WIMPless dark matter from non-Abelian hidden sectors with anomaly-mediated supersymmetry breaking

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In anomaly-mediated supersymmetry breaking models, superpartner masses are proportional to couplings squared. Their hidden sectors therefore naturally contain WIMPless dark matter, particles whose thermal relic abundance is guaranteed to be of the correct size, even though they are not weakly interacting massive particles. We study viable dark matter candidates in WIMPless anomaly-mediated supersymmetry breaking models with non-Abelian hidden sectors and highlight unusual possibilities that emerge in even the simplest models. In one example with a pure $SU(N)$ hidden sector, stable hidden gluinos freeze out with the correct relic density, but have an extremely low, but natural, confinement scale, providing a framework for self-interacting dark matter. In another simple scenario, hidden gluinos freeze out and decay to visible Winos with the correct relic density, and hidden glueballs may either be stable, providing a natural framework for mixed cold-hot dark matter, or may decay, yielding astrophysical signals. Last, we present a model with light hidden pions that may be tested with improved constraints on the number of nonrelativistic degrees of freedom. All of these scenarios are defined by a small number of parameters, are consistent with gauge coupling unification, preserve the beautiful connection between the weak scale and the observed dark matter relic density, and are natural, with relatively light visible superpartners. We conclude with comments on interesting future directions.

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I. INTRODUCTION

The thermal relic density of a dark matter candidate X is

$$\Omega_X \propto \frac{1}{\langle \sigma_{\rm an} v \rangle} \sim \frac{m_X^2}{g_X^4}, \qquad (1)$$

where $\langle \sigma_{\rm an} v \rangle$ is the thermally averaged annihilation cross section, and m_X and g_X are the characteristic mass scale and coupling determining this cross section. For weakly interacting massive particles (WIMPs), the characteristic values are $m_X \sim m_{\rm weak} \sim 100$ GeV and $g_X \sim g_{\rm weak} \simeq 0.65$, and the thermal relic density is roughly of the desired order of magnitude, $\Omega_X \sim 0.1$. This coincidence, the WIMP miracle, is a leading motivation for WIMPs and has guided many searches for dark matter in particle physics experiments.

At the same time, the relic density constrains only one combination of m_X and g_X. In the standard model (SM), the only possible value for g_X is $g_{\rm weak}$, since dark matter with significant electromagnetic or strong interactions is essentially excluded. However, if dark matter is in a hidden sector with its own interactions, other combinations of m_X and g_X can yield the correct thermal relic density. This is the possibility realized in WIMPless models [1,2], where dark matter is hidden, with no SM gauge interactions. In these models, the dark matter's mass m_X is not necessarily near $m_{\rm weak}$, and its hidden sector gauge couplings g_X are not necessarily near $g_{\rm weak}$, but

$$\frac{m_X^2}{g_X^4} \sim \frac{m_{\rm weak}^2}{g_{\rm weak}^4}. \qquad (2)$$

WIMPless dark matter particles therefore have the correct thermal relic density, but with a broad range of possible masses and couplings. In addition, their interaction strengths with SM particles may vary greatly, depending on the presence or absence of connector particles that induce dark matter-SM interactions through nongauge interactions. The WIMPless framework therefore preserves key virtues of WIMPs, but is far more general, leading to novel implications for direct [1,3] and indirect dark matter searches [4–7], precision experiments [8–11], high energy colliders [12,13], and cosmology [14].

Equation (2) is required for a thermal relic to match cosmological observations, but it is also motivated by particle physics considerations alone. For example, the new physics flavor and CP problems motivate supersymmetric (SUSY) models with gauge-mediated SUSY breaking (GMSB), where generation-blind superpartner masses are generated by gauge interactions. The resulting masses are $m_X \propto g_X^2$. If the constant of proportionality is similar in both the visible and hidden sectors, a stable hidden superpartner will satisfy Eq. (2) and be an excellent WIMPless dark matter candidate [1,2]. Beyond GMSB, however, the model-building possibilities for WIMPless dark matter have not been extensively studied.

In this work, we explore possible realizations of the WIMPless miracle in SUSY models with anomaly-mediated SUSY breaking (AMSB) [15,16]. As in the case of GMSB, AMSB models are motivated in large part by their potential to solve the new physics flavor and CP problems through generation-blind superpartner masses. These masses are again proportional to couplings squared, and so AMSB models are also natural homes for WIMPless dark matter. In the AMSB framework, one assumes that the minimal supersymmetric standard model (MSSM) is “sequestered” from the SUSY-breaking sector, that is, it does not have tree-level couplings to the SUSY-breaking sector. The visible sector’s superpartner masses are then generated purely by the Weyl anomaly,

$$m_v \sim \frac{g_v^2}{16\pi^2} m_{3/2}, \tag{3}$$

where $m_{3/2} \sim 100$ TeV is the gravitino mass. The same would hold for any hidden sector of the theory that is similarly sequestered from SUSY breaking, leading to

$$m_X \sim \frac{g_X^2}{16\pi^2} m_{3/2}. \tag{4}$$

As a result

$$\frac{m_X}{g_X^2} \sim \frac{1}{16\pi^2} m_{3/2} \sim \frac{m_v}{g_v^2}, \tag{5}$$

and, since $m_v \sim m_{\rm weak}$ and $g_v \sim g_{\rm weak}$, Eq. (2) holds.

In both GMSB and AMSB, the visible sector does not have a good thermal relic candidate. In GMSB, SM superpartners decay to the gravitino.[1] In AMSB, the Wino is typically the lightest supersymmetric particle (LSP) and is stable, but it typically annihilates too efficiently to have the correct thermal relic density. The possibility of a hidden dark matter candidate with the correct thermal relic density is therefore as welcome in AMSB as in GMSB. In fact, in several other aspects, AMSB models are more ideally suited for WIMPless dark matter than GMSB models. First, Eqs. (3) and (4) immediately imply Eq. (2), and so the WIMPless miracle does not require any “model building” to make the constant of proportionality similar in the visible and hidden sectors. And second, since $m_{3/2} \gg m_{\rm weak}$, hidden superpartners cannot decay to gravitinos, and WIMPless candidates are automatically stable, at least in the absence of couplings to the MSSM. Indeed, in some of our examples, the WIMPless dark matter is stable merely by virtue of spacetime symmetry and gauge symmetry.

Thanks to these properties, the models we will consider are extremely simple. The hidden sector is just an $SU(N)$ gauge theory with some number N_F of “quarks” in the fundamental representation. The simplest example is pure $SU(N)$, where the stable $SU(N)$ “gluino” is WIMPless dark matter, and the theory is completely specified by N and the hidden gauge coupling g_X. Even in this simplest example, we will find the possibility of interesting astrophysical implications. We will then consider slightly more complicated theories with $N_F > 0$ flavors and connector particles mediating hidden sector-visible sector interactions, again with unusual implications for experiments and observations. In all cases, however, the WIMPless miracle naturally preserves the beautiful connection between the weak scale and the correct dark matter density. Although we will not exhaustively explore the phenomenological consequences of these scenarios here, we will consider several qualitatively different model-building possibilities, highlight key constraints, and briefly mention some of the many possible implications for dark matter properties and the early Universe.

In Sec. II we derive results for AMSB superpartner spectra in the visible and hidden sectors, and we discuss relic densities and cosmological constraints in Sec. III. In Secs. IV, V, VI, and VII, we then present a number of models that satisfy these constraints, but have qualitatively different features. We summarize our conclusions and potentially interesting future directions in Sec. VIII.

II. SUPERPARTNER SPECTRA AND LSPS

A. Visible sector

In AMSB models the soft SUSY-breaking masses are determined by the gravitino mass $m_{3/2}$ and the values of the dimensionless couplings of the theory at the SUSY-breaking scale. These expressions are well known [15,16] and are given in Appendix A for a general SUSY model.

In the visible sector, which we assume has the low-energy field content of the MSSM, this implies that the superpartner spectrum is highly constrained. For gaugino masses, the β-function coefficients are $(b_1, b_2, b_3) = (33/5, 1, -3)$, and so the gaugino and gravitino mass parameters are in the ratio

$$M_1\colon M_2\colon M_3\colon m_{3/2} \simeq 3.31\colon 1\colon -10.5\colon 372. \tag{6}$$

Because $SU(2)$ is nearly conformal in the MSSM, the Wino is the lightest MSSM gaugino. The current bound from LEP2, $m_{\tilde{W}} \gtrsim 100$ GeV, implies $m_{3/2} \gtrsim 37$ TeV. In the scalar superpartner sector, the soft slepton masses squared turn out to be negative. There are many solutions to this problem [17–25]. Most of these do not modify the gaugino spectrum and therefore do not affect our discussion here.

As mentioned in Sec. I, the main assumption in AMSB is that the MSSM is sequestered from the SUSY-breaking sector. One way to achieve this is in the context of extra dimensions, with the two sectors localized on different branes and separated by an extra dimension [15]. If the hidden sector is localized on the same brane as the MSSM, it is likewise sequestered from the SUSY breaking. This

[1] Recall that in GMSB models, the gravitino mass is generically much smaller than the weak scale, whereas in AMSB models, the gravitino mass is roughly a loop factor above the weak scale [see Eq. (3)].

scenario allows for the presence of tree-level couplings of the hidden sector to the MSSM. Sequestering also results if the SUSY-breaking sector is near-conformal over some energy range [26]. In this case, not just the MSSM, but any other sector of the theory is sequestered from the SUSY breaking, so no extra assumptions are needed regarding the dark matter hidden sector.

B. Hidden sector

In contrast to the visible sector, there is a great deal of flexibility in defining the hidden sector's field content. For a gauge theory with matter, the results of Appendix A imply that the gaugino and scalar masses have the form

$$m_{1/2} \sim bg^2 \frac{1}{16\pi^2} m_{3/2} \tag{7}$$

$$m_0^2 \sim (y^4 - y^2 g^2 - bg^4)\left(\frac{1}{16\pi^2} m_{3/2}\right)^2, \tag{8}$$

where g and y represent gauge and Yukawa couplings, b represents one-loop β-function coefficients, and positive numerical coefficients have been neglected.

For a pure gauge theory, the theory must be non-Abelian so that the gauginos are massive and can potentially be WIMPless dark matter. For a gauge theory with matter there are many possibilities. We will consider only theories without Yukawa couplings and restrict our attention to theories without tachyonic scalars. These are not necessarily requirements, and there may well be interesting examples of WIMPless models in the cases we neglect. But given these assumptions, Eq. (8) shows that even with matter, we are led to consider only non-Abelian gauge groups. Non-Abelian gauge groups and strongly interacting dark matter have been explored previously [27–33], but we are led to this possibility for completely different reasons than those explored previously.

To be concrete, we focus in this work on hidden sectors that are $SU(N)$ gauge theories with $N_F \geq 0$ light flavors of matter in $N + \bar{N}$ representations, and no Yukawa couplings. We will refer to the hidden gauginos and gauge bosons as gluinos $\tilde{g}^h$ and gluons g^h, and the hidden matter as squarks $\tilde{q}^h$ and quarks q^h. In addition, in what follows, X will denote the hidden LSP (hLSP), and $\alpha_X \equiv g_X(m_X)^2/4\pi$ will denote the hidden sector's fine structure constant at the scale m_X.

Above the hidden gluino and squark mass scale, the one-loop β-function coefficient is $b_H = -3N + N_F$. The gluino and squark masses are then

$$m_{\tilde{g}^h} = (3N - N_F)\frac{\alpha_X}{4\pi} m_{3/2} \tag{9}$$

$$m_{\tilde{q}^h}^2 = (3N - N_F)\frac{N^2 - 1}{N}\left(\frac{\alpha_X}{4\pi} m_{3/2}\right)^2. \tag{10}$$

We require $N_F < 3N$ so that the supersymmetric theory is asymptotically-free and the squarks are nontachyonic. For $N_F \leq 2N$, the squark is the hLSP, and for $N_F > 2N$ (and, of course, for $N_F = 0$) the gluino is the hLSP.

In the absence of couplings to the MSSM, the hLSP is stable, because it is odd under the $SU(N)$ sector R parity. In fact, for some values of N, the stability follows just from spacetime symmetry and gauge symmetry. A particularly simple case is pure $SU(N)$ for which the gluino is clearly stable, since it is the lightest fermion. More generally, a gluino hLSP must decay to an odd number of quarks plus some number of gluons, but for even N, an odd number of fundamentals and antifundamentals does not contain the adjoint representation. Similarly, a squark hLSP cannot decay to quarks and gluons for even N.

Below the scale of the hLSP mass m_X, we are left with a nonsupersymmetric $SU(N)$ gauge theory with N_F flavors. For $N_F < N_*$, with $N_* \sim (2.5\text{–}3)N$, this theory is believed to confine [34–40]. As explained above, we need $N_F < 3N$, and so, at least for small values of N, the theory always confines. The confinement scale is

$$\Lambda \sim m_X \exp\left(\frac{2\pi}{b_L \alpha_X}\right), \tag{11}$$

where $b_L = -\frac{11}{3}N + \frac{2}{3}N_F$ is the β-function coefficient of the nonsupersymmetric theory. Below this scale, the quarks and gluons form color-neutral $SU(N)$ composites. Note that we always take $m_X > \Lambda_H$, where Λ_H is the strong coupling scale of the supersymmetric theory; otherwise, we would need to work directly in the low-energy effective theory of the $SU(N)$ composites.

III. COSMOLOGICAL CONSTRAINTS AND RELIC DENSITIES

We begin by outlining various requirements that all models must satisfy. This is not a complete list. In particular, there are important constraints from structure formation and halo profiles on self-interactions and charged dark matter, and from big bang nucleosynthesis (BBN) and other observations on scenarios where hidden sector particles decay to visible ones. We will discuss these where relevant when we present concrete models, starting in Sec. IV.

A. Relic density of visible LSPs

Given the assumption that the neutral Wino is the visible sector's LSP, it is natural to consider it as a dark matter candidate. Unfortunately, its thermal relic density is typically small, because it annihilates efficiently through the S-wave process $\tilde{W}\tilde{W} \rightarrow WW$ [41]. To obtain $\Omega_{\tilde{W}} \approx 0.23$ the Winos must be very heavy, with $m_{\tilde{W}} \sim 2$ TeV [16]. This problem is exacerbated in AMSB by the hierarchy in gaugino masses, as it implies $m_{\tilde{g}} \sim 20$ TeV, which is far above the weak scale and undermines the motivation of SUSY as a solution to the gauge hierarchy problem. To restore Wino dark matter as a possibility, previous attempts have abandoned the WIMP miracle and explored the possibility that Winos are produced not by thermal freeze-out, but through nonthermal mechanisms, such as the late decays of moduli [42] or Q balls [43], or by thermal freeze-out, but in a nonstandard cosmology [44].

B. Relic density of hidden LSPs

Whether the hLSP is the gluino or the squark, its annihilation cross section, just like the visible Wino's, is not helicity suppressed. Thanks to the WIMPless miracle, the two cross sections are very roughly comparable, irrespective of the hidden superpartner mass scale. However, there are N- and N_F-dependent factors that may enhance the hLSP thermal relic density significantly relative to the Wino case, because $SU(2)$ in the MSSM is nearly conformal and so the Wino thermal relic density may be thought of as accidentally low. Keeping track of these factors, we find

$$\frac{\Omega_{\tilde{g}^h}}{\Omega_{\tilde{W}}} \sim (3N - N_F)^2 \tag{12}$$

$$\frac{\Omega_{\tilde{q}^h}}{\Omega_{\tilde{W}}} \sim (3N - N_F)^2\left(\frac{N^2-1}{N}\right)^2. \tag{13}$$

The N- and N_F-dependent factors may be large. For a pure $SU(3)$ gauge theory, for example, we find enhancements of ~ 100, compensating for the too-low value of $\Omega_{\tilde{W}}$. We will also consider scenarios in which the hLSP decays to the Wino. In this case, the relic abundance will be diluted by $m_X/m_{\tilde{W}}$, but there is still a significant enhancement to the Wino relic density for a given $m_{3/2}$. Note that the gauge couplings factor out of the ratio of abundances, but do appear in mass ratios.

What happens to the hLSP relic density after freeze-out? Conventional dark matter candidates are neutral under preserved gauge symmetries. In this case, however, the hLSP is charged, leading to new phenomena. At temperatures $T \gtrsim \Lambda$, the hLSPs may annihilate through Sommerfeld-enhanced cross sections. At $T \lesssim \alpha_X^2 m_X$, they may also form hLSP-hLSP bound states, which then rapidly leads to hLSP pair annihilation. These effects have been analyzed previously in various contexts [14,29,45–47]. For the present scenario, they have a small $\sim\mathcal{O}(10\%)$ effect on the hLSP relic density, essentially because both Sommerfeld-enhanced and bound-state catalyzed annihilation rates are small compared to the Hubble expansion rate [14].

For $T \lesssim \Lambda$, however, the hLSPs will hadronize, potentially enhancing their annihilation [27–33]. In particular, the resulting "R hadrons" now have $\sim 1/\Lambda^2$ interactions, and pairs of R hadrons can form bound states, which potentially leads to rapid hLSP-hLSP annihilation [33]. This annihilation depends sensitively on the existence of light states with mass below Λ, since, for the two hLSPs to annihilate, the bound states of pairs of R hadrons must lose energy by radiating light particles. These issues were studied for the case of SM QCD, but their importance in the context of a general strongly interacting hidden sector merits further study. Note, however, that hadronization effects become irrelevant if, for example, the hLSP decays to the visible sector before $T \sim \Lambda$, or if Λ is so low that the hidden gluinos and gluons have never been cold enough to confine.

C. Relic density of hidden quark-gluon composites

At $T \sim \Lambda$, the hidden sector quarks and gluons form $SU(N)$ gauge-invariant composites, including "mesons," "glueballs," and "baryons," with masses of order Λ. The relic abundance of these composites is model-dependent, and it is useful to distinguish between three qualitatively different scenarios:

(c1) The hidden sector contains massless particles, such as Goldstone bosons or a photon associated with a new $U(1)$. These provide a thermal bath to allow the $SU(N)$ composites to annihilate to sufficiently low densities. Note that in this case, the composites have the usual thermal freeze-out, so that for $\Lambda \ll m_X$, their abundances are much smaller than the hLSP abundance. We will see an example of this type in Sec. VII.

(c2) There are connector fields that efficiently mediate decays of the *unstable* hidden $SU(N)$ composites to massless particles in the visible sector. This is realized in the models of Secs. V C 2 and VI C 2.

(c3) There are no massless fields in the hidden sector and no efficient decays to the visible sector. Some $SU(N)$ composites will then be stable, either because they are charged under some symmetry, or because they are the lightest states in the hidden sector. Requiring that the $SU(N)$ composites not overclose the Universe then places an upper bound on Λ. For example, consider the simplest case of pure $SU(N)$, whose lightest glueballs are stable. Let the visible sector's temperature be T, and assume the hidden temperature is similar. For $T \gtrsim \Lambda$ the gluons have thermal energy density $\rho_{\rm th} \propto T^4$, at $T \sim \Lambda$ the gluons form glueballs with mass $\sim\Lambda$, and for $T \lesssim \Lambda$, the glueball energy density is $(\Lambda/T)\rho_{\rm th} \propto \Lambda T^3$. The resulting glueball relic density now is $\Omega \sim \Lambda/100$ eV. Requiring that the glueballs not have relic density larger than the observed dark matter density, and, even more stringently, not be too large a contribution to hot dark matter [48,49] implies $\Lambda \lesssim 10$eV. The model of Sec. IV is an example of this type.

D. Hidden sector contributions to g_*

Light degrees of freedom contribute to the expansion rate of the Universe and are constrained by BBN [50,51]. In the context of hidden sectors, the current bound from BBN requires [2]

$$g_*^h\left[\frac{T^h_{\rm BBN}}{T^v_{\rm BBN}}\right]^4 \leq 2.52(95\%\text{C.L.}), \tag{14}$$

where g_*^h is the effective number of nonrelativistic degrees of freedom in the hidden sector at the time of BBN, and $T^h_{\rm BBN}$ and $T^v_{\rm BBN}$ are the temperatures of the hidden and visible sectors at the time of BBN, respectively.

If $T^h_{\rm BBN} = T^v_{\rm BBN}$, the constraint from BBN on g_*^h is stringent. For $m_X > T^h_{\rm BBN}$, the superpartners in the hidden sector are too heavy to contribute to g_*^h. However, if $\Lambda < T^h_{\rm BBN}$, the hidden quarks and gluons contribute $g_*^h = 2(N^2-1) + \frac{7}{2}N_F$. Even in the minimal case with $N = 2$ and $N_F = 0$, this exceeds the bound of Eq. (14) by more than a factor of 2.

The bound may be evaded in several ways, however, depending on the confinement scale Λ:

(d1) $\Lambda \lesssim T^h_{\rm BBN}(\sim\text{MeV})$. In this case, the counting above applies, and to evade the bound, the hidden sector must be colder than the visible sector at the time of BBN. If the hidden sector is completely hidden, it is quite natural for $T^h_{\rm BBN}$ and $T^v_{\rm BBN}$ to be different [52–54]. The model of Sec. IV is an example of this type. If, on the other hand, there are connector fields coupling the visible and hidden sectors, the reheat temperature must be below the mass of the connector fields. This possibility is realized in the models of Secs. V C 1 and VI C 1, where the connectors are very heavy, and this requirement is not very stringent. As an added bonus, in this scenario, the connector fields may be stable, as they are inflated away and not regenerated after reheating, avoiding overclosure constraints.

(d2) $\Lambda \gtrsim T^h_{\rm BBN}(\sim\text{MeV})$. At temperatures below Λ and above $T^h_{\rm BBN}$, the (unstable) $SU(N)$ composites decay to visible sector (MSSM) fields. We will see examples of this type in Secs. V C 2 and VI C 2.

(d3) $\Lambda \gtrsim T^h_{\rm BBN}(\sim\text{MeV})$. At temperatures below Λ and above $T^h_{\rm BBN}$, the (unstable) $SU(N)$ composites decay to light hidden sector fields. A simple realization of this possibility is decays to massless Goldstone bosons in the hidden sector. We will consider such an example in Sec. VII, with $N_F = 2$, so that there are 3 massless scalar Goldstone bosons, which is marginally consistent with Eq. (14).

IV. A PURE $SU(N)$ HIDDEN SECTOR WITHOUT CONNECTORS

We begin with a very simple model in which the hidden sector is a pure $SU(N)$ gauge theory without matter, and there are no connector fields coupling the visible and hidden sectors. The stable hidden gluino is WIMPless dark matter. The model is completely specified by $m_{3/2}$, m_X, and N. In terms of these, the hidden gauge coupling is determined by

$$m_X = 3N\frac{\alpha_X}{4\pi}m_{3/2}, \tag{15}$$

and the confinement scale is

$$\Lambda \sim m_X \exp\left(\frac{-6\pi}{11N\alpha_X}\right) = m_X \exp\left(\frac{-9m_{3/2}}{22m_X}\right) \simeq m_X 10^{-66m_{\tilde{W}}/m_X}. \tag{16}$$

Because there are no connectors, the hidden gluons g^h (and glueballs ($g^h g^h$), if they form) are also stable. As a result, the constraint (c3) on the glueball relic density discussed in Sec. III C applies, requiring $\Lambda \lesssim 10$ eV. The hidden sector is therefore weakly coupled at BBN and contributes $g_*^h = 2(N^2-1)$ relativistic degrees of freedom at BBN. The bound of Eq. (14) then implies

$$\xi_{\rm BBN} \equiv \frac{T^h_{\rm BBN}}{T^v_{\rm BBN}} \leq \left(\frac{1.26}{N^2-1}\right)^{1/4}; \tag{17}$$

although the hidden sector cannot be at the same temperature as the visible sector, the BBN constraint is satisfied if the hidden sector is just slightly colder. Note that, without connectors, it is quite natural for the visible and hidden sectors to be at different temperatures.

Hidden gluinos annihilate to hidden gluons through S-wave processes with cross section

$$\sigma(\tilde{g}^h\tilde{g}^h \to g^h g^h)v \simeq \sigma_0, \tag{18}$$

where

$$\sigma_0 = k\frac{\pi\alpha_X^2}{m_X^2}, \tag{19}$$

and k is an $\mathcal{O}(1)$ N-dependent coefficient. Using the results of Appendix B for thermal freeze-out in a hidden sector [2,55], the hidden gluino's thermal relic density is, then,

$$\Omega_X \simeq 0.23\xi_f\frac{1}{k}\left[\frac{0.025}{\alpha_X}\right]^2\left[\frac{m_X}{\text{TeV}}\right]^2 \simeq 0.23\xi_f\frac{N^2}{k}\left[\frac{m_{3/2}}{170\text{ TeV}}\right]^2, \tag{20}$$

where we have used Eq. (15). The relic density is independent of α_X and m_X, and is automatically of the right order of magnitude because the hierarchy problem implies $m_{3/2} \sim 100$ TeV; in short, this scenario realizes the WIMPless miracle. Of course, although Ω_X is insensitive to m_X and α_X, the dark matter's properties are not. In particular, the confinement scale Λ is extremely sensitive to these parameters.

As an example, consider $N = 3$. In this case, $k = 27/64$ [29], and $\xi_{\rm BBN}$, the hidden to visible temperature ratio at BBN, may be as large as 0.63. Taking this temperature ratio at freeze-out to be $\xi_f = 0.5$, the correct hidden gluino relic density is achieved for $m_{3/2} = 52$ TeV and $m_{\tilde{W}} = 140$ GeV. In terms of m_X, the coupling is

$$\alpha_X \simeq 0.027\frac{m_X}{\text{TeV}}, \tag{21}$$

and the confinement scale is

$$\Lambda \sim m_X (5.8 \times 10^{-10})^{\mathrm{TeV}/m_X}. \tag{22}$$

The constraint $\Lambda \lesssim 10$ eV implies $m_X \lesssim 850$ GeV. Hidden gluons have a temperature that is roughly similar to the cosmic microwave background temperature in the visible sector. Hidden gluinos have velocity dispersions that drop to $\sim 10^{-8}$, corresponding to temperatures $m_X v^2 \sim 10^{-4}$ eV, at redshifts $z \sim 100$ [14,56,57], before being sped back up to the current velocity $v \sim 10^{-3}$. For 0.1 eV $\lesssim \Lambda \lesssim 10$ eV, then, both the hidden gluons and gluinos cool to a temperature below Λ before redshift $z \sim 100$, and so form $(g^h \tilde{g}^h)$ bound states. If these remain intact, these bound states interact through a short-range force with cross section $\sigma \sim \Lambda^{-2}$. This violates Bullet Cluster bounds on dark matter self-interactions, which require $\sigma/m_X \lesssim 3000\,\mathrm{GeV}^{-3}$ [58,59]. On the other hand, when the bound states are sped back up to $v \sim 10^{-3}$, collisions may disassociate the bound states, and the relevant bound is on long-range interactions, as we now discuss.

For $\Lambda \lesssim 0.1$ eV ($m_X \lesssim 750$ GeV), there is never a time at which both hidden gluinos and gluons have a temperature below Λ, and so at least some of the hidden gluinos and gluons remain unbound. In this case, the result is a hidden gluon and gluino plasma, and the relevant bounds are not those on short-range interactions, but those on dark matter interacting through long-range forces [14,60–62]. The self-interactions are generically weak enough to avoid constraints from the Bullet Cluster, and for $m_X \sim 750$ GeV are marginally consistent with other bounds, such as those from the observation of elliptical halos [14]. Further work is required to determine if such scenarios are truly viable. Note, however, that extremely low values of Λ occur naturally in this scenario, and it is remarkable that this first example already leads to potentially interesting dark matter properties and provides an extremely simple framework for studying such phenomena.

V. A PURE $SU(N)$ HIDDEN SECTOR WITH CONNECTORS TO MSSM GAUGINOS

We now consider models with heavy connector fields that mediate interactions between the hidden and visible sectors. As in the previous section, we consider a hidden sector that is pure $SU(N)$, that is, without light flavors, so the hidden particle content consists of just gluons and gluinos, with the gluino mass of Eq. (15). The gluinos freeze out, but then decay to visible sector particles through connector-induced higher-dimension operators. Dark matter will be the conventional MSSM Winos, but, unlike in standard scenarios, these Winos will inherit their relic density from hidden gluinos, and this relic density will be naturally in the correct range because of the WIMPless miracle.

In these scenarios, the hidden gluons may form glueballs, and these, too, can in principle decay to MSSM fields via loops of connector fields. The decay times and final state are determined by the details of the connector fields. We will discuss two examples of connectors. In this section, we consider connectors that give rise to dimension-eight operators coupling the hidden and visible gauge sectors. In the next section, we will discuss a larger connector sector that couples the hidden gauge sector to the MSSM Higgs fields through dimension-six operators.

A. Connectors

To preserve the possibility of gauge coupling unification [63–67], we introduce connectors in complete multiplets of the MSSM $SU(5)$ gauge group. We will add N_Y vectorlike connectors Y and $\bar{Y}$ that transform as $(5, N)$ and $(\bar{5}, \bar{N})$ under $SU(5) \times SU(N)^h$, respectively, with a large supersymmetric mass M_Y. This scenario is therefore specified by 5 fundamental parameters: $m_{3/2}$, m_X, N, N_Y and M_Y.

As we will see below, we will need the hidden gluinos to be short-lived enough to avoid bounds from BBN. This can be arranged by having many light connectors. What are the bounds on M_Y and N_Y?

For M_Y above $m_{3/2}$, the connectors have no effect on the AMSB soft masses to leading order in the supersymmetry breaking. Their contributions to the soft masses are therefore suppressed by $m_{3/2}/M_Y$ compared to the AMSB soft masses. In fact, the size of these contributions is known, since the connectors behave just like the messengers of gauge mediation. We can obtain the connectors' spectrum by rescaling their superpotential mass term by the compensator, $M_Y Y\bar{Y} \to M_Y(1 + m_{3/2}\theta^2)Y\bar{Y}$, leading to fermion mass M_Y and scalar masses

$$m^2_{\tilde{Y}} = M^2_Y\left(1 \pm \frac{m_{3/2}}{M_Y}\right), \tag{23}$$

just like GMSB messengers with mass M_Y and a supersymmetry-breaking parameter $F = M_Y m_{3/2}$. Integrating out the connectors, we get loop corrections to the soft masses of the visible and hidden sectors. These are known for arbitrary F/M_Y^2 [68,69]. The leading-order term in F/M_Y^2 cancels the connectors' contributions to the AMSB soft masses above M_Y [17,19]. The higher-order terms give corrections to the leading-order AMSB soft masses that are less than 4% even for $M_Y = 2m_{3/2}$, and so we may take M_Y as light as $2m_{3/2}$ without distorting our other results.

As for N_Y, there is no strict upper bound, but the desire for perturbativity up to high scales and gauge coupling unification provides a strong motivation for low N_Y. For $M_Y \sim 100$ TeV, the requirement that gauge couplings remain perturbative up to the grand unified theory scale is that the effective number of $5 + \bar{5}$ multiplets satisfies $N_5 \leq 5$. In this case, $N_5 = NN_Y$; given $N \geq 2$, this implies $N_Y \leq 2$. For larger M_Y, this constraint is weaker.

B. Decay lifetimes

Box diagrams with Y particles in the loop mediate decays $\tilde{g}^h \to g^h g \tilde{g}$, $g^h W \tilde{W}$, $g^h B \tilde{B}$. At energies below M_Y, the box diagrams induce the operator

$$\frac{g_X^2 g_{SM}^2}{16\pi^2} \frac{2N_Y}{M_Y^4} \int d^4\theta \bar{W}^h_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} W^{h\alpha} W_\alpha = \alpha_X \alpha_{SM} \frac{2N_Y}{M_Y^4} [\bar{\lambda}^h (\sigma \cdot \partial) \lambda F^{h\rho\sigma} F_{\rho\sigma} + F^h_{\mu\nu} F^{\mu\nu} F^{h\rho\sigma} F_{\rho\sigma}], \tag{24}$$

where the bars stand for complex conjugation, and g_{SM} stands for the appropriate SM gauge coupling. *A priori*, decays to all SM gauge bosons are allowed. In some cases, some of these decays may be kinematically forbidden. For example, the constraint $\Lambda \lesssim 10$ eV, together with Eq. (16), implies $m_X \lesssim 6 m_{\tilde{W}}$, and so decays to MSSM gluinos are not allowed. In the following, we will focus on the decay to visible Winos, since this decay channel is always allowed if any of them are.[2]

For hidden gluinos and visible Winos that are comparable in mass, but not particularly degenerate, the decay width to Winos is

$$\Gamma(\tilde{g}^h \to g^h W \tilde{W}) \sim \frac{m_X}{8\pi} \frac{1}{16\pi^2} 3 \left(\alpha_X \alpha_2 \frac{2N_Y}{M_Y^4} \right)^2 m_X^8, \tag{25}$$

where $1/(16\pi^2)$ is the 3-body decay suppression factor, and the factor of 3 comes from summing over the 3 possible charge combinations of Winos and W bosons in the final state. Using $\alpha_2 \sim 1/30$ and Eq. (15), we find that the hidden gluino lifetime is

$$\tau(\tilde{g}^h \to g^h W \tilde{W}) \sim 0.3 \mathrm{s} \frac{N^2}{N_Y^2} \left[\frac{m_{3/2}}{100 m_X} \right]^{10} \left[\frac{M_Y}{2 m_{3/2}} \right]^8 \frac{\mathrm{TeV}}{m_X}, \tag{26}$$

where we have normalized M_Y to a fairly low value, as discussed in Sec. V A.

The operator of Eq. (24) also mediates glueball decay to pairs of MSSM gauge bosons (see also [70]). The dominant decay is to SM gluons, with a decay width

$$\Gamma((g^h g^h) \to gg) \sim \frac{\Lambda}{8\pi} 8 \left(\alpha_X \alpha_3 \frac{2N_Y}{M_Y^4} \right)^2 \Lambda^8, \tag{27}$$

implying a lifetime of roughly

$$\tau((g^h g^h) \to gg) \sim 10^{-4} \left[\frac{m_X}{\Lambda} \right]^9 \tau(\tilde{g}^h \to g^h W \tilde{W}). \tag{28}$$

Note that here we have not distinguished between the glueball mass and Λ. Glueball masses in pure glue theories have been calculated on the lattice and are typically larger than Λ; for example, see Ref. [71] for the case of $SU(3)$, which is a good example, since we will focus here on small N. The glueball lifetime of Eq. (28) is extremely sensitive to the glueball mass, so a more careful treatment of glueball masses would result in a significantly faster glueball decay than the estimate of Eq. (28). This would make it easier to satisfy the BBN constraints discussed below, but to be conservative, we will not include such refined estimates. Note, however, that a very small change in m_X or α_X may produce a large change in Λ to compensate for such missing factors, and so we expect the qualitatively distinct possibilities we identify below to remain in more detailed analyses.

The implications of the lifetime estimates of Eqs. (26) and (28) may be clarified if we further require that the Winos from $\tilde{g}^h$ decay have the correct relic density to be all of dark matter. To implement this constraint, it will be convenient to define the ratio of hidden gluino to Wino masses,

$$R \equiv \frac{m_X}{m_{\tilde{W}}}. \tag{29}$$

The Wino relic density is the $\tilde{g}^h$ relic density of Eq. (20) diluted by the ratio of masses, or

$$\Omega_{\tilde{W}} \simeq 0.23 \xi_f \frac{N^2}{k} \frac{1}{R} \left[\frac{m_{3/2}}{170 \text{ TeV}} \right]^2 \simeq 0.23 \xi_f \frac{N^2}{k} \frac{1}{R} \left[\frac{m_{\tilde{W}}}{460 \text{ GeV}} \right]^2. \tag{30}$$

For $\xi_f \sim k \sim 1$, Winos from hidden gluino decays are all of the dark matter for

$$m_{\tilde{W}} \sim \frac{\sqrt{R}}{N} 500 \text{ GeV}. \tag{31}$$

Assuming this, the $\tilde{g}^h$ lifetime is

$$\tau(\tilde{g}^h \to g^h W \tilde{W}) \sim 1 \text{ s} \frac{N^3}{N_Y^2} \left[\frac{M_Y}{2 m_{3/2}} \right]^8 \left[\frac{3.0}{R} \right]^{11.5}, \tag{32}$$

and the glueball lifetime is

$$\tau((g^h g^h) \to gg) \sim 10^{-4} 10^{594/R} \tau(\tilde{g}^h \to g^h W \tilde{W}). \tag{33}$$

C. Viable scenarios

What are the constraints on the $\tilde{g}^h$ and $(g^h g^h)$ lifetimes? For the hidden gluino, one might think that it must decay before temperature $\alpha_X^2 m_X$ to prevent gluino-gluino bound states from forming, thereby enhancing gluino annihilation and ruining the WIMPless miracle. As noted in Sec. III B, though, this is not required. The most stringent constraints are associated with BBN. The decay $\tilde{g}^h \to g^h W \tilde{W}$, followed by $W \to q \bar{q}'$, produces protons and neutrons, which are very dangerous for BBN. Hidden gluinos must therefore have lifetimes under ~ 1 s.

[2] We will discuss examples in which decays to gluinos are important in Sec. V C 2. Decays to Binos are negligible in all our examples.

For glueballs, there are two possibilities. If they are effectively stable, they must not contribute too much to hot dark matter, and so $\Lambda \lesssim 10$ eV. On the other hand, if they are unstable, their decays are also subject to constraints from BBN. These may be avoided if glueballs decay before 1 s. Of course, this may be too stringent a requirement: the constraints depend on whether the glueballs decay to SM gluons, W bosons or photons, and on the decay time. There are clearly many possibilities, leading to different constraints and also many possible signals.

For simplicity, however, here we consider only the two clearly viable possibilities in which either $\Lambda \lesssim 10$ eV and hidden gluons or glueballs are long lived, or $\Lambda \gtrsim$ MeV and glueballs decay before BBN.

1. Low Λ: $\Lambda \lesssim 10$ eV

For $\Lambda \lesssim 10$ eV, we need $R \lesssim 6$, where we have used Eq. (16). We then see immediately from Eq. (33) that glueballs are extraordinarily long-lived in this case. At the same time, for the hidden gluinos to decay before BBN, we need $R \gtrsim 3.0$. Given choices of N, N_Y and M_Y, and assuming the correct Wino relic density, there is then a one-parameter family of viable models parametrized by R in the range $3 \lesssim R \lesssim 6$.

As an example, consider $N = 3$, $N_Y = 1$, and $M_Y = 2m_{3/2}$. For $R = 5.5$, we find $m_{\tilde{W}} \sim 400$ GeV, $m_X \sim 2$ TeV, $\alpha_X \sim 0.02$, and $\Lambda \sim$ eV. Assuming $\xi_f \sim k \sim 1$, hidden gluinos freeze out with $\Omega_X \approx 1$, and then decay at 0.02 s to MSSM Winos with the right relic density to be all of dark matter. Because the hidden gluinos decay early, constraints on dark matter self-interactions do not apply.

Alternatively, taking $R = 4$ and $N = N_Y = 2$, we find very similar values for the masses of the Wino and the hidden gluino, but the confinement scale is much smaller, with $\Lambda \sim 10^{-5}$ eV. The hidden gluinos decay to Winos at 0.07 s, but the hidden gluons remain unbound and constitute a negligible fraction the Universe's energy density.

Note that the value of Λ may vary widely. For Λ near its upper bound, these scenarios predict mixed hot-cold dark matter, with observable implications for small-scale structure. Note also that some connectors are stable, as there are no gauge-invariant decays, so that the reheat temperature must be below M_Y. In this scenario, however, this is not a very stringent constraint, as the connectors are very heavy, with $M_Y \sim 100$ TeV, and it is already motivated by the BBN constraint on g_*^h.

2. Hidden glueballs decaying before BBN

For the glueballs to decay before BBN, we need $\tau((g^h g^h) \to gg) \lesssim 1$ s. At the same time, the hidden gluino must decay after Wino freeze out at $t \sim 10^{-10}$ s. As we will see, these requirements imply a large R, for which the hidden gluino can decay to the visible gluino. We therefore require

$$\frac{\tau((g^h g^h) \to gg)}{\tau(\tilde{g}^h \to g^h g \tilde{g})} \sim 30 \cdot 10^{-4} \times 10^{594/R} < 10^{10} \Rightarrow R \gtrsim 45, \tag{34}$$

where the factor of $30 \sim 8\alpha_3^2/(3\alpha_2^2)$ arises from the enhancement of the decay width to MSSM gluinos over the decay to MSSM Winos. As an example, consider $N = 6$, and $R = 55$, for which $m_{\tilde{W}} \sim 600$ GeV, $m_X \sim 30$ TeV, $\alpha_X \sim 0.1$, $m_{3/2} \sim 200$ TeV, and $\Lambda \sim 2$ TeV. The hidden gluino mass, m_X, is quite large, but it is below the unitarity bound for thermal relics [72]. Taking one set of connector fields, $N_Y = 1$, at $M_Y = 10m_{3/2}$,[3] one finds that the hidden gluino decays at $t \sim 10^{-8}$ s, and the glueball decays at $t \sim 1$ s, avoiding BBN constraints. The Wino thermal relic density is negligible, but nonthermal production from hidden gluino decays gives it the desired relic density.

Note that the hidden gluino decay occurs at temperatures somewhat below Λ, and so after the hidden gluino freezes out, it hadronizes and forms hidden R hadrons before it decays to the Wino. In principle, this could lead to renewed hidden gluino annihilations, since the cross section for R-hadron interactions is now raised to $\sim 1/\Lambda^2$. For these annihilations to occur, the R hadrons must first form bound states, and later lose energy so that the hidden gluino pair in the R-hadron bound state can actually annihilate; see, for example, the discussion in Ref. [33]. Both of these processes require the emission of light particles, which carry away the binding energy and the energy released when the initial excited R-hadron bound state relaxes to the ground state. These energies are characterized by two quantities: Λ and $\alpha_X^2 m_X$. In the scenarios given here, however, the lightest particles in the hidden sector are the glueballs, with masses $\gtrsim \Lambda$, and $\alpha_X^2 m_X < \Lambda$. Therefore, the hidden gluinos cannot annihilate effectively even after they hadronize, and they survive in R hadrons until they decay to Winos.

As in the previous case, this scenario has implications for observations. We expect that glueball decay times below 1 s are allowed, but for lifetimes near this upper bound, these scenarios predict astrophysical signals, in BBN or other observables sensitive to late decays. Finally, note that, also as in the previous case, the connector fields are stable, and the reheat temperature must again be below $M_Y \sim 1000$ TeV. However, $g_*^h = 0$ at BBN in this case, since the hidden glueballs decay to SM fields before BBN.

VI. A PURE $SU(N)$ HIDDEN SECTOR WITH CONNECTORS TO MSSM HIGGSINOS

A. Connectors

We now consider an alternative scenario in which the hidden gluinos decay not to SM gauge bosons, but to SM

[3]Note that for $N = 6$, the connectors constitute six additional flavors of the visible $SU(3)$, which still gives a perturbative coupling at the grand unified theory scale for $M_Y = 2000$ TeV.

Higgs bosons. We add one copy ($N_Y = 1$) of the same connector fields as before, as well as a vectorlike $SU(N)$ pair Q and $\bar{Q}$, which are singlets under the SM, so that, in all, the new heavy fields and their representations under $SU(5) \times SU(N)^h$ are

$$Y(5, N), \qquad \bar{Y}(\bar{5}, \bar{N}), \qquad Q(1, N), \qquad \bar{Q}(1, \bar{N}). \quad (35)$$

We couple the Q and the ($SU(2)$ doublets of the) Y connectors to the MSSM through the superpotential[4]

$$W = yY\bar{Q}H_d + \bar{y}\bar{Y}QH_u + M_Y Y\bar{Y} + M_Q Q\bar{Q}, \quad (36)$$

where H_u and H_d are the MSSM Higgs supermultiplets. For simplicity, we set $M_Y = M_Q \equiv M$ and $\bar{y} = y$. As in Sec. V, we expect $M \gtrsim 2m_{3/2}$ to be acceptable. The connector sector is effectively N pairs of $5 + \bar{5}$ (and 6 $SU(N)$ flavors), and so gauge coupling unification is preserved for $N \leq 5$.

B. Decay lifetimes

As in Sec. V, the connectors induce $\tilde{g}^h$ decay through a box diagram, this time with Q and Y connectors in the loop. Integrating out the connector fields yields the operator

$$\frac{g_X^2 y^2}{16\pi^2}\frac{2}{M^2}\int d^2\theta W^{h\alpha}W^h_\alpha H_u H_d = \frac{2\alpha_X\alpha_y}{M^2}(\bar{\lambda}^h\sigma^\mu\bar{\sigma}^\nu\tilde{H}_d F^h_{\mu\nu}H_u + F^h_{\mu\nu}F^{h\mu\nu}H_u H_d). \quad (37)$$

The decay width is roughly

$$\Gamma(\tilde{g}^h \to g^h H_u\tilde{H}_d) \sim \frac{m_X}{8\pi}\frac{1}{16\pi^2}2\left(\alpha_X\alpha_y\frac{2}{M^2}\right)^2 m_X^4, \quad (38)$$

where the loop factor is as in Eq. (25), and the factor of 2 accounts for the 2 possible charge assignments for the Higgs boson and Higgsino in the final state. Using Eq. (15), the $\tilde{g}^h$ lifetime is, then,

$$\tau(\tilde{g}^h \to g^h H_u\tilde{H}_d) \sim 1 \times 10^{-8}\ \mathrm{s}\left[\frac{N}{2}\right]^2\left[\frac{0.01}{\alpha_y}\right]^2\left[\frac{m_{3/2}}{100m_X}\right]^6 \times\left[\frac{M}{2m_{3/2}}\right]^4\frac{\mathrm{TeV}}{m_X}. \quad (39)$$

The operator of Eq. (37) also mediates glueball decay to two Higgs bosons. If kinematically accessible, the glueball decay width is

$$\Gamma((g^h g^h) \to H_u H_d) \sim \frac{\Lambda}{8\pi}2\left(\alpha_X\alpha_y\frac{2}{M^2}\right)^2\Lambda^4, \quad (40)$$

corresponding to a lifetime of

$$\tau((g^h g^h) \to H_u H_d) \sim 10^{-2}\left[\frac{m_X}{\Lambda}\right]^5\tau(\tilde{g}^h \to g^h H_u\tilde{H}_d), \quad (41)$$

subject to the same uncertainties discussed below Eq. (28).

As in Sec. V, we may include the constraint from the relic density. The relic density is again diluted by the hidden gluino decay to Winos, and so Eq. (30) again applies. Using Eq. (31), we find

$$\tau(\tilde{g}^h \to g^h H_u\tilde{H}_d) \sim 1 \times 10^{-4}\ \mathrm{s}\left[\frac{N}{2}\right]^3\left[\frac{0.01}{\alpha_y}\right]^2\left[\frac{M}{2m_{3/2}}\right]^4\frac{1}{R^{7.5}}, \quad (42)$$

and the glueball lifetime satisfies

$$\tau((g^h g^h) \to H_u H_d) \sim 10^{-2}10^{330/R}\tau(\tilde{g}^h \to g^h H_u\tilde{H}_d). \quad (43)$$

C. Viable scenarios

We may again identify two qualitatively different classes of viable scenarios, depending on whether the hidden gluons are effectively stable, or whether they form glueballs and decay before BBN. In contrast to Sec. V, however, where the operator of Eq. (24) was dimension 8, here the operator of Eq. (37) is only dimension 6. It is therefore easy to arrange for very small lifetimes, and the discrepancy between the gluino and glueball lifetimes is reduced.

1. Low Λ: $\Lambda \lesssim 10$ eV

If glueballs are effectively stable, we need $\Lambda \lesssim 10$ eV and $R \lesssim 6$. There are many possible choices of parameters that are viable. For example, let $N = 2$, $\alpha_y = 0.01$, and $R = 4$. The hidden gluino decays may be anywhere in the desired range 1ns $\lesssim t \lesssim 1$ s for M in the range $2m_{3/2} \lesssim M \lesssim 350m_{3/2}$. Hidden gluons are very long-lived. The other parameters are as in Sec. V C 1: $m_{\tilde{W}} = 400$ GeV, $m_X \sim 2$ TeV, $\alpha_X \sim 0.02$, and $\Lambda \sim$ eV. The hidden gluino freezes out with $\Omega_X \simeq 1$, and then decays to MSSM Winos with the right relic density to be all of dark matter. For Λ near its upper bound, this scenario provides a very simple framework for mixed dark matter, with both MSSM Wino cold and hidden glueball hot components.

The hidden sector gluons contribute $g_*^h = 6$ at BBN, and so the temperatures of the two sectors must be somewhat different. This is also motivated by the fact that some connectors are stable, and the reheat temperature must be below their mass.

2. Hidden glueballs decaying before BBN

For the ratio of glueball lifetime to $\tilde{g}^h$ lifetime not to exceed 10 orders of magnitude,

$$\frac{\tau((g^h g^h) \to gg)}{\tau(\tilde{g}^h \to g^h g\tilde{g})} \sim 10^{-2}10^{330/R} < 10^{10} \Rightarrow R \gtrsim 27. \quad (44)$$

[4]Note that once the MSSM Higgs bosons develop vacuum expectation values, the first two terms in the superpotential contribute to the connectors' masses, but these corrections are negligible.

Taking, for example, $N = 2$, $\alpha_y = 0.01$, $R = 30$, and $M = 37m_{3/2}$, we find that the hidden gluino lifetime is around 1 ns, the hidden glueball decays around 1 s, $m_{\tilde{W}} = 1.3$ TeV, $m_X \simeq 40$ TeV, $\alpha_X \sim 0.2$, and $\Lambda \sim 200$ GeV. The glueball decays may have observable effects in BBN or other astrophysical signals.

Once again the $\tilde{g}^h$ decays at temperatures a bit lower than Λ, but its abundance is not significantly diluted by hadronic effects. The hidden sector does not contribute to g_* at BBN, and so the visible and hidden sectors may be at the same temperature, but the reheat temperature must be below the mass of the stable connectors $M_Y \sim 10^3$ TeV.

VII. AN $SU(N)$ HIDDEN SECTOR WITH FLAVOR AND LIGHT GOLDSTONE BOSONS

So far we have studied three kinds of models. In one (Sec. IV), the hidden sector does not interact with the visible sector, so that both the hidden dark matter and the hidden composites are stable. In the second (Sec. V C 2 and VI C 2), the hidden dark matter candidate decays to the Wino, but the hidden $SU(N)$ composites (glueballs, for the case of pure Yang-Mills) decay to the visible sector. In the third (Sec. V C 1 and VI C 1) the hidden dark matter candidate again decays to the Wino, but the hidden composites are effectively stable. In this latter case, there is a stringent bound on the confinement scale $\Lambda \lesssim 10$ eV, since otherwise the hidden glueballs overclose the Universe or contribute too much to hot dark matter.

Here we will consider a qualitatively different example, in which the hidden sector contains light Goldstone bosons, with masses significantly below Λ. The hidden dark matter candidate will decay to the Wino through loops of connector fields, and the glueballs will decay to the hidden Goldstone bosons. Furthermore, the light Goldstone bosons provide a thermal bath for any stable $SU(N)$ composites, such as baryons, so that the resulting relic abundance of these composites is negligible.

For concreteness, we will focus on the simplest possibility, $N = 3$ and $N_F = 2$, that is, hidden $SU(3)$ with two massless flavors. Chiral symmetry breaking results in three Goldstone bosons, which is marginally consistent with BBN constraints on g_*^h, and is testable with future improvements of these constraints. The $SU(3)$ confinement scale is above an MeV, so that the only new light particles at BBN are the Goldstone bosons. We will also include connector fields so that the hLSP decays to the Wino shortly after Wino freeze-out. As we will see, the connector fields in this example are not stable, so that the hidden and visible sectors can be in thermal equilibrium.

Because $N_F \leq 2N$, the hLSP is now the hidden squark. Equation (10) implies that its mass is

$$m_X \simeq 0.34\alpha_X m_{3/2}, \tag{45}$$

and using also Eq. (11), we find

$$\Lambda \sim 10^{-36/R} m_X. \tag{46}$$

To get the correct Wino relic abundance from hidden squark decays, we need

$$m_{\tilde{W}} \sim \sqrt{R}\,300\ \text{GeV}, \tag{47}$$

so we can rewrite Λ as

$$\Lambda \sim 300\ \text{GeV} R^{3/2} 10^{-36/R}. \tag{48}$$

Requiring $\Lambda \gtrsim$ MeV, we find $R \gtrsim 5$.

We will now add connector fields to the theory, so that the hidden squark eventually decays to the Wino. As above, we take the connector fields to be vectorlike pairs transforming as bifundamentals under $SU(5) \times SU(3)^h$:

$$Y(5, 3), \qquad \bar{Y}(\bar{5}, \bar{3}). \tag{49}$$

We will need two such pairs, with the superpotential

$$W = yY_i^d \bar{q}_i^h H_d + \bar{y}\bar{Y}_i^d q_i^h H_u + M_Y Y\bar{Y}. \tag{50}$$

Here $i = 1, 2$, q^h, $\bar{q}^h$ are the hidden $SU(3)$ quarks, and the superscript d on the Y fields denotes the doublet fields of the 5 and $\bar{5}$. Note that the connectors are unstable: the doublet Y fields can decay to Higgs fields and hidden quark fields. Since running effects create a splitting between the doublet and triplet Y fields (see, for example, Ref. [68]), the triplets can decay to the doublets.

For simplicity, we will set $y = \bar{y}$. Integrating out the connector fields we have the following superpotential coupling of the hidden quarks to Higgs fields:

$$\frac{y^2}{M_Y} q_i^h \bar{q}_i^h H_u H_d, \tag{51}$$

which induces hidden squark decay into a hidden quark, Higgs and Higgsino with lifetime

$$\begin{aligned}\tau(\tilde{q}^h \to q^h H_u \tilde{H}_d) &\sim 3 \times 10^{-24}\ \text{s}\left[\frac{M_Y}{y^2 m_X}\right]^2\left[\frac{\text{TeV}}{m_X}\right] \\ &\simeq 1 \times 10^{-18}\ \text{s} R^{-3.5}\left[\frac{M_Y}{y^2 m_{3/2}}\right]^2.\end{aligned} \tag{52}$$

Thus, for example, for $R = 5$, we can have the hidden squark decay at 10^{-6} s for $M_Y = 10^7 y^2 m_{3/2}$.

Note that the operator of Eq. (51) induces a small Goldstone boson mass

$$m_\pi \sim y^2 \frac{\langle H_u H_d\rangle}{M_Y}. \tag{53}$$

As discussed in (c3), such masses are constrained by the bound on the amount of hot dark matter in the Universe. For $M_Y = 10^7 y^2 m_{3/2}$, $m_\pi \sim 10$ eV, which is consistent with these bounds.

VIII. CONCLUSIONS

Supersymmetric extensions of the SM contain a fundamental mass scale, the supersymmetry-breaking scale, which enters the masses of superpartners in the visible sector as well as in any hidden sector. Furthermore, if a hidden sector is truly hidden, with no interactions with the SM, it generically contains a stable superpartner, which is protected by the R parity of the hidden sector. These two features allow for the construction of dark matter models in which the dark matter relic abundance is related to the weak scale. In AMSB models, this abundance is actually the same as the usual WIMP abundance, since the dark matter mass is proportional to its coupling squared, and only their ratio enters in the abundance. These models thus offer a particularly simple realization of the WIMPless dark matter idea.

In this paper, we studied dark matter candidates from non-Abelian hidden sectors with AMSB. The hidden sectors we consider are very simple. They are $SU(N)$ gauge theories, with either no matter or a few fundamental flavors. In some of our examples, the hidden LSP is stable simply as a result of gauge symmetry and supersymmetry, and its relic abundance is automatically of the correct size by the WIMPless miracle. In other examples, the hidden and visible sectors interact through higher-dimension operators, so that the hidden LSP freezes out and then decays to a visible Wino. The result is Wino dark matter which, despite its large annihilation cross section, has the correct abundance, with favorable implications for indirect detection.

As we have seen, the phenomenology of these models is very rich, owing partly to the non-Abelian interactions of the dark matter candidate. As an example, some of these models have a confinement scale Λ that is naturally very small, as a result of renormalization group evolution, with a wealth of potentially interesting astrophysical implications. In the model of Sec. IV, the hidden LSP is the dark matter, and cannot be seen in any direct or indirect detection experiment. However, the confinement scale is very small, and the dark matter is self-interacting through a long-range non-Abelian force. In the examples of Secs. V and VI, hidden gluinos freeze out and decay to visible Winos with the correct relic density. The accompanying hidden glueballs may either be stable, as discussed in Secs. V C 1 and VI C 1, providing a natural framework for mixed cold-hot dark matter, or may decay, as discussed in Secs. V C 2 and VI C 2, yielding astrophysical signals. We have also presented in Sec. VII a model with 3 light hidden pions that contribute to the number of nonrelativistic degrees of freedom at BBN, and will be excluded or favored as constraints on this quantity improve. In all of these cases, the scenarios are defined by a small number of parameters, are consistent with gauge coupling unification, preserve the beautiful connection between the weak scale and the observed dark matter relic density, and are natural, with relatively light visible superpartners.

We have only outlined the main features of representative models here, and it would be interesting to explore specific models in more detail. The cosmology of (meta) stable particles with non-Abelian interactions was studied to some extent for the case of QCD, but even that case has many unsettled issues. It would also be interesting to study Abelian hidden sectors, or hidden sectors with no gauge interactions, but with Yukawa interactions. Such hidden sectors are theoretically less clean, because some model building is required to guarantee the stability of the hidden LSP, but their phenomenology is likely to be simpler.

Finally, the models we studied are very predictive, since, because the superpartner masses are determined by anomaly mediation, they depend on a very small number of parameters. They thus offer a particularly clean realization of WIMPless dark matter. We emphasize that the simplicity of the models is related to the fact that the hidden sector and visible sector are only coupled through higher-dimension operators, mediated by connector fields whose masses are much larger than the supersymmetry-breaking scale. The hidden LSP soft mass therefore only depends on the hidden sector gauge couplings and the gravitino mass, and does not involve any "μ terms." As a result, however, DM consists either of Winos, or of hidden LSPs, which cannot be detected directly or indirectly. It would be interesting to construct models in which the hidden LSP has stronger interactions with the visible sector, so that it might explain the DAMA and CoGeNT anomalies, in the spirit of Refs. [3,4]. This requires weak-scale connector fields, and therefore new μ terms. The hidden LSP soft mass would then generically depend on these new couplings and μ terms.

It would also be interesting to generalize the WIMPless idea to other frameworks of supersymmetry breaking, in which the hidden dark matter abundance does exhibit some dependence on the hidden dark matter mass and coupling, but is still related to the weak scale because of the underlying supersymmetry-breaking scale.

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APPENDIX A: AMSB SUPERPARTNER MASSES

In AMSB, the soft SUSY-breaking parameters are determined by the gravitino mass $m_{3/2}$ and the (weak-scale values of the) dimensionless couplings of the theory [15,16]. Consider a supersymmetric model with gauge group G, gauge coupling g and Yukawa couplings y^{ijk} defined by the superpotential

$$W = \tfrac{1}{6} y^{ijk} X_i X_j X_k, \tag{A1}$$

where the X_i are chiral superfields. The gauge and Yukawa coupling renormalization group equations are

$$\dot{g} = \frac{1}{16\pi^2} b g^3 \tag{A2}$$

$$\dot{y}^{\,ijk} = y^{pjk}\gamma_p^i + y^{ipk}\gamma_p^j + y^{ijp}\gamma_p^k, \tag{A3}$$

where $b = -3C(G) + \sum_i C(i)$, $\dot{()} \equiv d/d\ln(\mu/Q)$, and

$$\gamma_i^j = \frac{1}{16\pi^2}\left[\frac{1}{2} y_{imn} y^{jmn} - 2\delta_i^j g^2 C(i)\right]. \tag{A4}$$

The group theoretic constants are defined by

$$t^a t^a = C(G)\mathbf{1} \tag{A5}$$

$$\mathrm{Tr}\; t^a t^b = C(i)\delta^{ab}, \tag{A6}$$

where the matrices t^a are the generators for representation i. Note that in our conventions, asymptotically free theories have $b < 0$.

Defining soft SUSY-breaking terms

$$\mathcal{L}_{\text{soft}} = \{-\tfrac{1}{2} M_\lambda \lambda\lambda - \tfrac{1}{2}(m^2)_i^j \phi^{*i}\phi_j - \tfrac{1}{6} A^{ijk}\phi_i\phi_j\phi_k + \text{H.c.}\}, \tag{A7}$$

the AMSB soft SUSY-breaking parameters are

$$\begin{aligned} M_\lambda &= \frac{1}{16\pi^2} b g^2 m_{3/2} \\ (m^2)_i^j &= \frac{1}{2}\dot{\gamma}_i^j m_{3/2}^2 \\ A^{ijk} &= -(y^{pjk}\gamma_p^i + y^{ipk}\gamma_p^j + y^{ijp}\gamma_p^k) m_{3/2}. \end{aligned} \tag{A8}$$

APPENDIX B: THERMAL RELIC DENSITY IN A HIDDEN SECTOR WITH A DIFFERENT TEMPERATURE

Thermal freeze-out is modified if it occurs in a sector with a different temperature from the observable sector's [2,55]. Here we summarize the main results.

Assume that a particle X with mass m_X annihilates through S-wave processes with cross section

$$\sigma(XX \to \text{anything})v \approx \sigma_0. \tag{B1}$$

The particle then freezes out when the hidden and visible sector temperatures are T_f^h and T_f^v, respectively. The resulting thermal relic density is

$$\Omega_X \approx \frac{s_0}{\rho_c} \frac{3.79 x_f}{(g_{*S}/\sqrt{g_*^{\text{tot}}}) m_{\text{Pl}} \sigma_0}, \tag{B2}$$

where $s_0 \simeq 2970\ \text{cm}^{-3}$ is the visible sector's entropy density now, $\rho_c \simeq 0.527 \times 10^4\ \text{eV cm}^{-3}$ is the critical density, $x_f \equiv m_X/T_f^v$, $g_{*S} \sim 100$ and $g_*^{\text{tot}} \sim 100$ are the visible and total number of relativistic degrees of freedom at freeze-out, and $m_{\text{Pl}} \simeq 1.2 \times 10^{19}$ GeV is the Planck mass. The freeze-out temperature is given by

$$x_f = \xi_f \ln L - \tfrac{1}{2}\xi_f \ln(\xi_f \ln L), \tag{B3}$$

where

$$\xi_f \equiv \frac{T_f^h}{T_f^v}, \tag{B4}$$

and

$$L \approx 0.038 m_{\text{Pl}} m_X \sigma_0 (g/\sqrt{g_*^{\text{tot}}})\xi^{3/2}\delta(\delta + 2), \tag{B5}$$

where g is the number of X degrees of freedom, and the parameter δ is tuned to make these analytical results fit the numerical results. For $\xi \sim 0.3$–1, $\delta \sim 0.2$–0.5 gives a good fit [2].

As is well known, Ω_X is inversely proportional to σ_0 and only logarithmically sensitive to m_X. Note, however, that σ_0 is also only logarithmically sensitive to g. For example, for the case where X is a gluino of hidden $SU(N)$, the thermal relic density is not enhanced by $N^2 - 1$, as it would be for $N^2 - 1$ independent degrees of freedom, because the $N^2 - 1$ gluino degrees of freedom interact with each other. As a result, for a wide range of parameters, $x_f \approx 25\xi_f$ to a good approximation. We then find that the thermal relic density is

$$\begin{aligned} \Omega_X &\approx \xi_f \frac{0.17\ \text{pb}}{\sigma_0} \simeq \xi_f \frac{1}{\sigma_0}\left[\frac{0.021}{\text{TeV}}\right]^2 \\ &\simeq 0.23\xi_f \frac{1}{k}\left[\frac{0.025}{\alpha_X}\frac{m_X}{\text{TeV}}\right]^2, \end{aligned} \tag{B6}$$

where in the last step, we have parametrized the cross section as $\sigma_0 = k\pi\alpha_X^2/m_X^2$. The final result is, therefore, simple: for a thermal relic that freezes out in a hidden sector with a different temperature, the thermal relic density is modified by the factor $\xi_f \equiv T_f^h/T_f^v$ from the standard result.

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