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Emergence of a universal limiting speed

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We display several examples of how fields with different limiting velocities (the "speed of light") at a high energy scale can nevertheless have a common limiting velocity at low energies due to the effects of interactions. We evaluate the interplay of the velocities through the self-energy diagrams and use the renormalization group to evolve the system to low energy. The differences normally vanish only logarithmically, so that an exponentially large energy trajectory is required in order to satisfy experimental constraints. However, we also display a model in which the running is a power-law type, which could be more phenomenologically useful. The largest velocity difference should be in the system with the weakest interaction, which suggests that the study of the speed of gravitational waves would be the most stringent test of this phenomenon.

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I. INTRODUCTION

Many physical systems yield wavelike solutions which satisfy the wave equation with a speed of propagation c_i,

$$\left[\frac{\partial^2}{\partial t^2} - c_i^2\nabla^2\right]\phi(\mathbf{x}, t) = 0, \tag{1}$$

which is also the massless Klein-Gordon equation. To leading order, the Lagrangian of any such field obeys a Lorentz-like symmetry of Lorentz transformations scaled with the limiting speed c_i, even if the underlying system does not have that invariance. However, if there are multiple fields, they will, in general, have different limiting velocities, and there will not be a global Lorentz symmetry. If the fundamental interactions are emergent phenomenon from an underlying theory without Lorentz invariance [1–3], we might expect that particles would display different limiting speeds.

In this paper we show how interactions between the fields can lead to a universal limiting velocity, i.e. the speed of light, at low energies. We calculate how the different fields influence each other's propagation velocity through the self-energy diagrams, and then use the renormalization group (RG) to scale the results to low energy. Using several examples we show that the condition of equal velocities is the low energy endpoint of renormalization group evolution.[1] A heuristic explanation for this is that because fields can split into other types of fields, the propagation velocity of one field approaches that of the related other fields.

This result could be useful if the fields of the standard model (SM) are emergent from an underlying theory that is not Lorentz invariant. Of course, Lorentz invariance is conventionally taken as one of the foundational principles underlying all our fundamental interactions. However, the Weinberg-Witten [7] theorem is usually interpreted as telling us that non-Abelian gauge bosons and gravitons cannot be emergent fields arising from any underlying Lorentz-invariant nongauge theory. All known examples [1] satisfy this property. Therefore, if the idea of emergent fields has any application in the fundamental interactions, it appears to be required that Lorentz invariance is also emergent.

Our results show that a universal limiting velocity can be an emergent property in the low energy limit. However, in general, the difference in velocity runs towards zero only logarithmically. This means that the underlying scale of emergence needs to be exponentially far away, making it difficult to test any feature of that theory which is power suppressed. For example, we estimate that the scale where differences in the velocity are of order 10% would be beyond $10^{10^{13}}$ GeV. Because of this feature we propose a model that produces much faster power-law running. The model involves a large number of fields which accelerate the running. Another consequence of the running speeds is that observable differences in the velocities would be greatest if the interactions are the weakest. This suggests that the measurement of the velocity of gravitational waves would be the most sensitive test of this aspect of emergence.

This paper has the following structure. In the next section we give some general comments on our procedure. Then, in Secs. III, IV, V, and VI we calculate the beta functions for Yukawa theories, electrodynamics, Yang-Mills, and mixed theories, respectively. All cases yield beta functions such that the limiting velocities run towards

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[1]During the course of this work we found that this general approach has also been studied by S.-S. Lee [4] in the context of emergent supersymmetry. There is also some overlap of our work with the study of Lifshitz-type theories in Ref. [5]. The renormalization group running that we describe is also related to the study of the renormalization of Lorentz-violating electrodynamics [6].

each other at low energy. In Sec. VII, we analyze the general effects of logarithmic running and address the phenomenological constraints. Because of the difficulties posed by logarithmic running, we display a model with power-law running in Sec. VIII. We close with a summary and discussion. Some of the more technical details are described in a pair of appendixes.

II. SETUP

We assume that different species of fermions, scalars, and gauge fields emerge at some UV scale with different speeds of light. In condensed matter systems, phonons and magnons do not propagate at the same speed. Similarly, the same behavior is expected to carry on in an emergent theory of nature. In the absence of any form of interactions between particles, their speeds are expected to be frozen as we run down to the IR. However, these particles are interacting due to Yukawa and gauge forces. Hence, the total Lagrangian of such a system will be given by the sum of kinetic and interaction terms with certain *bare* coefficients specified initially at the UV. The parameter space of the system is spanned by the different speeds and interaction strengths. According to the principle of self-similarity and Wilsonian renormalization, the same Lagrangian will continue to describe the system at different energy scales, provided that we replace the bare parameters with the *renormalized* ones. This can be achieved by integrating out the high momentum modes as we run down from UV to IR. Quantum loops are sensitive to high momenta, and hence can be used to track the evolution of trajectories of the different speeds and interaction strengths in the parameter space. The evolution of these trajectories is encoded in the β functions that are given by the Gell-Mann Low equations

$$\beta(g_i) \equiv \mu \frac{dg_i}{d\mu} = f\{g_j\}, \tag{2}$$

where μ is the mass scale we introduce in dimensional regularization.

In theories with a universal limiting velocity, the Lorentz symmetry prevents the renormalization of the speed of light, and one can set $c = 1$ as a definition of natural units. However, if different species carry different limiting velocities, c_i, then these parameters also get renormalized and must be treated in the same manner as coupling constants. They carry a scale dependence through the renormalization procedure, and also generate their own beta function. We exploit this property to study the running of the limiting velocities.

Throughout the paper we use dimensional regularization (dim-reg). The high energy part of the quantum loops can be isolated by retaining only the $1/\epsilon$ pieces that arise upon using dim-reg. Finally, we notice that our treatment is limited to one-loop corrections, and that the β functions of the speeds require only self-energy corrections, while those of the couplings require vertex corrections as well (see Fig. 1).

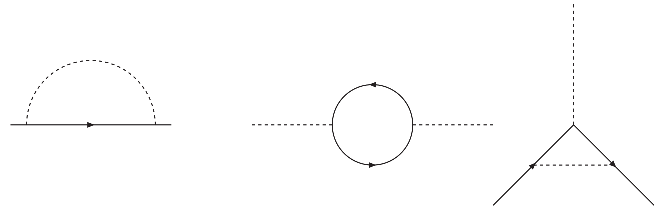

FIG. 1. The self-energy and vertex diagrams. Only self-energies will contribute to the running of the speeds, while the vertex is needed for the running of the coupling strength.

III. YUKAWA INTERACTIONS

We consider a two-species system, namely, scalars and fermions, having different speeds of light at the UV and coupled through Yukawa interaction. The Lagrangian density reads

$$\mathcal{L}_r = i\bar{\psi}_r\gamma^0\partial_0\psi_r - ic_f\bar{\psi}_r\vec{\gamma}\cdot\vec{\partial}\psi_r + \frac{1}{2}\partial_0\phi_r\partial_0\phi_r - \frac{c_b^2}{2}\vec{\partial}\phi_r\cdot\vec{\partial}\phi_r - g\phi_r\bar{\psi}_r\psi_r, \tag{3}$$

where the subscript r denotes the renormalized values of the fields. The momentum-space propagators for scalars and fermions are given by

$$D_b(p^0, \vec{p}) = \frac{i}{(p^0)^2 - c_b^2\vec{p}^2}, \qquad S_f(p^0, \vec{p}) = \frac{i}{p^0\gamma^0 - c_f\vec{p}\cdot\vec{\gamma}}. \tag{4}$$

The self-energies of fermions and scalars are, respectively,

$$-i\Sigma(p^0, \vec{p}) = (-ig)^2 \int \frac{d^4q}{(2\pi)^4} S_f(q^0, \vec{q}) \times D_b(p^0 - q^0, \vec{p} - \vec{q}) \tag{5}$$

and

$$i\Pi(p^0, \vec{p}) = -(-ig)^2 \int \frac{d^4q}{(2\pi)^4} \mathrm{tr}[S_f(q^0, \vec{q}) \times S_f(p^0 + q^0, \vec{q} + \vec{p})]. \tag{6}$$

In the following, we will be interested only in the divergent pieces of (5) and (6). The integral (6) is trivial to perform upon using the substitutions $k^0 = q^0/c_f$, $\vec{k} = \vec{q}$, $P^0 = p^0/c_f$, and $\vec{P} = \vec{p}$. Then, one readily finds

$$i\Pi(p^0, \vec{p}) = \frac{ig^2}{8\pi^2 c_f}\left[\frac{(p^0)^2}{c_f^2} - \vec{p}^2\right]\left(\frac{2}{\epsilon} + \text{finite}\right). \tag{7}$$

On the other hand, the integral (5) is more involved and it needs a bit more attention. Using the substitution $k^0 = q^0/c_f$ and $\vec{k} = \vec{q}$, we find

$$-i\Sigma = \frac{g^2}{c_b^2}\int \frac{d^4k}{(2\pi)^4}\frac{\not{k}}{k^2}\frac{1}{(p^0/c_b - c_f k^0/c_b)^2 - (\vec{k}-\vec{p})^2} = \frac{g^2}{c_b^2}[\gamma^0 I^0 - \vec{\gamma}\cdot\vec{I}], \tag{8}$$

where the integrals I^0 and $\vec{I}$ are given by (the details are in Appendix A)

$$I^0 = \frac{ip^0}{(4\pi)^2}\frac{2c_b}{c_f^2(1+a)^2}\left(\frac{2}{\epsilon} + \text{finite}\right) \tag{9}$$

and

$$\vec{I} = \frac{i\vec{p}}{(4\pi)^2}\frac{2a(1+2a)}{3(1+a)^2}\left(\frac{2}{\epsilon} + \text{finite}\right), \tag{10}$$

where $a = c_b/c_f$.

Now, we move to the vertex correction which, to one-loop order, reads

$$-igG = (-ig)^3\int \frac{d^4q}{(2\pi)^4} S_f(p_2^0 - q^0, \vec{p}_2 - \vec{q}) \times S_f(p_1^0 - q^0, \vec{p}_1 - \vec{q}) D_b(q^0, \vec{q}). \tag{11}$$

Using the change of variables $q^0/c_f = K^0$, $\vec{q} = \vec{K}$, $p_2^0/c_f = P_2^0$, $\vec{p}_2 = \vec{P}_2$, $p_1^0 = P_1^0/c_f$, and $\vec{p}_1 = \vec{P}_1$, and retaining only the divergent part of the integral, we obtain (see Appendix A)

$$-igG = \frac{ig^3}{(4\pi)^2}\frac{2}{c_f^2 c_b(1+a)}\left(\frac{2}{\epsilon} + \text{finite}\right). \tag{12}$$

At this point, the total Lagrangian including the one-loop effect is

$$\begin{aligned}\mathcal{L}_0 = \mathcal{L}_r + \mathcal{L}_c &+ \frac{2ig^2}{(4\pi)^2(1+a)^2} \times\left[\frac{1}{c_b c_f^2}\bar{\psi}_r\partial_0\gamma^0\psi_r - \frac{a(1+2a)}{3c_b^2}\bar{\psi}_r\vec{\gamma}\cdot\partial\psi_r\right]\left(\frac{2}{\epsilon}\right) \\ &+ \frac{g^2}{(4\pi)^2 c_f}\left[\frac{1}{c_f^2}\partial_0\phi_r\partial_0\phi_r - \vec{\partial}\phi_r\cdot\vec{\partial}\phi_r\right]\left(\frac{2}{\epsilon}\right) \\ &+ \frac{g^3}{(4\pi)^2 c_b c_f^2}\frac{2}{1+a}\left(\frac{2}{\epsilon}\right)\phi_r\bar{\psi}_r\psi_r,\end{aligned} \tag{13}$$

where $\mathcal{L}_c$ is the counter Lagrangian

$$\begin{aligned}\mathcal{L}_c = {}& i\delta_{Z_\psi}\bar{\psi}_r\partial_0\gamma^0\psi_r - i\delta_{Z_f}c_f\bar{\psi}_r\vec{\gamma}\cdot\vec{\partial}\psi_r + \frac{\delta_{Z_\phi}}{2}\partial_0\phi_r\partial_0\phi_r - \frac{\delta_{Z_b}}{2}c_b^2\vec{\partial}\phi_r\cdot\vec{\partial}\phi_r \\ &- g\delta_g\phi_r\bar{\psi}_r\psi_r.\end{aligned} \tag{14}$$

At this point, we can read off the different δs that are required to absorb the infinities. Furthermore, we define the bare fields $\phi_0 = Z_\phi^{1/2}\phi_r$ and $\psi_0 = Z_\psi^{1/2}\psi_r$, bare speeds c_{f_0} and c_{b_0}, and bare coupling g_0 such that the Lagrangian density reads

$$\mathcal{L}_0 = i\bar{\psi}_0\partial_0\gamma^0\psi_0 - c_{f0}\bar{\psi}_0\vec{\gamma}\cdot\vec{\partial}\psi_0 + \tfrac{1}{2}\partial_0\phi_0\partial_0\phi_0 - c_{b0}^2\vec{\partial}\phi_0\cdot\vec{\partial}\phi_0 - g_0\phi_0\bar{\psi}_0\psi_0. \tag{15}$$

Comparing (13) and (15) we find

$$\begin{aligned} c_{f_0} &= c_f Z_\psi^{-1} Z_f, \\ c_{b_0} &= c_b Z_\phi^{-1/2} Z_b^{1/2}, \\ g_0 &= g Z_g Z_\phi^{-1/2} Z_\psi^{-1}\mu^{\epsilon/2},\end{aligned} \tag{16}$$

where $Z = 1 + \delta$,

$$\begin{aligned} Z_\psi &= 1 - \frac{2g^2}{(4\pi)^2 c_b(c_f + c_b)^2}\left(\frac{2}{\epsilon}\right), \\ Z_\phi &= 1 - \frac{2g^2}{(4\pi)^2 c_f^3}\left(\frac{2}{\epsilon}\right), \\ Z_f &= 1 - \frac{2g^2(c_f + 2c_b)}{3(4\pi)^2 c_f c_b(c_f + c_b)^2}\left(\frac{2}{\epsilon}\right), \\ Z_b &= 1 - \frac{2g^2}{(4\pi)^2 c_f c_b^2}\left(\frac{2}{\epsilon}\right), \\ Z_g &= 1 + \frac{2g^2}{(4\pi)^2 c_f c_b(c_f + c_b)}\left(\frac{2}{\epsilon}\right).\end{aligned} \tag{17}$$

To proceed, we regard all the renormalized quantities above as functions of the scale μ that occurs in dim-reg. Then, we differentiate the system in Eq. (16) with respect to μ and solve simultaneously for $\beta(g)$, $\beta(c_b)$, and $\beta(c_f)$ to find

$$\begin{aligned}\beta(g) &= \frac{g^3(3c_b c_f^2 + 2c_b^2 c_f + c_b^3 + 4c_f^3)}{8\pi^2 c_b c_f^3(c_f + c_b)^2}, \\ \beta(c_b) &= \frac{g^2(c_b^2 - c_f^2)}{8\pi^2 c_b c_f^3}, \\ \beta(c_f) &= \frac{g^2(c_f - c_b)}{6\pi^2 c_b(c_f + c_b)^2}.\end{aligned} \tag{18}$$

Notice that the β functions of c_b and c_f do not depend on the vertex correction Z_g. Finally, we calculate the β function of the ratio $a = c_b/c_f$ to find

$$\beta(a) = \frac{\beta(c_b)}{c_f} - \frac{c_b}{c_f^2}\beta(c_f) = \frac{g^2}{48\pi^2}\frac{(a-1)[8a + 6(1+a)^3]}{c_b c_f^2(1+a)^2}, \tag{19}$$

from which we see that $c_b = c_f$ is an IR attractive line. We can also see that by studying the Jacobian $J = \partial\beta(c_i)/\partial c_j$, for $i, j = c_f, c_b$ at the fixed line $c_f = c_b$. The eigenvalues of J are $\{0, 7g^2/24\pi^2 c_f^3\}$. The positivity of the second value ensures that $c_b = c_f$ is an IR attractive fixed line.

We have seen the existence of an attractive IR fixed line corresponding to a common limiting speed. We will address more details about the running in Sec. VII.

IV. NONCOVARIANT ELECTRODYNAMICS

In this section, we study the RG flow of the limiting speeds of fermions and photons. The noncovariant and gauge-invariant Lagrangian density reads

$$\mathcal{L}_r = -\tfrac{1}{4}F_{r\mu\nu}F_r^{\mu\nu} + i\bar{\psi}_r(\partial_0 + iec_g A_{0r})\gamma^0\psi_r - i\bar{\psi}_r(c_f\vec{\partial} + iec_f\vec{A}_r)\cdot\vec{\gamma}\psi_r, \tag{20}$$

where $F_{r\mu\nu} = \partial_\mu A_{r\nu} - \partial_\nu A_{r\mu}$, $\partial_\mu = (\partial_0, c_g\vec{\partial})$, and c_g is the photon speed. The photon propagator in the Feynman gauge is given by

$$D_{g\mu\nu}(k^0, \vec{k}) = \frac{-i\eta_{\mu\nu}}{(k_0)^2 - c_g^2\vec{k}^2}. \tag{21}$$

To find the photon and fermion self-energies, it proves easier to write the interaction Lagrangian in the form $\mathcal{L}_I = -ec_{\mu\nu}\bar{\psi}_r A_r^\mu\gamma^\nu\psi_r$, where $c_{\mu\nu} = \mathrm{diag}(c_g, -c_f, -c_f, -c_f)$. Hence, the fermion self-energy is

$$\begin{aligned}-i\Sigma(p^0, \vec{p}) &= (-ie)^2 c_{\beta\mu}c_{\alpha\nu}\gamma^\mu \times\int\frac{d^4q}{(2\pi)^4}S_f(q^0,\vec{q})D_g^{\alpha\beta}(p^0-q^0,\vec{p}-\vec{q})\gamma^\nu,\\ &= -e^2\eta^{\alpha\beta}\frac{c_{\beta\mu}c_{\alpha\nu}}{c_g^2}\gamma^\mu[\gamma^0 I^0 - \vec{\gamma}\cdot\vec{I}]\gamma^\nu,\end{aligned} \tag{22}$$

where I_0 and I_1 are given in (9) and (10) after replacing c_b with c_g. The photon self-energy is given by

$$i\Pi_{\alpha\beta}(p^0, \vec{p}) = -(-ie)^2 c_{\alpha\nu}c_{\beta\mu}\int\frac{d^4q}{(2\pi)^4} \times \mathrm{tr}[\gamma^\nu S_f(q^0,\vec{q})\gamma^\mu S_f(p^0+q^0, \vec{p}+\vec{q})]. \tag{23}$$

Using the substitution $k_0 = q^0/c_f$, $\vec{k} = \vec{q}$, $P_0 = p^0/c_f$, and $\vec{P} = \vec{p}$, we can put $\Pi_{\alpha\beta}$ in a standard integral form. Hence,

$$i\Pi_{\alpha\beta}(p^0, \vec{p}) = \frac{4i}{3(4\pi)^2}\frac{e^2 c_{\alpha\nu}c_{\beta\mu}}{c_f}(P^\mu P^\nu - P^2\eta^{\mu\nu})\left(\frac{2}{\epsilon}\right), \tag{24}$$

where $P = (p^0/c_f, \vec{p})$. Explicit calculations shows that $\mathcal{P}^\alpha\Pi_{\alpha\beta} = 0$, where $\mathcal{P}^\alpha = (p^0, c_g\vec{p})$, and hence $\Pi_{\alpha\beta}$ is gauge invariant as expected.

The counter Lagrangian reads

$$\mathcal{L}_c = \mathcal{L}_{c\,\mathrm{gauge}} + i\delta_{Z_\psi}\bar{\psi}_r\partial_0\gamma^0\psi_r - i\delta_{Z_f}c_f\bar{\psi}_r\vec{\gamma}\cdot\vec{\partial}\psi_r, \tag{25}$$

and $\mathcal{L}_{c\,\mathrm{gauge}}$ is the counterterm for the gauge sector. Then, from (22) and (25), and after using the properties of γ matrices, we can immediately read Z_ψ and Z_f,

$$Z_\psi = 1 - \frac{2e^2(3c_f^2 - c_g^2)}{(4\pi)^2 c_g(c_f + c_g)^2}\left(\frac{2}{\epsilon}\right),$$
$$Z_f = 1 - \frac{2e^2(c_g^2 + c_f^2)(2c_g + c_f)}{3(4\pi)^2 c_f c_g(c_g + c_f)^2}\left(\frac{2}{\epsilon}\right).$$

Now we come to the counterterms in the gauge sector. A general counterterm written in momentum space takes the form

$$\begin{aligned}\mathcal{L}_{c\,\mathrm{gauge}}(p) = A_{0r}\delta_A[(p^0)^2 - \eta^{00}((p^0)^2 - c_g^2\vec{p}^2)]A_{0,r} &+ A_{ir}[c_g^2\delta_{g_B B}p^i p^j + \delta^{ij}(\delta_A(p^0)^2\\ &- c_g^2\delta_{g_B}\vec{p}^2)]A_{jr} - 2A_{ir}\delta_A c_g p^0 p^i A_{0r}.\end{aligned} \tag{26}$$

One can show that all the infinities in (24) can be absorbed using δ_A and δ_{g_B},

$$\begin{aligned}Z_A &= 1 - \frac{4}{3(4\pi)^2}\frac{e^2}{c_f}\left(\frac{2}{\epsilon}\right),\\ Z_{g_B} &= 1 - \frac{4}{3(4\pi)^2}\frac{e^2 c_f}{c_g^2}\left(\frac{2}{\epsilon}\right),\end{aligned} \tag{27}$$

where, as usual, $Z = 1 + \delta$. We write $\mathcal{L}_{c\,\mathrm{gauge}}(p)$ in the compact form

$$\mathcal{L}_{c\,\mathrm{gauge}}(p) = A_{\mu r}M^{\mu\nu}A_{\nu r}, \tag{28}$$

with

$$\begin{aligned}M^{00} &= c_g^2\delta_A\vec{p}^2,\\ M^{0i} &= -c_g\delta_A p^0 p^i,\\ M^{ij} &= c_g^2\delta_{g_B}p^i p^j + \delta^{ij}(\delta_A(p^0)^2 - c_g^2\delta_{g_B}\vec{p}^2).\end{aligned} \tag{29}$$

It is trivial to see that $\mathcal{P}_\alpha M^{\alpha\beta} = 0$, and hence $M_{\alpha\beta}$ is gauge invariant.

Now, defining the bare fields $\psi_0 = Z_\psi^{1/2}\psi_r$, $A_0^0 = Z_A^1 Z_{g_B}^{-1/2}A_r^0$, and $A_0^i = Z_A^{1/2}A_r^i$, and bare speeds c_{f0} and c_{g0}, the Lagrangian density reads

$$\mathcal{L}_0 = -\tfrac{1}{4}\mathcal{F}_{0\mu\nu}\mathcal{F}_0^{\mu\nu} + i\bar{\psi}_0\partial_0\gamma^0\psi_0 - ic_{f0}\bar{\psi}_0\vec{\gamma}\cdot\vec{\partial}\psi_0. \tag{30}$$

The bare gauge field Lagrangian in momentum space is given by $A_{0\mu}M_0^{\mu\nu}A_{0\nu}$, and

$$\begin{aligned}M_0^{00} &= c_{g0}^2\vec{p}^2,\\ M_0^{0i} &= -c_{g0}p^0 p^i,\\ M_0^{ij} &= c_{g0}^2 p^i p^j + \delta^{ij}((p^0)^2 - c_{g0}^2\vec{p}^2).\end{aligned} \tag{31}$$

The relations between the bare and renormalized speeds are

$$c_{f0} = c_f Z_\psi^{-1} Z_f, \qquad c_{g0} = c_g Z_A^{-1/2} Z_{g_B}^{1/2}, \tag{32}$$

from which we obtain

$$\begin{aligned} \beta(c_g) &= \frac{4e^2}{3(4\pi)^2}\frac{(c_g^2 - c_f^2)}{c_f c_g}, \\ \beta(c_f) &= \frac{8e^2}{3(4\pi)^2}\frac{(c_f - c_g)(4c_f^2 + 3c_f c_g + c_g^2)}{c_g(c_f + c_g)^2}. \end{aligned} \tag{33}$$

These β functions have the same structure as in the case of Yukawa interactions, and we immediately conclude that $c_f = c_g$ is an IR attractive line.

V. NONCOVARIANT YANG-MILLS THEORIES

In this section we generalize the results of QED to the case of non-Abelian gauge theories. We take the gauge group to be $SU(N)$, and the fermions in the fundamental representation

$$\begin{aligned} \mathcal{L}_r = \mathcal{L}_{r\,\text{free}} &+ g c_g \eta_{\mu\nu} \bar{\psi}_r A_r^{a\mu} \gamma^\nu \psi_r t^a \\ &- g c_g f^{abc} \partial_\kappa A^a_{r\lambda} A_r^{\kappa b} A_r^{\lambda c} \\ &- \tfrac{1}{4} g^2 c_g^2 f^{eab} f^{ecd} A^a_{r\kappa} A^b_{r\lambda} A_r^{c\kappa} A_r^{d\lambda}, \end{aligned} \tag{34}$$

where g is the coupling constant, t^a are the group generators, and f^{abc} are the group structure constants. $\mathcal{L}_{r\,\text{free}}$ is the free part of the Lagrangian,

$$\mathcal{L}_{r\,\text{free}} = -\tfrac{1}{4} F^a_{r\mu\nu} F_r^{a\mu\nu} + i\bar{\psi}_r \partial_0 \gamma^0 \psi_r - i c_f \bar{\psi}_r \vec{\partial}\cdot\vec{\gamma}\psi_r, \tag{35}$$

where as in the case of QED $F^a_{r\mu\nu} = \partial_\mu A^a_{r\nu} - \partial_\nu A^a_{r\mu}$, $\partial_\mu = (\partial_0, c_g \vec{\partial})$, and c_g is the gauge boson speed.

The fermion self-energy is identical to the case of QED; one just includes the quadratic Casimir operator in the fundamental representation $C_2(N) = (N^2 - 1)/2N$ into Eq. (22) to find

$$\begin{aligned} Z_\psi &= 1 - \frac{2C_2(N) g^2 (3c_f^2 - c_g^2)}{(4\pi)^2 c_g (c_f + c_g)^2}\left(\frac{2}{\epsilon}\right), \\ Z_f &= 1 - \frac{2C_2(N) g^2 (c_g^2 + c_f^2)(2c_g + c_f)}{3(4\pi)^2 c_f c_g (c_g + c_f)^2}\left(\frac{2}{\epsilon}\right). \end{aligned}$$

In calculating the gauge boson self-energy $\Pi^{ab}_{\alpha\beta}$, one encounters, in addition to the fermion loop, gauge boson and ghost loops,

$$\begin{aligned} i\Pi^{ab}_{\alpha\beta} = \bigg[&-i\frac{5C_2(G) g^2}{3(4\pi)^2 c_g}(\mathcal{P}_\alpha \mathcal{P}_\beta - \mathcal{P}^2 \eta_{\alpha\beta}) \\ &+ i\frac{4C(N) g^2 c_{\alpha\nu} c_{\beta\mu}}{3c_f}(P^\mu P^\nu - \eta^{\mu\nu} P^2)\bigg]\delta^{ab}\left(\frac{2}{\epsilon}\right), \end{aligned} \tag{36}$$

where $C_2(G) = N$ and $C(N) = 1/2$ are group factors, $\mathcal{P}^\mu = (p^0, c_g \vec{p})$, and $P^\mu = (p^0/c_f, \vec{p})$. As in QED, the infinities can be absorbed into the counterterm $A^a_{\mu r} M^{\mu\nu} A^a_{\nu r}$, where $M^{\mu\nu}$ are given in Eq. (29). Hence, we find

$$\begin{aligned} Z_A &= 1 + \left(-\frac{4C(N) g^2}{3(4\pi)^2 c_f} + \frac{5C_2(G) g^2}{3(4\pi)^2 c_g}\right)\left(\frac{2}{\epsilon}\right), \\ Z_{g_B} &= 1 + \left(-\frac{4C(N) g^2 c_f}{3(4\pi)^2 c_g^2} + \frac{5C_2(G) g^2}{3(4\pi)^2 c_g}\right)\left(\frac{2}{\epsilon}\right). \end{aligned} \tag{37}$$

Gluon loops will not modify their own propagation speed, due to the Lorentz-like symmetry of that sector when considered in isolation. This is visible in the formulas above. Since $\beta(c_g) \propto (Z_{g_B} - Z_A)$, the gauge bosons and ghost contributions cancel in obtaining $\beta(c_g)$. Overall, the β functions read

$$\begin{aligned} \beta(c_g) &= \frac{4C(N) g^2}{3(4\pi)^2}\frac{(c_g^2 - c_f^2)}{c_f c_g}, \\ \beta(c_f) &= \frac{8C_2(N) g^2}{3(4\pi)^2}\frac{(c_f - c_g)(4c_f^2 + 3c_f c_g + c_g^2)}{c_g(c_f + c_g)^2}, \end{aligned} \tag{38}$$

which, apart from group factors, are identical to the QED case.

VI. EMERGENCE OF LORENTZ SYMMETRY IN A MIXED SYSTEM

The emergence of a universal Lorentz symmetry in the above examples is intriguing to explore a more general setup consisting of multispecies and/or mixing between fermions, bosons, and gauge fields. Before delving into the most general case, we derive a general formula that enables us to calculate the β functions of such complex systems. This is done in Appendix B.

A. Yukawa-electrodynamics

Now, let us consider the more general case of Yukawa-electrodynamics. In this theory a fermion couples to a scalar through Yukawa interaction, and minimally to a $U(1)$ gauge field. The scalar is neutral under the $U(1)$ field. We assume that the fermion, scalar, and gauge field all have different speeds of light, c_f, c_b, and c_g, respectively. This is the simplest generalization of the above cases. The scalar and gauge field self-energies are identical to their expressions in Yukawa and QED sections, while the fermion self-energy is the sum of the contributions from the scalar and gauge field. The calculations of the corresponding Z renormalizations are very straightforward, and can be obtained directly from the previous two sections. Thus, Z_ϕ, Z_b, Z_A, and Z_{g_B} are given by their expressions in Eqs. (17) and (27), respectively, while

$$Z_\psi = 1 + \left(-\frac{2e^2(3c_f^2 - c_g^2)}{(4\pi)^2 c_g (c_f + c_g)^2} - \frac{2g^2}{(4\pi)^2 c_b (c_b + c_f)^2} \right)\left(\frac{2}{\epsilon}\right), \tag{39}$$

$$Z_f = 1 + \left(-\frac{2e^2(c_g^2 + c_f^2)(2c_g + c_f)}{3(4\pi)^2 c_f c_g (c_g + c_f)^2} - \frac{2g^2(c_f + 2c_b)}{3(4\pi)^2 c_f c_b (c_f + c_b)^2} \right)\left(\frac{2}{\epsilon}\right).$$

The relations between the bare and renormalized quantities are given as before,

$$c_{f0} = c_f Z_\psi^{-1} Z_f,$$
$$c_{b0} = c_b Z_\phi^{-1/2} Z_b^{1/2},$$
$$c_{g0} = c_g Z_A^{-1/2} Z_{g_B}^{1/2}.$$

In order to find the β functions of c_b, c_f, and c_g, we use Eq. (B11) to find

$$\beta_{c_f} = c_f\left[g\frac{\partial}{\partial g} + e\frac{\partial}{\partial e}\right](\rho_f - \rho_\psi),$$
$$\beta_{c_b} = -\frac{c_b g}{2}\frac{\partial}{\partial g}(\rho_\phi - \rho_b), \tag{40}$$
$$\beta_{c_g} = -\frac{c_g e}{2}\frac{\partial}{\partial e}(\rho_A - \rho_{g_B}),$$

where $Z = 1 + \rho(2/\epsilon)$. Finally, the β functions read

$$\beta(c_f) = \frac{g^2(c_f - c_b)}{6\pi^2 c_b (c_f + c_b)^2} + \frac{8e^2(c_f - cg)(4c_f^2 + 3c_f c_g + c_g^2)}{3(4\pi)^2 c_g (c_g + c_f)^2}, \tag{41}$$
$$\beta(c_b) = \frac{g^2(c_b^2 - c_f^2)}{8\pi^2 c_f^3 c_b},$$
$$\beta(c_g) = \frac{4e^2}{3(4\pi)^2}\frac{(c_g^2 - c_f^2)}{c_f c_g}.$$

This is exactly expected: since only fermions can couple to both scalars and gauge fields, we find that the photon and scalar speeds of light are identical to those found before, while the fermion speed gets contributions from both Yukawa and gauge sectors.

We can see that $c_f = c_b = c_g$ is an IR attractive fixed line by computing the eigenvalues of the Jacobian $J = (\partial\beta_i/\partial c_j)|_{c_f=c_b=c_g}$, where $i, j = c_f, c_b, c_g$,

$$\left\{ 0, \frac{(7g^2 + 12e^2) \pm \sqrt{(7g^2 + 12e^2)^2 - 304g^2e^2}}{48\pi^2} > 0 \right\}. \tag{42}$$

B. The general case

We consider N_f fermions interacting with N_b scalars or gauge bosons. Although we shall carry out the calculation in the case of Yukawa interactions, the Abelian and non-Abelian β functions have the same structure as we pointed out above.

The general Lagrangian density reads

$$\mathcal{L} = i\bar\psi_a\gamma^0\partial_0\psi_a - ic_{f_a}\bar\psi_a\vec\gamma\cdot\vec\partial\psi_a + \tfrac{1}{2}\partial_0\phi_i\partial_0\phi_i - \tfrac{1}{2}c_{b_i}^2\vec\partial\phi_i\cdot\vec\partial\phi_i - \bar\psi_a(u_{ab}^i + i\gamma^5 v_{ab}^i)\psi_b\phi_i, \tag{43}$$

where summation over repeated indices is implied. Denoting $z_{ab}^i = u_{ab} + iv_{ab}^i$ and noticing that $\mathbf{z^i} \to \mathbf{z^{i\dagger}}$, as we move $\mathbf{z^i}$ across the vertex $\phi\bar\psi\psi$ [8], we find

$$Z_{\psi_a} = 1 - \frac{2}{(4\pi)^2}\sum_{c,i}\frac{z_{ac}^i z_{ca}^{i*}}{c_{b_i}(c_{f_c} + c_{b_i})^2}\left(\frac{2}{\epsilon}\right),$$
$$Z_{f_a} = 1 - \frac{2}{3(4\pi)^2}\sum_{c,i}\frac{z_{ac}^i z_{ca}^{*i}(c_{f_c} + 2c_{b_i})}{c_{f_a}c_{b_i}(c_{f_c} + c_{b_i})^2}\left(\frac{2}{\epsilon}\right),$$
$$Z_{\phi_i} = 1 - \frac{16}{(4\pi)^2}\sum_{a,b}\frac{z_{ab}^i z_{ba}^{*i}}{(c_{f_a} + c_{f_b})^3}\left(\frac{2}{\epsilon}\right),$$
$$Z_{b_i} = 1 - \frac{8}{3(4\pi)^2}\sum_{a,b}\frac{z_{ab}^i z_{ba}^{*i}(c_{f_b}^2 + 4c_{f_a}c_{f_b} + c_{f_a}^2)}{c_{b_i}^2(c_{f_a} + c_{f_b})^3}\left(\frac{2}{\epsilon}\right). \tag{44}$$

Z_{f_a}, Z_{ψ_a} can be read directly from the Yukawa expressions in Eq. (17), while Z_{ϕ_i} and Z_{b_i} are obtained using a series of integrals similar to those given in Appendix A. Notice that quantum loops can also generate off-diagonal corrections $Z_{\psi_a\psi_b}$ if the couplings z_{ab}^i contain off-diagonal components. These corrections will induce kinetic mixing terms of the form $i\alpha_{ab}\bar\psi_a\partial^0\gamma^0\psi_b + i\beta_{ab}\bar\psi_a\partial^i\gamma^i\psi_b$. In Lorentz-invariant theories, where $\alpha_{ab} = \beta_{ab}$, we can always find a basis where $\alpha_{ab} = \beta_{ab}$ are diagonal by performing $SO(N_f)$ rotations. However, in the present case, and since, in general, $\alpha_{ab} \neq \beta_{ab}$, we have the freedom to diagonalize either the time-time or the space-space components. Diagonalizing the time-time component, and hence working in a basis where we have canonical kinetic terms, will always leave space-space mixing terms. We assume that these terms are always small compared to the diagonal speeds, i.e. $\beta_{ab}/c_{f_a} \ll 1$ for all a, and we ignore their evolution in the following analysis. The same thing can also be said about kinetic mixing terms for bosons.

To be able to use the grand formula (B11) we define

$$z^i_{ab} = g_{3iab}, \qquad z^{*i}_{ab} = g_{\bar{3}iab},$$
$$c_{f_a} = g_{1a}, \qquad c_{b_i} = g_{2i}, \tag{45}$$

and

$$Z_{\phi_i} = Z_{1i}, \qquad Z_{b_i} = Z_{2i}, \qquad Z_{z^i_{ab}} = Z_{3iab},$$
$$Z_{z^{*i}_{ab}} = Z_{\bar{3}iab}, \qquad Z_{f_a} = Z_{4a}, \qquad Z_{\psi_a} = Z_{5a}. \tag{46}$$

Now, we write (B11) as

$$\beta_{\mu M} = 2g_{\mu M}\sum_{\nu N} n_{\nu N,\mu M}\sum_{\alpha O} p_{\alpha O} g_{\alpha O}\frac{\partial \rho_{\nu N}}{\partial g_{\alpha O}}, \tag{47}$$

where the Greek indices run from 1 to 3 and $\bar{3}$, and the upper case Latin letters run over a and i. The nonzero values in (47) are $n_{1i',2i} = -\delta_{ii'}/2$, $n_{2i',2i} = \delta_{ii'}/2$, $n_{4a',1a} = \delta_{aa'}$, $n_{5a',1a} = -\delta_{aa'}$, $p_{3,abi} = 1/2$, and $p_{\bar{3},abi} = 1/2$. Hence, we obtain

$$\beta(c_{b_i}) = \frac{8}{3(4\pi)^2 c_{b_i}}\sum_{a,b}\frac{z^i_{ab}z^{*i}_{ba}[6c^2_{b_i} - (c^2_{f_a} + 4c_{f_a}c_{f_b} + c^2_{f_b})]}{(c_{f_a} + c_{f_b})^3},$$
$$\beta(c_{f_a}) = \frac{4}{3(4\pi)^2}\sum_{ib}\frac{z^i_{ab}z^{*i}_{ba}[3c_{f_a} - c_{f_b} - 2c_{b_i}]}{c_{b_i}(c_{f_b} + c_{b_i})^2}. \tag{48}$$

Again, we find in this general setup that $c_{f_a} = c_{b_i} = c$ for all i, and a is a fixed line. To study the nature of this line we perform a perturbation to the system about this line; i.e. we construct the Jacobian matrix at $c_{f_a} = c_{b_i} = c$. Defining $\Lambda^i_a = z^i_{aa}z^{*i}_{aa}$ and $\Gamma^i_a = \sum_{b\neq a} z^i_{ab}z^{*i}_{ba}$ we find

$$J_{(N_f+N_b)\times(N_f+N_b)} = \begin{bmatrix} J^1_{N_f\times N_f} & J^2_{N_f\times N_b} \\ J^3_{N_b\times N_f} & J^4_{N_b\times N_b}\end{bmatrix}, \tag{49}$$

where

$$J^1_{a,a} = \frac{\sum_i [2\Lambda^i_a + 3\Gamma^i_a]}{3(4\pi)^2 c^3},$$
$$J^1_{a,c} = -\frac{\sum_i z^i_{ac}z^{*i}_{ca}}{3(4\pi)^2 c^3},$$
$$J^2_{a,i} = \frac{-2[\Lambda^i_a + \Gamma^i_a]}{3(4\pi)^2 c^3}, \tag{50}$$
$$J^3_{i,a} = \frac{-4[\Lambda^i_a + \Gamma^i_a]}{(4\pi)^2 c^3},$$
$$J^4_{i,j} = \delta_{i,j}\frac{4\sum_a [\Lambda^i_a + \Gamma^i_a]}{(4\pi)^2 c^3}.$$

Although we were not able to diagonalize J analytically, numerical calculations show that we always have a spectrum of positive eigenvalues on the top of a zero mode. In Fig. 2 we plot the smallest eigenvalue λ of J, which governs the behavior of c_{f_a} and c_{b_i}, against the number of fermions N_f for a fixed number of bosons N_b. We find that $\lambda \propto N_f$ for large N_f. Hence, the effective running increases with the number of species as expected.

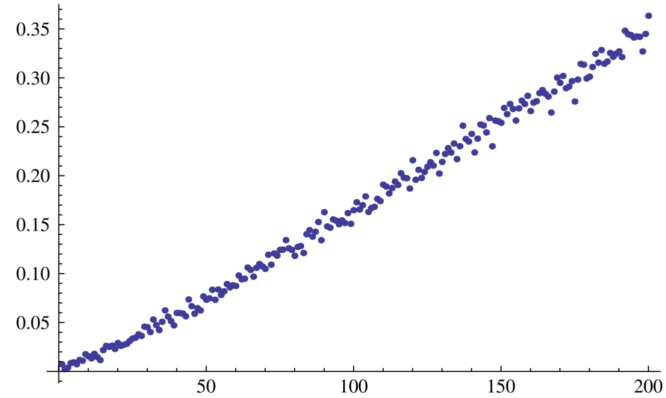

FIG. 2 (color online). The smallest eigenvalue of J (vertical) against the number of fermions N_f (horizontal) taking $N_b = 1$. The calculations are based on values of z^i_{ab} between 0 and 1 generated randomly at each N_f. The plot shows that $\lambda \propto N_f$ for large N_f.

VII. IMPLICATIONS OF LOGARITHMIC RUNNING

Let us consider how the differing speeds approach each other by studying the situation where the speeds are relatively close, but not identical, at some scale μ_*. We treat this problem to first order in the speed difference. If we define the relative speed difference as

$$\eta = \frac{c_b}{c_f} - 1, \tag{51}$$

the generic beta function has the form

$$\beta(\eta) = \mu\frac{\partial\eta}{\partial\mu} = \frac{bg^2}{4\pi^2 c^3}\eta + \mathcal{O}(\eta^2), \tag{52}$$

where b is a constant of order unity and where we denote the common low energy limit as $c_f \approx c_b \approx c$.

If the coupling g were to be treated as a constant, this running could be integrated to yield

$$\eta(\mu) = \eta_*\left(\frac{\mu}{\mu_*}\right)^{bg^2/4\pi^2c^3}. \tag{53}$$

The rescaling is of power-law form. If the coupling constant is small, the power-law exponent is also small and the running is slow. However, if the coupling is large (and constant) the running would be rapid with a power-law form, leading quickly to a universal speed of light.

However, the coupling itself also runs. For example, the Yukawa coupling beta function can be integrated to yield

$$\frac{g^2(\mu)}{4\pi c^3} = \frac{4\pi g^2_*}{5\log(\frac{\Lambda^2}{\mu^2})}, \tag{54}$$

with $\Lambda > \mu$. While this coupling could be large at high energy, it runs to smaller values at low energy.[2] This produces a quite different form for the running of the relative speeds. The correct form for the running of η is

$$\eta(\mu) = \eta_* \left[\frac{\log(\frac{\Lambda^2}{\mu_*^2})}{\log(\frac{\Lambda^2}{\mu^2})}\right]^{2b/5} = \eta_* \left[\frac{g^2(\mu)}{g^2(\mu_*)}\right]^{2b/5}. \tag{55}$$

This implies that the difference in the speeds runs only logarithmically.

There are tight constraints on the equality of the limiting velocities for the different particles. For direct measurement of the velocities, we can look at timing accomplished at high energy accelerators. For example, at CERN LEP, the electron beam travels at essentially the limiting velocity, since $E/m = \gamma \approx 10^5$. The timing of the accelerator relies on this limiting velocity being the speed of light. Because the timing of each bunch is recorded within ± 50 ns over about 1000 revolutions in the 27 km accelerator [9], we estimate that this constrains $\eta \leq 10^{-7}$ for electrons.

However, indirect constraints are more powerful, and these have been described by Altschul [10]. For $c_e > c$, energetic electrons traveling faster than the speed of light will radiate Cherenkov light, losing energy until they move at only the speed of light. This effect produces a maximum energy for subluminal motion, which is constrained by the observation of energetic electrons in astrophysics. For $c_e < c$, there is a constraint from the cutoff frequency in synchrotron emission. These constraints are more powerful than direct measurements because they bound factors of $\gamma_c = 1/\sqrt{1 - c_e^2/c^2} \approx 1/\sqrt{\eta}$ rather than the linear bounds on η from the velocity measurements. Altschul's bounds are $|\eta| \lesssim 10^{-14}$.

In order to achieve this close equality of the different speeds with logarithmic running, the running needs to occur over an exponentially large energy range. For example, even if we take $\eta_* \sim 10^{-1}$ and $\Lambda/\mu_* \sim 2$ (which barely allows perturbation theory to be used near the energy μ_*), we would need $\log(\Lambda/m_e) \sim 10^{13}$, where we have generously used m_e as the low energy scale. This clearly poses a problem for model building.

VIII. EMERGENT LORENTZ SYMMETRY: TOWARD MODEL BUILDING

In any realistic model of emergence without an intrinsic Lorentz invariance, we do not expect the different species to emerge with the same limiting speed. In the above sections we showed that there is a potential mechanism to overcome this problem in a class of models whenever we run the renormalization group down to lower energies. However, we found that the speeds of light are forced to run logarithmically along with the running coupling constants. This is a relatively slow running if we want to meet the stringent constraints on Lorentz violations without having to fine-tune the speeds at the UV. In this section, we propose a way out of this situation.

In order to increase the effect of RG running, there are two options. One is to keep the coupling constant large and unchanged with energy scale. Such a nearly conformal theory would convert logarithmic running into power-law running, as we saw in the last section. We also need the large coupling such that the exponent is large. Such theories are under active investigation [11] in the context of ‘‘walking technicolor,’’ where slowly running but strongly interacting theories are used to provide dynamical breaking of the electroweak theory while not producing excessive flavor changing processes. Should walking technicolor theories prove successful, it would be quite interesting to tie those results with the idea of an emergent limiting velocity. The other option is if there are a very large number of fields of different scales, such that the running is increased by a large (and energy dependent) factor. We explore this option below.

We introduce a large number N_f of hidden fermions in addition to the SM ones [12]. Moreover, we assume that all these fermions (hidden and SM) have the same origin, and hence all have the same initial speed of light $1 + c_{f_*}$, with $|c_{f_*}| \ll 1$, at some UV emergence scale μ_*. As a warm-up calculation, we assume that the fermions have a common initial charge e_* under a single $U(1)$ gauge sector. The gauge photon emerges with some initial speed $1 + c_{g_*}$, with $|c_{g_*}| \ll 1$, that is different from the speed of fermions. At the UV scale the fermions are taken to be massless and hence will participate in the running of the gauge coupling as well as photon and fermions speeds. As we run down our RG equations, some of the hidden fermions become massive and decouple from the RG equations. We model the dependence on the mass scale using a power law,

$$N_f(\mu) = \Gamma_f \left(\frac{\mu}{M_f}\right)^{\alpha_f}, \tag{56}$$

where Γ_f and α_f are positive constants, and M_f is an IR mass scale.[3] Since the fermions have a common initial speed and a common initial coupling strength, the evolution of the system can be modeled with a single c_f and e common to all fermions. Therefore, to one-loop order we have

[2]Clearly, for the running coupling in Yang-Mills theory, the coupling is small at high energies and becomes large only at low energy. However, the essential point is the same—that the coupling constant does not remain large at all energy scales.

[3]This exact behavior is also exhibited in models of large extra dimensions where the Kaluza-Klein modes (from the 4D point of view) obey a power law as in Eq. (56) [13]. In this context, M_f is the lowest Kaluza-Klein mode $M_f \sim 1/L$, where L is the size of the extra dimension, and $\alpha = d$ is the number of extra dimensions.

$$\mu\frac{de}{d\mu}=\frac{e^3}{12\pi^2}N_f(\mu),\qquad \mu\frac{dc_f}{d\mu}=\frac{e^2(c_f-c_g)}{3\pi^2},$$
$$\mu\frac{dc_g}{d\mu}=\frac{e^2(c_g-c_f)}{6\pi^2}N_f(\mu). \tag{57}$$

Integrating this system yields the running charge

$$e^2(\mu)=\frac{e_*^2}{1+\frac{e_*^2\Gamma_f}{6\pi^2\alpha_f}[(\frac{\mu_*}{M_f})^{\alpha_f}-(\frac{\mu}{M_f})^{\alpha_f}]}, \tag{58}$$

and speeds

$$\frac{c_g(\mu)-c_f(\mu)}{c_{g_*}-c_{f_*}}=\frac{e^2(\mu)}{e_*^2}\bigg[\bigg(\frac{e^2(\mu)}{e_*^2}\bigg)\bigg(\frac{\mu}{\mu_*}\bigg)^{\alpha_f}\bigg]^{e^2(0)/3\pi^2\alpha_f}\approx\frac{e^2(\mu)}{e_*^2}. \tag{59}$$

Hence, we see that both $e(\mu)$ and $c_g(\mu)-c_f(\mu)$ experience power-law running with IR values given by

$$\frac{c_g(0)-c_f(0)}{c_{g_*}-c_{f_*}}\approx\frac{e^2(0)}{e_*^2}\approx\frac{6\pi^2\alpha_f}{e_*^2\Gamma_f}\bigg(\frac{M}{\mu_*}\bigg)^{\alpha_f}. \tag{60}$$

Therefore, we can choose $\mu_*/M\sim 10^{14/\alpha_f}$ in order to meet the stringent requirement $\eta\sim 10^{-14}$. However, taking $e_*^2\lesssim 1$, so that we can trust our perturbation theory weakens the coupling strength to values $\sim 10^{-14}$. This is way below any interesting phenomenology.

In order to cure this problem, we introduce a large number of hidden $U(1)$ sectors in addition to the SM $U(1)_{\rm hyp}$. We also assume that all these gauge sectors emerge with the same initial speed of light. In addition, we take all fermions to be charged under the different $U(1)$'s with the same initial charge. As in the case of fermions, we assume that the hidden $U(1)$'s are massless at the UV scale; then they become massive and decouple as we run down the RG equations. Hence, the gauge sector obeys the scale-dependent relation

$$N_g(\mu)=\Gamma_g\bigg(\frac{\mu}{M_g}\bigg)^{\alpha_g}. \tag{61}$$

Under these assumptions the second equation in (57) is replaced by

$$\mu\frac{dc_f}{d\mu}=\frac{e^2(c_f-c_g)}{3\pi^2}N_g(\mu). \tag{62}$$

The solution of $e(\mu)$ is still given by Eq. (58), while that of $c_g(\mu)-c_f(\mu)$ can be expressed in terms of the hypergeometric function. Instead, we numerically integrate our system, setting appropriate values of the parameters α_f, α_g, Γ_f, and Γ_g. From Fig. 3 we see that introducing many copies of $U(1)$'s achieves power-law suppression of η in a very short interval. Since the many $U(1)$'s do not overwhelm the evolution of e, we can still get reasonable coupling strength in the IR.

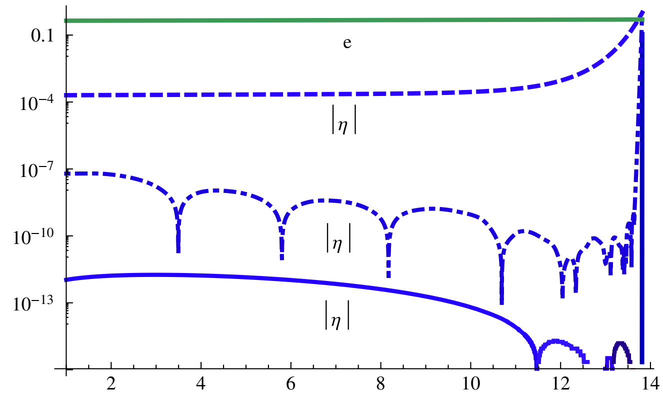

FIG. 3 (color online). Numerical simulation of the running of the charge e, and ratio $\eta=1-c_f/c_g$. The horizontal axis is in units of $\log(\mu/M_f)$. We take $M_g=M_f=1$, $\Gamma_g=10^{-3}$, $\Gamma_f=5$, $\alpha_f=10^{-2}$, $\mu_*/M_f=10^6$, and $\alpha_g=1$, 1.2, and 2 for the dashed, dot-dashed, and continuous lines, respectively. We also use the initial conditions $c_{f_*}=0.6$, $c_{g_*}=0.3$, and $e_*=0.5$. The running of e is logarithmic, while the running of η is power law. Very small values of η are achieved in a very short interval of running as we increase α_g, i.e. as we increase the number of gauge sectors. We also note that for $\alpha_g=2$ the value of η is saturated by the error tolerance of the code. More powerful computations may give smaller values.

To understand the choice of parameters used in the simulation in Fig. 3, it is instructive to calculate the total number of fermions and gauge fields as seen in the UV. Using Eqs. (56) and (61), and the numerical coefficients given in Fig. 3 (take $\alpha_g=2$), we find $N_g\approx 10^9$ and $N_f\approx 6$, which explains the above findings (we check our perturbative results below). Because the RG flow of e is enhanced only by a few fermionic species, the coupling constant runs only logarithmically, while the huge number of gauge fields that participate in the RG equation of speeds force the running of c_g-c_f to be extremely fast.

Because the freezing of the speeds sets in almost immediately, we can try to replace the power-law numbers in Eqs. (56) and (61) by constant numbers. The idea is that we just need to have $N_g\gg N_f\gtrsim 1$ for a short interval until the speeds freeze to their desired values. Then, we can immediately integrate the RG equations to find

$$e^2(\mu)=\frac{e_*^2}{1+\frac{N_fe_*^2}{6\pi^2}\log(\frac{\mu_*}{\mu})},$$
$$\frac{c_g(\mu)-c_f(\mu)}{c_{g_*}-c_{f_*}}\cong\bigg(\frac{e^2(\mu)}{e_*^2}\bigg)^{2N_g/N_f}. \tag{63}$$

At this point, one must make sure that higher loop corrections are suppressed; otherwise our perturbative expansion breaks down. Including the multiloop polarization graphs,

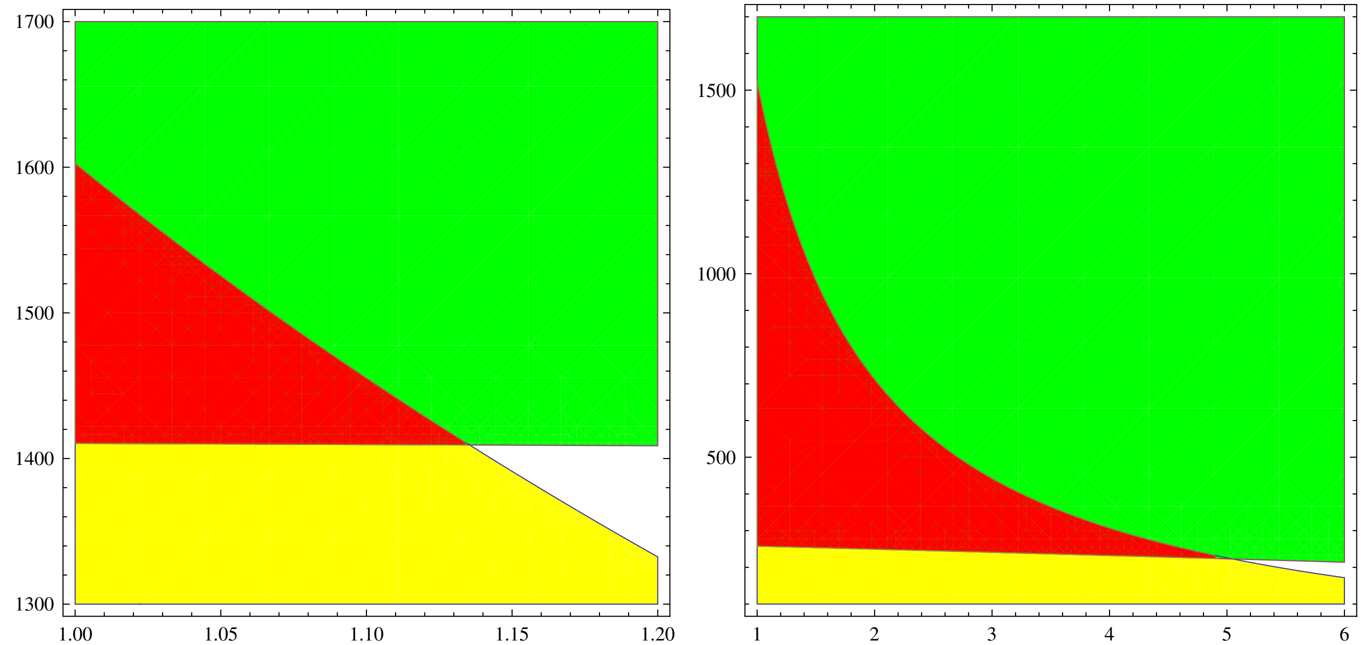

FIG. 4 (color online). The allowed parameter space (number of species) for $\mu_*/\mu_{\rm IR} = 10^3$ (left panel) and 10^{16} (right panel). The horizontal and vertical axes are, respectively, N_f and N_g. We take $e_{\rm IR}^2/4\pi \approx 1/129$, and then we solve for e_* from the first formula in Eq. (63). In all cases we check that $e_*^2 < 4\pi$ so we can trust our perturbation theory even in the UV. The red region is the intersection between the yellow and green areas. The green (upper) region represents the parameters that satisfy $\eta \cong |\frac{c_g(\mu_{\rm IR}) - c_f(\mu_{\rm IR})}{c_{g*} - c_{f*}}| \cong (\frac{e_{\rm IR}^2}{e_*^2})^{2N_g/N_f} < 10^{-14}$, while parameters in the yellow (lower) region ensure the validity of perturbation theory, i.e. $N_f N_g < 16\pi^2/e_*^2$.

and assuming $N_g \gg N_f \gtrsim 1$, we find that the RG running of the electric charge will be given by

$$\mu \frac{de}{d\mu} = e\bigg[\frac{4}{3}N_f\frac{e^2}{(4\pi)^2} + \mathcal{C}_2 N_g N_f\bigg(\frac{e^2}{(4\pi)^2}\bigg)^2 + \mathcal{C}_3 N_g^2 N_f^2\bigg(\frac{e^2}{(4\pi)^2}\bigg)^3 + \cdots\bigg], \tag{64}$$

where $\mathcal{C}_2$ and $\mathcal{C}_3$ are, respectively, the two-loop and three-loop numerical coefficients, and the dots represent higher order corrections.[4] These corrections continue the same trend as the lowest order ones. Hence, the perturbative expansion can be trusted as long as $N_g N_f < 16\pi^2/e^2$. We explored the parameter space, N_f and N_g, searching for the parameters that would allow the use of perturbation theory and, at the same time, give $\eta < 10^{-14}$. We found that $\mu_*/\mu_{\rm IR} \approx 500$ is a threshold value under which no parameters exist. In Fig. 4 we show the allowed parameter space (number of species) for $\mu_*/\mu_{\rm IR} = 10^3$ and 10^{16}. We see that the condition for validity of perturbation theory puts a severe constraint on the number of fermions. This constraint may be relaxed if we notice that the condition $N_f N_g < 16\pi^2/e^2$ is important mostly at the initial run, when e is relatively large. However, once e runs down the scale, for example, by means of nonperturbative RG treatment, the above condition may be satisfied with $N_f \sim \mathcal{O}(10)$ even for $\mu_*/\mu_{\rm IR} \lesssim 10^2$. So, as long as the nonperturbative result does not change the overall trend dramatically, fast running would be expected to still be obtained even if the perturbative analysis is not reliable in detail.

Assuming that the masses of the hidden sectors are larger than $\mu_{\rm IR}$, these masses decouple below $\mu_{\rm IR}$ and drop from the RG equations. Therefore, one needs only a constant large number of copies, $N_g \gg 1$ and $N_f \gtrsim 1$, to accomplish the emergence of an IR Lorentz-invariant fixed point in a relatively short interval of running. Moreover choosing the ratio $N_g/N_f \gg 1$, we can meet the stringent constraints on the parameter η. This opens up the possibility that many copies of hidden sectors may suppress Lorentz-violating effects already present at the TeV scale.

IX. DISCUSSION

Achieving a universal speed of light is a challenge for theories which do not postulate a fundamental Lorentz symmetry. This problem is visible in known emergence models [1] and also in Horava-Lifshitz gravity [2]. For emergent gauge fields, the Weinberg-Witten theorem [7] suggests that this will be a continual challenge, as a Lorentz-noninvariant initial theory may be required.

[4]In fact, at three loops, there are both $N_g^2 N_f$ and $N_g^2 N_f^2$ terms. The later dominate for $N_f \gtrsim 1$.

We have shown through several examples that a common limiting velocity can be emergent at low energies even if the original high energy theory involves fields satisfying the wave equation with different velocities. There is a heuristic rationale for this in that, since fields can transform into each other through interactions, the endpoint where all the fields travel in unison is preferred. The renormalization group treatment indeed produces this outcome.

Because the running is only logarithmic for simple systems, it would take an exponentially large amount of running in order that the limiting velocities be close enough to agree with experiment. We addressed the phenomenological constraints in Sec. VII. However, power-law running is also possible if the coupling is large and constant, or if there are a very large number of interacting degrees of freedom. We have reported on a model with this latter property.

It is important to note that not all forms of Lorentz violation disappear at low energies. The renormalization of a general parametrization of Lorentz violation of QED has been studied in Ref. [6], and some operators that grow at low energy are found. A well-behaved emergent theory must avoid those operators.

The running of the limiting velocities only happens due to the interactions that couple one type of particle to another. This implies that the running will be weakest if the coupling is weak. At low energies the gravitational coupling is by far the weakest of all the fundamental forces. This implies that the most plausible velocity difference would be that of gravity. While there have been some claims that the speed of gravity has been indirectly measured [14], the consensus appears to be that there is no experimental constraint on the speed of gravity [15]. However, indirectly there is a stringent limit at the 10^{-15} level on the difference of the speeds of gravity and that of light from gravitational Cherenkov radiation [16] which is valid if the speed of gravity is less than that of light. Future experiments with gravitational wave detectors provide the best opportunity to measure or constrain the difference if the speed of gravity is greater than that of light.

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APPENDIX A: USEFUL INTEGRALS

In this appendix we work out the details of the integrals I^0 and $\vec{I}$ appearing in Eq. (8). These integrals are given by

$$I^0 = \int \frac{dk^0}{2\pi} k^0 \int \frac{d\vec{k}}{(2\pi)^3} \frac{1}{(k^0)^2 - \vec{k}^2} \times \frac{1}{(p^0/c_b - c_f k^0/c_b)^2 - (\vec{k} - \vec{p})^2} \tag{A1}$$

and

$$\vec{I} = \int \frac{dk^0}{2\pi} \int \frac{d\vec{k}}{(2\pi)^3} \frac{\vec{k}}{(k^0)^2 - \vec{k}^2} \times \frac{1}{(p^0/c_b - c_f k^0/c_b)^2 - (\vec{k} - \vec{p})^2}. \tag{A2}$$

To perform the integral I^0, we first use the Feynman trick to find

$$I^0 = \int \frac{dk^0}{2\pi} k^0 \int \frac{d\vec{k}}{(2\pi)^3} \int_0^1 dx \frac{1}{(\vec{k}^2 - \Delta^2)^2}, \tag{A3}$$

where $\Delta^2 = -x(1-x)\vec{p}^2 + x(k^0)^2 + (1-x) \times (p^0/c_b - c_f k^0/c_b)^2$. Next, we interchange the integrals dx and $d\vec{k}$ and perform the integral over $d\vec{k}$ to find

$$I^0 = -\frac{i}{(4\pi)^{3/2}} \Gamma\left(\frac{1}{2}\right) \int \frac{dk^0}{2\pi} k^0 \int_0^1 dx \frac{1}{\sqrt{\Delta^2}}. \tag{A4}$$

Further, we exchange the integrals dx and dk^0, and rearrange the integrands to find

$$I^0 = -\frac{i}{(4\pi)^{3/2}} \Gamma\left(\frac{1}{2}\right) \int_0^1 dx \frac{1}{\sqrt{x(1 - c_f^2/c_b^2) + c_f^2/c_b^2}} \times \int \frac{dk^0}{2\pi} \frac{k^0}{\sqrt{k_0^2 + 2k_0 R_0 - M^2}}, \tag{A5}$$

where

$$R_0 = -\frac{(1-x)p^0 c_f/c_b^2}{x(1 - c_f^2/c_b^2) + c_f^2/c_b^2}, \qquad M^2 = \frac{-x(1-x)\vec{p}^2 + (1-x)(p^0)^2/c_b^2}{x(1 - c_f^2/c_b^2) + c_f^2/c_b^2}. \tag{A6}$$

Then, we perform the integral over dk^0, after analytically continuing from 1 to $d = 1 - \epsilon$ dimensions, to obtain

$$I^0 = \frac{i}{(4\pi)^2} \frac{c_f}{c_b^2} p^0 \int_0^1 dx \frac{(1-x)^{1-\epsilon/2}(-1)^{-\epsilon/2}}{[x(1 - c_f^2/c_b^2) + c_f^2/c_b^2]^{3/2-\epsilon}} \times \frac{\Gamma(\epsilon/2)}{\pi^{\epsilon/2}[x(p^0)^2/c_b^2 - x(x + (1-x)c_f^2/c_b^2)]^{\epsilon/2}}. \tag{A7}$$

Finally, we find

$$I^0 = \frac{ip^0}{(4\pi)^2}\frac{2c_b}{c_f^2(1+a)^2}\left(\frac{2}{\epsilon}+\text{finite}\right), \tag{A8}$$

where $a = c_b/c_f$. Similarly, we can show

$$\vec{I} = \frac{i\vec{p}}{(4\pi)^2}\frac{2a(1+2a)}{3(1+a)^2}\left(\frac{2}{\epsilon}+\text{finite}\right). \tag{A9}$$

The vertex correction in Eq. (11) results in the integral

$$-igG = \frac{g^3}{c_f c_b^2}\int\frac{dK^0}{2\pi}\int\frac{d\vec{K}}{(2\pi)^3}\frac{(K^0)^2-\vec{K}^2}{c_f^2(K^0)^2/c_b^2-\vec{K}^2} \times\frac{1}{[(P_1^0-K^0)^2-(\vec{P}_1-\vec{K})^2]} \times\frac{1}{[(P_2^0-K^0)^2-(\vec{P}_2-\vec{K})^2]}. \tag{A10}$$

Next, we use the Feynman trick to get

$$-igG = \frac{2g^3}{c_f c_b^2}\int\frac{dK^0}{2\pi}\int\frac{d\vec{K}}{(2\pi)^3}(\vec{K}^2-(K^0)^2) \times\int_0^1 dx\int_0^{1-x}dy\frac{1}{[\vec{K}^2-\Delta^2]^3}, \tag{A11}$$

where

$$\Delta^2 = [1+y(-1+c_f^2/c_b^2)](K^0)^2 - 2[(1-x-y)P_1^0 + P_2^0 x]K^0 + [(1-x-y)\vec{P}_1 + x\vec{P}_2]^2 + (1-x-y)P_1^2 + xP_2^2. \tag{A12}$$

Then, proceeding as we did before, we finally obtain the result in Eq. (12).

APPENDIX B: A GENERAL SETUP TO CALCULATE THE β FUNCTIONS

We assume that the parameter space is spanned by g_i, i, $j = 1, 2, \ldots, C$ couplings (these could be coupling strengths as well as speeds). Quantum loops will generate Z_m, m, $l = 1, 2, \ldots, D$ corrections to the original Lagrangian, and we restrict our treatment to one-loop order. In general, we may write

$$g_{i0} = g_i(\mu)\prod_{m=1}^{D} Z_m^{n_{m,i}}(\mu)\mu^{\epsilon p_i}. \tag{B1}$$

Taking the derivative of (B1) with respect to μ we obtain

$$g_i'(\mu)\prod_{m=1}^{D} Z_m^{n_{m,i}}(\mu)\mu^{\epsilon p_i} + g_i(\mu)\sum_{l=1}^{D} n_{l,i}Z_l'(\mu) \times\prod_{m\neq l}^{D} Z_m^{n_{m,i}}(\mu)\mu^{\epsilon p_i} + g_i(\mu)\epsilon p_i \times\prod_{m=1}^{D} Z_m^{n_{m,i}}(\mu)\mu^{\epsilon p_i - 1} = 0. \tag{B2}$$

Writing $Z_m(\mu) = 1 + \rho_m(\mu)\frac{2}{\epsilon}$ we find $Z_l'(\mu) = \frac{2}{\epsilon}\times \sum_{j=1}^{C}\frac{\partial\rho_l}{\partial g_j}g_j'(\mu)$. Also, using the definition $\beta_i(\mu) \equiv \mu\frac{\partial g_i(\mu)}{\partial\mu}$, Eq. (B2) becomes

$$\beta_i(\mu)\prod_{m=1}^{D}\left(1+\frac{2}{\epsilon}\rho_m(\mu)\right)^{n_{m,i}} + g_i(\mu) \times\sum_{l=1}^{D}\sum_{j=1}^{C}\frac{2}{\epsilon}n_{l,i}\frac{\partial\rho_l}{\partial g_j}\beta_j(\mu)\prod_{m\neq l}^{D}Z_m^{n_{m,i}}(\mu) + g_i(\mu)p_i\epsilon\prod_{m}^{D}\left(1+\frac{2}{\epsilon}\rho_m(\mu)\right)^{n_{m,i}} = 0. \tag{B3}$$

Since we are only interested in one-loop corrections, we can ignore all $\mathcal{O}(1/\epsilon^2)$ terms. Hence, Eq. (B3) reads

$$\beta_i(\mu)\left[1+\frac{2}{\epsilon}\sum_{m=1}^{D}\rho_m(\mu)n_{m,i}\right] + \frac{2}{\epsilon}g_i(\mu)\sum_{l=1}^{D}\sum_{j=1}^{C}n_{l,i}\beta_j(\mu) \times\frac{\partial\rho_l}{\partial g_j} + g_i(\mu)p_i\epsilon\left[1+\frac{2}{\epsilon}\sum_{m=1}^{D}\rho_m(\mu)n_{m,i}\right] = 0. \tag{B4}$$

Now, we can rewrite Eq. (B4) in the following simple expression:

$$\beta_i(\mu)\left[1+\frac{2}{\epsilon}\mathcal{A}_i(\mu)\right] + \frac{2}{\epsilon}\sum_{j\neq i}^{C}\mathcal{C}_{ij}(\mu)\beta_j(\mu) = -g_i(\mu)p_i\left[\epsilon + 2\sum_{m=1}^{D}\rho_m(\mu)n_{m,i}\right], \tag{B5}$$

where

$$\mathcal{A}_i(\mu) = \sum_{m=1}^{D}\left[\rho_m(\mu) + g_i(\mu)\frac{\partial\rho_m}{\partial g_i}\right]n_{m,i}, \qquad \mathcal{C}_{ij}(\mu) = g_i(\mu)\sum_{m=1}^{D}\frac{\partial\rho_m}{\partial g_j}n_{m,i}. \tag{B6}$$

Equation (B5) can also be written in the matrix form

$$\vec{\mathcal{M}}(\mu)\vec{\beta}(\mu) = -\vec{g}\vec{p}\left[\epsilon + 2\sum_{m=1}^{D}\rho_m n_{m,i}\right], \tag{B7}$$

where

$$\vec{\mathcal{M}}(\mu) = \begin{pmatrix} 1 + \frac{2}{\epsilon}\mathcal{A}_1 & \frac{2}{\epsilon}\mathcal{C}_{12} & \dots & \frac{2}{\epsilon}\mathcal{C}_{1C} \\ \frac{2}{\epsilon}\mathcal{C}_{21} & 1 + \frac{2}{\epsilon}\mathcal{A}_2 & \dots & \frac{2}{\epsilon}\mathcal{C}_{2C} \\ \dots & \dots & \dots & \dots \\ \frac{2}{\epsilon}\mathcal{C}_{C1} & \frac{2}{\epsilon}\mathcal{C}_{C2} & \dots & 1 + \frac{2}{\epsilon}\mathcal{A}_C \end{pmatrix}. \tag{B8}$$

The inverse of $\vec{\mathcal{M}}(\mu)$ is given by

$$\vec{\mathcal{M}}^{-1}(\mu) = \begin{pmatrix} 1 - \frac{2}{\epsilon}\mathcal{A}_1 & -\frac{2}{\epsilon}\mathcal{C}_{12} & \dots & -\frac{2}{\epsilon}\mathcal{C}_{1C} \\ -\frac{2}{\epsilon}\mathcal{C}_{21} & 1 - \frac{2}{\epsilon}\mathcal{A}_2 & \dots & -\frac{2}{\epsilon}\mathcal{C}_{2C} \\ \dots & \dots & \dots & \dots \\ -\frac{2}{\epsilon}\mathcal{C}_{C1} & -\frac{2}{\epsilon}\mathcal{C}_{C2} & \dots & 1 - \frac{2}{\epsilon}\mathcal{A}_C \end{pmatrix} + \mathcal{O}\left(\frac{1}{\epsilon^2}\right). \tag{B9}$$

Hence, solving for $\vec{\beta}$ from (B7) we obtain

$$\beta_i = 2g_i p_i \left(\mathcal{A}_i - \sum_{m=1}^{D} \rho_m n_{m,i}\right) + 2\sum_{j\neq i}^{C} \mathcal{C}_{ij} g_j p_j. \tag{B10}$$

Finally, we rearrange the terms to find

$$\beta_i = 2g_i \sum_{m=1}^{D} n_{m,i} \sum_{j=1}^{C} p_j g_j \frac{\partial \rho_m}{\partial g_j}. \tag{B11}$$

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