PHYSICAL REVIEW D **83,** 106007 (2011)

Gauge invariant coupling of fields to torsion: A string inspired model

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(Received 19 January 2011; published 16 May 2011)

In a consistent heterotic string theory, the Kalb-Ramond field, which is the source of space-time torsion, is augmented by Yang-Mills and gravitational Chern-Simons terms. When compactified to 4 dimensions and in the field theory limit, such additional terms give rise to interactions with interesting astrophysical predictions like rotation of plane of polarization for electromagnetic and gravitational waves. On the other hand, if one is also interested in coupling 2- or 3-form (Abelian or non-Abelian) gauge fields to torsion, one needs another class of interaction. In this paper, we shall study this interaction and offer some astrophysical and cosmological predictions. We explicitly calculate the Coleman-Weinberg potential for this theory. We also comment on the possibility of such terms in loop quantum gravity where, if the Barbero-Immirzi parameter is promoted to a field, acts as a source for torsion.

DOI: 10.1103/PhysRevD.83.106007 PACS numbers: 11.25.Wx, 04.60.Bc, 04.60.Cf

I. INTRODUCTION

The low energy physics of particle interactions is satisfactorily described by the standard model and general relativity. At higher energies available at the early universe or at astrophysical processes, it is expected that new degrees of freedom will emerge to play important role. Otherwise inaccessible at the present energy scale, these fields might interact with degrees of freedom of the standard model leading to some interesting theoretical predictions and observational signatures. Since string theory is a candidate for a unified description of field interactions even up to the Planck scale, we envisage that nature and the specific form of interaction of new fields with known degrees of freedom can be extracted from this theory in an unambiguous way. In this paper, we shall look for gauge invariant interactions of gauge fields (electromagnetic, gravitational and 2- and 3-form gauge fields) to torsion. In string theory, since the Kalb-Ramond (KR) field acts as a source of torsion, we shall have a look at possible gauge and gravitational interactions of a this KR field. The KR field is generic to any closed string spectrum but is *not* a degree of freedom of the standard model. One can anticipate that any observational effect involving the KR field, obtained using standard fields as probes, is then a window into the otherwise inaccessible world of very high energy physics supposedly predicted by string theories. On the other hand, loop quantum gravity (LQG) is also a candidate for quantum theory of gravity. In LQG, the Barbero-Immirzi parameter is a one-parameter ambiguity which describes various topological sectors. This parameter also comes up in the area spectrum and consequently in entropy of black holes, wherefrom its value is ascertained by comparing with the Bekenstein-Hawking entropy formula. If the Barbero-Immirzi parameter is promoted to a field, it acts as a source for torsion. It is then interesting to compare and contrast various interactions of fields with these two sources of torsion that arise in these two theories of quantum gravity. Since there are observational implications, the issue is even more satisfying.

In the context of the heterotic string theory, electromagnetic and gravitational interactions of KR fields arise quite naturally from the requirements of consistency. As is well known [1], the $(E_8 \otimes E_8)$ or $SO(32)$ heterotic strings are two anomaly-free gauge groups which can be coupled to $N = 1$ supergravity in 10 dimensions. Anomaly cancellation (the Green-Schwarz mechanism) requires that the KR 3-form field-strength is augmented by addition of $(E_8 \otimes E_8)$ Yang-Mills Chern-Simons 3-form and local Lorentz Chern-Simons 3-form [1]. This augmentation induces electromagnetic and gravitational interactions of the KR field which lead to potentially interesting physical effects showing up in the Maxwell and Einstein equations, when the theory is compactified to four dimensions. The electromagnetic effect mainly comprises a rotation of the polarization plane of electromagnetic waves from large red-shift sources, upon scattering from a homogeneous KR background [2–6]. This rotation is independent of the wavelength of the electromagnetic wave and cannot be explained by the Faraday effect, where the plane of polarization of the electromagnetic wave rotates depending quadratically on the wavelength while passing through some magnetized plasma. Similarly, the gravitational interaction leads to the result that the plane of polarization of gravitational waves rotates through an angle that is proportional to (a power of) the KR field-strength component [7]. Predictions of this kind can then be useful if some

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deviations from the traditional expectations are observed. For example, such interactions have been studied within the framework of the five-dimensional Randall-Sundrum braneworld model. When compactified to four dimensions, they lead to huge deviations from the expected results [7–11], which can be used to put bounds on the various parameters in the theory [12–14]. On the other hand, if the predictions are nonobservable, they lead to upper bounds on the presence of new fields which is important in our search for new theories and their couplings. To exemplify, in the present case of rotation of plane of polarization of electromagnetic waves, the magnitude of the effect is sensitive to the dimensional compactification of the underlying theory. For toroidal compactification (as well as for the Calabi-Yau compactification) of the theory (in the zero slope limit), the predicted rotation is proportional to the appropriate KR field-strength component (scaled by the inverse scale factor in a Friedmann universe), so that bounds on the observed rotation translate into a stringent upper bound on the size of the KR field-strength component. Moreover, if one uses the bounds on the KR field-strength obtained from the cosmic optical activity, the order of magnitude of the similar effect for gravitational waves can be calculated.

The interactions which give rise to the above-mentioned predictions arise very naturally in string theory and they have been well studied. Interestingly, one can also conceive of another class of interactions which has not been discussed in this context except for in [15,16], where only the electromagnetic interaction was considered. In this paper, we shall extend the study to non-Abelian gauge fields and discuss the effects of these possible new interactions in detail. Let us discuss the motivation for introducing such structures in brief (details will be in Sec. II). The issue originally arose during the study of Einstein-Cartan space-time. The idea was to construct a gauge invariant coupling of electromagnetic field (A_μ) to torsion—which is another geometrical property of the Einstein-Cartan space-time along with the metric. The field-strength ($F_{\mu\nu}$) for such a space-time also depends on torsion [17]. However, because the torsion does not have a transformation under $U(1)$ gauge transformation, the electromagnetic field-strength is not gauge invariant. This is dissatisfactory since we expect that field-strengths must be measurable even in space-times with torsion. This requirement on the field-strength demands that the torsion must also stay invariant under $U(1)$ gauge transformation. This situation implies that there is a nongravitational field, possibly massless, to function as the source of the torsion [2]. Since that field must be bosonic, one can opt for the KR antisymmetric second-rank tensor field $B_{\mu\nu}$ as a possible candidate. $B_{\mu\nu}$, being a massless antisymmetric field, is expected to be a gauge connection, as indeed it is, with the following gauge transformation $\delta_\lambda B_{\mu\nu} = \partial_{[\mu}\lambda_{\nu]}$, and this leaves its field-strength $H_{\mu\nu\lambda}$ gauge invariant. Moreover, for anomaly-free quantum theory, $H_{\mu\nu\lambda}$ must be modified with the addition of an electromagnetic Chern-Simons three-tensor and if $B_{\mu\nu}$ is endowed with a nontrivial electromagnetic gauge transformation along with Kalb-Ramond gauge transformations, the KR field-strength remains invariant under $U(1)$ gauge transformation. This is precisely what was needed: the torsion field is gauge invariant. Interactions of this type give rise to interactions in the form of rotation of plane of polarization of electromagnetic (and gravitational) waves as discussed in the previous paragraph.

What if one wants to couple a 2-form or a 3-form gauge field to torsion? Such fields arise in the perturbative and nonperturbative sector (D-branes) of string theory compactified to four dimensions and in supergravity. Again, field-strengths for such higher-rank tensor fields are also not invariant under their respective gauge transformations in presence of space-time torsion. Once we take the KR field as a source for torsion, there is a possible way out. We again demand the field-strengths of 2-form or 3-form gauge fields to be observable so that one again has to modify $H_{\mu\nu\lambda}$, but in a peculiar way. This extra term, instead being of the form $A \wedge F$ for the ($U(1)$) case above, is $A \wedge {}^*F$, where * denotes the Hodge-dual and A is a 1-, 2- or 3-form field. Again, if the field $B_{\mu\nu}$ has a nontrivial transformation under the gauge transformation of the form fields, its field-strength ($H_{\mu\nu\lambda}$) and hence torsion remain invariant under gauge transformations, as required (for this case, we shall work in order $O(\sqrt{G})$). It is also interesting to note that addition of such terms ($A \wedge {}^*F$) not only works for 2- and 3-form fields, but also for a 1-form field. Moreover, one gets an additional set of interaction for the electromagnetic fields and $H_{\mu\nu\lambda}$ field with observable consequences. These issues were first discussed in [15] and a possible embedding of such terms in $N = 1$ supersysymmetric theory was discussed in [16]. We should however point out that we do not attempt to derive these new interactions from any string theory but merely point to the possibility of such terms from the requirements of gauge invariance. One finds more evidence for existence of such terms: the gravitational counterpart of this new interaction contributes the Euler invariant to the effective action in four dimensions, which is well known in gravity and supergravity theories to come from stress-tensor anomaly in curved space-time [18–20]. In the sense of effective field theory [21], which does not assume any precise details of the fundamental interactions or short distance degrees of freedom, such kinds of terms are ubiquitous and lead to quantum corrections for general relativity. In the appendix of this paper we show one possible origin of such terms and in the discussion we shall detail our justification for such extra interactions from the effective field theory point of view. In this paper, we shall extend the formalism of [15] for non-Abelian gauge fields and also for gravity waves and look for observational predictions. Interestingly, because of the presence of the Hodge-dual, interactions of the latter kind violate spatial parity. With the cosmic microwave background (CMB) data and the Planck data available, it might be interesting to look for such ideas now. Indeed, observational implications of such terms have already been discussed [22–26], though the possible origin of terms have not been discussed in these papers and the coupling constant for such interactions are usually not pinned down. Moreover, it has been argued that in the presence of such new *anomalous* terms, the cosmological constant becomes dynamical and leads to changes in the CMB bi/trispectrum compared to simple inflationary models [27]. It is for this reason that we shall also study the effective potential, which is the first step in a detailed study of inflatory models. The Coleman-Weinberg potential calculated in Sec. V leads to the precise form of the potential from which many parameters can be compared and constrained from the known data.

The interest in LQG for such interactions and consequently its relation or differences with string theory/supergravity is due to some recent studies [28–34]. These papers deal with the consequences of promoting the Barbero-Immirzi (BI) parameter to a field. It turns out that the derivative of the BI field is the source for torsion. Moreover, since the BI field is pseudoscalar,[1] it is natural to compare and contrast this BI field with the axion [29]. If the BI field is an axion, its derivative is dual to the $H_{\mu\nu\lambda}$ field alluded to above and such fields might have interactions with electromagnetic and gravitational fields in the way very similar to the one discussed above in the context of string theory. We shall discuss these issues in detail below and point out to some observational implications.

The plan of the paper is as follows: In Sec. II, we discuss the gauge invariant coupling of various form fields to torsion and show how this can be achieved with special reference to electromagnetism and gravity. In Sec. III, we shall review the consequences of such interaction for the Maxwell fields and extend them to gravity in the next section. In Sec. V, we compute the quantum effective potential (Coleman-Weinberg potential) [35] for a theory of gravity by including the modified interactions. We will see that inclusion of a parity-violating scalar field (the BI field/axion) does not have any effect in the one-loop effective potential of a theory where higher curvature terms are present. We conclude in Sec. VI.

II. GAUGE INVARIANT INTERACTIONS OF FIELDS WITH TORSION

In the standard Einstein-Maxwell theory, the electromagnetic field-strength reduces to the flat space expression on account of the symmetric nature of the Christoffel connection. However, in the theory of gravity described by Einstein-Cartan theory, i.e. in the case where one has space-time torsion, the situation changes quite drastically, because the electromagnetic field-strength is no longer gauge invariant [17]. Indeed, it is easy to see that

$$F_{\mu\nu} = \partial_{[\mu}A_{\nu]} - T_{\mu\nu}{}^{\rho}A_{\rho}, \tag{1}$$

where $T_{\mu\nu}{}^{\rho}A_{\rho}$ is the torsion (antisymmetric combination of the Christoffel connection), is obviously not invariant under $U(1)$ gauge transformation $\delta_\lambda A = d\lambda$, λ being the gauge function. Since $F_{\mu\nu}$ and any field-strengths must be measurable quantities even in a curved space-time with torsion, the torsion tensor, a purely geometric quantity like curvature must also be gauge invariant. However, this implies that one must also have another geometrical quantity which might compensate for the loss of gauge invariance due to torsion. In absence of such compensating fields, it is natural to look for nongravitational fields to act as a source for torsion [2]. In the context of string theory, the Kalb-Ramond (KR) field seems to be an ideal candidate source [2]. Indeed, it also has all the desired gauge transformation properties required of torsion.

In this section, we shall first review the basic facts about the KR field as is known from string theory with special emphasis on its gauge transformation properties. The KR field is characterized by a 2-form potential B which has a 3-form field-strength $H \equiv dB$; the field-strength is invariant under the KR gauge transformation $\delta_{\bar{\lambda}}B = d\bar{\lambda}$, where $\bar{\lambda}$ is a 1-form gauge parameter. Immediately, one obtains the Bianchi identity for the KR field:

$$dH = 0 \tag{2}$$

In four-dimensional space-time, the free KR action is given by

$$S_H = \int_{\mathcal{M}_4} H \wedge {}^*H, \tag{3}$$

where ${}^*H$ is the Hodge-dual of the field-strength H. Varying this action with regard to B yields the KR field equation

$$d^*H = 0, \tag{4}$$

which has the local solution

$${}^*H = d\Phi_H, \tag{5}$$

where Φ_H is a scalar. Substituting this in Eq. (4)the one obtains for the field Φ_H

$$d^*d\Phi_H = 0. \tag{6}$$

Thus, on shell the Bianchi identity for the field B is the equation of motion for its Hodge-dual field. This is not surprising and is a feature of all Hodge-dual related fields.

Let us now point to the string theory connection. B occurs in the massless spectrum of the free string in 10-dimensional heterotic string theory. In the zero slope limit, this theory reduces to 10-dimensional $N = 1$ supergravity coupled to $N = 1 E_8 \otimes E_8$ super-Yang-Mills

[1]The expression for area spectrum in LQG depends on the BI parameter and as such must be a pseudoscalar for a well-defined transformation property of the area element.

theory. The requirement of 10-dimensional supersymmetry and that the quantum theory be free of all anomalies implies that the KR field-strength H be augmented as [1]

$$H = dB - \frac{1}{M_P}(\Omega_{\rm YM} - \Omega_L), \tag{7}$$

where

$$\Omega_{\rm YM} \equiv {\rm tr}\Big(A \wedge dA + \frac{2}{3} g A \wedge A \wedge A\Big) \tag{8}$$

is the Yang-Mills Chern-Simons 3-form with A the gauge connection 1-form and M_P is the Planck mass in four-dimensional space-time. Ω_L is the gravitational Chern-Simons 3-form obtained by replacing the Yang-Mills gauge connection A by the spin-connection 1-form ω, and the trace is taken over the local Lorentz indices. The augmentation in Eq. (7) has important consequences. The field H, being a field-strength, must remain gauge invariant under both Yang-Mills gauge transformations and under local Lorentz transformations. This implies that B must now transform nontrivially under both gauge transformations in spite of B being neutral. To simplify and to set the notations for the remaining part of the paper, let us say that the gauge field A is $U(1)$ valued. Then, the transformation of A is given by

$$\delta_\lambda A = d\lambda, \tag{9}$$

where, λ is the gauge parameter. The Chern-Simons term now only contains $A \wedge dA$. We shall now denote $\Omega_{\rm YM}$ by $\Omega_{\rm EM}$ and this term varies as

$$\delta_\lambda \Omega_{\rm EM} = d\lambda \wedge dA. \tag{10}$$

Thus, to achieve gauge invariance for the H field, the transformation law for B should include the 2-form in (10) so that under Yang-Mills gauge transformation

$$\delta_\lambda B = -\frac{1}{M_P}(\lambda dA). \tag{11}$$

Also, the gravitational field in the vielbein formalism can be treated very similarly to the Yang-Mills field. Specifically, the Yang-Mills potential A is analogous to the spin-connection 1-form ω_{AB}, where A, B are Lorentz indices. Under an infinitesimal Lorentz transformation with parameters given by an $SO(D-1, 1)$ matrix Θ, the transformation of ω is

$$\delta_L \omega = d\Theta + [\omega, \Theta], \tag{12}$$

The Lorentz Chern-Simons term varies as

$$\delta_L \Omega_L = {\rm tr}(d\Theta \wedge d\omega). \tag{13}$$

Similar to the argument above, transformation law for B should include the 2-form in (13) so that under Lorentz transformation

$$\delta_L B = -\frac{1}{M_P}\,{\rm tr}(\Theta d\omega). \tag{14}$$

Retaining the form of the KR action (3), it follows that the KR field equation does not change. Therefore, ${}^*H$ still has the local solution (5). However, the KR Bianchi identity certainly changes, leading to

$$d^* d\Phi_H = \frac{1}{M_P}\,{\rm tr}(F \wedge F - R \wedge R), \tag{15}$$

where $F(R)$ is the Yang-Mills (space-time) curvature 2-form. The Yang-Mills and Einstein equations change nontrivially. We shall consider these below in special situations, viz., the Maxwell part of the gauge interaction and linearized gravity.

This scenario works well for 1-form gauge fields. How about if we want a gauge invariant coupling of higher form fields to torsion? In [15], it was proposed that one needs additional terms to be augmented to the KR field-strength. For $U(1)$ gauge fields, it was proposed that an additional augmentation to H in the form of $M_P^{-1}(A \wedge {}^*F)$ is needed. But again, such an addition is not $U(1)$ gauge invariant. One needs to go further and we propose that the additional augmentation to be

$$H \to H + \frac{1}{M_P}(A \wedge {}^*F + \lambda d^* F) \tag{16}$$

The argument is obviously not based on any requirements arising from string theory and it is not known if one can embed such an interaction in any string theory. However, we shall discuss in Sec. VI how, in effective field theories, such terms are generic and lead to macroscopically observable results. Since effective field theories do not assume any precise details of microscopic inetraction, we expect such terms to exist in string theory or in its low energy effective action. In the appendix, we indicate the origin of such terms from a different perspective. It is also clear that in presence of such terms, the gauge transformation of B field changes from that obtained in Eq. (11) [2]:

$$\delta_\lambda B = -\frac{1}{M_P}(\lambda F + \lambda^* F). \tag{17}$$

We can also proceed further and add to Eq. (16) the spin-connection terms so that the augmentation takes the following form:

$$H \to H + \frac{\zeta}{M_P}(A \wedge {}^*F + \omega \wedge {}^*R), \tag{18}$$

where ζ is a parameter which takes values $+1$ or -1. We have introduced this parameter since we do not quite fix the coefficient. Now, instead of Eq. (15), the result of such

[2]An immediate consequence of this gauge transformation is that the $H_{\mu\nu\lambda}$ now can no longer be thought of as a parity eigenstate, and thus neither is its dual Φ_H. In other words, one can decompose $\Phi_H = \Phi_H^{(+)} + \Phi_H^{(-)}$ where $+$ indicates even parity and $-$ is for odd parity. However, we shall continue to use the generic term Φ_H for this field.

additional terms in Eq. (18) is (we consider terms only up to order M_P^{-1})

$$d^* d\Phi_H = \frac{1}{M_P}\,\mathrm{tr}(F\wedge F + \zeta F\wedge {}^*F - R\wedge R - \zeta R\wedge {}^*R). \tag{19}$$

In short, the upshot of the above analysis is that one can consider a gauge invariant action of the following form [2,3]:

$$S[g,T] = \int_{\mathcal{M}_4} d^4x\bigg[R(g,T) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}H_{\mu\nu\lambda}H^{\mu\nu\lambda} + T_{\mu\nu\lambda}H^{\mu\nu\lambda}\bigg], \tag{20}$$

where $H_{\mu\nu\lambda}$ is defined through Eq. (7) and the torsion tensor $T_{\mu\nu\lambda}$ is an auxiliary field satisfying the constraint $T_{\mu\nu\lambda} = H_{\mu\nu\lambda}$. Putting the local solution $H = -{}^*d\Phi_H$ from Eq. (5) in the action (20), we get the effective equation for the field Φ_H:

$$\begin{aligned} S[g,A,\Phi_H] &= \int_{\mathcal{M}_4} d^4x\bigg[R(g,T) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\partial_\mu\Phi_H\partial^\mu\Phi_H\bigg] \\ &\quad + \frac{\Phi_H}{M_P}(F\wedge F + \zeta F\wedge {}^*F - R\wedge R - \zeta R\wedge {}^*R), \end{aligned} \tag{21}$$

which is precisely the action for a pseudoscalar (Φ_H) coupled to gravity.[3] Note that contrary to the Maxwell fields, the gravitational interactions contribute higher derivative terms in the action. They are related to the gravitational axial current anomaly and stress-tensor anomaly respectively [18–20]. The equation of motion for this pseudoscalar is, however, given by Eq. (19). If the Barbero-Immirzi parameter is promoted to a field, the torsion is dual to the derivative of that pseudoscalar field (just like the Eq. (5)). In that case, one gets an effective action same as the first part of the action above [28,33]. In the following sections, we study the consequences of such interactions.

III. ELECTROMAGNETIC INTERACTIONS OF KR FIELD

In this section, we shall confine our study to the electromagnetic interactions of the KR field in four-dimensional Minkowski space-time. Let us first restrict ourselves to the interaction of the type $\Phi_H F_{\mu\nu}{}^*F^{\mu\nu}$. Observe that since the field Φ_H is a pseudoscalar, the interaction is parity conserving. The relevant four-dimensional field equations are

$$\partial_\mu H^{\mu\nu\rho} = 0 \qquad \partial_\mu F^{\mu\nu} = M_P^{-1}H^{\nu\rho\eta}F_{\rho\eta}. \tag{22}$$

The corresponding Bianchi identities are

$$\Box\Phi_H = M_P^{-1}F^{\mu\nu*}F_{\mu\nu} \qquad \partial_\mu{}^*F^{\mu\nu} = 0. \tag{23}$$

To simplify, let us assume that the "axion" field Φ_H is *homogeneous* and provides a background with which the Maxwell field interacts. We restrict our attention to lowest order in the inverse Planck mass M_P, so that terms on the right-hand side of the axion field Eq. (23) are ignored to a first approximation. Consequently, $\dot\Phi_H \equiv d\Phi_H/dt = f_0$, where f_0 is a constant of dimensionality of (mass)^2. Under these conditions, the Maxwell equations can be combined to yield the inhomogeneous wave equation for the magnetic field $\mathbf{B}$

$$\Box\mathbf{B} = -\frac{2f_0}{M_P}\nabla\times\mathbf{B}. \tag{24}$$

With the ansatz for a plane wave travelling in the z-direction, $\mathbf{B}(\mathbf{x},t) = \mathbf{B}_0(t)\exp ikz$, we obtain, for the left and the right circular polarization states $B_{0\pm} \equiv B_{0x} \pm iB_{0y}$,

$$\frac{d^2B_{0\pm}}{dt^2} + \bigg(k^2 \mp \frac{2f_0k}{M_P}\bigg)B_{0\pm} = 0. \tag{25}$$

We concentrate on the equation for magnetic field, as the conclusions will be same for that of electric field. The right and left circular polarization states have different angular frequencies (dispersion)

$$\omega_\pm^2 = k^2 \mp \frac{2kf_0}{M_P} \tag{26}$$

so that over a time interval Δt, the plane of polarization undergoes a rotation (for large k)

$$\Delta\Psi_{\rm op} \equiv |\omega_+ - \omega_-|\Delta t \simeq 2\frac{f_0}{M_P}\Delta t. \tag{27}$$

In Friedmann-Robertson-Walker space-time, the value of observed angle of rotation also incorporates the scale factor [3]. This means that $\Delta\Psi = \Delta\Psi(z)$, where z is the red-shift, and increases with red-shift. This rotation also differs from the better-understood Faraday rotation in that it is *achromatic* in the limit of high frequencies. Observationally, even for large red-shift sources, the angle of rotation is less than a degree, which imposes the restriction on the dimensionless quantity $f_0/M_P^2 < 10^{-20}$. In regard to astrophysical observations of optical activity, it appears that there is no definite evidence that the rotation of the plane of polarization travelling over cosmologically large distance is not entirely attributable to Faraday rotation due to magnetic fields present in the galactic plasma [36]. However, it is therefore not unlikely that the axion field will endow observable effect in CMB.

[3] Now, because ϕ_H can be both parity violating as well as parity conserving, each interaction is both parity conserving and parity violating. In what follows, we shall only consider the case where Φ_H is parity violating.

In contrast, if we consider only the extra augmentation, i.e the interaction $\Phi_H F_{\mu\nu} F^{\mu\nu}$, the resulting wave equation for the $\mathbf{B}$ field leads to entirely different results. Observe that this interaction violates spatial parity. The wave equation is simple to determine:

$$\frac{d^2\mathbf{B}}{dt^2} - 2\nabla\mathbf{B} + \frac{\zeta}{M_P} f_0 \frac{d\mathbf{B}}{dt} = 0, \qquad (28)$$

which eventually leads to the following equation for the left/right circularly polarized light [15]:

$$\frac{d^2 B_{+(-)}}{dt^2} + \frac{\bar{f}_0}{M_P}\frac{dB_{+(-)}}{dt} + k^2 dB_{+(-)} = 0, \qquad (29)$$

where $\bar{f}_0 = \zeta f_0$. The effect of parity violation is confined to the second term, which signifies either an enhancement or an attenuation of the intensity of the observed electromagnetic wave, depending on the sign of $\bar{f}_0$ [15]. We shall not go into the details of this calculation. Instead, we shall show that a similar effect also exists for gravity waves which might lead to some observational effects.

IV. BEHAVIOR OF GRAVITATIONAL WAVES

First, let us discuss the gravitational analogue of the rotation of plane of polarization (optical activity, Eq. (26)) discussed above [7]. This arises due to the parity-conserving term of the form $\Phi_H\,\mathrm{tr}(R \wedge R)$ in Eq. (15). First note that the augmentation of H in (7) implies that the $\mathrm{tr}(R \wedge R)$ term contributes an additional term to the Einstein equation over and above the energy-momentum tensor of the KR field. Formally,

$$\mathcal{G}_{\mu\nu} = \frac{8\pi}{M_P^2} T_{\mu\nu} + \frac{16\pi}{M_P^3}\frac{1}{\sqrt{-g}}\frac{\delta}{\delta g^{\mu\nu}}\int d^4x'\sqrt{-g}(x') \times \Phi_H(x') R_{\rho\lambda\sigma\eta}(x')^* R^{\rho\lambda\sigma\eta}(x'), \qquad (30)$$

where,

$$T_{\mu\nu} = H_{(\mu|\tau\rho} H_{\nu)}{}^{\tau\rho} - \frac{1}{6} g_{\mu\nu} H^2. \qquad (31)$$

It has been established in [7] that in the linearized approximation, the propagation of gravity waves in a homogeneous axion background is governed by (in large k limit, but in the Planckian regime $k < M_P$ with $16\pi k f_0/M_P^3 \ll 1$ as an expansion parameter)

$$\left[\frac{d^2}{dt^2} + k^2 + 8\pi f_0^2/M_P^2 \mp 1024\pi^2 k f_0^3/M_P^5\right]\varepsilon_\pm \simeq -8\pi f_0^2(1 \mp 16\pi k f_0/M_P^3)/M_P^2. \qquad (32)$$

We can now read off the dispersion relation

$$\omega_\pm^2 = k^2 + 4\pi f_0^2/M_P^2 \mp 1024\pi^2 k f_0^3/M_P^5, \qquad (33)$$

whence the group velocity is $v_{g\pm} = 1 + O(k^{-2})$ and the phase velocity is given by $v_{p\pm} = 1 \mp 512\pi^2(f_0^3/M_P^5 k)$. As in the electromagnetic case, the rotation of the polarization plane for gravitational waves is given by

$$\Delta\Psi_{\mathrm{grav}} \simeq 1024\pi^2\frac{f_0^3}{M_P^5}\Delta t. \qquad (34)$$

With the limits on f_0 given in the previous subsection, it is very small $O(10^{-30})$. However, since the tensor perturbations characterizing the gravitational wave do not get randomized, the effect is in principle observable.

Let us now restrict ourselves to the parity-violating term of the form $\Phi_H\,\mathrm{tr}(R \wedge {}^*R)$. Quite striking differences are seen with the new term. The electromagnetic analogue of this term has been discussed in [15,16] and reviewed in Eq. (29). In contrast to the rotation of plane of polarization for gravity waves as observed above, Eq. (33), we expect some new consequences. In fact, we expect to observe modulation for gravity waves. First, the effective action can be written as:

$$\mathcal{G}_{\mu\nu} = \frac{8\pi}{M_P^2} T_{\mu\nu} + \frac{16\pi}{M_P^3}\frac{1}{\sqrt{-g}}\frac{\delta}{\delta g^{\mu\nu}}\int d^4x'\sqrt{-g}(x') \times \Phi_H(x') R_{\rho\lambda\sigma\eta}(x') R^{\rho\lambda\sigma\eta}(x'), \qquad (35)$$

where

$$T_{\mu\nu} = H_{(\mu|\tau\rho} H_{\nu)}{}^{\tau\rho} - \frac{1}{6} g_{\mu\nu} H^2. \qquad (36)$$

We consider the Einstein equation in a linearized approximation. We decompose the metric $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ with the fluctuation $h_{\mu\nu}$ being considered small so that one need only retain terms of $O(h)$ in the Einstein equation. We further impose on the fluctuations $h_{\mu\nu}$ the Lorenz gauge $h_{\mu\nu}{}^{\nu}, = \frac{1}{2} h,_{\mu}$. we regard the axion field Φ_H as a homogeneous background satisfying Eq. (23) and consider its effect on a plane gravitational wave and we restrict to the lowest inverse power of the Planck mass for which a nontrivial effect is obtained. We ignore terms on the right-hand side of the axion field equation and set

$$\Box\Phi_H = 0 \qquad (37)$$

We have chosen the Lorenz gauge and not all components of $h_{\mu\nu}$ are independent. In fact, the only physical degrees of freedom of the spin 2 field are contained in h_{ij}, for which we choose a plane wave ansatz travelling in the z direction,

$$h_{ij} = \varepsilon_{ij}(t)\exp{-ikz}. \qquad (38)$$

The Latin indices above correspond to spatial directions. The other components of $h_{\mu\nu}$ can be gauged away, so that their field equation need not be considered. The only nonvanishing polarization components can be chosen to be $\varepsilon_{11} = -\varepsilon_{22}$, $\varepsilon_{12} = \varepsilon_{21}$; from these the circular polarization components can be constructed as in the Maxwell case: $\varepsilon_\pm \equiv \varepsilon_{11} \pm i\varepsilon_{12}$. Again, we shall assume that the scalar field is homogeneous and it has only time dependence so that $d\Phi/dt =: f_0$ is a constant. The equation of motion for the h_{ij} can be determined in a straightforward manner:

$$\Box h_{ij} = -\frac{16\pi}{M_P^2}[-(\eta_{ij} + h_{ij})f_0^2] - \frac{16\pi}{M_P^3}\zeta[f_0\Box h_{ij,t} + \Phi_H\Box\Box h_{ij}] \quad (39)$$

Now, to facilitate the calculation, let us make some simplified assumptions and notations. First, as seen from the previous section, let us define the dimensionless quantity $\alpha := (f_0/M_P^2) \ll 1$. Secondly, we shall remain in the Planckian regime but the wave number k is such that the dimensionless quantity $\beta := k/M_P$ is small (let us say $O(10^{-5})$). The modulus of the field Φ_H is taken to be order 1. The previous equation now reduces to:

$$\frac{d^2\epsilon_{ij}}{dt^2} + 16\pi\alpha\zeta\beta k\frac{d\epsilon_{ij}}{dt} + k^2\left(1 - \frac{16\pi\alpha^2}{\beta}\right)\epsilon_{ij} = \frac{16\pi f_0^2}{M_P^2}\eta_{ij} \quad (40)$$

This is an equation for a damped oscillator with a forcing term. The system can get damped or can sustain gravity waves. This depends on the value of the "$(b^2 - 4ac)$" term, which here is given by:

$$2ik\left[1 - \frac{16\pi\alpha^2}{\beta} + \frac{(16\pi\alpha\zeta\beta)^2}{4}\right]^{1/2} \quad (41)$$

Let us list the various possible cases. First, when $\alpha^2/\beta \geq 1$, i.e., small values of k (note that the third term in (41) is very small, with the value of β, it is of the order of 10^{-15} smaller compared to the second term and will not contribute appreciably), we get the scenario where the gravity waves dampen and are not observed:

$$h_{ij}(t, z) = \exp\left(-\frac{16\pi\alpha\zeta k}{M_P}\right)[A_{ij}e^{\bar{k}t - ikz} + B_{ij}e^{-\bar{k}t - ikz}]. \quad (42)$$

Second, consider the case where $\alpha^2/\beta < 1$ (i.e. large values of k). Then, the solutions of the Eq. (40) are

$$h_{ij}(t, z) = \exp\left(-\frac{16\pi\zeta\alpha k}{M_P}\right)[A_{ij}e^{ikt - ikz} + B_{ij}e^{-ikt - ikz}]. \quad (43)$$

This is the standard solution where the wave proceeds sinusoidally. It is clear that the solution to this equation can give attenuation/amplification of amplitude of gravity waves. To see this, choose $\zeta = +1$; then the Eq. (43) leads to attenuation of gravity waves, whereas for $\zeta = -1$ we get amplification of gravity waves. In short, in this case we do not see any rotation of plane of polarization of gravity wave, rather the attenuation/amplification of the wave during propagation is the result of such an interaction. Such phenomena for gravity waves was suggested in [22] which however was largely phenomenological. If such effects are present, they have implications for CMB spectrum. They lead to nonzero cross-correlation in multipole moments C_l^{TB} and C_l^{EB}. Such effects cannot be induced by Faraday rotation (if there is any intervening magnetic field). This is because it is an anisotropic effect which will also change l. With the Planck data coming up, we expect to see some of these effects or, if these are not seen, the experiments can be used to put bounds on the coupling constants for these interactions.

V. QUANTUM GRAVITY EFFECTS FOR THE HIGHER DERIVATIVE LAGRANGIAN

In this section we will study the effects of quantum fluctuations of different fields for a theory governed by the action (21) by calculating the one-loop effective potential using loop-expansion scheme [37]. We will concentrate on the gravitational part of the action only. Effective potential serves as a useful tool to investigate the vacuum structure of such a theory where one can define the theory to be valid up to an energy scale (Planck energy) through cutoff and make predictions treating it as an effective theory. As has been argued in [27], in a theory with *anomolous* terms like that considered in this paper, the cosmological constant may become a space-time dependent quantity. The quantum fluctuations also affect the CMB spectrum, which differs significantly from simple inflationary models leading to constraints from observational data. Indeed, the bispectrum, trispectrum and the non-Gaussianities of the calculated CMB spectrum can lead to newer understanding. For this reason, we devote this section to the calculation of the effective potential, which is the first step to the calculation of parameters in the inflatory models.

To keep the matters very general, we shall consider a theory of gravitation coupled with three different kinds of matter fields. The Einstein term is minimally coupled with a massive/massless scalar field ϕ_S which has a self-interacting potential. The action also contains an (axion) field ϕ_A coupled with a CP-odd term $R_{\mu\nu\alpha\beta}{}^*R^{\mu\nu\alpha\beta}$ and another field ϕ which is coupled to the CP-even term $R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$. In Euclidean signature, the Lagrangian of the theory is

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{g1} + \mathcal{L}_{g2} + \mathcal{L}_{g3} + \mathcal{L}_m \\ &= -\frac{1}{\kappa^2}R + a\phi R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} + b\phi_A R_{\mu\nu\alpha\beta}{}^*R^{\mu\nu\alpha\beta} \\ &\quad + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi_A\partial_\nu\phi_A \\ &\quad + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi_S\partial_\nu\phi_S + V(\phi_S),\end{aligned} \quad (44)$$

where $\kappa^2 = 16\pi G$ and a, b are coupling constants which can be specified later (they are M_P^{-3}). Let us now turn to calculate the effective potential. For that purpose, we first expand the metric $g_{\mu\nu}$ around a flat background:

$$g_{\mu\nu} = \delta_{\mu\nu} + \kappa h_{\mu\nu}, \tag{45}$$

where $\delta_{\mu\nu}$ is a flat background and the fluctuations $h_{\mu\nu}$ are small, $|h_{\mu\nu}| < 1$. For the decomposition (45), the inverse of the metric is

$$g^{\mu\nu} = \delta^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^{\mu}{}_{\lambda} h^{\lambda\nu} + \dots \tag{46}$$

Furthermore, the determinant of the metric, which will be needed in the following, will be given by

$$(g)^{1/2} = 1 + \frac{1}{2} h^{\alpha}{}_{\alpha} - \frac{1}{4} h^{\alpha}{}_{\beta} h^{\beta}{}_{\alpha} + \frac{1}{8} (h^{\alpha}{}_{\alpha})^2 + \dots \tag{47}$$

To calculate one-loop effective potential, we need to expand the Lagrangians only up to quadratic order in the $h_{\mu\nu}$. The expansions are listed below:

$$\begin{aligned}\sqrt{g}\mathcal{L}_{g1} &= \sqrt{g}R \\ &= -\frac{1}{4}\partial_\alpha h_{\mu\nu}\partial^\alpha h^{\mu\nu} + \frac{1}{4}\partial_\alpha h \partial^\alpha h - \frac{1}{2}\partial_\alpha h \partial_\beta h^{\alpha\beta} \\ &\quad + \frac{1}{2}\partial_\alpha h_{\mu\beta}\partial^\beta h^{\mu\alpha} + \text{total derivatives.}\end{aligned} \tag{48}$$

The expressions for the other two terms are long. However, we give them below. First,

$$\begin{aligned}\sqrt{g}\mathcal{L}_{g2} &= \sqrt{g} a \phi R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} \\ &= a\kappa^2(\partial_\nu\partial_\rho\phi h_{\mu\sigma}\partial^\nu\partial^\rho h^{\mu\sigma} + \partial_\rho\phi h_{\mu\sigma}\Box\partial^\rho h^{\mu\sigma} \\ &\quad + \phi h_{\mu\sigma}\Box\Box h^{\mu\sigma} + \partial_\nu\partial_\rho\phi h_{\mu\sigma}\partial^\mu\partial^\sigma h^{\nu\rho} \\ &\quad + \partial_\rho\phi h_{\mu\sigma}\partial^\mu\partial^\sigma\partial_\nu h^{\nu\rho} + \phi h_{\mu\sigma}\partial^\mu\partial^\sigma\partial_\nu\partial_\rho h^{\nu\rho} \\ &\quad - 2\partial_\nu\partial_\rho\phi h_{\mu\sigma}\partial^\nu\partial^\sigma h^{\mu\rho} - 2\partial_\rho\phi h_{\mu\sigma}\Box\partial^\sigma h^{\mu\rho} \\ &\quad - 2\phi h_{\mu\sigma}\Box\partial^\sigma\partial_\rho h^{\mu\rho})\end{aligned} \tag{49}$$

and

$$\begin{aligned}\sqrt{g}\mathcal{L}_{g3} &= \sqrt{g} b \phi_A R_{\mu\nu\alpha\beta}{}^{*}R^{\mu\nu\alpha\beta} \\ &= 2b\kappa^2\{\partial_\lambda\partial_\sigma\phi_A\partial_\alpha\partial^\lambda h^{\rho}_{\beta} h_{\rho\eta} + \partial_\sigma\phi_A h_{\rho\eta}\Box\partial_\alpha h^{\rho}_{\beta} \\ &\quad - \partial_\lambda\partial_\sigma h_{\rho\eta}\partial_\alpha\partial^\rho h^{\lambda}_{\beta}\}\epsilon^{\alpha\beta\sigma\eta}.\end{aligned} \tag{50}$$

Note that due to the presence of a Levi-Cività tensor, which is completely antisymmetric in its indices, only three terms will survive in the expansion of $\mathcal{L}_{g3}$, Since we are calculating one-loop effective potential, terms of order 2 in fluctuations will only contribute. To obtain one-loop effect, it is sufficient to choose space-time independent saddle points for the scalar (and pseudoscalar) fields;

$$\begin{aligned}\phi(x) &= \phi_0 + \Phi(x); \\ \phi_A(x) &= \phi_{A0} + \Phi_A(x); \\ \phi_S(x) &= \phi_{S0} + \Phi_S(x)\end{aligned}$$

With these choices, the derivative terms of the scalar fields will not contribute to the resulting Lagrangian (expanded about the saddle points). The Lagrangian relevant for calculating one-loop effective potential is by invoking the transverse-traceless gauge [38,39]. With $\partial_\mu h^{\mu\nu} = 0$ and $h = 0$, the relevant part of the Lagrangian becomes:

$$\begin{aligned}\mathcal{L}_{\text{rel}} &= \frac{1}{4} h_{\mu\nu}(-\Box_E) h^{\mu\nu} + a\kappa^2\phi_0 h_{\mu\nu}\Box_E\Box_E h^{\mu\nu} \\ &\quad - \frac{1}{2}\Phi_S(-\Box_E + V''(\phi_{S0}))\Phi_S - V(\phi_{S0}) \\ &\quad - \frac{1}{4}\kappa^2 h_{\mu\nu} V h^{\mu\nu} + \frac{1}{2}\Phi(-\Box_E)\Phi \\ &\quad + \frac{1}{2}\Phi_A(-\Box_E)\Phi_A,\end{aligned} \tag{51}$$

where $\Box_E$ is the operator in Euclidean space. Since we are perturbing around a flat background and made a choice of linear gauge, ghosts do not appear in this case [40–42]. However, although the higher derivative quantum gravity bare action contains massive negative norm states at tree level, whether they will spoil the unitarity of S matrix or not is inconclusive because quantum corrections may destabilize the ghosts [43,44]. Moreover, from the effective field theory description of gravity these issues can be sidelined [21,45]. The mass of the ghost fields are of the order of Planck mass, they will not be excited below the Planck scale [46,47] and here, we are dealing with a theory below that energy scale.

Note here that the (axion) field Φ_A has no contribution to the one-loop effective potential. Now, eqn (51) may be conveniently written as

$$\begin{aligned}\mathcal{L}_{\text{rel}} &= \frac{1}{2} h_{\mu\nu}\mathcal{O}^{\mu\nu\alpha\beta} h_{\alpha\beta} + \frac{1}{2}\Phi_S(-\Box_E + V''(\phi_{S0}))\Phi_S \\ &\quad + \frac{1}{2}\Phi(-\Box_E)\Phi + \frac{1}{2}\Phi_A(-\Box_E)\Phi_A,\end{aligned} \tag{52}$$

where the operator

$$\mathcal{O}^{\mu\nu\alpha\beta} = \frac{1}{2}\delta^{\mu\alpha}\delta^{\nu\beta}[-\Box_E + 2a\kappa^2\phi_0\Box_E\Box_E - \kappa^2 V(\phi_{S0})].$$

Now, we rewrite the Lagrangian in terms Ψ_i where $i = 1, 2, \dots 10$ denotes 10 independent components of $h_{\mu\nu}$ [48]

$$\mathcal{L}_{\text{rel}} = \frac{1}{2}\Phi(-\Box_E + V''(\phi_{S0}))\Phi + \frac{1}{2}\Psi_i M_{ij}\Psi_j, \tag{53}$$

where we have we have employed the following index correspondence: $\mu\nu \to i$ and $\alpha\beta \to j$. To get the one-loop effective potential we need to calculate the determinants of differential operators, which in this case reduces to calculate the eigenvalues of the 10×10 matrix M_{ij} [48].

The operator for scalar field is trivial. We write down the eigenvalues

$$\lambda_i = -\frac{1}{2}(k^2 + 4a\kappa^2\phi_0 k^4 - \kappa^2 V); \qquad (1 \le i \le 4)$$
$$\lambda_i = (k^2 + 4a\kappa^2\phi_0 k^4 - \kappa^2 V); \qquad (5 \le i \le 10). \tag{54}$$

The one-loop effective potential is given by

$$V_{\rm eff}^{(1)} = V(\phi_{S0}) + \frac{1}{2}\,{\rm Tr}\ln(k^2 + V'') + \sum_{i=1}^{10}\frac{1}{2}\,{\rm Tr}\ln\lambda_i, \tag{55}$$

where Tr is the functional trace. Performing the momentum space integrals and introducing a cutoff we obtain the unrenormalized one-loop effective potential

$$\begin{aligned} V_{\rm eff}(\phi_{S0}, \phi_0) &= \frac{5}{16\pi^2}\bigg[\bigg(\frac{\Lambda^4}{2} - \frac{1-2eg}{4e^2}\bigg)\ln\frac{e\Lambda^4}{g} + \frac{\Lambda^2}{2e} + \frac{g}{2e} \\ &\quad - \frac{1}{4e^2} + \frac{\sqrt{1-4eg}}{4e^2}\ln\bigg(\frac{1+\sqrt{1-4eg}}{1-\sqrt{1-4eg}}\bigg)\bigg] \\ &\quad + \frac{\Lambda^2 V''}{32\pi^2} + \frac{V''^2}{64\pi^2}\bigg(\ln\frac{V''}{\Lambda^2} - \frac{1}{2}\bigg) + V(\phi_{S0}), \end{aligned} \tag{56}$$

where $e = 4\phi_0 a\kappa^2$ and $g = -\kappa^2 V$, Λ^2 is the momentum cut-off. If we put the expressions of e and g back into the above expression the effective potential is seen to have an imaginary part:

$$\begin{aligned} V_{\rm eff}(\phi_{S0}, \phi_0) &= \frac{5}{16\pi^2}\bigg[\bigg(\frac{1+8\kappa^4\phi_0 aV}{64\kappa^4\phi_0^2a^2} - \frac{\Lambda^4}{2}\bigg)\ln\frac{V}{\Lambda^4} + \frac{\Lambda^2}{8\kappa^4\phi_0 a^2} - \frac{V}{2} - \frac{1}{64\kappa^4\phi_0^2a^2} + \frac{\sqrt{1+8\kappa^4\phi_0 aV}}{64\kappa^4\phi_0^2a^2} \\ &\quad \times \ln\bigg(\frac{1+\sqrt{1+8\kappa^4\phi_0 aV}}{1-\sqrt{1+8\kappa^4\phi_0 aV}}\bigg)\bigg] + \frac{5i}{16\pi}\bigg(\frac{1+8\kappa^4\phi_0 aV}{64\kappa^4\phi_0^2a^2} - \frac{\Lambda^4}{2}\bigg) + \frac{\Lambda^2V''}{32\pi^2} + \frac{V''^2}{64\pi^2}\bigg(\ln\frac{V''}{\Lambda^2} - \frac{1}{2}\bigg) + V(\phi_{S0}). \end{aligned} \tag{57}$$

It is interesting to see here that an imaginary part is generated in the effective potential. A similar kind of result was found in [38] for a theory where a single scalar field is coupled to gravity. The imaginary part of the effective potential signifies that we have chosen an unstable vacuum; in fact flat space is not a stable vacuum of this theory. The value of $V_{\rm eff}$ at t he asymmetric minimum serves as a cosmological constant at the tree level [38,41]. This interpretation can be explained as follows: Let $V_{\rm eff}$ develops an symmetry-breaking minima at the value of $\phi_{S0} = \phi_{S_{\rm min}}$ and $V_{\rm eff}(\phi_{S_{\rm min}}) \neq 0$ then $V_{\rm eff}(\phi_{S_{\rm min}})$ will act as a cosmological constant at the tree level. Now we include a cosmological constant to this theory, so now we have a different vacuum state not a flat space but a de Sitter space. The Lagrangian reads as

$$\begin{aligned} \mathcal{L} &= -\frac{1}{\kappa^2}(R - 2C) + a\phi R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} \\ &\quad + b\phi_A R_{\mu\nu\alpha\beta}{}^*R^{\mu\nu\alpha\beta} + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \\ &\quad + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi_A\partial_\nu\phi_A + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi_S\partial_\nu\phi_S + V(\phi_S), \end{aligned} \tag{58}$$

where C is the cosmological constant. If we repeat the calculation for the effective potential from (58), the imaginary part of the potential will be

$$\begin{aligned} &{\rm Im}[V_{\rm eff}(\phi_{S0}, \phi_0)] \\ &\quad = \frac{5}{16\pi}\bigg(\frac{1 + 2(\kappa^2 V + 2C)\phi_0 a\kappa^2}{64\kappa^4\phi_0^2a^2} - \frac{\Lambda^4}{2}\bigg). \end{aligned} \tag{59}$$

It is now obvious that we can fine tune the cosmological constant C such that the imaginary part of $V_{\rm eff}$ and the cosmological constant both vanish

$$\frac{1}{2}\kappa^2 V_{\rm eff}(\phi_{S_{\rm min}}) + \frac{1}{4\phi_0 a\kappa^2} + C = 0. \tag{60}$$

This makes the flat background a solution of the Einstein equation at the vacuum state. The calculation of effective potential here is done in a conventional approach which is not devoid of gauge ambiguities. However, it is well known that the Vilkovisky-DeWitt (VD) [49,50] approach of deriving effective potential is free from any ambiguities related to gauge-fixing condition or parametrization of the theory. We do not employ the method of VD here, although quite a number of papers have already been in the literature which calculate the effective potential in VD approach for ordinary and higher derivative gravity [42,51,52]. VD effective potential for the theory under consideration may be taken as a future project.

VI. DISCUSSIONS

Let us first recall the results of the paper. In string theory, the Kalb-Ramond field acts as a source term for torsion which has various interactions with gauge fields. In order that the interactions are gauge invariant, the Kalb-Ramond field $B_{\mu\nu}$ must be endowed with nontrivial transformations under gauge fields. This leads to some interesting interactions with observable consequences. One of them is the rotation of plane of polarization for electromagnetic and gravity waves. These had been studied earlier and have been matched with experimental results. However, these interactions are not the only possible ones. One can have additional ones which arise from the gauge invariant coupling of higher form fields to torsion. Such interaction was proposed in [15] and we correct and extend the formalism for non-Abelian fields and gravity. Observational consequences of such interactions are altogether different. They lead to amplification/attenuation of electromagnetic or gravity waves and have important implications for anisotropy of the cosmic microwave background by spatial parity violation [22]. For a such parity-breaking term, one can get certain nonvanishing multipole moment correlations between the temperature anisotropy and polarization of the CMB. In the CMB data, one usually observes correlations like $C_l^{\rm TT}$, $C_l^{\rm EE}$, $C_l^{\rm BB}$ and $C_l^{\rm TE}$ which arise from parity-conserving interactions. On the other hand, cross-correlations like $C_l^{\rm EB}$ and $C_l^{\rm TB}$ arise from parity-violating interactions from which bounds on the strength of such parity-violating terms can be ascertained. We also study the Coleman-Weinberg mechanism for such extended theory. This leads to a potential which might have some significance in the early universe and inflation. Initial studies with this potential show that one can generate the requisite number of e-foldings from such a theory near the Planck scale. Other consequences from such a potential require further study. We can list the new results of this paper: First, the construction of gauge invariant interaction for higher form gauge fields in [15] was incomplete which we have completed in (16). Second, we have extended the formalism to non-Abelian gauge fields and gravity. Third, we have studied the behavior of gravity waves which has important implications as discussed above and fourth, we have explicitly calculated the one-loop effectivepotential.

Let us now discuss the possible origin of the new augmented terms introduced in (16). Throughout the paper, we have presented various reasons which we list below. The first and probably the most compelling one is that such terms are necessary to form gauge invariant coupling of higher form gauge fields to torsion or Kalb-Ramond fields. We have not found in the literature any explicit reference to such terms in any low energy string effective action, though it has been shown that it is possible to embed such terms in a supersymmetric theory [16]. In the appendix, we derive the requisite new term from the boundary symplectic potential (associated with the standard first-order non-Abelian action) which shows that such terms can arise quite generically. The second argument is from the point of view of effective field theory which goes as follows: In our entire treatment, we have been dealing with cosmological or large distance scales. This is precisely the realm of effective field theory since we are interested in quantum effects at length scales much larger than the ultraviolet cutoff scale of gravity [21,27,53]. And thus, such effective theories, do not require the knowledge of precise details of the interaction of the newer degrees of freedom at the Planck scales. In spite of that, semiclassical effective theories capture the universality of interactions. The object of the semiclassical theory is to consider the space-time to be classical but the matter fields to be quantum mechanical. If the Planck's constant is not vanishing, the stress-energy tensor, which is now a quantum operator, has a quartic divergence. Upon renormalisation, it arises that general relativityis a effective quantum field theory if one augments the standard action by the the trace anomaly terms. Terms such as $R \wedge {}^*R$ are precisely the stress-tensor anomalies arising during quantization of massless scalar fields in curved space-time [54]. As it turns out, the quantum effective action is actually nonlocal but can be made local through introduction of scalar fields. Interestingly, the scalar fields in Eq. (44) play this role. In the bottom-up scenario of effective field theory, these scalar fields, which were absent in the original action, arise when we go up the energy scale. Conversely, when we come down in the energy scale from a Planck scale, which is what is done in string theory, we also expect to recover this action at some energy scale in four dimensions. For this very reason, we hope to find some way to generate the full action (21) from string theory.

ACKNOWLEDGMENTS

We would like to thank Arindam Mazumdar for useful discussions on cosmological implications of this work. We also thank Sandipan Sengupta for some discussions.

APPENDIX

In this appendix, we shall show the existence of the extra term of the form $(A \wedge {}^*F)$ added to the KR field in equation. The question is: where to look for such terms? To motivate, let us recall that the usual Chern-Simons term $(\Omega_{\rm YM})$ augmented to the KR field-strength H in Eq. (7) is actually a boundary term. In the $U(1)$ version, for example, the Chern-Simons term $(\Omega_{\rm YM})$ reduces to $(A \wedge F)$, which is precisely the conribution to boundary term corresponding to $(F \wedge F)$ in $U(1)$ gauge theory. By the same token, we shall look for the boundary terms for the action itself. Moreover, the usual Chern-Simons is an anomaly-canceling contribution just like the new terms which arise due to stress-tensor anomaly. More precisely, the gravitational Chern-Simons arise from the axial gravitational anomaly whereas the gravitational analogue of the new term is related to stress-tensor anomaly. Thus, we expect to find a derivation of the new contribution in a similar way to that of the Chern-Simons term.

Consider the Lagrangian 4-form for the *free* Yang-Mills theory

$$L = \mathrm{tr}(F \wedge {}_*F). \tag{A1}$$

The on-shell variation of the Lagrangian gives

$$\delta L = 2\,\mathrm{tr} d(\delta A \wedge {}^*F) := d\Theta(\delta). \tag{A2}$$

The term $\Theta(\delta)$ is a 3-form and is often called the symplectic potential. Now, consider the variation of the 1-form A through a parameter μ, $0 \leq \mu \leq 1$ and define:

$$\delta_\mu A =: A\delta\mu \quad \text{and} \quad A_{(\mu)} =: \mu A \quad \text{so that} \tag{A3}$$

$$^*F_{(\mu)} = \mu^* F + (\mu^2 - \mu)^*(A \wedge A). \tag{A4}$$

This implies that

$$\Theta(\delta_\mu) = 2\,\mathrm{tr}(A \wedge {}^*F_{(\mu)})\delta\mu. \tag{A5}$$

Thus, on shell, the above Eq. (A5) is equivalent to:

$$\frac{\delta}{\delta\mu}\,\mathrm{tr}(F \wedge {}^*F) = 2d\,\mathrm{tr}[\mu A \wedge {}^*F + (\mu^2 - \mu)A \wedge {}^*(A \wedge A)]. \tag{A6}$$

Integrating with respect to μ, we get

$$\mathrm{tr}\,(F \wedge {}^*F) = d\,\mathrm{tr}\Big[A \wedge {}^*F - \frac{1}{3}A \wedge {}^*(A \wedge A)\Big] = d\,\mathrm{tr}\Big[A \wedge {}^*dA + \frac{2}{3}A \wedge {}^*(A \wedge A)\Big]. \tag{A7}$$

Note that this term arises from a boundary contribution and is valid only on shell. In contrast, the usual Chern-Simons term, which can be derived in a similar fashion from the other boundary term $\mathrm{tr}(F \wedge F)$, only requires the Bianchi identity. In standard treatments, the boundary term vanishes by the boundary conditions on the fields. The above derivation is merely to show the existence of such terms in general when the field has all possible configurations.

Two comments are in order. First, in the equation above, we have considered only the *free* Yang-Mills theory. Now suppose that the Yang-Mills field is also coupled to other fields such as the KR field $H_{\mu\nu\lambda}$ in the present paper. In that case, the equation of motion for the Yang-Mills field is not merely $D^* F^i = 0$, but has contributions from the KR fields too. One then needs to look for the modification due to presence of such terms also. Second, as mentioned in the paper, we want not only to couple 1-form fields to H field but also 2- and 3-form fields. In those cases, the term $[A \wedge {}^*(A \wedge A)]$ does not arise (and is not a 3-form). For this reason, in what follows, we discard that term altogether. From above construction, we are led to the following term:

$$\mathrm{tr}\,(F \wedge {}^*F) = d\,\mathrm{tr}[\delta A \wedge {}^*F] + \mathrm{tr}(\delta A \wedge D^* F) \tag{A8}$$

For $\delta A^i = d\lambda^i + [A, \lambda]^i$, another term needs to be added to the first term. Thus, in total, we get the contribution to the total derivative to be

$$\mathrm{tr}\,(F \wedge {}^*F) = d\,\mathrm{tr}[A \wedge {}^*F + \lambda D^* F]. \tag{A9}$$

To understand the effect of this term, let us restrict to $U(1)$ gauge theory for simplicity. For $U(1)$ gauge fields, the effect of this augmentation leads to:

$$H \to H = dB + \frac{1}{M_P}(A \wedge {}^*F + \lambda d^* F). \tag{A10}$$

We want H to remain gauge invariant under $U(1)$ gauge transformation. Then, B must transform under $U(1)$ gauge transformation. This can be easily found from the above equation:

$$\delta_\lambda B = \lambda^* F \tag{A11}$$

In the whole setup, we have never explicitly used the equation of motion. Note that we have not added the term $\lambda d^* F$ in Eq. (19). That is because we want to look only for effects of order M_P^{-1} while the contribution of that term is of order M_P^{-2}.

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