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Dynamics of stringy congruence in the early universe

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We study twist and shear aspects of the stingy geodesic surface congruence. Under some natural conditions we derive the equations of the twist and shear in terms of the expansion of the Universe. We observe in this higher dimensional cosmology that, as the early universe evolves with expansion rate, the twist of the stringy congruence decreases exponentially and the initial twist value should be large enough to sustain the rotations of the ensuing universe, while the effects of the shear are negligible to produce the isotropic and homogeneous universe. We also investigate the twist and shear of the geodesic surface congruence of the null strings.

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I. INTRODUCTION

The Hawking-Penrose (HP) singularity [1] is assumed to exist at the beginning of the Universe. In the standard inflationary cosmology based on the HP singularity theorem and inflationary scenario, the Universe is believed to expand from the big bang. Assuming that the early universe was filled with a perfect fluid consisting of massive particles and/or massless particles and using the strong energy condition that was used to show the HP singularity theorem, one could find equations of state for each particle.

In the inflationary standard cosmology, it is believed that, after the big bang explosion, the radiation dominated phase occurred followed by the matter dominated one, even though there was a hot thermalization period of radiation and matter immediately after the big bang. Moreover, a phase transition exists between massive particle and massless particle phases in the Universe. The equation of state of the massive particle is different from that of the massless particle, and thus the massive particle phase is not the same as the massless particle one.

Recently, applying the string theory [2,3] to cosmology, both of us have studied the expansion of the Universe in terms of the HP singularity in geodesic surface congruences for the timelike and null strings [4,5]. Taking an ansatz that the expansion of the stringy congruence is constant along the string coordinate direction, we have derived the Raychaudhuri type equation, which is an evolution equation for the expansion, possessing correction terms associated with the stringy configurations. Assuming the stringy strong energy condition, we have the HP type inequality equation that produces the same inequality equation for both the timelike and null stringy congruences.

There have also been some progresses in geometrical approaches to the theoretical physics associated with the stringy congruence cosmology [4–6], the stringy Jacobi-Morse theory [7], the Sturm-Liouville theory [8], and the Gromov-Witten invariants [9]. The variation of the surface spanned by closed strings in a spacetime manifold has been considered to discuss conjugate strings on the geodesic surface and to induce the geodesic surface equation and the geodesic surface deviation equation, which yields a Jacobi field and the index form of a geodesic surface as in the case of point particles [7]. Later, after the geodesic equation and geodesic deviation equation with breaks on the path were formulated, the physical changes of the action have been investigated through the study of the geometry of the moduli space associated with the critical points of the action functional and the asymptotic boundary conditions in path space for point particles in a conservative physical system, where the particle motion on the n-sphere S^n was considered to discuss the moduli space of the path space, the corresponding homology groups, and the Sturm-Liouville operators [8]. Using symplectic cut-and-gluing formulas of the relative Gromov-Witten invariants, one of us obtained a recursive formula for the Hurwitz number of triple ramified geodesic surface coverings of a Riemann surface by a Riemann surface [9].

In this paper, we extend the previous results in the stringy cosmology to study the twist and shear features of the stringy geodesic congruences in the early universe. To do this, we exploit the paradigm that can delineate the stringy features of the HP singularity in the mathematical cosmology. Especially, we investigate the effects of the twist and shear of stringy congruence on the ensuing universe evolution.

This paper is organized as follows. In Sec. II, we introduce the formalism that describes the stringy congruence in the early universe. In Sec. III, we briefly recapitulate the expansion rate of the timelike stringy

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congruence by exploiting the Raychaudhuri type equation. In Sec. IV, we investigate the aspects of twist and shear of the stringy congruences in the early universe. Section V includes summary and discussions. In the Appendix, we treat the null stringy congruence in the early universe.

II. CONGRUENCE OF STRINGS

The action for a string is proportional to the area of the surface spanned in spacetime manifold M by the evolution of the string. In order to define the action on the curved manifold, we let (M, g_{ab}) be a D-dimensional manifold associated with the metric g_{ab}. Given g_{ab}, we can have a unique covariant derivative ∇_a satisfying [10]

$$\begin{aligned} \nabla_a g_{bc} &= 0, \\ \nabla_a \omega^b &= \partial_a \omega^b + \Gamma^b{}_{ac}\omega^c, \\ (\nabla_a\nabla_b - \nabla_b\nabla_a)\omega_c &= R_{abc}{}^d\omega_d. \end{aligned} \tag{2.1}$$

We parameterize the surface generated by the evolution of a string by two world sheet coordinates τ and σ, and then we have the corresponding vector fields $\xi^a = (\partial/\partial\tau)^a$ and $\zeta^a = (\partial/\partial\sigma)^a$. Since we have gauge degrees of freedom, we can choose the orthonormal gauge as follows [11]:

$$\xi \cdot \zeta = 0, \qquad \xi \cdot \xi + \zeta \cdot \zeta = 0, \tag{2.2}$$

where the plus sign in the second equation is due to the fact that $\xi \cdot \xi = -1$ is timelike and $\zeta \cdot \zeta = 1$ is spacelike. In the orthonormal gauge, we introduce tensor fields B_{ab} and $\bar{B}_{ab}$ defined as

$$B_{ab} = \nabla_b \xi_a, \qquad \bar{B}_{ab} = \nabla_b \zeta_a, \tag{2.3}$$

which satisfy the following identities:

$$B_{ab}\xi^a = 0, \qquad \bar{B}_{ab}\zeta^a = 0, \qquad -B_{ab}\xi^b + \bar{B}_{ab}\zeta^b = 0. \tag{2.4}$$

Here in the last equation, we have used the geodesic surface equation

$$-\xi^a\nabla_a\xi^b + \zeta^a\nabla_a\zeta^b = 0. \tag{2.5}$$

In particular, the timelike curves of the strings are geodesic, then the geodesic surface equation holds.

We let the vector field $\eta^a = (\partial/\partial\alpha)^a$ be the deviation vector which represents the displacement to an infinitesimally nearby world sheet, and we let Σ denote the three-dimensional submanifold spanned by the world sheets $\gamma_\alpha(\tau, \sigma)$. We then may choose τ, σ, and α as coordinates of Σ to yield the commutator relations,

$$\begin{aligned} \mathcal{L}_\xi \eta^a &= \xi^b\nabla_b\eta^a - \eta^b\nabla_b\xi^a = 0, \\ \mathcal{L}_\zeta \eta^a &= \zeta^b\nabla_b\eta^a - \eta^b\nabla_b\zeta^a = 0, \\ \mathcal{L}_\xi \zeta^a &= \xi^b\nabla_b\zeta^a - \zeta^b\nabla_b\xi^a = 0. \end{aligned} \tag{2.6}$$

Using the above relations, we obtain

$$\xi^a\nabla_a\eta^b - \zeta^a\nabla_a\eta^b = (B^b{}_a - \bar{B}^b{}_a)\eta^a. \tag{2.7}$$

Next we introduce the metrics h_{ab} and $\bar{h}_{ab}$,

$$h_{ab} = g_{ab} + \xi_a\xi_b, \qquad \bar{h}_{ab} = g_{ab} - \zeta_a\zeta_b, \tag{2.8}$$

which satisfy

$$\begin{aligned} &h_{ab}\xi^a = 0, &&h_{ab}\xi^b = 0, &&h_{ab}g^{bc}h_{cd} = h_{ad}, \\ &\bar{h}_{ab}\zeta^a = 0, &&\bar{h}_{ab}\zeta^b = 0, &&\bar{h}_{ab}g^{bc}\bar{h}_{cd} = \bar{h}_{ad}, \\ &h_{ab}h^{ab} = D-1, &&\bar{h}_{ab}\bar{h}^{ab} = D-1, &&h_{ab}\bar{h}^{ab} = D-2. \end{aligned} \tag{2.9}$$

Here we note that h_{ab} and $\bar{h}_{ab}$ are the metrics on the hypersurfaces orthogonal to ξ^a and ζ^a, respectively. Moreover, we can define projection operators $h^a{}_b$ and $\bar{h}^a{}_b$ as follows:

$$h^a{}_b = g^{ac}h_{cb}, \qquad \bar{h}^a{}_b = g^{ac}\bar{h}_{cb}. \tag{2.10}$$

These operators fulfil

$$\begin{aligned} &h^a{}_b h^b{}_c = h^{ab}h_{bc} = h^a{}_c, &&\bar{h}^a{}_b\bar{h}^b{}_c = \bar{h}^{ab}\bar{h}_{bc} = \bar{h}^a{}_c, \\ &h_{ab}h^{bc}h_{cd} = h_{ad}, &&\bar{h}_{ab}\bar{h}^{bc}\bar{h}_{cd} = \bar{h}_{ad}. \end{aligned} \tag{2.11}$$

Now, we decompose B_{ab} into three pieces

$$B_{ab} = \frac{1}{D-1}\theta h_{ab} + \sigma_{ab} + \omega_{ab}, \tag{2.12}$$

where the expansion θ, the shear σ_{ab}, and the twist ω_{ab} of the stringy congruence are given by

$$\begin{aligned} \theta &= B^{ab}h_{ab}, \\ \sigma_{ab} &= B_{(ab)} - \frac{1}{D-1}\theta h_{ab}, \\ \omega_{ab} &= B_{[ab]}. \end{aligned} \tag{2.13}$$

Similarly, $\bar{B}_{ab}$ is also decomposed into three parts

$$\bar{B}_{ab} = \frac{1}{D-1}\bar{\theta}\bar{h}_{ab} + \bar{\sigma}_{ab} + \bar{\omega}_{ab}, \tag{2.14}$$

where

$$\begin{aligned} \bar{\theta} &= \bar{B}^{ab}\bar{h}_{ab}, \\ \bar{\sigma}_{ab} &= \bar{B}_{(ab)} - \frac{1}{D-1}\bar{\theta}\bar{h}_{ab}, \\ \bar{\omega}_{ab} &= \bar{B}_{[ab]}. \end{aligned} \tag{2.15}$$

We then find

$$\begin{aligned} \sigma_{ab}h^{ab} &= 0, \quad \omega_{ab}h^{ab} = 0, \\ \bar{\sigma}_{ab}\bar{h}^{ab} &= 0, \quad \bar{\omega}_{ab}\bar{h}^{ab} = 0, \\ -\sigma_{ab}\xi^b + \bar{\sigma}_{ab}\zeta^b &= 0, \quad -\omega_{ab}\xi^b + \bar{\omega}_{ab}\zeta^b = 0, \end{aligned} \tag{2.16}$$

and

$$-\xi^c\nabla_c B_{ab} + \zeta^c\nabla_c \bar{B}_{ab} = B^c{}_b B_{ac} - \bar{B}^c{}_b \bar{B}_{ac} - R_{cbad}(\xi^c\xi^d - \zeta^c\zeta^d). \tag{2.17}$$

Exploiting (2.17) one arrives at

$$-\xi^a\nabla_a\theta + \zeta^a\nabla_a\bar{\theta} = \frac{1}{D-1}(\theta^2 - \bar{\theta}^2) + \sigma_{ab}\sigma^{ab} - \bar{\sigma}_{ab}\bar{\sigma}^{ab} - \omega_{ab}\omega^{ab} + \bar{\omega}_{ab}\bar{\omega}^{ab} + R_{ab}(\xi^a\xi^b - \zeta^a\zeta^b), \tag{2.18}$$

$$-\xi^c\nabla_c\omega_{ab} + \zeta^c\nabla_a\bar{\omega}_{ab} = \frac{2}{D-1}\theta(\omega_{ab} - \xi^c\xi_{[a}\omega_{b]c}) - \frac{2}{D-1}\bar{\theta}(\bar{\omega}_{ab} + \zeta^c\zeta_{[a}\bar{\omega}_{b]c}) + 2(\sigma^c_{[b}\omega_{a]c} - \bar{\sigma}^c_{[b}\bar{\omega}_{a]c}), \tag{2.19}$$

$$\begin{aligned}-\xi^c\nabla_c\sigma_{ab} + \zeta^c\nabla_a\bar{\sigma}_{ab} = {}& \frac{1}{(D-1)^2}(\theta^2\xi_a\xi_b + \bar{\theta}^2\zeta_a\zeta_b) + \frac{2}{D-1}(\theta h^c{}_{(a} - \bar{\theta}\bar{h}^c{}_{(a})\sigma_{b)c} + \sigma_{ac}\sigma^c{}_b - \bar{\sigma}_{ac}\bar{\sigma}^c{}_b \\ & + \omega_{ac}\omega^c{}_b - \bar{\omega}_{ac}\bar{\omega}^c{}_b - \left(R_{c(ab)d} + \frac{1}{D-1}g_{ab}R_{cd}\right)(\xi^a\xi^b - \zeta^a\zeta^b) \\ & - \frac{1}{D-1}g_{ab}(\sigma_{cd}\sigma^{cd} - \bar{\sigma}_{cd}\bar{\sigma}^{cd} - \omega_{cd}\omega^{cd} + \bar{\omega}_{cd}\bar{\omega}^{cd}) + \frac{1}{D-1}\theta\xi^c\xi_{(a}\nabla_{|c|}\xi_{b)} \\ & + \frac{1}{D-1}\bar{\theta}\zeta^c\zeta_{(a}\nabla_{|c|}\zeta_{b)} + \frac{1}{D-1}\xi_a\xi_b\xi^c\nabla_c\theta + \frac{1}{D-1}\zeta_a\zeta_b\zeta^c\nabla_c\bar{\theta}.\end{aligned} \tag{2.20}$$

III. EXPANSION OF STRINGY CONGRUENCE

The motion types of stringy congruence can be described in terms of expansion, twist, and shear. In this section, we will pedagogically summarize the previous results [4,5] on the expansion rate of stringy congruence in the early universe for the sake of completeness. We will consider the twist and shear motions in the next section.

Taking an ansatz that the expansion $\bar{\theta}$ is constant along the σ direction, from (2.20), one obtains a Raychaudhuri type equation

$$\begin{aligned}\frac{d\theta}{d\tau} = {}& -\frac{1}{D-1}(\theta^2 - \bar{\theta}^2) - \sigma_{ab}\sigma^{ab} + \bar{\sigma}_{ab}\bar{\sigma}^{ab} + \omega_{ab}\omega^{ab} \\ & - \bar{\omega}_{ab}\bar{\omega}^{ab} - R_{ab}(\xi^a\xi^b - \zeta^a\zeta^b).\end{aligned} \tag{3.1}$$

We now assume that $\omega_{ab} = \bar{\omega}_{ab}$, $\sigma_{ab} = \bar{\sigma}_{ab}$ and a stringy strong energy condition

$$\begin{aligned}& R_{ab}(\xi^a\xi^b - \zeta^a\zeta^b) \\ & \quad = 8\pi\left(T_{ab}(\xi^a\xi^b - \zeta^a\zeta^b) + \frac{2}{D-2}T\right) \geq 0,\end{aligned} \tag{3.2}$$

where T_{ab} and T are the energy-momentum tensor and its trace, respectively. The Raychaudhuri type equation (3.1) then has a solution of the form

$$\frac{1}{\theta(\tau)} \geq \frac{1}{\theta(0)} + \frac{1}{D-1}\left(\tau - \int_0^\tau d\tau\left(\frac{\bar{\theta}}{\theta}\right)^2\right). \tag{3.3}$$

We assume that $\theta(0)$ is negative so that the congruence is initially converging as in the point-particle case. The inequality (3.3) implies that $\theta(\tau)$ must pass through the singularity within a proper time

$$\tau \leq \frac{D-1}{|\theta(0)|} + \int_0^\tau d\tau\left(\frac{\bar{\theta}}{\theta}\right)^2. \tag{3.4}$$

For a perfect fluid, the energy-momentum tensor given by

$$T_{ab} = \rho u_a u_a + P(g_{ab} + u_a u_b), \tag{3.5}$$

where ρ and P are the mass-energy density and pressure of the fluid as measured in its rest frame, respectively, and u^a is the timelike D-velocity in its rest frame [10,12], the stringy strong energy condition (3.2) yields only one inequality equation

$$\frac{D-4}{D-2}\rho + \frac{D}{D-2}P \geq 0. \tag{3.6}$$

Now, we consider the point-particle limit of the timelike stringy congruence. If the fiber space F in the fibration π: $M \to N_4$ is a point, then the total space M is the same as the base spacetime four manifold N_4. In this case, the geodesic surfaces are geodesic in N_4, the congruence of timelike geodesic surfaces is a congruence of timelike geodesics, and so $\bar{B}_{ab} = \bar{\theta} = \bar{\sigma}_{ab} = \bar{\omega}_{ab} = 0$. If the congruence is hypersurface orthogonal, then we have $\omega_{ab} = 0$. Suppose that the strong energy condition $R_{ab}\xi^a\xi^b \geq 0$ is satisfied to yield two inequalities [1,10,13]

$$\rho + 3P \geq 0, \qquad \rho + P \geq 0. \tag{3.7}$$

We then have the differential inequality equation

$$\frac{d\theta}{d\tau} + \frac{1}{3}\theta^2 \leq 0, \tag{3.8}$$

which has a solution in the following form:

$$\frac{1}{\theta(\tau)} \geq \frac{1}{\theta(0)} + \frac{1}{3}\tau. \tag{3.9}$$

If we assume that $\theta(0)$ is negative, the expansion $\theta(\tau)$ must go to the negative infinity along that geodesic within a proper time

$$\tau \leq \frac{3}{|\theta(0)|}, \quad (3.10)$$

whose consequence coincides with that of Hawking and Penrose [1].

Next, we consider the expansion of the null stringy congruence in the early universe, which is explicitly described in the Appendix. Taking the ansatz that the expansion $\bar{\theta}$ is constant along the σ direction as in the timelike case, we have another Raychaudhuri type equation (A20). With the assumption that $\omega_{ab} = \bar{\omega}_{ab}$, $\sigma_{ab} = \bar{\sigma}_{ab}$, and a stringy strong energy condition (A21) for null case, exploiting the energy-momentum tensor of the perfect fluid we reproduce the inequality (3.6) in the timelike congruence of strings. We assume again that $\theta(0)$ is negative. The inequality (A22) then implies that $\theta(\lambda)$ must pass through the singularity within an affine length

$$\lambda \leq \frac{D-2}{|\theta(0)|} + \frac{D-2}{D-1}\int_0^\lambda d\lambda\left(\frac{\bar{\theta}}{\theta}\right)^2$$

as in (A23).

In the point-particle limit with the strong energy condition

$$R_{ab}k^a k^b \geq 0$$

in (A29), one can obtain the equation of state

$$\rho + P \geq 0$$

in (A30) for the null point congruence [1,10,13]. If we assume that the initial value is negative, the expansion $\theta(\lambda)$ must go to the negative infinity along that geodesic within a finite affine length

$$\lambda \leq \frac{2}{|\theta(0)|}$$

as in (A31) [1].

Moreover, the stringy universe evolves without any phase transition, since there exists only one equation of state (3.6) both for the radiation and matter, differently from the point-particle inflationary cosmology with two equations of state in (3.7) and (A30) for matter and radiation, respectively.

IV. TWIST AND SHEAR OF STRINGY CONGRUENCE

In this section we will consider the twist and shear of stringy congruence in the early universe. First, we investigate the twist feature of the stringy congruence. Taking in (2.20) an ansatz that the twist $\bar{\omega}_{ab}$ is constant along the σ direction, we obtain an evolution equation for the twist

$$\begin{aligned}\frac{d\omega_{ab}}{d\tau} = &-\frac{2}{D-1}\theta(\omega_{ab} - \xi^c\xi_{[a}\omega_{b]c}) \\ &+\frac{2}{D-1}\bar{\theta}(\bar{\omega}_{ab} + \zeta^c\zeta_{[a}\bar{\omega}_{b]c}) \\ &- 2(\sigma^c{}_{[b}\omega_{a]c} - \bar{\sigma}^c{}_{[b}\bar{\omega}_{a]c}).\end{aligned} \quad (4.1)$$

We now assume that $\omega_{ab} = \bar{\omega}_{ab}$, $\sigma_{ab} = \bar{\sigma}_{ab}$, and $\theta \gg \bar{\theta}$ to obtain[1]

$$\frac{d\omega_{ab}}{d\tau} = -\frac{2}{D-1}\theta(\omega_{ab} - \xi^c\xi_{[a}\omega_{b]c}). \quad (4.2)$$

Here one notes that the twist ω_{bc} is orthogonal to the timelike vector field ξ^c so that their inner product contraction $\xi^c\omega_{bc}$ in (4.2) vanishes. The above equation then becomes

$$\frac{d\omega_{ab}}{d\tau} = -\frac{2}{D-1}\theta\omega_{ab}, \quad (4.3)$$

which has solution of the form

$$\omega_{ab}(\tau) = \omega_{ab}(0)\exp\left(-\frac{2}{D-1}\int_0^\tau d\tau\theta\right). \quad (4.4)$$

This solution indicates that, as the early universe evolves with the expansion rate θ, θ increases and the twist of the stringy congruence ω_{ab} decreases exponentially. Moreover, the initial twist $\omega_{ab}(0)$ should be enormously large enough to support the whole rotations of the ensuing universe later.

It is worthy to note that in the higher D-dimensional stringy cosmology, one can have the condition $\omega_{ab} = \bar{\omega}_{ab} \neq 0$, where the nonvanishing ω_{ab} initiates the rotational degrees of freedom in the Universe such as the rotational motions of galaxies, stars, planets, and moons. Moreover the nonvanishing $\bar{\omega}_{ab}$ could explain the rotational degrees of freedom of the strings or physical particles themselves [2,3,11]. Next, we consider the point-particle limit of the timelike stringy congruence where $\omega_{ab} = \bar{\omega}_{ab} = 0$. We can then have the Hawking and Penrose limit with $\omega_{ab} = 0$ in the $D = 4$ point-particle congruence cosmology [1].

Second, we study the shear of the stringy congruence. Taking an ansatz that the shear $\bar{\sigma}_{ab}$ is constant along the σ direction, from (2.20) we obtain an evolution equation for the shear:

[1]In deriving (3.3), we did not neglect the $\bar{\theta}$ correction terms. However, from now on, we will keep the zeroth order term of $\bar{\theta}$ with respect to θ to see the twist and shear features of the stringy congruence.

$$\frac{d\sigma_{ab}}{d\tau} = -\frac{1}{(D-1)^2}(\theta^2\xi_a\xi_b + \bar{\theta}^2\zeta_a\zeta_b) - \frac{2}{D-1}(\theta h^c{}_{(a} - \bar{\theta}\bar{h}^c{}_{(a})\sigma_{b)c} - \sigma_{ac}\sigma^c{}_b + \bar{\sigma}_{ac}\bar{\sigma}^c{}_b - \omega_{ac}\omega^c{}_b + \bar{\omega}_{ac}\bar{\omega}^c{}_b + \left(R_{c(ab)d} + \frac{1}{D-1}g_{ab}R_{cd}\right)(\xi^a\xi^b - \zeta^a\zeta^b) + \frac{1}{D-1}g_{ab}(\sigma_{cd}\sigma^{cd} - \bar{\sigma}_{cd}\bar{\sigma}^{cd} - \omega_{cd}\omega^{cd} + \bar{\omega}_{cd}\bar{\omega}^{cd}) - \frac{1}{D-1}\theta\xi^c\xi_{(a}\nabla_{|c|}\xi_{b)} - \frac{1}{D-1}\bar{\theta}\zeta^c\zeta_{(a}\nabla_{|c|}\zeta_{b)} - \frac{1}{D-1}\xi_a\xi_b\xi^c\nabla_c\theta - \frac{1}{D-1}\zeta_a\zeta_b\zeta^c\nabla_c\bar{\theta}. \tag{4.5}$$

We again assume that $\omega_{ab} = \bar{\omega}_{ab}$, $\sigma_{ab} = \bar{\sigma}_{ab}$, and $\theta \gg \bar{\theta}$ to yield

$$\frac{d\sigma_{ab}}{d\tau} = -\frac{1}{(D-1)^2}\theta^2\xi_a\xi_b - \frac{2}{D-1}\theta h^c{}_{(a}\sigma_{b)c} + \left(R_{c(ab)d} + \frac{1}{D-1}g_{ab}R_{cd}\right)(\xi^a\xi^b - \zeta^a\zeta^b) - \frac{1}{D-1}\theta\xi^c\xi_{(a}\nabla_{|c|}\xi_{b)} - \frac{1}{D-1}\xi_a\xi_b\xi^c\nabla_c\theta. \tag{4.6}$$

At this point, we digress to carefully consider the shear tensor field σ_{ab} of the stringy congruence. In the D-dimensional spacetime manifold (M, g_{ab}), we considered the metrics h_{ab} and $\bar{h}_{ab}$ in (2.8) on the hypersurfaces orthogonal to the timelike direction and to the string direction, respectively. The metrics g_{ab}, h_{ab}, and $\bar{h}_{ab}$ have signatures $(1, D-1)$, $(0, D-1)$, and $(1, D-2)$, respectively. In particular, h_{ab} is positive definite and may have an Euclidean metric on the $(D-1)$-dimensional hypersurface N_{D-1}, which is orthogonal to the time direction. We may now choose orthogonal basis for the hypersurface N_{D-1}.

The symmetric part $B_{(ab)}$ of the tensor field B_{ab} on the hypersurface N_{D-1} is given by a $(D-1)\times(D-1)$ matrix which can be split into two pieces as follows:

$$B_{(ab)} = \frac{1}{D-1}\theta h_{ab} + \sigma_{ab}, \tag{4.7}$$

where

$$\frac{1}{D-1}\theta h_{ab} = \begin{pmatrix} \frac{\theta}{D-1} & & \\ & \cdots & \\ & & \frac{\theta}{D-1} \end{pmatrix}, \quad \sigma_{ab} = \begin{pmatrix} \theta_1 - \frac{\theta}{D-1} & & \sigma_{ij} \\ & \cdots & \\ \sigma_{ji} & & \theta_{D-1} - \frac{\theta}{D-1} \end{pmatrix}. \tag{4.8}$$

Here σ_{ij} are off-diagonal elements of the matrix σ_{ab}. It is well known in astrophysics that the Universe is homogeneously and isotropically expanding. Exploiting the fact that the Universe is homogeneously expanding, one can see that the off-diagonal part of the shear tensor vanishes, $\sigma_{ij} = 0$, to yield

$$\sigma_{ab} = \mathrm{diag}\left(\theta_1 - \frac{\theta}{D-1}, \cdots, \theta_{D-1} - \frac{\theta}{D-1}\right). \tag{4.9}$$

Next, since the Universe is isotropically expanding, one can observe that all the diagonal elements of the shear tensor are the same so that we can arrive at

$$\theta_1 = \cdots = \theta_{D-1}. \tag{4.10}$$

Moreover, by definition the shear tensor field σ_{ab} is traceless and symmetric to yield

$$\theta_a = \frac{\theta}{D-1}, \qquad (a = 1, 2, \cdots, D-1), \tag{4.11}$$

which indicates that all the shear tensor components vanish,

$$\sigma_{ab} = 0. \tag{4.12}$$

This result on the Euclidean manifold can be extended to the more general curved manifold case without loss of generality. One can thus conclude that there are no shear features in the homogeneous and isotropic universe regardless of the dynamic equation for the shear σ_{ab} in (4.6). Next, we consider the point-particle limit of the timelike stringy congruence in which $\sigma_{ab} = \bar{\sigma}_{ab} = 0$. In this case we can have the Hawking and Penrose limit with $\sigma_{ab} = 0$ in the point-particle congruence cosmology [1].

Now, we consider the twist and shear of the null stringy congruence in the early universe, which is systematically delineated in the Appendix. Exploiting the fact that the twist ω_{bc} is orthogonal to the null tangent vector field k^c, one can arrive at the evolution equation

$$\omega_{ab}(\lambda) = \omega_{ab}(0)\exp\left(-\frac{2}{D-2}\int_0^\lambda d\lambda\theta\right)$$

as in (A26) of the twist ω_{ab} of the null stringy congruence along the affine parameter λ. This shows that ω_{ab} decreases exponentially with the modified factor associated with the dimensionality, with respect to the timelike stringy congruence in (4.4).

Next, in order to consider the point-particle limit of the null stringy congruence, we first assume that $\omega_{ab} = \bar{\omega}_{ab}$, $\sigma_{ab} = \bar{\sigma}_{ab}$, and $\theta \gg \bar{\theta}$ to yield (A27). As in the case of the timelike stringy congruence, all the shear tensor components again vanish as in (A28), so that there are no shear features in the homogeneous and isotropic universe regardless of the dynamic equation for the shear σ_{ab} in (A27). As for the point-particle case of the twist of the null congruence, we have $\omega_{ab} = \bar{\omega}_{ab} = \sigma_{ab} = \bar{\sigma}_{ab} = 0$ so that there are no twist and shear motions in the homogeneous and isotropic universe.

V. CONCLUSIONS

In summary, the stringy universe evolves without any phase transition, because there is only one equation of state both for the radiation and matter, differently from the point-particle inflationary cosmology with two equations of state for matter and radiation, respectively. By exploiting the fact that there is only one equation of state in evolution of the Universe, it was also shown that the stringy cosmology is cyclic, similar to the brane cyclic cosmology, but modified: big bang, radiation-matter mixture phase, dark energy dominated phase, big crunch, and again big bang [5].

In the higher dimensional stringy cosmology, as the early universe evolves with the expansion rate θ, θ increases and the twist of the stringy congruence ω_{ab} decreases exponentially, and the initial twist $\omega_{ab}(0)$ should be extremely large enough to support the whole rotation of the ensuing universe. It is worthy to note that in the stringy cosmology one can have the condition $\omega_{ab} = \bar{\omega}_{ab} \neq 0$. Here the nonvanishing ω_{ab} initiates the rotational degrees of freedom in the Universe such as the rotational motions of galaxies, stars, planets, and moons, while the nonvanishing $\bar{\omega}_{ab}$ could explain the rotational degrees of freedom of the strings or physical particles themselves. On the other hand, the effects of the shear of the stringy congruence on the ensuing universe evolution are negligible to produce the isotropic and homogeneous universe features, regardless of the details of the dynamic equations of motions for the shear of the stringy congruence.

Next, for the null stringy congruence corresponding to the massless photons in the higher dimensional cosmology, through the evolution of the early universe, the expansion rate θ increases and the twist ω_{ab} of the null stringy congruence decreases exponentially, and the initial twist is extremely large enough to generate the whole rotation of the ensuing universe, similar to the case of the timelike stringy congruence corresponding to the massive physical particles. In the null stringy cosmology one can also have the condition $\omega_{ab} = \bar{\omega}_{ab} \neq 0$. ω_{ab} initiates the rotational degrees of freedom in the Universe such as the celestial body rotational motions, while the nonvanishing $\bar{\omega}_{ab}$ could explain the rotational degrees of freedom of the strings or physical photons themselves. On the other hand, there exist no effects of the shear of the null stringy congruence on the ensuing universe evolution to produce the isotropic and homogeneous universe features, regardless of the details of the dynamic equations of motions for the shear of the stringy congruence.

Recently, the Alice detector of the Large Hadron Collider (LHC) is scheduled to detect the so-called quark-gluon plasma state, which is assumed to exist in an extremely hot soup of massive quarks and massless gluons. Both in the standard and stringy cosmologies, this quark-gluon plasma state is supposed to occur immediately after the big bang of the tiny early universe manufactured in the LHC. In the point-particle standard cosmology, the quark-gluon plasma state can exist shortly and disappear eventually to enter the radiation dominated phase, while in the stringy higher dimensional cosmology the quark-gluon plasma state can develop into particles such as protons and neutrons and sustain the radiation and matter mixture phase. It is expected that the Alice will be able to detect the procedure of particle states along with the evolution of the tiny universe planned to occur at the LHC, and it will be able to determine which cosmology is viable. We recall that as far as radiation and matter are concerned, the mixture of these two exists together in the current universe.

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APPENDIX: NULL STRINGY CONGRUENCE

In this section, we will investigate the congruence of the null strings, where the tangent vector of a null curve is normal to itself. See Refs. [14–16] for the proper definition and propagation of the classical null strings. We consider the evolution of vectors in a $(D-2)$-dimensional subspace of spatial vectors normal to the null tangent vector field $k^a = (\partial/\partial\lambda)^a$, where λ is the affine parameter, and to an auxiliary null vector l^a that points in the opposite spatial direction to k^a, normalized by [13]

$$l^a k_a = -1 \tag{A1}$$

and is parallel transported, namely,

$$k^a \nabla_a l^b = 0. \tag{A2}$$

The spatial vectors in the $(D-2)$-dimensional subspace are then orthogonal to both k^a and l^a.

We now introduce the metrics n_{ab} and $\bar{h}_{ab}$ that are defined in (2.8),

$$n_{ab} = g_{ab} + k_a l_b + l_a k_b, \quad \bar{h}_{ab} = g_{ab} - \zeta_a \zeta_b. \tag{A3}$$

Similarly to the timelike case, we introduce tensor fields

$$B_{ab} = \nabla_b k_a, \qquad \bar{B}_{ab} = \nabla_b \zeta_a, \tag{A4}$$

satisfying the identities

$$B_{ab} k^a = \bar{B}_{ab} \zeta^a = 0, \quad -B_{ab} k^b + \bar{B}_{ab} \zeta^b = 0. \tag{A5}$$

We also define the deviation vector $\eta^a = (\partial/\partial\alpha)^a$ representing the displacement to an infinitesimally nearby world sheet so that we can choose λ, σ, and α as coordinates of the three-dimensional submanifold spanned by the world sheets. We then have the commutator relations

$$\mathcal{L}_k \eta^a = \mathcal{L}_\zeta \eta^a = \mathcal{L}_k \zeta^a = 0,$$
$$k^a \nabla_a \eta^b - \zeta^a \nabla_a \eta^b = (B^b{}_a - \bar{B}^b{}_a)\eta^a. \tag{A6}$$

We decompose B_{ab} into three pieces

$$B_{ab} = \frac{1}{D-2}\theta n_{ab} + \sigma_{ab} + \omega_{ab}, \tag{A7}$$

where the expansion, shear, and twist of the stringy congruence along the affine direction are defined as

$$\theta = B^{ab} n_{ab},$$
$$\sigma_{ab} = B_{(ab)} - \frac{1}{D-2}\theta n_{ab}, \tag{A8}$$
$$\omega_{ab} = B_{[ab]}.$$

It is noteworthy that even though we have the same notations for B_{ab}, θ, σ_{ab}, and ω_{ab} in (2.12) and (A7), the differences of these notations among the timelike string cases and null string cases are understood in the context. The metric n_{ab} also satisfies the identities

$$\sigma_{ab} n^{ab} = \omega_{ab} n^{ab} = 0, \tag{A9}$$

and

$$n_{ab} k^a = n_{ab} k^b = n_{ab} l^a = n_{ab} l^b = 0,$$
$$n_{ab} g^{bc} n_{cd} = n_{ad},$$
$$n_{ab} n^{ab} = D-2, \tag{A10}$$
$$n_{ab} \bar{h}^{ab} = D-3.$$

We define $n^a{}_b$ as

$$n^a{}_b = g^{ac} n_{cb} = \delta^a{}_b + k^a l_b + l^a k_b, \tag{A11}$$

which fulfills the following identities:

$$k^c \nabla_c n^a{}_b = 0, \tag{A12}$$

and

$$n^a{}_b k^b = n^a{}_b k_a = n^a{}_b l^b = n^a{}_b l_a = 0,$$
$$n^a{}_b n^b{}_c = n^a{}_c, \tag{A13}$$
$$n_{ab} n^{ac} = n_b{}^c,$$
$$n_a{}^b n_{bc} = n_{ac}.$$

Similarly, we decompose $\bar{B}_{ab}$ into three parts as in the timelike case

$$\bar{B}_{ab} = \frac{1}{D-1}\bar{\theta}\bar{h}_{ab} + \bar{\sigma}_{ab} + \bar{\omega}_{ab}, \tag{A14}$$

where $\bar{\theta}$, $\bar{\sigma}_{ab}$, and $\bar{\omega}_{ab}$ are given by (2.15). We then have the identities

$$B_{ab} k^a = \bar{B}_{ab} \zeta^a = 0,$$
$$-\sigma_{ab} k^b + \bar{\sigma}_{ab} \zeta^b = 0, \tag{A15}$$
$$-\omega_{ab} k^b + \bar{\omega}_{ab} \zeta^b = 0,$$

and

$$-k^c \nabla_c B_{ab} + \zeta^c \nabla_c \bar{B}_{ab} = B^c{}_b B_{ac} - \bar{B}^c{}_b \bar{B}_{ac} - R_{cbad}(k^c k^d - \zeta^c \zeta^d). \tag{A16}$$

Using (A16) we find

$$-k^a \nabla_a \theta + \zeta^a \nabla_a \bar{\theta} = \frac{1}{D-2}\theta^2 - \frac{1}{D-1}\bar{\theta}^2 + \sigma_{ab}\sigma^{ab} - \bar{\sigma}_{ab}\bar{\sigma}^{ab} - \omega_{ab}\omega^{ab} + \bar{\omega}_{ab}\bar{\omega}^{ab} + R_{ab}(k^a k^b - \zeta^a \zeta^b), \tag{A17}$$

$$-k^c \nabla_c \omega_{ab} + \zeta^c \nabla_a \bar{\omega}_{ab} = \frac{2}{D-2}\theta(\omega_{ab} - k^c k_{[a}\omega_{b]c}) - \frac{2}{D-1}\bar{\theta}(\bar{\omega}_{ab} + \zeta^c \zeta_{[a}\bar{\omega}_{b]c}) + 2(\sigma^c{}_{[b}\omega_{a]c} - \bar{\sigma}^c{}_{[b}\bar{\omega}_{a]c}), \tag{A18}$$

$$\begin{aligned}-k^c \nabla_c \sigma_{ab} + \zeta^c \nabla_a \bar{\sigma}_{ab} = {}& \frac{1}{(D-2)^2}\theta^2 k_a k_b + \frac{1}{(D-1)^2}\bar{\theta}^2 \zeta_a \zeta_b + \frac{2}{D-2}\theta h^c{}_{(a}\sigma_{b)c} - \frac{2}{D-1}\bar{\theta}\bar{h}^c{}_{(a}\sigma_{b)c} \\ & + \sigma_{ac}\sigma^c{}_b - \bar{\sigma}_{ac}\bar{\sigma}^c{}_b + \omega_{ac}\omega^c{}_b - \bar{\omega}_{ac}\bar{\omega}^c{}_b - \left(R_{c(ab)d} + \frac{1}{D-2}g_{ab}R_{cd}\right)k^c k^d \\ & + \left(R_{c(ab)d} + \frac{1}{D-1}g_{ab}R_{cd}\right)\zeta^c \zeta^d - \frac{1}{D-2}g_{ab}(\sigma_{cd}\sigma^{cd} - \omega_{cd}\omega^{cd}) \\ & + \frac{1}{D-1}g_{ab}(\bar{\sigma}_{cd}\bar{\sigma}^{cd} - \bar{\omega}_{cd}\bar{\omega}^{cd}) + \frac{1}{D-2}\theta k^c k_{(a}\nabla_{|c|}k_{b)} + \frac{1}{D-1}\bar{\theta}\zeta^c \zeta_{(a}\nabla_{|c|}\zeta_{b)} \\ & + \frac{1}{D-2}k_a k_b k^c \nabla_c \theta + \frac{1}{D-1}\zeta_a \zeta_b \zeta^c \nabla_c \bar{\theta}.\end{aligned} \tag{A19}$$

Taking the ansatz that the expansion $\bar{\theta}$ is constant along the σ direction as in the timelike case, we have another Raychaudhuri type equation,

$$\frac{d\theta}{d\lambda} = -\frac{1}{D-2}\theta^2 + \frac{1}{D-1}\bar{\theta}^2 - \sigma_{ab}\sigma^{ab} + \bar{\sigma}_{ab}\bar{\sigma}^{ab} + \omega_{ab}\omega^{ab} - \bar{\omega}_{ab}\bar{\omega}^{ab} - R_{ab}(k^a k^b - \zeta^a\zeta^b). \quad \text{(A20)}$$

Assuming $\omega_{ab} = \bar{\omega}_{ab}$, $\sigma_{ab} = \bar{\sigma}_{ab}$, and a stringy strong energy condition for null case

$$R_{ab}(k^a k^b - \zeta^a\zeta^b) \geq 0, \quad \text{(A21)}$$

and exploiting the energy-momentum tensor of the perfect fluid, we reproduce the inequality (3.6) in the timelike congruence of strings. The Raychaudhuri type equation (A20) for the null strings then has a solution in the following form:

$$\frac{1}{\theta(\lambda)} \geq \frac{1}{\theta(0)} + \frac{1}{D-2}\left(\lambda - \frac{D-2}{D-1}\int_0^\lambda d\lambda\left(\frac{\bar{\theta}}{\theta}\right)^2\right), \quad \text{(A22)}$$

where $\theta(0)$ is the initial value of θ at $\lambda = 0$. We assume again that $\theta(0)$ is negative. The inequality (A22) then implies that θ must pass through the singularity within an affine length [1]

$$\lambda \leq \frac{D-2}{|\theta(0)|} + \frac{D-2}{D-1}\int_0^\lambda d\lambda\left(\frac{\bar{\theta}}{\theta}\right)^2. \quad \text{(A23)}$$

Similarly, we assume that $\omega_{ab} = \bar{\omega}_{ab}$, $\sigma_{ab} = \bar{\sigma}_{ab}$, and $\theta \gg \bar{\theta}$ to obtain

$$\frac{d\omega_{ab}}{d\lambda} = -\frac{2}{D-2}\theta(\omega_{ab} - k^c k_{[a}\omega_{b]c}). \quad \text{(A24)}$$

Here one notes that the twist ω_{bc} is orthogonal to the null tangent vector field k^c so that their inner product contraction $k^c\omega_{bc}$ in (A24) vanishes. The above equation is then reduced to the following form:

$$\frac{d\omega_{ab}}{d\lambda} = -\frac{2}{D-2}\theta\omega_{ab}, \quad \text{(A25)}$$

whose solution is given by

$$\omega_{ab}(\lambda) = \omega_{ab}(0)\exp\left(-\frac{2}{D-2}\int_0^\lambda d\lambda\theta\right). \quad \text{(A26)}$$

As in the timelike case, as the early universe evolves with the expansion rate θ, θ increases and the twist of the null stringy congruence ω_{ab} decreases exponentially.

Next, we assume that the shear $\bar{\sigma}_{ab}$ is constant along the σ direction as in the timelike case and $\omega_{ab} = \bar{\omega}_{ab}$, $\sigma_{ab} = \bar{\sigma}_{ab}$, and $\theta \gg \bar{\theta}$ to yield

$$\begin{aligned}\frac{d\sigma_{ab}}{d\lambda} = &-\frac{1}{(D-2)^2}\theta^2 k_a k_b - \frac{2}{D-2}\theta h^c{}_{(a}\sigma_{b)c} \\ &+\left(R_{c(ab)d} + \frac{1}{D-2}g_{ab}R_{cd}\right)k^c k^d \\ &-\left(R_{c(ab)d} + \frac{1}{D-1}g_{ab}R_{cd}\right)\zeta^c\zeta^d \\ &+\frac{1}{(D-1)(D-2)}g_{ab}(\sigma_{cd}\sigma^{cd} - \omega_{cd}\omega^{cd}) \\ &-\frac{1}{D-2}\theta k^c k_{(a}\nabla_{|c|}k_{b)} - \frac{1}{D-2}k_a k_b k^c\nabla_c\theta. \quad \text{(A27)}\end{aligned}$$

As in the case of the timelike stringy congruence, all the shear tensor components again vanish,

$$\sigma_{ab} = 0, \quad \text{(A28)}$$

so that there are no shear features in the homogeneous and isotropic universe regardless of the dynamic equation for the shear σ_{ab} in (A27).

Finally, we consider the point-particle case of the null congruence with $\bar{B}_{ab} = \bar{\theta} = \bar{\sigma}_{ab} = \bar{\omega}_{ab} = 0$ and $\omega_{ab} = 0$. We assume that the strong energy condition

$$R_{ab}k^a k^b \geq 0 \quad \text{(A29)}$$

is satisfied, then we obtain [1,10,13]

$$\rho + P \geq 0. \quad \text{(A30)}$$

If we assume that $\theta(0)$ is negative, the expansion $\theta(\lambda)$ must go to the negative infinity along that geodesic within a finite affine length to yield [1]

$$\lambda \leq \frac{2}{|\theta(0)|}. \quad \text{(A31)}$$

As for the point-particle case of the twist of the null congruence, we have $\omega_{ab} = \bar{\omega}_{ab} = \sigma_{ab} = \bar{\sigma}_{ab} = 0$ so that there are no twist and no shear motions in the homogeneous and isotropic universe.

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