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Testing the *CP*-violating MSSM in stau decays at the LHC and ILC

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We study *CP* violation in the two-body decay of a scalar tau into a neutralino and a tau, which should be probed at the LHC and ILC. From the normal tau polarization, a *CP* asymmetry is defined which is sensitive to the *CP* phases of the trilinear scalar coupling parameter A_τ, the gaugino mass parameter M_1, and the Higgsino mass parameter μ, in the stau-neutralino sector of the minimal supersymmetric standard model. Asymmetries of more than 70% are obtained in scenarios with strong stau mixing. As a result, detectable *CP* asymmetries in stau decays at the LHC are found, motivating further detailed experimental studies for probing the supersymmetry *CP* phases.

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I. INTRODUCTION

The surplus of matter over antimatter within the Universe can only be explained with a thorough understanding of *CP* violation. The *CP* phase in the quark mixing matrix of the standard model, which has been confirmed by B-meson experiments [1], is not sufficient to understand the baryon asymmetry of the Universe [2]. However, the minimal supersymmetric standard model (MSSM) [3] provides new physical phases that are manifestly *CP* sensitive. After absorbing nonphysical phases, we chose the complex parameters to be the Higgsino mass parameter μ, the U(1) and SU(3) gaugino mass parameters M_1 and M_3, and the trilinear scalar coupling parameters A_f of the third generation sfermions ($f = b, t, \tau$). The corresponding phases violate *CP*, and are generally constrained by experimental bounds on electric dipole moments (EDMs) [4]. However, these restrictions are strongly model dependent [5–7], such that additional measurements outside the low energy EDM sector are required.

Many *CP* observables have been proposed and studied in order to measure *CP* violation. Total cross sections [8], masses [9], and branching ratios [10], are *CP*-even quantities. For a direct evidence of *CP* violation, however, *CP*-odd (T-odd) observables are required. Examples are rate asymmetries of either branching ratios [11], cross sections [12], or angular distributions [13]. Since these rate asymmetries require the presence of absorptive phases, they are typically small, of the order of $<10\%$, if they are not resonantly enhanced [14]. Larger *CP*-odd observables which already appear at tree-level are desirable. These are T-odd triple products of momenta and/or spins, from which *CP*-odd asymmetries can be constructed. Such triple product asymmetries are highly *CP* sensitive, and have been intensively studied both at lepton and hadron colliders [15,16].

Third generation sfermions have a rich phenomenology at high energy colliders like the LHC [17] or ILC [18] due to a sizable mixing of left and right states. In addition, the *CP* phases of the trilinear coupling parameters A_f are rather unconstrained by the EDMs [7,19,20]. The phases of A_b and A_t have been studied in stop [21–24] and sbottom [25,26] decays, respectively. Since these are decays of a scalar particle, the spin-spin correlations have to be taken into account. The triple product asymmetries can then be up to 40%, for sizable squark mixing. Similarly for probing the *CP*-violating phase of A_τ in the stau vertex, $\tilde\tau$-$\tilde\chi^0$-τ, it is essential to include the tau spin. Only then is there a sensitivity to the phase of A_τ [27,28]. If the spin of the tau is summed over, this crucial information is lost. Triple product asymmetries including the tau polarization have been studied in neutralino decays $\tilde\chi_i^0 \to \tilde\tau\tau$ [28], and also in chargino decays $\tilde\chi_i^\pm \to \tilde\nu_\tau \tau^\pm$ [29]. It was shown that the normal tau polarization itself is *CP* sensitive, and that the asymmetries are large and of the order of 60% to 70%.

We are thus motivated to study *CP* violation, including the tau polarization, in the two-body decay of a stau

$$\tilde\tau_m \to \tau + \tilde\chi_i^0, \qquad m = 1, 2, \qquad i = 2, 3, 4, \tag{1}$$

followed by the subsequent chain of two-body decays

$$\tilde\chi_i^0 \to \ell_1 + \tilde\ell_n; \tag{2a}$$

$$\tilde\ell_n \to \tilde\chi_1^0 + \ell_2; \qquad n = L, R, \qquad \ell = e, \mu. \tag{2b}$$

See Fig. 1 for a schematic picture of the entire stau decay. This process is kinematically open for a mass hierarchy

$$m_{\tilde\tau} > m_{\tilde\chi_i^0} > m_{\tilde e} = m_{\tilde\mu}, \tag{3}$$

where the staus are heavier than the smuons and selectrons. We thus work in MSSM scenarios with heavier stau soft supersymmetry (SUSY) breaking parameters

$$M_{\tilde E_\tau} > M_{\tilde E_e} = M_{\tilde E_\mu}, \tag{4}$$

$$M_{\tilde L_\tau} > M_{\tilde L_e} = M_{\tilde L_\mu}. \tag{5}$$

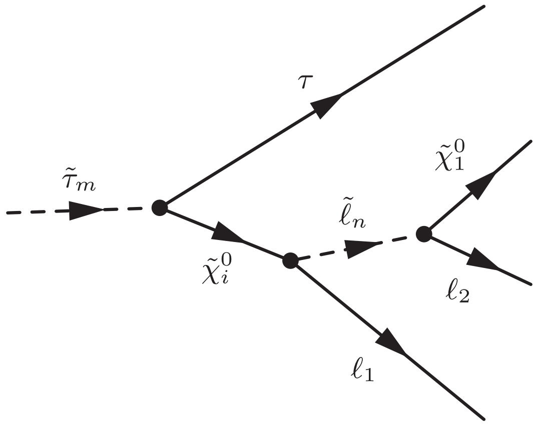


FIG. 1. Schematic picture of stau decay.

We show that the normal tau polarization, with respect to the plane spanned by the τ and ℓ_1 momentum, is a triple product asymmetry which is sensitive to the phases of A_τ, M_1, and μ in the stau-neutralino sector. For nearly degenerate stau masses, $M_{\tilde{E}_\tau} \approx M_{\tilde{L}_\tau}$, a strong stau mixing is obtained which results in tau polarization asymmetries of more than 70%. This should be measurable at colliders.[1] Since the stau is a scalar particle, its particular production does not contribute to CP-sensitive spin-spin correlations, and can thus be considered separately. This allows a collider-independent study, where we only discuss the boost dependence of the CP asymmetries.

The paper is organized as follows. In Sec. II we review stau mixing and the stau-neutralino Lagrangian with complex couplings. We calculate the amplitude squared for the entire stau decay in the spin-density matrix formalism [30]. We construct the CP asymmetry from the normal tau polarization, and discuss its MSSM parameter dependence, as well its boost dependence for colliders like the ILC and LHC. In Sec. III, we numerically study the phase and parameter dependence of the asymmetry, and the stau and neutralino branching ratios. We comment on the impact of the $\tilde{\tau}_2$ decay in scenarios with nearly degenerate stau masses. We summarize and conclude in Sec. IV. The Appendices contain the definitions of momenta and spin vectors, the analytical expressions for the stau decay amplitudes in the spin-density matrix formalism, and formulae for the stau decay widths.

II. FORMALISM

A. Stau mixing

In the complex MSSM, the stau mixing matrix in the $(\tilde{\tau}_L, \tilde{\tau}_R)$ basis is [3,31]

$$\mathcal{M}_{\tilde{\tau}} = \begin{pmatrix} m^2_{\tilde{\tau}_L} & e^{-i\phi_{\tilde{\tau}}} m_\tau |\Lambda_{\tilde{\tau}}| \\ e^{i\phi_{\tilde{\tau}}} m_\tau |\Lambda_{\tilde{\tau}}| & m^2_{\tilde{\tau}_R} \end{pmatrix}. \quad (6)$$

CP violation is parameterized by the physical phase

$$\phi_{\tilde{\tau}} = \arg[\Lambda_{\tilde{\tau}}], \quad (7)$$

$$\Lambda_{\tilde{\tau}} = A_\tau - \mu^* \cot\beta, \quad (8)$$

with the complex trilinear scalar coupling parameter A_τ, the complex Higgsino mass parameter μ, and $\tan\beta = v_1/v_2$, the ratio of the vacuum expectation values of the two neutral Higgs fields. The left and right stau masses are

$$m^2_{\tilde{\tau}_L} = M^2_{\tilde{L}_\tau} + \left(-\tfrac{1}{2} + \sin^2\theta_w\right) m_Z^2 \cos(2\beta) + m_\tau^2, \quad (9)$$

$$m^2_{\tilde{\tau}_R} = M^2_{\tilde{R}_\tau} - \sin^2\theta_w m_Z^2 \cos(2\beta) + m_\tau^2, \quad (10)$$

with the real soft SUSY breaking parameters $M^2_{\tilde{L}_\tau,\tilde{E}_\tau}$, the electroweak mixing angle θ_w, and the masses of the Z boson m_Z, and of the tau lepton, m_τ.

In the mass basis, the stop eigenstates are [3,31]

$$\begin{pmatrix} \tilde{\tau}_1 \\ \tilde{\tau}_2 \end{pmatrix} = \mathcal{R}^{\tilde{\tau}} \begin{pmatrix} \tilde{\tau}_L \\ \tilde{\tau}_R \end{pmatrix}, \quad (11)$$

with the diagonalization matrix

$$\mathcal{R}^{\tilde{\tau}} = \begin{pmatrix} e^{i\phi_{\tilde{\tau}}} \cos\theta_{\tilde{\tau}} & \sin\theta_{\tilde{\tau}} \\ -\sin\theta_{\tilde{\tau}} & e^{-i\phi_{\tilde{\tau}}} \cos\theta_{\tilde{\tau}} \end{pmatrix}, \quad (12)$$

and the stau mixing angle

$$\cos\theta_{\tilde{\tau}} = \frac{-m_\tau |\Lambda_{\tilde{\tau}}|}{\sqrt{m_\tau^2 |\Lambda^2_{\tilde{\tau}}| + (m^1_{\tilde{\tau}_1} - m^2_{\tilde{\tau}_2})^2}}, \quad (13)$$

$$\sin\theta_{\tilde{\tau}} = \frac{m^2_{\tilde{\tau}_L} - m^2_{\tilde{\tau}_1}}{\sqrt{m_\tau^2 |\Lambda^2_{\tilde{\tau}}| + (m^2_{\tilde{\tau}_1} - m^2_{\tilde{\tau}_2})^2}}. \quad (14)$$

The stau mass eigenvalues are

$$m^2_{\tilde{\tau}_{1,2}} = \frac{1}{2}\Big[(m^2_{\tilde{\tau}_L} + m^2_{\tilde{\tau}_R}) \mp \sqrt{(m^2_{\tilde{\tau}_L} - m^2_{\tilde{\tau}_R})^2 + 4 m^2_{\tilde{\tau}} |\Lambda_{\tilde{\tau}}|^2}\Big]. \quad (15)$$

B. Lagrangian and complex couplings

The relevant Lagrangian terms for the stau decay $\tilde{\tau}_m \to \tau \tilde{\chi}_i^0$ are [3,31]

$$\mathcal{L}_{\tau\tilde{\tau}\tilde{\chi}^0} = g \bar{\tau} (a^{\tilde{\tau}}_{mi} P_R + b^{\tilde{\tau}}_{mi} P_L) \tilde{\chi}_i^0 \tilde{\tau}_m + \text{h.c.}, \quad (16)$$

with $P_{L,R} = (1 \mp \gamma_5)/2$, and the weak coupling constant $g = e/\sin\theta_w$, $e > 0$. The couplings are defined as [31]

[1]Note that we do not include the tau decay in our calculations. However, some of the decay products of the tau have to be reconstructed in order to measure the tau spin. The main goal of our work is to motivate such an experimental study, to address the feasibility of measuring the CP phases at the LHC or ILC.

$$a^{\tilde{\tau}}_{mi} \equiv \sum_{n=1}^{2} (\mathcal{R}^{\tilde{\tau}}_{mn})^* \mathcal{A}^{\tau}_{in}, \qquad b^{\tilde{\tau}}_{mi} \equiv \sum_{n=1}^{2} (\mathcal{R}^{\tilde{\tau}}_{mn})^* \mathcal{B}^{\tau}_{in}. \tag{17}$$

The stau diagonalization matrix $\mathcal{R}^{\tilde{\tau}}$ is given in Eq. (2), and

$$\mathcal{A}^{\tau}_{i} \equiv \begin{pmatrix} f^L_{\tau i} \\ h^R_{\tau i} \end{pmatrix}, \qquad \mathcal{B}^{\tau}_{i} \equiv \begin{pmatrix} h^L_{\tau i} \\ f^R_{\tau i} \end{pmatrix}. \tag{18}$$

In the photino, zino, Higgsino basis $(\tilde{\gamma}, \tilde{Z}, \tilde{H}^0_a, \tilde{H}^0_b)$, we have

$$f^L_{\tau i} = \sqrt{2}\left[\frac{1}{\cos\theta_w}\left(\frac{1}{2} - \sin^2\theta_w\right) N_{i2} + \sin\theta_w N_{i1}\right], \tag{19}$$

$$f^R_{\tau i} = \sqrt{2}\sin\theta_w(\tan\theta_w N^*_{i2} - N^*_{i1}), \tag{20}$$

$$h^L_{\tau i} = (h^R_{\tau i})^* = -Y_\tau(N^*_{i3}\cos\beta + N^*_{i4}\sin\beta), \tag{21}$$

$$Y_\tau = \frac{m_\tau}{\sqrt{2} m_W \cos\beta}, \tag{22}$$

with m_W the mass of the W boson, and N the complex, unitary 4×4 matrix that diagonalizes the neutralino mass matrix [3]

$$N^* \cdot \mathcal{M}_{\tilde{\chi}^0} \cdot N^\dagger = \mathrm{diag}(m_{\tilde{\chi}^0_1}, \ldots, m_{\tilde{\chi}^0_4}). \tag{23}$$

The interaction Lagrangian relevant for the neutralino decay $\tilde{\chi}^0_i \to \tilde{\ell}^{\pm}_{R,L}\ell^{\mp}$, for $\ell = e, \mu$ is [3]

$$\mathcal{L}_{\ell\tilde{\ell}\tilde{\chi}^0} = g\bar{\ell} f^L_{\ell i} P_R \tilde{\chi}^0_i \tilde{\ell}_L + g\bar{\ell} f^R_{\ell i} P_L \tilde{\chi}^0_i \tilde{\ell}_R + \text{h.c.}, \tag{24}$$

with the couplings $f^{L,R}_{\ell i}$ given in Eqs. (19) and (20).

C. Tau spin-density matrix

The unnormalized, 2×2, Hermitian, τ spin-density matrix for stau decay, Eqs. (1) and (2), reads

$$\rho^{\lambda_\tau \lambda'_\tau} \equiv \int (|\mathcal{M}|^2)^{\lambda_\tau \lambda'_\tau} d\mathcal{L}\text{ips}, \tag{25}$$

with the amplitude $\mathcal{M}$, and the Lorentz invariant phase-space element $d\mathcal{L}$ips, for details see Appendix B. The τ helicities are denoted by λ_τ and λ'_τ. In the spin-density matrix formalism [30], the amplitude squared is given by

$$(|\mathcal{M}|^2)^{\lambda_\tau \lambda'_\tau} = |\Delta(\tilde{\chi}^0_i)|^2 |\Delta(\tilde{\ell})|^2 \times \sum_{\lambda_i \lambda'_i} \rho_D(\tilde{\tau})^{\lambda_\tau \lambda'_\tau}_{\lambda_i \lambda'_i} \rho_{D_1}(\tilde{\chi}^0_i)^{\lambda_i \lambda'_i} D_2(\tilde{\ell}), \tag{26}$$

with the neutralino helicities λ_i, λ'_i. The amplitude squared decomposes into the remnants of the propagators

$$\Delta(j) = \frac{i}{s_j - m_j^2 + i m_j \Gamma_j}, \tag{27}$$

with mass m_j, and width Γ_j of particle $j = \tilde{\chi}^0_i$ or $\tilde{\ell}$, and the unnormalized spin-density matrices for stau decay $\rho_D(\tilde{\tau})$, and neutralino decay $\rho_{D_1}(\tilde{\chi}^0_i)$. The decay matrix of the spinless slepton is a factor since the polarizations of the final lepton and lightest supersymmetric particle (LSP) are not accessible. The corresponding amplitude is denoted by $D_2(\tilde{\ell})$. Defining a set of spin basis vectors s^a_τ for the tau, see Eqs. (A10) in Appendix A, and $s^b_{\tilde{\chi}^0_i}$ for the neutralino [32], the spin-density matrices can be expanded in terms of the Pauli matrices σ

$$\rho_\mathrm{D}(\tilde{\tau})^{\lambda_\tau\lambda'_\tau}_{\lambda_i\lambda'_i} = \mathrm{D}\delta^{\lambda_\tau\lambda'_\tau}\delta_{\lambda_i\lambda'_i} + \Sigma^a_\mathrm{D}(\sigma^a)^{\lambda_\tau\lambda'_\tau}\delta_{\lambda_i\lambda'_i} + \Sigma^b_\mathrm{D}\delta^{\lambda_\tau\lambda'_\tau}(\sigma^b)_{\lambda_i\lambda'_i} + \Sigma^{ab}_\mathrm{D}(\sigma^a)^{\lambda_\tau\lambda'_\tau}(\sigma^b)_{\lambda_i\lambda'_i}, \tag{28}$$

$$\rho_{\mathrm{D}_1}(\tilde{\chi}^0_i)^{\lambda'_i\lambda_i} = \mathrm{D}_1\delta^{\lambda'_i\lambda_i} + \Sigma^b_{\mathrm{D}_1}(\sigma^b)^{\lambda'_i\lambda_i}, \tag{29}$$

with an implicit sum over a, $b = 1, 2, 3$, respectively. The real expansion coefficients D, D_1, Σ^a_D, Σ^b_D, $\Sigma^b_{\mathrm{D}_1}$ and Σ^{ab}_D contain the physical information of the process. D denotes the unpolarized part of the amplitude for stau decay $\tilde{\tau}_m \to \chi^0_i\tau$, D_1 denotes the unpolarized part for neutralino decay $\chi^0_i \to \tilde{\ell}_R\ell_1$, respectively. Σ^a_D gives the tau polarization, Σ^b_D, and $\Sigma^b_{\mathrm{D}_1}$ describe the contributions from the neutralino polarization, and Σ^{ab}_D is the spin-spin correlation term, which contains the *CP*-sensitive parts. We give the expansion coefficients explicitly in Appendix C.

Inserting the density matrices, Eqs. (28) and (29), into Eq. (26), we get for the amplitude squared

$$(|\mathcal{M}|^2)^{\lambda_\tau\lambda'_\tau} = 2|\Delta(\tilde{\chi}^0_i)|^2|\Delta(\tilde{\ell})|^2\Big[(\mathrm{DD}_1 + \Sigma^b_\mathrm{D}\Sigma^b_{\mathrm{D}_1})\delta^{\lambda_\tau\lambda'_\tau} + (\Sigma^a_\mathrm{D}\mathrm{D}_1 + \Sigma^{ab}_\mathrm{D}\Sigma^b_{\mathrm{D}_1})(\sigma^a)^{\lambda_\tau\lambda'_\tau}\Big]\mathrm{D}_2, \tag{30}$$

with an implicit sum over a, $b = 1, 2, 3$. The amplitude squared $(|\mathcal{M}|^2)^{\lambda_\tau\lambda'_\tau}$ is now decomposed into an unpolarized part (first summand), and into the part for the tau polarization (second summand), in Eq. (30). By using the completeness relations for the neutralino spin vectors, Eq. (A12), the products in Eq. (30) can be written,[2]

$$\Sigma^b_\mathrm{D}\Sigma^b_{\mathrm{D}_1} = \underset{(-)}{+}\frac{g^4}{2}(|a^{\tilde{\tau}}_{mi}|^2 - |b^{\tilde{\tau}}_{mi}|^2)|f^R_{\ell i}|^2 \times \left[m^2_{\tilde{\chi}^0_i}(p_\tau \cdot p_{\ell_1}) - (p_{\tilde{\chi}^0_i} \cdot p_\tau)(p_{\ell_1} \cdot p_{\tilde{\chi}^0_i})\right], \tag{31}$$

[2]The formulas are given for the decay of a negatively charged stau $\tilde{\tau}_m \to \tau^- \tilde{\chi}^0_i$, followed by $\tilde{\chi}^0_i \to \ell^+_1 \tilde{\ell}^-_R$. The signs in parentheses in Eqs. (31) and (32) hold for the charge conjugated stau decay $\tilde{\tau}^*_m \to \tau^+ \tilde{\chi}^0_i$; $\tilde{\chi}^0_i \to \ell^+_1 \tilde{\ell}^-_R$. In order to obtain the terms for the decay $\tilde{\tau}^{(*)}_m \to \tau^{\mp}\tilde{\chi}^0_i$, however followed by the neutralino decay into a positively charged slepton, $\tilde{\chi}^0_i \to \ell^-_1 \tilde{\ell}^+_R$, one has to reverse the signs of Eqs. (31) and (32). This is due to the sign change of $\Sigma^b_{\mathrm{D}_1}$, see Eqs. (C6)C6. In Appendix C, we also give the terms for the neutralino decay into a left slepton, $\tilde{\chi}^0_i \to \ell^{\pm}_1 \tilde{\ell}^{\mp}_L$. Note that the term proportional to m_τ in Eq. (32) is negligible at high particle energies $E \gg m_\tau$.

$$\Sigma_{\rm D}^{ab}\Sigma_{\rm D_1}^{b} = \overset{+}{(-)}\frac{g^4}{2}(|a^{\tilde\tau}_{mi}|^2+|b^{\tilde\tau}_{mi}|^2)|f^R_{\ell i}|^2 m_\tau\Big[(s^a_\tau\cdot p_{\tilde\chi^0_i})(p_{\tilde\chi^0_i}\cdot p_{\ell_1}) - m^2_{\tilde\chi^0_i}(s^a_\tau\cdot p_{\ell_1})\Big]$$
$$\overset{+}{(-)} g^4\Re\mathfrak{e}\{a^{\tilde\tau}_{mi}(b^{\tilde\tau}_{mi})^*\}|f^R_{\ell i}|^2 m_{\tilde\chi^0_i}\Big[(p_\tau\cdot p_{\tilde\chi^0_i})(s^a_\tau\cdot p_{\ell_1}) - (p_\tau\cdot p_{\ell_1})(s^a_\tau\cdot p_{\tilde\chi^0_i})\Big]$$
$$\overset{+}{(-)} g^4|f^R_{\ell i}|^2 m_{\tilde\chi^0_i}\Im\mathfrak{m}\{a^{\tilde\tau}_{mi}(b^{\tilde\tau}_{mi})^*\}\Big[p_{\tilde\tau}, p_{\ell_1}, p_\tau, s^a_\tau\Big]. \tag{32}$$

The spin-spin correlation term $\Sigma_{\rm D}^{ab}\Sigma_{\rm D_1}^{b}$, Eq. (32), explicitly depends on the imaginary part $\Im\mathfrak{m}\{a^{\tilde\tau}_{mi}(b^{\tilde\tau}_{mi})^*\}$ of the stau-tau-neutralino couplings, Eq. (16). Thus this term is manifestly *CP* sensitive, i.e., it depends on the phases ϕ_{A_τ}, ϕ_1, ϕ_μ of the stau-tau-neutralino sector. The imaginary part is multiplied by the totally antisymmetric (epsilon) product,

$$\mathcal{E}^a \equiv [p_{\tilde\tau}, p_{\ell_1}, p_\tau, s^a_\tau] \equiv \epsilon_{\mu\nu\rho\sigma} p^\mu_{\tilde\tau} p^\nu_{\ell_1} p^\rho_\tau s^{a,\sigma}_\tau, \tag{33}$$

with the convention $\epsilon_{0123} = 1$. Since each of the spatial components of the four-momenta p, or the spin vectors s^a_τ, changes sign under a time transformation, $t \to -t$, the epsilon product $\mathcal{E}^a$ is T-odd. In the stau rest frame, $p^\mu_{\tilde\tau} = (m_{\tilde\tau}, \mathbf{0})$, the epsilon product reduces to the T-odd triple product $\mathcal{T}^a$

$$[p_{\tilde\tau}, p_{\ell_1}, p_\tau, s^a_\tau] = m_{\tilde\tau}(\mathbf{p}_{\ell_1}\times\mathbf{p}_\tau)\cdot\mathbf{s}^a_\tau \equiv m_{\tilde\tau}\mathcal{T}^a. \tag{34}$$

The task in the next section is to define an observable, that projects out from the amplitude squared the part proportional to $\mathcal{E}^a$ (or $\mathcal{T}^a$), in order to probe the *CP*-sensitive coupling combination $\Im\mathfrak{m}\{a^{\tilde\tau}_{mi}(b^{\tilde\tau}_{mi})^*\}$.

D. Normal tau polarization and *CP* asymmetry

The τ polarization is given by the expectation value of the Pauli matrices $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ [33]

$$\mathcal{P} = \frac{{\rm Tr}\{\rho\boldsymbol{\sigma}\}}{{\rm Tr}\{\rho\}}, \tag{35}$$

with the τ spin-density matrix ρ, as given in Eq. (25). In our convention for the polarization vector $\mathcal{P} = (\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3)$, the components $\mathcal{P}_1$ and $\mathcal{P}_3$ are the transverse and longitudinal polarizations in the plane spanned by $\mathbf{p}_{\ell_1}$ and $\mathbf{p}_\tau$, respectively, and $\mathcal{P}_2$ is the polarization normal to that plane. See our definition of the tau spin basis vectors s^a_τ in Appendix A.

The normal τ polarization is equivalently defined as

$$\mathcal{P}_2 \equiv \frac{N(\uparrow) - N(\downarrow)}{N(\uparrow) + N(\downarrow)}, \tag{36}$$

with the number of events N with the τ spin up ($\uparrow$) or down ($\downarrow$), with respect to the quantization axis $\mathbf{p}_{\ell_1}\times\mathbf{p}_\tau$, see Eq. (A10). The normal τ polarization can thus also be regarded as an asymmetry

$$\mathcal{P}_2 = \frac{\sigma(\mathcal{T}>0) - \sigma(\mathcal{T}<0)}{\sigma(\mathcal{T}>0) + \sigma(\mathcal{T}<0)}, \tag{37}$$

of the triple product

$$\mathcal{T} = (\mathbf{p}_{\ell_1}\times\mathbf{p}_\tau)\cdot\boldsymbol{\xi}_\tau, \tag{38}$$

where $\boldsymbol{\xi}_\tau$ is the direction of the τ spin vector for each event. The triple product $\mathcal{T}$ is included in the spin-spin correlation term $\Sigma_{\rm D}^{ab}\Sigma_{\rm D_1}^{b}$, Eq. (32), cf. Eq. (34), and the asymmetry thus probes the term which contains the *CP*-sensitive coupling combination $\Im\mathfrak{m}\{a^{\tilde\tau}_{mi}(b^{\tilde\tau}_{mi})^*\}$.

Since under naive time reversal, $t\to -t$, the triple product $\mathcal{T}$ changes sign, the tau polarization $\mathcal{P}_2$, Eq. (37), is T-odd. Because of *CPT* invariance [34], $\mathcal{P}_2$ would thus be *CP*-odd at tree level. In general, $\mathcal{P}_2$ also has contributions from absorptive phases, e.g. from intermediate s-state resonances or final-state interactions, which do not signal *CP* violation. Although such absorptive contributions are a higher order effect, and thus expected to be small, they can be eliminated in the true *CP* asymmetry [28]

$$\mathcal{A}^{CP}_\tau = \tfrac{1}{2}(\mathcal{P}_2 - \bar{\mathcal{P}}_2), \tag{39}$$

where $\bar{\mathcal{P}}_2$ is the normal tau polarization for the charged conjugated process $\tilde\tau^*_m \to \tau^+\tilde\chi^0_i$. For our analysis at tree level, where no absorptive phases are present, we find $\bar{\mathcal{P}}_2 = -\mathcal{P}_2$, see the sign change in Eqs. (31) and (32), and thus $\mathcal{A}^{CP}_\tau = \mathcal{P}_2$. We study $\mathcal{A}^{CP}_\tau$ in the following, which is however equivalent to $\mathcal{P}_2$ at tree level.

Inserting now the explicit form of the density matrix ρ, Eq. (25), into Eq. (35), together with Eq. (30), we obtain the *CP* asymmetry

$$\mathcal{A}^{CP}_\tau = \mathcal{P}_2 = \frac{\int \Sigma_{\rm D}^{a=2,b}\Sigma_{\rm D_1}^{b}\, d\mathcal{L}{\rm ips}}{\int {\rm DD_1}\, d\mathcal{L}{\rm ips}}, \tag{40}$$

where we have used the narrow width approximation for the propagators in the phase-space element $d\mathcal{L}{\rm ips}$, see Eq. (D9). Note that in the denominator of $\mathcal{A}^{CP}_\tau$, Eq. (40), the spin correlation terms vanish, $\int \Sigma_{\rm D}^{b}\Sigma_{\rm D_1}^{b} d\mathcal{L}{\rm ips} = 0$, see Eq. (31), when integrated over phase space. In the numerator only the spin-spin correlation term $\Sigma_{\rm D}^{ab}\Sigma_{\rm D_1}^{b}$, for $a=2$ contributes, which contains the T-odd epsilon product $\mathcal{E}^a$, see Eq. (33).

E. Parameter dependence of the *CP* asymmetry

To qualitatively understand the dependence of the asymmetry $\mathcal{A}^{CP}_\tau$, Eq. (40), on the MSSM parameters, we study in some detail its dependence on the $\tilde\tau_m$-τ-$\tilde\chi^0_i$ couplings, $a^{\tilde\tau}_{mi}$ and $b^{\tilde\tau}_{mi}$, see Eq. (D3). From the explicit form of the decay terms $\Sigma_{\rm D}^{b}\Sigma_{\rm D_1}^{b}$ Eq. (31), and D, D$_1$, Eqs. (C1) and (C5), respectively, we find that the asymmetry

$$\mathcal{A}_\tau^{CP} = \eta_{mi} \frac{m_{\tilde\chi_i^0} \int [p_{\tilde\tau}, p_{\ell_1}, p_\tau, s_\tau^{a=2}] d\mathcal{L}\mathrm{ips}}{(p_{\tilde\chi_i^0}\cdot p_\tau)(p_{\tilde\chi_i^0}\cdot p_{\ell_1}) \int d\mathcal{L}\mathrm{ips}}, \quad (41)$$

with $(p_{\tilde\chi_i^0}\cdot p_\tau) = (m_{\tilde\tau}^2 - m_{\tilde\chi_i^0}^2)/2$, and $(p_{\tilde\chi_i^0}\cdot p_{\ell_1}) = (m_{\tilde\chi_i^0}^2 - m_{\tilde\ell}^2)/2$, is proportional to the decay coupling factor

$$\eta_{mi} = \frac{\Im\mathfrak{m}\{a_{mi}^{\tilde\tau}(b_{mi}^{\tilde\tau})^*\}}{\frac{1}{2}(|a_{mi}^{\tilde\tau}|^2 + |b_{mi}^{\tilde\tau}|^2)}, \quad (42)$$

with $\eta_{mi} \in [-1, 1]$. We thus expect maximal asymmetries for equal moduli of left and right couplings, $|a_{mi}^{\tilde\tau}| \approx |b_{mi}^{\tilde\tau}|$, which have a phase difference of about $\pi/2$, where the coupling factor can be maximal $\eta_{mi} = \pm 1$, see Eq. (42).

To study the dependence of η on the *CP* phase $\phi_{\tilde\tau}$ of the stau sector, and the stau mixing angle $\theta_{\tilde\tau}$, we expand the imaginary part of the product of $\tilde\tau_m$-τ-$\tilde\chi_i^0$ couplings

$$\Im\mathfrak{m}\Big\{a_{mi}^{\tilde\tau}(b_{mi}^{\tilde\tau})^*\Big\} = \Im\mathfrak{m}\Big\{|\mathcal{R}_{m1}|^2 f_{\tau i}^L h_{\tau i}^R + |\mathcal{R}_{m2}|^2 f_{\tau i}^{*R} h_{\tau i}^R + \mathcal{R}_{m1}\mathcal{R}_{m2}^*[(h_{\tau i}^R)^2 - f_{\tau i}^R f_{\tau i}^{*L}]\Big\}, \quad (43)$$

in terms of the stau mixing matrix $\mathcal{R}$, the gauge couplings $f_{\tau i}^{L,R}$ and the Higgs couplings $h_{\tau i}^{L,R}$. In particular, for a *CP*-conserving neutralino sector, $\phi_1 = \phi_\mu = 0$, we have

$$\Im\mathfrak{m}\{a_{mi}^{\tilde\tau}(b_{mi}^{\tilde\tau})^*\} = \underset{(-)}{+} \sin\phi_{\tilde\tau}\sin(2\theta_{\tilde\tau})\frac{1}{2}[(h_{\tau i}^R)^2 - f_{\tau i}^R f_{\tau i}^L], \quad (44)$$

for $m = 1$, and the sign in parentheses holds for $m = 2$. Thus we expect a maximal η and thus maximal asymmetries for maximal stau mixing,[3] $\theta_{\tilde\tau} \approx \pm\pi/4$, and a maximal *CP* phase in the stau mixing matrix, $\phi_{\tilde\tau} \approx \pm\pi/2$. Note that, in particular, the dependence of $\phi_{\tilde\tau}$ on ϕ_{A_τ} is strong for $|A_\tau| > |\mu|\tan\beta$. We will study numerically the phase and parameter dependence on $\mathcal{A}_\tau^{CP}$ and η further in Sec. III.

F. Boost dependence

The triple product asymmetry $\mathcal{A}_\tau^{CP}$, Eq. (40), is not Lorentz invariant but depends on the boost of the decaying stau,

$$\beta_{\tilde\tau} = \frac{|\mathbf{p}_{\tilde\tau}|}{E_{\tilde\tau}}. \quad (45)$$

In Fig. 2, we show the boost dependence of the asymmetry $\mathcal{A}_\tau^{CP}(\beta_{\tilde\tau})$, normalized by $\mathcal{A}_\tau^{CP}(\beta_{\tilde\tau} = 0)$. The SUSY parameters are given in Table I, and we have chosen three sets of different $\tilde\tau$ soft-breaking parameters $\{M_{\tilde E_\tau}, M_{\tilde L_\tau}\} = \{195, 200\}$ GeV (solid, red); $\{395, 400\}$ GeV (dashed, green); and $\{998, 1000\}$ GeV (dotted, blue). The corresponding stau masses are $\{m_{\tilde\tau_1}, m_{\tilde\tau_2}\} = \{194, 209\}$; $\{395, 404\}$; $\{998, 1002\}$ GeV, respectively. The corresponding asymmetries in the stau rest frame are $\mathcal{A}_\tau^{CP}(\beta_{\tilde\tau}) = -66\%$; -72%, -71%. Note that we have chosen nearly degenerate stau masses which lead to an enhanced stau mixing and thus to maximal asymmetries; see also the discussion in Sec. III.

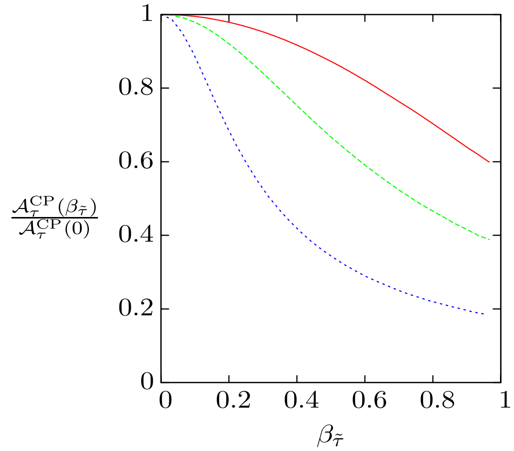


FIG. 2 (color online). Boost distributions of the τ polarization asymmetry $\mathcal{A}_\tau^{CP}$, Eq. (39), normalized by $\mathcal{A}_\tau^{CP}(\beta_{\tilde\tau} = 0)$, for three different sets of stau masses, $m_{\tilde\tau_{1,2}} \approx 200$ GeV (solid, red), 400 GeV (dashed, green), and 1000 TeV (dotted, blue), see text, for stau decay $\tilde\tau_1 \to \tau\tilde\chi_2^0$, followed by $\tilde\chi_2^0 \to \ell_1\tilde\ell_R$, and $\tilde\ell_R \to \tilde\chi_1^0\ell_2$ ($\ell = e$ or μ), see Fig. 1, The SUSY parameters are given in Table I.

For the stau masses $\{m_{\tilde\tau_1}, m_{\tilde\tau_2}\} = \{194, 209\}$ GeV, the staus can be produced at the ILC with $\sqrt{s} = 500$ GeV, and have a fixed boost of $\beta_{\tilde\tau} = 0.63$. The corresponding asymmetry is then reduced to $\mathcal{A}_\tau^{CP} = -53\%$ if the stau rest frame cannot be reconstructed. Typical ILC cross section for these masses are of the order of some 20 fb [35].

If the staus are produced at the LHC, they will have a distinct boost distribution, depending on their mass, which typically peaks at high values $\beta_{\tilde\tau} \approx 0.9$, for stau masses of the order of a few 100 GeV up to a 1 TeV, see e.g. Refs [21,26]. Then the normal tau polarization in the laboratory frame is obtained by folding the boost dependent polarization $\mathcal{A}_\tau^{CP}$ with the normalized stau boost distribution [21],

$$\mathcal{A}_{\tau\ \mathrm{lab}}^{CP} = \frac{1}{\sigma_P}\int_0^1 \frac{d\sigma_P}{d\beta_{\tilde\tau}}\mathcal{A}_\tau^{CP}(\beta_{\tilde\tau}) d\beta_{\tilde\tau}, \quad (46)$$

TABLE I. Benchmark scenario. The mass parameters M_2, $|\mu|$, A_τ, $M_{\tilde E}$, $M_{\tilde L}$ $M_{\tilde E_\tau}$, and $M_{\tilde L_\tau}$ are given in GeV.

ϕ_1	ϕ_μ	ϕ_{A_τ}	M_2	$\lvert\mu\rvert$	A_τ	$\tan\beta$
0	0	$\pi/2$	250	250	2000	3

$M_{\tilde E_\tau}$	$M_{\tilde L_\tau}$	$M_{\tilde E}$	$M_{\tilde L}$
495	500	150	200

[3] Note that a maximal mixing is naturally achieved for nearly degenerate staus. However then the asymmetries for $\tilde\tau_1$ and $\tilde\tau_2$ decay typically have similar magnitude but opposite sign, and thus might cancel. See the discussion at the end of the numerics in Sec. III D.

with the production cross section $\sigma_P = \sigma(pp \to \tilde{\tau}^+\tilde{\tau}^-)$. The typical reduction of the normal tau polarization $\mathcal{A}_\tau^{CP}{}_{\rm lab}$ is of the order of two-thirds of the asymmetry compared to that in the stau rest frame $\mathcal{A}_\tau^{CP}(0)$. However, it has been recently shown (for similar asymmetries in stop decays at the LHC), that the rest frame can be partly reconstructed event by event using on-shell mass conditions, see Ref. [22]. The LHC cross section for stau pair production, $\sigma(pp \to \tilde{\tau}_1^+\tilde{\tau}_1^-)$, also sensitively depends on the stau masses, e.g., for our benchmark scenario in Table I, we find cross sections up to 10 fb at $\sqrt{s} = 14$ TeV [35].

III. NUMERICAL RESULTS

We quantitatively study the tau polarization asymmetry, and the branching ratios for the two-body decay chain

$$\tilde{\tau}_1 \to \tau + \tilde{\chi}_2^0; \quad \tilde{\chi}_2^0 \to \ell_1^+ + \tilde{\ell}_R^-; \quad \tilde{\ell}_R^- \to \tilde{\chi}_1^0 + \ell_2^-, \tag{47}$$

for $\ell = e, \mu$. The asymmetry probes the MSSM phases ϕ_1, ϕ_μ and ϕ_{A_τ}, of the neutralino and stau sector. We center our numerical discussion around a general MSSM benchmark scenario, see Table I. We choose heavier soft-breaking parameters in the stau sector than in the $\tilde{e}$, $\tilde{\mu}$ sector, to enable the mass hierarchy

$$m_{\tilde{\tau}_m} > m_{\tilde{\chi}_i^0} > m_{\tilde{\ell}_R} > m_{\tilde{\chi}_1^0}. \tag{48}$$

Further we choose almost degenerate staus which enhances their mixing, leading to maximal asymmetries. We choose a large value of the trilinear scalar coupling parameter, $|A_\tau| > |\mu| \tan\beta$,[4] to enhance the impact of ϕ_{A_τ} in the stau sector. Finally, to reduce the number of MSSM parameters, we use the (grand unified theory inspired) relation $|M_1| = 5/3 M_2 \tan^2\theta_w$ [3] for the gaugino mass parameters. The resulting masses of the staus, neutralinos and charginos are summarized in Table II.

A. Phase dependence

For the benchmark scenario given in Table I, we study the phase dependence of the asymmetry $\mathcal{A}_\tau^{CP}$ in the stau rest frame. In Fig. 3(a), we show the dependence on the *CP* phases in the neutralino sector, ϕ_1 and ϕ_μ. In Fig. 3(b), we show the dependence on the phases in the stau sector ϕ_{A_τ} and ϕ_μ. The asymmetry strongly depends on $\phi_{A_\tau} \approx \phi_{\tilde{\tau}}$, which we expect for $|A_\tau| \gg |\mu| \tan\beta$ as in our benchmark scenario, see Table I. In particular for $\phi_\mu = 0$ in Fig. 3(b), the asymmetry follows the approximation formula Eq. (44), and attains its maximal values at $\phi_{\tilde{\tau}} \approx \phi_{A_\tau} \approx \pm\pi/2$.

[4]The value of $|A_\tau|$ is restricted by the vacuum stability condition as $|A_\tau|^2 < 3(m_{\tilde{\tau}}^2 + m_{\tilde{\nu}_\tau}^2 + M_H^2 + \mu^2)$ [36].

TABLE II. Mass spectrum for the scenario in Table I.

$\tilde{\ell}$	m [GeV]	$\tilde{\chi}$	m [GeV]
$\tilde{e}_R$, $\tilde{\mu}_R$	155	$\tilde{\chi}_1^0$	112
$\tilde{e}_L$, $\tilde{\mu}_L$	204	$\tilde{\chi}_2^0$	190
$\tilde{\nu}_e$, $\tilde{\nu}_\mu$	192	$\tilde{\chi}_3^0$	254
$\tilde{\nu}_\tau$	497	$\tilde{\chi}_4^0$	327
$\tilde{\tau}_1$	495	$\tilde{\chi}_1^\pm$	181
$\tilde{\tau}_2$	504	$\tilde{\chi}_2^\pm$	325

B. $|A_\tau|$-tanβ dependence and stau mixing

In Fig. 4(a), we show the $|A_\tau|$ and tanβ dependence of the asymmetry $\mathcal{A}_\tau^{CP}$ in the stau rest frame. We can observe that the asymmetry obtains its maximum, $\mathcal{A}_\tau^{CP} \approx -77\%$, where also the coupling factor is maximal, $\eta \approx 0.95$, see Fig. 4(b). As discussed in Subsection II E, the imaginary part of the product of the stau couplings $\Im\mathrm{m}\{a_{mi}^{\tilde{\tau}}(b_{mi}^{\tilde{\tau}})^*\}$ is maximal for a maximal *CP* phase $\phi_{\tilde{\tau}} = \pi/2$ in the stau sector, which we show in Fig. 4(c). Note that the location of the maximum of $\mathcal{A}_\tau^{CP}$ is not at maximal stau mixing, $\sin(\theta_{\tilde{\tau}}) = 1/\sqrt{2} \approx 0.7$, since $\eta \propto \sin(2\theta_{\tilde{\tau}})/(|a^{\tilde{\tau}}|^2 + |b^{\tilde{\tau}}|^2)$ starts to decrease for increasing A_τ and tanβ.

To study the stau mixing, we show the $M_{\tilde{E}_\tau} - M_{\tilde{L}_\tau}$ dependence of the asymmetry $\mathcal{A}_\tau^{CP}$ in Fig. 5(a). In the entire $M_{\tilde{E}_\tau} - M_{\tilde{L}_\tau}$ plane, the *CP* phase in the stau sector is almost maximal, $\phi_{\tilde{\tau}} = 0.61\pi$. However, the asymmetry obtains its maxima in the small corridor $M_{\tilde{E}_\tau} \approx M_{\tilde{L}_\tau}$, where the stau mixing is maximal, $\theta_{\tilde{\tau}} = \pi/4$.

C. $|\mu| - M_2$ dependence and branching ratios

We show the $|\mu| - M_2$ dependence of the asymmetry $\mathcal{A}_\tau^{CP}$ in Fig. 5(b). The maxima of $\mathcal{A}_\tau^{CP}$ are obtained where the coupling factor η is also maximal, see Eq. (42).

In Fig. 6(a), we show the corresponding stau branching ratio, $\mathrm{BR}(\tilde{\tau}_1 \to \tau\tilde{\chi}_2^0)$, which can be as large as 40%. Other competing channels can reach $\mathrm{BR}(\tilde{\tau}_1 \to \tau\tilde{\chi}_1^0) \approx 65\%$, and $\mathrm{BR}(\tilde{\tau}_1 \to \nu_\tau\tilde{\chi}_{1(2)}^\pm) \approx 20(10)\%$. The stau decay into the chargino $\tilde{\chi}_1^\pm$ is always open since typically the second lightest neutralino and the lightest chargino are almost degenerate, $m_{\tilde{\chi}_2^0} \approx m_{\tilde{\chi}_1^\pm}$. The neutralino branching ratio $\mathrm{BR}(\tilde{\chi}_2^0 \to \ell\tilde{\ell}_R)$, summed over $\ell = e, \mu$, is shown in Fig. 6(b), which reaches up to 100%. The other important competing decay channels are $\mathrm{BR}(\tilde{\chi}_2^0 \to \nu_\ell\tilde{\nu}_\ell)$, and $\mathrm{BR}(\tilde{\chi}_2^0 \to \ell\tilde{\ell}_L)$, which open around $\mu \approx 250$ GeV and $\mu \approx 300$ GeV, respectively, for $M_2 = 250$ GeV. Note that in our benchmark scenario, see Table I, we have $\mathrm{BR}(\tilde{\ell}_R \to \tilde{\chi}_1^0\ell) = 1$.

D. Impact of $\tilde{\tau}_2$ decay

As we discussed in Sec. III B, we find large asymmetries for nearly degenerate staus, where we naturally obtain a maximal stau mixing. However, then typically the

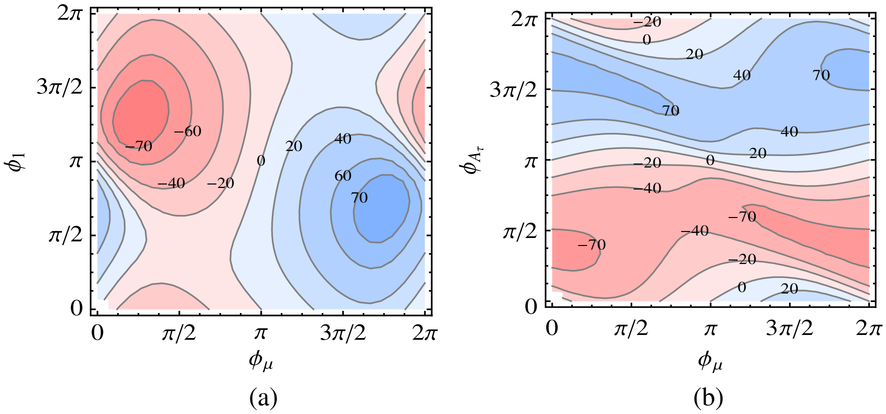


FIG. 3 (color online). Phase dependence of (a) the τ polarization asymmetry $\mathcal{A}_\tau^{CP}$, Eq. (39), in percent, in the $\phi_1 - \phi_\mu$ plane (for $\phi_{A_\tau} = 0$), and (b) in the $\phi_{A_\tau} - \phi_\mu$ plane (for $\phi_1 = 0$), in the stau rest frame. We consider the decay $\tilde{\tau}_1 \to \tau\tilde{\chi}_2^0$, followed by $\tilde{\chi}_2^0 \to \ell_1^+\tilde{\ell}_R^-$, and $\tilde{\ell}_R^- \to \tilde{\chi}_1^0\ell_2^-$ where $\ell = e$ or μ, cf. Figure 1. The other MSSM parameters are defined in Table I.

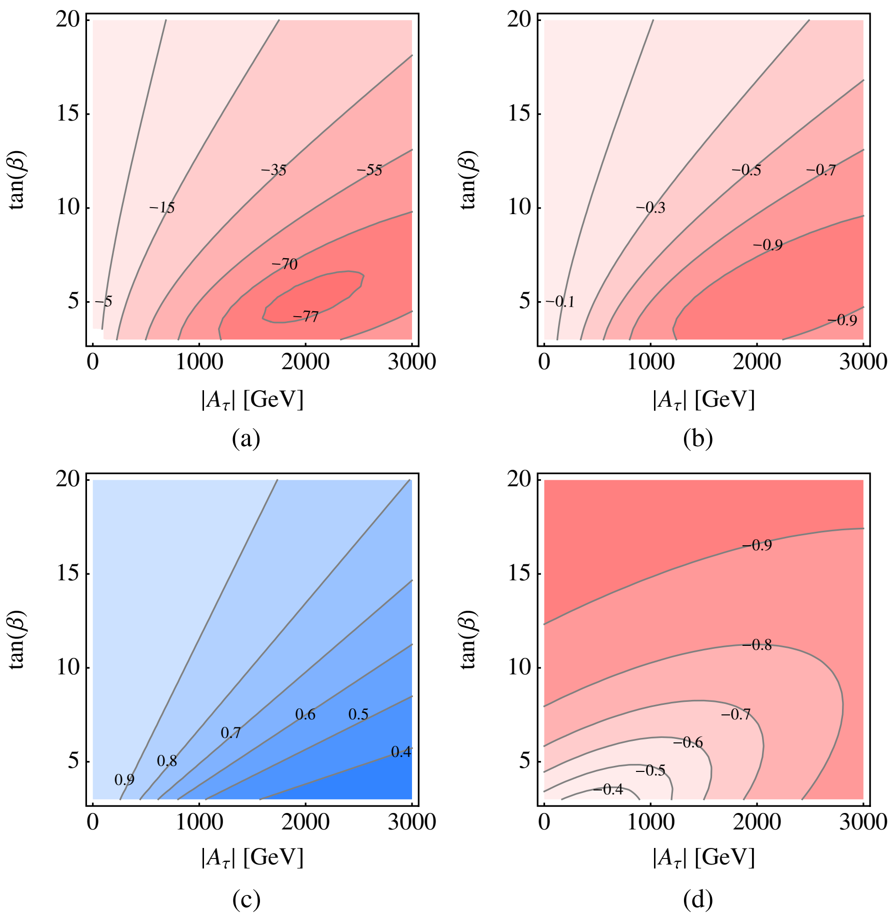
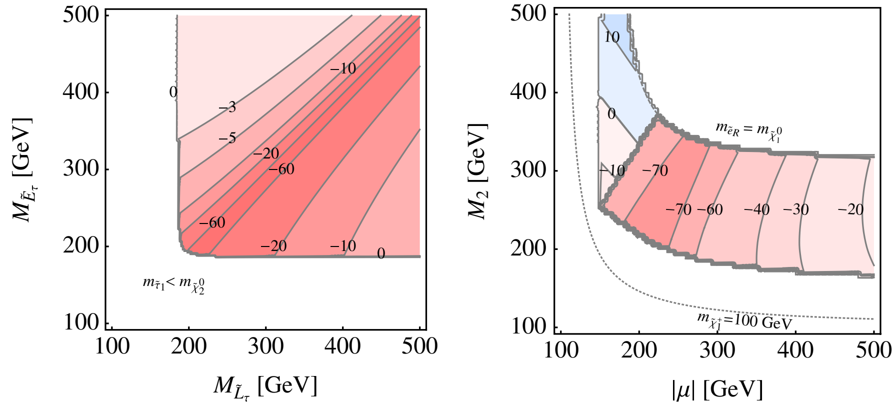


FIG. 4 (color online). $|A_\tau| - \tan\beta$ dependence of (a) the τ polarization asymmetry $\mathcal{A}_\tau^{CP}$, Eq. (39), in percent, in the stau rest frame [for the decay $\tilde{\tau}_1 \to \tau\tilde{\chi}_2^0$, followed by $\tilde{\chi}_2^0 \to \ell_1^+\tilde{\ell}_R^-$, and $\tilde{\ell}_R^- \to \tilde{\chi}_1^0\ell_2^-$ for $\ell = e$ or μ, cf. Fig. 1], (b) the coupling factor η, Eq. (42), (c) the phase $\phi_{\tilde{\tau}}$ in the stau sector, Eq. (7), and (d) $\sin(2\theta_{\tilde{\tau}})$, with $\theta_{\tilde{\tau}}$ the stau mixing angle, Eqs. (13) and (14). The plots are for $\phi_{A_\tau} = \pi/4$, the other MSSM parameters are given in Table I.

asymmetries for $\tilde{\tau}_1$ and $\tilde{\tau}_2$ decay are similar in magnitude, but opposite in sign. For example in our benchmark scenario we find $\mathcal{A}_\tau^{CP} = -71\%$ for $\tilde{\tau}_1$ decay, but $\mathcal{A}_\tau^{CP} = +32\%$ for the decay of $\tilde{\tau}_2$. If the production and decay process of $\tilde{\tau}_1$ cannot be experimentally disentangled from that of $\tilde{\tau}_2$ properly, the two asymmetries might cancel. We show their sum in Fig. 7(a) in the $M_{\tilde{E}_\tau} - M_{\tilde{L}_\tau}$ plane. In Fig. 7(b), we show the corresponding stau mass splitting.



(a) (b)

FIG. 5 (color online). Dependence of the τ polarization asymmetry $\mathcal{A}_\tau^{CP}$, Eq. (39), in percent, in the stau rest frame (for the decay $\tilde{\tau}_1 \to \tau\tilde{\chi}_2^0$, followed by $\tilde{\chi}_2^0 \to \ell_1^+\tilde{\ell}_R^-$, and $\tilde{\ell}_R^- \to \tilde{\chi}_1^0\ell_2^-$ for $\ell = e$ or μ, see Fig. 1), on (a) the soft-breaking parameters in the stau sector $M_{\tilde{\tau}_R}$, $M_{\tilde{\tau}_L}$, Eqs. (9) and (10). In (b) the dependence of $\mathcal{A}_\tau^{CP}$ on the gaugino and Higgsino parameters $|\mu|$, M_2. Below the contour $m_{\tilde{e}_R} = m_{\tilde{\chi}_2^0}$ the two-body decay $\tilde{\chi}_2^0 \to \ell\tilde{\ell}_R$ is kinematically forbidden, above the contour $m_{\tilde{e}_R} = m_{\tilde{\chi}_1^0}$ the lightest neutralino is no longer the LSP since $m_{\tilde{e}_R} < m_{\tilde{\chi}_1^0}$. Below the contour $m_{\tilde{\chi}_1^\pm} = 100$ GeV the lightest chargino is lighter than 100 GeV. The MSSM parameters are given in Table I.

Note that also the stau branching ratios are similar in size; for example, in our benchmark scenario we have $\mathrm{BR}(\tilde{\tau}_1 \to \tau\tilde{\chi}_2^0) = 18\%$, and $\mathrm{BR}(\tilde{\tau}_2 \to \tau\tilde{\chi}_2^0) = 30\%$. For the $M_{\tilde{E}_\tau} - M_{\tilde{L}_\tau}$ plane shown in Fig. 5, the decay branching ratio $\mathrm{BR}(\tilde{\tau}_1 \to \tau\tilde{\chi}_2^0)$ is at least 10%, and that of $\tilde{\tau}_2$ is larger by roughly a factor of 2 to 4.

IV. SUMMARY AND CONCLUSIONS

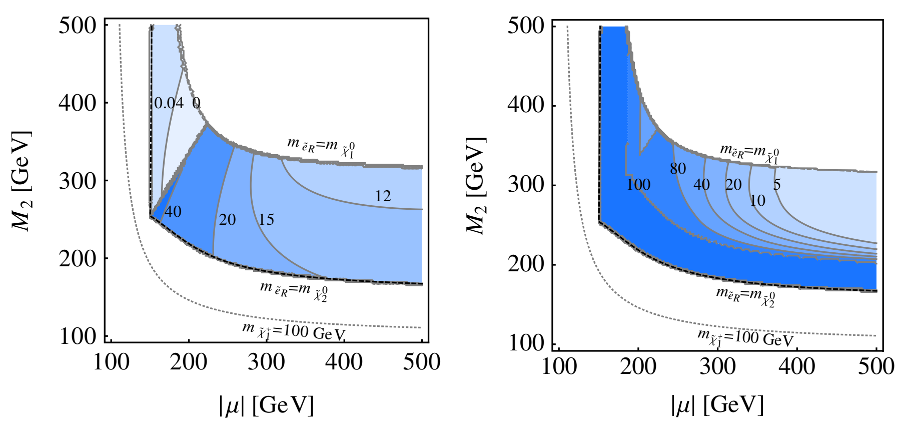
We have analyzed the normal tau polarization and the corresponding *CP* asymmetry in the two-body decay chain of a stau

$$\tilde{\tau}_1 \to \tau + \tilde{\chi}_2^0. \tag{49}$$

The *CP*-sensitive parts appear only in the spin-spin correlations, which can be probed by the subsequent neutralino decay

$$\tilde{\chi}_2^0 \to \ell_1 + \tilde{\ell}_R; \qquad \tilde{\ell}_R \to \tilde{\chi}_1^0 + \ell_2, \tag{50}$$

for $\ell = e$, μ. The T-odd tau polarization normal to the plane spanned by the τ and ℓ_1 momenta, can then be used to define a *CP*-odd tau polarization asymmetry. It is based on a triple product, which probes the *CP* phases of the trilinear scalar coupling parameter A_τ, the Higgsino mass parameter μ, and the U(1) gaugino mass parameter M_1.



(a) (b)

FIG. 6 (color online). Contour lines in the $|\mu| - M_2$ plane of (a) the stau branching ratio $\mathrm{BR}(\tilde{\tau}_1 \to \tau\tilde{\chi}_2^0)$ in percent, and (b) the neutralino branching ratio $\mathrm{BR}(\tilde{\chi}_2^0 \to \ell\tilde{\ell}_R)$, in percent, summed over both lepton flavors $\ell = e$, μ and charges, for the MSSM parameters as given in Table I. Below the contours $m_{\tilde{e}_R} = m_{\tilde{\chi}_2^0}$ in Figs. 6(a) and 6(b) the two-body decay $\tilde{\chi}_2^0 \to \ell\tilde{\ell}_R$ is kinematically forbidden, above the contours $m_{\tilde{e}_R} = m_{\tilde{\chi}_1^0}$ the lightest neutralino is no longer the LSP since $m_{\tilde{e}_R} < m_{\tilde{\chi}_1^0}$. Below the contours $m_{\tilde{\chi}_1^\pm} = 100$ GeV the lightest chargino is lighter than 100 GeV.

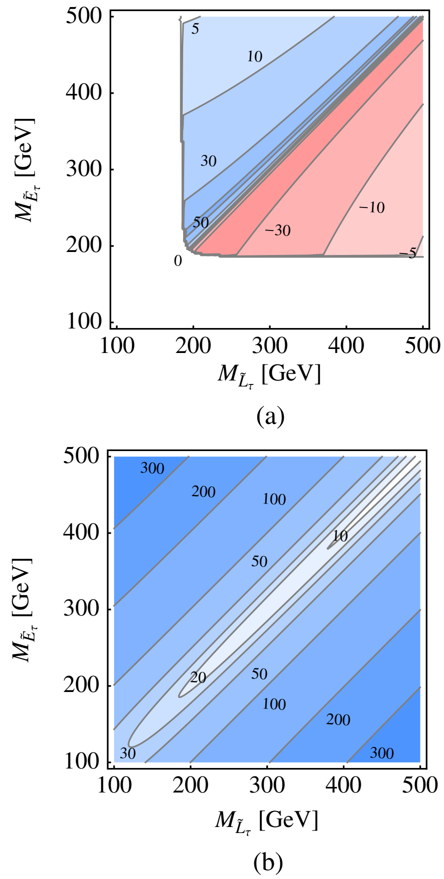


FIG. 7 (color online). Contour lines of (a) the sum of the τ polarization asymmetries $\mathcal{A}_\tau^{CP}$, Eq. (39), in percent, for the decays $\tilde{\tau}_1 \to \tau\tilde{\chi}_2^0$ and $\tilde{\tau}_2 \to \tau\tilde{\chi}_2^0$, each in the stau rest frame and followed by $\tilde{\chi}_2^0 \to \ell_1^+ \tilde{\ell}_R^-$, $\tilde{\ell}_R^- \to \tilde{\chi}_1^0 \ell_2^-$, for $\ell = e$ or μ, see Fig. 1, and (b) the stau mass splitting $m_{\tilde{\tau}_2} - m_{\tilde{\tau}_1}$ in GeV. Both plots are shown in the plane of the soft-breaking parameters of the stau sector, $M_{\tilde{E}_\tau} - M_{\tilde{L}_\tau}$, see Eqs. (9) and (10). The other MSSM parameters are given in Table I.

We have analyzed the analytical and numerical dependence of the asymmetry on these parameters in detail. In particular, for nearly degenerate staus where the stau mixing is strong, the asymmetry obtains its maxima and can be larger than 70%. The normal tau polarization can thus be considered as an ideal *CP* observable to probe the *CP* phases in the stau and neutralino sector of the MSSM.

Since the *CP*-sensitive parts appear only in the subsequent stau decay products the stau production process can be separated. Thus both, ILC, and LHC collider studies are possible. Concerning the kinematical dependence, the asymmetry is not Lorentz invariant, since it is based on a triple product. At the LHC, staus are produced with a distinct boost distribution. Evaluated in the laboratory frame, the resulting tau polarization asymmetries get typically reduced by a factor of two-thirds, compared to the stau rest frame.

We want to stress that a thorough experimental analysis, addressing background processes, detector properties, and event rate reconstruction efficiencies, will be needed in order to explore the measurability of *CP* phases in the stau sector at the LHC or ILC. We hope that our work motivates such a study.

ACKNOWLEDGMENTS

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APPENDIX A: MOMENTA AND SPIN VECTORS

For the stau decay $\tilde{\tau}_m \to \tau\tilde{\chi}_i^0$, we choose the coordinate frame in the laboratory (lab) system, such that the momentum of decaying $\tilde{\tau}$ points in the z direction.

$$p_{\tilde{\tau}}^\mu = (E_{\tilde{\tau}}, 0, 0, |\mathbf{p}_{\tilde{\tau}}|), \tag{A1}$$

$$p_\tau^\mu = E_\tau(1, \sin\theta_\tau, 0, \cos\theta_\tau), \tag{A2}$$

with the decay angle $\theta_\tau = \sphericalangle(\mathbf{p}_{\tilde{\tau}}, \mathbf{p}_\tau)$, and

$$E_\tau \approx |\mathbf{p}_\tau| \approx \frac{(m_{\tilde{\tau}}^2 - m_{\tilde{\chi}_i^0}^2)}{2(E_{\tilde{\tau}} - |\mathbf{p}_{\tilde{\tau}}|\cos\theta_\tau)}, \tag{A3}$$

in the limit $m_\tau \to 0$. The momenta of the leptons from the subsequent neutralino decay $\tilde{\chi}_i^0 \to \ell_1\tilde{\ell}$; $\tilde{\ell} \to \tilde{\chi}_1^0\ell_2$ (1), can be parameterized by

$$p_{\ell_1}^\mu = E_{\ell_1}(1, \sin\theta_1\cos\phi_1, \sin\theta_1\sin\phi_1, \cos\theta_1), \tag{A4}$$

$$p_{\ell_2}^\mu = E_{\ell_2}(1, \sin\theta_2\cos\phi_2, \sin\theta_2\sin\phi_2, \cos\theta_2), \tag{A5}$$

with the energies

$$E_{\ell_1} = \frac{m_{\tilde{\chi}_i^0}^2 - m_{\tilde{\ell}}^2}{2(E_{\tilde{\chi}_i^0} - |\mathbf{p}_{\tilde{\chi}_i^0}|\cos\theta_{D_1})}, \tag{A6}$$

$$E_{\ell_2} = \frac{m_{\tilde{\ell}}^2 - m_{\tilde{\chi}_i^0}^2}{2(E_{\tilde{\ell}} - |\mathbf{p}_{\tilde{\ell}}|\cos\theta_{D_2})}, \tag{A7}$$

and the decay angles $\theta_{D_1} = \sphericalangle(\mathbf{p}_{\tilde{\chi}_i^0}, \mathbf{p}_{\ell_1})$, $\theta_{D_2} = \sphericalangle(\mathbf{p}_{\tilde{\ell}}, \mathbf{p}_{\ell_2})$, that is,

$$\cos\theta_{D_1} = \frac{(\mathbf{p}_{\tilde{\tau}} - \mathbf{p}_\tau)\cdot\hat{\mathbf{p}}_{\ell_1}}{|\mathbf{p}_{\tilde{\tau}} - \mathbf{p}_\tau|}, \tag{A8}$$

$$\cos\theta_{D_2} = \frac{(\mathbf{p}_{\tilde{\tau}} - \mathbf{p}_\tau - \mathbf{p}_{\ell_1}) \cdot \hat{\mathbf{p}}_{\ell_2}}{|\mathbf{p}_{\tilde{\tau}} - \mathbf{p}_\tau - \mathbf{p}_{\ell_1}|}, \tag{A9}$$

with the unit momentum vector $\hat{\mathbf{p}} = \mathbf{p}/|\mathbf{p}|$. We define the tau spin vectors by

$$s_\tau^{1,\mu} = \left(0, \frac{\mathbf{s}_\tau^2 \times \mathbf{s}_\tau^3}{|\mathbf{s}_\tau^2 \times \mathbf{s}_\tau^3|}\right), \qquad s_\tau^{2,\mu} = \left(0, \frac{\mathbf{p}_{\ell_1} \times \mathbf{p}_\tau}{|\mathbf{p}_{\ell_1} \times \mathbf{p}_\tau|}\right),$$
$$s_\tau^{3,\mu} = \frac{1}{m_\tau}\left(|\mathbf{p}_\tau|, \frac{E_\tau}{|\mathbf{p}_\tau|}\mathbf{p}_\tau\right). \tag{A10}$$

The spin vectors s_τ^a, $a = 1, 2, 3$, for the tau, and $s_{\tilde{\chi}_i^0}^b$, $b = 1, 2, 3$, for the neutralino $\tilde{\chi}_i^0$, fulfil completeness relations

$$\sum_a s_\tau^{a,\mu} s_\tau^{a,\nu} = -g^{\mu\nu} + \frac{p_\tau^\mu p_\tau^\nu}{m_\tau^2}, \tag{A11}$$

$$\sum_b s_{\tilde{\chi}_i^0}^{b,\mu} s_{\tilde{\chi}_i^0}^{b,\nu} = -g^{\mu\nu} + \frac{p_{\tilde{\chi}_i^0}^\mu p_{\tilde{\chi}_i^0}^\nu}{m_{\tilde{\chi}_i^0}^2}, \tag{A12}$$

and they form orthonormal sets

$$s_\tau^a \cdot s_\tau^c = -\delta^{ac}, \qquad s_\tau^a \cdot \hat{p}_\tau = 0, \tag{A13}$$

$$s_{\tilde{\chi}_i^0}^b \cdot s_{\tilde{\chi}_i^0}^c = -\delta^{bc}, \qquad s_{\tilde{\chi}_i^0}^b \cdot \hat{p}_{\tilde{\chi}_i^0} = 0, \tag{A14}$$

with $\hat{p}^\mu = p^\mu/m$. Note that the asymmetry $\mathcal{A}_\tau^{CP}$, Eq. (40), does not depend on the explicit form of the neutralino spin vectors, since they are summed in the amplitude squared, see Eq. (31), using the completeness relation.

APPENDIX B: PHASE SPACE

The Lorentz invariant phase-space element for the stau decay chain, see Eqs. (1) and (2), can be decomposed into two-body phase-space elements [37]

$$d\mathcal{L}\mathrm{ips}(s_{\tilde{\tau}}; p_{\ell_1}, p_{\ell_2}, p_{\tilde{\chi}_1^0}) = \frac{1}{(2\pi)^2} d\mathcal{L}\mathrm{ips}(s_{\tilde{\tau}}; p_\tau, p_{\tilde{\chi}_i^0})$$
$$\times ds_{\tilde{\chi}_i^0} d\mathcal{L}\mathrm{ips}(s_{\tilde{\chi}_i^0}; p_{\ell_1}, p_{\tilde{\ell}}) ds_{\tilde{\ell}} d\mathcal{L}\mathrm{ips}(s_{\tilde{\ell}}; p_{\ell_2}, p_{\tilde{\chi}_1^0}). \tag{B1}$$

The different contributions are

$$d\mathcal{L}\mathrm{ips}(s_{\tilde{\tau}}; p_\tau, p_{\tilde{\chi}_i^0}) = \frac{1}{4\pi} \frac{|\mathbf{p}_\tau|^2}{m_{\tilde{\tau}}^2 - m_{\tilde{\chi}_i^0}^2} \sin\theta_\tau d\theta_\tau, \tag{B2}$$

$$d\mathcal{L}\mathrm{ips}(s_{\tilde{\chi}_i^0}; p_{\ell_1}, p_{\tilde{\ell}}) = \frac{1}{2(2\pi)^2} \frac{|\mathbf{p}_{\ell_1}|^2}{m_{\tilde{\chi}_i^0}^2 - m_{\tilde{\ell}}^2} d\Omega_1, \tag{B3}$$

$$d\mathcal{L}\mathrm{ips}(s_{\tilde{\ell}}; p_{\ell_2}, p_{\tilde{\chi}_1^0}) = \frac{1}{2(2\pi)^2} \frac{|\mathbf{p}_{\ell_2}|^2}{m_{\tilde{\ell}}^2 - m_{\tilde{\chi}_1^0}^2} d\Omega_2, \tag{B4}$$

with $s_j = p_j^2$ and $d\Omega_j = \sin\theta_j d\theta_j d\phi_j$.

APPENDIX C: DENSITY MATRIX FORMALISM

The coefficients of the stau decay matrix, Eq. (28), are

$$\mathrm{D} = \frac{g^2}{2}(|a_{mi}^{\tilde{\tau}}|^2 + |b_{mi}^{\tilde{\tau}}|^2)(p_{\tilde{\chi}_i^0} \cdot p_\tau)$$
$$- g^2 \Re\mathrm{e}\{a_{mi}^{\tilde{\tau}}(b_{mi}^{\tilde{\tau}})^*\} m_{\tilde{\chi}_i^0} m_\tau, \tag{C1}$$

$$\Sigma_\mathrm{D}^a = \underset{(+)}{-} \frac{g^2}{2}(|a_{mi}^{\tilde{\tau}}|^2 - |b_{mi}^{\tilde{\tau}}|^2) m_\tau (p_{\tilde{\chi}_i^0} \cdot s_\tau^a), \tag{C2}$$

$$\Sigma_\mathrm{D}^b = \underset{(+)}{-} \frac{g^2}{2}(|a_{mi}^{\tilde{\tau}}|^2 - |b_{mi}^{\tilde{\tau}}|^2) m_{\tilde{\chi}_i^0} (p_\tau \cdot s_{\tilde{\chi}_i^0}^b), \tag{C3}$$

$$\Sigma_\mathrm{D}^{ab} = \frac{g^2}{2}(|a_{mi}^{\tilde{\tau}}|^2 + |b_{mi}^{\tilde{\tau}}|^2)(s_\tau^a \cdot s_{\tilde{\chi}_i^0}^b) m_\tau m_{\tilde{\chi}_i^0} + g^2 \Re\mathrm{e}\{a_{mi}^{\tilde{\tau}}(b_{mi}^{\tilde{\tau}})^*\}$$
$$\times \left[(s_\tau^a \cdot p_{\tilde{\chi}_i^0})(s_{\tilde{\chi}_i^0}^b \cdot p_\tau) - (s_\tau^a \cdot s_{\tilde{\chi}_i^0}^b)(p_{\tilde{\chi}_i^0} \cdot p_\tau)\right]$$
$$- g^2 \Im\mathrm{m}\{a_{mi}^{\tilde{\tau}}(b_{mi}^{\tilde{\tau}})^*\}[s_\tau^a, p_\tau, s_{\tilde{\chi}_i^0}^b, p_{\tilde{\chi}_i^0}]. \tag{C4}$$

The formulas are given for the decay of a negatively charged stau, $\tilde{\tau}_m \to \tau^- \tilde{\chi}_i^0$. The signs in parentheses hold for the charge conjugated decay $\tilde{\tau}_m^* \to \tau^+ \tilde{\chi}_i^0$.

Note that the terms proportional to m_τ in Eqs. (C1), (C2), and (C4), are negligible at high particle energies $E \gg m_\tau$, in particular Σ_D^a can be neglected.

The coefficients of the $\tilde{\chi}_1^0$ decay matrix, Eq. (29), are [32]

$$\mathrm{D}_1 = \frac{g^2}{2}|f_{\ell i}^R|^2 (m_{\tilde{\chi}_i^0}^2 - m_{\tilde{\ell}}^2), \tag{C5}$$

$$\Sigma_{\mathrm{D}_1}^b = \underset{(-)}{+} g^2 |f_{\ell i}^R|^2 m_{\tilde{\chi}_i^0} (s_{\tilde{\chi}_i^0}^b \cdot p_{\ell_1}), \tag{C6}$$

and the selectron decay factor is

$$\mathrm{D}_2 = g^2 |f_{\ell_1}^R|^2 (m_{\tilde{\ell}}^2 - m_{\tilde{\chi}_1^0}^2). \tag{C7}$$

The signs in parentheses hold for the charge conjugated processes, that is $\tilde{\chi}_i^0 \to \ell_1^- \tilde{\ell}_R^+$ in Eq. (C6).

For the decay into a left slepton $\tilde{\chi}_i^0 \to \ell_1^+ \tilde{\ell}_L^-$, Eqs. (C5)–(C7) read [32]

$$\mathrm{D}_1 = \frac{g^2}{2}|f_{\ell i}^L|^2 (m_{\tilde{\chi}_i^0}^2 - m_{\tilde{\ell}}^2), \tag{C8}$$

$$\Sigma_{\mathrm{D}_1}^b = \underset{(+)}{-} g^2 |f_{\ell i}^L|^2 m_{\tilde{\chi}_i^0} (s_{\tilde{\chi}_i^0}^b \cdot p_{\ell_1}), \tag{C9}$$

$$D_2 = g^2|f^L_{\ell 1}|^2(m^2_{\tilde{\ell}} - m^2_{\tilde{\chi}^0_1}), \quad \text{(C10)}$$

respectively. The expressions for Eqs. (31) and (32) have to be changed accordingly. The sign in parenthesis in Eq. (C9) holds for the charge conjugated process $\tilde{\chi}^0_i \to \ell^-_1 \tilde{\ell}^+_L$.

APPENDIX D: STAU DECAY WIDTHS

The partial decay width for the decay $\tilde{\tau}_m \to \tau\tilde{\chi}^0_i$ in the stau rest frame is [31]

$$\Gamma(\tilde{\tau}_m \to \tau\tilde{\chi}^0_i) = \frac{m^2_{\tilde{\tau}} - m^2_{\tilde{\chi}^0_i}}{4\pi m^3_{\tilde{\tau}}} \mathrm{D}, \quad \text{(D1)}$$

with the decay function D given in Eqs. (C1), and the approximation $m_\tau = 0$. For the decay $\tilde{\tau}_m \to \nu_\tau \tilde{\chi}^\pm_j$ the width is [31]

$$\Gamma(\tilde{\tau}_m \to \nu_\tau \tilde{\chi}^\mp_j) = \frac{(m^2_{\tilde{\tau}} - m^2_{\tilde{\chi}^\pm_j})^2}{16\pi m^3_{\tilde{\tau}}} g^2 |l^{\tilde{\tau}}_{mj}|^2, \quad \text{(D2)}$$

with the stau-chargino-neutrino coupling [3,31]

$$l^{\tilde{\tau}}_{mj} = -(\mathcal{R}^{\tilde{\tau}}_{m1})^* U_{j1} + Y_\tau (\mathcal{R}^{\tilde{\tau}}_{m2})^* U_{j2}, \quad \text{(D3)}$$

and the stau diagonalization matrix $\mathcal{R}^{\tilde{\tau}}$, Eq. (12), the Yukawa coupling Y_τ, Eq. (22), and the matrix U, that diagonalizes the chargino matrix [3],

$$U^* \cdot \mathcal{M}_{\tilde{\chi}^\pm} \cdot V^\dagger = \mathrm{diag}(m_{\tilde{\chi}^\pm_1}, m_{\tilde{\chi}^\pm_2}). \quad \text{(D4)}$$

The stau decay width for the entire decay chain, Eqs. (1) and (2), is then given by

$$\Gamma(\tilde{\tau} \to \tau \ell_1 \ell_2 \tilde{\chi}^0_1) = \frac{1}{2m_{\tilde{\tau}}} \int |\mathcal{M}|^2 d\mathcal{L}\mathrm{ips}(s_{\tilde{\tau}}; p_\tau, p_{\ell_1}, p_{\ell_2}, p_{\tilde{\chi}^0_1}) \quad \text{(D5)}$$

$$= \Gamma(\tilde{\tau}) \times \mathrm{BR}(\tilde{\tau} \to \tau\tilde{\chi}^0_i) \times \mathrm{BR}(\tilde{\chi}^0_i \to \ell_1 \tilde{\ell}) \mathrm{BR}(\tilde{\ell} \to \ell_2 \tilde{\chi}^0_1), \quad \text{(D6)}$$

$$\Gamma(\tilde{\tau} \to \tau \ell_1 \ell_2 \tilde{\chi}^0_1) = [2mm] = \frac{1}{2m_{\tilde{\tau}}} \int |\mathcal{M}|^2 (s; p_\tau, p_{\ell_1}, p_{\ell_2}, p_{\tilde{\chi}^0_1})[2mm] = \Gamma(\tilde{\tau}) \times \mathrm{BR}(\tilde{\tau} \to \tau\tilde{\chi}^0_i) \times \mathrm{BR}(\tilde{\chi}^0_i \to \ell_1 \tilde{\ell}) \times \mathrm{BR}(\tilde{\ell} \to \ell_2 \tilde{\chi}^0_1) \quad \text{(D7)}$$

with the phase-space element $d\mathcal{L}$ips, as given in the Appendix A, the amplitude squared

$$|\mathcal{M}|^2 = 4|\Delta(\tilde{\chi}^0_i)|^2 |\Delta(\tilde{\ell})|^2 \mathrm{D}\mathrm{D}_1\mathrm{D}_2, \quad \text{(D8)}$$

obtained from Eqs. (30) by summing the tau helicities λ_τ, λ'_τ. The neutralino branching ratios are given, for example, in Ref. [32], and we assume $\mathrm{BR}(\tilde{\ell} \to \ell_2 \tilde{\chi}^0_1) = 1$. We use the narrow width approximation for the propagators

$$\int |\Delta(j)|^2 ds_j = \frac{\pi}{m_j \Gamma_j}, \quad \text{(D9)}$$

which is justified for $\Gamma_j/m_j \ll 1$, which holds in our case with $\Gamma_j \lesssim \mathcal{O}(1\ \mathrm{GeV})$. Note, however, that in principle the naive $\mathcal{O}(\Gamma/m)$ expectation of the error can easily receive large off-shell corrections of an order of magnitude, and more, in particular, at threshold, or due to interferences with other resonant, or nonresonant processes [38].

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