

Hidden conformal symmetry of a rotating black hole with four charges

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Kerr/CFT correspondence exhibits many remarkable connections between the near-horizon Kerr black hole and a conformal field theory (CFT). Recently, Castro, Maloney, and Strominger showed that a hidden conformal symmetry exists in the solution space of a Kerr black hole. In this paper we investigate a rotating black hole with four independent $U(1)$ charges derived from string theory which is known as the four-dimensional Cvetič-Youm solution, and we prove that the same hidden conformal symmetry also holds. We obtain the exact black hole entropy using the temperatures derived. The entropy and absorption cross section agree with the previous results [M. Cvetič and F. Larsen, *Nucl. Phys.* **B506**, 107 (1997).] and [M. Cvetič and F. Larsen, *J. High Energy Phys.* **09** (2009) 088.]. In addition, we clarify a previous explanation on the temperatures of the Cvetič-Youm solution's dual CFT. This work provides more robust derivation of the hidden conformal symmetry of Kerr-like black holes and as well as Kerr/CFT correspondence.

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I. INTRODUCTION

In [1], a duality between a rotating black hole and conformal field theory, known as Kerr/CFT correspondence, was shown by Guica, Hartman, Song, and Strominger. The derivation of Kerr/CFT correspondence depends highly on the parameters describing the black hole. To obtain this correspondence there should be at least one more parameter other than the black hole mass M . Lots of evidences show that black holes parametrized by more than one variable share this duality. The extension from a Kerr black hole to black holes parametrized by more than angular momentum J corresponds to the ideas in [2]. For more works on Kerr/CFT correspondence and its generalizations, see [3].

Recently, a hidden conformal symmetry of a Kerr black hole was found by Castro, Maloney, and Strominger [4]. Through inspection on the equation of motion of a test particle propagating in the Kerr black hole background, a $SL(2, \mathbb{R})_L \times SL(2, \mathbb{R})_R$ symmetry was found in the radial part of the solution. Although this symmetry is broken by angular coordinate identification, the correct entropy and absorption cross section can still be given by the temperatures T_L and T_R , which implies Kerr/CFT correspondence. And it shows that the hidden conformal symmetry, although not globally defined, has provided enough information supporting the duality.

After this interesting evidence was found, several works considering different types of Kerr-like black holes have been done [5–9]. Without any violation, the hidden

conformal symmetry exists in all of these black holes, and the conformal field theory (CFT) interpretations all give the right results. In this paper, we consider the four-dimensional Cvetič-Youm solution, a four-dimensional rotating black hole with four independent charges derived from toroidally compactified string theory [10]. The details of this solution were given in [11,12]. We show that the hidden conformal symmetry still holds by the solution space of a black hole of this type, and the CFT description is correct. A comparison between the temperatures we derived and the previous ones is also given. From that we make a clarification about an interpretation in [11,12].

This paper is organized as follows. In Sec. II we give a description of the parametrization of a rotating black hole with four charges and the wave equation of a particle propagating in the black hole background. In Sec. III we prove the existence of $SL(2, \mathbb{R})_L \times SL(2, \mathbb{R})_R$ symmetry in the wave equation. In Sec. IV we give the CFT interpretation. Section V is for general discussions.

II. MASSLESS SCALAR WAVE EQUATION IN THE BLACK HOLE BACKGROUND

The Cvetič-Youm solution is a rotating black hole with four charges derived from toroidally compactified string theory [10–12]. It is parametrized by the mass M , angular momentum J , and four independent charges Q_i , $i = 1, 2, 3, 4$. We adopt the convention in [11,12] and use μ , δ_i , and l as the new parametrization. From this the old parameters are given, respectively, as

$$8G_4M = \frac{1}{2} \mu \sum_{i=1}^4 \cosh 2\delta_i, \quad (1)$$

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$$8G_4 Q_i = \frac{1}{2} \mu \sinh 2\delta_i, \quad i = 1, 2, 3, 4, \quad (2)$$

$$8G_4 J = \frac{1}{2} \mu l \left(\prod_{i=1}^4 \cosh \delta_i - \prod_{i=1}^4 \sinh \delta_i \right). \quad (3)$$

The gravitational coupling constant in four dimensions is $G_4 = \frac{1}{8}$, which corresponds to $(2\pi)^6(\alpha')^4/V_6$ in string units.

To check the conformal symmetry in the solution space, we set up a neutral, massless scalar particle propagating in the black hole background. The wave equation is given by the Klein-Gordon equation

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \Phi) = 0, \quad (4)$$

where the metric was given in [10]. This equation is more conveniently expressed in term of the dimensionless radial coordinate

$$x \equiv \frac{r - \frac{1}{2}(r_+ + r_-)}{r_+ - r_-}. \quad (5)$$

Here r is the radial coordinate, r_+ and r_- are radii of the outer and inner horizons. The horizons can be written as $x = \pm \frac{1}{2}$, respectively, and the asymptotic space corresponds to the large x region. In terms of μ , l , δ_i , the surface accelerations of two horizons κ_\pm and the angular velocity Ω are

$$\begin{aligned} \frac{1}{\kappa_\pm} &= \frac{\mu^2}{2\sqrt{\mu^2 - l^2}} \left(\prod_i \cosh \delta_i + \prod_i \sinh \delta_i \right) \\ &\pm \frac{1}{2} \mu \left(\prod_i \cosh \delta_i - \prod_i \sinh \delta_i \right), \end{aligned} \quad (6)$$

$$\frac{\Omega}{\kappa_+} = \frac{l}{\sqrt{\mu^2 - l^2}}, \quad (7)$$

and in spherical coordinates the wave function reads

$$\Phi \equiv \Phi_r(x) \chi(\theta) e^{-i\omega t + im\phi} = \Phi(x, \theta) e^{-i\omega t + im\phi}. \quad (8)$$

The wave equation can be written as [11,12]

$$\begin{aligned} \frac{\partial}{\partial x} \left(x^2 - \frac{1}{4} \right) \frac{\partial}{\partial x} \Phi(x, \theta) + \frac{1}{4} \left[x \Delta^2 \omega^2 + x M \Delta \omega^2 \right. \\ \left. - 4\tilde{\Lambda} \frac{1}{x - \frac{1}{2}} \left(\frac{\omega}{\kappa_+} - m \frac{\Omega}{\kappa_+} \right)^2 \right. \\ \left. - \frac{1}{x + \frac{1}{2}} \left(\frac{\omega}{\kappa_-} - m \frac{\Omega}{\kappa_+} \right)^2 \right] \Phi(x, \theta) = 0, \end{aligned} \quad (9)$$

where $\tilde{\Lambda}$ is the angular part of the equation

$$\begin{aligned} \tilde{\Lambda} &= -\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \sin \theta \frac{\partial}{\partial \theta} + \frac{m^2}{\sin^2 \theta} - \frac{1}{16} l^2 \omega^2 \cos^2 \theta \\ &- \frac{1}{16} \mu^2 \omega^2 \left(1 + \sum_{i < j} \cosh 2\delta_i \cosh 2\delta_j \right). \end{aligned} \quad (10)$$

Note that by taking the four charges to zero, which is realized by taking $\delta_i = 0$, the angular equation reduces to the form in Kerr's case.

To investigate the equation, we take the near-region limit introduced in [4]. The near region is where the conformal structure appears. When the wavelength of the test particle is large enough compared to the radius curvature

$$\omega M \ll 1, \quad (11)$$

the near region is defined as

$$r \ll \frac{1}{\omega}. \quad (12)$$

Note that the ‘‘near’’ should not be confused with the one in ‘‘near horizon.’’ Actually, the near region could be arbitrarily large, as discussed in [4]. By taking this near-region limit we can neglect the higher order ω^2 terms in (9), and therefore the angular part of the equation reduces to the Laplacian in spherical coordinates

$$\left[\frac{1}{\sin \theta} \partial_\theta (\sin \theta \partial_\theta) - \frac{m^2}{\sin^2 \theta} \right] \chi(\theta) = -\Lambda \chi(\theta),$$

with the constant

$$\Lambda = \tilde{j}(\tilde{j} + 1).$$

The radial part of Eq. (9) becomes

$$\begin{aligned} \frac{\partial}{\partial x} \left(x^2 - \frac{1}{4} \right) \frac{\partial}{\partial x} \Phi_r + \frac{1}{4} \left[\frac{1}{x - \frac{1}{2}} \left(\frac{\omega}{\kappa_+} - m \frac{\Omega}{\kappa_+} \right)^2 \right. \\ \left. - \frac{1}{x + \frac{1}{2}} \left(\frac{\omega}{\kappa_-} - m \frac{\Omega}{\kappa_+} \right)^2 \right] \Phi_r = \tilde{j}(\tilde{j} + 1) \Phi_r. \end{aligned} \quad (13)$$

One should notice that, after taking the near-region limit, although the expressions do not contain δ_i explicitly, the variables ω , Ω , κ_\pm , and r_\pm above are determined by parameters μ , l , and the four charge parameters δ_i . At this stage, the form of the equation is similar to the one in Kerr's case. By this observation, we use the formula in [4] to check the operator.

III. THE $SL(2, \mathbb{R})_L \times SL(2, \mathbb{R})_R$ SYMMETRY IN THE SOLUTION SPACE

Following Ref. [4], we reconstruct the operator in the above wave equation by showing that this operator could be viewed as the Casimir of a $SL(2, \mathbb{R})$ group. First, one should employ the conformal coordinates

$$\begin{aligned} \omega^+ &= \sqrt{\frac{2x-1}{2x+1}} e^{2\pi T_R \phi}, & \omega^- &= \sqrt{\frac{2x-1}{2x+1}} e^{2\lambda_L t + 2\pi T_L \phi}, \\ y &= \sqrt{\frac{2}{2x+1}} e^{\lambda_L t + \pi(T_L + T_R)\phi}, \end{aligned} \quad (14)$$

where

$$T_R = \frac{\kappa_+}{2\pi\Omega}, \quad (15)$$

$$T_L = \frac{\kappa_+(\kappa_- + \kappa_+)}{2\pi\Omega(\kappa_- - \kappa_+)}, \quad (16)$$

and

$$\lambda_L = \frac{\kappa_+\kappa_-}{\kappa_- - \kappa_+}. \quad (17)$$

These three variables contain the δ_i -dependence implicitly, and are different from the ones in a Kerr black hole. T_L and T_R are the temperatures of the dual CFT. Defining the left and right temperatures is the key point in the whole formula. The right decision which presents the hidden conformal symmetry will also match the CFT description, and vice versa. This is an important feature of the hidden conformal symmetry.

Then one can define the local vectors

$$H_1 = i\partial_+, \quad H_0 = i\left(\omega^+\partial_+ + \frac{1}{2}y\partial_y\right), \quad (18)$$

$$H_{-1} = i(\omega^+\partial_+ + \omega^+y\partial_y - y^2\partial_-),$$

and

$$\bar{H}_1 = i\partial_-, \quad \bar{H}_0 = i\left(\omega^-\partial_- + \frac{1}{2}y\partial_y\right), \quad (19)$$

$$\bar{H}_{-1} = i(\omega^-\partial_- + \omega^-y\partial_y - y^2\partial_+).$$

Clearly, they satisfy two sets of $SL(2, \mathbb{R})$ Lie algebra, respectively,

$$\begin{aligned} [H_0, H_{\pm 1}] &= \mp iH_{\pm 1}, & [H_{-1}, H_1] &= -2iH_0, \\ [\bar{H}_0, \bar{H}_{\pm 1}] &= \mp i\bar{H}_{\pm 1}, & [\bar{H}_{-1}, \bar{H}_1] &= -2i\bar{H}_0. \end{aligned} \quad (20)$$

The quadratic Casimir of this algebra is

$$\begin{aligned} H^2 &= \bar{H}^2 = -H_0^2 + \frac{1}{2}(H_1H_{-1} + H_{-1}H_1) \\ &= \frac{1}{4}(y^2\partial_y^2 - y\partial_y) + y^2\partial_+\partial_-, \end{aligned} \quad (21)$$

which we identify as the operator that appears in the left-hand side of Eq. (13). To see this, one needs to rewrite the variables in terms of the (t, x, ϕ) coordinates. In our case the vector fields are

$$\begin{aligned} H_1 &= \frac{ie^{-2\pi T_R\phi}\sqrt{4x^2-1}}{2}\partial_x + \frac{ie^{-2\pi T_R\phi}x}{\pi T_R\sqrt{4x^2-1}}\partial_\phi \\ &\quad - \frac{ie^{-2\pi T_R\phi}(T_R + 2T_Lx)}{2T_R\sqrt{4x^2-1}\lambda_L}\partial_t, \\ H_0 &= \frac{i(\lambda_L\partial_\phi - \pi T_L\partial_t)}{2\pi T_R\lambda_L}, \\ H_{-1} &= -\frac{ie^{2\pi T_R\phi}\sqrt{4x^2-1}}{2}\partial_x + \frac{ie^{2\pi T_R\phi}x}{\pi T_R\sqrt{4x^2-1}}\partial_\phi \\ &\quad - \frac{ie^{2\pi T_R\phi}(T_R + 2T_Lx)}{2T_R\sqrt{4x^2-1}\lambda_L}\partial_t, \end{aligned} \quad (22)$$

and the antiholomorphic part is

$$\begin{aligned} \bar{H}_1 &= \frac{ie^{-2(\pi T_L\phi + \lambda_L t)}\sqrt{4x^2-1}}{2}\partial_x - \frac{ie^{-2(\pi T_L\phi + \lambda_L t)}}{2\pi\sqrt{4x^2-1}T_R}\partial_\phi \\ &\quad + \frac{ie^{-2(\pi T_L\phi + \lambda_L t)}(T_L + 2xT_R)}{2\sqrt{4x^2-1}T_R\lambda_L}\partial_t, \\ \bar{H}_0 &= \frac{i}{2\lambda_L}\partial_t, \\ \bar{H}_{-1} &= -\frac{ie^{2\pi T_L\phi + 2\lambda_L t}\sqrt{4x^2-1}}{2}\partial_x - \frac{ie^{2\pi T_L\phi + 2\lambda_L t}}{2\pi\sqrt{4x^2-1}T_R}\partial_\phi \\ &\quad + \frac{ie^{2\pi T_L\phi + 2\lambda_L t}(T_L + 2xT_R)}{2\sqrt{4x^2-1}T_R\lambda_L}\partial_t. \end{aligned} \quad (23)$$

Thus here H^2 on the left-hand side of the wave equation is exactly the $SL(2, \mathbb{R})$

$$\begin{aligned} H^2 &= -\frac{1}{4\pi^2 T_R^2 (4x^2 - 1)}\partial_\phi^2 - \frac{T_L^2 + T_R^2 + 4T_L T_R x}{4T_R^2 (4x^2 - 1)\lambda_L^2}\partial_t^2 \\ &\quad + \frac{T_L + 2T_R x}{2\pi T_R^2 \lambda_L (4x^2 - 1)}\partial_t\partial_\phi + \left(x^2 - \frac{1}{4}\right)\partial_x^2 + 2x\partial_x. \end{aligned} \quad (24)$$

The original scalar field wave equation (13) can be rewritten as

$$H^2\Phi = \bar{H}^2\Phi = \tilde{j}(\tilde{j} + 1)\Phi. \quad (25)$$

By writing the wave equation we see that the hidden conformal symmetry is now manifest. The vectors $\{H_{0,\pm 1}\}_L$ and $\{\bar{H}_{0,\pm 1}\}_R$ together generate a $SL(2, \mathbb{R})_L \times SL(2, \mathbb{R})_R$ algebra. The weights of the fields are then

$$h_L = \tilde{j}, \quad h_R = \tilde{j}. \quad (26)$$

IV. CFT INTERPRETATION OF T_L AND T_R

A. Temperature and entropy

Having proven that, after taking the near-region limit (12), the operator in the radial part of wave equation can be identified as the Casimir of a $SL(2, \mathbb{R})_L \times SL(2, \mathbb{R})_R$ algebra. A reasonable next step is to consider that the near region of the black hole should be dual to a (T_L, T_R) finite temperature state in a CFT. As an examination, we calculate the black hole's entropy microscopically using the Cardy formula,

$$S = \frac{\pi^2}{3}(c_L T_L + c_R T_R). \quad (27)$$

However, the $SL(2, \mathbb{R})_L \times SL(2, \mathbb{R})_R$ is a Virasoro algebra without central charge. This situation is the same as the case of Kerr black hole. In the original Kerr/CFT correspondence, the central charges are derived in the near-horizon region of an extremal Kerr black hole [1]. In the same sense as in [4], we conjecture that the central charges will keep valid both from an extreme black hole to a

nonextreme one, and from near horizon to the near region defined above. Based on that, we will use the central charges derived previously and temperatures obtained in this paper. For extreme Kerr, from the asymptotic symmetry group the central charges are given as

$$c_R = c_L = 12J. \quad (28)$$

Cooperating with the temperatures given in (15) and (16), from the Cardy formula the entropy of the black hole is given as

$$S = \frac{\pi^2}{3}(c_L T_L + c_R T_R) = 4\pi J \left(\frac{\kappa_+}{\Omega} \right) \left(\frac{\kappa_-}{\kappa_- - \kappa_+} \right). \quad (29)$$

Using (6) and (7) we have

$$S = \frac{2\pi}{8G_4} \left[\frac{1}{2} \mu^2 \left(\prod_i \cosh \delta_i + \prod_i \sinh \delta_i \right) + \frac{1}{2} \mu \sqrt{\mu^2 - l^2} \left(\prod_i \cosh \delta_i - \prod_i \sinh \delta_i \right) \right]. \quad (30)$$

According to [11,12], the Bekenstein-Hawking entropy of Cvetič-Youm solution is

$$S = \frac{A}{4G_N} = 2\pi \left[\frac{1}{2} \mu^2 \left(\prod_i \cosh \delta_i + \prod_i \sinh \delta_i \right) + \frac{1}{2} \mu \sqrt{\mu^2 - l^2} \left(\prod_i \cosh \delta_i - \prod_i \sinh \delta_i \right) \right]. \quad (31)$$

Apparently, the two entropies are equal to each other after setting $G_4 = \frac{1}{8}$. Thus as we promised, T_L and T_R indeed give the correct entropy, and the near region of the black hole should be dual to a CFT.

Additionally, we would like to point out that there is an interesting comparison between the L and R temperature defined in [11,12] and the T_L and T_R we used in this paper. These two pairs of temperatures differ by a factor \mathcal{R}_4 . T_L and T_R in this paper emerge as a direct result of the hidden conformal symmetry, and they match the Cardy formula perfectly. In [12], to get the correct entropy microscopically, the factor

$$\mathcal{R}_4 = \mu \left(\prod_{i=1}^4 \cosh \delta_i + \prod_{i=1}^4 \sinh \delta_i \right)$$

is embedded into the Cardy formula. In their paper [12], Cvetič and Larsen considered the result derived in this way as a phenomenological model describing the black hole. Based on the result we interpret it in another way: by putting this factor \mathcal{R}_4 into the L and R temperature instead, they become the exact T_L and T_R in this paper, and then the Cardy formula will make sense without any modification. Thus, if the Cardy formula gives the correct entropy, then

the temperatures will imply the hidden conformal symmetry.

B. Absorption probabilities

Next we analyze the black hole's absorption probabilities for a massless neutral scalar in the near region. A systematic way to exam the CFT interpretation is to check the behavior of the test particle's solution [4]. In our case, it is

$$\begin{aligned} \Phi_0^{\text{in}} &= \left(\frac{x - \frac{1}{2}}{x + \frac{1}{2}} \right)^{-(i\beta_H(\omega - m\Omega)/4\pi)} \left(x + \frac{1}{2} \right)^{-1-\tilde{j}} \\ &\times F \left(1 + \tilde{j} - i \frac{\beta_R \omega - 2\beta_H m \Omega}{4\pi}, 1 + \tilde{j} - i \frac{\beta_L \omega}{4\pi}, 1 - i \frac{\beta_H}{4\pi} (\omega - m\Omega), \frac{x - \frac{1}{2}}{x - \frac{1}{2}} \right). \end{aligned} \quad (32)$$

Here the symbols above are defined as

$$\beta_H = \frac{2\pi}{\kappa_+}, \quad \beta_R = \frac{2\pi}{\kappa_+} + \frac{2\pi}{\kappa_-}, \quad \beta_L = \frac{2\pi}{\kappa_+} - \frac{2\pi}{\kappa_-}. \quad (33)$$

From [11,12], the asymptotic form of Φ is

$$\begin{aligned} \Phi_0^{\text{in}} &\sim Ax^{\tilde{j}} + Bx^{-1-\tilde{j}} \\ &\sim x^{-1-\tilde{j}} \frac{\Gamma(1 - i\frac{\beta_H(\omega - m\Omega)}{2\pi})\Gamma(-1 - 2\tilde{j})}{\Gamma(\tilde{j} - i\frac{\beta_L \omega}{2\pi})\Gamma(\tilde{j} - i\frac{\beta_R \omega - 2\beta_H m \Omega}{2\pi})} \\ &+ x^{\tilde{j}} \frac{\Gamma(1 - i\frac{\beta_H(\omega - m\Omega)}{2\pi})\Gamma(1 + 2\tilde{j})}{\Gamma(1 + \tilde{j} - i\frac{\beta_L \omega}{2\pi})\Gamma(1 + \tilde{j} - i\frac{\beta_R \omega - 2\beta_H m \Omega}{2\pi})}. \end{aligned} \quad (34)$$

At the outer boundary of the matching region

$$\Phi_0^{\text{in}} \sim x^{\tilde{j}} \frac{\Gamma(1 - i\frac{\beta_H(\omega - m\Omega)}{2\pi})\Gamma(1 + 2\tilde{j})}{\Gamma(1 + \tilde{j} - i\frac{\beta_L \omega}{2\pi})\Gamma(1 + \tilde{j} - i\frac{\beta_R \omega - 2\beta_H m \Omega}{2\pi})}, \quad (35)$$

and thus, from properties of Γ function we obtain

$$P_{\text{abs}} \sim |A|^{-2} \sim \sinh \left(\frac{\beta_H(\omega - m\Omega)}{2\pi} \right) \left| \Gamma \left(1 + \tilde{j} - i\frac{\beta_L \omega}{2\pi} \right) \right|^2 \times \left| \Gamma \left(1 + \tilde{j} - i\frac{\beta_R \omega - 2\beta_H m \Omega}{2\pi} \right) \right|^2. \quad (36)$$

From the first law of thermodynamics

$$T_H \delta S = \delta M - \Omega \delta J, \quad (37)$$

we wish to find the conjugate charges δE_R and δE_L such that

$$\delta S = \frac{\delta E_L}{T_L} + \frac{\delta E_R}{T_R}. \quad (38)$$

With the identification chosen as

$$\delta M = \omega, \quad \delta J = m,$$

the solution is

$$\delta E_L = \frac{\kappa_- + \kappa_+}{2\Omega\kappa_-} \delta M, \quad \delta E_R = \frac{\kappa_- - \kappa_+}{2\Omega\kappa_-} \delta M - \delta J.$$

We chose the identification of left and right frequencies as

$$\delta E_L \equiv \omega_L, \quad \delta E_R \equiv \omega_R, \quad (39)$$

and get the result

$$\omega_L = \frac{\omega}{2\Omega} \frac{\kappa_- + \kappa_+}{\kappa_-}, \quad \omega_R = \frac{\omega}{2\Omega} \frac{\kappa_- - \kappa_+}{\kappa_-} - m. \quad (40)$$

Putting these back into (36), the expression of absorption probability turns out to be

$$P_{\text{abs}} \sim T_L^{2h_L-1} T_R^{2h_R-1} \sinh\left(\frac{\omega_L}{2T_L} + \frac{\omega_R}{2T_R}\right) \times \left| \Gamma\left(h_L + \frac{\omega_L}{2\pi T_L}\right) \right|^2 \left| \Gamma\left(h_R + \frac{\omega_R}{2\pi T_R}\right) \right|^2, \quad (41)$$

which is exactly the form of finite temperature cross section for CFT.

$$\left[\frac{1}{\sin\theta} \frac{d}{d\theta} \left(\sin\theta \frac{d}{d\theta} \right) + \left(\Lambda_{lm}^s - a^2 \omega^2 \sin^2\theta - 2a\omega s \cos\theta - \frac{m^2 + s^2 + 2ms \cos\theta}{\sin^2\theta} \right) \right] \chi^s(\theta) = 0$$

$$\left[\Delta^{-s} \frac{d}{dr} \left(\Delta^{s+1} \frac{d}{dr} \right) + \left(\frac{H^2 - 2is(r-M)H}{\Delta} + 4is\omega r + 2am\omega + s(s+1) - \Lambda_{lm}^s \right) \right] \Phi^s(r) = 0,$$

where Δ and H are determined by the concrete black hole configuration, and s is the spin of the test particle. One can find the condition that $s \neq 0$ will make the radial part more complicated even after neglecting the ω^2 terms by the near-region limit. Modification of the conformal coordinates seems to be a solution, and we leave this to future works. One would also consider the case involving the cosmology constant. Discussions on multiple-charge rotating black hole in anti-de Sitter (AdS) space were given by [14]. Searching the hidden conformal symmetry in these Kerr-like AdS black holes' backgrounds could be another interesting topic waiting to be explored. Results of Kerr-Newman AdS have been given in [8].

On the other hand, as for the near region, it is actually not so near. According to the holographic principle, the black hole's dual field theory should be constructed on the horizon. Thus it is interesting to ask what will happen if we

V. DISCUSSION

In this paper, we prove the existence of the hidden conformal symmetry of a rotating black hole with four charges known as the four-dimensional Cvetič-Youm solution. The duality to a conformal field theory is also discussed, with the calculations on entropy and absorption cross section being the same as previous results. The only particle type we consider in this paper is a scalar, which has a relatively simple wave function. If the formula used to derive the hidden conformal symmetry is consistent, it should be also valid in the cases when higher spin particles are taken into consideration. A calculation on the correlator of photons and gravitons has been given in [6], while evidences more than these are expected. The key to show the hidden conformal symmetry is to construct the operator in the wave function as a Casimir induced by the conformal coordinates. Wave functions of higher spin particles in a black hole background were studied many years ago [13]. These works show that, in general the angular and radial part satisfies the Teukolsky function,

applied more (or different) constraints to distinguish the regions (which give rise to various limits we can apply on the wave function), and whether symmetries would appear in these regions.

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