

Composite and elementary natures of $a_1(1260)$ mesonHideko Nagahiro,^{1,2} Kanabu Nawa,² Sho Ozaki,² Daisuke Jido,³ and Atsushi Hosaka²¹*Department of Physics, Nara Women's University, Nara 630-8506, Japan*²*Research Center for Nuclear Physics (RCNP), Osaka University, Ibaraki, Osaka 567-0047, Japan*³*Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan*

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We develop a practical method to analyze the mixing structure of hadrons consisting of two components of quark composite and hadronic composite. As an example, we investigate the properties of the axial vector meson $a_1(1260)$ and discuss its mixing properties quantitatively. We also make reference to the large N_c procedure and its limitation for the classification of such a mixed state.

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Hadrons, interacting with the strong force, are composite objects of quarks and gluons. One of recent interests in the hadron structure is whether hadrons are made up of quarks and gluons confined in a single-particle potential as described in the conventional quark model, or rather develop subcomponents of quark-clusters inside hadrons. A typical example of the latter is the deuteron, which is composed of a proton and a neutron, not of six quarks in a single confining potential [1]. It has been also suggested that some hadronic resonances could have substantially large components of hadronic composites [2,3]. Such recursive (nesting-box) structures are also seen in nuclear physics. For instance, the first 0^+ excited state of ^{12}C is described as a three- α cluster state [4].

In general, it is not easy to clearly identify the quark-cluster components, because the strong interaction scales for quarks and hadrons are not well separated. To simplify the situation, it would be a good starting point to set up a model space of a state described by a single-particle potential and one of several quark-clusters. The former may be identified with the “elementary” component¹ while the latter with the (hadronic) composite, the situation which was studied by Weinberg [1]. If hadronic resonant states are unavoidably mixtures of hadronic and quark-composites, an important issue is to clarify how these components are mixed in a hadron.

One good example to study the two features and their mixing is provided by the low-lying axial vector meson $a_1(1260)$. The a_1 meson is a candidate of the chiral partner of the ρ meson [5–7] described as a $q\bar{q}$ -composite, for example, in the Nambu-Jona-Lasinio model [8,9] and in the Lattice calculation [10]. It can also appear as a gauge boson of the hidden local symmetry [11], which is recently reconciled with the five-dimensional gauge field of the holographic QCD [12,13]. On the other hand, in coupled-channel approaches based on the chiral effective theory [14,15], the a_1 meson has been described as a dynamically generated resonance in the $\pi\rho$ scattering

without introducing its explicit pole term. The a_1 nature has been studied by calculating physical observables such as the radiative decay width [16] or the τ decay spectrum into three pions [17,18]. Yet, the internal structure of the a_1 meson is not well understood.

In this paper, we focus on hadron structure having two components of quark-composite (we refer to it as the elementary component) and hadronic composite. We propose a method to disentangle their mixture appearing in the physically observed state by taking the a_1 meson as an example. The method provided here can be generally applied to other mixed systems if the interaction is given between the elementary component and possible constituents making the composite state. Our ingredients for the study of a_1 are therefore π and ρ mesons², which have potential to generate the *composite* a_1 meson with a suitable $\pi\rho$ interaction, and the *elementary* component of the a_1 meson (dominated by $q\bar{q}$), which couples to the $\pi\rho$ pair by a three-point interaction. We first solve the $\pi\rho$ scattering amplitude to find the poles corresponding to the physical a_1 , and then develop a method to clarify the mixing nature of the two components by introducing appropriate bases for pure composite and elementary components. As an extension, we apply our method to the study of the large N_c behavior of the a_1 state. The large N_c limit is usually considered to give a guiding principle for the classification of state, in which the mass of the $q\bar{q}$ bound state scales as $\mathcal{O}(1)$ in powers of N_c while that of a meson-meson molecule scales with higher order of N_c [19,20]. Reference [21], however, has brought caution for the use of large N_c argument for the dynamically generated scalar resonances. We discuss the validity of the classification for the mixed states of hadronic composite and elementary components.

Let us start with the composite a_1 meson, which is dynamically generated in the s -wave $\pi\rho$ scattering through the nonperturbative dynamics. The scattering amplitude t satisfies the Bethe-Salpeter equation,

¹The elementary component was referred to as the CDD pole or genuine quark state in the literature [3].

²We regard the ρ meson as stable particles in the present model setting.

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$t = v + vGt$, where v is a four-point $\pi\rho$ interaction and G the $\pi\rho$ two-body propagator and its formal solution is given by

$$t = \frac{v}{1 - vG}. \quad (1)$$

If the potential v is sufficiently attractive, the amplitude develops a pole corresponding to a composite bound or resonant state of the scattering system at the energy satisfying $1 - vG = 0$. In the present case of a_1 , the potential v can be obtained from the s -wave projection of the Weinberg-Tomozawa interaction [14], and the pole in the $\pi\rho$ scattering amplitude can appear above the $\pi\rho$ threshold as a resonance in the second Riemann sheet, which is a consequence of the energy-dependent interaction. This pole corresponds to the $\pi\rho$ -composite a_1 meson [14] without $q\bar{q}$ quark core [3,22].

Because the elementary a_1 meson has a coupling to the π and ρ mesons, it also contributes to the $\pi\rho$ scattering amplitude in the form of an effective $\pi\rho$ interaction going through the elementary a_1 pole:

$$v_{a_1} = g \frac{1}{s - m_{a_1}^2 + i\epsilon} g, \quad (2)$$

where g is the coupling to $\pi\rho$ that can depend on s , and m_{a_1} the bare mass of the elementary a_1 meson. The full scattering amplitude T having both the $\pi\rho$ four-point interaction v and the a_1 pole term v_{a_1} is then written by

$$T = \frac{v + v_{a_1}}{1 - (v + v_{a_1})G}. \quad (3)$$

This amplitude generates poles corresponding to physical resonant states of the problem. They are expressed as a superposition of the basis states associated with the two poles of composite a_1 in Eq. (1) and elementary a_1 in Eq. (2), respectively.

Now let us study the mixing nature of the physical states. To this end, we first express equivalently the amplitude t in Eq. (1) as

$$t \equiv g_R(s) \frac{1}{s - s_p} g_R(s), \quad (4)$$

where s_p is the pole position of the amplitude t in Eq. (1). In this form, we can interpret $(s - s_p)^{-1}$ as the one-particle propagator of the composite a_1 meson as shown in Fig. 1 by taking an analogy with the conventional discussion of the bound-state problem [22]. The vertex function $g_R(s)$ that is defined so as to reproduce Eq. (1) exactly is interpreted as the effective coupling of the composite a_1 to $\pi\rho$. This interpretation works well in the neighborhood of the pole, $s \sim s_p$. As s is further away from s_p , $g_R(s)$ receives more contributions from the nonresonant background [23].

Having the form of Eq. (4), we now rewrite the scattering amplitude T in Eq. (3) as

$$T = (g_R, g) \frac{1}{\hat{D}_0^{-1} - \hat{\Sigma}} \begin{pmatrix} g_R \\ g \end{pmatrix}, \quad (5)$$

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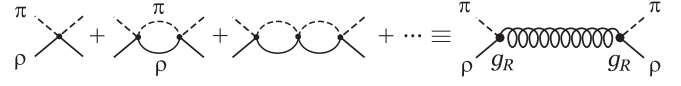


FIG. 1. Infinite set of diagrams that contributes to the meson-meson scattering amplitude in Eq. (1) and the definition of the propagator of the composite a_1 meson in Eq. (4).

where

$$\hat{D}_0^{-1} = \begin{pmatrix} s - s_p & \\ & s - m_{a_1}^2 \end{pmatrix}, \quad \hat{\Sigma} = \begin{pmatrix} & g_R G g \\ g G g_R & g G g \end{pmatrix}. \quad (6)$$

The diagonal elements of the matrix \hat{D}_0 are the free propagators of the two a_1 's, one for the composite and the other for the elementary ones having the proper normalization, and the matrix $\hat{\Sigma}$ expresses the self-energy and interactions for these modes. One can prove that Eq. (5) is identical with Eq. (3) after some algebra.

We emphasize that the expression of Eq. (5) makes it possible to analyze the mixing nature of the physical a_1 in terms of the original two bases. Having the amplitude in the form of Eq. (5), the matrix $\hat{D} \equiv (\hat{D}_0^{-1} - \hat{\Sigma})^{-1}$ is identified with the propagators of the physical states represented by the bases of the elementary and composite a_1 's. The diagonal elements D^{ii} indicate the dressed propagators of the composite and the elementary a_1 's as shown in Fig. 2, which express the a_1 mesons acquiring the quantum effects: e.g., the elementary a_1 introduced without width in Eq. (2) now has a width owing to its decay into $\pi\rho$ system. The important features of the propagators are that they have poles exactly at the same positions as the full amplitude T in Eq. (3), and the residues of the diagonal elements D^{ii} ($i = 1, 2$) defined by

$$D^{ii} = \frac{z_a^{ii}}{s - M_a^2} + \frac{z_b^{ii}}{s - M_b^2} + (\text{regular term}) \quad (7)$$

have the meaning of the wave function renormalization and then carry the information on the mixing rate of the physical resonant states. For instance, the residue z_a^{11} means the probability of finding the original composite a_1 component in the resulting state having the mass M_a .

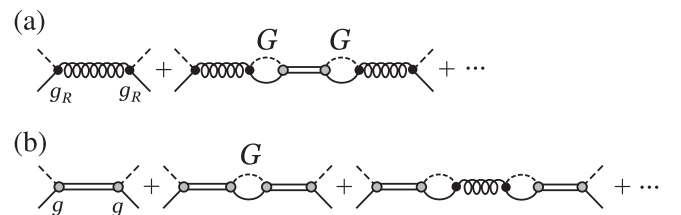


FIG. 2. Diagrams that contribute to the full amplitude in Eq. (5) going through (a) the component of the dressed propagator D^{11} and (b) that of D^{22} . The dashed and solid internal lines indicate the π and ρ propagators, while the curved and double lines are those of the composite and elementary a_1 's.

The above discussions have close analogy to a two-level problem, which is well realized when the energy s is not very far away from the original two poles s_ρ and $m_{a_1}^2$. As s is getting away from them, the orthogonality of the physical two-resonant states is lost and, furthermore, the residues can exceed unity. These problems arise owing to the energy dependence of $\hat{\Sigma}$, especially those of g_R and g in $\hat{\Sigma}$. We note that the energy dependence of g_R is unavoidable for the resonant states generated dynamically.

So far, we have given a general framework to investigate the mixing properties of hadrons consisting of multiple components having different origins. To perform a concrete calculation, we shall employ a suitable model for a_1 . Here we take only the single $\pi\rho$ channel into account for the composite a_1 because other channels have been found not important [14], but we can easily extend our framework to coupled-channel cases. Our ingredients for the study of the a_1 system are, therefore, the π and ρ and (elementary) a_1 mesons, for which a suitable model is given.

In this paper, as for interaction Lagrangians, we adopt the chiral Lagrangians induced by the holographic QCD approach as the Sakai-Sugimoto model [12] that is constructed in the strong-coupling limit of large- N_c QCD. The relevant Lagrangians [12] are given by

$$\mathcal{L}_{\text{WT}} = \frac{1}{f_\pi^2} \text{tr}([\rho^\mu, \partial^\nu \rho_\mu][\pi, \partial_\nu \pi]), \quad (8)$$

$$\begin{aligned} \mathcal{L}_{a_1\pi\rho} = & -ig_{a_1\pi\rho} \frac{4}{f_\pi} \{ \text{tr}((\partial_\mu a_{1\nu} - \partial_\nu a_{1\mu})[\partial^\mu \pi, \rho^\nu]) \\ & + \text{tr}((\partial_\mu \rho_\nu - \partial_\nu \rho_\mu)[\partial^\mu \pi, a_1^\nu]) \}, \quad (9) \end{aligned}$$

where $\rho^\mu \equiv \vec{\rho}^\mu \cdot \frac{\vec{\tau}}{2}$, $\pi \equiv \vec{\pi} \cdot \frac{\vec{\tau}}{2}$, and $a_1^\mu \equiv \vec{a}_1^\mu \cdot \frac{\vec{\tau}}{2}$. The first Lagrangian gives the four-point Weinberg-Tomozawa (WT) interaction and the second gives the three-point vertex of $a_1\pi\rho$.

The advantage of using the concept of the holographic QCD approach is that the large- N_c condition ensures that the a_1 field in the Lagrangian does not contain hadronic composite components, and hence we can avoid the double-counting in the analysis of the mixing nature. The model contains two inputs, $f_\pi = 92.4$ MeV and $m_\rho = 776$ MeV, giving the mass of the elementary a_1 meson $m_{a_1} = 1189$ MeV and the $a_1\pi\rho$ coupling constant $g_{a_1\pi\rho} = 0.26$. In the present study, we consider the a_1 field appearing in the Lagrangian with these physical constants to be the quark composite that survives in the large N_c limit. For the mass of the pion, we employ the physical value $m_\pi = 138$ MeV that is isospin-averaged. These interactions in Eqs. (8) and (9) are essentially the same as those of the hidden-local symmetry [11], except for the actual values of m_{a_1} and $g_{a_1\pi\rho}$.

Using these interactions, we take the WT potential V_{WT} [14] and the a_1 pole term V_{a_1} as

$$V_{\text{WT}} = \frac{\epsilon \cdot \epsilon'}{4f_\pi^2} \left\{ 3s - 2(m_\rho^2 + m_\pi^2) - \frac{1}{s}(m_\rho^2 - m_\pi^2)^2 \right\}, \quad (10)$$

$$V_{a_1} = -\frac{8}{f_\pi^2} g_{a_1\pi\rho}^2 \frac{\epsilon \cdot \epsilon'}{s - m_{a_1}^2 + i\epsilon} (s - m_\rho^2)^2, \quad (11)$$

after the s -wave projection with on-shell energies for external π and ρ [14]. In actual calculations, we should treat the polarization vectors ϵ and ϵ' for the ρ meson appropriately. As reported in detail in Ref. [14], by substituting the coefficient $v_{\text{WT}}(v_{a_1})$ defined by $V_{\text{WT}}(V_{a_1}) \equiv -\epsilon \cdot \epsilon' v_{\text{WT}}(v_{a_1})$ for the potentials in Eq. (3), we can obtain the scattering amplitude for the transverse polarization mode, in which we can find poles dynamically generated.

In solving the scattering equation, we need to calculate the propagator function G . Because the potentials in Eqs. (10) and (11) are separable, the formal solution of Eq. (1) becomes algebraic, and the function G is then given by

$$G(\sqrt{s}) = i \int \frac{d^4q}{(2\pi)^4} \frac{1}{(P-q)^2 - m_\pi^2 + i\epsilon} \frac{1}{q^2 - m_\rho^2 + i\epsilon}, \quad (12)$$

where P is the total four-momentum as $P^2 = s$. The integral (12) diverges, and hence, we regularize it by the dimensional regularization, and for the finite part we introduce a subtraction constant $a(\mu)$ as

$$\begin{aligned} G(\sqrt{s}) = & \frac{1}{16\pi^2} \left\{ a(\mu) + \ln \frac{m_\rho^2}{\mu^2} + \frac{m_\pi^2 - m_\rho^2 + s}{2s} \ln \frac{m_\pi^2}{m_\rho^2} \right. \\ & + \frac{q'}{\sqrt{s}} [\ln(s - (m_\rho^2 - m_\pi^2)) + 2q'\sqrt{s}] \\ & + \ln(s + (m_\rho^2 - m_\pi^2)) + 2q'\sqrt{s} \\ & - \ln(s - (m_\rho^2 - m_\pi^2)) - 2q'\sqrt{s} \\ & \left. - \ln(s + (m_\rho^2 - m_\pi^2)) - 2q'\sqrt{s} - 2\pi i \right\}, \quad (13) \end{aligned}$$

where μ is the scale parameter that is set to be 900 MeV in this paper and $q' = \lambda^{1/2}(s, m_\rho^2, m_\pi^2)/2\sqrt{s}$. A crucial point here is that the constant $a(\mu)$ is chosen to be a *natural* value [3], which ensures that the resulting resonant states, if they exist, are interpreted as a purely hadronic composite. Following the prescription in Ref. [3], we choose $a(\mu) = -0.2$ at the renormalization scale μ , which satisfies the matching condition $\text{Re} G(\sqrt{s} = m_\rho) = 0$. A choice of the subtraction constant $a(\mu)$ different from the natural value is equivalent to the introduction of the CDD (or elementary) pole that is not included in the model space of the scattering problem [3].

The polarization vector of the intermediate ρ meson is treated in the same way as reported in Ref. [14].

Now we can evaluate the scattering amplitude (3) or (5) numerically. We find two poles at (a) $\sqrt{s} = 1033 - 107i$ MeV and at (b) $1728 - 313i$ MeV, corresponding to the physical states in the present model. These pole positions are significantly different from those of the two basis states, $\sqrt{s}_\rho = 1012 - 221i$ MeV and

$m_{a_1} = 1189$ MeV, because of the mixing effect. For further investigation, we vary the coupling strength of $a_1\pi\rho$ by introducing a parameter x ($0 \leq x \leq 1$) and $g_{a_1\pi\rho} \rightarrow xg_{a_1\pi\rho}$, which controls the mixing strength.

In Fig. 3(a), we show the resulting pole-flow in the complex-energy plane by changing the mixing parameter x . At $x = 0$, the poles corresponding to the basis states of the composite and elementary a_1 's are found at the positions indicated by open square and open circle in Fig. 3(a), respectively. When the mixing is turned on, the pole starting from the composite a_1 (we refer to it as ‘‘pole-a’’) approaches the real axis, ending at $1033 - 107i$ MeV when $x = 1$ (solid square), while that from the elementary a_1 pole (‘‘pole-b’’) goes far from the real axis and reaches $1728 - 313i$ MeV when $x = 1$ (solid circle).

In Fig. 3(b), we show the squared amplitude $|T|^2$ in Eq. (3) (or (5)) at $x = 1$. As shown in the figure, a peak structure is dominated by the pole-a, while a signal of the pole-b cannot be seen clearly because of its huge width (~ 600 MeV). Therefore, the pole expected to be observed in experiments is the pole-a located at lower energy position [solid square in Fig. 3(a)] that comes from the composite a_1 pole. Indeed, the contribution of the pole-b is found to interfere destructively with the tail of the pole-a (around $\text{Re}\sqrt{s} \sim 1.7\text{--}1.8$ MeV) as shown in Fig. 3(b). Our present model setting, however, is rather simple and may not be applicable to such a higher energy region where

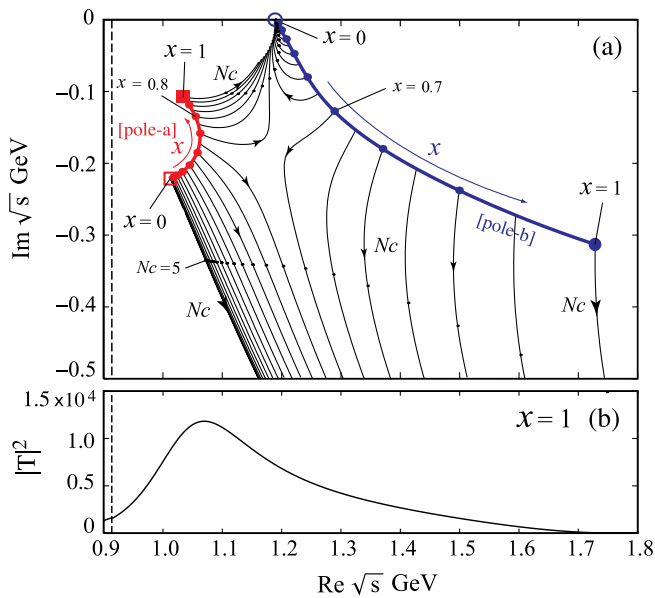


FIG. 3 (color online). (a) Trajectories of the poles in the full scattering amplitude in Eq. (3) by changing the mixing parameter x (thick lines). The open square indicates the pole position of the composite a_1 and open circle indicates the elementary a_1 pole ($x = 0$). The other end points of solid circle and square correspond to the physical points ($x = 1$). Thin lines represent the pole-flows as N_c is increased from $N_c = 3$ for fixed x with small dots at $N_c = 5$. The vertical dashed line denotes the $\pi\rho$ threshold energy. (b) Squared amplitude $|T|^2$ of $\pi\rho \rightarrow \pi\rho$ process on the real energy axis for the mixing parameter $x = 1$.

contributions from higher coupled-channels neglected here should be important. A detailed investigation for an evidence of the second state of the a_1 meson, namely, the pole-b, is an interesting future work.

To study the mixing properties more quantitatively, we show the absolute values of the residues as functions of the mixing parameter x in Fig. 4 (left panel). We can verify that, before turning on the mixing, the pole-a is purely the composite a_1 ($z_a^{11} = 1$ and $z_a^{22} = 0$ at $x = 0$) and the pole-b is the elementary a_1 as we intended. One of the most important messages can be read from the magnitude of z_a^{11} and z_a^{22} at $x = 1$, which are the residues of the possibly observed a_1 state. While we should carefully discuss the meaning of the residues of complex-value for resonant states, we can say that the pole-a at $x = 1$, although its location is close to the composite a_1 pole, has a component of the elementary a_1 meson comparable to that of the composite a_1 . This conclusion can be drawn only after we look into the wave function by the residues z .

As for the pole-b, we find that the magnitude of the residues z_b^{22} and z_b^{11} interchange at around $x \sim 0.7$, meaning that the nature of the resonant state of pole-b changes from the elementary particle-like structure to the composite one. For larger x (≥ 0.8), z_b^{11} and z_b^{22} become larger than unity because the energy dependence of the potentials become stronger at the higher energy region where the pole-b is located. Once again, in such a higher energy region ($\text{Re}\sqrt{s} \gtrsim 1.5$ GeV), our present model may not be applicable.

Next, we test the large N_c dependence of the pole positions according to the scaling law of the pion-decay constant f_π as $f_\pi \rightarrow f_\pi\sqrt{N_c/3}$. In Fig. 3(a), we also show the trajectories of the pole positions by changing the N_c value for fixed mixing strength. At $x = 0$, as reported in Ref. [24], we see the composite a_1 pole tends to have a heavier mass and a wider width as N_c is increased. For small mixing parameter $x \lesssim 0.6$, we find a similar behavior for the pole-a while the pole-b goes back to the elementary a_1 position. At $x \sim 0.7$, we find that the two N_c -trajectories flip simultaneously. For $x \gtrsim 0.7$, the pole-b goes away from the real axis as the composite a_1 does, while the pole-a approaches the elementary pole as N_c is increased. In this way, the result of the large N_c classification depends

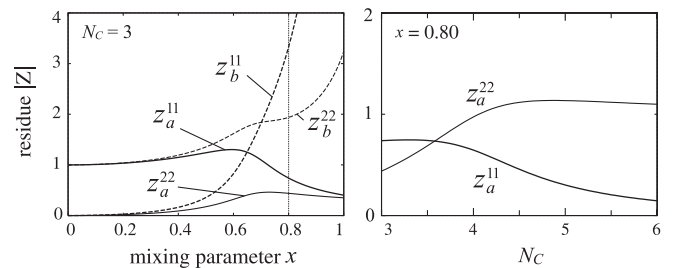


FIG. 4. Absolute value of the residues defined in Eq. (7). The left panel shows the mixing parameter x dependence at $N_c = 3$ while the right panel is the N_c dependence at $x = 0.8$. The meaning of each line is indicated in the figure.

strongly on the mixing parameter x , although the component of the composite a_1 is always larger than that of elementary ($z_a^{11} > z_a^{22}$) at $N_c = 3$ as shown in Fig. 4 (left panel).

In Fig. 4 (right panel), we show the N_c dependence of the residues of pole-a at $x = 0.8$. There we find that the magnitudes of the residues of the pole-a, z_a^{11} and z_a^{22} , interchange at $N_c \sim 3.5$. This indicates that the nature of the resonance *changes* as N_c is varied. Thus, for the mixed system of elementary and composite components, the large N_c limit does not always reflect the world at $N_c = 3$. This is a consequence of there being two sources of the N_c dependence, one from the $a_1\pi\rho$ three-point vertex, which controls the mixing strength between the two basis states, and the other from the WT interaction, which determines the pole position (s_p) of the basis state for the composite a_1 . Their competition determines the nature of the a_1 states. Actually, the $a_1\pi\rho$ interaction in Eq. (9) is of order $N_c^{-1/2}$ and vanishes in the large N_c limit, where the two states of $q\bar{q}$ meson and hadronic composite states decouple. In the real world of $N_c = 3$, however, the interaction remains finite and gives an important contribution to the mixing dynamics as we showed. Therefore, we conclude that the large N_c classification, which is often used to identify the character of resonances, does not necessarily work for such a mixed systems. In Ref. [25], we provide a general discussion of a two-level effective model of large N_c behavior.

We have developed a general method to analyze the mixing structure of hadrons consisting of two components of quark and hadronic composites. As an example, the nature of the $a_1(1260)$ axial-vector meson has been explored. The present analysis points out theoretically that the a_1 meson has comparable amounts of the elementary a_1 component to the $\pi\rho$ composite a_1 . Quest for evidences of the mixing nature in physical observables is an interesting future work.

The method proposed in the present paper makes it possible to discuss the validity of the classification in the large N_c limit, which is considered to give a guiding principle to identify the nature of hadronic resonances. We have shown explicitly that the mixing nature of hadrons in the large N_c limit could differ from that at finite $N_c = 3$. We conclude that the simple classification does not always work when admixture of components having different origins is important.

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- [1] S. Weinberg, *Phys. Rev.* **130**, 776 (1963); **137**, B672 (1965).
 - [2] V. Baru, J. Haidenbauer, C. Hanhart, Y. Kalashnikova, and A. E. Kudryavtsev, *Phys. Lett. B* **586**, 53 (2004).
 - [3] T. Hyodo, D. Jido, and A. Hosaka, *Phys. Rev. C* **78**, 025203 (2008).
 - [4] Y. Funaki, A. Tohsaki, H. Horiuchi, P. Schuck, and G. Ropke, *Phys. Rev. C* **67**, 051306 (2003).
 - [5] S. Weinberg, *Phys. Rev. Lett.* **18**, 507 (1967).
 - [6] C. W. Bernard, A. Duncan, J. LoSecco, and S. Weinberg, *Phys. Rev. D* **12**, 792 (1975).
 - [7] G. Ecker, J. Gasser, A. Pich, and E. de Rafael, *Nucl. Phys. B* **321**, 311 (1989).
 - [8] A. Dhar and S.R. Wadia, *Phys. Rev. Lett.* **52**, 959 (1984).
 - [9] A. Hosaka, *Phys. Lett. B* **244**, 363 (1990).
 - [10] M. Wingate, T. A. DeGrand, S. Collins, and U. M. Heller, *Phys. Rev. Lett.* **74**, 4596 (1995).
 - [11] M. Bando, T. Kugo, and K. Yamawaki, *Phys. Rep.* **164**, 217 (1988); N. Kaiser and U.G. Meissner, *Nucl. Phys. A* **519**, 671 (1990).
 - [12] T. Sakai and S. Sugimoto, *Prog. Theor. Phys.* **113**, 843 (2005); **114**, 1083 (2005).
 - [13] K. Nawa, H. Suganuma, and T. Kojo, *Phys. Rev. D* **75**, 086003 (2007).
 - [14] L. Roca, E. Oset, and J. Singh, *Phys. Rev. D* **72**, 014002 (2005).
 - [15] M. F. M. Lutz and E. E. Kolomeitsev, *Nucl. Phys. A* **730**, 392 (2004).
 - [16] L. Roca, A. Hosaka, and E. Oset, *Phys. Lett. B* **658**, 17 (2007); H. Nagahiro, L. Roca, and E. Oset, *Phys. Rev. D* **77**, 034017 (2008); H. Nagahiro, L. Roca, A. Hosaka, and E. Oset, *Phys. Rev. D* **79**, 014015 (2009).
 - [17] M. Wagner and S. Leupold, *Phys. Rev. D* **78**, 053001 (2008).
 - [18] D. Gomez Dumm, A. Pich, and J. Portoles, *Phys. Rev. D* **69**, 073002 (2004); D. Dumm, P. Roig, A. Pich, and J. Portoles, *Phys. Lett. B* **685**, 158 (2010).
 - [19] E. Witten, *Nucl. Phys. B* **160**, 57 (1979).
 - [20] R.L. Jaffe, *AIP Conf. Proc.* **964**, 1 (2007).
 - [21] J. R. Peláez and G. Ríos, *Acta Phys. Pol. B* **2**, 215 (2009).
 - [22] D. Lurie and A.J. Macfarlane, *Phys. Rev.* **136**, B816 (1964); D. Lurie, *Particle and Fields* (Interscience Publishers, New York, 1968).
 - [23] M. Doring, C. Hanhart, F. Huang, S. Krewald, and U.G. Meissner, *Phys. Lett. B* **681**, 26 (2009).
 - [24] L. S. Geng, E. Oset, J. R. Pelaez, and L. Roca, *Eur. Phys. J. A* **39**, 81 (2009).
 - [25] K. Nawa, H. Nagahiro, S. Ozaki, D. Jido, and A. Hosaka (unpublished).