they can arise only for the weak boson couplings to quarks as described by CKM matrices. They are necessarily unitary, since all quark masses are given by a single vacuum expectation value of a neutral Higgs multiplied by numbers, not a matrix. The concept of "weak universality" was first put forward by Cabibbo in 1967 [67]. Afterwards, it was scrutinized experimentally. Later, it was understood if the weak forces are embedded—as it applied to the SM—in a single non-Abelian gauge theory, weak universality has to hold. There is the singular value decomposition theorem, which tell us that the matrices relating mass and flavor left *up* and *down* quarks are unitary and therefore their matrix product—the CKM matrix—is also unitary. For $N = 2$ families, their phases can be transformed away, for $N = 3$ there is one irreducible phase that is therefore observable. CP violation can surface in processes where quarks from three families can contribute as real or virtual entities. The latter happens in SM, since FCNCs arise effectively through quantum corrections, namely, box and penguin diagrams [68]. There are box diagrams in only one kind, the WW box. Penguins come in three varieties: the Z_L, the γ, and the gluon penguins are particularly essential for direct CP violation.

For N families, the unitarity of V_{CKM} translates into [69]

$$\sum_{i=1}^{N} \lambda_i = \delta_{jk}, \quad \text{with} \quad \lambda_i = V_{ji}^* V_{ki}, \qquad (27)$$

with $|\lambda_i| > 0$ for $i = 1, 2 \ldots N$ families, with $\{i\}$ coming from the down-type sector and $\{j, k\}$ coming from the up-type sector; i.e., a triangle equation in the complex plane: CP violation arises at the mass generation. If two of the quark were mass degenerate, CP invariance would survive mass generation—yet it is not the case on our world.

If the SM contained a fourth family, it could—and probably would—have two more observable phases with more *independent* CP asymmetries.

That very fact that in the SM one has to study transitions where quarks from three families contribute reduces the number of "interesting" cases for CP violation very significantly. The observation that absolute values of off-diagonal elements of V_{CKM} are small, even tiny, in most such cases are experimentally "challenging" at least—except in beauty decays, since leading decays are so suppressed and therefore make them experimentally challenging for a different reason. It was expected that absolute values of the off-diagonal elements are small, but it was surprising that $|V_{bc}|$, $|V_{bu}|$, etc. are so tiny, namely, much smaller than $|V_{su}|$. Originally, it was conjectured—based on no good theoretical reason—that also the CKM phase is small. Now, we know that the CKM phase is not small. Therefore, NP is likely to exhibit also sizable phases—like LHT.

The size of CP asymmetries depends on the phases, the absolute values of the quark mixing matrix elements, and the masses of the internal virtual gauge bosons and fermions and their mass hierarchies. Let us analyze to what degree each criterion can be fulfilled in this class of NP models we call LHT-like for box and penguins. The range of masses of mirror quarks and W_H and Z_H is rather limited—for two very different reasons: (a) None have been found up to ~ 200 GeV. (b) They could be much higher like 10 TeV or even 100 TeV. Then they could not be directly produced at the LHC. Therefore, we stop our analysis at the 1–2 TeV scale. In that case, some of the new states could mix significantly. The gauge boson mass is also of the $O(1 \text{ TeV})$. Large angles and phases are possible but limited by experimental constraints from B and K physics. Also, quite important to our analysis, hierarchies in fermionic masses are very small, a lot smaller that what we find in the SM.

In LHT-like models, the operators with new degrees of freedom are similar to the SM-SD ones. Hence, they scale similarly to the SM-SD operators, which has been established for decades now [35] and has been used extensively to qualitatively judge the size of flavor dynamics ever since. Let us reexamine this scaling behavior. Of course, this scaling is only an approximation of the detailed form factors, however, most of the time, such approximations are enough to estimate the size of many effects in flavor physics. The mass hierarchy in the new fermionic sector can be established with

$$m_i^a = m_1^a h_i^a \quad \forall \; i = 1 \ldots N. \qquad (28)$$

Here, a distinguishes amongst the members of each family which defines the sector to which the fermion belongs and it is *not* summed over. It is possible to set $h_1^a \neq 1$ and choose m_1^a to be any finite mass representative of the physics in consideration. The form factors in flavor physics can be approximately expressed as

$$F(x) = f_n(x)(\log(x))^m \qquad f_n(x) = x^n,$$
$$n \in \mathbb{Z}, \qquad m = 0, 1. \qquad (29)$$

Ignoring QCD (or QCD-like) corrections, any matrix elements involving the processes mentioned above are of the form

$$\mathcal{M} \sim \sum_{i=1}^{N} \lambda_i F(x_i). \qquad (30)$$

Here, $x_i^a = (m_i^a/m_G)^2$ is the commonly used square of the ratio of the internal fermion mass to the mass scale of the massive gauge bosons. The superscript a has been dropped as the matrix element usually involves just one sector. Using the unitarity relation in Eq. (27), it can be shown that under the hierarchy defined in Eqs. (28) and (30) scales as

$$\sum_{i=1}^{N} \lambda_i F(x_i) = f_n(x_1) \sum_{i=1}^{N} \lambda_i F(h_i^2) + F(x_1) \sum_{i=1}^{N} \lambda_i f_n(h_i^2). \quad (31)$$

Note that the mass scale of the fermions does not suffer from unitarity suppression, but the hierarchy does. In other words, the only two ways of getting large matrix elements are either to start at a very high mass scale for fermions or to build a very strong hierarchy that will illude the unitarity suppression, or both.

Now, let us have a look at boxes and penguins.
Box diagrams: Box diagrams scale as x for both large and small x. Setting $n = 1$ and $m = 0$ in Eq. (29), we see that contributions from these will scale as

$$\mathcal{M} \sim x_1 \sum_{i=1}^{N} \lambda_i h_i^2. \qquad (32)$$

This clearly tells us that box diagrams are sensitive to both mass scales and large hierarchies. Hence, any NP model containing either fermions with masses comparable or greater than the gauge bosons or having a large hierarchy amongst the families or both will make large contributions through box diagrams. Z_L penguins: Z_L Penguins scale as $x \log(x)$ for small x and as x for large x. In the regime in which they scale as x, the conclusion is the same as above. Setting $n = 1$ and $m = 1$ in Eq. (29), we see that if, for some reason, NP offers small x, the contribution will scale as

$$\mathcal{M} \sim x_1 \sum_{i=1}^{N} \lambda_i h_i^2 \log(h_i^2) + x_1 \log(x_1) \sum_{i=1}^{N} \lambda_i h_i^2. \quad (33)$$

Hence, in addition to the conclusions drawn for large x, Z_L penguins will get large contributions from NP if the fermionic masses are light compared to the gauge boson mass and shows large hierarchies. However, such scenarios will be very unusual if not unheard of in the NP models currently under consideration. γ and chromomagnetic penguins: Electromagnetic and chromomagnetic penguins scale as x for small values of x but are asymptotic to a constant for large values of x. Hence, they can only benefit from large hierarchies at fermionic mass scale smaller than the gauge boson. Even if NP has large fermionic masses, even with large hierarchies to offer, it will show up only as moderate enhancements in this class of penguins.

On the other hand, photonic penguins scale as $\log(x)$. Hence, $n = 0$ and $m = 1$ in Eq. (29), we see its contribution to matrix elements will scale as

$$\mathcal{M} \sim \sum_{i=1}^{N} \lambda_i \log(h_i^2). \qquad (34)$$

NP intervention will fail to produce any enhancement in photonic penguins unless it has very sharp hierarchies to offer in its fermionic masses. Large mass scales will have no effect on photonic penguins.

A look at the mass spectrum of the heavy fermions, Fig. 1, which we used in the previous study [30] and in the present one, show that we have heavy mass scales in the spectrum, but not large hierarchies. As a result, we get large enhancements to purely electroweak processes, which involve box diagrams and Z_L penguins. Such is the case for $D^0 \to \mu^+ \mu^-$ and A_{FB}, where we see orders of magnitude enhancement to SM-SD rates and also in $D^0 - \bar{D}^0$ oscillations. A_{FB}^{CP} too benefits from this effect. For electromagnetic penguins and chromomagnetic penguins, NP intervention can only be moderate. This explains the small enhancements that we saw in $D^0 \to \gamma\gamma$, which is primarily sensitive to O_7. However, in $D \to X_u l^+ l^-$, the dominant contribution, by orders of magnitude is the photonic penguins in O_9 if we ignore QCD corrections which are anyways blind to NP. Even if LHT manages to enhance the other contributions, it falls short of the SM contribution to C_9. Even the LHT contributions to the box diagrams and Z_L penguins in O_9 fails to overcome the SM contribution to the photonic penguins. Moreover, the photonic penguins do not see much NP intervention, as we have already argued.

This gives us a qualitative way of understanding why LHT fails to enhance processes driven by $\Delta C = 1$ dynamics involving the D meson system unless QCD effects are completely absent. This argument can potentially be extended to any other model which shares the same flavor structure as LHT, what we have previously defined as LHT-like. Of course, we always have to keep in mind that we are heavily constrained from B and K physics and that limits us in the parameter space that we can choose to work with.

VII. CONCLUSIONS

Charm hadrons, stable under strong forces, were predicted to keep the SM consistent with the observed suppression of strangeness changing neutral currents, as was their preference to decay into strange hadrons, with decays into nonstrange hadrons being Cabibbo suppressed, and to preserve renormalizability; they were even found in the expected mass range. Therefore, decays of charm hadrons were hardly seen as worth probing for manifestations of NP. Furthermore, one realized that kaon and even more, beauty hadrons could clearly exhibit NP signals, since their leading SM decays are Cabibbo and KM suppressed. A minority of authors argue that the probe for NP should not

be given up in charm decays, since the SM weak phenomenology is "dull" and one needs "only" many more statistics there.

The case was somewhat strengthened after the observation of $D^0 - \bar{D}^0$ oscillations, although it is not outside some SM estimates. Furthermore, it was found that a class of NP models like LHT that is motivated from outside the flavor dynamics can yet produce a sizable contribution to the observed $D^0 - \bar{D}^0$ signal and can create much stronger indirect CP violation [29].

$D \rightarrow X_u l^+ l^-$ is a good laboratory for NP, as it is rich in its operator structure, and involves almost anything that FCNCs have to offer, including direct CP violation. However, we find that LHT fails to create a significant dent to SM-SD contributions to this channel other than significantly enhancing A_{FB}^c and A_{FB}^{CP} for reasons we have explained above. Moreover, the SM-LD contribution dominates over both SM-SD and LHT contributions to the branching fractions. We reiterate the conclusion that we had come to in our previous work [30]: while LHT can contribute significantly to $\Delta C = 2$ processes, it fails to dominate in $\Delta C = 1$ processes with orders of magnitude enhancements unless the contribution appears through box diagrams and are bereft of relatively large QCD effects.

We also go ahead and have a second look at what we previously defined as LHT-like models: again we found that certain conclusions can be drawn about NP's flavor structure by studying weak decays independent of the other details of the underlying model. The absence of large hierarchies and unitarity of the new mass mixing matrices within such NP models heavily limits the new FCNCs. Also, we have shown that experimental limits in B and K physics can be directly ported to give constraints on NP intervention in D physics.

During our analysis, we developed a more general conjecture: if NP models affect the dynamics of both the up-type and down-type quarks in a tightly correlated way—as shown in Eq. (24) in the case of LHT—they will mostly fail to contribute significantly in $\Delta C = 1$ dynamics, even if they play a major role in $\Delta C = 2$ processes. For constraints from $\Delta S = 1$ and $\Delta B = 1$, reactions will suppress $\Delta C = 1$ coupling greatly, since sensitivity for NP is often greater in K and B decays because of their leading SM transitions are Cabibbo or KM suppressed; otherwise it would oversaturate for $\Delta C = 2$ effects. The crucial feature on the right-hand side of Eq. (24) is only that the CKM matrix is very close to the identity matrix; the same conclusion should apply for any matrix in the right-hand side that is close to the identity matrix. Of course, the validity of this conjecture is yet to be tested in other $\Delta C = 1$ processes like direct CP violations in nonleptonic charm decays, keeping in mind that charm transitions could still produce surprises for us—and about SM's "ability" to cope with them. In the event that future experiments reveal a clear manifestation of NP through enhancements in the $\Delta C = 1$ processes, it is unlikely that any LHT-like model can be a "culprit."

ACKNOWLEDGMENTS

APPENDIX A: QCD CORRECTIONS

Here, we include some of the numbers and functions which appear in the QCD correction of the operators. The function $f(z)$ in Eq. (A1) is that which appears in the two-loop $O(\alpha_s)$ correction to C_7 given by [38]

$$
f(z) = -\frac{1}{243}[576\pi^2 z^{3/2} + (3672 - 288\pi^2 - 1296\zeta(3) + (1944 - 324\pi^2)\log(z) + 108\log(z)^2 + 36\log(z)^3)z
$$

$$
+ (324 - 576\pi^2 + (1728 - 216\pi^2)\log(z) + 324\log(z)^2 + 36\log(z)^3)z^2
$$

$$
+ (1296 - 12\pi^2 + 1776\log(z) - 2052\log(z)^2)z^3] - \frac{4\pi i}{81}[(144 - 6\pi^2 + 18\log(z) + 18\log(z)^2)z
$$

$$
+ (-54 - 6\pi^2 + 108\log(z) + 18\log(z)^2)z^2 + (116 - 96\log(z))z^3] + O(z^4) \tag{A1}
$$

$$
\mathbf{a} = \left\{ \tfrac{14}{23}, \tfrac{16}{23}, \tfrac{6}{23}, -\tfrac{12}{23}, 0.4086, -0.4230, -0.8994, 0.1456 \right\} \tag{A2}
$$

$$
\mathbf{z} = \left\{ \tfrac{14}{25}, \tfrac{16}{25}, \tfrac{6}{25}, -\tfrac{12}{25}, 0.3469, -0.4201, -0.8451, 0.1317 \right\} \tag{A3}
$$

$$
\mathbf{X} = \begin{pmatrix}
-3.5687 & 2.5813 & 0.40 & 0.6524 & -0.0532 & -0.0034 & -0.0084 \\
-4.0742 & 2.7827 & 0.40 & 0.8461 & 0.0444 & 0.0068 & -0.0059 \\
-22.423 & 18.290 & 0.0 & 4.3019 & -0.1241 & 0.0001 & -0.0452 \\
-23.434 & 18.693 & 0.0 & 4.6894 & 0.071 & 0.0206 & -0.0402 \\
9.8081 & -8.8366 & 0.0 & -0.7779 & 0.0289 & -0.0486 & -0.1739 \\
2.8271 & -3.2361 & 0.0 & 0.4903 & 0.0433 & -0.1303 & 0.0056
\end{pmatrix}. \tag{A4}
$$

The vectors **a** and **z** [38], which appear in C_7, are given by Eqs. (A2) and (A3). The matrix **X** is given by Eq. (A4). The Wilson coefficients of the operators O_{1-6} at $\mu = m_b$ are given by [70]

$$C_j(m_b) = \sum_{i=3}^{8} h_{j(i-2)} \eta_b^{a_i} \quad \forall \; j = 1 \ldots 6 \qquad \text{(A5)}$$

with the initial condition

$$\mathbf{C}(m_W) = \{0, 1, 0, 0, 0, 0\}. \qquad \text{(A6)}$$

The coefficient matrix h_{ji} given by

$$\begin{pmatrix} 1/2 & -1/2 & 0 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 & 0 \\ 1/6 & -1/14 & 0.0510 & -0.1403 & -0.0113 & 0.0054 \\ -1/6 & -1/14 & 0.0984 & 0.1214 & 0.0156 & 0.0026 \\ 0 & 0 & -0.0397 & 0.0117 & -0.0025 & 0.0304 \\ 0 & 0 & 0.0335 & 0.0239 & -0.0462 & -0.0112 \end{pmatrix}.$$
$$\text{(A7)}$$

The Wilson coefficients given in Eq. (A5) can be calculated using the anomalous dimension matrix given in Ref. [38], with five active flavors for $\mu \geq m_b$. To calculate $\mathbf{C}(m_c)$, the same procedure with four active flavors for $m_b \geq \mu \geq m_c$ will yield the necessary results.

APPENDIX B: AUXILIARY FUNCTIONS

The auxiliary functions that come into the LHT contributions to $D^0 \rightarrow X_u l^+ l^-$ are listed below. They can also be found in [61]. The function $S(x)$ is a contribution from the Z_L penguin diagrams with internal mirror quarks, which was pointed out in [71] and subsequently in [72]. It replaces the divergence mentioned in [61], which was later updated in [62]

$$G_Z(x, y) = -\frac{3}{4} U(x, y, 1) \quad G_A(x, y) = \frac{1}{25a} G_Z(x, y)$$

$$G_\eta(x, y) = -\frac{3}{10a} U(x, y, \eta_a)$$

$$F_W(x, y) = \frac{3}{2} x - F_5(x, y) + 7 F_6(x, y) + 3 U(x, y, 1)$$

$$F_2(x) = -\frac{1}{2} \left(\frac{x^2 \log(x)}{(1-x)^2} + \frac{1}{1-x} \right)$$

$$F_5(x, y) = \frac{x^3 \log(x)}{(1-x)(y-x)} - \frac{y^3 \log(y)}{(1-y)(x-y)}$$

$$F_6(x, y) = \frac{z^2 \log(x)}{(1-x)(y-x)} + \frac{y^2 \log(y)}{(1-y)(x-y)}$$

$$R_2(x) = -\left(\frac{x \log(x)}{(1-x)^2} + \frac{1}{1-x} \right)$$

$$S(x) = x \left(\frac{x^2 - 2x + 4}{(1-x)^2} \log(z) + \frac{7-x}{2(1-x)} \right)$$

$$U(x, y, \eta) = \frac{x^2 \log(x)}{(1-x)(\eta-x)(x-y)} + \frac{y^2 \log(y)}{(1-y)(\eta-y)(y-x)}$$

$$+ \frac{\eta^2 \log(\eta)}{(1-\eta)(x-\eta)(\eta-y)}. \qquad \text{(B1)}$$

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