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Gauge invariant approach to low-spin anomalous conformal currents and shadow fields

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Conformal low-spin anomalous currents and shadow fields in flat space-time of dimensions greater than or equal to four are studied. The gauge invariant formulation for such currents and shadow fields is developed. Gauge symmetries are realized by involving Stueckelberg and auxiliary fields. The gauge invariant differential constraints for anomalous currents and shadow fields and the realization of global conformal symmetries are obtained. Gauge invariant two-point vertices for anomalous shadow fields are also obtained. In the Stueckelberg gauge frame, these gauge invariant vertices become the standard two-point vertices of conformal field theory. Light-cone gauge two-point vertices of the anomalous shadow fields are derived. The AdS/CFT correspondence for anomalous currents and shadow fields and the respective normalizable and non-normalizable solutions of massive low-spin anti–de Sitter fields is studied. The bulk fields are considered in a modified de Donder gauge that leads to decoupled equations of motion. We demonstrate that leftover on-shell gauge symmetries of bulk massive fields correspond to gauge symmetries of boundary anomalous currents and shadow fields, while the modified (Lorentz) de Donder gauge conditions for bulk massive fields correspond to differential constraints for boundary anomalous currents and shadow fields.

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I. INTRODUCTION

In space-time of dimension $d \geq 4$, fields of conformal field theory (CFT) can be separated into two groups: conformal currents and shadow fields. The field having Lorentz algebra spin s and conformal dimension $\Delta = s + d - 2$, is referred to as conformal current with canonical dimension, while field having the Lorentz algebra spin s and conformal dimension $\Delta > s + d - 2$ is referred to as anomalous conformal current. Accordingly, the field having Lorentz algebra spin s and conformal dimension $\Delta = 2 - s$, is referred to as shadow field with canonical dimension,[1] while field having Lorentz algebra spin s and conformal dimension $\Delta < 2 - s$ is referred to as anomalous shadow field.

In Refs. [8,9], we developed the gauge invariant (Stueckelberg) approach to the conformal currents and shadow fields having canonical conformal dimensions. In the framework of AdS/CFT correspondence such currents and shadow fields are related to *massless* anti–de Sitter (AdS) fields. The purpose of this paper is to develop gauge invariant approach to the anomalous conformal currents and shadow fields which, in the framework of AdS/CFT correspondence, are related to *massive* AdS fields. The examples of spin-1 and spin-2 conformal fields demonstrate all characteristic features of our approach. In this paper, because these examples are very important in their own right, we discuss spin-1 and spin-2 anomalous conformal currents and shadow fields. Arbitrary spin anomalous conformal currents and shadow fields will be considered in a forthcoming publication. Our approach can be summarized as follows.

(i) Starting with the field content of the standard formulation of anomalous conformal currents (and anomalous shadow fields), we introduce Stueckelberg fields and auxiliary fields, i.e., we extend space of fields entering the standard CFT.

(ii) On the extended space of currents (and shadow fields), we introduce differential constraints, gauge transformations, and conformal algebra transformations. These differential constraints are invariant under the gauge transformations and the conformal algebra transformations.

(iii) The gauge symmetries and the differential constraints make it possible to match our approach and the standard one, i.e., by appropriate gauge fixing to exclude the Stueckelberg fields and by solving differential constraints to exclude the auxiliary fields we obtain the standard formulation of anomalous conformal currents and shadow fields.

We apply our approach to the study of AdS/CFT correspondence between massive AdS fields and corresponding boundary anomalous conformal currents and shadow fields. We demonstrate that normalizable modes of massive AdS fields are related to anomalous conformal currents, while non-normalizable modes of massive AdS fields are related to anomalous shadow fields. In the earlier literature, the correspondence between non-normalizable bulk modes and shadow fields was studied in Ref. [10] (for spin-1 fields) and in Ref. [11] (for spin-2 fields). To our

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[1]It is the shadow fields having canonical dimension that are used to discuss conformal invariant equations of motion and Lagrangian formulations (see, e.g., Refs. [1–6]). In earlier literature, discussion of shadow field dualities may be found in Ref. [7].

knowledge, the AdS/CFT correspondence between normalizable massive modes and anomalous conformal currents has not been considered in the earlier literature. As compared to the studies in Refs. [10,11], our approach involves large amount of gauge symmetries. Therefore the results of these references are obtained from the ones in this paper by using some particular gauge condition, which we refer to as Stueckelberg gauge fixing. We note also that our approach provides quick access to the light-cone gauge formulation of CFT. Perhaps, one of the main advantages of our approach is that this approach gives easy access to the study of AdS/CFT correspondence in light-cone gauge frame. This is very important for the future application of our approach to studying string/gauge theory dualities because one expects that string theory in AdS/Ramond-Ramond background can be quantized only in light-cone gauge.

Our approach to the study of AdS/CFT correspondence can be summarized as follows.

(i) We use a CFT adapted gauge invariant approach to the AdS field dynamics developed in Ref. [12]. For spin-1 and spin-2 massive AdS fields, we use the respective modified Lorentz gauge and modified de Donder gauge. A remarkable property of these gauges is that they lead to the simple *decoupled* bulk equations of motion which can be solved in terms of the Bessel function and this simplifies considerably the study of AdS/CFT correspondence. Also, using these gauges, we demonstrate that the two-point gauge invariant vertex of the anomalous shadow field does indeed emerge from massive AdS field action when it is evaluated on solution of the Dirichlet problem. AdS field action evaluated on the solution of the Dirichlet problem will be referred to as effective action in this paper.

(ii) The number of boundary gauge fields involved in our gauge invariant approach to the anomalous conformal current (or anomalous shadow field) coincides with the number of bulk massive gauge AdS fields involved in the standard gauge invariant Stueckelberg approach to massive field. Note however that, instead of the standard gauge invariant approach to massive field, we use the CFT adapted formulation of massive AdS field developed in Ref. [12].[2]

(iii) Our modified Lorentz gauge (for spin-1 massive AdS field) and modified de Donder gauge (for spin-2 massive AdS field) turn out to be related to the differential constraints we obtained in the framework of gauge invariant approach to the anomalous conformal currents and shadow fields.

(iv) *Leftover on-shell* gauge symmetries of massive bulk AdS fields are related to the gauge symmetries of boundary anomalous conformal currents (or anomalous shadow fields).

The rest of the paper is organized as follows.

In Sec. II, we summarize the notation used in this paper.

In Secs. III and IV, we start with the respective examples of the spin-1 anomalous conformal current and spin-1 anomalous shadow field. We illustrate our gauge invariant approach to describing the anomalous conformal current and shadow field. For the spin-1 anomalous shadow field, we obtain the gauge invariant two-point vertex and discuss how our gauge invariant approach is related to the standard approach to CFT. Also, using our gauge invariant approach we obtain a light-cone gauge description of the spin-1 anomalous conformal current and shadow field.

Sections V and VI are devoted to spin-2 anomalous conformal current and spin-2 anomalous shadow field, respectively. In these sections we generalize results of Secs. III and IV to the case of the spin-2 anomalous conformal current and shadow field.

In Sec. VII, we discuss the two-point current-shadow field interaction vertex.

In Sec. VIII, because the use of a modified Lorentz (de Donder) gauge makes the study of AdS/CFT correspondence for the spin-1 (spin-2) field similar to the one for the scalar field, we briefly review the AdS/CFT correspondence for the scalar field.

Section IX is devoted to the study of AdS/CFT correspondence for the bulk spin-1 massive AdS field and boundary spin-1 anomalous conformal current and shadow field, while in Sec. X we extend results of Sec. IX to the case of spin-2 fields.

We collect various technical details in two appendices. In Appendices A and B we present details of the derivation of the CFT adapted gauge invariant Lagrangian for the respective spin-1 and spin-2 massive AdS fields.

II. PRELIMINARIES

A. Notation

Our conventions are as follows. x^a denotes coordinates in d-dimensional flat space-time, while ∂_a denotes derivatives with respect to x^a, $\partial_a \equiv \partial/\partial x^a$. Vector indices of the Lorentz algebra $so(d-1,1)$ take the values $a,b,c,e = 0,1,\ldots,d-1$. We use the mostly positive flat metric tensor η^{ab}. To simplify our expressions we drop η_{ab} in scalar products, i.e., we use $X^aY^a \equiv \eta_{ab}X^aY^b$. Throughout this paper we use operators constructed out of the derivatives and coordinates,

$$\Box = \partial^a\partial^a, \qquad x\partial \equiv x^a\partial^a, \qquad x^2 = x^a x^a. \qquad (2.1)$$

Sometimes we use a light-cone frame. In the light-cone frame, space-time coordinates are decomposed as

[2] We note also that the number of gauge transformation parameters involved in our gauge invariant approach to the anomalous current (or anomalous shadow field) coincides with the number of gauge transformation parameters of bulk massive gauge AdS field involved in the standard gauge invariant approach to massive field.

$x^a = x^+, x^-, x^i$, where light-cone coordinates in $\pm$ directions are defined as $x^\pm = (x^{d-1} \pm x^0)/\sqrt{2}$ and x^+ is taken to be a light-cone time. $so(d-2)$ algebra vector indices take values $i, j = 1, \ldots, d-2$. We adopt the conventions:

$$\partial^i = \partial_i \equiv \partial/\partial x^i, \qquad \partial^\pm = \partial_\mp \equiv \partial/\partial x^\mp. \tag{2.2}$$

B. Global conformal symmetries

In d-dimensional flat space-time, the conformal algebra $so(d, 2)$ consists of translation generators P^a, a dilatation generator D, conformal boost generators K^a, and generators of the $so(d-1, 1)$ Lorentz algebra J^{ab}. We assume the following normalization for commutators of the conformal algebra:

$$\begin{aligned}
&[D, P^a] = -P^a, \qquad [P^a, J^{bc}] = \eta^{ab}P^c - \eta^{ac}P^b,\\
&[D, K^a] = K^a, \qquad [K^a, J^{bc}] = \eta^{ab}K^c - \eta^{ac}K^b,\\
&[P^a, K^b] = \eta^{ab}D - J^{ab},\\
&[J^{ab}, J^{ce}] = \eta^{bc}J^{ae} + 3 \text{ terms}.
\end{aligned} \tag{2.3}$$

Let ϕ denotes conformal current (or shadow field) in the flat space-time of dimension $d \geq 4$. Under conformal algebra transformations the ϕ transforms as

$$\delta_{\hat G}\phi = \hat G \phi, \tag{2.4}$$

where the realization of the conformal algebra generators $\hat G$ in terms of differential operators acting on the ϕ takes the form

$$P^a = \partial^a, \tag{2.5}$$

$$J^{ab} = x^a\partial^b - x^b\partial^a + M^{ab}, \tag{2.6}$$

$$D = x\partial + \Delta, \tag{2.7}$$

$$K^a = K^a_{\Delta,M} + R^a, \tag{2.8}$$

$$K^a_{\Delta,M} \equiv -\tfrac{1}{2}x^2\partial^a + x^a D + M^{ab}x^b. \tag{2.9}$$

In (2.6), (2.7), and (2.8), Δ is an operator of conformal dimension, M^{ab} is a spin operator of the Lorentz algebra. The action of M^{ab} on the fields of the Lorentz algebra is well known and for the rank-2 tensor, vector, and scalar fields considered in this paper is given by

$$\begin{aligned}
&M^{ab}\phi^{ce} = \eta^{ae}\phi^{cb} + \eta^{ac}\phi^{be} - (a \leftrightarrow b),\\
&M^{ab}\phi^c = \eta^{ac}\phi^b - (a \leftrightarrow b),\\
&M^{ab}\phi = 0.
\end{aligned} \tag{2.10}$$

These relations imply that action of operator $K^a_{M,\Delta}$ (2.9) on the fields can be presented as

$$\begin{aligned}
&K^a_{\Delta,M}\phi^{bc} = K^a_\Delta\phi^{bc} + M^{abf}\phi^{fc} + M^{acf}\phi^{bf},\\
&K^a_{\Delta,M}\phi^b = K^a_\Delta\phi^b + M^{abf}\phi^f,\\
&K^a_{\Delta,M}\phi = K^a_\Delta\phi,
\end{aligned} \tag{2.11}$$

$$K^a_\Delta \equiv -\tfrac{1}{2}x^2\partial^a + x^a(x\partial + \Delta), \tag{2.12}$$

$$M^{abc} \equiv \eta^{ab}x^c - \eta^{ac}x^b. \tag{2.13}$$

In (2.8), R^a is an operator depending, in general, on the derivatives with respect to the space-time coordinates[3] and not depending on the space-time coordinates x^a. In the standard formulation of conformal currents and shadow fields, the operator R^a is equal to zero, while in the gauge invariant approach that we develop in this paper, the operator R^a is nontrivial. This implies that, in the framework of the gauge invariant approach, the complete description of the conformal currents and shadow fields requires, among other things, finding the operator R^a.

III. SPIN-1 ANOMALOUS CONFORMAL CURRENT

In this section, we develop a gauge invariant approach to the spin-1 anomalous conformal current. Besides the gauge invariant formulation, we discuss two gauge conditions which can be used for studying the anomalous conformal currents—the Stueckelberg gauge and light-cone gauge. We would like to discuss these gauges because of the following reasons.

(i) It turns out that the Stueckelberg gauge reduces our approach to the standard formulation of CFT. Therefore, the use of the Stueckelberg gauge allows us to demonstrate how the standard approach to anomalous conformal currents is obtained from our gauge invariant approach.

(ii) Motivation for considering the light-cone gauge frame comes from the conjectured duality of the supersymmetric Yang-Mills theory theory and the theory of the superstring in AdS background [14]. By analogy with flat space, we expect that a quantization of the Green-Schwarz AdS superstring [15] will be straightforward only in the light-cone gauge [16,17]. Therefore, it seems that from the stringy perspective of AdS/CFT correspondence, the light-cone approach to CFT is the fruitful direction to go.

A. Gauge invariant formulation

To discuss the gauge invariant formulation of the spin-1 anomalous conformal current in the flat space of dimension $d \geq 4$ we use one vector field $\phi^a_{\text{cur},0}$ and two scalar fields $\phi_{\text{cur},1}$, $\phi_{\text{cur},-1}$:

$$\phi^a_{\text{cur},0}, \qquad \phi_{\text{cur},-1}, \qquad \phi_{\text{cur},1}. \tag{3.1}$$

[3]For the conformal currents and shadow fields studied in this paper, the operator R^a does not depend on the derivatives. The dependence of R^a on derivatives appears, e.g., in an ordinary-derivative approach to conformal fields [13].

The fields $\phi^a_{\rm cur,0}$ and $\phi_{\rm cur,\pm1}$ transform in the respective vector and scalar irreps of the Lorentz algebra $so(d-1,1)$. We note that fields (3.1) have the conformal dimensions

$$\Delta_{\phi^a_{\rm cur,0}} = \frac{d}{2} + \kappa, \qquad \Delta_{\phi_{\rm cur,\pm1}} = \frac{d}{2} + \kappa \pm 1, \tag{3.2}$$

where κ is a dimensionless parameter. In the framework of AdS/CFT correspondence, κ is related to the mass parameter m of a spin-1 massive AdS field as

$$\kappa \equiv \sqrt{m^2 + \frac{(d-2)^2}{4}}. \tag{3.3}$$

We now introduce the following differential constraint:

$$\partial^a \phi^a_{\rm cur,0} + r^{00}_z \Box \phi_{\rm cur,-1} + r^{00}_\zeta \phi_{\rm cur,1} = 0, \tag{3.4}$$

$$r^{00}_z \equiv \left(\frac{2\kappa + d - 2}{4\kappa}\right)^{1/2}, \quad r^{00}_\zeta \equiv \left(\frac{2\kappa - d + 2}{4\kappa}\right)^{1/2}. \tag{3.5}$$

One can make sure that this constraint is invariant under the gauge transformations

$$\delta\phi^a_{\rm cur,0} = \partial^a \xi_{\rm cur,0}, \tag{3.6}$$

$$\delta\phi_{\rm cur,-1} = -r^{00}_z \xi_{\rm cur,0}, \tag{3.7}$$

$$\delta\phi_{\rm cur,1} = -r^{00}_\zeta \Box \xi_{\rm cur,0}, \tag{3.8}$$

where $\xi_{\rm cur,0}$ is a gauge transformation parameter.

To complete our gauge invariant formulation we provide the realization of the operator R^a on the space of gauge fields (3.1),

$$\begin{aligned} R^a \phi^b_{\rm cur,0} &= -2\kappa r^{00}_z \eta^{ab} \phi_{\rm cur,-1}, \\ R^a \phi_{\rm cur,-1} &= 0, \\ R^a \phi_{\rm cur,1} &= -2\kappa r^{00}_\zeta \phi^a_{\rm cur,0}. \end{aligned} \tag{3.9}$$

Using (3.9), we make sure that constraint (3.4) is invariant under transformations of the conformal algebra (2.4).

We obtained the differential constraint, gauge transformations, and realization of the operator R^a by generalizing our results for the spin-1 conformal current with the canonical dimension which we obtained in Ref. [8]. We note that results in this section can also be obtained by using the framework of the tractor approach in Ref. [18] (see also Refs. [19–21]).[4] Our fields $\phi_{\rm cur,-1}$ and $\phi_{\rm cur,1}$ are identified with the respective fields V^+ and V^- in Ref. [18]. Doing so, one can make sure that constraint and gauge transformations given in Eqs. (34) and (36) in Ref. [18] can be represented as our differential constraint (3.4) and gauge transformations (3.6), (3.7), and (3.8). To summarize, our fields (3.1) can be written as a tractor vector subject to a Thomas-D divergence type constraint in Ref. [18]. A similar construction was used to describe the bulk massive spin-1 field in Ref. [18].[5] Note that, in our approach, we use our fields (3.1) for the discussion of the spin-1 anomalous conformal current.

B. Stueckelberg gauge frame

We now discuss the spin-1 anomalous conformal current in the Stueckelberg gauge frame. From (3.7), we see that the scalar field $\phi_{\rm cur,-1}$ transforms as the Stueckelberg field, i.e., this field can be gauged away via Stueckelberg gauge fixing,

$$\phi_{\rm cur,-1} = 0. \tag{3.10}$$

Using this gauge in constraint (3.4), we see that the remaining scalar field $\phi_{\rm cur,1}$ can be expressed in terms of the vector field $\phi^a_{\rm cur,0}$,

$$\phi_{\rm cur,1} = -\frac{1}{r^{00}_\zeta} \partial^a \phi^a_{\rm cur,0}, \tag{3.11}$$

i.e., making use of the gauge symmetry and differential constraint (3.4) we reduce the field content of our approach (3.1) to the one in the standard approach. In other words, the gauge symmetry and differential constraint make it possible to match our approach and the standard formulation of the spin-1 anomalous conformal current.[6]

C. Light-cone gauge frame

For the spin-1 anomalous conformal current, the light-cone gauge frame is achieved through the use of differential constraint (3.4) and the light-cone gauge condition. Using the gauge symmetry of the spin-1 anomalous conformal current (3.6), we impose the light-cone gauge on the field $\phi^a_{\rm cur,0}$,

$$\phi^+_{\rm cur,0} = 0. \tag{3.12}$$

Using this gauge in differential constraint (3.4), we find

$$\phi^-_{\rm cur,0} = -\frac{\partial^j}{\partial^+} \phi^j_{\rm cur,0} - \frac{r^{00}_z}{\partial^+} \Box \phi_{\rm cur,-1} - \frac{r^{00}_\zeta}{\partial^+} \phi_{\rm cur,1}. \tag{3.13}$$

[4] In mathematical literature, discussion of the tractor approach may be found in Ref. [22].

[5] In earlier literature, use of the conformal symmetries for a discussion of massive field can be found in Ref. [23]. Discussion of interrelations between the gauge invariant formulation of the currents and shadow fields and the gauge invariant formulation of massive fields via breaking conformal symmetries may be found in Ref. [8]. We thank M. A. Vasiliev for pointing us to Ref. [23].

[6] As in the standard approach to CFT, our currents can be considered either as fundamental field degrees of freedom or as composite operators. At the group theoretical level that we study in this paper, this distinction is immaterial. A discussion of interesting methods for building conformal currents as composite operators may be found in Refs. [24,25].

We see that we are left with the vector field $\phi^i_{\rm cur,0}$ and two scalar fields $\phi_{{\rm cur},\pm 1}$. These fields constitute the field content of the light-cone gauge frame.

IV. SPIN-1 ANOMALOUS SHADOW FIELD

A. Gauge invariant formulation

To discuss the gauge invariant formulation of the spin-1 anomalous shadow field in space of dimension $d \geq 4$ we use one vector field $\phi^a_{\rm sh,0}$ and two scalar fields $\phi_{{\rm sh},-1}$, $\phi_{\rm sh,1}$:

$$\phi^a_{\rm sh,0}, \qquad \phi_{{\rm sh},-1}, \qquad \phi_{\rm sh,1}. \tag{4.1}$$

The fields $\phi^a_{\rm sh,0}$ and $\phi_{{\rm sh},\pm 1}$ transform in the respective vector and scalar representations of the Lorentz algebra $so(d-1,1)$. We note that these fields have the conformal dimensions

$$\Delta_{\phi^a_{\rm sh,0}} = \frac{d}{2} - \kappa, \qquad \Delta_{\phi_{{\rm sh},\pm 1}} = \frac{d}{2} - \kappa \pm 1. \tag{4.2}$$

In the framework of the AdS/CFT correspondence, κ is related to the mass parameter m of spin-1 massive AdS field as in (3.3).

We now introduce the following differential constraint:

$$\partial^a \phi^a_{\rm sh,0} + r^{00}_\zeta \Box \phi_{{\rm sh},-1} + r^{00}_z \phi_{\rm sh,1} = 0, \tag{4.3}$$

where r^{00}_z, r^{00}_ζ are given in (3.5). We make sure that constraint (4.3) is invariant under the gauge transformations

$$\delta \phi^a_{\rm sh,0} = \partial^a \xi_{\rm sh,0}, \tag{4.4}$$

$$\delta \phi_{{\rm sh},-1} = -r^{00}_\zeta \xi_{\rm sh,0}. \tag{4.5}$$

$$\delta \phi_{\rm sh,1} = -r^{00}_z \Box \xi_{\rm sh,0}, \tag{4.6}$$

where $\xi_{\rm sh,0}$ is a gauge transformation parameter.

To complete our gauge invariant formulation of the spin-1 anomalous shadow field we provide the realization of the operator R^a on the space of gauge fields (4.1),

$$\begin{aligned} R^a \phi^b_{\rm sh,0} &= 2\kappa r^{00}_\zeta \eta^{ab} \phi_{{\rm sh},-1}, \\ R^a \phi_{{\rm sh},-1} &= 0, \\ R^a \phi_{\rm sh,1} &= 2\kappa r^{00}_z \phi^a_{\rm sh,0}. \end{aligned} \tag{4.7}$$

We proceed with the discussion of two-point vertex for the spin-1 anomalous shadow field. The gauge invariant two-point vertex we find takes the form

$$\Gamma = \int d^d x_1 d^d x_2 \Gamma_{12}, \tag{4.8}$$

$$\Gamma_{12} = \frac{\phi^a_{\rm sh,0}(x_1)\phi^a_{\rm sh,0}(x_2)}{2|x_{12}|^{2\kappa+d}} + \sum_{\lambda=\pm 1} \frac{\omega_\lambda}{2|x_{12}|^{2\kappa+d-2\lambda}} \phi_{{\rm sh},\lambda}(x_1)\phi_{{\rm sh},\lambda}(x_2), \tag{4.9}$$

$$\omega_1 = \frac{1}{2\kappa(2\kappa+d-2)}, \qquad \omega_{-1} = 2(\kappa+1)(2\kappa+d), \tag{4.10}$$

$$|x_{12}|^2 \equiv x^a_{12} x^a_{12}, \qquad x^a_{12} = x^a_1 - x^a_2. \tag{4.11}$$

One can check that this vertex is invariant under the gauge transformations of the spin-1 anomalous shadow field given in (4.4), (4.5), and (4.6). Also, we check that the vertex is invariant under the conformal algebra transformations.

The kernel of the vertex Γ is related to a two-point correlation function of the spin-1 anomalous conformal current. In our approach, the spin-1 anomalous conformal current is described by gauge fields given in (3.1). Therefore, in order to discuss the correlation function of the anomalous conformal current in a proper way, we should impose a gauge condition on the gauge fields in (3.1).[7] We have considered the spin-1 anomalous conformal current in the Stueckelberg and light-cone gauge frames. This is to say that the correlation function of the spin-1 anomalous conformal current in the Stueckelberg and light-cone gauge frames can be obtained from the two-point vertex Γ taken in the respective Stueckelberg and light-cone gauge frames. To this end we now discuss the spin-1 anomalous shadow field in the Stueckelberg and light-cone gauge frames.

B. Stueckelberg gauge frame

For the spin-1 anomalous shadow field, the Stueckelberg gauge frame is achieved through the use of differential constraint (4.3) and the Stueckelberg gauge condition. From (4.5), we see that the scalar field $\phi_{{\rm sh},-1}$ transforms as the Stueckelberg field, i.e., this field can be gauged away via Stueckelberg gauge fixing,

$$\phi_{{\rm sh},-1} = 0. \tag{4.12}$$

Using this gauge in (4.3), we see that the remaining scalar field $\phi_{\rm sh,1}$ can be expressed in terms of the vector field $\phi^a_{\rm sh,0}$,

$$\phi_{\rm sh,1} = -\frac{1}{r^{00}_z} \partial^a \phi^a_{\rm sh,0}. \tag{4.13}$$

Thus we see that the use of gauge symmetry and differential constraint reduces field content of our approach (4.1) to the one in the standard approach. In other words, the gauge symmetry and differential constraint make it possible to

[7] We note that, in the gauge invariant approach, correlation functions of the conformal current can be studied without gauge fixing. To do that one needs to construct gauge invariant field strengths for the gauge potentials $\phi^a_{\rm cur,0}$, $\phi_{{\rm cur},\pm 1}$. The study of field strengths for the conformal current is beyond the scope of this paper. A recent interesting discussion of the method for building field strengths may be found in Refs. [26,27].

match our approach and the standard formulation of the spin-1 anomalous shadow field.

We proceed with the discussion of Stueckelberg gauge-fixed two-point vertex of the spin-1 anomalous shadow field, i.e., we relate our vertex (4.8) with the one in the standard approach to CFT. To this end we note that vertex of the standard approach to CFT is obtained from our gauge invariant vertex (4.8) by plugging the Stueckelberg gauge condition (4.12) and the solution to differential constraint (4.13) into (4.9). Doing so, we find that the two-point density Γ_{12} (4.9) takes the form (up to total derivative)

$$\Gamma_{12}^{\text{Stuck.g.fram}} = k_1 \Gamma_{12}^{\text{stand}}, \tag{4.14}$$

$$\Gamma_{12}^{\text{stand}} = \frac{\phi_{\text{sh}}^a(x_1) O_{12}^{ab} \phi_{\text{sh}}^b(x_2)}{|x_{12}|^{2\kappa+d}}, \tag{4.15}$$

$$O_{12}^{ab} \equiv \eta^{ab} - \frac{2x_{12}^a x_{12}^b}{|x_{12}|^2}, \tag{4.16}$$

$$k_1 \equiv \frac{2\kappa + d}{2(2\kappa + d - 2)}, \tag{4.17}$$

where $\Gamma_{12}^{\text{stand}}$ (4.15) stands for the two-point vertex of the spin-1 anomalous shadow field in the standard approach to CFT. From (4.14), we see that our gauge invariant vertex taken to be in the Stueckelberg gauge frame coincides, up to the normalization factor k_1, with the two-point vertex in the standard approach to CFT. As we have demonstrated in Sec. III B, in the Stueckelberg gauge frame, we are left with the vector field $\phi_{\text{cur},0}^a$. The two-point correlation function of this vector field is defined by the kernel of vertex Γ^{stand} (4.15).

C. Light-cone gauge frame

For the spin-1 anomalous shadow field, the light-cone gauge frame is achieved through the use of the light-cone gauge and differential constraint (4.3). Taking into account the gauge transformation of the field $\phi_{\text{sh},0}^a$ (4.4), we impose the light-cone gauge,

$$\phi_{\text{sh},0}^+ = 0. \tag{4.18}$$

Using this gauge in differential constraint (4.3), we obtain a solution for ϕ_{sh}^-,

$$\phi_{\text{sh},0}^- = -\frac{\partial^j}{\partial^+}\phi_{\text{sh},0}^j - \frac{r_z^{00}}{\partial^+}\phi_{\text{sh},1} - \frac{r_\zeta^{00}}{\partial^+}\Box\phi_{\text{sh},-1}. \tag{4.19}$$

We see that we are left with the vector field $\phi_{\text{sh},0}^i$ and the scalar fields $\phi_{\text{sh},\pm 1}$. These fields constitute the field content of the light-cone gauge frame. Note that, in contrast to the Stueckelberg gauge frame, the scalar fields $\phi_{\text{sh},\pm 1}$ become independent field degrees of freedom (D.o.F.) in the light-cone gauge frame.

Using (4.18) in (4.9) leads to a light-cone gauge-fixed vertex

$$\Gamma_{12}^{(\text{l.c.})} = \frac{\phi_{\text{sh},0}^i(x_1)\phi_{\text{sh},0}^i(x_2)}{2|x_{12}|^{2\kappa+d}} + \sum_{\lambda=\pm1} \frac{\omega_\lambda}{2|x_{12}|^{2\kappa+d-2\lambda}}\phi_{\text{sh},\lambda}(x_1)\phi_{\text{sh},\lambda}(x_2), \tag{4.20}$$

where ω_λ are given in (4.10). As in the case of the gauge invariant vertex (4.9), the light-cone vertex (4.20) is diagonal with respect to the fields $\phi_{\text{sh},0}^i$ and $\phi_{\text{sh},\pm1}$. Note, however, that in contrast to the gauge invariant vertex, the light-cone vertex is constructed out of the fields which are not subject to any constraints.

Thus, as we have promised, *our gauge invariant vertex gives easy and quick access to the light-cone gauge vertex*. All that is required to get the light-cone gauge vertex (4.20) is to replace the $so(d-1,1)$ Lorentz algebra vector indices appearing in the gauge invariant vertex (4.9) by the vector indices of the $so(d-2)$ algebra.

The kernel of the light-cone vertex gives the two-point correlation function of the spin-1 anomalous conformal current taken to be in the light-cone gauge. Defining the two-point correlation functions of the fields $\phi_{\text{cur},0}^i$, $\phi_{\text{cur},\pm1}$ in a usual way,

$$\begin{aligned}\langle\phi_{\text{cur},0}^i(x_1), \phi_{\text{cur},0}^j(x_2)\rangle &= \frac{\delta^2\Gamma^{(\text{l.c.})}}{\delta\phi_{\text{sh},0}^i(x_1)\delta\phi_{\text{sh},0}^j(x_2)},\\ \langle\phi_{\text{cur},\lambda}(x_1), \phi_{\text{cur},\lambda}(x_2)\rangle &= \frac{\delta^2\Gamma^{(\text{l.c.})}}{\delta\phi_{\text{sh},-\lambda}(x_1)\delta\phi_{\text{sh},-\lambda}(x_2)},\end{aligned} \tag{4.21}$$

$\lambda = \pm1$, and using (4.20), we obtain the two-point light-cone gauge correlation functions of the spin-1 anomalous conformal current,

$$\begin{aligned}\langle\phi_{\text{cur},0}^i(x_1), \phi_{\text{cur},0}^j(x_2)\rangle &= \frac{\delta^{ij}}{|x_{12}|^{2\kappa+d}},\\ \langle\phi_{\text{cur},\lambda}(x_1), \phi_{\text{cur},\lambda}(x_2)\rangle &= \frac{\omega_{-\lambda}}{|x_{12}|^{2\kappa+d+2\lambda}},\end{aligned} \tag{4.22}$$

$\lambda = \pm1$, where ω_λ are given in (4.10).

V. SPIN-2 ANOMALOUS CONFORMAL CURRENT

A. Gauge invariant formulation

To discuss gauge invariant formulation of spin-2 anomalous conformal current in flat space of dimension $d \geq 4$ we use one rank-2 tensor field, two vector fields, and three scalar fields,

$$\begin{gathered}\phi_{\text{cur}}^{ab},\\ \phi_{\text{cur},-1}^a, \qquad \phi_{\text{cur},1}^a,\\ \phi_{\text{cur},-2}, \qquad \phi_{\text{cur},0}, \qquad \phi_{\text{cur},2}.\end{gathered} \tag{5.1}$$

The fields ϕ_{cur}^{ab}, $\phi_{\text{cur},\pm1}^a$ and $\phi_{\text{cur},0}$, $\phi_{\text{cur},\pm2}$ transform in the respective rank-2 tensor, vector and scalar representations of the Lorentz algebra $so(d-1,1)$. Note that the tensor field $\phi_{\rm cur,0}^{ab}$ is symmetric $\phi_{\rm cur,0}^{ab}=\phi_{\rm cur,0}^{ba}$ and traceful $\phi_{\rm cur,0}^{aa}\neq 0$. We note that fields (5.1) have the conformal dimensions

$$\Delta_{\phi_{\rm cur,0}^{ab}}=\frac{d}{2}+\kappa,\qquad \Delta_{\phi_{\rm cur,\lambda}^{a}}=\frac{d}{2}+\kappa+\lambda,\qquad \lambda=\pm1,$$
$$\Delta_{\phi_{\rm cur,\lambda}}=\frac{d}{2}+\kappa+\lambda,\qquad \lambda=0,\pm2, \tag{5.2}$$

where κ is a dimensionless parameter. In the framework of AdS/CFT correspondence κ is related to the mass parameter m of the spin-2 massive AdS field as[8]

$$\kappa=\sqrt{m^2+\frac{d^2}{4}}. \tag{5.3}$$

We now introduce the following differential constraints:

$$\partial^b\phi_{\rm cur,0}^{ab}-\tfrac{1}{2}\partial^a\phi_{\rm cur,0}^{bb}+r_z^{00}\Box\phi_{\rm cur,-1}^{a}+r_\zeta^{00}\phi_{\rm cur,1}^{a}=0, \tag{5.4}$$

$$\partial^a\phi_{\rm cur,-1}^{a}+\tfrac{1}{2}r_z^{00}\phi_{\rm cur,0}^{aa}+\sqrt{2}r_z^{01}\Box\phi_{\rm cur,-2}+r_\zeta^{01}\phi_{\rm cur,0}=0, \tag{5.5}$$

$$\partial^a\phi_{\rm cur,1}^{a}+\tfrac{1}{2}r_\zeta^{00}\Box\phi_{\rm cur,0}^{aa}+r_z^{10}\Box\phi_{\rm cur,0}+\sqrt{2}r_\zeta^{10}\phi_{\rm cur,2}=0, \tag{5.6}$$

$$\begin{aligned}
r_z^{00}&\equiv\left(\frac{2\kappa+d}{4\kappa}\right)^{1/2},\\
r_z^{10}&\equiv\left(\frac{(2\kappa+d)(\kappa-1)d}{4\kappa(\kappa+1)(d-2)}\right)^{1/2},\\
r_z^{01}&\equiv\left(\frac{2\kappa+d-2}{4(\kappa-1)}\right)^{1/2},\\
r_\zeta^{00}&\equiv\left(\frac{2\kappa-d}{4\kappa}\right)^{1/2},\\
r_\zeta^{10}&\equiv\left(\frac{2\kappa-d+2}{4(\kappa+1)}\right)^{1/2},\\
r_\zeta^{01}&\equiv\left(\frac{(2\kappa-d)(\kappa+1)d}{4\kappa(\kappa-1)(d-2)}\right)^{1/2}.
\end{aligned} \tag{5.7}$$

One can make sure that these differential constraints are invariant under the gauge transformations

$$\begin{aligned}
\delta\phi_{\rm cur,0}^{ab}&=\partial^a\xi_{\rm cur,0}^{b}+\partial^b\xi_{\rm cur,0}^{a}+\frac{2r_z^{00}}{d-2}\eta^{ab}\Box\xi_{\rm cur,-1}\\
&\quad+\frac{2r_\zeta^{00}}{d-2}\eta^{ab}\xi_{\rm cur,1},\\
\delta\phi_{\rm cur,-1}^{a}&=\partial^a\xi_{\rm cur,-1}-r_z^{00}\xi_{\rm cur,0}^{a},\\
\delta\phi_{\rm cur,1}^{a}&=\partial^a\xi_{\rm cur,1}-r_\zeta^{00}\Box\xi_{\rm cur,0}^{a},\\
\delta\phi_{\rm cur,-2}&=-\sqrt{2}r_z^{01}\xi_{\rm cur,-1},\\
\delta\phi_{\rm cur,0}&=-r_\zeta^{01}\Box\xi_{\rm cur,-1}-r_z^{10}\xi_{\rm cur,1},\\
\delta\phi_{\rm cur,2}&=-\sqrt{2}r_\zeta^{10}\Box\xi_{\rm cur,1},
\end{aligned} \tag{5.8}$$

where $\xi_{\rm cur,0}^{a}$, $\xi_{\rm cur,\pm1}$ are gauge transformation parameters.

To complete our gauge invariant formulation we find the realization of the operator R^a on the space of gauge fields (5.1),

$$\begin{aligned}
R^a\phi_{\rm cur,0}^{bc}&=-2\kappa r_z^{00}(\eta^{ab}\phi_{\rm cur,-1}^{c}+\eta^{ac}\phi_{\rm cur,-1}^{b})\\
&\quad+\frac{4(\kappa-1)r_z^{00}}{d-2}\eta^{bc}\phi_{\rm cur,-1}^{a},\\
R^a\phi_{\rm cur,-1}^{b}&=-2\sqrt{2}(\kappa-1)r_z^{01}\eta^{ab}\phi_{\rm cur,-2},\\
R^a\phi_{\rm cur,1}^{b}&=-r_\zeta^{00}(2\kappa\phi_{\rm cur,0}^{ab}+\eta^{ab}\phi_{\rm cur,0}^{cc})\\
&\quad-2(\kappa+1)r_z^{10}\eta^{ab}\phi_{\rm cur,0},\\
R^a\phi_{\rm cur,-2}&=0,\\
R^a\phi_{\rm cur,0}&=-2(\kappa-1)r_\zeta^{01}\phi_{\rm cur,-1}^{a},\\
R^a\phi_{\rm cur,2}&=-2\sqrt{2}(\kappa+1)r_\zeta^{10}\phi_{\rm cur,1}^{a}.
\end{aligned} \tag{5.9}$$

Using (5.9), we check that the constraints (5.4), (5.5), and (5.6) are invariant under conformal algebra transformations (2.4).

As in the case of the spin-1 anomalous conformal current, we obtained the differential constraints, gauge transformations, and the realization of the operator R^a by generalizing our results for the spin-2 conformal current with the canonical dimension which we obtained in Ref. [8]. Differential constraints and gauge transformations for spin-2 anomalous conformal current can also be obtained by using the framework of tractor approach [see formulas (103) and (110) in Ref. [18]]. Obviously, our constraints (5.4), (5.5), and (5.6) and gauge transformations (5.8) can be matched with the ones in Ref. [18] by using appropriate field redefinitions. The basis of the fields we use in this paper turns out to be more convenient for the study of AdS/CFT correspondence. To summarize, our fields (5.1) can be written as a tractor rank-2 tensor field subject to a Thomas-D divergence type constraint in Ref. [18]. A similar construction was used to describe the bulk massive spin-2 field in Ref. [18]. Note that, in our approach, we use our fields (5.1) for the discussion of spin-2 anomalous conformal current.

[8]The parameter κ for the spin-2 field (5.3) should not be confused with the one for the spin-1 field (3.3).

B. Stueckelberg gauge frame

For the spin-2 anomalous conformal current, the Stueckelberg gauge frame is achieved through the use of differential constraints (5.4), (5.5), and (5.6) and the Stueckelberg gauge condition. From (5.8), we see that the vector field $\phi_{\rm cur,-1}^a$ and the scalar fields $\phi_{\rm cur,-2}$, $\phi_{\rm cur,0}$ transform as Stueckelberg fields, i.e., these fields can be gauged away via Stueckelberg gauge fixing,

$$\phi_{\rm cur,-1}^a = 0, \qquad \phi_{\rm cur,-2} = 0, \qquad \phi_{\rm cur,0} = 0. \qquad (5.10)$$

Using gauge conditions (5.10) in the constraint (5.5), we find that the field $\phi_{\rm cur,0}^{ab}$ becomes traceless, while using gauge conditions (5.10) in the constraints (5.4) and (5.6), we find that the remaining vector field $\phi_{\rm cur,1}^a$ and the scalar field $\phi_{\rm cur,2}$ can be expressed in terms of the rank-2 tensor field $\phi_{\rm cur,0}^{ab}$,

$$\begin{aligned}
\phi_{\rm cur,0}^{aa} &= 0,\\
\phi_{\rm cur,1}^a &= -\frac{1}{r_\zeta^{00}}\partial^b\phi_{\rm cur,0}^{ab}, \qquad (5.11)\\
\phi_{\rm cur,2} &= \frac{1}{\sqrt{2}r_\zeta^{00}r_\zeta^{10}}\partial^a\partial^b\phi_{\rm cur,0}^{ab}.
\end{aligned}$$

Relations (5.10) and (5.11) provide the complete description of the Stueckelberg gauge frame for the spin-2 anomalous conformal current. We note that the traceless rank-2 tensor $\phi_{\rm cur,0}^{ab}$ can be identified with the one in the standard approach to CFT.

Thus, we see that the gauge symmetries and the differential constraints make it possible to match our approach and the standard one, i.e., by gauging away the Stueckelberg fields (5.10) and by solving the differential constraints (5.4), (5.5), and (5.6) we obtain the standard formulation of the spin-2 anomalous conformal current.

C. Light-cone gauge frame

For the spin-2 anomalous conformal current, the light-cone gauge frame is achieved through the use of the differential constraints (5.4), (5.5), and (5.6) and light-cone gauge condition.

Using the gauge transformations of the fields $\phi_{\rm cur,0}^{ab}$, $\phi_{\rm cur,\pm1}^a$ (5.8), we impose the light-cone gauge,

$$\phi_{\rm cur,0}^{+a} = 0, \qquad \phi_{\rm cur,\lambda}^{+} = 0, \qquad \lambda = \pm 1. \qquad (5.12)$$

Plugging this gauge in the differential constraints (5.4), (5.5), and (5.6), we find

$$\begin{aligned}
\phi_{\rm cur,0}^{ii} &= 0,\\
\phi_{\rm cur,0}^{-i} &= -\frac{\partial^j}{\partial^+}\phi_{\rm cur,0}^{ij} - \frac{r_z^{00}}{\partial^+}\Box\phi_{\rm cur,-1}^i - \frac{r_\zeta^{00}}{\partial^+}\phi_{\rm cur,1}^i,\\
\phi_{\rm cur,0}^{--} &= \frac{\partial^i\partial^j}{\partial^+\partial^+}\phi_{\rm cur,0}^{ij} + \frac{2r_z^{00}\partial^i}{\partial^+\partial^+}\Box\phi_{\rm cur,-1}^i + \frac{2r_\zeta^{00}\partial^i}{\partial^+\partial^+}\phi_{\rm cur,1}^i\\
&\quad + \frac{\sqrt{2}r_z^{00}r_z^{10}}{\partial^+\partial^+}\Box^2\phi_{\rm cur,-2} + \frac{\sqrt{2}r_\zeta^{00}r_\zeta^{01}}{\partial^+\partial^+}\phi_{\rm cur,2}\\
&\quad + \frac{r_z^{00}r_\zeta^{01} + r_\zeta^{00}r_z^{10}}{\partial^+\partial^+}\Box\phi_{\rm cur,0},\\
\phi_{\rm cur,-1}^- &= -\frac{\partial^j}{\partial^+}\phi_{\rm cur,-1}^j - \frac{\sqrt{2}r_z^{01}}{\partial^+}\Box\phi_{\rm cur,-2} - \frac{r_\zeta^{01}}{\partial^+}\phi_{\rm cur,0},\\
\phi_{\rm cur,1}^- &= -\frac{\partial^j}{\partial^+}\phi_{\rm cur,1}^j - \frac{r_z^{10}}{\partial^+}\Box\phi_{\rm cur,0} - \frac{\sqrt{2}r_\zeta^{10}}{\partial^+}\phi_{\rm cur,2}. \qquad (5.13)
\end{aligned}$$

We see that we are left with $so(d-2)$ algebra traceless rank-2 tensor field, two vector fields, and three scalar fields,

$$\begin{array}{ccc}
 & \phi_{\rm cur,0}^{ij} & \\
\phi_{\rm cur,-1}^i, & & \phi_{\rm cur,1}^i, \\
\phi_{\rm cur,-2}, & \phi_{\rm cur,0}, & \phi_{\rm cur,2},
\end{array} \qquad (5.14)$$

which constitute the field content of the light-cone gauge frame.

VI. SPIN-2 ANOMALOUS SHADOW FIELD

A. Gauge invariant formulation

To discuss gauge invariant formulation of spin-2 anomalous shadow field in the flat space of dimension $d \geq 4$ we use one rank-2 tensor field, two vector fields, and three scalars fields,

$$\begin{array}{ccc}
 & \phi_{\rm sh,0}^{ab}, & \\
\phi_{\rm sh,-1}^a, & & \phi_{\rm sh,1}^a, \\
\phi_{\rm sh,-2}, & \phi_{\rm sh,0}, & \phi_{\rm sh,2}.
\end{array} \qquad (6.1)$$

The fields $\phi_{\rm sh,0}^{ab}$, $\phi_{\rm sh,\pm1}^a$ and $\phi_{\rm sh,0}$, $\phi_{\rm sh,\pm2}$ transform in the respective rank-2 tensor, vector and scalar representations of the Lorentz algebra $so(d-1,1)$. Note that the tensor field $\phi_{\rm sh,0}^{ab}$ is symmetric $\phi_{\rm sh,0}^{ab} = \phi_{\rm sh,0}^{ba}$ and traceful $\phi_{\rm sh,0}^{aa} \neq 0$. Conformal dimensions of the fields are given by

$$\begin{aligned}
&\Delta_{\phi_{\rm sh,0}^{ab}} = \frac{d}{2} - \kappa, \qquad \Delta_{\phi_{\rm sh,\lambda}^{a}} = \frac{d}{2} - \kappa + \lambda, \qquad \lambda = \pm 1,\\
&\Delta_{\phi_{\rm sh,\lambda}} = \frac{d}{2} - \kappa + \lambda, \qquad \lambda = 0, \pm 2. \qquad (6.2)
\end{aligned}$$

In the framework of AdS/CFT correspondence, κ is related to the mass parameter m of the spin-2 massive AdS field as in (5.3).

We now introduce the following differential constraints:

$$\partial^b \phi_{\mathrm{sh},0}^{ab} - \tfrac{1}{2}\partial^a \phi_{\mathrm{sh},0}^{bb} + r_\zeta^{00}\Box\phi_{\mathrm{sh},-1}^a + r_z^{00}\phi_{\mathrm{sh},1}^a = 0, \quad (6.3)$$

$$\partial^a \phi_{\mathrm{sh},-1}^a + \tfrac{1}{2}r_\zeta^{00}\phi_{\mathrm{sh},0}^{aa} + \sqrt{2}r_\zeta^{10}\Box\phi_{\mathrm{sh},-2} + r_z^{10}\phi_{\mathrm{sh},0} = 0, \quad (6.4)$$

$$\partial^a \phi_{\mathrm{sh},1}^a + \tfrac{1}{2}r_z^{00}\Box\phi_{\mathrm{sh},0}^{aa} + r_\zeta^{01}\Box\phi_{\mathrm{sh},0} + \sqrt{2}r_z^{01}\phi_{\mathrm{sh},2} = 0, \quad (6.5)$$

where the parameters r_ζ^{mn} and r_z^{mn} are given in (5.7). One can make sure that these constraints are invariant under the gauge transformations

$$\begin{aligned}
\delta\phi_{\mathrm{sh},0}^{ab} &= \partial^a\xi_{\mathrm{sh},0}^b + \partial^b\xi_{\mathrm{sh},0}^a + \frac{2r_z^{00}}{d-2}\eta^{ab}\xi_{\mathrm{sh},1} + \frac{2r_\zeta^{00}}{d-2}\eta^{ab}\Box\xi_{\mathrm{sh},-1},\\
\delta\phi_{\mathrm{sh},-1}^a &= \partial^a\xi_{\mathrm{sh},-1} - r_\zeta^{00}\xi_{\mathrm{sh},0}^a,\\
\delta\phi_{\mathrm{sh},1}^a &= \partial^a\xi_{\mathrm{sh},1} - r_z^{00}\Box\xi_{\mathrm{sh},0}^a,\\
\delta\phi_{\mathrm{sh},-2} &= -\sqrt{2}r_\zeta^{10}\xi_{\mathrm{sh},-1},\\
\delta\phi_{\mathrm{sh},0} &= -r_\zeta^{01}\xi_{\mathrm{sh},1} - r_z^{10}\Box\xi_{\mathrm{sh},-1},\\
\delta\phi_{\mathrm{sh},2} &= -\sqrt{2}r_z^{01}\Box\xi_{\mathrm{sh},1},
\end{aligned}\quad (6.6)$$

where $\xi_{\mathrm{sh},0}^a$, $\xi_{\mathrm{sh},\pm1}$ are gauge transformation parameters.

We then find that a realization of the operator R^a on fields (6.1) takes the following form:

$$\begin{aligned}
R^a\phi_{\mathrm{sh},0}^{bc} &= 2\kappa r_\zeta^{00}(\eta^{ab}\phi_{\mathrm{sh},-1}^c + \eta^{ac}\phi_{\mathrm{sh},-1}^b) - \frac{4(\kappa+1)r_\zeta^{00}}{d-2}\eta^{bc}\phi_{\mathrm{sh},-1}^a,\\
R^a\phi_{\mathrm{sh},-1}^b &= 2\sqrt{2}(\kappa+1)r_\zeta^{10}\eta^{ab}\phi_{\mathrm{sh},-2},\\
R^a\phi_{\mathrm{sh},1}^b &= r_z^{00}(2\kappa\phi_{\mathrm{sh},0}^{ab} - \eta^{ab}\phi_{\mathrm{sh},0}^{cc}) + 2(\kappa-1)r_\zeta^{01}\eta^{ab}\phi_{\mathrm{sh},0},\\
R^a\phi_{\mathrm{sh},-2} &= 0,\\
R^a\phi_{\mathrm{sh},0} &= 2(\kappa+1)r_z^{10}\phi_{\mathrm{sh},-1}^a,\\
R^a\phi_{\mathrm{sh},2} &= 2\sqrt{2}(\kappa-1)r_z^{01}\phi_{\mathrm{sh},1}^a.
\end{aligned}\quad (6.7)$$

Using (6.7), we check that the constraints (6.3), (6.4), and (6.5) are invariant under transformations of the conformal algebra.

We proceed with the discussion of the two-point vertex for the spin-2 anomalous shadow field. The gauge invariant two-point vertex we find takes the form given (4.8), where the two-point density Γ_{12} is given by

$$\begin{aligned}
\Gamma_{12} = {}& \frac{1}{4|x_{12}|^{2\kappa+d}}\Big(\phi_{\mathrm{sh},0}^{ab}(x_1)\phi_{\mathrm{sh},0}^{ab}(x_2) - \frac{1}{2}\phi_{\mathrm{sh},0}^{aa}(x_1)\phi_{\mathrm{sh},0}^{bb}(x_2)\Big)\\
&+ \sum_{\lambda=\pm1}\frac{\omega_\lambda}{2|x_{12}|^{2\kappa+d-2\lambda}}\phi_{\mathrm{sh},\lambda}^a(x_1)\phi_{\mathrm{sh},\lambda}^a(x_2)\\
&+ \sum_{\lambda=0,\pm2}\frac{\omega_\lambda}{2|x_{12}|^{2\kappa+d-2\lambda}}\phi_{\mathrm{sh},\lambda}(x_1)\phi_{\mathrm{sh},\lambda}(x_2),
\end{aligned}\quad (6.8)$$

$$\begin{aligned}
\omega_1 &= \frac{1}{2\kappa(2\kappa+d-2)},\\
\omega_0 &= 1,\\
\omega_{-1} &= 2(\kappa+1)(2\kappa+d),\\
\omega_2 &= \frac{1}{4\kappa(\kappa-1)(2\kappa+d-2)(2\kappa+d-4)},\\
\omega_{-2} &= 4(\kappa+1)(\kappa+2)(2\kappa+d)(2\kappa+d+2).
\end{aligned}\quad (6.9)$$

We check that this vertex is invariant under both gauge transformations (6.6) and global conformal transformations of the spin-2 anomalous shadow field. A remarkable feature of the vertex is its diagonal form with respect to the gauge fields entering the field content (6.1).

B. Stueckelberg gauge frame

For the spin-2 anomalous shadow field, the Stueckelberg gauge frame is achieved though the use of differential constraints (6.3), (6.4), and (6.5) and a Stueckelberg gauge condition. From gauge transformations (6.6), we see that the vector field $\phi_{\mathrm{sh},-1}^a$ and the scalar fields $\phi_{\mathrm{sh},-2}$, $\phi_{\mathrm{sh},0}$ transform as Stueckelberg fields, i.e., these fields can be gauged away via Stueckelberg gauge fixing,

$$\phi_{\mathrm{sh},-1}^a = 0, \qquad \phi_{\mathrm{sh},-2} = 0, \qquad \phi_{\mathrm{sh},0} = 0. \quad (6.10)$$

Using gauge conditions (6.10) in the constraint (6.4), we find that the field $\phi_{\mathrm{sh},0}^{ab}$ becomes traceless, while using gauge conditions (6.10) in the constraints (6.3) and (6.5) we find that the remaining vector field $\phi_{\mathrm{sh},1}^a$ and the scalar field $\phi_{\mathrm{sh},2}$ can be expressed in terms of the rank-2 tensor field $\phi_{\mathrm{sh},0}^{ab}$,

$$\begin{aligned}
\phi_{\mathrm{sh},0}^{aa} &= 0,\\
\phi_{\mathrm{sh},1}^a &= -\frac{1}{r_z^{00}}\partial^b\phi_{\mathrm{sh},0}^{ab},\\
\phi_{\mathrm{sh},2} &= \frac{1}{\sqrt{2}r_z^{00}r_z^{01}}\partial^a\partial^b\phi_{\mathrm{sh},0}^{ab}.
\end{aligned}\quad (6.11)$$

Relations (6.10) and (6.11) provide the complete description of the Stueckelberg gauge frame for the spin-2 anomalous shadow field.

Plugging (6.10) and (6.11) in (6.8), we find that our Γ_{12} (6.8) takes the form (up to the total derivative),

$$\Gamma_{12}^{\mathrm{Stuck.g.fram}} = k_2\Gamma_{12}^{\mathrm{stand}}, \quad (6.12)$$

$$\Gamma_{12}^{\rm stand} = \phi_{{\rm sh},0}^{a_1a_2}(x_1)\frac{O_{12}^{a_1b_1}O_{12}^{a_2b_2}}{|x_{12}|^{2\kappa+d}}\phi_{{\rm sh},0}^{b_1b_2}(x_2), \qquad (6.13)$$

$$k_2 \equiv \frac{2\kappa+d+2}{4(2\kappa+d-2)}, \qquad (6.14)$$

where O_{12}^{ab} is defined in (4.16), while $\Gamma_{12}^{\rm stand}$ (6.13) stands for the two-point vertex of the spin-2 anomalous shadow field in the standard approach to CFT. From (6.12), we see that our gauge invariant vertex taken to be in the Stueckelberg gauge frame coincides, up to normalization factor k_2, with the two-point vertex in the standard approach to CFT. The kernel of vertex $\Gamma^{\rm stand}$ (6.13) defines the two-point correlation function of the spin-2 conformal conformal current taken to be in the Stueckelberg gauge frame.

C. Light-cone gauge frame

For the spin-2 anomalous shadow field, the light-cone gauge frame is achieved through the use of differential constraints (6.3), (6.4), and (6.5) and the light-cone gauge. Taking into account the gauge transformations of the fields $\phi_{{\rm sh},0}^{ab}$, $\phi_{{\rm sh},\pm1}^{a}$ given in (6.6), we impose the light-cone gauge condition,

$$\phi_{{\rm sh},0}^{+a} = 0, \qquad \phi_{{\rm sh},\lambda}^{+} = 0, \qquad \lambda = \pm 1. \qquad (6.15)$$

Plugging this gauge condition in the constraints (6.3), (6.4), and (6.5), we find

$$\begin{aligned}
\phi_{{\rm sh},0}^{ii} &= 0,\\
\phi_{{\rm sh},0}^{-i} &= -\frac{\partial^j}{\partial^+}\phi_{{\rm sh},0}^{ij} - \frac{r_\zeta^{00}}{\partial^+}\Box\phi_{{\rm sh},-1}^{i} - \frac{r_z^{00}}{\partial^+}\phi_{{\rm sh},1}^{i},\\
\phi_{{\rm sh},0}^{--} &= \frac{\partial^i\partial^j}{\partial^+\partial^+}\Box\phi_{{\rm sh},0}^{ij} + \frac{2r_\zeta^{00}\partial^i}{\partial^+\partial^+}\phi_{{\rm sh},-1}^{i} + \frac{2r_z^{00}\partial^i}{\partial^+\partial^+}\Box^2\phi_{{\rm sh},1}^{i}\\
&\quad + \frac{\sqrt{2}r_\zeta^{00}r_\zeta^{10}}{\partial^+\partial^+}\phi_{{\rm sh},-2} + \frac{\sqrt{2}r_z^{00}r_z^{01}}{\partial^+\partial^+}\phi_{{\rm sh},2}\\
&\quad + \frac{r_z^{00}r_\zeta^{01} + r_\zeta^{00}r_z^{10}}{\partial^+\partial^+}\Box\phi_{{\rm sh},0},\\
\phi_{{\rm sh},-1}^{-} &= -\frac{\partial^j}{\partial^+}\phi_{{\rm sh},-1}^{j} - \frac{\sqrt{2}r_\zeta^{10}}{\partial^+}\Box\phi_{{\rm sh},-2} - \frac{r_z^{10}}{\partial^+}\phi_{{\rm sh},0},\\
\phi_{{\rm sh},1}^{-} &= -\frac{\partial^j}{\partial^+}\phi_{{\rm sh},1}^{j} - \frac{r_\zeta^{01}}{\partial^+}\Box\phi_{{\rm sh},0} - \frac{\sqrt{2}r_z^{01}}{\partial^+}\phi_{{\rm sh},2}.
\end{aligned} \qquad (6.16)$$

We see that we are left with the $so(d-2)$ algebra traceless rank-2 tensor field, two vector fields, and three scalar fields,

$$\begin{gathered}
\phi_{{\rm sh},0}^{ij},\\
\phi_{{\rm sh},-1}^{i}, \qquad \phi_{{\rm sh},1}^{i},\\
\phi_{{\rm sh},-2}, \qquad \phi_{{\rm sh},0}, \qquad \phi_{{\rm sh},2},
\end{gathered} \qquad (6.17)$$

which constitute a field content of the spin-2 anomalous shadow field in the light-cone gauge frame. Note that, in contrast to the Stueckelberg gauge frame, the vector fields and the scalar fields become independent field D.o.F. in the light-cone gauge frame.

Using (6.15) in (6.8) leads to the light-cone gauge-fixed vertex

$$\begin{aligned}
\Gamma_{12}^{(\rm l.c.)} &= \frac{1}{4|x_{12}|^{2\kappa+d}}\phi_{{\rm sh},0}^{ij}(x_1)\phi_{{\rm sh},0}^{ij}(x_2)\\
&\quad + \sum_{\lambda=\pm1}\frac{\omega_\lambda}{2|x_{12}|^{2\kappa+d-2\lambda}}\phi_{{\rm sh},\lambda}^{i}(x_1)\phi_{{\rm sh},\lambda}^{i}(x_2)\\
&\quad + \sum_{\lambda=0,\pm2}\frac{\omega_\lambda}{2|x_{12}|^{2\kappa+d-2\lambda}}\phi_{{\rm sh},\lambda}(x_1)\phi_{{\rm sh},\lambda}(x_2),
\end{aligned} \qquad (6.18)$$

where ω_λ are defined in (6.9). We see that, as in the case of the gauge invariant vertex (6.8), the light-cone vertex (6.18) is diagonal with respect to the fields entering the field content of the light-cone gauge frame (6.17). Note however that, in contrast to the gauge invariant vertex, the light-cone vertex is constructed out of the fields (6.17) which are not subject to any differential constraints.

As before, we see that *our gauge invariant vertex gives easy and quick access to the light-cone gauge vertex*. Namely, all that is required to get the light-cone gauge vertex (6.18) is to remove the trace of the tensor field $\phi_{{\rm sh},0}^{ab}$ and replace the $so(d-1,1)$ Lorentz algebra vector indices appearing in the gauge invariant vertex (6.8) by the vector indices of the $so(d-2)$ algebra.

The kernel of the light-cone vertex (6.18) gives the two-point correlation function of the spin-2 anomalous conformal current taken to be in the light-cone gauge. Defining two-point correlation functions for light-cone fields of the anomalous conformal current (5.14) in the usual way

$$\begin{aligned}
\langle\phi_{{\rm cur},0}^{ij}(x_1),\phi_{{\rm cur},0}^{kl}j(x_2)\rangle &\equiv \frac{\delta^2\Gamma^{(\rm l.c.)}}{\delta\phi_{{\rm sh},0}^{ij}(x_1)\delta\phi_{{\rm sh},0}^{kl}(x_2)},\\
\langle\phi_{{\rm cur},\lambda}^{i}(x_1),\phi_{{\rm cur},\lambda}^{j}(x_2)\rangle &\equiv \frac{\delta^2\Gamma^{(\rm l.c.)}}{\delta\phi_{{\rm sh},-\lambda}^{i}(x_1)\delta\phi_{{\rm sh},-\lambda}^{j}(x_2)},\\
\langle\phi_{{\rm cur},\lambda}(x_1),\phi_{{\rm cur},\lambda}(x_2)\rangle &\equiv \frac{\delta^2\Gamma^{(\rm l.c.)}}{\delta\phi_{{\rm sh},-\lambda}(x_1)\delta\phi_{{\rm sh},-\lambda}(x_2)},
\end{aligned} \qquad (6.19)$$

we obtain

$$\begin{aligned}
\langle\phi_{{\rm cur},0}^{ij}(x_1),\phi_{{\rm cur},0}^{kl}(x_2)\rangle &= \frac{1}{|x_{12}|^{2\kappa+d}}\Pi^{ij;kl},\\
\langle\phi_{{\rm cur},\lambda}^{i}(x_1),\phi_{{\rm cur},\lambda}^{j}(x_2)\rangle &= \frac{\omega_{-\lambda}}{|x_{12}|^{2\kappa+d+2\lambda}}\delta^{ij},\\
\langle\phi_{{\rm cur},\lambda}(x_1),\phi_{{\rm cur},\lambda}(x_2)\rangle &= \frac{\omega_{-\lambda}}{|x_{12}|^{2\kappa+d+2\lambda}},
\end{aligned} \qquad (6.20)$$

where ω_λ are defined in (6.9) and we use the notation

$$\Pi^{ij;kl} = \frac{1}{2}\Big(\delta^{ik}\delta^{jl} + \delta^{il}\delta^{jk} - \frac{2}{d-2}\delta^{ij}\delta^{kl}\Big). \qquad (6.21)$$

VII. TWO-POINT CURRENT-SHADOW FIELD INTERACTION VERTEX

We now discuss the two-point current-shadow field interaction vertex. In the gauge invariant approach, the interaction vertex is determined by requiring the vertex to be invariant under both gauge transformations of currents and shadow fields. Also, the interaction vertex should be invariant under conformal algebra transformations.

Spin-1.—We begin with spin-1 fields. Let us consider the following vertex:

$$\mathcal{L} = \phi_{\mathrm{cur},0}^a \phi_{\mathrm{sh},0}^a + \phi_{\mathrm{cur},-1}\phi_{\mathrm{sh},1} + \phi_{\mathrm{cur},1}\phi_{\mathrm{sh},-1}. \qquad (7.1)$$

Denoting the left-hand side of (4.3) by C_{sh} we find that under gauge transformations of the current (3.6), (3.7), and (3.8) the variation of the vertex (7.1) takes the form (up to total derivative)

$$\delta_{\xi_{\mathrm{cur},0}}\mathcal{L} = -\xi_{\mathrm{cur},0}C_{\mathrm{sh}}. \qquad (7.2)$$

From this expression, we see that the vertex $\mathcal{L}$ is invariant under gauge transformations of the current provided the shadow field satisfies the differential constraint (4.3). Denoting the left-hand side of (3.4) by C_{cur} we find that under gauge transformations of the shadow field (4.4), (4.5), and (4.6) the variation of the vertex (7.1) takes the form (up to total derivative)

$$\delta_{\xi_{\mathrm{sh}}}\mathcal{L} = -\xi_{\mathrm{sh},0}C_{\mathrm{cur}}, \qquad (7.3)$$

i.e., the vertex $\mathcal{L}$ is invariant under gauge transformations of the shadow field provided the current satisfies the differential constraint (3.4).

Making use of the realization of the conformal algebra symmetries obtained in Sections III and IV, we check that vertex $\mathcal{L}$ (7.1) is invariant under the conformal algebra transformations.

Spin-2.—We proceed with spin-2 fields. One can make sure that the following vertex:

$$\mathcal{L} = \frac{1}{2}\phi_{\mathrm{cur},0}^{ab}\phi_{\mathrm{sh},0}^{ab} - \frac{1}{4}\phi_{\mathrm{cur},0}^{aa}\phi_{\mathrm{sh},0}^{bb} + \sum_{\lambda=\pm1}\phi_{\mathrm{cur},\lambda}^a\phi_{\mathrm{sh},-\lambda}^a + \sum_{\lambda=0,\pm2}\phi_{\mathrm{cur},\lambda}\phi_{\mathrm{sh},-\lambda} \qquad (7.4)$$

is invariant under gauge transformations of the spin-2 shadow field (6.6) provided the spin-2 current satisfies the differential constraints (5.4), (5.5), and (5.6). This vertex (7.4) is also invariant under gauge transformations of the spin-2 anomalous current (5.8) provided the spin-2 shadow field satisfies the differential constraints (6.3), (6.4), and (6.5). Using the representation for generators of the conformal algebra obtained in Sections V and VI, we check that vertex $\mathcal{L}$ (7.4) is invariant under the conformal algebra transformations.

VIII. ADS/CFT CORRESPONDENCE. PRELIMINARIES

We now study AdS/CFT correspondence for free massive AdS fields and boundary anomalous conformal currents and shadow fields. To this end we use the gauge invariant CFT adapted description of AdS massive fields and the modified Lorentz and de Donder gauges found in Ref. [12]. *It is the use of our fields and the modified Lorentz and de Donder gauges that leads to the decoupled form of gauge-fixed equations of motion and a surprisingly simple Lagrangian*.[9] Owing these properties of our fields and the modified (Lorentz) de Donder gauge, we simplify significantly the computation of the effective action.[10] Note that the modified (Lorentz) de Donder gauge turns out to be invariant under on-shell leftover gauge symmetries of bulk AdS fields. Also note that, in our approach, we have gauge symmetries not only at the AdS side but also at the boundary CFT. Therefore, in the framework of our approach, the study of AdS/CFT correspondence implies the matching of:

(i) Lorentz (de Donder) gauge conditions for bulk massive fields and differential constraints for boundary anomalous conformal currents and shadow fields;
(ii) leftover on-shell gauge symmetries for bulk massive fields and gauge symmetries of boundary anomalous conformal currents and shadow fields;
(iii) on-shell global symmetries of bulk massive fields and global symmetries of boundary anomalous conformal currents and shadow fields;
(iv) an effective action evaluated on the solution of equations of motion with the Dirichlet problem corresponding to the boundary anomalous shadow field and boundary two-point gauge invariant vertex for the anomalous shadow field.

Global AdS symmetries in CFT adapted approach.—Relativistic symmetries of the AdS_{d+1} field dynamics are described by the $so(d,2)$ algebra. In d-dimensional space, global symmetries of anomalous conformal currents and shadow fields are also described by the $so(d,2)$ algebra. To discuss global symmetries of anomalous conformal currents and shadow fields we have used conformal basis of the $so(d,2)$ algebra [see (2.3)]. Therefore, for the application to the study of AdS/CFT correspondence, it is convenient to realize the relativistic bulk $so(d,2)$ algebra symmetries by using the basis of the conformal algebra. The most convenient way to achieve the conformal basis

[9]Our massive gauge fields are obtained from gauge fields used in the standard gauge invariant approach to massive fields by the invertible transformation. Details of the transformation may be found in Appendices A and B. A discussion of interesting methods for solving AdS field equations of motion without gauge fixing may be found in Refs. [28,29].

[10]We recall that the bulk action evaluated on the solution of the Dirichlet problem is referred to as effective action in this paper.

realization of bulk $so(d,2)$ symmetries is to use the Poincaré parametrization of AdS space,[11]

$$ds^2 = \frac{1}{z^2}(dx^a dx^a + dzdz). \tag{8.1}$$

In this parametrization, the $so(d,2)$ algebra transformations of the massive arbitrary spin AdS field ϕ take the form $\delta_{\hat{G}}\phi = \hat{G}\phi$, where the realization of the $so(d,2)$ algebra generators $\hat{G}$ in terms of differential operators acting on ϕ is given by

$$P^a = \partial^a, \tag{8.2}$$

$$J^{ab} = x^a\partial^b - x^b\partial^a + M^{ab}, \tag{8.3}$$

$$D = x\partial + \Delta, \qquad \Delta = z\partial_z + \frac{d-1}{2}, \tag{8.4}$$

$$K^a = K^a_{\Delta,M} + R^a, \tag{8.5}$$

$$K^a_{\Delta,M} = -\tfrac{1}{2}x^2\partial^a + x^a D + M^{ab}x^b, \tag{8.6}$$

$$R^a = R^a_{(0)} + R^a_{(1)}, \tag{8.7}$$

$$R^a_{(1)} = -\tfrac{1}{2}z^2\partial^a. \tag{8.8}$$

The operator $R^a_{(0)}$ (8.7) does not depend on boundary coordinates x^a, boundary derivatives ∂^a, and the derivative with respect to the radial coordinate, ∂_z. The operator $R^a_{(0)}$ acting on spin D.o.F. depends only on the radial coordinate z. Thus, we see all that is required to complete description of the global symmetries of AdS field dynamics is to find realization of the operator $R^a_{(0)}$ on space of gauge AdS fields.

AdS/CFT correspondence for spin-0 anomalous current and normalizable modes of scalar massive AdS field[12].—Because use of modified Lorentz (de Donder) gauge makes the study of AdS/CFT correspondence for the spin-1 (spin-2) field similar to the one the for scalar field we begin with a brief review of the AdS/CFT correspondence for the scalar field.

The action and Lagrangian for the massive scalar field in the AdS_{d+1} background take the form[13]

$$S = \int d^d x dz \mathcal{L}, \tag{8.9}$$

$$\mathcal{L} = \tfrac{1}{2}\sqrt{|g|}(g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + m^2\Phi^2). \tag{8.10}$$

[11]In our approach only $so(d-1,1)$ symmetries are realized manifestly. The $so(d,2)$ symmetries could be realized manifestly by using ambient space approach (see, e.g., [30–32]).

[12]Also see Refs. [33].

[13]From now on we use, unless otherwise specified, the Euclidian signature.

In terms of the canonical normalized field ϕ defined by relation $\Phi = z^{(d-1)/2}\phi$, the Lagrangian takes the form (up to total derivative)

$$\mathcal{L} = \tfrac{1}{2}|d\phi|^2 + \tfrac{1}{2}|\mathcal{T}_{\nu-(1/2)}\phi|^2, \tag{8.11}$$

$$\mathcal{T}_\nu \equiv \partial_z + \frac{\nu}{z}, \tag{8.12}$$

$$\nu = \sqrt{m^2 + \frac{d^2}{4}}. \tag{8.13}$$

The equation of motion obtained from Lagrangian (8.11) takes the form

$$\Box_\nu\phi = 0, \tag{8.14}$$

$$\Box_\nu \equiv \Box + \partial_z^2 - \frac{1}{z^2}\left(\nu^2 - \frac{1}{4}\right). \tag{8.15}$$

The normalizable solution of Eq. (8.14) is given by

$$\phi(x,z) = U_\nu^{\rm sc}\phi_{\rm cur}(x), \tag{8.16}$$

$$U_\nu^{\rm sc} \equiv h_\nu\sqrt{zq}J_\nu(zq)q^{-(\nu+(1/2))}, \tag{8.17}$$

$$h_\nu \equiv 2^\nu\Gamma(\nu+1), \qquad q^2 \equiv \Box, \tag{8.18}$$

where J_ν stands for the Bessel function. The asymptotic behavior of solution (8.16) is given by

$$\phi(x,z) \stackrel{z\to 0}{\longrightarrow} z^{\nu+(1/2)}\phi_{\rm cur}(x), \tag{8.19}$$

i.e., we see that spin-0 current $\phi_{\rm cur}$ is indeed boundary value of the normalizable solution.

In the case under consideration, we have no gauge symmetries and gauge conditions. Therefore, all that is required to complete the AdS/CFT correspondence is to match bulk global symmetries of the AdS field $\phi(x,z)$ and boundary global symmetries of the current $\phi_{\rm cur}(x)$. Global symmetries on the AdS side are described in (8.2)-(8.8), and those on the CFT side are described in (2.5)-(2.8), respectively. We see that the Poincaré symmetries match automatically. Using the notation $D_{\rm AdS}$ and $D_{\rm CFT}$ to indicate the respective realizations of the D symmetry on bulk fields (8.4) and conformal currents (2.7) we obtain the relation

$$D_{\rm AdS}\phi(x,z) = U_\nu^{\rm sc}D_{\rm CFT}\phi_{\rm cur}(x), \tag{8.20}$$

where the expressions for $D_{\rm CFT}$ corresponding to $\phi_{\rm cur}$ can be obtained from (2.7) by using $\Delta = \frac{d}{2} + \nu$ with ν given in (8.13). Thus, D symmetries of $\phi(x,z)$ and $\phi_{\rm cur}(x)$ also match. To match the K^a symmetries in (2.8) and (8.5) we note that the respective operators $R^a_{(0)}$ and R^a act trivially, $R^a_{(0)}\phi(x,z) = 0$, $R^a\phi_{\rm cur}(x) = 0$ and then make sure that the K^a symmetries also match.

AdS/CFT correspondence for spin-0 shadow field and non-normalizable modes of scalar massive AdS field.—Following the procedure in Ref. [34], we note that non-normalizable solution of Eq. (8.14) with the Dirichlet problem corresponding to the boundary shadow scalar field $\phi_{\rm sh}(x)$ takes the form

$$\phi(x,z) = \sigma \int d^d y G_\nu(x-y,z)\phi_{\rm sh}(y), \qquad (8.21)$$

$$G_\nu(x,z) = \frac{c_\nu z^{\nu+(1/2)}}{(z^2+|x|^2)^{\nu+(d/2)}}, \qquad (8.22)$$

$$c_\nu \equiv \frac{\Gamma(\nu+\frac{d}{2})}{\pi^{d/2}\Gamma(\nu)}. \qquad (8.23)$$

To be flexible, we use normalization factor σ in (8.21). For the case of the scalar field, a commonly used normalization in (8.21) is achieved by setting $\sigma = 1$. Asymptotic behaviors of the Green function (8.22) and solution (8.21) are well known,

$$G_\nu(x,z) \stackrel{z\to 0}{\longrightarrow} z^{-\nu+(1/2)}\delta^d(x), \qquad (8.24)$$

$$\phi(x,z) \stackrel{z\to 0}{\longrightarrow} z^{-\nu+(1/2)}\sigma\phi_{\rm sh}(x). \qquad (8.25)$$

From (8.25), we see that our solution has indeed asymptotic behavior corresponding to the shadow scalar field.

Using equations of motion (8.14) in the bulk action (8.9) with Lagrangian (8.11) we obtain the effective action given by[14]

$$-S_{\rm eff} = \int d^d x \mathcal{L}_{\rm eff}|_{z\to 0}, \qquad (8.26)$$

$$\mathcal{L}_{\rm eff} = \tfrac{1}{2}\phi \mathcal{T}_{\nu-(1/2)}\phi. \qquad (8.27)$$

Plugging the solution of the Dirichlet problem (8.21) into (8.26) and (8.27), we obtain the effective action

$$-S_{\rm eff} = \nu c_\nu \sigma^2 \int d^d x_1 d^d x_2 \frac{\phi_{\rm sh}(x_1)\phi_{\rm sh}(x_2)}{|x_{12}|^{2\nu+d}}. \qquad (8.28)$$

Using the commonly used value of σ, $\sigma = 1$, in (8.28), we obtain the properly normalized effective action found in Refs. [35,36]. An interesting novelty of our computation of $S_{\rm eff}$ is that we use the Fourier transform of the Green function. The details of our computation may be found in Appendix C in Ref. [9].

[14]Following a commonly used setup, we consider the solution of the Dirichlet problem which tends to zero as $z\to\infty$. Therefore, in (8.26), we ignore contribution to $S_{\rm eff}$ when $z=\infty$.

IX. ADS/CFT CORRESPONDENCE FOR SPIN-1 FIELDS

We now discussthe AdS/CFT correspondence for the bulk spin-1 massive AdS field and boundary spin-1 anomalous conformal current and shadow field. To this end we are going to use the CFT adapted gauge invariant Lagrangian and the modified Lorentz gauge condition [12].[15] Because our approach is closely related to the gauge invariant approach to the massive field we start with a brief review of the latter approach.

Gauge invariant approach to spin-1 massive field in AdS_{d+1} *space.*—In the gauge invariant approach, the spin-1 massive field is described by fields

$$\Phi^A, \qquad \Phi, \qquad (9.1)$$

which transform in the respective vector and scalar representations of $so(d,1)$ algebra. In the Lorentzian signature, the Lagrangian given by

$$\begin{aligned} e^{-1}\mathcal{L} &= -\tfrac{1}{4}F^{AB}F^{AB} - \tfrac{1}{2}F^A F^A, \\ F^{AB} &\equiv \mathcal{D}^A\Phi^B - \mathcal{D}^B\Phi^A, \\ F^A &\equiv \mathcal{D}^A\Phi + m\Phi^A, \end{aligned} \qquad (9.2)$$

is invariant under the gauge transformations

$$\delta\Phi^A = \mathcal{D}^A\Xi, \qquad \delta\Phi = -m\Xi. \qquad (9.3)$$

Details of our notation may be found in Appendix A. Lagrangian (9.2) can be cast into the form which is more convenient for our purposes,

$$e^{-1}\mathcal{L} = \tfrac{1}{2}\Phi^A(\mathcal{D}^2 - m^2 + d)\Phi^A + \tfrac{1}{2}\Phi(\mathcal{D}^2 - m^2)\Phi + \tfrac{1}{2}C_{\rm st}^2, \qquad (9.4)$$

$$C_{\rm st} \equiv \mathcal{D}^C\Phi^C + m\Phi. \qquad (9.5)$$

A. CFT adapted gauge invariant approach to spin-1 massive field in AdS_{d+1}

In our approach, the spin-1 massive AdS field is described by fields

$$\phi^a, \qquad \phi_{-1}, \qquad \phi_1, \qquad (9.6)$$

which are the respective vector and scalar fields of the $so(d)$ algebra. Fields in (9.6) are related by invertible transformation with fields in (9.1) (see Appendix A). The CFT adapted gauge invariant action and Lagrangian for fields (9.6) take the form,

$$S = \int d^d x dz \mathcal{L}, \qquad (9.7)$$

[15]For the spin-1 massless field, the modified Lorentz gauge was found in Ref. [37], while for the massless arbitrary spin field the modified de Donder gauge was discovered in Ref. [38].

$$\mathcal{L} = \frac{1}{2}|d\phi^a|^2 + \frac{1}{2}|\mathcal{T}_{\kappa-(1/2)}\phi^a|^2 + \frac{1}{2}\sum_{\lambda=\pm1}(|d\phi_\lambda|^2 + |\mathcal{T}_{\kappa-(1/2)+\lambda}\phi_\lambda|^2) - \frac{1}{2}C^2, \quad (9.8)$$

$$C \equiv \partial^a\phi^a + r_\zeta^{00}\mathcal{T}_{\kappa+(1/2)}\phi_1 + r_z^{00}\mathcal{T}_{-\kappa+(1/2)}\phi_{-1}, \quad (9.9)$$

where $\mathcal{T}_\nu$ is given in (8.12), while κ and r_z^{00}, r_ζ^{00} are defined in (3.3) and (3.5), respectively. Lagrangian (9.8) is invariant under gauge transformations

$$\delta\phi^a = \partial^a\xi, \quad (9.10)$$

$$\delta\phi_{-1} = r_z^{00}\mathcal{T}_{\kappa-(1/2)}\xi, \quad (9.11)$$

$$\delta\phi_1 = r_\zeta^{00}\mathcal{T}_{-\kappa-(1/2)}\xi, \quad (9.12)$$

where ξ is a gauge transformation parameter. Details of the derivation of Lagrangian (9.8) from the one in (9.4) may be found in Appendix A.

Gauge invariant equations of motion obtained from Lagrangian (9.8) take the form

$$\Box_\kappa\phi^a - \partial^a C = 0,$$
$$\Box_{\kappa-1}\phi_{-1} - r_z^{00}\mathcal{T}_{\kappa-(1/2)}C = 0, \quad (9.13)$$
$$\Box_{\kappa+1}\phi_1 - r_\zeta^{00}\mathcal{T}_{-\kappa-(1/2)}C = 0,$$

where the operator $\Box_\nu$ is given in (8.15).

Global AdS symmetries in CFT adapted approach.—The general form of the realization of global symmetries for arbitrary spin AdS field was given in (8.2), (8.3), (8.4), and (8.5). All that is required to complete the description of the global symmetries is to find the realization of the operator $R_{(0)}^a$ on the space of gauge fields. For the case of the spin-1 massive field, the realization of the operator $R_{(0)}^a$ on the space of gauge fields (9.6) is given by

$$R_{(0)}^a\phi^b = z\eta^{ab}r_\zeta^{00}\phi_1 + z\eta^{ab}r_z^{00}\phi_{-1},$$
$$R_{(0)}^a\phi_{-1} = -zr_z^{00}\phi^a, \quad (9.14)$$
$$R_{(0)}^a\phi_1 = -zr_\zeta^{00}\phi^a.$$

Modified Lorentz gauge.—Modified Lorentz gauge is defined to be

$$C = 0, \qquad \text{modified Lorentz gauge}, \quad (9.15)$$

where C is given in (9.9). Using this gauge condition in equations of motion (9.13) gives simple gauge-fixed equations of motion,

$$\Box_\kappa\phi^a = 0, \qquad \Box_{\kappa+\lambda}\phi_\lambda = 0, \qquad \lambda = \pm1. \quad (9.16)$$

Thus, we see that the gauge-fixed equations of motion are decoupled.

We note that the modified Lorentz gauge and gauge-fixed equations have leftover on-shell gauge symmetry. Namely, modified Lorentz gauge (9.15) and gauge-fixed equations (9.16) are invariant under gauge transformations given in (9.10), (9.11), and (9.12) provided the gauge transformation parameter satisfies the equation

$$\Box_\kappa\xi = 0. \quad (9.17)$$

B. AdS/CFT correspondence for anomalous current and normalizable modes of massive AdS field

We now ready to discuss AdS/CFT correspondence for the spin-1 massive AdS field and spin-1 anomalous conformal current. We begin with an analysis of the normalizable solution of Eqs. (9.16). The normalizable solution of Eqs. (9.16) takes the form

$$\phi^a(x,z) = U_\kappa\phi_{\text{cur},0}^a(x),$$
$$\phi_{-1}(x,z) = -U_{\kappa-1}\phi_{\text{cur},-1}(x), \quad (9.18)$$
$$\phi_1(x,z) = U_{\kappa+1}\phi_{\text{cur},1}(x),$$

$$U_\nu \equiv h_\kappa\sqrt{zq}J_\nu(zq)q^{-(\nu+(1/2))}, \quad (9.19)$$

$$h_\kappa \equiv 2^\kappa\Gamma(\kappa+1), \qquad q^2 \equiv \Box. \quad (9.20)$$

Note that we do not show explicitly the dependence of U_ν on parameter κ (3.3). The asymptotic behavior of solution (9.18) is given by

$$\phi^a(x,z) \stackrel{z\to0}{\longrightarrow} z^{\kappa+(1/2)}\phi_{\text{cur},0}^a(x),$$
$$\phi_{-1}(x,z) \stackrel{z\to0}{\longrightarrow} -2\kappa z^{\kappa-(1/2)}\phi_{\text{cur},-1}(x), \quad (9.21)$$
$$\phi_1(x,z) \stackrel{z\to0}{\longrightarrow} \frac{z^{\kappa+(3/2)}}{2(\kappa+1)}\phi_{\text{cur},1}(x).$$

From (9.21), we see that $\phi_{\text{cur},0}^a$, $\phi_{\text{cur},\pm1}$ are indeed boundary values of the normalizable solution. In the right-hand side of (9.18) we use the notation $\phi_{\text{cur},0}^a$, $\phi_{\text{cur},\pm1}$ since we are going to demonstrate that these boundary values are indeed the gauge fields entering the gauge invariant formulation of the spin-1 anomalous conformal current in Sec. III. Namely, one can prove the following statements:

(i) For normalizable solution (9.18), modified Lorentz gauge condition (9.15) leads to the differential constraint (3.4) of the spin-1 anomalous conformal current.

(ii) *Leftover on-shell* gauge transformations (9.10), (9.11), and (9.12) of normalizable solution (9.18) lead to gauge transformations (3.6), (3.7), and (3.8) of the spin-1 anomalous conformal current.[16]

[16]Transformations given in (9.10), (9.11), and (9.12) are off-shell gauge transformations. Leftover on-shell gauge transformations are obtained from (9.10), (9.11), and (9.12) by using the gauge transformation parameter which satisfies Eq. (9.17).

(iii) On-shell global $so(d,2)$ symmetries of the normalizable modes of the spin-1 massive AdS_{d+1} field become global $so(d,2)$ conformal symmetries of the spin-1 anomalous conformal current.

These statements can easily be proved by using the following relations for the operator U_ν:

$$\mathcal{T}_{\nu-(1/2)}U_\nu = U_{\nu-1}, \tag{9.22}$$

$$\mathcal{T}_{-\nu-(1/2)}U_\nu = -U_{\nu+1}\Box, \tag{9.23}$$

$$\mathcal{T}_{-\nu+(1/2)}(zU_\nu) = -zU_{\nu+1}\Box + 2U_\nu, \tag{9.24}$$

$$\Box_\nu(zU_{\nu+1}) = 2U_\nu, \tag{9.25}$$

which, in turn, can be obtained by using the following well-known identities for the Bessel function:

$$\mathcal{T}_\nu J_\nu = J_{\nu-1}, \qquad \mathcal{T}_{-\nu}J_\nu = -J_{\nu+1}. \tag{9.26}$$

Matching of the bulk modified Lorentz gauge and boundary constraint.—As an illustration, we demonstrate how the differential constraint for the anomalous conformal current (3.4) can be obtained from the modified Lorentz gauge condition (9.15). To this end, adapting relations (9.22) and (9.23) for the respective $\nu = \kappa + 1$ and $\nu = \kappa - 1$ we obtain the relations

$$\mathcal{T}_{\kappa+(1/2)}U_{\kappa+1} = U_\kappa, \qquad \mathcal{T}_{-\kappa+(1/2)}U_{\kappa-1} = -U_\kappa. \tag{9.27}$$

Plugging solutions ϕ^a, $\phi_{\pm1}$ (9.18) in C (9.9) and using (9.27) we obtain the relation

$$C = U_\kappa C_{\mathrm{cur}}, \tag{9.28}$$

where C_{cur} stands for the left-hand side of (3.4). From (9.28), we see that our modified Lorentz gauge condition $C = 0$ (9.15) leads indeed to a differential constraint for the anomalous conformal current (3.4).

Matching of bulk and boundary gauge symmetries.—As the second illustration, we demonstrate how gauge transformations of the anomalous conformal current (3.6), (3.7), and (3.8) can be obtained from leftover on-shell gauge transformations of the massive AdS field (9.10), (9.11), and (9.12). To this end we note that the corresponding normalizable solution of the equation for gauge transformation parameter (9.17) takes the form

$$\xi(x,z) = U_\kappa \xi_{\mathrm{cur},0}(x). \tag{9.29}$$

Plugging ϕ^a (9.18) and ξ (9.29) in (9.10), we see that (9.10) leads indeed to (3.6). To match boundary gauge transformation (3.7) and bulk gauge transformation (9.11) we plug the solution for ξ (9.29) in bulk gauge transformation (9.11) and adapt relation (9.22) for $\nu = \kappa$ to obtain

$$\delta\phi_{-1}(x,z) = r_z^{00}\mathcal{T}_{\kappa-(1/2)}U_\kappa\xi_{\mathrm{cur},0}(x) = U_{\kappa-1}r_z^{00}\xi_{\mathrm{cur},0}(x) \tag{9.30}$$

on the one hand. On the other hand, the solution for ϕ_{-1} (9.18) implies

$$\delta\phi_{-1}(x,z) = -U_{\kappa-1}\delta\phi_{\mathrm{cur},-1}(x). \tag{9.31}$$

Comparing (9.30) and (9.31) we see that boundary gauge transformation (3.7) and bulk gauge transformation (9.11) match. In the same way one can make sure that the remaining boundary gauge transformation (3.8) and bulk gauge transformation (9.12) also match.

Matching of bulk and boundary global symmetries.—We note that the representation for generators given in (8.2), (8.3), (8.4), and (8.5) is valid for the gauge invariant theory of AdS fields. This to say that our modified Lorentz gauge respects the Poincaré and dilatation symmetries, but breaks the conformal boost symmetries (K^a symmetries). In other words, expressions for generators P^a, J^{ab}, and D given in (8.2), (8.3), and (8.4) are still valid for the gauge-fixed AdS fields, while the expression for the generator K^a (8.5) should be modified to restore K^a symmetries for the gauge-fixed AdS fields. Therefore, let us first demonstrate the matching of the Poincaré and dilatation symmetries. What is required is to demonstrate the matching of the $so(d,2)$ algebra generators for bulk AdS field given in (8.2), (8.3), and (8.4) and the ones for the boundary conformal current given in (2.5), (2.6), and (2.7). As for generators of the Poincaré algebra, P^a, J^{ab}, they already coincide on both sides [see formulas (2.5) and (2.6) and the respective formulas (8.2) and (8.3)]. Next, consider the dilatation generator D. Here we need an explicit form of the solution to the bulk theory equations of motion given in (9.18). Using the notations D_{AdS} and D_{CFT} to indicate the respective realizations of the dilatation generator D on bulk field (8.4) and boundary current (2.7), we obtain the relations

$$\begin{aligned} D_{\mathrm{AdS}}\phi^a(x,z) &= U_\kappa D_{\mathrm{CFT}}\phi^a_{\mathrm{cur},0}(x),\\ D_{\mathrm{AdS}}\phi_{-1}(x,z) &= -U_{\kappa-1}D_{\mathrm{CFT}}\phi_{\mathrm{cur},-1}(x),\\ D_{\mathrm{AdS}}\phi_1(x,z) &= U_{\kappa+1}D_{\mathrm{CFT}}\phi_{\mathrm{cur},1}(x), \end{aligned} \tag{9.32}$$

where D_{CFT} corresponding to $\phi^a_{\mathrm{cur},0}$, $\phi_{\mathrm{cur},-1}$, $\phi_{\mathrm{cur},1}$ can be obtained from (2.7) and the respective conformal dimensions (3.2). Thus, the generators D_{AdS} and D_{CFT} also match.

We now turn to the matching of the K^a symmetries. As we have already said, our modified Lorentz gauge breaks the K^a symmetries. To demonstrate this we note that K^a transformations of gauge fields (9.6) are given by

$$\begin{aligned} K^a\phi^b &= K^a_\Delta\phi^b + M^{abe}\phi^e + z\eta^{ab}r_\zeta^{00}\phi_1 + z\eta^{ab}r_z^{00}\phi_{-1}\\ &\quad - \tfrac{1}{2}z^2\partial^a\phi^b,\\ K^a\phi_1 &= K^a_\Delta\phi_1 - zr_\zeta^{00}\phi^a - \tfrac{1}{2}z^2\partial^a\phi_1,\\ K^a\phi_{-1} &= K^a_\Delta\phi_{-1} - zr_z^{00}\phi^a - \tfrac{1}{2}z^2\partial^a\phi_{-1}, \end{aligned} \tag{9.33}$$

where K^a_Δ and M^{abc} are defined in (2.12) and (2.13), while Δ is given in (8.4). Using these transformation rules we find that C (9.9) transforms as

$$K^a C = K^a_{\Delta+1} C - \tfrac{1}{2} z^2 \partial^a C - 2\phi^a, \tag{9.34}$$

i.e., we see that the modified Lorentz gauge condition $C = 0$ is not invariant under the K^a transformations,

$$K^a C|_{C=0} = -2\phi^a. \tag{9.35}$$

This implies that generator K^a given in (8.5) should be modified to restore the K^a symmetries of the gauge-fixed AdS field theory. To restore these broken K^a symmetries we should, following standard procedure, add compensating gauge transformations to maintain the K^a symmetries. Thus, in order to find improved $K^a_{\rm impr}$ transformations of the gauge-fixed AdS fields (9.6) we start with the generic global K^a transformations (9.33) supplemented by the appropriate compensating gauge transformations

$$\begin{aligned} K^a_{\rm impr} \phi^b &= K^a \phi^b + \partial^b \xi^{K^a}, \\ K^a_{\rm impr} \phi_{-1} &= K^a \phi_{-1} + r_z^{00} \mathcal{T}_{\kappa-(1/2)} \xi^{K^a}, \\ K^a_{\rm impr} \phi_1 &= K^a \phi_1 + r_\zeta^{00} \mathcal{T}_{-\kappa-(1/2)} \xi^{K^a}, \end{aligned} \tag{9.36}$$

where ξ^{K^a} stands for the parameter of the compensating gauge transformations. Computing the $K^a_{\rm impr}$ transformation of C

$$K^a_{\rm impr} C = K^a_{\Delta+1} C - \tfrac{1}{2} z^2 \partial^a C - 2\phi^a + \Box_\kappa \xi^{K^a}, \tag{9.37}$$

and requiring the $K^a_{\rm impr}$ transformation to maintain the gauge condition $C = 0$,

$$K^a_{\rm impr} C|_{C=0} = 0, \tag{9.38}$$

we get the equation for ξ^{K^a}

$$\Box_\kappa \xi^{K^a} - 2\phi^a = 0. \tag{9.39}$$

Thus, we obtain the nonhomogeneous second-order differential equation for the compensating gauge transformation parameter ξ^{K^a}. Plugging normalizable solution (9.18) in (9.39) we obtain the equation:

$$\Box_\kappa \xi^{K^a}(x, z) = 2U_\kappa \phi^a_{{\rm cur},0}(x). \tag{9.40}$$

Using (9.25), the solution to Eq. (9.40) is easily found to be

$$\xi^{K^a}(x, z) = z U_{\kappa+1} \phi^a_{{\rm cur},0}(x). \tag{9.41}$$

Plugging (9.18) and (9.41) in (9.36), we make sure that improved $K^a_{\rm impr}$ transformations lead to the conformal boost transformations for the spin-1 anomalous conformal current given in (2.4) and (2.8) with operator R^a defined in (3.9).

C. AdS/CFT correspondence for anomalous shadow field and non-normalizable mode of massive AdS field

We proceed to a discussion of AdS/CFT correspondence for the bulk spin-1 massive AdS field and boundary spin-1 anomalous shadow field.

Matching of the effective action and boundary two-point vertex.—In order to find the bulk effective action $S_{\rm eff}$ we should, following the standard strategy, solve the bulk equations of motion with the Dirichlet problem corresponding to the boundary anomalous shadow field and plug the solution into the bulk action. Using gauge invariant equations of motion (9.13) in bulk action (9.7), we obtain the following effective action:

$$S_{\rm eff} = -\int d^d x \mathcal{L}_{\rm eff}|_{z\to 0}, \tag{9.42}$$

$$\begin{aligned} \mathcal{L}_{\rm eff} = {}& \frac{1}{2}\phi^a \mathcal{T}_{\kappa-(1/2)} \phi^a + \frac{1}{2}\sum_{\lambda=\pm 1} \phi_\lambda \mathcal{T}_{\kappa-(1/2)+\lambda} \phi_\lambda \\ & - \frac{1}{2}(r_z^{00}\phi_{-1} + r_\zeta^{00}\phi_1) C. \end{aligned} \tag{9.43}$$

As we have already seen, the use of the modified Lorentz gauge considerably simplifies the equations of motion. Now, using modified Lorentz gauge (9.15) in (9.43), we obtain

$$\mathcal{L}_{\rm eff}|_{C=0} = \frac{1}{2}\phi^a \mathcal{T}_{\kappa-(1/2)} \phi^a + \frac{1}{2}\sum_{\lambda=\pm 1} \phi_\lambda \mathcal{T}_{\kappa-(1/2)+\lambda} \phi_\lambda, \tag{9.44}$$

i.e. we see that $\mathcal{L}_{\rm eff}$ is also simplified. In order to find $S_{\rm eff}$ we should solve gauge-fixed equations of motion (9.16) with the Dirichlet problem corresponding to the boundary anomalous shadow field and plug the solution into (9.44). We now discuss the solution to equations of motion (9.16).

Because gauge-fixed equations of motion (9.16) are similar to the ones for the scalar AdS field (8.14) we can simply apply the result in Sec. VIII. This is to say that the solution of Eqs. (9.16) with the Dirichlet problem corresponding to the spin-1 anomalous shadow field takes the form

$$\begin{aligned} \phi^a(x, z) &= \sigma_{1,0} \int d^d y G_\kappa(x-y, z) \phi^a_{{\rm sh},0}(y), \\ \phi_{-1}(x, z) &= \sigma_{0,-1} \int d^d y G_{\kappa-1}(x-y, z) \phi_{{\rm sh},1}(y), \\ \phi_1(x, z) &= \sigma_{0,1} \int d^d y G_{\kappa+1}(x-y, z) \phi_{{\rm sh},-1}(y), \end{aligned} \tag{9.45}$$

$$\sigma_{1,0} \equiv 1, \tag{9.46}$$

$$\sigma_{0,-1} \equiv -\frac{1}{2(\kappa-1)}, \qquad \sigma_{0,1} \equiv 2\kappa, \tag{9.47}$$

where the Green function is given in (8.22).

Using the asymptotic behavior of the Green function G_ν (8.24), we find the asymptotic behavior of our solution

$$\phi^a(x,z) \stackrel{z\to 0}{\longrightarrow} z^{-\kappa+(1/2)}\phi^a_{\mathrm{sh},0}(x),$$
$$\phi_{-1}(x,z) \stackrel{z\to 0}{\longrightarrow} -\frac{z^{-\kappa+(3/2)}}{2(\kappa-1)}\phi_{\mathrm{sh},1}(x), \tag{9.48}$$
$$\phi_1(x,z) \stackrel{z\to 0}{\longrightarrow} 2\kappa z^{-\kappa-(1/2)}\phi_{\mathrm{sh},-1}(x).$$

From these expressions, we see that our solution has indeed asymptotic behavior corresponding to the spin-1 anomalous shadow field. Note that because the solution has non-integrable asymptotic behavior (9.48), such a solution is referred to as the non-normalizable solution in the literature.

We now explain the choice of the normalization factors $\sigma_{1,0}$, $\sigma_{0,\pm 1}$ in (9.46) and (9.47). The choice of $\sigma_{1,0}$ is a matter of convention. Following commonly used convention, we set this normalization factor to be equal to 1. The remaining normalization factors $\sigma_{0,\pm 1}$ are then determined uniquely by requiring that the modified Lorentz gauge condition for the spin-1 massive AdS field (9.15) amount to the differential constraint for the spin-1 anomalous shadow field (4.3). With the choice made in (9.46) and (9.47) we find the relations

$$\partial^a\phi^a = \int d^d y G_\kappa(x-y,z)\partial^a\phi^a_{\mathrm{sh},0}(y),$$
$$\mathcal{T}_{-\kappa+(1/2)}\phi_{-1} = \int d^d y G_\kappa(x-y,z)\phi_{\mathrm{sh},1}(y), \tag{9.49}$$
$$\mathcal{T}_{\kappa+(1/2)}\phi_1 = \int d^d y G_\kappa(x-y,z)\Box\phi_{\mathrm{sh},-1}(y).$$

From these relations and (9.9), we see that our choice of $\sigma_{1,\pm 1}$ (9.47) allows us to match the modified Lorentz gauge for the spin-1 massive AdS field (9.15) and the differential constraint for the spin-1 anomalous shadow field given in (4.3). We note the helpful relations for the Green function which we use for the derivation of relations (9.49),

$$\mathcal{T}_{-\kappa+(1/2)}G_{\kappa-1} = -2(\kappa-1)G_\kappa,$$
$$\mathcal{T}_{\kappa+(1/2)}G_{\kappa+1} = \frac{1}{2\kappa}\Box G_\kappa, \tag{9.50}$$

where $G_\nu \equiv G_\nu(x-y,z)$.

All that remains to obtain S_{eff} is to plug the solution of the Dirichlet problem for the AdS field (9.45) into (9.42) and (9.44). Using the general formula given in (8.28), we obtain

$$-S_{\mathrm{eff}} = 2\kappa c_\kappa \Gamma, \tag{9.51}$$

where κ and c_κ are defined in (3.3) and (8.23), respectively, and Γ is the gauge invariant two-point vertex of the spin-1 anomalous shadow field given in (4.8) and (4.9).

Thus we see that imposing the modified Lorentz gauge on the spin-1 massive AdS field and computing the bulk action on the solution of equations of motion with the Dirichlet problem corresponding to the boundary anomalous shadow field, we obtain the gauge invariant two-point vertex of the spin-1 anomalous shadow field.

Because in the literature S_{eff} is expressed in terms of the two-point vertex taken in the Stueckelberg gauge frame, Γ^{stand} (4.15), we use (4.14) and represent our result (9.51) as

$$-S_{\mathrm{eff}} = \frac{\kappa(2\kappa+d)}{2\kappa+d-2}c_\kappa\Gamma^{\mathrm{stand}}. \tag{9.52}$$

This relation was obtained in Ref. [10]. The fact that S_{eff} is proportional to Γ^{stand} is expected because of the conformal symmetry, but for the systematical study of AdS/CFT correspondence it is important to know the normalization factor in front of Γ^{stand} (9.52). Our normalization factor coincides with the one found in Ref. [10].[17]

Note that we have obtained the more general relation given in (9.51), while relation (9.52) is obtained from (9.51) by using the Stueckelberg gauge frame. An attractive feature of our approach is that it provides the possibility to use other gauge conditions which might be preferable in certain applications. This is to say that, in the light-cone gauge frame, relation (9.51) takes the form

$$-S_{\mathrm{eff}} = 2\kappa c_\kappa \Gamma^{(\mathrm{l.c.})}. \tag{9.53}$$

Note that the transformation of relation (9.52) to the one in (9.53) requires cumbersome computations because the Stueckelberg gauge frame removes the scalar field entering the light-cone gauge frame (see Secs. IV B and IV C). It is relation (9.53) that seems to be the most suitable for the study of the duality of the light-cone gauge Green-Schwarz AdS superstring and the corresponding boundary gauge theory.

Matching of bulk and boundary gauge symmetries.—The modified Lorentz gauge (9.15) and gauge-fixed equations (9.16) are invariant under gauge transformations given in (9.10), (9.11), and (9.12) provided the gauge transformation parameter satisfies Eq. (9.17). The non-normalizable solution to this equation is given by

$$\xi(x,z) = \int d^d y G_\kappa(x-y,z)\xi_{\mathrm{sh}}(y). \tag{9.54}$$

We now note that, on the one hand, plugging (9.54) in (9.10), (9.11), and (9.12) and using relations (9.50) we represent the on-shell gauge transformations of $\phi^a(x,z)$, $\phi_{-1}(x,z)$, and $\phi_1(x,z)$ as

[17]The computation of S_{eff} for the spin-1 massless field may be found in Ref. [36] and, in the framework of our approach, in Ref. [9].

$$\delta\phi^a = \int d^d y G_\kappa(x-y,z)\partial^a \xi_{\rm sh}(y),$$
$$\delta\phi_{-1} = \frac{r_z^{00}}{2(\kappa-1)}\int d^d y G_{\kappa-1}(x-y,z)\Box\xi_{\rm sh}(y), \qquad (9.55)$$
$$\delta\phi_1 = -2\kappa r_\zeta^{00}\int d^d y G_{\kappa+1}(x-y,z)\xi_{\rm sh}(y).$$

On the other hand, relations (9.45) imply

$$\delta\phi^a(x,z) = \sigma_{1,0}\int d^d y G_\kappa(x-y,z)\delta\phi_{{\rm sh},0}^a(y),$$
$$\delta\phi_{-1}(x,z) = \sigma_{1,-1}\int d^d y G_{\kappa-1}(x-y,z)\delta\phi_{{\rm sh},1}(y), \qquad (9.56)$$
$$\delta\phi_1(x,z) = \sigma_{1,1}\int d^d y G_{\kappa+1}(x-y,z)\delta\phi_{{\rm sh},-1}(y).$$

Comparing (9.55) with (9.56) we see that the on-shell leftover gauge symmetries of the solution of the Dirichlet problem for the spin-1 massive AdS field amount to gauge symmetries of the spin-1 anomalous shadow field (4.4), (4.5), and (4.6).

Matching of bulk and boundary global symmetries.—The matching can be demonstrated by following the procedure we used for the spin-1 anomalous current in Sec. IX B. Therefore to avoid repetitions we briefly discuss some necessary details. The matching of bulk and boundary Poincaré symmetries is obvious. Using conformal dimensions for the spin-1 anomalous shadow field given in (4.2), the solution for bulk fields in (9.45), and the bulk dilatation operator (8.4) we make sure that the dilatation bulk and boundary symmetries also match. In order to match K^a symmetries we consider improved $K_{\rm impr}^a$ transformations with compensating gauge transformation parameters satisfying Eqs. (9.39). Using the relation for the Green function

$$\Box_\nu(zG_{\nu-1}) = -4(\nu-1)G_\nu, \qquad (9.57)$$

it is easy to see that the solution to Eq. (9.39) with ϕ^a as in (9.45) is given by

$$\xi^{K^a}(x,z) = z\sigma_{1,0}^\xi\int d^d y G_{\kappa-1}(x-y,z)\phi_{{\rm sh},0}^a(y), \qquad (9.58)$$

$$\sigma_{1,0}^\xi \equiv -\frac{1}{2(\kappa-1)}. \qquad (9.59)$$

Using (9.45) and (9.58) in (9.36), we make sure that improved bulk $K_{\rm impr}^a$ symmetries amount to K^a symmetries of the spin-1 anomalous shadow field given in (2.8) and (4.7).

To summarize, we note that it is the matching of the bulk on-shell leftover gauge symmetries of the solution to the Dirichlet problem and bulk global symmetries and the respective boundary gauge symmetries of the anomalous shadow field and boundary global symmetries that explains why the effective action coincides with the gauge invariant two-point vertex for the boundary anomalous shadow field [see (9.51)].

X. ADS/CFT CORRESPONDENCE FOR SPIN-2 FIELDS

Before discussing AdS/CFT correspondence for the spin-2 massive AdS field and spin-2 anomalous conformal current and shadow field we present our CFT adapted gauge invariant approach to the spin-2 massive AdS field. Because our approach is closely related to the gauge invariant approach to the massive field we start with a brief review of the latter approach.

Gauge invariant approach to the spin-2 massive field in AdS$_{d+1}$ *space.*—In the gauge invariant approach, the spin-2 massive field is described by gauge fields

$$\Phi^{AB}, \qquad \Phi^A, \qquad \Phi, \qquad (10.1)$$

which transform in the respective rank-2 tensor, vector, and scalar representations of $so(d,1)$ algebra. In the Lorentzian signature, the Lagrangian found in Ref. [39] takes the form[18]

$$\frac{1}{e}\mathcal{L} = \frac14\Phi^{AB}E_{\rm EH}\Phi^{AB} + \frac12\Phi^A E_{\rm Max}\Phi^A + \frac12\Phi\mathcal{D}^2\Phi + m\Phi^A(\mathcal{D}^B\Phi^{BA} - \mathcal{D}^A\Phi^{BB}) + f\Phi\mathcal{D}^A\Phi^A - \frac{m^2-2}{4}\Phi^{AB}\Phi^{AB} + \frac{m^2+d-2}{4}\Phi^{AA}\Phi^{BB} + \frac{fm}{2}\Phi^{AA}\Phi - \frac{d}{2}\Phi^A\Phi^A + \frac{(d+1)m^2}{2(d-1)}\Phi^2, \qquad (10.2)$$

$$f \equiv \Big(\frac{2d}{d-1}m^2 + 2d\Big)^{1/2}, \qquad (10.3)$$

where the respective second-derivative Einstein-Hilbert and Maxwell operators $E_{\rm EH}$, $E_{\rm Max}$ are given by

$$E_{\rm EH}\Phi^{AB} = \mathcal{D}^2\Phi^{AB} - \mathcal{D}^A\mathcal{D}^C\Phi^{CB} - \mathcal{D}^B\mathcal{D}^C\Phi^{CA} + \mathcal{D}^A\mathcal{D}^B\Phi^{CC} + \eta^{AB}(\mathcal{D}^C\mathcal{D}^E\Phi^{CE} - \mathcal{D}^2\Phi^{CC}),$$
$$E_{\rm Max}\Phi^A = \mathcal{D}^2\Phi^A - \mathcal{D}^A\mathcal{D}^B\Phi^B. \qquad (10.4)$$

Lagrangian (10.2) is invariant under gauge transformations

$$\delta\Phi^{AB} = \mathcal{D}^A\Xi^B + \mathcal{D}^B\Xi^A + \frac{2m}{d-1}\eta^{AB}\Xi,$$
$$\delta\Phi^A = \mathcal{D}^A\Xi - m\Xi^A, \qquad (10.5)$$
$$\delta\Phi = -f\Xi,$$

where Ξ^A, Ξ are gauge transformation parameters. In Ref. [12], we found new representation for Lagrangian (10.2),

[18] A recent interesting discussion of massive AdS fields may be found in [40].

$$\frac{1}{e}\mathcal{L} = \frac{1}{4}\Phi^{AB}(\mathcal{D}^2 - m^2 + 2)\Phi^{AB} - \frac{1}{8}\Phi^{AA}(\mathcal{D}^2 - m^2 - 2d + 4)\Phi^{BB} + \frac{1}{2}\Phi^A(\mathcal{D}^2 - m^2 - d)\Phi^A + \frac{1}{2}\Phi(\mathcal{D}^2 - m^2 - 2d)\Phi + \frac{1}{2}C_{\rm st}^A C_{\rm st}^A + \frac{1}{2}C_{\rm st}^2, \tag{10.6}$$

$$C_{\rm st}^A = \mathcal{D}^B\Phi^{BA} - \frac{1}{2}\mathcal{D}^A\Phi^{BB} + m\Phi^A, \qquad C_{\rm st} = \mathcal{D}^A\Phi^A + \frac{m}{2}\Phi^{AA} + f\Phi. \tag{10.7}$$

From (10.6), we see that it is the use of quantities $C_{\rm st}^A$ and $C_{\rm st}$ that simplifies the structure of the gauge invariant Lagrangian. We note also that the relations $C_{\rm st}^A = 0$, $C_{\rm st} = 0$ define the standard de Donder gauge condition for the spin-2 massive field.[19]

Interrelation of the gauge invariant Lagrangian and Pauli-Fierz Lagrangian.—As is well known, the spin-2 massive AdS field can be described by the Pauli-Fierz Lagrangian given by

$$\frac{1}{e}\mathcal{L}_{\rm PF} = \frac{1}{4}\Phi_{\rm PF}^{AB}(E_{\rm EH}\Phi_{\rm PF})^{AB} - \frac{m^2-2}{4}\Phi_{\rm PF}^{AB}\Phi_{\rm PF}^{AB} + \frac{m^2+d-2}{4}\Phi_{\rm PF}^{AA}\Phi_{\rm PF}^{BB}, \tag{10.8}$$

where $\Phi_{\rm PF}^{AB}$ is the rank-2 tensor field of $so(d,1)$ algebra. The Pauli-Fierz Lagrangian can be obtained from the gauge invariant Lagrangian (10.2) in an obvious way. Namely, gauge transformations (10.5) allow us to gauge away the fields Φ^A and Φ. Doing so and identifying the rank-2 tensor field in (10.1) with $\Phi_{\rm PF}^{AB}$, we get the Pauli-Fierz Lagrangian from the gauge invariant Lagrangian (10.2),

$$\mathcal{L}_{\rm PF} = \mathcal{L}|_{\Phi^{AB}\equiv\Phi_{PF}^{AB},\Phi^A=0,\Phi=0}. \tag{10.9}$$

For the case of flat space, it is well known that the gauge invariant Lagrangian can be obtained from the Pauli-Fierz Lagrangian. It turns out that this interrelation is still valid in AdS space too. Namely, introducing the following representation of the Pauli-Fierz field in terms of gauge fields (10.1):

$$\Phi_{\rm PF}^{AB} = \Phi^{AB} + \frac{1}{m}(\mathcal{D}^A\Phi^B + \mathcal{D}^B\Phi^A) + \frac{2}{mf}\mathcal{D}^A\mathcal{D}^B\Phi + \frac{2m}{(d-1)f}\eta^{AB}\Phi, \tag{10.10}$$

and plugging such $\Phi_{\rm PF}^{AB}$ (10.10) into the Pauli-Fierz Lagrangian (10.8), we obtain the gauge invariant Lagrangian (10.2).[20]

A. CFT adapted gauge invariant approach to the spin-2 massive field in AdS$_{d+1}$

We now discuss our CFT adapted approach to the spin-2 massive AdS field. For details of the derivation of the CFT adapted gauge invariant Lagrangian, see Appendix B.

In our approach, the spin-2 massive field is described by the gauge fields

$$\begin{array}{ccc} & \phi^{ab}, & \\ \phi_{-1}^a, & & \phi_1^a, \\ \phi_{-2}, & \phi_0, & \phi_2. \end{array} \tag{10.11}$$

The fields ϕ^{ab}, $\phi_{\pm1}^a$ and ϕ_0, $\phi_{\pm2}$ are the respective rank-2 tensor, vector and scalar fields of the $so(d)$ algebra. The CFT adapted gauge invariant Lagrangian for these fields takes the form [12]

$$\begin{aligned}\mathcal{L} = {}& \frac{1}{4}|d\phi^{ab}|^2 - \frac{1}{8}|d\phi^{aa}|^2 + \frac{1}{4}|\mathcal{T}_{\kappa-(1/2)}\phi^{ab}|^2 - \frac{1}{8}|\mathcal{T}_{\kappa-(1/2)}\phi^{aa}|^2 \\ & + \frac{1}{2}\sum_{\lambda=\pm1}(|d\phi_\lambda^a|^2 + |\mathcal{T}_{\kappa-(1/2)+\lambda}\phi_\lambda^a|^2) \\ & + \frac{1}{2}\sum_{\lambda=0,\pm2}(|d\phi_\lambda|^2 + |\mathcal{T}_{\kappa-(1/2)+\lambda}\phi_\lambda|^2) \\ & - \frac{1}{2}C^aC^a - \frac{1}{2}C_1C_1 - \frac{1}{2}C_{-1}C_{-1},\end{aligned} \tag{10.12}$$

where we use the notation

$$\begin{aligned} C^a \equiv {}& \partial^b\phi^{ab} - \tfrac{1}{2}\partial^a\phi^{bb} + r_z^{00}\mathcal{T}_{-\kappa+(1/2)}\phi_{-1}^a + r_\zeta^{00}\mathcal{T}_{\kappa+(1/2)}\phi_1^a, \\ C_1 \equiv {}& \partial^a\phi_1^a - \tfrac{1}{2}r_\zeta^{00}\mathcal{T}_{-\kappa-(1/2)}\phi^{aa} + r_z^{10}\mathcal{T}_{-\kappa-(1/2)}\phi_0 + \sqrt{2}r_\zeta^{10}\mathcal{T}_{\kappa+(3/2)}\phi_2, \\ C_{-1} \equiv {}& \partial^a\phi_{-1}^a - \tfrac{1}{2}r_z^{00}\mathcal{T}_{\kappa-(1/2)}\phi^{aa} + \sqrt{2}r_z^{01}\mathcal{T}_{-\kappa+(3/2)}\phi_{-2} + r_\zeta^{01}\mathcal{T}_{\kappa-(1/2)}\phi_0,\end{aligned} \tag{10.13}$$

and $\mathcal{T}_\nu$ is given in (8.12), while κ and r_z^{mn} r_ζ^{mn} are defined in (5.3) and (5.7), respectively. Lagrangian (10.12) is invariant under the gauge transformations

[19] A recent discussion of the *standard* de Donder-Feynman gauge for massless fields may be found in Refs. [41–43]. To our knowledge the explicit form of $C_{\rm st}^A$, $C_{\rm st}$ (10.7) has not been discussed in the earlier literature.

[20] To our knowledge, formula (10.10) is new and has not been discussed in the earlier literature. For 4d flat space, formula (10.10) was given in Ref. [44], while for flat space with $d > 4$, in Ref. [8].

$$\begin{aligned}
\delta\phi^{ab} &= \partial^a\xi^b + \partial^b\xi^a + \frac{2r_\zeta^{00}}{d-2}\eta^{ab}\mathcal{T}_{\kappa+(1/2)}\xi_1 \\
&\quad + \frac{2r_z^{00}}{d-2}\eta^{ab}\mathcal{T}_{-\kappa+(1/2)}\xi_{-1}, \\
\delta\phi_{-1}^a &= \partial^a\xi_{-1} + r_z^{00}\mathcal{T}_{\kappa-(1/2)}\xi^a, \\
\delta\phi_1^a &= \partial^a\xi_1 + r_\zeta^{00}\mathcal{T}_{-\kappa-(1/2)}\xi^a, \\
\delta\phi_{-2} &= \sqrt{2}r_z^{01}\mathcal{T}_{\kappa-(3/2)}\xi_{-1}, \\
\delta\phi_0 &= r_z^{10}\mathcal{T}_{\kappa+(1/2)}\xi_1 + r_\zeta^{01}\mathcal{T}_{-\kappa+(1/2)}\xi_{-1}, \\
\delta\phi_2 &= \sqrt{2}r_\zeta^{10}\mathcal{T}_{-\kappa-(3/2)}\xi_1,
\end{aligned} \tag{10.14}$$

where ξ^a, $\xi_{\pm1}$ are gauge transformation parameters.

The gauge invariant equations of motion obtained from Lagrangian (10.12) take the form

$$\begin{aligned}
&\Box_\kappa\phi^{ab} - \partial^a C^b - \partial^b C^a - \frac{2r_z^{00}\eta^{ab}}{d-2}\mathcal{T}_{-\kappa+(1/2)}C_{-1} \\
&\quad - \frac{2r_\zeta^{00}\eta^{ab}}{d-2}\mathcal{T}_{\kappa+(1/2)}C_1 = 0, \\
&\Box_\kappa\phi_{-1}^a - \partial^a C_{-1} - r_z^{00}\mathcal{T}_{\kappa-(1/2)}C^a = 0, \\
&\Box_{\kappa+1}\phi_1^a - \partial^a C_1 - r_\zeta^{00}\mathcal{T}_{-\kappa-(1/2)}C^a = 0, \\
&\Box_{\kappa-2}\phi_{-2} - \sqrt{2}r_z^{01}\mathcal{T}_{\kappa-(3/2)}C_{-1} = 0, \\
&\Box_\kappa\phi_0 - r_\zeta^{01}\mathcal{T}_{-\kappa+(1/2)}C_{-1} - r_z^{10}\mathcal{T}_{\kappa+(1/2)}C_1 = 0, \\
&\Box_{\kappa+2}\phi_2 - \sqrt{2}r_\zeta^{10}\mathcal{T}_{-\kappa-(3/2)}C_1 = 0,
\end{aligned} \tag{10.15}$$

where $\Box_\nu$ is defined in (8.15). We see that the gauge invariant equations of motion are coupled.

Global AdS symmetries.—We now discuss the realization of the global AdS symmetries on the space of gauge fields (10.11). The realization of the global AdS symmetries is already given in (8.2)-(8.8). All that remains to complete the description of these symmetries is to find the realization of the operator $R_{(0)}^a$ on the space of gauge fields (10.11). The action of the operator $R_{(0)}^a$ on the space of gauge fields (10.11) is found to be

$$\begin{aligned}
R_{(0)}^a\phi^{bc} &= zr_\zeta^{00}\Big(\eta^{ab}\phi_1^c + \eta^{ac}\phi_1^b - \frac{2\eta^{bc}}{d-2}\phi_1^a\Big) \\
&\quad + zr_z^{00}\Big(\eta^{ab}\phi_{-1}^c + \eta^{ac}\phi_{-1}^b - \frac{2\eta^{bc}}{d-2}\phi_{-1}^a\Big), \\
R_{(0)}^a\phi_1^b &= -zr_\zeta^{00}\phi^{ab} + z\eta^{ab}(\sqrt{2}r_\zeta^{10}\phi_2 + r_z^{10}\phi_0), \\
R_{(0)}^a\phi_{-1}^b &= -zr_z^{00}\phi^{ab} + z\eta^{ab}(\sqrt{2}r_z^{01}\phi_{-2} + r_\zeta^{01}\phi_0), \\
R_{(0)}^a\phi_2 &= -z\sqrt{2}r_\zeta^{10}\phi_1^a, \\
R_{(0)}^a\phi_0 &= -zr_z^{10}\phi_1^a - zr_\zeta^{01}\phi_{-1}^a, \\
R_{(0)}^a\phi_{-2} &= -z\sqrt{2}r_z^{01}\phi_{-1}^a.
\end{aligned} \tag{10.16}$$

Modified de Donder gauge.—The modified de Donder gauge is defined to be

$$C^a = 0, \quad C_{-1} = 0, \quad C_1 = 0, \quad \text{modified de Donder gauge}, \tag{10.17}$$

where C^a, $C_{\pm1}$ are given in (10.13). Using this gauge in equations of motion (10.15) gives the surprisingly simple gauge-fixed equations of motion,

$$\begin{aligned}
&\Box_\kappa\phi^{ab} = 0, \qquad \Box_{\kappa+\lambda}\phi_\lambda^a = 0, \qquad \lambda = \pm1, \\
&\Box_{\kappa+\lambda}\phi_\lambda = 0, \qquad \lambda = 0, \pm2.
\end{aligned} \tag{10.18}$$

We see that the gauge-fixed equations are decoupled.

The modified de Donder gauge and gauge-fixed equations have leftover on-shell gauge symmetry. Namely, the modified de Donder gauge (10.17) and gauge-fixed equations (10.18) are invariant under the gauge transformations given in (10.14) provided the gauge transformation parameters satisfy the equations

$$\Box_\kappa\xi^a = 0, \qquad \Box_{\kappa+\lambda}\xi_\lambda = 0, \qquad \lambda = \pm1. \tag{10.19}$$

B. AdS/CFT correspondence for anomalous current and normalizable modes of massive AdS field

We are now ready to discuss the AdS/CFT correspondence for the bulk spin-2 massive AdS field and boundary spin-2 anomalous conformal current.[21] To this end we use our CFT adapted approach to AdS field dynamics and the modified de Donder gauge.

First of all, we note that the normalizable solution of equations of motion (10.18) is given by

$$\begin{aligned}
\phi^{ab}(x,z) &= U_\kappa\phi_{\text{cur},0}^{ab}(x), \\
\phi_{-1}^a(x,z) &= -U_{\kappa-1}\phi_{\text{cur},-1}^a(x), \\
\phi_1^a(x,z) &= U_{\kappa+1}\phi_{\text{cur},1}^a(x), \\
\phi_{-2}(x,z) &= U_{\kappa-2}\phi_{\text{cur},-2}(x), \\
\phi_0(x,z) &= -U_\kappa\phi_{\text{cur},0}(x), \\
\phi_2(x,z) &= U_{\kappa+2}\phi_{\text{cur},2}(x),
\end{aligned} \tag{10.20}$$

where U_ν is defined in (9.19). From (10.20), we find the asymptotic behavior of the normalizable solution

[21] To our knowledge AdS/CFT correspondence for the bulk spin-2 massive AdS field and boundary spin-2 anomalous conformal current has not been studied in the literature.

$$\begin{aligned}
&\phi^{ab}(x,z)\stackrel{z\to 0}{\longrightarrow} z^{\kappa+(1/2)}\phi^{ab}_{\mathrm{cur},0}(x),\\
&\phi^a_{-1}(x,z)\stackrel{z\to 0}{\longrightarrow} -2\kappa z^{\kappa-(1/2)}\phi^a_{\mathrm{cur},-1}(x),\\
&\phi^a_1(x,z)\stackrel{z\to 0}{\longrightarrow} \frac{z^{\kappa+(3/2)}}{2(\kappa+1)}\phi^a_{\mathrm{cur},1}(x),\\
&\phi_{-2}(x,z)\stackrel{z\to 0}{\longrightarrow} 4\kappa(\kappa+1)z^{\kappa-(3/2)}\phi_{\mathrm{cur},-2}(x),\\
&\phi_0(x,z)\stackrel{z\to 0}{\longrightarrow} -z^{\kappa+(1/2)}\phi_{\mathrm{cur},0}(x),\\
&\phi_2(x,z)\stackrel{z\to 0}{\longrightarrow} \frac{z^{\kappa+(5/2)}}{4\kappa(\kappa-1)}\phi_{\mathrm{cur},2}(x).
\end{aligned} \tag{10.21}$$

From (10.21), we see that the fields $\phi^{ab}_{\mathrm{cur},0}$, $\phi^a_{\mathrm{cur},\pm1}$, $\phi_{\mathrm{cur},0}$, $\phi_{\mathrm{cur},\pm2}$ are indeed boundary values of the normalizable solution. Moreover, in the right-hand side of (10.20), we use the notation $\phi^{ab}_{\mathrm{cur},0}$, $\phi^a_{\mathrm{cur},\pm1}$, $\phi_{\mathrm{cur},0}$, $\phi_{\mathrm{cur},\pm2}$ because these boundary values turn out to be the gauge fields entering our gauge invariant formulation of the spin-2 anomalous conformal current in Sec. VA. Namely, one can prove the following statements:

(i) *Leftover on-shell* gauge transformations (10.14) of the normalizable solution (10.20) lead to gauge transformations of the anomalous conformal current (5.8).[22]

(ii) For the normalizable solution (10.20), the modified de Donder gauge condition (10.17) leads to the differential constraints (5.4), (5.5), and (5.6) of the anomalous conformal current.

(iii) On-shell global $so(d,2)$ bulk symmetries of the normalizable spin-2 massive modes in AdS_{d+1} become global $so(d,2)$ boundary conformal symmetries of the spin-2 anomalous conformal current.

These statements can be proved following the procedure we demonstrated for the spin-1 fields in Sec. IX B. Therefore, to avoid repetitions we briefly discuss some necessary details.

Matching of bulk and boundary gauge symmetries.—To match gauge symmetries we analyze leftover on-shell gauge symmetries which are described by the solutions of equations given in (10.19). The normalizable solution to these equations takes the form,

$$\begin{aligned}
\xi^a(x,z) &= U_\kappa \xi^a_{\mathrm{cur},0}(x),\\
\xi_{-1}(x,z) &= -U_{\kappa-1}\xi_{\mathrm{cur},-1}(x),\\
\xi_1(x,z) &= U_{\kappa+1}\xi_{\mathrm{cur},1}(x).
\end{aligned} \tag{10.22}$$

Plugging (10.20) and (10.22) into bulk gauge transformations (10.14) we make sure that the leftover on-shell bulk gauge transformations amount to boundary gauge transformations of the spin-2 anomalous conformal current given in (5.8).

Matching of bulk de Donder gauge and boundary differential constraints.—Plugging the solution to equations for AdS fields (10.20) into the modified de Donder gauge and using relations (9.22) and (9.23), we make sure that the modified de Donder gauge (10.17) amounts to differential constrains (5.4), (5.5), and (5.6).

Matching of bulk and boundary global symmetries.—The matching of bulk and boundary Poincaré symmetries is obvious. Using conformal dimensions for the spin-2 anomalous current given in (5.2), the solution for bulk fields in (10.20), and the bulk dilatation operator (8.4) we make sure that the dilatation bulk and boundary symmetries also match. As before, what is nontrivial is to match K^a symmetries. As in the case of the modified Lorentz gauge, the modified de Donder gauge breaks bulk K^a symmetries. In order to restore these broken K^a symmetries we add compensating gauge transformations to the generic K^a symmetries,

$$K^a_{\mathrm{impr}} = K^a + \delta_{\xi^{K^a}}. \tag{10.23}$$

The compensating gauge transformation parameters can usually be found by requiring improved transformations (10.23) to maintain the modified de Donder gauge (10.17),

$$K^a_{\mathrm{impr}}C^b = 0, \quad K^a_{\mathrm{impr}}C_{-1} = 0, \quad K^a_{\mathrm{impr}}C_1 = 0. \tag{10.24}$$

Doing so, we make sure that Eqs. (10.24) amount to the equations for the compensating gauge transformation parameters,

$$\begin{aligned}
\Box_\kappa \xi^{bK^a} &= 2\phi^{ab} - \eta^{ab}\phi^{cc},\\
\Box_{\kappa-1}\xi^{K^a}_{-1} &= 2\phi^a_{-1},\\
\Box_{\kappa+1}\xi^{K^a}_1 &= 2\phi^a_1.
\end{aligned} \tag{10.25}$$

Using (9.25) and (10.20), we find the solution for the compensating gauge transformation parameters,

$$\begin{aligned}
\xi^{bK^a}(x,z) &= zU_{\kappa+1}(\phi^{ab}_{\mathrm{cur},0}(x) - \tfrac{1}{2}\eta^{ab}\phi^{cc}_{\mathrm{cur},0}(x)),\\
\xi^{K^a}_{-1}(x,z) &= -zU_\kappa \phi^a_{\mathrm{cur},-1}(x),\\
\xi^{K^a}_1(x,z) &= zU_{\kappa+2}\phi^a_{\mathrm{cur},1}(x),
\end{aligned} \tag{10.26}$$

where operator U_ν is given in (9.19). Plugging (10.20) and (10.26) in (10.23), we make sure that the improved bulk K^a_{impr} symmetries of the spin-2 massive AdS field amount to K^a symmetries of the spin-2 anomalous conformal current given in (2.8) and (5.9).

C. AdS/CFT correspondence for anomalous shadow field and non-normalizable mode of massive AdS field

We proceed to a discussion of AdS/CFT correspondence for the bulk spin-2 massive AdS field and boundary spin-2 anomalous shadow field.

[22] Transformations given in (10.14) are off-shell gauge transformations. Leftover on-shell gauge transformations are obtained from (10.14) by using gauge transformation parameters which satisfy Eqs. (10.19).

Matching of the effective action and boundary two-point vertex. In order to find $S_{\rm eff}$ we should solve the equations of motion with the Dirichlet problem corresponding to the boundary anomalous shadow field and plug the solution into action. Using equations of motion (10.15) in the bulk action (9.7) with Lagrangian (10.12), we obtain the boundary effective action (9.42) with $\mathcal{L}_{\rm eff}$ given by

$$\begin{aligned}\mathcal{L}_{\rm eff} = & \frac{1}{4}\phi^{ab}\mathcal{T}_{\kappa-(1/2)}\phi^{ab} - \frac{1}{8}\phi^{aa}\mathcal{T}_{\kappa-(1/2)}\phi^{bb} \\ & + \frac{1}{2}\sum_{\lambda=\pm1}\phi_\lambda^a\mathcal{T}_{\kappa+\lambda-(1/2)}\phi_\lambda^a \\ & + \frac{1}{2}\sum_{\lambda=0,\pm2}\phi_\lambda\mathcal{T}_{\kappa+\lambda-(1/2)}\phi_\lambda, \\ & - \frac{1}{2}(r_z^{00}\phi_{-1}^a + r_\zeta^{00}\phi_1^a)C^a \\ & + \Big(\frac{r_z^{00}}{4}\phi^{aa} - \frac{r_z^{01}}{\sqrt{2}}\phi_{-2} - \frac{r_\zeta^{01}}{2}\phi_0\Big)C_{-1} \\ & + \Big(\frac{r_\zeta^{00}}{4}\phi^{aa} - \frac{r_z^{10}}{2}\phi_0 - \frac{r_\zeta^{10}}{\sqrt{2}}\phi_2\Big)C_1. \end{aligned} \qquad (10.27)$$

We have demonstrated that the use of the modified de Donder gauge considerably simplifies the equations of motion. Now using modified de Donder gauge (10.17) in (10.27), we obtain

$$\begin{aligned}\mathcal{L}_{\rm eff}\big|_{C^a=0,\ C_{\pm1}=0} = & \frac{1}{4}\phi^{ab}\mathcal{T}_{\kappa-(1/2)}\phi^{ab} - \frac{1}{8}\phi^{aa}\mathcal{T}_{\kappa-(1/2)}\phi^{bb} \\ & + \frac{1}{2}\sum_{\lambda=\pm1}\phi_\lambda^a\mathcal{T}_{\kappa-(1/2)+\lambda}\phi_\lambda^a \\ & + \frac{1}{2}\sum_{\lambda=0,\pm2}\phi_\lambda\mathcal{T}_{\kappa-(1/2)+\lambda}\phi_\lambda, \end{aligned} \qquad (10.28)$$

i.e., we see that $\mathcal{L}_{\rm eff}$ is also considerably simplified. To find $S_{\rm eff}$ we should solve the gauge-fixed equations of motion (10.18) with the Dirichlet problem corresponding to the boundary anomalous shadow field and plug the solution into $\mathcal{L}_{\rm eff}$. To this end we discuss the solution of the equations of motion (10.18).

Our equations of motion take the decoupled form and similar to the equations of motion for the massive scalar AdS field. Therefore, we can apply the procedure described in Sec. VIII. Doing so, we obtain the solution of Eq. (10.18) with the Dirichlet problem corresponding to the spin-2 anomalous shadow field,

$$\phi^{ab}(x,z) = \sigma_{2,0}\int d^dyG_\kappa(x-y,z)\phi_{\rm sh,0}^{ab}(y), \qquad (10.29)$$

$$\phi_\lambda^a(x,z) = \sigma_{1,\lambda}\int d^dyG_{\kappa+\lambda}(x-y,z)\phi_{{\rm sh},-\lambda}^a(y), \qquad \lambda=\pm1, \qquad (10.30)$$

$$\phi_\lambda(x,z) = \sigma_{0,\lambda}\int d^dyG_{\kappa+\lambda}(x-y,z)\phi_{{\rm sh},-\lambda}(y), \qquad \lambda=0,\pm2, \qquad (10.31)$$

$$\sigma_{2,0} = 1, \qquad (10.32)$$

$$\begin{aligned}&\sigma_{1,-1} = -\frac{1}{2(\kappa-1)}, \qquad \sigma_{1,1} = 2\kappa, \\ &\sigma_{0,-2} = \frac{1}{4(\kappa-1)(\kappa-2)}, \\ &\sigma_{0,0} = -1, \qquad \sigma_{0,2} = 4\kappa(\kappa+1),\end{aligned} \qquad (10.33)$$

where the Green function G_ν is given in (8.22), while κ is defined in (5.3). The choice of normalization factor $\sigma_{2,0}$ (10.32) is a matter of convention. The remaining normalization factors given in (10.33) are uniquely determined by requiring that the modified de Donder gauge (10.17) amount to the differential constraints for the spin-2 anomalous shadow field.

Using the asymptotic behavior of the Green function given in (8.24), we find the asymptotic behavior of our solution

$$\begin{aligned}&\phi^{ab}(x,z)\overset{z\to0}{\longrightarrow} z^{-\kappa+(1/2)}\phi_{\rm sh,0}^{ab}(x), \\ &\phi_{-1}^a(x,z)\overset{z\to0}{\longrightarrow} -\frac{z^{-\kappa+(3/2)}}{2(\kappa-1)}\phi_{\rm sh,1}^a(x), \\ &\phi_1^a(x,z)\overset{z\to0}{\longrightarrow} 2\kappa z^{-\kappa-(1/2)}\phi_{{\rm sh},-1}^a(x), \\ &\phi_{-2}(x,z)\overset{z\to0}{\longrightarrow} \frac{z^{-\kappa+(5/2)}}{4(\kappa-1)(\kappa-2)}\phi_{\rm sh,2}(x), \\ &\phi_0(x,z)\overset{z\to0}{\longrightarrow} -z^{-\kappa+(1/2)}\phi_{\rm sh,0}(x), \\ &\phi_2(x,z)\overset{z\to0}{\longrightarrow} 4\kappa(\kappa+1)z^{-\kappa-(3/2)}\phi_{{\rm sh},-2}(x),\end{aligned} \qquad (10.34)$$

which tells us that the solution (10.29), (10.30), and (10.31) has indeed asymptotic behavior corresponding to the anomalous shadow field.

Finally, to obtain the effective action we plug the solution of the Dirichlet problem for AdS fields, (10.29), (10.30), and (10.31) into (9.42) and (10.28). Using the general formula given in (8.28), we obtain

$$-S_{\rm eff} = 2\kappa c_\kappa\Gamma, \qquad (10.35)$$

where κ and c_κ are defined in (5.3) and (8.23), respectively, and Γ is the gauge invariant two-point vertex of the spin-2 anomalous shadow field given in (4.8) and (6.8).

Thus, using the modified de Donder gauge for the spin-2 massive AdS field and computing the bulk action on the solution of equations of motion with the Dirichlet problem corresponding to the boundary anomalous shadow field, we obtain the gauge invariant two-point vertex of the spin-2 anomalous shadow field.

Using (6.12), we can represent our result (10.35) in the Stueckelberg gauge frame

$$-S_{\text{eff}} = \frac{\kappa(2\kappa + d + 2)}{2(2\kappa + d - 2)} c_\kappa \Gamma^{\text{stand}}, \tag{10.36}$$

while, in the light-cone gauge frame, our result is represented as

$$-S_{\text{eff}} = 2\kappa c_\kappa \Gamma^{(\text{l.c.})}, \tag{10.37}$$

where $\Gamma^{(\text{l.c.})}$ is given in (6.18). Relation (10.36) with the normalization factor in front of Γ^{stand} as in (10.36) was obtained in Ref. [11].[23] Note that we have obtained the more general relation given in (10.35), while relation (10.36) is obtained from (10.35) by using the Stueckelberg gauge frame. It is our general relation (10.35) that provides the possibility for the derivation of all other relations like the ones in (10.36) and (10.37) just by choosing appropriate gauge conditions. Note that the transformation of relation (10.36) to the one in (10.37) requires cumbersome computations because the Stueckelberg gauge frame removes the vector and scalar field entering the light-cone gauge frame (see Secs. VI B and VI C).

Matching of bulk and boundary gauge symmetries.—The modified de Donder gauge (10.17) and gauge-fixed equations (10.18) are invariant under the gauge transformations given in (10.14) provided the gauge transformation parameters satisfy Eqs. (10.19). The non-normalizable solution to Eqs. (10.19) is given by

$$\begin{aligned} \xi^a(x,z) &= \int d^d y G_\kappa(x-y,z)\xi_{\text{sh},0}(y), \\ \xi_\lambda(x,z) &= \sigma_{1,\lambda}\int d^d y G_{\kappa+\lambda}(x-y,z)\xi_{\text{sh},-\lambda}(y), \end{aligned} \tag{10.38}$$

$\lambda = \pm 1$, where $\sigma_{1,\pm 1}$ are given in (10.33). Plugging (10.38), (10.29), (10.30), and (10.31) in (10.14) we make sure the on-shell leftover gauge symmetries of the solution of the Dirichlet problem for the spin-2 massive AdS field amount to the gauge symmetries of the spin-2 anomalous shadow field (6.6).

Matching of bulk and boundary global symmetries.—The matching can be demonstrated by following the procedure we used for the spin-2 anomalous current in Sec. X B. Therefore to avoid repetitions we briefly discuss some necessary details. The matching of bulk and boundary Poincaré symmetries is obvious. Using conformal dimensions for the spin-2 anomalous shadow given in (6.2), the solution for bulk fields in (10.29), (10.30), and (10.31), and the bulk dilatation operator (8.4), we make sure that dilatation bulk and boundary symmetries also match. In order to match K^a symmetries we consider improved K^a_{impr} transformations (10.23) with gauge transformation parameters that satisfy Eqs. (10.25). Using (9.57), we see that the solution to Eqs. (10.25) with the right-hand sides as in (10.29) and (10.30) is given by

$$\begin{aligned} \xi^{bK^a}(x,z) &= z\sigma^\xi_{2,0}\int d^d y G_{\kappa-1}(x-y,z) \\ &\quad\times\Big(\phi^{ab}_{\text{sh},0}(y) - \frac{1}{2}\eta^{ab}\phi^{cc}_{\text{sh},0}(y)\Big), \\ \xi^{K^a}_{-1}(x,z) &= z\sigma^\xi_{1,-1}\int d^d y G_{\kappa-2}(x-y,z)\phi^a_{\text{sh},1}(y), \\ \xi^{K^a}_{1}(x,z) &= z\sigma^\xi_{1,1}\int d^d y G_{\kappa}(x-y,z)\phi^a_{\text{sh},-1}(y), \end{aligned} \tag{10.39}$$

$$\sigma^\xi_{2,0} \equiv -\frac{1}{2(\kappa-1)}, \tag{10.40}$$

$$\sigma^\xi_{1,-1} \equiv \frac{1}{4(\kappa-1)(\kappa-2)}, \qquad \sigma^\xi_{1,1} \equiv -1, \tag{10.41}$$

where the Green function is given in (8.22). Using these compensating gauge transformation parameters in improved bulk K^a_{impr} symmetries (10.23) we make sure that these K^a_{impr} symmetries amount to K^a symmetries of the spin-2 anomalous shadow field given in (2.8) and (6.7).

To summarize, it is the matching of the bulk on-shell leftover gauge symmetries of the solution to the Dirichlet problem and bulk global symmetries and the respective boundary gauge symmetries of the anomalous shadow field and boundary global symmetries that explains why the effective action coincides with the gauge invariant two-point vertex for the boundary anomalous shadow field [see (10.35)].

Comparing our results for the spin-1 and spin-2 fields given in (9.51) and (10.35), respectively, we see that our approach gives a uniform description of the interrelation between the effective action of massive AdS fields and the two-point gauge invariant vertex of shadow fields. Note however that the value of κ for the spin-1 field (3.3) should not be confused with the one for the spin-2 field (5.3). For the case of the arbitrary spin-s field, the κ was found in Refs. [12,48],

$$\kappa = \sqrt{m^2 + \Big(s + \frac{d-4}{2}\Big)^2}. \tag{10.42}$$

All that is required to generalize relation (10.35) to arbitrary spin-s fields is to plug κ (10.42) in (10.35). A detailed study of arbitrary spin fields will be given in forthcoming publication.

XI. CONCLUSIONS

In this paper, we extend the gauge invariant Stueckelberg approach to the CFT initiated in Refs. [8,9] to the study of anomalous conformal currents and shadow

[23] The computation of S_{eff} for the spin-2 massless field may be found in Refs. [45–47]. In the framework of our approach, S_{eff} was studied in Ref. [12].

fields. In the framework of the AdS/CFT correspondence the anomalous conformal currents and shadow fields are related to massive fields of AdS string theory. It is well known that all Lorentz covariant approaches to string field theory involve large amount of Stueckelberg fields and the corresponding gauge symmetries (see, e.g., [49]). Because our approach to anomalous conformal currents and shadow fields also involves Stueckelberg fields we believe that our approach will be helpful to understand string/gauge theory duality better. Note also that we obtain the gauge invariant vertex for anomalous shadow fields which provides quick and easy access to the light-cone gauge vertex. In the framework of AdS/CFT correspondence this vertex is related to the AdS field action evaluated on the solution of the Dirichlet problem. Because one expects that the quantization of the AdS superstring is straightforward only in the light-cone gauge we believe that our light-cone gauge vertex will also be helpful in various studies of AdS/CFT duality. The results obtained should have a number of the following interesting applications and generalizations.

(i) In this paper, we considered the gauge invariant approach for spin-1 and spin-2 anomalous conformal currents and shadow fields. It would be interesting to generalize our approach to the case of arbitrary spin anomalous conformal currents and shadow fields.

(ii) In this paper we studied the two-point gauge invariant vertex of anomalous shadow fields. A generalization of our approach to the case of 3-point and 4-point gauge invariant vertices will give us the possibility to study various applications of our approach along the lines of Refs. [50–52]

(iii) Because our modified de Donder gauge leads to a considerably simplified analysis of AdS field dynamics we believe that this gauge might also be useful to better understand various aspects of AdS/QCD correspondence which are discussed, e.g., in Refs. [53,54].

(iv) Dirac's idea of arranging d-dimensional conformal physics in $d+2$ dimensional multiplets was heavily pushed recently (see Refs. [55–57]). We think that the use of the methods and approaches developed in Refs. [55–57] may be very useful for the study of AdS/CFT correspondence.

(v) The Becchi-Rouet-Stora-Tyutin approach is one of powerful approaches to the analysis of various aspects of relativistic dynamics (see, e.g., Refs. [58–63]. We think that an extension of this approach to the case anomalous conformal currents and shadow fields should be relatively straightforward.

(vi) In the last few years, there were interesting developments in studying the mixed symmetry fields [64–68]. It would be interesting to apply methods developed in these references to studying anomalous conformal currents and shadow fields. There are other various interesting approaches in the literature which could be used to discuss the gauge invariant formulation of anomalous conformal currents and shadow fields. This is to say that various recently developed interesting formulations of field dynamics in terms of unconstrained fields in flat space may be found in Refs. [69–71].

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APPENDIX A: DERIVATION OF THE CFT ADAPTED LAGRANGIAN FOR A SPIN-1 MASSIVE FIELD IN AdS$_{d+1}$

In this Appendix, we explain some details of the derivation of the CFT adapted gauge invariant Lagrangian for the spin-1 massive field given in (9.8). The presentation in this Appendix is given by using the Lorentzian signature. The Euclidean signature Lagrangian in Sec. IX A is obtained from the Lorentzian signature Lagrangian by simple substitution $\mathcal{L} \to -\mathcal{L}$.

Spin-1 massive field.—We use the field Φ^A carrying flat Lorentz algebra $so(d,1)$ vector indices $A, B = 0, 1, \ldots, d-1, d$. The field Φ^A is related to the field carrying the base manifold indices Φ^μ, $\mu = 0, 1, \ldots, d$, in the standard way $\Phi^A = e^A_\mu \Phi^\mu$, where e^A_μ is vielbein of AdS$_{d+1}$ space. For the Poincaré parametrization of AdS$_{d+1}$ space (8.1), vielbein $e^A = e^A_\mu dx^\mu$ and Lorentz connection, $de^A + \omega^{AB} \wedge e^B = 0$, are given by

$$e^A_\mu = \frac{1}{z}\delta^A_\mu, \qquad \omega^{AB}_\mu = \frac{1}{z}(\delta^A_z \delta^B_\mu - \delta^B_z \delta^A_\mu), \tag{A1}$$

where δ^A_μ is Kronecker delta symbol. We use a covariant derivative with the flat indices $\mathcal{D}^A$,

$$\mathcal{D}_A \equiv e^\mu_A \mathcal{D}_\mu, \qquad \mathcal{D}^A = \eta^{AB}\mathcal{D}_B, \tag{A2}$$

where e^μ_A is the inverse of AdS vielbein, $e^A_\mu e^\mu_B = \delta^A_B$ and η^{AB} is the flat metric tensor. With a choice made in (A1), the covariant derivative takes the form

$$\mathcal{D}^A \Phi^B = \hat{\partial}^A \Phi^B + \delta^B_z \Phi^A - \eta^{AB}\Phi^z, \qquad \hat{\partial}^A \equiv z\partial^A, \tag{A3}$$

where we adopt the following conventions for the derivatives and coordinates: $\partial^A = \eta^{AB}\partial_B$, $\partial_A = \partial/\partial x^A$, $x^A \equiv \delta^A_\mu x^\mu$, $x^A = x^a$, x^d with the identification $x^d \equiv z$.

In an arbitrary parametrization of AdS, the Lagrangian of the spin-1 massive field is given in (9.4). We now use the Poincaré parametrization of AdS and introduce the following quantity:

$$\mathbf{C} \equiv \mathcal{D}^C \Phi^C + m\Phi + 2\Phi^z. \tag{A4}$$

We note that it is the relation $\mathbf{C} = 0$ that defines the modified Lorentz gauge. Using the relations (up to total derivative)

$$e\Phi^A \mathcal{D}^2 \Phi^A = e(\Phi^A(\Box_{0\,\mathrm{AdS}} - 1)\Phi^A + 4\Phi^z\mathbf{C} + (d-7)\Phi^z\Phi^z - 4m\Phi\Phi^z), \tag{A5}$$

$$e\Phi\mathcal{D}^2\Phi = e\Phi\Box_{0\,\mathrm{AdS}}\Phi \tag{A6}$$

$$C_{\mathrm{st}}^2 = \mathbf{C}^2 - 4\Phi^z\mathbf{C} + 4\Phi^z\Phi^z, \tag{A7}$$

$$\Box_{0\,\mathrm{AdS}} \equiv z^2(\Box + \partial_z^2) + (1-d)z\partial_z, \tag{A8}$$

$e \equiv \det e_\mu^A$, we represent Lagrangian (9.4) and $\mathbf{C}$ (A4) as

$$e^{-1}\mathcal{L} = \frac{1}{2}\Phi^A(\Box_{0\,\mathrm{AdS}} - m^2 + d - 1)\Phi^A + \frac{1}{2}\Phi(\Box_{0\,\mathrm{AdS}} - m^2)\Phi + \frac{d-3}{2}\Phi^z\Phi^z - 2m\Phi\Phi^z + \frac{1}{2}\mathbf{C}^2, \tag{A9}$$

$$\mathbf{C} = \hat\partial^A\Phi^A + (2-d)\Phi^z + m\Phi. \tag{A10}$$

Using canonically normalized fields $\tilde\Phi^A$, $\tilde\Phi$, and C defined by

$$\Phi^A = z^{(d-1)/2}\tilde\Phi^A, \qquad \Phi = z^{(d-1)/2}\tilde\Phi, \qquad \mathbf{C} = z^{(d+1)/2}C, \tag{A11}$$

we obtain

$$\mathcal{L} = \frac{1}{2}\tilde\Phi^A\Big(\Box + \partial_z^2 - \frac{1}{z^2}\Big(m^2 + \frac{d^2-1}{4} + 1 - d\Big)\Big)\tilde\Phi^A + \frac{1}{2}\tilde\Phi\Big(\Box + \partial_z^2 - \frac{1}{z^2}\Big(m^2 + \frac{d^2-1}{4}\Big)\Big)\tilde\Phi + \frac{d-3}{2z^2}\tilde\Phi^z\tilde\Phi^z - \frac{2m}{z^2}\tilde\Phi^z\tilde\Phi + \frac{1}{2}C^2, \tag{A12}$$

$$C = \partial^A\tilde\Phi^A + \frac{3-d}{2z}\tilde\Phi^z + \frac{m}{z}\tilde\Phi. \tag{A13}$$

In terms of the $so(d-1,1)$ tensorial components of the field $\tilde\Phi^A$ given by $\tilde\Phi^a$, $\tilde\Phi^z$, Lagrangian (A12) and C (A13) take the form

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_0 + \tfrac{1}{2}C^2, \tag{A14}$$

$$\mathcal{L}_1 = \tfrac{1}{2}\tilde\Phi^a\hat K_0\tilde\Phi^a \tag{A15}$$

$$\mathcal{L}_0 = \tfrac{1}{2}\tilde\Phi^z\hat K_{3-d}\tilde\Phi^z + \tfrac{1}{2}\tilde\Phi\hat K_{d-1}\tilde\Phi - \frac{2m}{z^2}\tilde\Phi^z\tilde\Phi, \tag{A16}$$

$$C = \partial^a\tilde\Phi^a + \mathcal{T}_{(3-d)/2}\tilde\Phi^z + \frac{m}{z}\tilde\Phi, \tag{A17}$$

$$\hat K_\omega = \Box + \partial_z^2 - \frac{1}{z^2}\Big(\kappa^2 - \frac{1}{4} + \omega\Big), \tag{A18}$$

where κ and $\mathcal{T}_\nu$ are defined in (3.3) and (8.12), respectively. In terms of fields (9.6) defined by

$$\tilde\Phi^a = \phi^a, \quad \tilde\Phi^z = r_z^{00}\phi_{-1} + r_\zeta^{00}\phi_1, \quad \tilde\Phi = -r_\zeta^{00}\phi_{-1} + r_z^{00}\phi_1, \tag{A19}$$

where r_z^{00}, r_ζ^{00} are defined in (3.5), we represent $\mathcal{L}_1$ (A15) and $\mathcal{L}_0$ (A16) as

$$\mathcal{L}_1 = \frac{1}{2}\phi^a\Box_\kappa\phi^a, \qquad \mathcal{L}_0 = \frac{1}{2}\sum_{\lambda=\pm1}\phi_\lambda\Box_{\kappa+\lambda}\phi_\lambda, \tag{A20}$$

while C (A17) takes the desired form given in (9.9). Noticing the relation

$$\mathcal{T}^\dagger_{\nu-(1/2)}\mathcal{T}_{\nu-(1/2)} = -\partial_z^2 + \frac{1}{z^2}\Big(\nu^2 - \frac{1}{4}\Big), \tag{A21}$$

and taking into account expressions for $\Box_\nu$ (8.15) and $\mathcal{L}_1$, $\mathcal{L}_0$ (A20), we see that Lagrangian (A14) takes the form of the CFT adapted gauge invariant Lagrangian (9.8).

Lagrangian (9.4) is invariant under gauge transformations (9.3). Making the rescaling $\Xi = z^{(d-3)/2}\xi$, we check that these gauge transformations lead to the ones given in (9.10), (9.11), and (9.12).

APPENDIX B: DERIVATION OF THE CFT ADAPTED LAGRANGIAN FOR THE SPIN-2 MASSIVE FIELD IN AdS$_{d+1}$

We present details of the derivation of the CFT adapted gauge invariant Lagrangian and the respective gauge transformations of the spin-2 massive field given in (10.12) and (10.14).

In an arbitrary parametrization of AdS, the Lagrangian for the spin-2 massive field is given in (10.6). We now use the Poincaré parametrization of AdS and introduce the following quantities:

$$\mathbf{C}^A \equiv C_{\mathrm{st}}^A + 2\Phi^{zA} - \delta_z^A\Phi^{BB}, \qquad \mathbf{C} \equiv C_{\mathrm{st}} + 2\Phi^z. \tag{B1}$$

We note that it is the relations $\mathbf{C}^A = 0$, $\mathbf{C} = 0$ that define the modified de Donder gauge. Using the relations (up to total derivative)

$$\frac{1}{4}e\Phi^{AB}\mathcal{D}^2\Phi^{AB} = e\Big(\frac{1}{4}\Phi^{AB}(\Box_{0\,\mathrm{AdS}} - 2)\Phi^{AB} + \frac{d-5}{2}\Phi^{zA}\Phi^{zA} + 2\Phi^{zz}\Phi^{AA} - \frac{d}{4}\Phi^{AA}\Phi^{BB} + 2\Phi^{zA}\mathbf{C}^A - \Phi^{AA}\mathbf{C}^z - 2m\Phi^{zA}\Phi^A + m\Phi^{AA}\Phi^z\Big), \tag{B2}$$

$$\tfrac{1}{2}C^A_{\rm st}C^A_{\rm st} = \tfrac{1}{2}\mathbf{C}^A\mathbf{C}^A - 2\Phi^{zA}\mathbf{C}^A + \Phi^{AA}\mathbf{C}^z + 2\Phi^{zA}\Phi^{zA} - 2\Phi^{zz}\Phi^{AA} + \tfrac{1}{2}\Phi^{AA}\Phi^{BB}, \qquad \text{(B3)}$$

$$e\Phi^A\mathcal{D}^2\Phi^A = e(\Phi^A(\Box_{0\,\rm AdS} - 1)\Phi^A + 4\Phi^z\mathbf{C} + (d-7)\Phi^z\Phi^z - 2m\Phi^{AA}\Phi^z - 4f\Phi\Phi^z), \qquad \text{(B4)}$$

$$C^2_{\rm st} = \mathbf{C}^2 - 4\Phi^z\mathbf{C} + 4\Phi^z\Phi^z, \qquad \text{(B5)}$$

where $\Box_{0\,\rm AdS}$ is given in (A8), we represent Lagrangian (10.6) and $\mathbf{C}^A$, $\mathbf{C}$ (B1) as

$$\begin{aligned} e^{-1}\mathcal{L} =& \frac{1}{4}\Phi^{AB}(\Box_{0\,\rm AdS} - m^2)\Phi^{AB} - \frac{1}{8}\Phi^{AA}(\Box_{0\,\rm AdS} - m^2)\Phi^{BB} \\ &+ \frac{d-1}{2}\Phi^{zA}\Phi^{zA} - 2m\Phi^{zA}\Phi^A \\ &+ \frac{1}{2}\Phi^A(\Box_{0\,\rm AdS} - m^2 - d - 1)\Phi^A + \frac{d-3}{2}\Phi^z\Phi^z \\ &- 2f\Phi\Phi^z + \frac{1}{2}\Phi(\Box_{0\,\rm AdS} - m^2 - 2d)\Phi \\ &+ \frac{1}{2}\mathbf{C}^A\mathbf{C}^A + \frac{1}{2}\mathbf{C}\mathbf{C}, \end{aligned} \qquad \text{(B6)}$$

$$\begin{aligned} \mathbf{C}^A &= \hat{\partial}^B\Phi^{AB} - \frac{1}{2}\hat{\partial}^A\Phi^{BB} + (1-d)\Phi^{zA} + m\Phi^A, \\ \mathbf{C} &= \hat{\partial}^A\Phi^A + (2-d)\Phi^z + \frac{m}{2}\Phi^{AA} + f\Phi. \end{aligned} \qquad \text{(B7)}$$

Using canonically normalized fields and quantities $\tilde{\mathbf{C}}^A$, $\tilde{\mathbf{C}}$,

$$\begin{aligned} &\Phi^{AB} = z^{(d-1)/2}\tilde{\Phi}^{AB}, \quad \Phi^A = z^{(d-1)/2}\tilde{\Phi}^A, \quad \Phi = z^{(d-1)/2}\tilde{\Phi}, \\ &\mathbf{C}^A = z^{(d+1)/2}\tilde{\mathbf{C}}^A, \quad \mathbf{C} = z^{(d+1)/2}\tilde{\mathbf{C}}, \end{aligned} \qquad \text{(B8)}$$

we obtain

$$\begin{aligned} \mathcal{L} =& \frac{1}{4}\tilde{\Phi}^{AB}\hat{K}_0\tilde{\Phi}^{AB} - \frac{1}{8}\tilde{\Phi}^{AA}\hat{K}_0\tilde{\Phi}^{BB} + \frac{1}{2}\tilde{\Phi}^A\hat{K}_{d+1}\tilde{\Phi}^A \\ &+ \frac{1}{2}\tilde{\Phi}\hat{K}_{2d}\tilde{\Phi} + \frac{d-1}{2z^2}\tilde{\Phi}^{zA}\tilde{\Phi}^{zA} - \frac{2m}{z^2}\tilde{\Phi}^{zA}\tilde{\Phi}^A \\ &+ \frac{d-3}{2z^2}\tilde{\Phi}^z\tilde{\Phi}^z - \frac{2f}{z^2}\tilde{\Phi}^z\tilde{\Phi} + \frac{1}{2}\tilde{\mathbf{C}}^A\tilde{\mathbf{C}}^A + \frac{1}{2}\tilde{\mathbf{C}}\tilde{\mathbf{C}}, \end{aligned} \qquad \text{(B9)}$$

$$\begin{aligned} \tilde{\mathbf{C}}^a &= \partial^b\tilde{\Phi}^{ab} - \frac{1}{2}\partial^a\tilde{\Phi}^{BB} + \mathcal{T}_{-(d-1)/2}\tilde{\Phi}^{za} + \frac{m}{z}\tilde{\Phi}^a, \\ \tilde{\mathbf{C}}^z &= \partial^a\tilde{\Phi}^{za} - \frac{1}{2}\mathcal{T}_{(d-1)/2}\tilde{\Phi}^{BB} + \mathcal{T}_{-(d-1)/2z}\tilde{\Phi}^{zz} + \frac{m}{z}\tilde{\Phi}^z, \\ \tilde{\mathbf{C}} &= \partial^a\tilde{\Phi}^a + \mathcal{T}_{-(d-3)/2}\tilde{\Phi}^z + \frac{m}{2z}\tilde{\Phi}^{AA} + \frac{f}{z}\tilde{\Phi}, \end{aligned} \qquad \text{(B10)}$$

where κ and $\hat{K}_\omega$ are defined in (5.3) and (A18), respectively. In terms of new fields defined by the relations

$$\begin{aligned} &\phi^{ab} = \tilde{\Phi}^{ab} + \frac{1}{d-2}\eta^{ab}\tilde{\Phi}^{zz}, \quad \phi^{za} = \tilde{\Phi}^{za}, \quad \phi^a = \tilde{\Phi}^a, \\ &\phi^{zz} = \frac{u}{2}\tilde{\Phi}^{zz}, \quad \phi^z = \tilde{\Phi}^z, \quad \phi = \tilde{\Phi}, \end{aligned} \qquad \text{(B11)}$$

Lagrangian $\mathcal{L}$ (B9) and $\tilde{\mathbf{C}}^A$, $\tilde{\mathbf{C}}$ (B10) take the form

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_1 + \mathcal{L}_0 + \tfrac{1}{2}\tilde{\mathbf{C}}^A\tilde{\mathbf{C}}^A + \tfrac{1}{2}\tilde{\mathbf{C}}\tilde{\mathbf{C}}, \qquad \text{(B12)}$$

$$\mathcal{L}_2 = \tfrac{1}{4}\phi^{ab}\hat{K}_0\phi^{ab} - \tfrac{1}{8}\phi^{aa}\hat{K}_0\phi^{bb}, \qquad \text{(B13)}$$

$$\mathcal{L}_1 = \frac{1}{2}\phi^{za}\hat{K}_{1-d}\phi^{za} + \frac{1}{2}\phi^a\hat{K}_{1+d}\phi^a - \frac{2m}{z^2}\phi^{za}\phi^a, \qquad \text{(B14)}$$

$$\begin{aligned} \mathcal{L}_0 =& \frac{1}{2}\phi^{zz}\hat{K}_{4-2d}\phi^{zz} + \frac{1}{2}\phi^z\hat{K}_4\phi^z + \frac{1}{2}\phi\hat{K}_{2d}\phi \\ &- \frac{2g}{z^2}\phi^{zz}\phi^z - \frac{2f}{z^2}\phi^z\phi, \end{aligned} \qquad \text{(B15)}$$

$$\begin{aligned} \tilde{\mathbf{C}}^a &= \partial^b\phi^{ab} - \frac{1}{2}\partial^a\phi^{bb} + \mathcal{T}_{(1-d)/2}\phi^{za} + \frac{m\phi^a}{z}, \\ \tilde{\mathbf{C}}^z &= \partial^a\phi^{za} - \frac{1}{2}\mathcal{T}_{(d-1)/2}\phi^{aa} + u\mathcal{T}_{(3-d)/2}\phi^{zz} + \frac{m\phi^z}{z}, \end{aligned} \qquad \text{(B16)}$$

$$\begin{aligned} &\tilde{\mathbf{C}} = \partial^a\phi^a + \mathcal{T}_{(3-d)/2}\phi^z + \frac{m\phi^{aa}}{2z} - \frac{g\phi^{zz}}{(d-2)z} + \frac{f\phi}{z}, \\ &g \equiv m\Big(2\frac{d-2}{d-1}\Big)^{1/2}, \qquad u \equiv \Big(2\frac{d-1}{d-2}\Big)^{1/2}, \end{aligned} \qquad \text{(B17)}$$

where f is defined in (10.3). We proceed as follows.

(i) First, we note that $\mathcal{L}_2$ (B13) can be represented as

$$\mathcal{L}_2 = \tfrac{1}{4}\phi^{ab}\Box_\kappa\phi^{ab} - \tfrac{1}{8}\phi^{aa}\Box_\kappa\phi^{bb}, \qquad \text{(B18)}$$

where κ and $\Box_\kappa$ are given in (5.3) and (8.15), respectively.

(ii) Introducing vector fields $\phi^a_{\pm 1}$ by the orthogonal transformation

$$\begin{aligned} \phi^{za} &= r^{00}_z\phi^a_{-1} + r^{00}_\zeta\phi^a_1, \\ \phi^a &= -r^{00}_\zeta\phi^a_{-1} + r^{00}_z\phi^a_1, \end{aligned} \qquad \text{(B19)}$$

where r^{00}_z, r^{00}_ζ are given in (5.7) we cast $\mathcal{L}_1$ (B14) into the form

$$\mathcal{L}_1 = \frac{1}{2}\sum_{\lambda=\pm1}\phi^a_\lambda\Box_{\kappa+\lambda}\phi^a_\lambda. \qquad \text{(B20)}$$

We note that the inverse of the transformation (B19) is given by

$$\begin{aligned} \phi^a_{-1} &= r^{00}_z\phi^{za} - r^{00}_\zeta\phi^a, \\ \phi^a_1 &= r^{00}_\zeta\phi^{za} + r^{00}_z\phi^a. \end{aligned} \qquad \text{(B21)}$$

(iii) Introducing scalar fields ϕ_0, $\phi_{\pm 2}$ by the orthogonal transformation

$$\phi^{zz} = s_{11}\phi_{-2} + s_{12}\phi_0 + s_{13}\phi_2,$$
$$\phi^z = s_{21}\phi_{-2} + s_{22}\phi_0 + s_{23}\phi_2, \qquad \text{(B22)}$$
$$\phi = s_{31}\phi_{-2} + s_{32}\phi_0 + s_{33}\phi_2,$$

$$s_{11} = \Big(\frac{(2\kappa+d)(2\kappa+d-2)(d-2)}{16\kappa(\kappa-1)(d-1)}\Big)^{1/2},$$
$$s_{12} = \Big(\frac{(2\kappa+d)(2\kappa-d)d}{8(\kappa^2-1)(d-1)}\Big)^{1/2},$$
$$s_{13} = \Big(\frac{(2\kappa-d)(2\kappa-d+2)(d-2)}{16\kappa(\kappa+1)(d-1)}\Big)^{1/2},$$
$$s_{21} = -\Big(\frac{(2\kappa-d)(2\kappa+d-2)}{8\kappa(\kappa-1)}\Big)^{1/2},$$
$$s_{22} = \Big(\frac{d(d-2)}{4(\kappa^2-1)}\Big)^{1/2},$$
$$s_{23} = \Big(\frac{(2\kappa+d)(2\kappa-d+2)}{8\kappa(\kappa+1)}\Big)^{1/2},$$
$$s_{31} = \Big(\frac{(2\kappa-d)(2\kappa-d+2)d}{16\kappa(\kappa-1)(d-1)}\Big)^{1/2},$$
$$s_{32} = -\Big(\frac{(2\kappa+d-2)(2\kappa-d+2)(d-2)}{8(\kappa^2-1)(d-1)}\Big)^{1/2},$$
$$s_{33} = \Big(\frac{(2\kappa+d)(2\kappa+d-2)d}{16\kappa(\kappa+1)(d-1)}\Big)^{1/2}, \qquad \text{(B23)}$$

we cast $\mathcal{L}_0$ (B15) into the form

$$\mathcal{L}_0 = \frac{1}{2}\sum_{\lambda=-2,0,2}\phi_\lambda\Box_{\kappa+\lambda}\phi_\lambda. \qquad \text{(B24)}$$

For the reader's convenience, we note that the inverse of the transformation (B22) is given by

$$\phi_{-2} = s_{11}\phi^{zz} + s_{21}\phi^z + s_{31}\phi,$$
$$\phi_0 = s_{12}\phi^{zz} + s_{22}\phi^z + s_{32}\phi, \qquad \text{(B25)}$$
$$\phi_2 = s_{13}\phi^{zz} + s_{23}\phi^z + s_{33}\phi.$$

(iv) Representing $\tilde{\mathbf{C}}^a$, $\tilde{\mathbf{C}}^z$, $\tilde{\mathbf{C}}$ in terms of the vector fields $\phi^a_{\pm 1}$ and the scalar fields ϕ_0, $\phi_{\pm 2}$ and introducing C^a, $C_{\pm 1}$ by relations

$$C^a = \tilde{\mathbf{C}}^a,$$
$$C_1 = r_\zeta^{00}\tilde{\mathbf{C}}^z + r_z^{00}\tilde{\mathbf{C}}, \qquad \text{(B26)}$$
$$C_{-1} = r_z^{00}\tilde{\mathbf{C}}^z - r_\zeta^{00}\tilde{\mathbf{C}},$$

we find that these C^a, $C_{\pm 1}$ take the form given in (10.13). We note the helpful relation

$$\tilde{\mathbf{C}}^A\tilde{\mathbf{C}}^A + \tilde{\mathbf{C}}\,\tilde{\mathbf{C}} = C^aC^a + C_{-1}C_{-1} + C_1C_1. \qquad \text{(B27)}$$

(v) Making use of relation (A21) and taking into account expressions for $\mathcal{L}_2$ (B18), $\mathcal{L}_1$ (B20), $\mathcal{L}_0$ (B24) and formula (B27), we see that Lagrangian (B12) takes the form of the CFT adapted gauge invariant Lagrangian (10.12).

We now present some details of the derivation of gauge transformations given in (10.14). Lagrangian (10.6) is invariant under gauge transformations given in (10.5). In terms of canonically normalized fields (B8), these gauge transformations take the form

$$\delta\tilde{\Phi}^{ab} = \partial^a\xi^b + \partial^b\xi^a - \frac{2}{z}\eta^{ab}\xi^z + \frac{2m\eta^{ab}}{(d-1)z}\xi,$$
$$\delta\tilde{\Phi}^{za} = \partial^a\xi^z + \mathcal{T}_{(d-1)/2}\xi^a,$$
$$\delta\tilde{\Phi}^{zz} = 2\mathcal{T}_{(d-3)/2}\xi^z + \frac{2m}{(d-1)z}\xi,$$
$$\delta\tilde{\Phi}^a = \partial^a\xi - \frac{m}{z}\xi^a, \qquad \text{(B28)}$$
$$\delta\tilde{\Phi}^z = \mathcal{T}_{(d-3)/2}\xi - \frac{m}{z}\xi^z,$$
$$\delta\tilde{\Phi} = -\frac{f}{z}\xi.$$

In terms of fields defined in (B11), gauge transformations (B28) take the form

$$\delta\phi^{ab} = \partial^a\xi^b + \partial^b\xi^a + \frac{2\eta^{ab}}{d-2}\mathcal{T}_{-(d-1)/2}\xi^z + \frac{2m\eta^{ab}}{d-2}\xi,$$
$$\delta\phi^{za} = \partial^a\xi^z + \mathcal{T}_{(d-1)/2}\xi^a,$$
$$\delta\phi^{zz} = u\mathcal{T}_{(d-3)/2}\xi^z + \frac{mu}{(d-1)z}\xi,$$
$$\delta\phi^a = \partial^a\xi - \frac{m}{z}\xi^a,$$
$$\delta\phi^z = \mathcal{T}_{(d-3)/2}\xi - \frac{m}{z}\xi^z,$$
$$\delta\phi = -\frac{f}{z}\xi. \qquad \text{(B29)}$$

Introducing new gauge transformation parameters by the orthogonal transformation

$$\xi^z = r_z^{00}\xi_{-1} + r_\zeta^{00}\xi_1, \qquad \xi = -r_\zeta^{00}\xi_{-1} + r_z^{00}\xi_1, \qquad \text{(B30)}$$

and using vector fields $\phi^a_{\pm 1}$ (B21) and scalar fields ϕ_0, $\phi_{\pm 2}$ (B25), we find that gauge transformations (B29) take desired form given in (10.14).

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