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Vacuum fluctuations in a supersymmetric model in FRW spacetime

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We study a noninteracting supersymmetric model in an expanding FRW spacetime. A soft supersymmetry breaking induces a nonzero contribution to the vacuum energy density. A short distance cutoff of the order of Planck length provides a scale for the vacuum energy density comparable with the observed cosmological constant. Assuming the presence of a dark energy substance in addition to the vacuum fluctuations of the field, an effective equation of state is derived in a self-consistent approach. The effective equation of state is sensitive to the choice of the cutoff but no fine-tuning is needed.

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I. INTRODUCTION

It is generally accepted that the cosmological constant term which was introduced *ad hoc* in the Einstein-Hilbert action is actually related to the vacuum energy density of matter fields. Observational evidence for an accelerating expansion [1–3] implies that the vacuum energy density dominates the total energy density today. The vacuum energy density estimated in a simple quantum field theory is by about 120 orders of magnitude larger than the value required by astrophysical and cosmological observations [4] so that extreme fine-tuning is needed in order to make a cancellation up to 120 decimal places. Theoretically, it is possible that the cosmological constant is precisely zero and the acceleration of the universe expansion is attributed to the so-called *dark energy* (DE), a fluid with sufficiently negative pressure, such that its magnitude exceeds $1/3$ of the energy density. Nevertheless, even if such a substance exists, it is extremely difficult to tune the vacuum energy to be exactly zero. Hence, the fine-tuning problem persists unless there exists a symmetry principle that forbids a nonzero vacuum energy. Such a principle is indeed provided by supersymmetry [5]. In field theory with exact supersymmetry, the contributions of fermions and bosons to vacuum energy precisely cancel [6]. However, the supersymmetry in the real world is not exact.

A nonzero cosmological constant implies the de Sitter symmetry group of spacetime rather than the Poincaré group which is the spacetime symmetry group of an exact supersymmetry. Hence, the structure of de Sitter spacetime automatically breaks the supersymmetry. Conversely, a low energy supersymmetry breaking could in principle generate a nonzero cosmological constant of an acceptable magnitude. Unfortunately, the scale of supersymmetry breaking required by the particle physics phenomenology must be of the order of 1 TeV or larger implying a cosmological constant too large by about 60 orders of magnitude. Some nonsupersymmetric models with an equal number of boson and fermion degrees of freedom have been constructed [7] so that all the divergent contributions to the vacuum energy density cancel and a small finite contribution can be made comparable with the observed value of the cosmological constant.

In this paper we investigate the fate of vacuum energy when an unbroken supersymmetric model is embedded in spatially flat, homogeneous and isotropic spacetime. In addition, we assume the presence of a dark energy type of substance obeying the equation of state $p_{\rm DE} = w\rho_{\rm DE}$, with $w < 0$. Unlike in flat spacetime, the vacuum energy density turns out to be nonzero depending on background metric. Hence, the expansion is caused by a combined effect of both DE and vacuum fluctuations of the supersymmetric field. Solving the Friedman equations self-consistently, we find the effective equation of state of DE. In particular, we find the conditions for which the effective expansion becomes of de Sitter type. The contribution of the supersymmetric field fluctuations is found to be of the same order of magnitude as DE and no fine-tuning is needed.

We do not claim that our model describes a realistic scenario but it is tempting to speculate along the lines described in an earlier paper [8] where a naive model of supersymmetry in de Sitter spacetime has been considered. Our working assumption is that the universe today contains DE and no matter apart from fluctuations of a supersymmetric vacuum as a relict of symmetry breaking in the early universe. Since the global geometry is nonflat, the lack of Poincaré symmetry will lift the Fermi-Bose degeneracy and the energy density of vacuum fluctuations will be nonzero. This type of "soft" supersymmetry breaking is similar to the supersymmetry breaking at finite temperature where the Fermi-Bose degeneracy is lifted by quantum statistics ([9] and references therein).

The remainder of the paper is organized as follows. In Sec. II we introduce a supersymmetric model in an expanding FRW universe. The calculations and results are presented in Sec. III. In Sec. IV we discuss the effective equation of state of DE. Concluding remarks are given in Sec. V. In the Appendix we review the covariant regularization schemes of the vacuum expectation value of the energy-momentum tensor in flat spacetime.

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II. THE MODEL

Here we consider a noninteracting Wess-Zumino supersymmetric model with N species and calculate the energy density of vacuum fluctuations in de Sitter spacetime. In general, the supersymmetric Lagrangian $\mathcal{L}$ for N chiral superfields has the form [10]

$$\mathcal{L} = \sum_i \Phi_i^\dagger \Phi_i|_D + W(\Phi)|_F + \text{H.c.}, \tag{1}$$

where the index i distinguishes the various left chiral superfields Φ_i and $W(\Phi)$ denotes the superpotential for which we take

$$W(\Phi) = \frac{1}{2}\sum_i m_i \Phi_i \Phi_i. \tag{2}$$

Eliminating auxiliary fields by equations of motion, the Lagrangian (1) may be recast in the form

$$\mathcal{L} = \partial_\mu \phi_i^\dagger \partial^\mu \phi_i - m_i^2 |\phi_i|^2 + \frac{i}{2}\bar{\Psi}_i \gamma^\mu \partial_\mu \Psi_i - \frac{1}{2} m_i \bar{\Psi}_i \Psi_i, \tag{3}$$

where ϕ_i and Ψ_i are the complex scalar and the Majorana spinor fields, respectively, and summation over the species index i is understood. For simplicity, from now on we suppress the dependence on i.

Next we assume a curved background spacetime geometry with metric $g_{\mu\nu}$. Spinors in curved spacetime are conveniently treated using the so-called vierbein formalism. The metric is decomposed as

$$g_{\mu\nu}(x) = \eta_{ab} e^a{}_\mu e^b{}_\nu; \qquad g^{\mu\nu}(x) = \eta^{ab} e_a{}^\mu e_b{}^\nu, \tag{4}$$

where the set of coefficients $e^a{}_\mu$ is called the *vierbein* and

$$e_a{}^\mu = \eta_{ab} g^{\mu\nu} e^b{}_\nu \tag{5}$$

is the inverse of the vierbein. Obviously,

$$g \equiv \det g_{\mu\nu} = -(\det e^a{}_\mu)^2. \tag{6}$$

The action may be written as

$$S = \int d^4x \sqrt{-g}(\mathcal{L}_B + \mathcal{L}_F), \tag{7}$$

where $\mathcal{L}_B$ and $\mathcal{L}_F$ are the boson and fermion Lagrangians, respectively. The Lagrangian for a complex scalar field may be expressed as the sum of the Lagrangians for two real fields:

$$\mathcal{L}_B = \frac{1}{2}\sum_{i=1}^{2} (g^{\mu\nu} \varphi^i_{,\mu} \varphi^i_{,\nu} - m^2 \varphi^{i2}). \tag{8}$$

The fermion part is given by [11]

$$\mathcal{L}_F = \frac{i}{4}(\bar{\Psi}\tilde{\gamma}^\mu \Psi_{;\mu} - \bar{\Psi}_{;\mu}\tilde{\gamma}^\mu \Psi) - \frac{1}{2} m \bar{\Psi}\Psi, \tag{9}$$

where $\tilde{\gamma}^\mu$ are the curved spacetime gamma matrices,

$$\tilde{\gamma}^\mu = e_a{}^\mu \gamma^a, \tag{10}$$

with ordinary Dirac gamma matrices denoted by γ^a. Variation of (7) with respect to $\bar{\Psi}$ yields the Dirac equation in curved spacetime:

$$i\tilde{\gamma}^\mu \Psi_{;\mu} - m\Psi = 0. \tag{11}$$

The covariant derivatives of the spinor are defined as

$$\Psi_{;\mu} = \Psi_{,\mu} - \Gamma_\mu \Psi, \tag{12}$$

$$\bar{\Psi}_{;\mu} = \bar{\Psi}_{,\mu} + \bar{\Psi}\Gamma_\mu, \tag{13}$$

where

$$\Gamma_\mu = \tfrac{1}{8}\omega_\mu{}^{ab}[\gamma^a, \gamma^b], \tag{14}$$

with the spin connection [12]

$$\omega_\mu{}^{ab} = -\eta^{bc} e_c{}^\nu (e^a{}_{\nu,\mu} - \Gamma^\lambda_{\mu\nu} e^a{}_\lambda). \tag{15}$$

In FRW metric the vierbein is diagonal and in spatially flat FRW spacetime takes a simple form:

$$e^a{}_\mu = \text{diag}(1, a, a, a), \tag{16}$$

where $a = a(t)$ is the cosmological expansion scale.

III. CALCULATION OF THE VACUUM ENERGY DENSITY AND PRESSURE

A spatially flat FRW metric is given by

$$ds^2 = dt^2 - a(t)^2 d\vec{x}^2. \tag{17}$$

It is convenient to work in the conformal frame with metric

$$ds^2 = a(\eta)^2 (d\eta^2 - d\vec{x}^2), \tag{18}$$

where the proper time t of the isotropic observers, or cosmic time, is related to the conformal time η as

$$dt = a(\eta) d\eta. \tag{19}$$

In order to calculate the energy density and pressure of the vacuum fluctuations, we need the vacuum expectation value of the energy-momentum tensor. The energy-momentum tensor is derived from S as [11]

$$T_{\mu\nu} = \frac{\eta_{ab} e^b{}_\mu}{\sqrt{-g}} \frac{\delta S}{\delta e_a{}^\nu} = T^F_{\mu\nu} + T^B_{\mu\nu}, \tag{20}$$

where the boson and fermion parts are derived from the respective scalar and spinor Lagrangians:

$$T^B_{\mu\nu} = \sum_{i=1}^{2} \partial_\mu \varphi^i \partial_\nu \varphi^i - g_{\mu\nu}\mathcal{L}_B, \tag{21}$$

$$T^F_{\mu\nu} = \frac{i}{4}(\bar{\psi}\tilde{\gamma}_{(\mu}\psi_{;\nu)} - \bar{\psi}_{(;\mu}\tilde{\gamma}_{\nu)}\psi). \tag{22}$$

Owing to the assumed homogeneity and isotropy of spacetime, the calculation of the density and pressure requires the $T^0{}_0$ component and the trace $T^\mu{}_\mu$. Specifically for the metric (18) we obtain

$$T^{B0}{}_0 = \sum_{i=1}^{2}\left(\frac{1}{2a^2}(\partial_\eta\varphi^i)^2 + \frac{1}{2a^2}(\nabla\varphi^i)^2 + \frac{1}{2}m^2\varphi^{i2}\right), \quad (23)$$

$$T^{B\mu}{}_\mu = \sum_{i=1}^{2}\left(-\frac{1}{a^2}(\partial_\eta\varphi^i)^2 + \frac{1}{a^2}(\nabla\varphi^i)^2 + 2m^2\varphi^{i2}\right), \quad (24)$$

$$T^{F0}{}_0 = -\frac{i}{4a^4}(\bar\psi\gamma^j\partial_j\psi - (\partial_j\bar\psi)\gamma^j\psi) + \frac{1}{2a^3}m\bar\psi\psi, \quad (25)$$

$$T^{F\mu}{}_\mu = \frac{1}{2a^3}m\bar\psi\psi. \quad (26)$$

Assuming a general perfect fluid form of the vacuum expectation value of $T_{\mu\nu}$,

$$\langle T_{\mu\nu}\rangle = (\rho + p)u_\mu u_\nu - p g_{\mu\nu}, \quad (27)$$

the energy density and pressure of the vacuum fluctuations are given by

$$\rho = u^\mu u^\nu\langle T_{\mu\nu}\rangle, \quad (28)$$

$$p = \tfrac{1}{3}(\rho - \langle T^\mu{}_\mu\rangle), \quad (29)$$

where u_μ is the velocity of the fluid and $\langle A\rangle$ denotes the vacuum expectation value of an operator A. In particular, for vacuum energy we expect

$$\langle T^{\mu\nu}_\Lambda\rangle = \rho_\Lambda g^{\mu\nu}, \quad (30)$$

in accord with Lorentz invariance. In this case we have

$$p_\Lambda = -\rho_\Lambda. \quad (31)$$

With this equation of state we reproduce empty-space Einstein's equations with a cosmological constant equal to

$$\Lambda = 8\pi G\rho_\Lambda. \quad (32)$$

In the following sections we make the calculations in comoving coordinates. In comoving coordinates Eqs. (28) and (29) simplify to

$$\rho_{\rm vac} = \langle T^0{}_0\rangle, \quad (33)$$

$$p_{\rm vac} = \tfrac{1}{3}\langle T^0{}_0 - T^\mu{}_\mu\rangle. \quad (34)$$

A. Scalar fields

Next we consider quantum scalar fields in a spatially flat FRW spacetime with metric (18). Each real scalar field operator is decomposed as

$$\varphi(\eta,\vec x) = \sum_{\vec k} a^{-1}(\chi_k(\eta)e^{i\vec k\vec x}a_k + \chi_k(\eta)^* e^{-i\vec k\vec x}a_k^\dagger), \quad (35)$$

in full analogy with the standard flat-spacetime expression (A3) considered in the Appendix. The function χ_k satisfies the field equation

$$\chi_k'' + (m^2a^2 + k^2 - a''/a)\chi_k = 0, \quad (36)$$

where the prime $'$ denotes a derivative with respect to the conformal time η. In the massless case, the exact solutions to this equation may easily be found [11]. In particular, in de Sitter spacetime $a''/a = 1/\eta^2$, and one finds positive frequency solutions:

$$\chi_k = \frac{1}{\sqrt{2Vk}}e^{-ik\eta}\left(1 - \frac{i}{k\eta}\right). \quad (37)$$

The operators a_k associated with these solutions annihilate the adiabatic vacuum in the asymptotic past (Bunch-Davies vacuum) [11,13].

If $m \neq 0$ solutions to (36) may be constructed by making use of the WKB ansatz

$$\chi_k(\eta) = \frac{1}{\sqrt{2VaW_k(\eta)}}e^{-i\int^\eta aW_k(\tau)d\tau}, \quad (38)$$

where the function W_k is found by solving (36) iteratively up to an arbitrary order in adiabatic expansion [12]. For our purpose we need the solution up to the 2nd order only which reads

$$W_k = \omega_k + \omega^{(2)}, \quad (39)$$

where

$$\omega_k = \sqrt{m^2 + k^2/a^2}. \quad (40)$$

The general expression for the second order term is [12]

$$\omega^{(2)} = -\frac{3}{8}\frac{1}{\omega_k}\frac{\dot a^2}{a^2} - \frac{3}{4}\frac{1}{\omega_k}\frac{\ddot a}{a} - \frac{3}{4}\frac{k^2}{a^2\omega_k^3}\frac{\dot a^2}{a^2} + \frac{1}{4}\frac{k^2}{a^2\omega_k^3}\frac{\ddot a}{a} + \frac{5}{8}\frac{k^4}{a^4\omega_k^5}\frac{\dot a^2}{a^2}, \quad (41)$$

where the overdot denotes a derivative with respect to the cosmic time t. Then, Eq. (39) may be written as

$$W_k = \omega_k - \frac{1}{2\omega_k}\left(\frac{\dot a^2}{a^2} + \frac{\ddot a}{a}\right)[1 + \mathcal{O}(m^2/\omega_k^2)], \quad (42)$$

or, using (19), as

$$W_k = \omega_k - \frac{1}{\omega_k}\frac{a''}{a^3}[1 + \mathcal{O}(m^2/\omega_k^2)]. \quad (43)$$

We can calculate now the vacuum expectation value of the 0-0 component and the trace of the boson energy-momentum tensor. Using (A6) and the commutation properties of a_k and $a_k^\dagger$, from (23) and (24) with (35) we find

$$\langle T^{B0}{}_0\rangle = \frac{V}{a^4}\int\frac{d^3k}{(2\pi)^3}(|\chi_k'|^2 + a^2\omega_k^2|\chi_k|^2), \quad (44)$$

$$\langle T^{B\mu}{}_\mu\rangle = -2\frac{V}{a^4}\int\frac{d^3k}{(2\pi)^3}[|\chi_k'|^2 - a^2(\omega_k^2 + m^2)|\chi_k|^2]. \quad (45)$$

Using (33) and (38) with (43) we obtain

$$\rho^B = \frac{1}{a^3}\int\frac{d^3k}{(2\pi)^3\omega_k}\bigg[\omega_k^2 + \frac{1}{2}\frac{a'^2}{a^4} + \frac{1}{2}\frac{a'^2}{a^4}\frac{m^2}{\omega_k^2} + \frac{1}{4}\bigg(2\frac{a'^2a''}{a^7} - \frac{a'a'''}{a^6}\bigg)\frac{1}{\omega_k^2} + \mathcal{O}(\omega_k^{-4})\bigg]. \tag{46}$$

The first term in square brackets is identical to the flat-spacetime result. The second term is a quadratically divergent contribution due to a nonflat geometry, the next two terms are logarithmically divergent, and the rest is finite. Similarly, with the help of (34) we find the boson contribution to the pressure:

$$p^B = \frac{1}{a^3}\int\frac{d^3k}{(2\pi)^3\omega_k}\bigg[\frac{k^2}{3a^2} + \frac{1}{6}\bigg(3\frac{a'^2}{a^4} - 2\frac{a''}{a^3}\bigg) + \frac{1}{6}\bigg(3\frac{a'^2}{a^4} - \frac{a''}{a^3}\bigg)\frac{m^2}{\omega_k^2} + \frac{1}{4}\bigg(2\frac{a'^2a''}{a^7} - \frac{a'a'''}{a^6}\bigg)\frac{1}{\omega_k^2} + \mathcal{O}(\omega_k^{-4})\bigg]. \tag{47}$$

B. Spinor fields

Next we proceed to quantize the fermions. The Dirac equation in curved spacetime may be derived from (9). Specifically for a spatially flat FRW metric, we obtain

$$i\gamma^0\bigg(\partial_0 + \frac{3}{2}\frac{\dot{a}}{a}\bigg)\Psi + i\frac{1}{a}\gamma^j\partial_j\Psi - m\Psi = 0. \tag{48}$$

Rescaling the Majorana fermion field Ψ as

$$\Psi = a^{-3/2}\psi, \tag{49}$$

and introducing the conformal time, we obtain for ψ the usual flat-spacetime Dirac equation

$$i\gamma^0\partial_\eta\psi + i\gamma^j\partial_j\psi - am\psi = 0, \tag{50}$$

with time dependent effective mass am. The quantization of ψ is now straightforward [14,15]. The Majorana field ψ may be decomposed as usual:

$$\psi(\eta,\vec{x}) = \sum_{\vec{k},s}(u_{ks}(\eta)e^{i\vec{k}\vec{x}}b_{ks} + v_{ks}(\eta)e^{-i\vec{k}\vec{x}}b_{ks}^\dagger), \tag{51}$$

where the spinor u_{ks} may be expressed as

$$u_{ks} = \frac{1}{\sqrt{V}}\begin{pmatrix}(i\zeta_k' + am\zeta_k)\phi_s\\ \vec{\sigma}\vec{k}\zeta_k\phi_s\end{pmatrix}. \tag{52}$$

Here, the two-spinors ϕ_s are the helicity eigenstates which may be chosen as

$$\phi_+ = \begin{pmatrix}1\\0\end{pmatrix}; \qquad \phi_- = \begin{pmatrix}0\\1\end{pmatrix}. \tag{53}$$

The spinor v_{ks} is related to u_{ks} by charge conjugation

$$v_{ks} = i\gamma^0\gamma^2(\bar{u}_{ks})^T. \tag{54}$$

The norm of the spinors may be easily calculated

$$\bar{u}_{ks}u_{ks} = -\bar{v}_{ks}v_{ks} = \frac{1}{V}(am\zeta_k^* - i\zeta_k^{*\prime})(am\zeta_k + i\zeta_k') - \frac{1}{V}k^2|\zeta_k|^2. \tag{55}$$

The mode functions ζ_k satisfy the equation

$$\zeta_k'' + (m^2a^2 + k^2 - ima')\zeta_k = 0. \tag{56}$$

In addition, the functions ζ_k satisfy the condition [15]

$$k^2|\zeta_k|^2 + (am\zeta_k^* - i\zeta_k^{*\prime})(am\zeta_k + i\zeta_k') = C_1. \tag{57}$$

It may be easily verified that the left-hand side of this equation is a constant of motion of Eq. (56). The constant C_1 is fixed by the normalization of the spinors and by the initial conditions. A natural assumption is that at $t = 0$ ($\eta = -1/H$, $a = 1$) the solution behaves as a plane wave $\zeta_k = C_2e^{-iE_kt}$, where $E_k = \sqrt{k^2 + m^2}$. This gives $\zeta_k(0) = C_2$, $\zeta_k'(0) = -iC_2E_k$, and hence $C_1 = 2C_2^2E_k(m + E_k)$. From (55) and (57) we obtain

$$\bar{u}_{ks}u_{ks} = -\bar{v}_{ks}v_{ks} = \frac{1}{V}(C_1 - 2k^2|\zeta_k|^2), \tag{58}$$

which at $t = 0$ reads

$$\bar{u}_{ks}u_{ks} = -\bar{v}_{ks}v_{ks} = C_1\frac{m}{VE_k}. \tag{59}$$

For $C_1 = 1$ this coincides with the standard flat-spacetime normalization [11].

In the massless case the solutions to (56) are plane waves. For $m \neq 0$ two methods have been used to solve (56) for a general spatially flat FRW spacetime: (a) expanding in negative powers of E_k and solving a recursive set of differential equations [14]; (b) using a WKB ansatz similar to (38) and the adiabatic expansion [15].

By making use of the decomposition (51) and the standard anticommuting properties of the creation and annihilation operators, the vacuum expectation value of the 0-0 component (25) and of the trace (26) of the fermion energy-momentum tensor may be written as

$$\langle T^{F0}{}_0\rangle = \frac{1}{2a^4}\sum_{\vec{k},s}\bar{v}_{ks}(am - \vec{k}\vec{\gamma})v_{ks}, \tag{60}$$

$$\langle T^{F\mu}{}_\mu\rangle = \frac{1}{2a^4}\sum_{\vec{k},s}am\bar{v}_{ks}v_{ks}. \tag{61}$$

Evaluating the expression under the sum and replacing the sum with an integral as in (A6), we obtain

$$\langle T^{F0}{}_0\rangle = \frac{1}{a^4}\int\frac{d^3k}{(2\pi)^3}[ik^2(\zeta_k\zeta_k^{*\prime} - \zeta_k^*\zeta_k') - am], \tag{62}$$

$$\langle T^{F\mu}{}_\mu\rangle = -\frac{1}{a^4}\int\frac{d^3k}{(2\pi)^3}am(1 - 2k^2|\zeta_k|^2). \tag{63}$$

The expressions under the integral sign in (62) and (63) were calculated by Baacke and Patzold [14]. We quote their results for the divergent contributions:

$$\langle T^{F0}{}_0\rangle_{\rm div} = \frac{1}{a^4}\int\frac{d^3k}{(2\pi)^3}\Big[-E_k - \frac{(a^2-1)m^2}{2E_k} + \frac{(a^2-1)^2m^4}{8E_k^3} + \frac{a'^2m^2}{8E_k^3}\Big], \tag{64}$$

$$\langle T^{F\mu}{}_\mu\rangle_{\rm div} = -\frac{1}{a^4}\int\frac{d^3k}{(2\pi)^3}\Big[\frac{a^2m^2}{E_k} - \frac{aa''m^2}{4E_k^3} - \frac{a^4m^4}{2E_k^3} + \frac{a^2m^4}{2E_k^3}\Big]. \tag{65}$$

Note that the first three terms in square brackets in (64) are identical to the first three terms in the expansion of $a\omega_k = \sqrt{E_k^2 + a^2m^2 - m^2}$ in powers of E_k^{-2}. Hence, we can write

$$\rho^F = \langle T^{F0}{}_0\rangle = \frac{1}{a^3}\int\frac{d^3k}{(2\pi)^3\omega_k}\Big[-\omega_k^2 + \frac{1}{8}\frac{a'^2}{a^4}\frac{m^2}{\omega_k^2} + \mathcal{O}(\omega_k^{-4})\Big]. \tag{66}$$

The first term in square brackets is precisely the flat-spacetime vacuum energy of the fermion field. The second term is a logarithmically divergent contribution due to the FRW geometry and the last term is finite and vanishes in the flat-spacetime limit $a' \to 0$. Note that, as opposed to bosons, there is no quadratic divergence of the type a'^2/ω_k.

Similarly, from (65) we obtain

$$\langle T^{F\mu}{}_\mu\rangle = \frac{1}{a^3}\int\frac{d^3k}{(2\pi)^3\omega_k}\Big[-m^2 + \frac{1}{4}\frac{a''}{a^3}\frac{m^2}{\omega_k^2} + \mathcal{O}(\omega_k^{-4})\Big], \tag{67}$$

and using (34) we find the fermion contribution to the pressure

$$p^F = \frac{1}{a^3}\int\frac{d^3k}{(2\pi)^3\omega_k}\Big[-\frac{1}{3}\frac{k^2}{a^2} + \frac{1}{24}\frac{a'^2}{a^4}\frac{m^2}{\omega_k^2} - \frac{1}{12}\frac{a''}{a^3}\frac{m^2}{\omega_k^2} + \mathcal{O}(\omega_k^{-4})\Big]. \tag{68}$$

C. Putting it all together

Assembling the boson and fermion contributions, the final expressions for the vacuum energy density and pressure of each chiral supermultiplet are

$$\rho = \rho^B + \rho^F = \frac{1}{a^3}\int\frac{d^3k}{(2\pi)^3\omega_k}\Big[\frac{1}{2}\frac{a'^2}{a^4} + \frac{5}{8}\frac{a'^2}{a^4}\frac{m^2}{\omega_k^2} + \frac{1}{4}\Big(2\frac{a'^2a''}{a^7} - \frac{a'a'''}{a^6}\Big)\frac{1}{\omega_k^2} + \mathcal{O}(\omega_k^{-4})\Big], \tag{69}$$

$$p = p^B + p^F = \frac{1}{a^3}\int\frac{d^3k}{(2\pi)^3\omega_k}\Big[\frac{1}{6}\Big(3\frac{a'^2}{a^4} - 2\frac{a''}{a^3}\Big) + \frac{1}{24}\Big(13\frac{a'^2}{a^4} - 6\frac{a''}{a^3}\Big)\frac{m^2}{\omega_k^2} + \frac{1}{4}\Big(2\frac{a'^2a''}{a^7} - \frac{a'a'''}{a^6}\Big)\frac{1}{\omega_k^2} + \mathcal{O}(\omega_k^{-4})\Big]. \tag{70}$$

The dominant contributions in (69) and (70) come from the leading terms in square brackets which diverge quadratically. Note that these quadratically divergent terms are due to bosons; fermions only provide a cancellation of all divergent and finite terms in the respective flat-spacetime contributions of bosons or fermions.

To make the results finite we need to regularize the integrals. We will use a simple 3-dim momentum cutoff regularization (recently dubbed "brute force" cutoff regularization [16]) which, as shown in the Appendix, may be regarded as a covariant regularization in a preferred Lorentz frame defined by the DE fluid.

The advantage of this approach is a clear physical meaning of the regularization scheme: one discards the part of the momentum integral over those momenta where a different, yet unknown physics should occur. In this scheme a preferred Lorentz frame is invoked which is natural in a cosmological context where a preferred reference frame exists: the frame fixed by the cosmic microwave background or large scale matter distribution. A similar standpoint was advocated by Maggiore [17] and Mangano [18]. Furthermore, as we have already demonstrated, a supersymmetry provides a cancellation of all flat-spacetime contributions irrespective of the regularization method one uses.

We change the integration variable to the physical momentum $p = k/a$ and introduce a cutoff of the order of the Planck mass $\Lambda_{\rm cut} \sim m_{\rm Pl}$. The leading terms yield

$$\rho = \frac{N}{4\pi^2}\frac{a'^2}{a^4}\int_0^{\Lambda_{\rm cut}} pdp(1 + \mathcal{O}(p^{-2})) \cong \frac{N\Lambda_{\rm cut}^2}{8\pi^2}\frac{a'^2}{a^4}(1 + \mathcal{O}(\Lambda_{\rm cut}^{-2}\ln\Lambda_{\rm cut})), \tag{71}$$

$$p \cong \frac{N\Lambda_{\rm cut}^2}{24\pi^2}\Big(3\frac{a'^2}{a^4} - 2\frac{a''}{a^3}\Big)(1 + \mathcal{O}(\Lambda_{\rm cut}^{-2}\ln\Lambda_{\rm cut})), \tag{72}$$

where N is the number of chiral species. Clearly, we do not obtain the vacuum equation of state (31) as may have been expected as a consequence of a regularization that assumes the existence of a preferred Lorentz frame.

In order to estimate the cutoff we first neglect background DE and assume that the total energy density ρ is given by (71). If we compare the first Friedman equation with (71) keeping the leading term on the right-hand side, we find that our cutoff should satisfy

$$\Lambda_{\rm cut} \cong \sqrt{\frac{3\pi}{N}} m_{\rm Pl}. \tag{73}$$

It is worthwhile to note that several approaches [17,19–21] with substantially different underlying philosophy have led to results similar to (71). In particular, Cohen, Kaplan, and Nelson [19] have employed a cosmological horizon radius $R_H = 1/H$ as a long distance cutoff and derived an upper bound

$$\rho \cong \Lambda_{\rm UV}^4 \leq \frac{3}{8\pi} \frac{m_{\rm Pl}^2}{L^2} \tag{74}$$

from a holographic principle. Here, $\Lambda_{\rm UV}$ and L denote the ultraviolet and long distance cutoffs, respectively. Our result would saturate the holographic bound (74) if we identify $a'/a^2 = 1/L$.

The closest approach to ours is that of Maggiore [17] and Sloth [21], who present a similar calculation of zero-point energy using massless boson fields only. The main difference in [17] with respect to ours is that the cancellation of the quartic contributions was done by hand on the basis of the procedure used previously in the literature with the so-called ADM mass. In our model, the cancellation of all (not only quartically divergent) flat-spacetime contributions is naturally provided by supersymmetry. Another difference is that our results (71) and (72) are sufficiently general to allow a self-consistent approach.

The above consideration gives only an estimate for the cutoff. In the next section we give a self-consistent treatment of the supersymmetric vacuum fluctuations in the presence of DE.

IV. EFFECTIVE EQUATION OF STATE

Since there is no way to precisely determine the cutoff, it is convenient to introduce a free dimensionless cutoff parameter λ of order $\lambda \lesssim 1$ such that

$$\Lambda_{\rm cut} = \lambda \sqrt{\frac{3\pi}{N}} m_{\rm Pl}. \tag{75}$$

The factor $1/\sqrt{N}$ is introduced to make the result independent of the number of species. If we reinstate the cosmic time t, Eqs. (71) and (72) become

$$\rho = \lambda \frac{3}{8\pi G} \frac{\dot{a}^2}{a^2}, \tag{76}$$

$$p = \lambda \frac{1}{8\pi G} \left(\frac{\dot{a}^2}{a^2} - 2\frac{\ddot{a}}{a} \right). \tag{77}$$

Obviously, the pressure is negative if $\dot{a}^2 < 2a\ddot{a}$. For example, for a de Sitter expansion we find $\dot{a}^2 = a\ddot{a}$ and $p = -\rho/3$. This case was considered by Maggiore [17], who concluded that the vacuum fluctuations cannot (at least in his approach) be interpreted as a part of the cosmological constant because in the second Friedman equation the accelerating effects of pressure are canceled by those from the density. We shall see shortly that this conclusion is slightly altered in a self-consistent approach to the effective equation of state.

In addition to vacuum fluctuations of matter fields, we assume existence of DE characterized by the equation of state $p_{\rm DE} = w\rho_{\rm DE}$. The Friedman equations then take the form

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi}{3} G\rho_{\rm DE} + \lambda \frac{\dot{a}^2}{a^2}, \tag{78}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3} G(\rho_{\rm DE} + 3p_{\rm DE}) - \lambda \left(\frac{\dot{a}^2}{a^2} - \frac{\ddot{a}}{a} \right). \tag{79}$$

Introducing the effective equation of state

$$p_{\rm eff} = w_{\rm eff}\rho_{\rm eff}, \tag{80}$$

where

$$\rho_{\rm eff} = \frac{\rho_{\rm DE}}{1-\lambda}, \tag{81}$$

$$w_{\rm eff} = w + \frac{2}{3}\frac{\lambda}{1-\lambda}, \tag{82}$$

Eqs. (78) and (79) may be recast in the standard FRW form:

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi}{3} G\rho_{\rm eff}, \tag{83}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3} G(1 + 3w_{\rm eff})\rho_{\rm eff}. \tag{84}$$

Three remarks are in order. First, it is clear from (81) why we have chosen the cutoff parameter λ less than 1. Second, it follows from (82) that the contribution of the vacuum fluctuations to the effective equation of state is always positive and, hence, it goes against acceleration! The third remark concerns the Bianchi identity which would not be respected if the vacuum fluctuations were the only source of gravity in Einstein's equations. However, because of the additional contribution to the energy-momentum tensor coming from DE, it is not necessary to have both contributions separately conserved. Since the effective pressure and energy density satisfy Einstein's field equations (83) and (84), the combined energy momentum is conserved and therefore the Bianchi identity is respected. In this way an interaction between the vacuum fluctuations and DE is implicitly assumed in the spirit of the two component model of Grande, Sola, and Štefančić [22].

It is worthwhile to analyze interesting cosmological solutions to Eqs. (83) and (84) depending on the nature of DE given by the equation of state $p_{\rm DE} = w\rho_{\rm DE}$.

(1) Consider first the case when there is no DE, i.e., when $p_{\rm DE} = \rho_{\rm DE} = 0$. In this case Eqs. (78) and (79) admit only a trivial solution $a =$ const. Clearly, if $\lambda = 1$, Eq. (78) becomes a trivial identity and

Eq. (79) implies $\dot{a} = 0$. If $\lambda \neq 1$, Eqs. (78) and (79) are satisfied if and only if $\dot{a} = 0$. Therefore, $a =$ const is the only solution to (78) and (79) for any choice of λ. In other words, FRW spacetime cannot be generated by vacuum fluctuations alone in an empty background.

(2) Another interesting special case is DE represented by a cosmological constant, i.e., for the equation of state $p_{\rm DE} = -\rho_{\rm DE}$. It follows from (82) that an accelerated expansion ($w_{\rm eff} < -1/3$) is achieved for any value of the cutoff parameter in the range $0 < \lambda < 1/2$. This case has also been discussed in [17,18].

(3) A more general case is obtained if we only require accelerating expansion, i.e., if the effective equation of state satisfies $w_{\rm eff} < -1/3$. Then Eq. (82) implies that the range $-1 < w < -1/3$ is compatible with $0 < \lambda < 1/2$, whereas $w < -1$ would imply $\lambda > 1/2$. In the latter case the DE equation of state violates the dominant energy condition. The fluid of which the equation of state violates the dominant energy condition was dubbed *phantom energy* [23,24] and has recently become a popular alternative to quintessence and cosmological constant [25].

(4) In the last example, we require that the background be de Sitter, i.e., $w_{\rm eff} = -1$. In other words the effective equation of state describes an effective cosmological constant. From (82) we find

$$w = -\frac{2}{3}\frac{\lambda}{1-\lambda} - 1. \tag{85}$$

Hence, this case may be realized only for a fluid with $w < -1$, i.e., for the phantom energy. We see that in a self-consistent approach, unlike in the example discussed in [17], a de Sitter expansion can be achieved as a result of a combined effect of DE and vacuum fluctuations.

V. CONCLUSION

We have calculated the contribution of supersymmetric fields to vacuum energy in spatially flat, homogeneous and isotropic spacetime. In addition to supersymmetric fields we have assumed existence of a substance obeying the equation of state $p_{\rm DE} = w\rho_{\rm DE}$, with $w < 0$. Unlike in flat spacetime, the vacuum fluctuations turn out to be nonzero depending on background metric. Combining effects of both dark energy and vacuum fluctuations of the supersymmetric field in a self-consistent way, we have found the effective equation of state. In particular, we have found the conditions for which the effective expansion becomes of de Sitter type. The contribution of the supersymmetric field fluctuations is of the same order of magnitude as DE and no fine-tuning is needed.

We have found that if we impose a UV cutoff of the order $m_{\rm Pl}$ the leading term in the energy density of vacuum fluctuations is of the order $H^2 m_{\rm Pl}^2$, where $H = \dot{a}/a$. In this way, if we identify the expansion parameter H with the Hubble parameter today, the model provides a phenomenologically acceptable value of the vacuum energy density. We have also found that a consistency with the Friedman equations implies that a natural cutoff must be inversely proportional to $\sqrt{N}$. A similar natural cutoff has been recently proposed in order to resolve the so-called species problem of black-hole entropy [26].

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APPENDIX: COVARIANT REGULARIZATION OF $T_{\mu\nu}$ IN FLAT SPACETIME

To illustrate problems related to the field theoretical calculation of vacuum energy, we review the well-known results for the scalar field in flat spacetime [27–29]. Consider a single noninteracting real scalar field described by the Lagrangian

$$\mathcal{L} = \tfrac{1}{2}\eta^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu} - \tfrac{1}{2}m^2\varphi^2, \tag{A1}$$

with the corresponding energy-momentum tensor

$$T_{\mu\nu} = \partial_\mu\varphi\partial_\nu\varphi - \eta_{\mu\nu}\mathcal{L}. \tag{A2}$$

The field operator is decomposed as

$$\varphi(t, \vec{x}) = \sum_{\vec{k}} \frac{1}{\sqrt{2VE_k}}(e^{-iE_kt+i\vec{k}\vec{x}}a_k + e^{iE_kt-i\vec{k}\vec{x}}a_k^\dagger), \tag{A3}$$

where

$$E_k = \sqrt{m^2 + k^2} \tag{A4}$$

and a_k and $a_k^\dagger$ are the annihilation and creation operators, respectively, associated with the plane wave solutions with the standard commutation properties,

$$[a_k, a_{k'}^\dagger] = \delta_{\vec{k}\vec{k}'}. \tag{A5}$$

From (A1)–(A3) with (A5) and replacing the sum over momenta by an integral in the usual way,

$$\sum_{\vec{k}} = V\int\frac{d^3k}{(2\pi)^3}, \tag{A6}$$

we find the vacuum expectation value of $T_{\mu\nu}$:

$$\langle T_{\mu\nu}\rangle = \frac{1}{2}\int\frac{d^3k}{(2\pi)^3E_k}k_\mu k_\nu, \tag{A7}$$

where $k_\mu = (E_k, \vec{k})$. The right-hand side of (A7) may be expressed in a manifestly covariant way [27]:

$$\langle T_{\mu\nu}\rangle = \int \frac{d^4k}{(2\pi)^3} k_\mu k_\nu \delta(k^\rho k_\rho - m^2)\theta(k_0). \tag{A8}$$

The delta function under the integral restricts the integration to the hypersurface defined by

$$k^\mu k_\mu - m^2 = 0; \qquad k_0 > 0, \tag{A9}$$

with the invariant measure d^3k/E_k on the hypersurface. Performing the integral over k_0 in (A8), one recovers (A7). However, if one assumes the vacuum expectation value of $T_{\mu\nu}$ to be of the form

$$\langle T_{\mu\nu}\rangle = \rho_{\rm vac} g_{\mu\nu}, \tag{A10}$$

as dictated by Lorentz invariance of the vacuum, one encounters inconsistency since different results for $\rho_{\rm vac}$ are obtained depending on which component of $T_{\mu\nu}$ one calculates. For example, using T_{00}, one finds

$$\rho_{\rm vac} = \langle T_{00}\rangle = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3} E_k. \tag{A11}$$

On the other hand, using the trace, one finds

$$\rho_{\rm vac} = \frac{1}{4} T^\mu{}_\mu = \frac{m^2}{8}\int \frac{d^3k}{(2\pi)^3 E_k}, \tag{A12}$$

which does not agree with (A11). One must conclude that the assumption (A10) is not compatible with (A8). The reason for this inconsistency is that the integrals in expressions (A7) and (A8) are divergent and make sense only if they are regularized.

One way to covariantly regularize (A7) or (A8) is to cut the hypersurface (A9) by a spacelike hyperplane defined by

$$f(k_\mu) \equiv u^\mu k_\mu - \sqrt{K^2 + m^2} = 0, \tag{A13}$$

where K is an arbitrary constant of dimension of mass and u_μ is a general future directed timelike unit vector, which may be parametrized as

$$u_0 = \cosh\alpha, \qquad u_1 = \sinh\alpha \sin\theta \cos\phi,$$
$$u_2 = \sinh\alpha \sin\theta \sin\phi, \qquad u_3 = \sinh\alpha \cos\theta. \tag{A14}$$

Clearly, the vector u_μ is normal to the hypersurface $f =$ const because $\partial f/\partial k^\mu = u_\mu$. In this way, one effectively introduces a preferred Lorentz frame defined by the vector u_μ as if the vacuum fluctuations are embedded in a homogeneous fluid moving with the velocity u_μ. The special form of the constant in (A13) is chosen for convenience.

The hyperplane cuts the hypersurface (A9) at a two-dimensional intersection defined by (A13) together with (A9). This gives a quadratic equation the solutions of which define a two-dimensional closed surface as a boundary of the integration domain Σ defined by

$$\sqrt{K^2 + m^2} - u^\mu k_\mu > 0 \tag{A15}$$

together with (A9). Hence, the regularized expression for $\langle T_{\mu\nu}\rangle$ is given by

$$\langle T_{\mu\nu}\rangle = \frac{1}{2}\int_\Sigma \frac{d^3k}{(2\pi)^3 E_k} k_\mu k_\nu, \tag{A16}$$

or in a manifestly covariant form

$$\langle T_{\mu\nu}\rangle = \int \frac{d^4k}{(2\pi)^3} k_\mu k_\nu \delta(k^\rho k_\rho - m^2) \times \theta(u^\rho k_\rho)\theta(\sqrt{K^2 + m^2} - u^\rho k_\rho). \tag{A17}$$

Using a general perfect fluid form (27), ρ and p are given by the invariant expressions

$$\rho = \frac{1}{2}\int_\Sigma \frac{d^3k}{(2\pi)^3 E_k} (u^\mu k_\mu)^2, \tag{A18}$$

$$p = \frac{1}{6}\int_\Sigma \frac{d^3k}{(2\pi)^3 E_k} [(u^\mu k_\mu)^2 - m^2]. \tag{A19}$$

In comoving frame ($\alpha = 0$), the integration domain Σ becomes a ball of radius K and we obtain

$$\rho = \langle T_{00}\rangle = \frac{1}{2}\int_{k<K} \frac{d^3k}{(2\pi)^3} E_k, \tag{A20}$$

$$p = \langle T_{ii}\rangle = \frac{1}{6}\int_{k<K} \frac{d^3k}{(2\pi)^3 E_k} k^2. \tag{A21}$$

Hence, the described covariant regularization is equivalent to a simple 3-dim momentum cutoff procedure. The integration yields

$$\rho = \frac{K^4}{16\pi^2} + \frac{m^2K^2}{16\pi^2} - \frac{1}{64\pi^2}\ln\frac{K^2}{m^2} + \cdots, \tag{A22}$$

$$p = \frac{1}{3}\frac{K^4}{16\pi^2} - \frac{1}{3}\frac{m^2K^2}{16\pi^2} + \frac{1}{64\pi^2}\ln\frac{K^2}{m^2} + \cdots, \tag{A23}$$

where the ellipses denote the finite terms.

This result reveals two problems. The first one concerns the fine-tuning. Assuming that the ordinary field theory is valid up to the scale of quantum gravity, i.e. the Planck scale, the leading term in (A22) yields

$$\rho \approx \frac{m_{\rm Pl}^4}{16\pi^2} \approx 10^{73}\ {\rm GeV}^4, \tag{A24}$$

compared with the observed value

$$\rho_{\rm cr} \approx 10^{-47}\ {\rm GeV}^4. \tag{A25}$$

This huge discrepancy may be easily rectified in flat spacetime simply by subtracting all divergent contributions and redefining the vacuum to have its energy exactly zero. However, as soon as we demand that vacuum energy or cosmological constant is nonzero, the calculations should be repeated in curved spacetime (e.g. de Sitter spacetime) and a simple subtraction of vacuum energy by fiat cannot be done.

If, in addition to the vacuum fluctuations of the field, one assumes that there exists an independent cosmological constant term Λ, as a result one would find an effective vacuum energy

$$\rho_{\rm eff} = \rho + \rho_\Lambda. \tag{A26}$$

In order to reproduce the observed value, one needs a cancellation of the two terms on the right-hand side up to 120 decimal places! The problem is actually much more severe as there are many contributions to vacuum energy from different fields with different interactions and all these contributions must somehow cancel to give the observed vacuum energy density.

The second problem is related to the equation of state. Obviously, Eqs. (A22) and (A23) do not reproduce the expected vacuum energy equation of state (31), as required by Lorentz invariance. Instead we find $p = \rho/3$ for the quartic term, $p = -\rho/3$ for the quadratic term, and only the logarithmic term satisfies (31). This violation of Lorentz invariance is not surprising since the adopted covariant regularization procedure assumes existence of a preferred Lorentz frame.

In principle, it is possible to regularize the energy-momentum tensor by imposing (30) and ignoring the mentioned inconsistency of the derived covariant expression (A7). Then, using the manifestly covariant form (A8) of the energy-momentum tensor, one can calculate the components using covariant regularization schemes which do not invoke a preferred Lorentz frame. For example, the dimensional regularization with the $\overline{\rm MS}$ prescription gives [28]

$$\rho_{\rm dim} = -p_{\rm dim} = -\frac{m^4}{64\pi^2}\left(\ln\frac{K^2}{m^2} + \frac{3}{2}\right), \tag{A27}$$

and one would conclude that a covariant regularization removes the Lorentz violating quartic and quadratic divergences and retains only the logarithmically divergent term which agrees with the logarithmic term of the 3-dim cutoff procedure in (A22) and (A23). However, in the Pauli Villars regularization, one finds [27]

$$\rho_{\rm PV} = -p_{\rm PV} = \frac{1}{64\pi^2}\left[-\frac{1}{2}K^4 + 2m^2K^2 - m^4\left(\ln\frac{K^2}{m^2} + \frac{3}{2}\right)\right], \tag{A28}$$

so in this covariant procedure the quartic and quadratic divergences are present with coefficients different from those of the 3-dim cutoff procedure. Again, the logarithmic term agrees with that of (A22) and (A23). Both dimensional and Pauli Villars regularizations have an unpleasant property that the leading term contribution yields $\rho < 0$. This property is unphysical since $\rho \equiv \langle T^0{}_0\rangle$ should be positive for the scalar field as follows from (23). Ossola and Sirlin have argued [27] that the quartic term in (A28) may be removed by demanding strict scale invariance in the limit $m \to 0$ or by invoking the Feynman regulator.

Two other Lorentz invariant regularization schemes were considered by Andrianov *et al.* [29]: ζ-function regularization and the UV cutoff regularization of the large wave-number field modes. It was concluded that the former method is not adequate in treating the cosmological constant problem as it redirects the problem from the UV to the IR region. The latter method with a suitable choice of the large wave-number cutoff reproduces the Pauli Villars regularization result (A28). With the choice advocated in [29], one can get rid of the quartic term but then the coefficient of the quadratic term changes.

We see from the above analysis that a covariant regularization is ambiguous although in all mentioned covariant methods the logarithmic term comes with the same coefficient as in the 3-dim cutoff procedure. With the exception of the dimensional regularization where the power low divergences are absent by definition, the quadratic term is always present with a coefficient that depends on the regularization method.

[1] A. G. Reiss *et al.*, Astron. J. **116**, 1009 (1998); S. Perlmutter *et al.*, Astrophys. J. **517**, 565 (1999); S. Perlmutter, M. S. Turner, and M. J. White, Phys. Rev. Lett. **83**, 670 (1999).

[2] C. L. Bennett *et al.*, Astrophys. J. Suppl. Ser. **148**, 1 (2003); D. N. Spergel *et al.*, Astrophys. J. Suppl. Ser. **148**, 175 (2003); H. V. P. Peiris *et al.*, Astrophys. J. Suppl. Ser. **148**, 213 (2003).

[3] D. N. Spergel *et al.*, Astrophys. J. Suppl. Ser. **170**, 377 (2007).

[4] S. Weinberg, arXiv:astro-ph/0005265.

[5] J. Wess and B. Zumino, Nucl. Phys. **B70**, 39 (1974).

[6] S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989).

[7] A. Y. Kamenshchik, A. Tronconi, G. P. Vacca, and G. Venturi, Phys. Rev. D **75**, 083514 (2007); G. L. Alberghi, A. Tronconi, G. P. Vacca, G. Venturi, and A. Y. Kamenshchik, JETP Lett. **88**, 705 (2008).

[8] N. Bilić, arXiv:1004.4984.

[9] K. Kratzert, Ann. Phys. (N.Y.) **308**, 285 (2003).

[10] D. Bailin and A. Love, *Supersymmetric Gauge Field Theory and String Theory* (IOP Publishing, Bristol and Philadelphia, 1999).

[11] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, England, 1992).

[12] L. E. Parker and D. J. Toms, *Quantum Field Theory in Curved Spacetime* (Cambridge University Press, Cambridge, England, 2009).

[13] T. Jacobson, arXiv:gr-qc/0308048.

[14] J. Baacke and C. Patzold, Phys. Rev. D **62**, 084008 (2000).

[15] S. L. Cherkas and V. L. Kalashnikov, J. Cosmol. Astropart. Phys. 01 (2007) 028.
[16] W. Xue, K. Dasgupta, and R. Brandenberger, arXiv:1103.0285.
[17] M. Maggiore, Phys. Rev. D **83**, 063514 (2011).
[18] G. Mangano, Phys. Rev. D **82**, 043519 (2010).
[19] A. G. Cohen, D. B. Kaplan, and A. E. Nelson, Phys. Rev. Lett. **82**, 4971 (1999).
[20] I. L. Shapiro and J. Sola, J. High Energy Phys. 02 (2002) 006; I. L. Shapiro, J. Sola, and H. Štefančić, J. Cosmol. Astropart. Phys. 01 (2005) 012; J. Sola, J. Phys. A **41**, 164066 (2008).
[21] M. S. Sloth, arXiv:1005.3241.
[22] J. Grande, J. Sola, and H. Stefancic, J. Cosmol. Astropart. Phys. 08 (2006) 011.
[23] R. R. Caldwell, Phys. Lett. B **545**, 23 (2002).
[24] R. R. Caldwell, M. Kamionkowski, and N. N. Weinberg, Phys. Rev. Lett. **91**, 071301 (2003).
[25] Odintsov, Phys. Lett. B **562**, 147 (2003); L. P. Chimento and R. Lazkoz, Phys. Rev. Lett. **91**, 211301 (2003); P. Singh, M. Sami, and N. Dadhich, Phys. Rev. D **68**, 023522 (2003); J. G. Hao and X. Z. Li, Phys. Rev. D **67**, 107303 (2003); V. Faraoni, Phys. Rev. D **68**, 063508 (2003); V. B. Johri, Phys. Rev. D **70**, 041303 (2004); E. Elizalde, S. Nojiri, and S. D. Odintsov, Phys. Rev. D **70**, 043539 (2004); H. Štefančić, Phys. Lett. B **586**, 5 (2004).
[26] G. Dvali and C. Gomez, Phys. Lett. B **674**, 303 (2009); G. Dvali, Fortschr. Phys. **58**, 528 (2010); G. Dvali and S. N. Solodukhin, arXiv:0806.3976.
[27] G. Ossola and A. Sirlin, Eur. Phys. J. C **31**, 165 (2003).
[28] E. K. Akhmedov, arXiv:hep-th/0204048.
[29] A. A. Andrianov, F. Cannata, P. Giacconi, A. Y. Kamenshchik, and R. Soldati, Phys. Lett. B **651**, 306 (2007).