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Proposal and theoretical formalism for studying baryon radiative decays from $J/\psi \to B^*\bar{B} + \bar{B}^*B \to \gamma B\bar{B}$

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With accumulation of high statistics data at BESIII, one may study many new interesting channels. Among them, $J/\psi \to B^*\bar{B} + \bar{B}^*B \to \gamma B\bar{B}$ processes may provide valuable information of the radiative decays of the excited baryons $B^*(N^*, \Lambda^*, \Sigma^*, \Xi^*)$, and may shed light on their internal quark-gluon structure. Our estimation for the branching ratios of the nucleon excitations $N^*(1440)$, $N^*(1535)$ and $N^*(1520)$ from the reaction $J/\psi \to N^*\bar{p} + \bar{N}^*p \to p\bar{p}\gamma$, indicates that these processes can be studied at BESIII with $10^{10} J/\psi$ events. Explicit theoretical formulas for the partial wave analysis (PWA) of the $J/\psi \to B^*\bar{B} + \bar{B}^*B$ with $B^* \to B\gamma$ and $\bar{B}^* \to \bar{B}\gamma$ within covariant L-S Scheme are provided.

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I. INTRODUCTION

Baryons $B(N, \Lambda, \Sigma, \Xi, \cdots)$ and their excited states $B^*(N^*, \Lambda^*, \Sigma^*, \Xi^*, \cdots)$ are complex systems of confined quarks and gluons. Excited baryons are sensitive to details of quark confinement [1], which is poorly understood within the fundamental theory of strong interactions—Quantum Chromodynamics (QCD). Thus, understanding their structure and determining their properties (masses, decay widths, branching ratios, spins, parities, electromagnetic form factors, magnetic moments, polarizabilities) will provide a better understanding of how confinement works in baryons. Concerning the internal quark-gluon structure of baryons there are various proposed configurations: (a) the classical constituent three quark (qqq) states; (b) $qqqg$ hybrid states [2]; (c) diquark-quark states [3,4]; (d) meson-baryon states [5–8]; (e) pentaquark with diquark clusters [9–13], etc. A series of new experiments on excited nucleon N^* physics with electromagnetic probes have been started at modern facilities such as TJNAF [14], ELSA [15], GRAAL [16], SPRING8 [17] and BEPC [18,19]. In last few years these facilities provided a considerable amount of precise data for various excited nucleon production and decay channels and opened a great opportunity to make quantitative investigations of the baryon structure. To extract properties of N^* resonances partial wave analysis (PWA) is necessary. In this paper, we first show that the radiative decays of baryons can be studied at BESIII with expected 10^{10} J/ψ events. Then we provide PWA formulas within covariant L-S Scheme [20] for multistep chain processes $J/\psi \to B^*\bar{B} + \bar{B}^*B \to \gamma B\bar{B}$ with $B^*(N^*, \Lambda^*, \Sigma^*, \Xi^*, \cdots)$. Because electromagnetic transition rates of excited baryons to their respective ground states offer a stringent test on the quark model dynamics [21,22], it is therefore highly desirable to study the electromagnetic decay rates from excited baryon states in order to refine the quark model description of the baryons. To date, very few electromagnetic transition rates have been measured for the excited baryon resonances [23]. For a detailed discussion of the experimental and theoretical status of the excited baryons and their electromagnetic decays, see the review by Landsberg [21].

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II. ESTIMATION OF BRANCHING RATIOS FOR $J/\psi \to B^*\bar{B} + \bar{B}^*B \to B\bar{B}\gamma$

In hadron spectroscopy, the ground states of the hadron spectrum are now well understood. However, the excited states still prove a significant challenge. The first excited state $N^*(1440)P_{11}$ with positive parity $J^P = 1/2^+$, and the adjacent excited state $N^*(1535)S_{11}$ with negative parity $J^P = 1/2^-$, as well as $N^*(1520)D_{13}$ with $J^P = 3/2^-$, have been identified by using various techniques. Although these four-star resonances are within the energy region of many modern research facilities, their properties including radiative decays are still not well determined. Previous BES experiments already clearly observed these resonances in $J/\psi \to p\bar{p}\eta$, $\bar{p}n\pi^+ + c.c.$, $\bar{p}p\pi^0$ [18,19]. With 2 orders of magnitude and higher statistics at BESIII, the radiative decays of these N^* may also be studied in $J/\psi \to \gamma p\bar{p}$. In fact, this decay channel has already been studied by the BESII experiment. A strong narrow peak, $X(1860)$, near the threshold in the invariant mass spectrum of proton-antiproton pairs was observed [24]. The branching ratio for $J/\psi \to \gamma p\bar{p}$ is about 3.8×10^{-4} [23], among which the contribution of $J/\psi \to \gamma X(1860) \to \gamma p\bar{p}$ is about 7.0×10^{-5} [24]. The PWA formulas for determining quantum numbers of intermediate resonances decaying to $p\bar{p}$ are given in Ref. [25]. Because of limited statistics and a large background from $J/\psi \to \bar{p}p\pi^0$ channel, no observation of $N^* \to p\gamma$ was reported.

TABLE I. The mass (MeV), widths (MeV), and branching ratios (10^{-6}) for $J/\psi \to N^*\bar{p} + \bar{N}^* p \to p\gamma\bar{p}$ through intermediate N^* states.

m_{N^*}	Γ	$Br(J/\psi \to N^*\bar{p} + \bar{N}^* p)$	$Br(N^* \to p\gamma)$	$Br(J/\psi \to p\gamma\bar{p})$
938		$210 \sim 224$ [23]		$19.8 \sim 21.0$
1440	300	$133 \sim 354$ [19]	$350 \sim 480$ [23]	$0.046 \sim 0.170$
1535	150	$92 \sim 210$ [19]	$1500 \sim 3500$ [23]	$0.138 \sim 0.735$
1520	115	$34 \sim 154$ [19]	$4600 \sim 5600$ [23]	$0.156 \sim 0.862$

Based on the branching ratios for the reaction $J/\psi \to N^*\bar{p} + \bar{N}^* p$ measured by BESII [19] and branching ratios of $N^* \to p\gamma$ given by PDG [23], we give the estimation of branching ratios for the reaction $J/\psi \to N^*\bar{p} + \bar{N}^* p \to p\bar{p}\gamma$ through the intermediate $N^* = p(938)$, $N^*(1440)$, $N^*(1535)$ and $N^*(1520)$ states, as shown in Table I.

In the estimation of the contribution from the off shell nucleon pole, we use the following effective Lagrangian for the vertex γpp [26]:

$$\mathcal{L}_{\gamma pp} = -e\bar{\psi}_p(\gamma^\mu A_\mu - \frac{\kappa_p}{2m_p}\sigma^{\mu\nu}\partial_\nu A_\mu)\psi_p, \quad (1)$$

where $\kappa_p = 2.739$ and m_p are the proton magnetic moment and mass, respectively. The following off shell form factor is assumed:

$$F = \frac{\Lambda^4}{\Lambda^4 + (p_{N^*}^2 - m_{N^*}^2)^2}, \quad (2)$$

where $\Lambda = 0.8$ GeV, p_{N^*} and m_{N^*} are the N^* four momentum and mass. Here we also use the experimental photon energy cut condition $E_\gamma > 25$ MeV. Because of the zero width of proton, the main contribution for $J/\psi \to p\bar{p} \to p\bar{p}\gamma$ is from the low energy photon, for example, the branching ratio will be reduced to 6.7×10^{-6} for the photon energy cut $E_\gamma > 100$ MeV. The contribution from the off shell proton pole contribution is well separated from N^* contributions on the Dalitz plot.

Because of flavor SU(3) symmetry, the excited hyperons are produced at a similar rate, so the typical branching ratio for the $J/\psi \to B^*\bar{B} + \bar{B}^* B \to \gamma B\bar{B}$ processes is about $10^{-7} \sim 10^{-6}$. With expected $10^{10} J/\psi$ events and much improved photon detection at BESIII, these processes can definitely be studied in order to provide unique information on the structure of various excited nucleon and hyperon states, and to give substantial insight into the nonperturbative aspects of the QCD.

III. FORMALISM

Now we present the necessary tools for the construction of the covariant L-S scheme for the $B^*\bar{B}M(\bar{B}^*BM)$ and $B^*B\gamma(\bar{B}^*\bar{B}\gamma)$ couplings. The partial wave amplitudes $U_i^{\mu\nu}$ in the covariant L-S scheme can be constructed by using pure orbital angular momentum covariant tensors $\tilde{t}^{(L_{bc})}_{\mu_1\cdots\mu_{L_{bc}}}$, covariant spin wave functions ψ (Ψ) or ϕ (Φ), metric tensor $g^{\mu\nu}$, totally antisymmetric Levi-Civita tensor $\epsilon_{\mu\nu\lambda\sigma}$ and momentum of the parent particle.

For a given hadronic decay process $a \to bc$, in the L-S scheme on hadronic level, the initial state is described by its four momentum p_μ and its spin state $\mathbf{S}_a$; the final state is described by the relative orbital angular momentum state of bc system $\mathbf{L}_{bc}$ and their spin states ($\mathbf{S}_b$, $\mathbf{S}_c$). The spin states ($\mathbf{S}_a$, $\mathbf{S}_b$, $\mathbf{S}_c$) can be well represented by the relativistic Rarita-Schwinger spin wave functions for particles of arbitrary spin [27–30]. The spin-$\frac{1}{2}$ wave function is the standard Dirac spinor $u(p, S)$ or $v(p, S)$ and the spin-1 wave function is the standard spin-1 polarization four-vector $\varepsilon^\mu(p, S)$ for a particle with momentum p and spin projection S:

$$\sum_{S=0,\pm1} \varepsilon_\mu(p, S)\varepsilon_\nu^*(p, S) = -g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2} \equiv -\tilde{g}_{\mu\nu}(p), \quad (3)$$

with $p^\mu\varepsilon_\mu(p, S) = 0$, which states that the spin-1 wave function is orthogonal to its own momentum. Here the Minkowsky metric tensor has the form

$$g_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1). \quad (4)$$

Spin wave functions for particles of higher spins are constructed from these two basic spin wave functions with C-G coefficients $(J_1, J_{1z}; J_2, J_{2z}|J, J_z)$ as

$$\varepsilon_{\mu_1\mu_2\cdots\mu_n}(p, n, S) = \sum_{S_{n-1},S_n} (n-1, S_{n-1}; 1, S_n|n, S)\varepsilon_{\mu_1\mu_2\cdots\mu_{n-1}}(p, n-1, S_{n-1})\varepsilon_{\mu_n}(p, S_n) \quad (5)$$

for a particle with integer spin $n \geq 2$, and

$$u_{\mu_1\mu_2\cdots\mu_n}\left(p, n+\frac{1}{2}, S\right) = \sum_{S_n,S_{n+1}} \left(n, S_n; \frac{1}{2}, S_{n+1}\Big|n+\frac{1}{2}, S\right)\varepsilon_{\mu_1\mu_2\cdots\mu_n}(p, n, S_n)u(p, S_{n+1}) \quad (6)$$

for a particle with half integer spin $n + \frac{1}{2}$ of $n \geq 1$. For an antiparticle with half integer spin $n + \frac{1}{2}$ of $n \geq 1$, one has

$$v_{\mu_1\mu_2\cdots\mu_n}(p, n+\frac{1}{2}, S) = \sum_{S_n, S_{n+1}} (n, S_n; \frac{1}{2}, S_{n+1}|n+\frac{1}{2}, S)\varepsilon_{\mu_1\mu_2\cdots\mu_n}(p, n, S_n)v(p, S_{n+1}). \tag{7}$$

For a process $a \to b + c$, if there exists a relative orbital angular momentum $\mathbf{L}_{bc}$ between the particle B and c, then the orbital angular momentum $\mathbf{L}_{bc}$ state can be represented by covariant tensor wave functions $\tilde{t}^{(L)}_{\mu_1\cdots\mu_L}$, which are the same as for the meson decay [20,29,31]

$$\tilde{t}^{(0)} = 1, \tag{8}$$

$$\tilde{t}^{(1)}_{\mu} = \tilde{g}_{\mu\nu}(p_a)r^{\nu} \equiv \tilde{r}_{\mu}, \tag{9}$$

$$\tilde{t}^{(2)}_{\mu\nu} = \tilde{r}_{\mu}\tilde{r}_{\nu} - \frac{1}{3}(\tilde{r}\cdot\tilde{r})\tilde{g}_{\mu\nu}, \tag{10}$$

$$\tilde{t}^{(3)}_{\mu\nu\lambda} = \tilde{r}_{\mu}\tilde{r}_{\nu}\tilde{r}_{\lambda} - \frac{1}{5}(\tilde{r}\cdot\tilde{r})(\tilde{g}_{\mu\nu}\tilde{r}_{\lambda} + \tilde{g}_{\nu\lambda}\tilde{r}_{\mu} + \tilde{g}_{\lambda\mu}\tilde{r}_{\nu}), \tag{11}$$

$$\begin{aligned}\tilde{t}^{(4)}_{\mu\nu\lambda\sigma} = {}& \tilde{r}_{\mu}\tilde{r}_{\nu}\tilde{r}_{\lambda}\tilde{r}_{\sigma} - \frac{1}{7}(\tilde{r}\cdot\tilde{r})(\tilde{g}_{\mu\nu}\tilde{r}_{\lambda}\tilde{r}_{\sigma} + \tilde{g}_{\nu\lambda}\tilde{r}_{\mu}\tilde{r}_{\sigma} + \tilde{g}_{\lambda\mu}\tilde{r}_{\nu}\tilde{r}_{\sigma} + \tilde{g}_{\mu\sigma}\tilde{r}_{\nu}\tilde{r}_{\lambda} + \tilde{g}_{\nu\sigma}\tilde{r}_{\lambda}\tilde{r}_{\mu} + g_{\lambda\sigma}\tilde{r}_{\mu}\tilde{r}_{\nu}) \\ &+ \frac{1}{35}(\tilde{r}\cdot\tilde{r})^2(\tilde{g}_{\mu\nu}\tilde{g}_{\lambda\sigma} + \tilde{g}_{\nu\lambda}\tilde{g}_{\mu\sigma} + \tilde{g}_{\lambda\mu}\tilde{g}_{\nu\sigma}),\end{aligned} \tag{12}$$

$$\begin{aligned}\tilde{t}^{(5)}_{\mu\nu\lambda\sigma\delta} = {}& \tilde{r}_{\mu}\tilde{r}_{\nu}\tilde{r}_{\lambda}\tilde{r}_{\sigma}\tilde{r}_{\delta} - \frac{1}{9}(\tilde{r}\cdot\tilde{r})(\tilde{g}_{\mu\nu}\tilde{r}_{\lambda}\tilde{r}_{\sigma}\tilde{r}_{\delta} + \tilde{g}_{\nu\lambda}\tilde{r}_{\mu}\tilde{r}_{\sigma}\tilde{r}_{\delta} + \tilde{g}_{\lambda\mu}\tilde{r}_{\nu}\tilde{r}_{\sigma}\tilde{r}_{\delta} + \tilde{g}_{\mu\sigma}\tilde{r}_{\nu}\tilde{r}_{\lambda}\tilde{r}_{\delta} + \tilde{g}_{\nu\sigma}\tilde{r}_{\lambda}\tilde{r}_{\mu}\tilde{r}_{\delta} + g_{\lambda\sigma}\tilde{r}_{\mu}\tilde{r}_{\nu}\tilde{r}_{\delta} \\ &+ \tilde{g}_{\delta\mu}\tilde{r}_{\lambda}\tilde{r}_{\mu}\tilde{r}_{\sigma} + \tilde{g}_{\delta\nu}\tilde{r}_{\lambda}\tilde{r}_{\mu}\tilde{r}_{\sigma} + \tilde{g}_{\delta\sigma}\tilde{r}_{\lambda}\tilde{r}_{\mu}\tilde{r}_{\nu} + \tilde{g}_{\delta\lambda}\tilde{r}_{\nu}\tilde{r}_{\mu}\tilde{r}_{\sigma}) + \frac{1}{63}(\tilde{r}\cdot\tilde{r})^2(\tilde{g}_{\mu\nu}\tilde{g}_{\lambda\sigma}\tilde{r}_{\delta} + \tilde{g}_{\nu\lambda}\tilde{g}_{\mu\sigma}\tilde{r}_{\delta} + \tilde{g}_{\lambda\mu}\tilde{g}_{\nu\sigma}\tilde{r}_{\delta} \\ &+ \tilde{g}_{\mu\nu}\tilde{g}_{\lambda\delta}\tilde{r}_{\sigma} + \tilde{g}_{\nu\lambda}\tilde{g}_{\mu\delta}\tilde{r}_{\sigma} + \tilde{g}_{\lambda\mu}\tilde{g}_{\nu\delta}\tilde{r}_{\sigma} + \tilde{g}_{\mu\nu}\tilde{g}_{\delta\sigma}\tilde{r}_{\lambda} + \tilde{g}_{\nu\delta}\tilde{g}_{\mu\sigma}\tilde{r}_{\lambda} + \tilde{g}_{\delta\mu}\tilde{g}_{\nu\sigma}\tilde{r}_{\lambda} + \tilde{g}_{\lambda\nu}\tilde{g}_{\delta\sigma}\tilde{r}_{\mu} + \tilde{g}_{\nu\delta}\tilde{g}_{\lambda\sigma}\tilde{r}_{\mu} \\ &+ \tilde{g}_{\delta\lambda}\tilde{g}_{\nu\sigma}\tilde{r}_{\mu} + \tilde{g}_{\lambda\mu}\tilde{g}_{\delta\sigma}\tilde{r}_{\nu} + \tilde{g}_{\mu\delta}\tilde{g}_{\lambda\sigma}\tilde{r}_{\nu} + \tilde{g}_{\delta\lambda}\tilde{g}_{\mu\sigma}\tilde{r}_{\nu}),\end{aligned} \tag{13}$$

$$\cdots$$

$$\begin{aligned}\tilde{t}^{(L)}_{\mu_{i_1}\mu_{i_2}\cdots\mu_{i_L}} = {}& \tilde{r}_{\mu_{i_1}}\tilde{r}_{\mu_{i_1}}\cdots\tilde{r}_{\mu_{i_L}} + \sum_{l=1}^{[L/2]}\sum_{i_1<i_2<\cdots<i_{2l}=1}^{L}\frac{(-\tilde{r}\cdot\tilde{r})^l}{(2L-1)(2L-3)\cdots(2L-2l+1)}\frac{1}{2^l l!} \\ &\times(\tilde{g}_{\mu_{i_1}\mu_{i_2}}\tilde{g}_{\mu_{i_3}\mu_{i_4}}\cdots\tilde{g}_{\mu_{i_{2l-1}}\mu_{i_{2l}}} + \mu_{i_1}, \mu_{i_2}, \cdots\mu_{i_{2l}} \quad \text{permutation}, (2l)!\text{term}) \\ &(\tilde{r}_{\mu_1}\tilde{r}_{\mu_2}\cdots\tilde{r}_{\mu_{i_1-1}}\tilde{r}_{\mu_{i_1+1}}\cdots\tilde{r}_{\mu_{i_2-1}}\tilde{r}_{\mu_{i_2+1}}\cdots\tilde{r}_{\mu_{i_{2l}-1}}\tilde{r}_{\mu_{i_{2l}+1}}\cdots\tilde{r}_{\mu_L}),\end{aligned} \tag{14}$$

where $r = p_b - p_c$ is the relative four momentum of the two decay products in the parent particle rest frame; $(\tilde{r}\cdot\tilde{r}) = -\mathbf{r}^2$; $[L/2] = n$ when $L = 2n$ and $L = 2n+1$; and

$$p_a^{\mu}\tilde{t}^{(1)}_{\mu} = p_a^{\mu}\tilde{t}^{(2)}_{\mu\nu} = p_a^{\mu}t^{(3)}_{\mu\nu\lambda} = 0, \quad \tilde{g}^{\mu\nu}(p_a) = g^{\mu\nu} - \frac{p_a^{\mu}p_a^{\nu}}{p_a^2}.$$

In the L-S scheme, we need to use the conservation relation of total angular momentum

$$\mathbf{S}_a = \mathbf{S}_b + \mathbf{S}_c + \mathbf{L}_{bc} \quad \text{or} \quad -\mathbf{S}_a + \mathbf{S}_b + \mathbf{S}_c + \mathbf{L}_{bc} = 0. \tag{15}$$

Besides, the parity should be conserved, which means

$$\eta_a = \eta_b\eta_c(-1)^L, \tag{16}$$

where η_a, η_b and η_c are the intrinsic parities of particles a, b and c, respectively. From this relation, one knows whether L should be even or odd. Then from Eq. (15) one can figure out how many different L-S combinations there are, which determines the number of independent couplings.

Comparing with the pure meson case [29], here we need to introduce the concept of relativistic total spin of two fermions. For the case of a as a vector meson, b as excited baryons (B^*) with spin $n+\frac{1}{2}$ and c as antibaryons ($\bar{B}$) with spin-$\frac{1}{2}$, the total spin of bc ($\mathbf{S}_{bc}$) can be either n or $n+1$. The two $\mathbf{S}_{bc}$ states can be represented as

$$\psi^{(n)}_{\mu_1\cdots\mu_n} = \bar{u}_{\mu_1\cdots\mu_n}(p_b, S_b)\gamma_5 v(p_c, S_c), \quad (17)$$

$$\Psi^{(n+1)}_{\mu_1\cdots\mu_{n+1}} = \bar{u}_{\mu_1\cdots\mu_n}(p_b, S_b)\left(\gamma_{\mu_{n+1}} - \frac{r_{\mu_{n+1}}}{m_a + m_b + m_c}\right) \times v(p_c, S_c) + (\mu_1 \leftrightarrow \mu_{n+1}) + \cdots + (\mu_n \leftrightarrow \mu_{n+1}) \quad (18)$$

for $\mathbf{S}_{bc}$ of n and $n+1$, respectively. As a special case of $n = 0$, we have

$$\psi^{(0)} = \bar{u}(p_b, S_b)\gamma_5 v(p_c, S_c), \quad (19)$$

$$\Psi^{(1)}_\mu = \bar{u}(p_b, S_b)(\gamma_\mu - \frac{r_\mu}{m_a + m_b + m_c})v(p_c, S_c). \quad (20)$$

Here the r_μ term is necessary to cancel out the $\mathbf{p}$-dependent component in the $\bar{u}\gamma_\mu v$ expression.

For the case of a as a vector meson, b as excited antibaryons ($\bar{B}^*$) with spin $n + \frac{1}{2}$ and c as baryons (B) with spin-$\frac{1}{2}$, the above equations can be written as

$$\psi^{C(n)}_{\mu_1\cdots\mu_n} = -\bar{u}(p_c, S_c)\gamma_5 v_{\mu_1\cdots\mu_n}(p_b, S_b), \quad (21)$$

$$\Psi^{C(n+1)}_{\mu_1\cdots\mu_{n+1}} = \bar{u}(p_c, S_c)(\gamma_{\mu_{n+1}} - \frac{r_{\mu_{n+1}}}{m_a + m_b + m_c}) \times v_{\mu_1\cdots\mu_n}(p_b, S_b) + (\mu_1 \leftrightarrow \mu_{n+1}) + \cdots + (\mu_n \leftrightarrow \mu_{n+1}) \quad (22)$$

for $\mathbf{S}_{bc}$ of n and $n+1$, respectively. As a special case of $n = 0$, we have

$$\psi^{C(0)} = -\bar{u}(p_c, S_c)\gamma_5 v(p_b, S_b), \quad (23)$$

$$\Psi^{C(1)}_\mu = \bar{u}(p_c, S_c)(\gamma_\mu - \frac{r_\mu}{m_a + m_b + m_c})v(p_b, S_b). \quad (24)$$

For the case of a as excited baryons (B^*) with spin $n + \frac{1}{2}$, b as baryons (B) and c as a meson, one needs to couple $-\mathbf{S}_a$ and $\mathbf{S}_b$ first to get $\mathbf{S}_{ab} \equiv -\mathbf{S}_a + \mathbf{S}_b$ states, which are

$$\phi^{(n)}_{\mu_1\cdots\mu_n} = \bar{u}(p_b, S_b)u_{\mu_1\cdots\mu_n}(p_a, S_a), \quad (25)$$

$$\Phi^{(n+1)}_{\mu_1\cdots\mu_{n+1}} = \bar{u}(p_b, S_b)\gamma_5\tilde{\gamma}_{\mu_{n+1}}u_{\mu_1\cdots\mu_n}(p_a, S_a) + (\mu_1 \leftrightarrow \mu_{n+1}) + \cdots + (\mu_n \leftrightarrow \mu_{n+1}) \quad (26)$$

for $\mathbf{S}_{ab}$ of n and $n+1$, respectively, and

$$\phi^{(0)} = \bar{u}(p_b, S_b)u(p_a, S_a), \quad (27)$$

$$\Phi^{(1)}_\mu = \bar{u}(p_b, S_b)\gamma_5\tilde{\gamma}_\mu u(p_a, S_a) \quad (28)$$

with $\tilde{\gamma}_\mu = \tilde{g}_{\mu\nu}(p_a)\gamma^\nu$.

For the case of a as excited antibaryons ($\bar{B}^*$) with spin $n + \frac{1}{2}$, b as an antibaryon ($\bar{B}$) and c as a meson, as before one needs to couple $-\mathbf{S}_a$ and $\mathbf{S}_b$ first to get $\mathbf{S}_{ab} \equiv -\mathbf{S}_a + \mathbf{S}_b$ states, which are

$$\phi^{C(n)}_{\mu_1\cdots\mu_n} = \bar{v}_{\mu_1\cdots\mu_n}(p_a, S_a)v(p_b, S_b), \quad (29)$$

$$\Phi^{C(n+1)}_{\mu_1\cdots\mu_{n+1}} = \bar{v}_{\mu_1\cdots\mu_n}(p_a, S_a)\gamma_5\tilde{\gamma}_{\mu_{n+1}}v(p_b, S_b) + (\mu_1 \leftrightarrow \mu_{n+1}) + \cdots + (\mu_n \leftrightarrow \mu_{n+1}) \quad (30)$$

for $\mathbf{S}_{ab}$ of n and $n+1$, respectively. As a special case of $n = 0$, we have

$$\phi^{C(0)} = \bar{v}(p_a, S_a)v(p_b, S_b), \quad (31)$$

$$\Phi^{C(1)}_\mu = \bar{v}(p_a, S_a)\gamma_5\tilde{\gamma}_\mu v(p_b, S_b). \quad (32)$$

IV. PARTIAL WAVE AMPLITUDES

We consider the following process:

$$J/\psi \to B^*\bar{B} + \bar{B}^*B \to \gamma B\bar{B}. \quad (33)$$

The possible J^P for B^* is $\frac{1}{2}^\pm, \frac{3}{2}^\pm, \frac{5}{2}^\pm, \frac{7}{2}^\pm$. We denote the four momenta of J/ψ, $B^*(\bar{B}^*)$ and γ by p_μ, $p_{B^*\mu}(p_{\bar{B}^*\mu})$ and q_μ. The orbital spin tensor describing the first and second steps will be denoted by $\tilde{T}^{(L)}_{\mu_1\cdots\mu_L}$ and $\tilde{t}^{(l)}_{\mu_1\cdots\mu_l}$. For the process (33) the general form of the decay amplitude is

$$M = \varepsilon_\mu(p, S_{J/\psi})e^*_\nu(q, S_\gamma)M^{\mu\nu} = \varepsilon_\mu(p, S_{J/\psi})e^*_\nu(q, S_\gamma)\sum_{i,j}U^{\mu\nu}_{i,j}, \quad (34)$$

where $\varepsilon_\mu(p, S_{J/\psi})$ is the polarization four-vector of the J/ψ; $e_\nu(q, S_\gamma)$ is the polarization four-vector of the photon; $S_{J/\psi}$ and S_γ are the spin third components of J/ψ and photon, respectively; $U^{\mu\nu}_{i,j}$ is the ith B^* and $\bar{B}^*$; jth partial wave amplitude with complex coupling constants to be determined by the experiment. The spin-1 polarization four-vector $\varepsilon_\mu(p, S_{J/\psi})$ for J/ψ with four momentum p_μ satisfies the relation in Eq. (3). For J/ψ production from e^+e^- annihilation, the electrons are highly relativistic, with the result that $S_{J/\psi} = \pm 1$ for the J/ψ spin projection takes the beam direction as the z-axis. This limits $S_{J/\psi}$ to $+1$ and -1. Then one has the following relation:

$$\sum_{S_{J/\psi}=\pm 1}\varepsilon_\mu(p, S_{J/\psi})\varepsilon^*_{\mu'}(p, S_{J/\psi}) = \delta_{\mu\mu'}(\delta_{\mu 1} + \delta_{\mu 2}). \quad (35)$$

For the photon polarization four-vector, there is the usual Lorentz orthogonality conditions. Namely, the polarization four-vector $e_\nu(q, S_\gamma)$ of the photon with momenta q satisfies

$$q^\nu e_\nu(q, S_\gamma) = 0, \quad (36)$$

which states that spin-1 wave function is orthogonal to its own momentum. The above relation is the same as for a massive vector meson. However, for the photon, there is the additional gauge invariance condition

$$\sum_{S_\gamma} e^*_\mu(q, S_\gamma) e_\nu(q, S_\gamma) = -g_{\mu\nu} + \frac{q_\mu K_\nu + K_\mu q_\nu}{q \cdot K} - \frac{K \cdot K}{(q \cdot K)^2} q_\mu q_\nu \equiv -g^{(\perp\perp)}_{\mu\nu} \quad (37)$$

with $K = p - q$ and $K^\nu e_\nu = 0$.

Although $B^*(\frac{1}{2}^\pm, \frac{3}{2}^\pm, \frac{5}{2}^\pm, \frac{7}{2}^\pm)B\omega$ couplings have the same structure as the $B^*(\frac{1}{2}^\pm, \frac{3}{2}^\pm, \frac{5}{2}^\pm, \frac{7}{2}^\pm)B\gamma$ couplings, the gauge invariance requirement for the $B^*B\gamma$ couplings reduces the number of independent amplitudes. For example, the partial wave amplitudes for the process $B^*(\frac{3}{2}^\pm) \to B(\frac{1}{2}^+)\gamma$ can be written as

$$M = (g_1 M_1^\nu + g_2 M_2^\nu + g_3 M_3^\nu) e^*_\nu(q, S_\gamma), \quad (38)$$

where

$$M_1^\nu = i\phi^{(1)}_\mu \epsilon^{\mu\nu\lambda\sigma} \tilde{t}^{(1)}_\lambda \hat{p}_{B^*\sigma}, \quad M_2^\nu = \Phi^{(2)\nu\mu} \tilde{t}^{(1)}_\mu, \quad M_3^\nu = \Phi^{(2)}_{\mu\lambda} \tilde{t}^{(3)\mu\lambda\nu}, \quad (39)$$

here $\hat{p}_{B^*\sigma} = p_{B^*\sigma}/m_{B^*}$. Because of the gauge invariance requirement

$$(g_1 M_1^\nu + g_2 M_2^\nu + g_3 M_3^\nu) q_\nu = 0, \quad (40)$$

we get the relation

$$g_2 = -\frac{3}{5}(\tilde{r} \cdot \tilde{r}) g_3 = \frac{3}{5} \frac{(m^2_{B^*} - m^2_B)^2}{m^2_{B^*}} g_3, \quad (41)$$

which means that there are two independent partial wave amplitudes. References [32,33] also provided basically equivalent partial wave amplitude formulas for the vertex $B^*B\gamma$ in the spin-orbital approach. In order to be able to compare our results with conventional helicity amplitudes for the radiative decays of baryon resonances [23], we also give the relation between our coupling constants and helicity amplitudes in the Appendix.

To compute decay width, we need an expression for $|M|^2$. Note that the square modulus of the decay amplitude, which gives the decay probability of a certain configuration, should be independent of any particular frame, that is, a Lorentz scalar. Thus by using Eqs. (35) and (37), we have

$$d\Gamma = \frac{(2\pi)^4}{2M_{J/\psi}} |M|^2 d\Phi_3(p; q_\gamma, p_B, p_{\bar{B}}), \quad (42)$$

where $M_{J/\psi}$ is the mass of the J/ψ, and the general form of the matrix element square is

$$|M|^2 = \frac{1}{2} \sum_{S_{J/\psi}=\pm 1} \sum_{S_\gamma=\pm 1} \sum_{S_B} \sum_{S_{\bar{B}}} |\varepsilon_\mu(p, S_{J/\psi}) e^*_\nu(q, S_\gamma) M^{\mu\nu}|^2 = \frac{1}{2} \sum_{\mu=\pm 1} \sum_{S_B} \sum_{S_{\bar{B}}} M^{\mu\nu}(-g^{(\perp\perp)}_{\nu\nu'}) M^{*\mu\nu'} = \frac{1}{2} \sum_{i,i'} \sum_{j,j'} \sum_{\mu=1}^{2} \sum_{S_B} \sum_{S_{\bar{B}}} U^{\mu\nu}_{i,j}(-g^{(\perp\perp)}_{\nu\nu'}) U^{*\mu\nu'}_{i',j'}, \quad (43)$$

and the standard Lorentz invariant 3-body phase space element $d\Phi_3$ is given by

$$d\Phi_3(p; q, p_B, p_{\bar{B}}) = \delta^4(p - q - p_B - p_{\bar{B}}) \frac{d^3\mathbf{q}}{(2\pi)^3 2E_\gamma} \times \frac{2m_B d^3\mathbf{p}_B}{(2\pi)^3 2E_B} \frac{2m_{\bar{B}} d^3\mathbf{p}_{\bar{B}}}{(2\pi)^3 2E_{\bar{B}}}. \quad (44)$$

From (33) we see that B^* and $\bar{B}^*$ are the intermediate resonances decaying into $B\gamma$ and $\bar{B}^*\gamma$, respectively, therefore we need to introduce into the amplitude the following propagators denoted by G_{B^*} and $G_{\bar{B}^*}$ [34,35]:

$$G_{B^*}\left(\frac{1}{2}\right) = f^{B^*}_{B\gamma} \sum_{S_{B^*}} u(p_{B^*}, S_{B^*}) \bar{u}(p_{B^*}, S_{B^*}) = f^{B^*}_{B\gamma} \frac{(\not{p}_{B^*} + m_{B^*})}{2m_{B^*}},$$
$$G_{\bar{B}^*}\left(\frac{1}{2}\right) = \bar{f}^{\bar{B}^*}_{\bar{B}\gamma} \sum_{S_{\bar{B}^*}} v(p_{\bar{B}^*}, S_{\bar{B}^*}) \bar{v}(p_{\bar{B}^*}, S_{\bar{B}^*}) = \bar{f}^{\bar{B}^*}_{\bar{B}\gamma} \frac{(\not{p}_{\bar{B}^*} - m_{\bar{B}^*})}{2m_{\bar{B}^*}}, \quad (45)$$

$$G^{\mu\nu}_{B^*}\left(\frac{3}{2}\right) = f^{B^*}_{B\gamma} \sum_{S_{B^*}} u^\mu(p_{B^*}, S_{B^*}) \bar{u}^\nu(p_{B^*}, S_{B^*}) = f^{B^*}_{B\gamma} \frac{(\not{p}_{\bar{B}^*} + m_{B^*})}{2m_{B^*}} P^{\mu\nu}_{B^*}\left(\frac{3}{2}\right),$$
$$G^{\mu\nu}_{\bar{B}^*}\left(\frac{3}{2}\right) = \bar{f}^{\bar{B}^*}_{\bar{B}\gamma} \sum_{S_{\bar{B}^*}} v^\mu(p_{\bar{B}^*}, S_{\bar{B}^*}) \bar{v}^\nu(p_{\bar{B}^*}, S_{\bar{B}^*}) = \bar{f}^{\bar{B}^*}_{\bar{B}\gamma} \frac{(\not{p}_{B^*} - m_{\bar{B}^*})}{2m_{\bar{B}^*}} P^{\mu\nu}_{\bar{B}^*}\left(\frac{3}{2}\right), \quad (46)$$

$$G^{\mu\nu\alpha\beta}_{B^*}\left(\frac{5}{2}\right) = f^{B^*}_{B\gamma} \sum_{S_{B^*}} u^{\mu\nu}(p_{B^*}, S_{B^*}) \bar{u}^{\alpha\beta}(p_{B^*}, S_{B^*}) = f^{B^*}_{B\gamma} \frac{(\not{p}_{B^*} + m_{B^*})}{2m_{B^*}} P^{\mu\nu}_{B^*}\left(\frac{5}{2}\right),$$
$$G^{\mu\nu\alpha\beta}_{\bar{B}^*}\left(\frac{5}{2}\right) = \bar{f}^{\bar{B}^*}_{\bar{B}\gamma} \sum_{S_{\bar{B}^*}} v^{\mu\nu}(p_{\bar{B}^*}, S_{\bar{B}^*}) \bar{v}^{\alpha\beta}(p_{\bar{B}^*}, S_{\bar{B}^*}) = \bar{f}^{\bar{B}^*}_{\bar{B}\gamma} \frac{(\not{p}_{\bar{B}^*} - m_{\bar{B}^*})}{2m_{\bar{B}^*}} P^{\mu\nu}_{\bar{B}^*}\left(\frac{5}{2}\right), \quad (47)$$

$$G^{\mu\nu\alpha\beta\lambda\sigma}_{B^*}\left(\frac{7}{2}\right) = f^{B^*}_{\bar{B}\gamma}\sum_{S_{B^*}} u^{\mu\nu\alpha}(p_{B^*}, S_{B^*})\bar{u}^{\beta\lambda\sigma}(p_{B^*}, S_{B^*})$$
$$= f^{B^*}_{\bar{B}\gamma}\frac{(\not{p}_{B^*} + m_{B^*})}{2m_{B^*}} P^{\mu\nu\alpha\beta\lambda\sigma}_{B^*}\left(\frac{7}{2}\right),$$
$$G^{\mu\nu\alpha\beta\lambda\sigma}_{\bar{B}^*}\left(\frac{7}{2}\right) = \bar{f}^{\bar{B}^*}_{\bar{B}\gamma}\sum_{S_{\bar{B}^*}} v^{\mu\nu\alpha}(p_{\bar{B}^*}, S_{\bar{B}^*})\bar{v}^{\beta\lambda\sigma}(p_{\bar{B}^*}, S_{\bar{B}^*}) \quad (48)$$
$$= \bar{f}^{\bar{B}^*}_{\bar{B}\gamma}\frac{(\not{p}_{\bar{B}^*} - m_{\bar{B}^*})}{2m_{\bar{B}^*}} P^{\mu\nu\alpha\beta\lambda\sigma}_{\bar{B}^*}\left(\frac{7}{2}\right),$$

where

$$f^{B^*}_{\bar{B}\gamma} = \frac{2m_{B^*}}{p^2_{B^*} - m^2_{B^*} + im_{B^*}\Gamma_{B^*}},$$
$$\bar{f}^{\bar{B}^*}_{\bar{B}\gamma} = \frac{2m_{\bar{B}^*}}{p^2_{\bar{B}^*} - m^2_{\bar{B}^*} + im_{\bar{B}^*}\Gamma_{\bar{B}^*}}, \quad (49)$$

here m_{B^*}, $m_{\bar{B}^*}$ and Γ_{B^*}, $\Gamma_{\bar{B}^*}$ are the resonances masses and widths;

$$P^{\mu\nu}_{B^*}\left(\frac{3}{2}\right) = -g^{\mu\nu} + \frac{1}{3}\gamma^\mu\gamma^\nu + \frac{2}{3}\frac{p^\mu_{B^*}p^\nu_{B^*}}{m^2_{B^*}} + \frac{1}{3m_{B^*}}(\gamma^\mu p^\nu_{B^*} - \gamma^\nu p^\mu_{B^*}),$$
$$P^{\mu\nu}_{\bar{B}^*}\left(\frac{3}{2}\right) = -g^{\mu\nu} + \frac{1}{3}\gamma^\mu\gamma^\nu + \frac{2}{3}\frac{p^\mu_{\bar{B}^*}p^\nu_{\bar{B}^*}}{m^2_{\bar{B}^*}} - \frac{1}{3m_{\bar{B}^*}}(\gamma^\mu p^\nu_{\bar{B}^*} - \gamma^\nu p^\mu_{\bar{B}^*}),$$
$$P^{\mu\nu\alpha\beta}_{B^*/\bar{B}^*}\left(\frac{5}{2}\right) = \frac{1}{2}(\tilde{g}^{\mu\nu}\tilde{g}^{\alpha\beta} + \tilde{g}^{\mu\beta}\tilde{g}^{\alpha\nu}) - \frac{1}{5}\tilde{g}^{\mu\alpha}\tilde{g}^{\nu\beta} - \frac{1}{10}(\tilde{\gamma}^\mu\tilde{\gamma}^\nu\tilde{g}^{\alpha\beta} + \tilde{\gamma}^\mu\tilde{\gamma}^\beta\tilde{g}^{\alpha\nu} + \tilde{\gamma}^\alpha\tilde{\nu}\tilde{g}^{\mu\beta} + \tilde{\gamma}^\alpha\tilde{\gamma}^\beta\tilde{g}^{\mu\nu}),$$
$$P^{\mu\nu\alpha\beta\lambda\sigma}_{B^*/\bar{B}^*}\left(\frac{7}{2}\right) = \frac{4}{9}\gamma_\tau\gamma_\rho P^{(4)\tau\mu\nu\alpha\rho\beta\lambda\sigma}, \quad (50)$$

and where

$$P^{(4)\tau\mu\nu\alpha\rho\beta\lambda\sigma} = \frac{1}{24}(\tilde{g}^{\tau\rho}\tilde{g}^{\mu\beta}\tilde{g}^{\nu\lambda}\tilde{g}^{\alpha\sigma} + (\rho, \beta, \lambda, \sigma \quad \text{permutation}, \quad 24 \quad \text{terms}))$$
$$- \frac{1}{84}(\tilde{g}^{\tau\mu}\tilde{g}^{\rho\beta}\tilde{g}^{\nu\lambda}\tilde{g}^{\alpha\sigma} + (\tau, \mu, \nu, \alpha \quad \text{permutation}, \quad \rho, \beta, \lambda, \sigma \quad \text{permutation}, \quad 72 \quad \text{terms})$$
$$+ \frac{1}{105}(\tilde{g}^{\tau\mu}\tilde{g}^{\nu\alpha} + \tilde{g}^{\tau\nu}\tilde{g}^{\mu\alpha} + \tilde{g}^{\tau\alpha}\tilde{g}^{\mu\nu})(\tilde{g}^{\rho\beta}\tilde{g}^{\lambda\sigma} + \tilde{g}^{\rho\lambda}\tilde{g}^{\beta\sigma} + \tilde{g}^{\rho\sigma}\tilde{g}^{\beta\lambda}). \quad (51)$$

For the different partial wave amplitudes, we use the following notation and label our amplitudes:

$$(S_{B^*\bar{B}/\bar{B}^*B}, L_{B^*\bar{B}/\bar{B}^*B}, S_{B^*B/\bar{B}^*\bar{B}}), \quad (52)$$

where $\mathbf{S}_{B^*\bar{B}/\bar{B}^*B} = \mathbf{S}_{B^*} + \mathbf{S}_{\bar{B}}$ or $\mathbf{S}_{\bar{B}^*} + \mathbf{S}_B$; $\mathbf{L}_{B^*\bar{B}/\bar{B}^*B}$ is the relative orbital angular momentum between B^* and $\bar{B}$ or $\bar{B}^*$ and B; $\mathbf{S}_{B^*B/\bar{B}^*\bar{B}} = -\mathbf{S}_{B^*} + \mathbf{S}_B$ or $-\mathbf{S}_{\bar{B}^*} + \mathbf{S}_{\bar{B}}$. By considering the parity and angular momentum conservations in the following we provide all relevant covariant amplitudes for the process (33). In these amplitudes, 1, 2 and 3 denote the three final state particles B, $\bar{B}$ and γ.

For $J/\psi(1^-) \rightarrow B^*(\frac{1}{2}^+)\bar{B}(\frac{1}{2}^-) + \bar{B}^*(\frac{1}{2}^-)B(\frac{1}{2}^+) \rightarrow \gamma B(\frac{1}{2}^+)\bar{B}(\frac{1}{2}^-)$, we find two independent covariant amplitudes for a vector meson $J/\psi(1^-)$ decaying into the $B^*(\frac{1}{2}^+)\bar{B}(\frac{1}{2}^-)$ and $\bar{B}^*(\frac{1}{2}^-)B(\frac{1}{2}^+)$ states, and one independent covariant amplitude for excited baryon resonances $B^*(\frac{1}{2}^+)$ and $\bar{B}^*(\frac{1}{2}^-)$ decaying into $\gamma B(\frac{1}{2}^+)$ and $\gamma\bar{B}(\frac{1}{2}^-)$. All in all we get the following two covariant amplitudes with two independent coupling constants $g^{i,a}$ and $g^{i,b}$ which are determined by the experiment:
(1, 0, 1)

$$U^{\mu\nu}_{i,1} = g^{i,a}\left(\sum_{S_{B^*}} i\Phi^{(1)}_\beta\Psi^{(1)\mu}\tilde{t}^{(1)}_{(13)\lambda}\epsilon^{\beta\nu\lambda\sigma}\hat{p}_{B^*\sigma}f^{B^*}_{\bar{B}\gamma} - \sum_{S_{\bar{B}^*}} i\Psi^{C(1)\mu}\Phi^{C(1)}_\beta\tilde{t}^{(1)}_{(23)\lambda}\epsilon^{\beta\nu\lambda\sigma}\hat{p}_{\bar{B}^*\sigma}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma}\right), \quad (53)$$

(1, 2, 1)

$$U^{\mu\nu}_{i,2} = g^{i,b}\left(\sum_{S_{B^*}} i\Phi^{(1)}_\beta\Psi^{(1)}_\alpha\tilde{T}^{(2)\alpha\mu}_{(B^*2)}\tilde{t}^{(1)}_{(13)\lambda}\epsilon^{\beta\nu\lambda\sigma}\hat{p}_{B^*\sigma}f^{B^*}_{\bar{B}\gamma} - \sum_{S_{\bar{B}^*}} i\Psi^{C(1)}_\alpha\Phi^{C(1)}_\beta\tilde{T}^{(2)\alpha\mu}_{(\bar{B}^*1)}\tilde{t}^{(1)}_{(23)\lambda}\epsilon^{\beta\nu\lambda\sigma}\hat{p}_{\bar{B}^*\sigma}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma}\right). \quad (54)$$

(Note that these are according to Eq. (52), where we labeled our amplitudes as (1, 0, 1) and (1, 2, 1).)

For $J/\psi(1^-) \rightarrow B^*(\frac{1}{2}^-)\bar{B}(\frac{1}{2}^-) + \bar{B}^*(\frac{1}{2}^+)B(\frac{1}{2}^+) \rightarrow \gamma B(\frac{1}{2}^+)\bar{B}(\frac{1}{2}^-)$ we get the following two covariant amplitudes with two independent coupling constants $g^{i,a}$ and $g^{i,\bar{b}}$ which are determined by the experiment:

(0, 1, 1)

$$U_{i,1}^{\mu\nu} = g^{i,a}\bigg(-\frac{2}{3}C_{B^*B\gamma}\sum_{S_{B^*}}\Phi^{(1)\nu}\psi^{(0)}\tilde{T}^{(1)\mu}_{(B^*2)}f^{B^*}_{B\gamma} - \frac{2}{3}C_{\bar{B}^*\bar{B}\gamma}\sum_{S_{\bar{B}^*}}\psi^{C(0)}\Phi^{C(1)\nu}\tilde{T}^{(1)\mu}_{(\bar{B}^*1)}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma} + \sum_{S_{B^*}}\Phi^{(1)}_{\beta}\psi^{(0)}\tilde{T}^{(1)\mu}_{(B^*2)}\tilde{t}^{(2)\beta\nu}_{(13)}f^{B^*}_{B\gamma} + \sum_{S_{\bar{B}^*}}\psi^{C(0)}\tilde{\Phi}^{C(1)}_{\beta}T^{(1)\mu}_{(\bar{B}^*1)}\tilde{t}^{(2)\beta\nu}_{(23)}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma}\bigg), \tag{55}$$

(1, 1, 1)

$$U_{i,2}^{\mu\nu} = g^{i,b}\bigg(-\frac{2}{3}C_{B^*B\gamma}\sum_{S_{B^*}}i\Phi^{(1)\nu}\Psi^{(1)}_{\alpha}\tilde{T}^{(1)}_{(B^*2)\delta}\epsilon^{\alpha\mu\delta\tau}\hat{p}_{\tau}f^{B^*}_{B\gamma} + \frac{2}{3}C_{\bar{B}^*\bar{B}\gamma}\sum_{S_{\bar{B}^*}}i\Psi^{C(1)}_{\alpha}\Phi^{C(1)\nu}\tilde{T}^{(1)}_{(\bar{B}^*1)\delta}\epsilon^{\alpha\mu\delta\tau}\hat{p}_{\tau}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma}\bigg) + \sum_{S_{B^*}}i\Phi^{(1)}_{\beta}\Psi^{(1)}_{\alpha}\tilde{T}^{(1)}_{(B^*2)\delta}\tilde{t}^{(2)\beta\nu}_{(13)}\epsilon^{\alpha\mu\delta\tau}\hat{p}_{\tau}f^{B^*}_{B\gamma} - \sum_{S_{\bar{B}^*}}i\Psi^{C(1)}_{\alpha}\Phi^{C(1)}_{\beta}\tilde{T}^{(1)}_{(\bar{B}^*1)\delta}\tilde{t}^{(2)\beta\nu}_{(23)}\epsilon^{\alpha\mu\delta\tau}\hat{p}_{\tau}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma}. \tag{56}$$

One may note that for $J/\psi \to B^*\bar{B} + \bar{B}^*B \to \gamma B\bar{B}$ with $J^P(B^*(\bar{B}^*)) = \frac{3}{2}^{\pm}, \frac{5}{2}^{\pm}, \frac{7}{2}^{\pm}$, we get the six covariant amplitudes for i-th $B^*(\bar{B}^*)$ with five independent coupling constants $g^{i,a}_{J/\psi}$, $g^{i,b}_{J/\psi}$, $g^{i,c}_{J/\psi}$, $g^{i,a}_{\gamma}$ and $g^{i,b}_{\gamma}$, which are determined by the experiment. Thus for $J/\psi(1^-) \to B^*(\frac{3}{2}^+)\bar{B}(\frac{1}{2}^-) + \bar{B}^*(\frac{3}{2}^-)B(\frac{1}{2}^+) \to \gamma B(\frac{1}{2}^+)\bar{B}(\frac{1}{2}^-)$ we get the following six covariant amplitudes:

(1, 0, 1)

$$U_{i,1}^{\mu\nu} = g^{i,a}_{J/\psi}g^{i,a}_{\gamma}\bigg(\sum_{S_{B^*}}i\phi^{(1)}_{\beta}\psi^{(1)\mu}\tilde{t}^{(1)}_{(13)\lambda}\epsilon^{\beta\nu\lambda\sigma}\hat{p}_{B^*\sigma}f^{B^*}_{B\gamma} - \sum_{S_{\bar{B}^*}}i\psi^{C(1)\mu}\phi^{C(1)}_{\beta}\tilde{t}^{(1)}_{(23)\lambda}\epsilon^{\beta\nu\lambda\sigma}\hat{p}_{\bar{B}^*\sigma}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma}\bigg), \tag{57}$$

(1, 2, 1)

$$U_{i,2}^{\mu\nu} = g^{i,b}_{J/\psi}g^{i,a}_{\gamma}\bigg(\sum_{S_{B^*}}i\phi^{(1)}_{\beta}\psi^{(1)}_{\alpha}\tilde{T}^{(2)\alpha\mu}_{(B^*2)}\tilde{t}^{(1)}_{(13)\lambda}\epsilon^{\beta\nu\lambda\sigma}\hat{p}_{B^*\sigma}f^{B^*}_{B\gamma} - \sum_{S_{\bar{B}^*}}i\psi^{C(1)}_{\alpha}\phi^{C(1)}_{\beta}\tilde{T}^{(2)\alpha\mu}_{(\bar{B}^*1)}\tilde{t}^{(1)}_{(23)\lambda}\epsilon^{\beta\nu\lambda\sigma}\hat{p}_{\bar{B}^*\sigma}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma}\bigg), \tag{58}$$

(2, 2, 1)

$$U_{i,3}^{\mu\nu} = g^{i,c}_{J/\psi}g^{i,a}_{\gamma}\bigg(-\sum_{S_{B^*}}\phi^{(1)}_{\beta}\Psi^{(2)}_{\alpha\rho}\epsilon^{\alpha\mu\delta\tau}\tilde{T}^{(2)\rho}_{(B^*2)\delta}\hat{p}_{\tau}\epsilon^{\beta\nu\lambda\sigma}\tilde{t}^{(1)}_{(13)\lambda}\hat{p}_{B^*\sigma}f^{B^*}_{B\gamma} - \sum_{S_{\bar{B}^*}}\Psi^{C(2)}_{\alpha\rho}\phi^{C(1)}_{\beta}\epsilon^{\alpha\mu\delta\tau}\tilde{T}^{C(2)\rho}_{(\bar{B}^*1)\delta}\hat{p}_{\tau}\epsilon^{\beta\nu\lambda\sigma}\tilde{t}^{C(1)}_{(23)\lambda}\hat{p}_{\bar{B}^*\sigma}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma}\bigg), \tag{59}$$

(1, 0, 2)

$$U_{i,4}^{\mu\nu} = g^{i,a}_{J/\psi}g^{i,b}_{\gamma}\bigg(-\frac{3}{5}C_{B^*B\gamma}\sum_{S_{B^*}}\Phi^{(2)\nu\beta}\psi^{(1)\mu}\tilde{t}^{(1)}_{(13)\beta}f^{B^*}_{B\gamma} - \frac{3}{5}C_{\bar{B}^*\bar{B}\gamma}\sum_{S_{\bar{B}^*}}\psi^{C(1)\mu}\Phi^{C(2)\nu\beta}\tilde{t}^{C(1)}_{(23)\beta}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma} + \sum_{S_{B^*}}\Phi^{(2)}_{\beta\lambda}\psi^{(1)\mu}\tilde{t}^{(3)\beta\lambda\nu}_{(13)}f^{B^*}_{B\gamma} + \sum_{S_{\bar{B}^*}}\psi^{C(1)\mu}\Phi^{C(2)}_{\beta\lambda}\tilde{t}^{(3)\beta\lambda\nu}_{(23)}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma}\bigg), \tag{60}$$

(1, 2, 2)

$$U_{i,5}^{\mu\nu} = g^{i,b}_{J/\psi}g^{i,b}_{\gamma}\bigg(-\frac{3}{5}C_{B^*B\gamma}\sum_{S_{B^*}}\Phi^{(2)\nu\beta}\psi^{(1)}_{\alpha}\tilde{T}^{(2)\alpha\mu}_{(B^*2)}\tilde{t}^{(1)}_{(13)\beta}f^{B^*}_{B\gamma} - \frac{3}{5}C_{\bar{B}^*\bar{B}\gamma}\sum_{S_{\bar{B}^*}}\psi^{C(1)}_{\alpha}\Phi^{C(2)\nu\beta}\tilde{T}^{(2)\alpha\mu}_{(\bar{B}^*1)}\tilde{t}^{(1)}_{(23)\beta}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma}\bigg) + \sum_{S_{B^*}}\Phi^{(2)}_{\beta\lambda}\psi^{(1)}_{\alpha}\tilde{T}^{(2)\alpha\mu}_{(B^*2)}\tilde{t}^{(3)\beta\lambda\nu}_{(13)}f^{B^*}_{B\gamma} + \sum_{S_{\bar{B}^*}}\psi^{C(1)}_{\alpha}\Phi^{C(2)}_{\beta\lambda}\tilde{T}^{(2)\alpha\mu}_{(\bar{B}^*1)}\tilde{t}^{(3)\beta\lambda\nu}_{(23)}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma}\bigg), \tag{61}$$

(2, 2, 2)

$$U_{i,6}^{\mu\nu} = g^{i,c}_{J/\psi}g^{i,b}_{\gamma}\bigg(-\frac{3}{5}C_{B^*B\gamma}\sum_{S_{B^*}}i\Phi^{(2)\nu\beta}\Psi^{(2)}_{\alpha\rho}\tilde{T}^{(2)\rho}_{(B^*2)\delta}\tilde{t}^{(1)}_{(13)\beta}\epsilon^{\alpha\mu\delta\tau}\hat{p}_{\tau}f^{B^*}_{B\gamma} + \frac{3}{5}C_{\bar{B}^*\bar{B}\gamma}\sum_{S_{\bar{B}^*}}i\Psi^{C(2)}_{\alpha\rho}\Phi^{C(2)\nu\beta}\tilde{T}^{(2)\rho}_{(\bar{B}^*1)\delta}\tilde{t}^{(1)}_{(23)\beta}\epsilon^{\alpha\mu\delta\tau}\hat{p}_{\tau}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma}\bigg) + \sum_{S_{B^*}}i\Phi^{(2)}_{\beta\lambda}\Psi^{(2)}_{\alpha\rho}\tilde{T}^{(2)\rho}_{(B^*2)\delta}\tilde{t}^{(3)\beta\lambda\nu}_{(13)}\epsilon^{\alpha\mu\delta\tau}\hat{p}_{\tau}f^{B^*}_{B\gamma} - \sum_{S_{\bar{B}^*}}i\Psi^{C(2)}_{\alpha\rho}\Phi^{C(2)}_{\beta\lambda}\tilde{T}^{(2)\rho}_{(\bar{B}^*1)\delta}\tilde{t}^{(3)\beta\lambda\nu}_{(23)}\epsilon^{\alpha\mu\delta\tau}\hat{p}_{\tau}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma}\bigg). \tag{62}$$

For $J/\psi(1^-) \to B^*(\frac{3}{2}^-)\bar{B}(\frac{1}{2}^-) + \bar{B}^*(\frac{3}{2}^+)B(\frac{1}{2}^+) \to \gamma B(\frac{1}{2}^+)\bar{B}(\frac{1}{2}^-)$ we get the following six covariant amplitudes:

(1, 1, 1)

$$U_{i,1}^{\mu\nu} = g_{J/\psi}^{i,a} g_{\gamma}^{i,a} \bigg(-\frac{2}{3} C_{B^* B\gamma} \sum_{S_{B^*}} i\phi^{(1)\nu} \psi_{\alpha}^{(1)} \tilde{T}_{(B^*2)\delta}^{(1)} \epsilon^{\alpha\mu\delta\tau} \hat{p}_{\tau} f_{B\gamma}^{B^*} + \frac{2}{3} C_{\bar{B}^* \bar{B}\gamma} \sum_{S_{\bar{B}^*}} i\psi_{\alpha}^{C(1)} \phi^{C(1)\nu} \tilde{T}_{(\bar{B}^*1)\delta}^{(1)} \epsilon^{\alpha\mu\delta\tau} \hat{p}_{\tau} \bar{f}_{\bar{B}\gamma}^{\bar{B}^*} + \sum_{S_{B^*}} i\phi_{\beta}^{(1)} \psi_{\alpha}^{(1)} \tilde{T}_{(B^*2)\delta}^{(1)} \tilde{t}_{(13)}^{(2)\beta\nu} \epsilon^{\alpha\mu\delta\tau} \hat{p}_{\tau} f_{B\gamma}^{B^*} - \sum_{S_{\bar{B}^*}} i\psi_{\alpha}^{C(1)} \phi_{\beta}^{C(1)} \tilde{T}_{(\bar{B}^*1)\delta}^{(1)} \tilde{t}_{(23)}^{(2)\beta\nu} \epsilon^{\alpha\mu\delta\tau} \hat{p}_{\tau} \bar{f}_{\bar{B}\gamma}^{\bar{B}^*} \bigg), \tag{63}$$

(2, 1, 1)

$$U_{i,2}^{\mu\nu} = g_{J/\psi}^{i,b} g_{\gamma}^{i,a} \bigg(-\frac{2}{3} C_{B^* B\gamma} \sum_{S_{B^*}} \phi^{(1)\nu} \Psi^{(2)\mu\alpha} \tilde{T}_{(B^*2)\alpha}^{(1)} f_{B\gamma}^{B^*} - \frac{2}{3} C_{\bar{B}^* \bar{B}\gamma} \sum_{S_{\bar{B}^*}} \Psi^{C(2)\mu\alpha} \phi^{C(1)\nu} \tilde{T}_{(\bar{B}^*1)\alpha}^{(1)} \bar{f}_{\bar{B}\gamma}^{\bar{B}^*} + \sum_{S_{B^*}} \phi_{\beta}^{(1)} \Psi^{(2)\mu\alpha} \tilde{T}_{(B^*2)\alpha}^{(1)} \tilde{t}_{(13)}^{(2)\beta\nu} f_{B\gamma}^{B^*} + \sum_{S_{\bar{B}^*}} \Psi^{C(2)\mu\alpha} \tilde{\phi}_{\beta}^{C(1)} T_{(\bar{B}^*1)\alpha}^{(1)} \tilde{t}_{(23)}^{(2)\beta\nu} \bar{f}_{\bar{B}\gamma}^{\bar{B}^*} \bigg), \tag{64}$$

(2, 3, 1)

$$U_{i,3}^{\mu\nu} = g_{J/\psi}^{i,c} g_{\gamma}^{i,a} \bigg(\sum_{S_{B^*}} -\frac{2}{3} C_{B^* B\gamma} \phi^{(1)\nu} \Psi_{\alpha\delta}^{(2)} \tilde{T}_{(B^*2)}^{(3)\alpha\delta\mu} f_{B\gamma}^{B^*} - \frac{2}{3} C_{\bar{B}^* \bar{B}\gamma} \sum_{S_{\bar{B}^*}} \Psi_{\alpha\delta}^{C(2)} \phi^{C(1)\nu} \tilde{T}_{(\bar{B}^*1)}^{(3)\alpha\delta\mu} \bar{f}_{\bar{B}\gamma}^{\bar{B}^*} + \sum_{S_{B^*}} \phi_{\beta}^{(1)} \Psi_{\alpha\delta}^{(2)} \tilde{T}_{(B^*2)}^{(3)\alpha\delta\mu} \tilde{t}_{(13)}^{(2)\beta\nu} f_{B\gamma}^{B^*} + \sum_{S_{\bar{B}^*}} \Psi_{\alpha\delta}^{C(2)} \phi_{\beta}^{C(1)} \tilde{T}_{(\bar{B}^*1)}^{(3)\alpha\delta\mu} \tilde{t}_{(23)}^{(2)\beta\nu} \bar{f}_{\bar{B}\gamma}^{\bar{B}^*} \bigg), \tag{65}$$

(1, 1, 2)

$$U_{i,4}^{\mu\nu} = g_{J/\psi}^{i,a} g_{\gamma}^{i,b} \bigg(-\sum_{S_{B^*}} \Phi_{\beta\xi}^{(2)} \psi_{\alpha}^{(1)} \epsilon^{\alpha\mu\delta\tau} \tilde{T}_{(B^*2)\delta}^{(1)} \hat{p}_{\tau} \epsilon^{\beta\nu\lambda\sigma} \tilde{t}_{(13)\lambda}^{(2)\xi} \hat{p}_{B^*\sigma} f_{B\gamma}^{B^*} - \sum_{S_{\bar{B}^*}} \psi_{\alpha}^{C(1)} \Phi_{\beta\xi}^{C(2)} \epsilon^{\alpha\mu\delta\tau} \tilde{T}_{(\bar{B}^*1)\delta}^{(1)} \hat{p}_{\tau} \epsilon^{\beta\nu\lambda\sigma} \tilde{t}_{(23)\lambda}^{(2)\xi} \hat{p}_{\bar{B}^*\sigma} \bar{f}_{\bar{B}\gamma}^{\bar{B}^*} \bigg), \tag{66}$$

(2, 1, 2)

$$U_{i,5}^{\mu\nu} = g_{J/\psi}^{i,b} g_{\gamma}^{i,b} \bigg(\sum_{S_{B^*}} i\Phi_{\beta\xi}^{(2)} \Psi^{(2)\mu\alpha} \tilde{T}_{(B^*2)\alpha}^{(1)} \epsilon^{\beta\nu\lambda\sigma} \tilde{t}_{(13)\lambda}^{(2)\xi} \hat{p}_{B^*\sigma} f_{B\gamma}^{B^*} - \sum_{S_{\bar{B}^*}} i\Psi^{C(2)\mu\alpha} \Phi_{\beta\xi}^{C(2)} \tilde{T}_{(\bar{B}^*1)\alpha}^{(1)} \epsilon^{\beta\nu\lambda\sigma} \tilde{t}_{(23)\lambda}^{(2)\xi} \hat{p}_{\bar{B}^*\sigma} \bar{f}_{\bar{B}\gamma}^{\bar{B}^*} \bigg), \tag{67}$$

(2, 3, 2)

$$U_{i,6}^{\mu\nu} = g_{J/\psi}^{i,c} g_{\gamma}^{i,b} \bigg(\sum_{S_{B^*}} i\Phi_{\beta\xi}^{(2)} \Psi_{\alpha\delta}^{(2)} \tilde{T}_{(B^*2)}^{(3)\alpha\delta\mu} \epsilon^{\beta\nu\lambda\sigma} \tilde{t}_{(13)\lambda}^{(2)\xi} \hat{p}_{B^*\sigma} f_{B\gamma}^{B^*} - \sum_{S_{\bar{B}^*}} i\Psi_{\alpha\delta}^{C(2)} \Phi_{\beta\xi}^{C(2)} \tilde{T}_{(\bar{B}^*1)}^{(3)\alpha\delta\mu} \epsilon^{\beta\nu\lambda\sigma} \tilde{t}_{(23)\lambda}^{(2)\xi} \hat{p}_{\bar{B}^*\sigma} \bar{f}_{\bar{B}\gamma}^{\bar{B}^*} \bigg). \tag{68}$$

For $J/\psi(1^-) \rightarrow B^*(\frac{5}{2}^+)\bar{B}(\frac{1}{2}^-) + \bar{B}^*(\frac{5}{2}^-)B(\frac{1}{2}^+) \rightarrow \gamma B(\frac{1}{2}^+)\bar{B}(\frac{1}{2}^-)$ we get the following six covariant amplitudes:

(2, 2, 2)

$$U_{i,1}^{\mu\nu} = g_{J/\psi}^{i,a} g_{\gamma}^{i,a} \bigg(-\frac{3}{5} C_{B^* B\gamma} \sum_{S_{B^*}} i\phi^{(2)\nu\beta} \psi_{\alpha\rho}^{(2)} \epsilon^{\alpha\nu\delta\tau} \tilde{T}_{(B^*2)\delta}^{(2)\rho} \hat{p}_{\tau} \tilde{t}_{(13)\beta}^{(1)} f_{B\gamma}^{B^*} + \frac{3}{5} C_{\bar{B}^* \bar{B}\gamma}; \sum_{S_{\bar{B}^*}} i\psi_{\alpha\rho}^{C(2)} \phi^{C(2)\nu\beta} \epsilon^{\alpha\nu\delta\tau} \tilde{T}_{(\bar{B}^*1)\delta}^{(2)\rho} \hat{p}_{\tau} \tilde{t}_{(23)\beta}^{(1)} \bar{f}_{\bar{B}\gamma}^{\bar{B}^*} + \sum_{S_{B^*}} i\phi_{\beta\lambda}^{(2)} \psi_{\alpha\rho}^{(2)} \epsilon^{\alpha\nu\delta\tau} \tilde{T}_{(B^*2)\delta}^{(2)\rho} \hat{p}_{\tau} \tilde{t}_{(13)}^{(3)\beta\lambda\nu} f_{B\gamma}^{B^*} - \sum_{S_{\bar{B}^*}} i\psi_{\alpha\rho}^{C(2)} \phi_{\beta\lambda}^{C(2)} \epsilon^{\alpha\nu\delta\tau} \tilde{T}_{(\bar{B}^*1)\delta}^{(2)\rho} \hat{p}_{\tau} \tilde{t}_{(23)}^{(3)\beta\lambda\nu} \bar{f}_{\bar{B}\gamma}^{\bar{B}^*} \bigg),$$

(3, 2, 2)

$$U_{i,2}^{\mu\nu} = g_{J/\psi}^{i,b} g_{\gamma}^{i,a} \bigg(-\frac{3}{5} C_{B^* B\gamma} \sum_{S_{B^*}} \phi^{(2)\nu\beta} \Psi^{(3)\mu\alpha\delta} \tilde{T}_{(B^*2)\alpha\delta}^{(2)} \tilde{t}_{(13)\beta}^{(1)} f_{B\gamma}^{B^*} - \frac{3}{5} C_{\bar{B}^* \bar{B}\gamma} \sum_{S_{\bar{B}^*}} \Psi^{C(3)\mu\alpha\delta} \phi^{C(2)\nu\beta} \tilde{T}_{(\bar{B}^*1)\alpha\delta}^{(2)} \tilde{t}_{(23)\beta}^{(1)} \bar{f}_{\bar{B}\gamma}^{\bar{B}^*} + \sum_{S_{B^*}} \phi_{\beta\lambda}^{(2)} \Psi^{(3)\mu\alpha\delta} \tilde{T}_{(B^*2)\alpha\delta}^{(2)} \tilde{t}_{(13)}^{(3)\beta\lambda\nu} f_{B\gamma}^{B^*} + \sum_{S_{\bar{B}^*}} \Psi^{C(3)\mu\alpha\delta} \phi_{\beta\lambda}^{C(2)} \tilde{T}_{(\bar{B}^*1)\alpha\delta}^{(2)} \tilde{t}_{(23)}^{(3)\beta\lambda\nu} \bar{f}_{\bar{B}\gamma}^{\bar{B}^*} \bigg), \tag{69}$$

(3, 4, 2)

$$U_{i,3}^{\mu\nu} g_{J/\psi}^{i,c} g_{\gamma}^{i,a}\bigg(-\frac{3}{5}C_{B^*B\gamma}\sum_{S_{B^*}}\phi^{(2)\nu\beta}\Psi^{(3)}_{\alpha\delta\tau}\tilde{T}^{(4)\alpha\delta\tau\mu}_{(B^*2)}\tilde{t}^{(1)}_{(13)\beta}f^{B^*}_{B\gamma} - \frac{3}{5}C_{\bar{B}^*\bar{B}\gamma}\sum_{S_{\bar{B}^*}}\Psi^{C(3)}_{\alpha\delta\tau}\phi^{C(2)\nu\beta}\tilde{T}^{(4)\alpha\delta\tau\mu}_{(\bar{B}^*1)}\tilde{t}^{(1)}_{(23)\beta}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma} + \sum_{S_{B^*}}\phi^{(2)}_{\beta\lambda}\Psi^{(3)}_{\alpha\delta\tau}\tilde{T}^{(4)\alpha\delta\tau\mu}_{(B^*2)}\tilde{t}^{(3)\beta\lambda\nu}_{(13)}f^{B^*}_{B\gamma} + \sum_{S_{\bar{B}^*}}\Psi^{C(3)}_{\alpha\delta\tau}\phi^{C(2)}_{\beta\lambda}\tilde{T}^{(4)\alpha\delta\tau\mu}_{(\bar{B}^*1)}\tilde{t}^{(3)\beta\lambda\nu}_{(23)}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma}\bigg), \tag{70}$$

(2, 2, 3)

$$U_{i,4}^{\mu\nu} = g_{J/\psi}^{i,a} g_{\gamma}^{i,b}\bigg(-\sum_{S_{B^*}}\Phi^{(3)}_{\beta\eta\xi}\psi^{(2)}_{\alpha\rho}\epsilon^{\alpha\nu\delta\tau}\tilde{T}^{(2)\rho}_{(B^*2)\delta}\hat{p}_{\tau}\epsilon^{\beta\nu\lambda\sigma}\tilde{t}^{(3)\eta\xi}_{(13)\lambda}\hat{p}_{B^*\sigma}f^{B^*}_{B\gamma} - \sum_{S_{\bar{B}^*}}\psi^{C(2)}_{\alpha\rho}\Phi^{C(3)}_{\beta\eta\xi}\epsilon^{\alpha\nu\delta\tau}\tilde{T}^{(2)\rho}_{(\bar{B}^*1)\delta}\hat{p}_{\tau}\epsilon^{\beta\nu\lambda\sigma}\tilde{t}^{(3)\eta\xi}_{(23)\lambda}\hat{p}_{\bar{B}^*\sigma}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma}\bigg), \tag{71}$$

(3, 2, 3)

$$U_{i,5}^{\mu\nu} = g_{J/\psi}^{i,b} g_{\gamma}^{i,b}\bigg(\sum_{S_{B^*}}i\Phi^{(3)}_{\beta\eta\xi}\Psi^{(3)\mu\alpha\delta}\tilde{T}^{(2)}_{(B^*2)\alpha\delta}\epsilon^{\beta\nu\lambda\sigma}\tilde{t}^{(3)\eta\xi}_{(13)\lambda}\hat{p}_{B^*\sigma}f^{B^*}_{B\gamma} - \sum_{S_{\bar{B}^*}}i\Psi^{C(3)\mu\alpha\delta}\Phi^{C(3)}_{\beta\eta\xi}\tilde{T}^{(2)}_{(\bar{B}^*1)\alpha\delta}\epsilon^{\beta\nu\lambda\sigma}\tilde{t}^{(3)\eta\xi}_{(23)\lambda}\hat{p}_{\bar{B}^*\sigma}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma}\bigg), \tag{72}$$

(3, 4, 3)

$$U_{i,6}^{\mu\nu} = g_{J/\psi}^{i,c} g_{\gamma}^{i,b}\bigg(\sum_{S_{B^*}}i\Phi^{(3)}_{\beta\eta\xi}\Psi^{(3)}_{\alpha\delta\tau}\tilde{T}^{(4)\alpha\delta\tau\mu}_{(B^*2)}\epsilon^{\beta\nu\lambda\sigma}\tilde{t}^{(3)\eta\xi}_{(13)\lambda}\hat{p}_{B^*\sigma}f^{B^*}_{B\gamma} - \sum_{S_{\bar{B}^*}}i\Psi^{C(3)}_{\alpha\delta\tau}\Phi^{C(3)}_{\beta\eta\xi}\tilde{T}^{(4)\alpha\delta\tau\mu}_{(\bar{B}^*1)}\epsilon^{\beta\nu\lambda\sigma}\tilde{t}^{(3)\eta\xi}_{(23)\lambda}\hat{p}_{\bar{B}^*\sigma}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma}\bigg). \tag{73}$$

For $J/\psi(1^-) \to B^*(\frac{5}{2}^-)\bar{B}(\frac{1}{2}^-) + \bar{B}^*(\frac{5}{2}^+)\bar{B}(\frac{1}{2}^+) \to \gamma B(\frac{1}{2}^+)\bar{B}(\frac{1}{2}^-)$ we get the following six covariant amplitudes:

(2, 1, 2)

$$U_{i,1}^{\mu\nu} = g_{J/\psi}^{i,a} g_{\gamma}^{i,a}\bigg(\sum_{S_{B^*}}i\phi^{(2)}_{\beta\eta}\psi^{(2)\mu\alpha}\tilde{T}^{(1)}_{(B^*2)\alpha}\epsilon^{\beta\nu\lambda\sigma}\tilde{t}^{(2)\eta}_{(13)\lambda}\hat{p}_{B^*\sigma}f^{B^*}_{B\gamma} - \sum_{S_{\bar{B}^*}}i\psi^{C(2)\mu\alpha}\phi^{C(2)}_{\beta\eta}\tilde{T}^{(1)}_{(\bar{B}^*1)\alpha}\epsilon^{\beta\nu\lambda\sigma}\tilde{t}^{(2)\eta}_{(23)\lambda}\hat{p}_{\bar{B}^*\sigma}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma}\bigg), \tag{74}$$

(2, 3, 2)

$$U_{i,2}^{\mu\nu} = g_{J/\psi}^{i,b} g_{\gamma}^{i,a}\bigg(\sum_{S_{B^*}}i\phi^{(2)}_{\beta\eta}\psi^{(2)}_{\alpha\delta}\tilde{T}^{(3)\alpha\delta\mu}_{(B^*2)}\epsilon^{\beta\nu\lambda\sigma}\tilde{t}^{(2)\eta}_{(13)\lambda}\hat{p}_{B^*\sigma}f^{B^*}_{B\gamma} - \sum_{S_{\bar{B}^*}}i\psi^{C(2)}_{\alpha\delta}\phi^{C(2)}_{\beta\eta}\tilde{T}^{(3)\alpha\delta\mu}_{(\bar{B}^*1)}\epsilon^{\beta\nu\lambda\sigma}\tilde{t}^{(2)\eta}_{(23)\lambda}\hat{p}_{\bar{B}^*\sigma}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma}\bigg), \tag{75}$$

(3, 3, 2)

$$U_{i,3}^{\mu\nu} = g_{J/\psi}^{i,c} g_{\gamma}^{i,a}\bigg(-\sum_{S_{B^*}}\phi^{(2)}_{\beta\eta}\Psi^{(3)}_{\alpha\rho\zeta}\epsilon^{\alpha\mu\delta\tau}\tilde{T}^{(3)\rho\zeta}_{(B^*2)\delta}\hat{p}_{\tau}\epsilon^{\beta\nu\lambda\sigma}\tilde{t}^{(2)\eta}_{(13)\lambda}\hat{p}_{B^*\sigma}f^{B^*}_{B\gamma} - \sum_{S_{\bar{B}^*}}\Psi^{C(3)}_{\alpha\rho\zeta}\phi^{C(2)}_{\beta\eta}\epsilon^{\alpha\mu\delta\tau}\tilde{T}^{(3)\rho\zeta}_{(\bar{B}^*1)\delta}\hat{p}_{\tau}\epsilon^{\beta\nu\lambda\sigma}\tilde{t}^{(2)\eta}_{(23)\lambda}\hat{p}_{\bar{B}^*\sigma}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma}\bigg), \tag{76}$$

(2, 1, 3)

$$U_{i,4}^{\mu\nu} = g_{J/\psi}^{i,a} g_{\gamma}^{i,b}\bigg(-\frac{4}{7}C_{B^*B\gamma}\sum_{S_{B^*}}\Phi^{(3)\nu\lambda\sigma}\psi^{(2)\mu\alpha}\tilde{T}^{(1)}_{(B^*2)\alpha}\tilde{t}^{(2)}_{(13)\lambda\sigma}f^{B^*}_{B\gamma} - \frac{4}{7}C_{\bar{B}^*\bar{B}\gamma}\sum_{S_{\bar{B}^*}}\psi^{C(2)\mu\alpha}\Phi^{C(3)\nu\lambda\sigma}\tilde{T}^{(1)}_{(\bar{B}^*1)\alpha}\tilde{t}^{(2)}_{(23)\lambda\sigma}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma} + \sum_{S_{B^*}}\Phi^{(3)}_{\beta\lambda\sigma}\psi^{(2)\mu\alpha}\tilde{T}^{(1)}_{(B^*2)\alpha}\tilde{t}^{(4)\beta\lambda\sigma\nu}_{(13)}f^{B^*}_{B\gamma} + \sum_{S_{\bar{B}^*}}\psi^{C(2)\mu\alpha}\Phi^{C(3)}_{\beta\lambda\sigma}\tilde{T}^{(1)}_{(\bar{B}^*1)\alpha}\tilde{t}^{(4)\beta\lambda\sigma\nu}_{(23)}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma}\bigg), \tag{77}$$

(2, 3, 3)

$$U_{i,5}^{\mu\nu} = g_{J/\psi}^{i,b} g_{\gamma}^{i,b}\bigg(-\frac{4}{7}C_{B^*B\gamma}\sum_{S_{B^*}}\Phi^{(3)\nu\lambda\sigma}\psi^{(2)}_{\alpha\delta}\tilde{T}^{(3)\alpha\delta\mu}_{(B^*2)}\tilde{t}^{(2)}_{(13)\lambda\sigma}f^{B^*}_{B\gamma} - \frac{4}{7}C_{\bar{B}^*\bar{B}\gamma}\sum_{S_{\bar{B}^*}}\psi^{C(2)}_{\alpha\delta}\Phi^{C(3)\nu\lambda\sigma}\tilde{T}^{(3)\alpha\delta\mu}_{(\bar{B}^*1)}\tilde{t}^{(2)}_{(23)\lambda\sigma}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma} + \sum_{S_{B^*}}\Phi^{(3)}_{\beta\lambda\sigma}\psi^{(2)}_{\alpha\delta}\tilde{T}^{(3)\alpha\delta\mu}_{(B^*2)}\tilde{t}^{(4)\beta\lambda\sigma\nu}_{(13)}f^{B^*}_{B\gamma} + \sum_{S_{\bar{B}^*}}\psi^{C(2)}_{\alpha\delta}\Phi^{C(3)}_{\beta\lambda\sigma}\tilde{T}^{(3)\alpha\delta\mu}_{(\bar{B}^*1)}\tilde{t}^{(4)\beta\lambda\sigma\nu}_{(23)}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma}\bigg), \tag{78}$$

(3, 3, 3)

$$U_{i,6}^{\mu\nu} = g_{J/\psi}^{i,c} g_{\gamma}^{i,b} \bigg(-\frac{4}{7} C_{B^* B\gamma} \sum_{S_{B^*}} i\Phi^{(3)\nu\lambda\sigma} \Psi^{(3)}_{\alpha\rho\zeta} \epsilon^{\alpha\mu\delta\tau} \tilde{T}^{(3)\rho\zeta}_{(B^*2)\delta} \hat{p}_\tau \tilde{t}^{(2)}_{(13)\lambda\sigma} f^{B^*}_{B\gamma} - \frac{4}{7} C_{\bar{B}^*\bar{B}\gamma} \sum_{S_{\bar{B}^*}} i\Psi^{C(3)}_{\alpha\rho\zeta} \Phi^{C(3)\nu\lambda\sigma} \epsilon^{\alpha\mu\delta\tau} \tilde{T}^{(3)\rho\zeta}_{(\bar{B}^*1)\delta} \hat{p}_\tau \tilde{t}^{(2)}_{(23)\lambda\sigma} \bar{f}^{\bar{B}^*}_{\bar{B}\gamma}$$
$$+ \sum_{S_{B^*}} i\Phi^{(3)}_{\beta\lambda\sigma} \Psi^{(3)}_{\alpha\rho\zeta} \epsilon^{\alpha\mu\delta\tau} \tilde{T}^{(3)\rho\zeta}_{(B^*2)\delta} \hat{p}_\tau \tilde{t}^{(4)\beta\lambda\sigma\nu}_{(13)} f^{B^*}_{B\gamma} + \sum_{S_{\bar{B}^*}} i\Psi^{C(3)}_{\alpha\rho\zeta} \Phi^{C(3)}_{\beta\lambda\sigma} \epsilon^{\alpha\mu\delta\tau} \tilde{T}^{(3)\rho\zeta}_{(\bar{B}^*1)\delta} \hat{p}_\tau \tilde{t}^{(4)\beta\lambda\sigma\nu}_{(23)} \bar{f}^{\bar{B}^*}_{\bar{B}\gamma} \bigg). \tag{79}$$

For $J/\psi(1^-) \to B^*(\frac{7}{2}^+)\bar{B}(\frac{1}{2}^-) + \bar{B}^*(\frac{7}{2}^-)B(\frac{1}{2}^+) \to \gamma B(\frac{1}{2}^+)\bar{B}(\frac{1}{2}^-)$ we get the following six covariant amplitudes:

(3, 2, 3)

$$U_{i,1}^{\mu\nu} = g_{J/\psi}^{i,a} g_{\gamma}^{i,a} \bigg(\sum_{S_{B^*}} i\phi^{(3)}_{\beta\eta\xi} \psi^{(3)\mu\alpha\delta} \tilde{T}^{(2)}_{(B^*2)\alpha\delta} \epsilon^{\beta\nu\lambda\sigma} \tilde{t}^{(3)\eta\xi}_{(13)\lambda} \hat{p}_{B^*\sigma} f^{B^*}_{B\gamma} - \sum_{S_{\bar{B}^*}} i\psi^{C(3)\mu\alpha\delta} \phi^{C(3)}_{\beta\eta\xi} \tilde{T}^{(2)}_{(\bar{B}^*1)\alpha\delta} \epsilon^{\beta\nu\lambda\sigma} \tilde{t}^{(3)\eta\xi}_{(23)\lambda} \hat{p}_{\bar{B}^*\sigma} \bar{f}^{\bar{B}^*}_{\bar{B}\gamma} \bigg), \tag{80}$$

(3, 4, 3)

$$U_{i,2}^{\mu\nu} = g_{J/\psi}^{i,b} g_{\gamma}^{i,a} \bigg(\sum_{S_{B^*}} i\phi^{(3)}_{\beta\eta\xi} \psi^{(3)}_{\alpha\delta\tau} \tilde{T}^{(4)\alpha\delta\tau\mu}_{(B^*2)} \epsilon^{\beta\nu\lambda\sigma} \tilde{t}^{(3)\eta\xi}_{(13)\lambda} \hat{p}_{B^*\sigma} f^{B^*}_{B\gamma} - \sum_{S_{\bar{B}^*}} i\psi^{C(3)}_{\alpha\delta\tau} \phi^{C(3)}_{\beta\eta\xi} \tilde{T}^{(4)\alpha\delta\tau\mu}_{(\bar{B}^*1)} \epsilon^{\beta\nu\lambda\sigma} \tilde{t}^{(3)\eta\xi}_{(23)\lambda} \hat{p}_{\bar{B}^*\sigma} \bar{f}^{\bar{B}^*}_{\bar{B}\gamma} \bigg), \tag{81}$$

(4, 4, 3)

$$U_{i,3}^{\mu\nu} = g_{J/\psi}^{i,c} g_{\gamma}^{i,a} \bigg(-\sum_{S_{B^*}} \phi^{(3)}_{\beta\eta\xi} \Psi^{(4)}_{\alpha\rho\zeta\gamma} \epsilon^{\alpha\mu\delta\tau} \tilde{T}^{(4)\rho\zeta\gamma}_{(B^*2)\delta} \hat{p}_\tau \epsilon^{\beta\nu\lambda\sigma} \tilde{t}^{(3)\eta\xi}_{(13)\lambda} \hat{p}_{B^*\sigma} f^{B^*}_{B\gamma}$$
$$- \sum_{S_{\bar{B}^*}} \Psi^{C(4)}_{\alpha\rho\zeta\gamma} \phi^{C(3)}_{\beta\eta\xi} \epsilon^{\alpha\mu\delta\tau} \tilde{T}^{(4)\rho\zeta\gamma}_{(\bar{B}^*1)\delta} \hat{p}_\tau \epsilon^{\beta\nu\lambda\sigma} \tilde{t}^{(3)\eta\xi}_{(23)\lambda} \hat{p}_{\bar{B}^*\sigma} \bar{f}^{\bar{B}^*}_{\bar{B}\gamma} \bigg), \tag{82}$$

(3, 2, 4)

$$U_{i,4}^{\mu\nu} = g_{J/\psi}^{i,a} g_{\gamma}^{i,b} \bigg(-\frac{5}{9} C_{B^* B\gamma} \sum_{S_{B^*}} \Phi^{(4)\nu\lambda\sigma\eta} \psi^{(3)\mu\alpha\delta} \tilde{T}^{(2)}_{(B^*2)\alpha\delta} \tilde{t}^{(3)}_{(13)\lambda\sigma\eta} f^{B^*}_{B\gamma} - \frac{5}{9} C_{\bar{B}^*\bar{B}\gamma} \sum_{S_{\bar{B}^*}} \psi^{C(3)\mu\alpha\delta} \Phi^{C(4)\nu\lambda\sigma\eta} \tilde{T}^{(2)}_{(\bar{B}^*1)\alpha\delta} \tilde{t}^{(3)}_{(23)\lambda\sigma\eta} \bar{f}^{\bar{B}^*}_{\bar{B}\gamma}$$
$$+ \sum_{S_{B^*}} \Phi^{(4)}_{\beta\lambda\sigma\eta} \psi^{(3)\mu\alpha\delta} \tilde{T}^{(2)}_{(B^*2)\alpha\delta} \tilde{t}^{(5)\beta\lambda\sigma\eta\nu}_{(13)} f^{B^*}_{B\gamma} + \sum_{S_{\bar{B}^*}} \psi^{C(3)\mu\alpha\delta} \Phi^{C(4)}_{\beta\lambda\sigma\eta} \tilde{T}^{(2)}_{(\bar{B}^*1)\alpha\delta} \tilde{t}^{(5)\beta\lambda\sigma\eta\nu}_{(23)} \bar{f}^{\bar{B}^*}_{\bar{B}\gamma} \bigg), \tag{83}$$

(3, 4, 4)

$$U_{i,5}^{\mu\nu} = g_{J/\psi}^{i,b} g_{\gamma}^{i,b} \bigg(-\frac{5}{9} C_{B^* B\gamma} \sum_{S_{B^*}} \Phi^{(4)\nu\lambda\sigma\eta} \psi^{(3)}_{\alpha\delta\tau} \tilde{T}^{(4)\alpha\delta\tau\mu}_{(B^*2)} \tilde{t}^{(3)}_{(13)\lambda\sigma\eta} f^{B^*}_{B\gamma} - \frac{5}{9} C_{\bar{B}^*\bar{B}\gamma} \sum_{S_{\bar{B}^*}} \psi^{C(3)}_{\alpha\delta\tau} \Phi^{C(4)\nu\lambda\sigma\eta} \tilde{T}^{(4)\alpha\delta\tau\mu}_{(\bar{B}^*1)} \tilde{t}^{(3)}_{(23)\lambda\sigma\eta} \bar{f}^{\bar{B}^*}_{\bar{B}\gamma}$$
$$+ \sum_{S_{B^*}} \Phi^{(4)}_{\beta\lambda\sigma\eta} \psi^{(3)}_{\alpha\delta\tau} \tilde{T}^{(4)\alpha\delta\tau\mu}_{(B^*2)} \tilde{t}^{(5)\beta\lambda\sigma\eta\nu}_{(13)} f^{B^*}_{B\gamma} + \sum_{S_{\bar{B}^*}} \psi^{C(3)}_{\alpha\delta\tau} \Phi^{C(4)}_{\beta\lambda\sigma\eta} \tilde{T}^{(4)\alpha\delta\tau\mu}_{(\bar{B}^*1)} \tilde{t}^{(5)\beta\lambda\sigma\eta\nu}_{(23)} \bar{f}^{\bar{B}^*}_{\bar{B}\gamma} \bigg), \tag{84}$$

(4, 4, 4)

$$U_{i,6}^{\mu\nu} = g_{J/\psi}^{i,c} g_{\gamma}^{i,b} \bigg(-\frac{5}{9} C_{B^* B\gamma} \sum_{S_{B^*}} i\Phi^{(4)\nu\lambda\sigma\eta} \Psi^{(4)}_{\alpha\rho\zeta\gamma} \epsilon^{\alpha\mu\delta\tau} \tilde{T}^{(4)\rho\zeta\gamma}_{(B^*2)\delta} \hat{p}_\tau \tilde{t}^{(3)}_{(13)\lambda\sigma\eta} f^{B^*}_{B\gamma}$$
$$+ \frac{5}{9} C_{\bar{B}^*\bar{B}\gamma} \sum_{S_{\bar{B}^*}} i\Psi^{C(4)}_{\alpha\rho\zeta\gamma} \Phi^{C(4)\nu\lambda\sigma\eta} \epsilon^{\alpha\mu\delta\tau} \tilde{T}^{(4)\rho\zeta\gamma}_{(\bar{B}^*2)\delta} \hat{p}_\tau \tilde{t}^{(3)}_{(23)\lambda\sigma\eta} \bar{f}^{\bar{B}^*}_{\bar{B}\gamma} + \sum_{S_{B^*}} i\Phi^{(4)}_{\beta\lambda\sigma\eta} i\Psi^{(4)}_{\alpha\rho\zeta\gamma} \epsilon^{\alpha\mu\delta\tau} \tilde{T}^{(4)\rho\zeta\gamma}_{(B^*2)\delta} \hat{p}_\tau \tilde{t}^{(5)\beta\lambda\sigma\eta\nu}_{(13)} f^{B^*}_{B\gamma} \tag{85}$$
$$- \sum_{S_{\bar{B}^*}} i\Psi^{C(4)}_{\alpha\rho\zeta\gamma} \Phi^{C(4)}_{\beta\lambda\sigma\eta} \epsilon^{\alpha\mu\delta\tau} \tilde{T}^{(4)\rho\zeta\gamma}_{(\bar{B}^*2)\delta} \hat{p}_\tau \tilde{t}^{(5)\beta\lambda\sigma\eta\nu}_{(23)} \bar{f}^{\bar{B}^*}_{\bar{B}\gamma} \bigg).$$

For $J/\psi(1^-) \to B^*(\frac{7}{2}^-)\bar{B}(\frac{1}{2}^-) + \bar{B}^*(\frac{7}{2}^+)B(\frac{1}{2}^+) \to \gamma B(\frac{1}{2}^+)\bar{B}(\frac{1}{2}^-)$ we get the following six covariant amplitudes:

(3, 3, 3)

$$U^{\mu\nu}_{i,1} = g^{i,a}_{J/\psi} g^{i,a}_{\gamma}\bigg(-\frac{4}{7}C_{B^*B\gamma}\sum_{S_{B^*}} i\phi^{(3)\nu\beta\lambda}\psi^{(3)}_{\alpha\rho\zeta}\epsilon^{\alpha\mu\delta\tau}\tilde{T}^{(3)\rho\zeta}_{(B^*2)\delta}\hat{p}_\tau \tilde{t}^{(2)}_{(13)\beta\lambda} f^{B^*}_{B\gamma} + \frac{4}{7}C_{\bar{B}^*\bar{B}\gamma}\sum_{S_{\bar{B}^*}} i\psi^{C(3)}_{\alpha\rho\zeta}\phi^{C(3)\nu\beta\lambda}\epsilon^{\alpha\mu\delta\tau}\tilde{T}^{(3)\rho\zeta}_{(\bar{B}^*1)\delta}\hat{p}_\tau \tilde{t}^{(2)}_{(23)\beta\lambda}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma}$$
$$+ \sum_{S_{B^*}}\phi^{(3)}_{\beta\lambda\sigma} i\psi^{(3)}_{\alpha\rho\zeta}\epsilon^{\alpha\mu\delta\tau}\tilde{T}^{(3)\rho\zeta}_{(B^*2)\delta}\hat{p}_\tau \tilde{t}^{(4)\beta\lambda\sigma\nu}_{(13)} f^{B^*}_{B\gamma} - \sum_{S_{\bar{B}^*}} i\psi^{C(3)}_{\alpha\rho\zeta}\phi^{C(3)}_{\beta\lambda\sigma}\epsilon^{\alpha\mu\delta\tau}\tilde{T}^{(3)\rho\zeta}_{(\bar{B}^*1)\delta}\hat{p}_\tau \tilde{t}^{(4)\beta\lambda\sigma\nu}_{(23)}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma}\bigg), \tag{86}$$

(4, 3, 3)

$$U^{\mu\nu}_{i,2} = g^{i,b}_{J/\psi} g^{i,a}_{\gamma}\bigg(-\frac{4}{7}C_{B^*B\gamma}\sum_{S_{B^*}}\phi^{(3)\nu\beta\lambda}\Psi^{(4)\mu\alpha\delta\tau}\tilde{T}^{(3)}_{(B^*2)\alpha\delta\tau}\tilde{t}^{(2)}_{(13)\beta\lambda} f^{B^*}_{B\gamma} - \frac{4}{7}C_{\bar{B}^*\bar{B}\gamma}\sum_{S_{\bar{B}^*}}\Psi^{C(4)\mu\alpha\delta\tau}\phi^{C(3)\nu\beta\lambda}\tilde{T}^{(3)}_{(\bar{B}^*1)\alpha\delta\tau}\tilde{t}^{(2)}_{(23)\beta\lambda}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma}$$
$$+ \sum_{S_{B^*}}\phi^{(3)}_{\beta\lambda\sigma}\Psi^{(4)\mu\alpha\delta\tau}\tilde{T}^{(3)}_{(B^*2)\alpha\delta\tau}\tilde{t}^{(4)\beta\lambda\sigma\nu}_{(13)} f^{B^*}_{B\gamma} + \sum_{S_{\bar{B}^*}}\Psi^{C(4)\mu\alpha\delta\tau}\phi^{C(3)}_{\beta\lambda\sigma}\tilde{T}^{(3)}_{(\bar{B}^*1)\alpha\delta\tau}\tilde{t}^{(4)\beta\lambda\sigma\nu}_{(23)}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma}\bigg), \tag{87}$$

(4, 5, 3)

$$U^{\mu\nu}_{i,3} = g^{i,c}_{J/\psi} g^{i,a}_{\gamma}\bigg(-\frac{4}{7}C_{B^*B\gamma}\sum_{S_{B^*}}\phi^{(3)\nu\beta\lambda}\Psi^{(4)}_{\alpha\delta\tau\rho}\tilde{T}^{(5)\alpha\delta\tau\rho\mu}_{(B^*2)}\tilde{t}^{(2)}_{(13)\beta\lambda} f^{B^*}_{B\gamma} - \frac{4}{7}C_{\bar{B}^*\bar{B}\gamma}\sum_{S_{\bar{B}^*}}\Psi^{C(4)}_{\alpha\delta\tau\rho}\phi^{C(3)\nu\beta\lambda}\tilde{T}^{(5)\alpha\delta\tau\rho\mu}_{(\bar{B}^*1)}\tilde{t}^{(2)}_{(23)\beta\lambda}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma}$$
$$+ \sum_{S_{B^*}}\phi^{(3)}_{\beta\lambda\sigma}\Psi^{(4)}_{\alpha\delta\tau\rho}\tilde{T}^{(5)\alpha\delta\tau\rho\mu}_{(B^*2)}\tilde{t}^{(4)\beta\lambda\sigma\nu}_{(13)} f^{B^*}_{B\gamma} + \sum_{S_{\bar{B}^*}}\Psi^{C(4)}_{\alpha\delta\tau\rho}\phi^{C(3)}_{\beta\lambda\sigma}\tilde{T}^{(5)\alpha\delta\tau\rho\mu}_{(\bar{B}^*1)}\tilde{t}^{(4)\beta\lambda\sigma\nu}_{(23)}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma}\bigg), \tag{88}$$

(3, 3, 4)

$$U^{\mu\nu}_{i,4} g^{i,a}_{J/\psi} g^{i,b}_{\gamma}\bigg(-\sum_{S_{B^*}}\Phi^{(4)}_{\beta\eta\xi\varrho}\psi^{(3)}_{\alpha\rho\zeta}\epsilon^{\alpha\mu\delta\tau}\tilde{T}^{(3)\rho\zeta}_{(B^*2)\delta}\hat{p}_\tau\epsilon^{\beta\nu\lambda\sigma}\tilde{t}^{(4)\eta\xi\varrho}_{(13)\lambda}\hat{p}_{B^*\sigma} f^{B^*}_{B\gamma}$$
$$- \sum_{S_{\bar{B}^*}}\psi^{C(3)}_{\alpha\rho\zeta}\Phi^{C(4)}_{\beta\eta\xi\varrho}\epsilon^{\alpha\mu\delta\tau}\tilde{T}^{(3)\rho\zeta}_{(\bar{B}^*1)\delta}\hat{p}_\tau\epsilon^{\beta\nu\lambda\sigma}\tilde{t}^{(4)\eta\xi\varrho}_{(23)\lambda}\hat{p}_{\bar{B}^*\sigma}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma}\bigg), \tag{89}$$

(4, 4, 4)

$$U^{\mu\nu}_{i,5} = g^{i,b}_{J/\psi} g^{i,b}_{\gamma}\bigg(\sum_{S_{B^*}} i\Phi^{(4)}_{\beta\eta\xi\varrho}\Psi^{(4)\mu\alpha\delta\tau}\tilde{T}^{(3)}_{(B^*2)\alpha\delta\tau}\epsilon^{\beta\nu\lambda\sigma}\tilde{t}^{(4)\eta\xi\varrho}_{(13)\lambda}\hat{p}_{B^*\sigma} f^{B^*}_{B\gamma} - \sum_{S_{\bar{B}^*}} i\Psi^{C(4)\mu\alpha\delta\tau}\Phi^{C(4)}_{\beta\eta\xi\varrho}\tilde{T}^{(3)}_{(\bar{B}^*2)\alpha\delta\tau}\epsilon^{\beta\nu\lambda\sigma}\tilde{t}^{(4)\eta\xi\varrho}_{(23)\lambda}\hat{p}_{\bar{B}^*\sigma}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma}\bigg), \tag{90}$$

(4, 5, 4)

$$U^{\mu\nu}_{i,6} = g^{i,c}_{J/\psi} g^{i,b}_{\gamma}\bigg(\sum_{S_{B^*}} i\Phi^{(4)}_{\beta\eta\xi\varrho}\Psi^{(4)}_{\alpha\delta\tau\rho}\tilde{T}^{(5)\alpha\delta\tau\rho\mu}_{(B^*2)}\epsilon^{\beta\nu\lambda\sigma}\tilde{t}^{(4)\eta\xi\varrho}_{(13)\lambda}\hat{p}_{B^*\sigma} f^{B^*}_{B\gamma} - \sum_{S_{\bar{B}^*}} i\Psi^{C(4)}_{\alpha\delta\tau\rho}\Phi^{C(4)}_{\beta\eta\xi\varrho}\tilde{T}^{(5)\alpha\delta\tau\rho\mu}_{(\bar{B}^*1)}\epsilon^{\beta\nu\lambda\sigma}\tilde{t}^{(4)\eta\xi\varrho}_{(23)\lambda}\hat{p}_{\bar{B}^*\sigma}\bar{f}^{\bar{B}^*}_{\bar{B}\gamma}\bigg). \tag{91}$$

Note that where

$$\psi^{(n)}_{\mu_1\cdots\mu_n} = \psi^{(n)}_{\mu_1\cdots\mu_n}(p_{B^*}, S_{B^*}; p_{\bar{B}}, S_{\bar{B}}), \quad \Psi^{(n+1)}_{\mu_1\cdots\mu_{n+1}} = \Psi^{(n+1)}_{\mu_1\cdots\mu_{n+1}}(p_{B^*}, S_{B^*}; p_{\bar{B}}, S_{\bar{B}}), \quad \phi^{(n)}_{\mu_1\cdots\mu_n} = \phi^{(n)}_{\mu_1\cdots\mu_{n+}}(p_{B^*}, S_{B^*}; p_B, S_B),$$
$$\Phi^{(n+1)}_{\mu_1\cdots\mu_{n+}} = \Phi^{(n+1)}_{\mu_1\cdots\mu_{n+}}(p_{B^*}, S_{B^*}; p_B, S_B), \quad \psi^{C(n)}_{\mu_1\cdots\mu_n} = \psi^{C(n)}_{\mu_1\cdots\mu_n}(p_{\bar{B}^*}, S_{\bar{B}^*}; p_B, S_B), \quad \Psi^{C(n+1)}_{\mu_1\cdots\mu_{n+1}} = \Psi^{C(n+1)}_{\mu_1\cdots\mu_{n+1}}(p_{\bar{B}^*}, S_{\bar{B}^*}; p_B, S_B),$$
$$\phi^{C(n)}_{\mu_1\cdots\mu_n} = \phi^{C(n)}_{\mu_1\cdots\mu_n}(p_{\bar{B}^*}, S_{\bar{B}^*}; p_{\bar{B}}, S_{\bar{B}}), \quad \Phi^{C(n+1)}_{\mu_1\cdots\mu_{n+1}} = \Phi^{C(n+1)}_{\mu_1\cdots\mu_{n+1}}(p_{\bar{B}^*}, S_{\bar{B}^*}; p_{\bar{B}}, S_{\bar{B}}).$$

and

$$C_{B^*B\gamma} = -\frac{(m^2_{B^*} - m^2_B)^2}{m^2_{B^*}}, \qquad C_{\bar{B}^*\bar{B}\gamma} = -\frac{(m^2_{\bar{B}^*} - m^2_{\bar{B}})^2}{m^2_{\bar{B}^*}}.$$

For the reaction $J/\psi \to p\bar{p} \to \gamma p\bar{p}$ we get following two covariant amplitudes with two independent coupling constants $g^{p,a}$ and $g^{p,b}$ which are determined by the experiment:

(1, 0, 1)

$$U^{\mu\nu}_{p,1} = g^{p,a}\bigg(\sum_{S_{p^*}} \Phi^{\nu}_{p^*}\Psi^{(1)\mu} f^{p^*}_{p\gamma} - \sum_{S_{\bar{p}^*}} \Psi^{C(1)\mu}\Phi^{C\nu}_{\bar{p}^*} f^{\bar{p}^*}_{\bar{p}\gamma}\bigg), \tag{92}$$

(1, 2, 1)

$$U^{\mu\nu}_{p,2} = g^{p,b}\bigg(\sum_{S_{p^*}} \Phi^{\nu}_{p^*}\Psi^{(1)}_{\alpha}\tilde{T}^{(2)\alpha\mu}_{(p^*2)} f^{p^*}_{p\gamma} - \sum_{S_{\bar{p}^*}} \Psi^{C(1)}_{\alpha}\Phi^{C\nu}_{\bar{p}^*}\tilde{T}^{(2)\alpha\mu}_{(\bar{p}^*1)} \bar{f}^{\bar{p}^*}_{\bar{p}\gamma}\bigg). \tag{93}$$

Here we use p^* and $\bar{p}^*$ as intermediate states instead of p and $\bar{p}$; one can read $f^{p^*}_{p\gamma}$ and $\bar{f}^{\bar{p}^*}_{\bar{p}\gamma}$ from Eq. (49), by setting Γ_{p^*} and $\Gamma_{\bar{p}^*}$ to zero; moreover

$$\Phi^{\nu}_{p^*}(p_p, S_p, p_{p^*}, S_{p^*}) = -e\bar{u}(p_p, S_p)\bigg(\gamma^{\nu} - i\frac{\kappa_N}{2m_N}\sigma^{\nu\mu}p_{3\mu}\bigg)u(p_{p^*}, S_{p^*}),$$

$$\Phi^{C\nu}_{\bar{p}^*}(p_{\bar{p}^*}, S_{\bar{p}^*}, p_{\bar{p}}, S_{\bar{p}}) = -e\bar{v}(p_{\bar{p}^*}, S_{\bar{p}^*})\bigg(\gamma^{\nu} - i\frac{\kappa_N}{2m_N}\sigma^{\nu\mu}p_{3\mu}\bigg)v(p_{\bar{p}}, S_{\bar{p}}),$$

which are obtained from the effective Lagrangian of $NN\gamma$ in Eq. (1).

V. CONCLUSION

To provide a consistent and complete picture of baryon resonances, the various possible production and decay channels need to be explored. With estimated branching ratios for contribution of the $N^*(1440)$, $N^*(1535)$ and $N^*(1520)$ to the process $J/\psi \to \gamma p\bar{p}$, we propose to study radiative decays of excited nucleon and hyperon states through $J/\psi \to B^*\bar{B} + \bar{B}^*B \to \gamma B\bar{B}$ processes at BESIII. We provide explicit partial wave amplitude formulas for these processes where J^P for B^* is $\frac{1}{2}^{\pm}$, $\frac{3}{2}^{\pm}$, $\frac{5}{2}^{\pm}$, and $\frac{7}{2}^{\pm}$. These formulas can be used to perform partial wave analysis of forthcoming high statistics data from BESIII on these channels to extract varied useful information on the excited baryons. The BESIII can produce ground states (N, Λ, Σ, Ξ) and excited baryon states (N^*,Λ^*,Σ^*,Ξ^*) via $J/\psi \to B^*\bar{B} + \bar{B}^*B \to \gamma B\bar{B}$, as well as do further investigations into the dynamics of the excited baryons. We hope that our knowledge about the structure of the excited baryon resonances and the mechanisms of nucleon and hyperon production will be clarified by the near future studies at BESIII.

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APPENDIX: THE RELATION WITH THE HELICITY AMPLITUDE

In this Appendix, we discuss the relation between amplitudes in the L-S and helicity formalism for $B^* \to B\gamma$. From Ref. [36], the radiative decay width is related to the helicity amplitudes $A_{1/2}$ and $A_{3/2}$ as

$$\Gamma_{\gamma} = \frac{k^2}{4\pi}\frac{m_B}{m_{B^*}}\frac{8}{2J+1}(|A_{3/2}|^2 + |A_{1/2}|^2), \tag{A1}$$

where k is the photon three momentum magnitude, and J is the total spin of B^*. Let us consider that the photon is moving along the z-axis, and the photon is right-handed polarized, in other words, the spin of the photon is along the z-axis. $A_{3/2}$ is the spin-3/2 helicity amplitude of the initial B^* in a state with $|J, 3/2>$ and final B in a state with $|1/2, 1/2>$, and $A_{1/2}$ denotes the spin-1/2 helicity amplitude of the B^* with $|J, 1/2>$ and final B with $|1/2, -1/2>$.

In the L-S Scheme the decay amplitude formulas for $B^* \to B\gamma$ are

$$\begin{aligned}\Gamma_{\gamma} &= \frac{k}{2\pi}\frac{m_B}{m_{B^*}}\frac{1}{2J+1}\sum_{S_{B^*},S_B,S_{\gamma}}|M|^2 \\ &= \frac{k}{2\pi}\frac{m_B}{m_{B^*}}\frac{1}{2J+1}(|M_{(3/2,1/2,1)}|^2 + |M_{(1/2,-1/2,1)}|^2 \\ &\quad + |M_{(-1/2,1/2,-1)}|^2 + |M_{(-3/2,-1/2,-1)}|^2) \\ &= \frac{k}{\pi}\frac{m_B}{m_{B^*}}\frac{1}{2J+1}(|M_{(3/2,1/2,1)}|^2 + |M_{(1/2,-1/2,1)}|^2).\end{aligned} \tag{A2}$$

By comparing Eq. (A1) with Eq. (A2), we can have the relation between the helicity and L-S amplitudes as follows:

$$|A_{3/2}|^2 = \frac{1}{2k}|M_{(3/2,1/2,1)}|^2 = \frac{1}{2k}|(g_1 M_{1(3/2,1/2,1)} + g_2 M_{2(3/2,1/2,1)})|^2, \quad \text{(A3)}$$

$$|A_{1/2}|^2 = \frac{1}{2k}|M_{(1/2,-1/2,1)}|^2 = \frac{1}{2k}|(g_1 M_{1(1/2,-1/2,1)} + g_2 M_{2(1/2,-1/2,1)})|^2. \quad \text{(A4)}$$

As an example, now we calculate the $M(S_{B^*}, S_B, S_\gamma)$ for $B^*(\frac{3}{2}) \to B\gamma$. From Eq. (40), the two dependent amplitudes can be written as

$$M_1 = i\phi_\mu^{(1)}\epsilon^{\mu\nu\lambda\sigma}\tilde{t}_\lambda^{(1)}\hat{p}_{B^*\sigma}\epsilon_\nu^*, \quad \text{(A5)}$$

$$M_2 = -\frac{2}{3}(\tilde{r}\cdot\tilde{r})\Phi^{(2)\nu\mu}\tilde{t}_\mu^{(1)}\epsilon_\nu^* + \Phi_{\mu\lambda}^{(2)}\tilde{t}^{(3)\mu\lambda\nu}\epsilon_\nu^*. \quad \text{(A6)}$$

Before starting our calculation of $M(S_{B^*}, S_B, S_\gamma)$, we should define wave functions of particles

$$\bar{u}(p_B, 1/2) = \sqrt{\frac{E_B + m_B}{2m_B}}\left(1, 0, \frac{k}{E_B + m_B}, 0\right), \quad \text{(A7)}$$

$$\bar{u}(p_B, -1/2) = \sqrt{\frac{E_B + m_B}{2m_B}}\left(0, 1, 0, \frac{-k}{E_B + m_B}\right), \quad \text{(A8)}$$

$$u(m_{B^*}, 1/2) = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}, \qquad u(m_{B^*}, -1/2) = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}, \quad \text{(A9)}$$

and

$$\epsilon^*(p_\gamma, 1, 1) = \left(0, -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}i, 0\right), \quad \epsilon(m_{B^*}, 1, 1) = \left(0, -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}i, 0\right), \quad \text{(A10)}$$

$$\epsilon(m_{B^*}, 1, 0) = (0, 0, 0, 1), \quad \epsilon(m_{B^*}, 1, -1) = \left(0, \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}i, 0\right), \quad \text{(A11)}$$

where B^* is at rest, in other words, $p_{B^*} = (m_{B^*}, 0, 0, 0)$, $p_B = (E_B, 0, 0, -k)$ and $p_\gamma = (k, 0, 0, k)$. By using Eqs. (5) and (6), the different states of $\phi_\mu^{(1)}$ and $\Phi_{\mu\nu}^{(2)}$ can be obtained:

$$\phi_\mu^{(1)}(S_{B^*} = 1/2, S_B = -1/2) = \sqrt{\frac{E_B + m_B}{2m_B}}\sqrt{\frac{1}{3}}\epsilon_\mu(m_{B^*}, 1, 1), \quad \text{(A12)}$$

$$\phi_\mu^{(1)}(S_{B^*} = 3/2, S_B = 1/2) = \sqrt{\frac{E_B + m_B}{2m_B}}\epsilon_\mu(m_{B^*}, 1, 1), \quad \text{(A13)}$$

$$\Phi_{\mu\nu}^{(2)}(S_{B^*} = 1/2, S_B = -1/2) = \sqrt{\frac{E_B + m_B}{2m_B}}\sqrt{3}(\epsilon_\mu(m_{B^*}, 1, 1)\epsilon_\nu(m_{B^*}, 1, 0) + \epsilon_\mu(m_{B^*}, 1, 0)\epsilon_\nu(m_{B^*}, 1, 1)), \quad \text{(A14)}$$

$$\Phi_{\mu\nu}^{(2)}(S_{B^*} = 3/2, S_B = 1/2) = -\sqrt{\frac{E_B + m_B}{2m_B}}(\epsilon_\mu(m_{B^*}, 1, 1)\epsilon_\nu(m_{B^*}, 1, 0) + \epsilon_\mu(m_{B^*}, 1, 0)\epsilon_\nu(m_{B^*}, 1, 1)). \quad \text{(A15)}$$

At last, we get the following amplitudes $M(S_{B^*}, S_B, S_\gamma)$:

$$M_1(1/2, -1/2, 1) = \frac{2k}{\sqrt{3}}\sqrt{\frac{E_B + m_B}{2m_B}}, \quad \text{(A16)}$$

$$M_1(3/2, 1/2, 1) = 2k\sqrt{\frac{E_B + m_B}{2m_B}}, \quad \text{(A17)}$$

$$M_2(1/2, -1/2, 1) = \frac{-24k^3}{\sqrt{3}}\sqrt{\frac{E_B + m_B}{2m_B}}, \quad \text{(A18)}$$

$$M_2(3/2, 1/2, 1) = 8k^3\sqrt{\frac{E_B + m_B}{2m_B}}, \quad \text{(A19)}$$

where we have used the following relations:

$$i\epsilon_{\mu abc} = \gamma_5(\gamma_\mu\gamma_a\gamma_b\gamma_c - \gamma_\mu\gamma_a g_{bc} + \gamma_\mu\gamma_b g_{ac} - \gamma_\mu\gamma_c g_{ab} - \gamma_a\gamma_b g_{\mu c} + \gamma_a\gamma_c g_{\mu b} - \gamma_b\gamma_c g_{\mu a} + g_{\mu a}g_{bc} - g_{\mu b}g_{ac} + g_{\mu c}g_{ab}), \quad \text{(A20)}$$

$$\gamma^5 = \begin{pmatrix}0 & \sigma_0\\ \sigma_0 & 0\end{pmatrix}, \qquad \gamma^0 = \begin{pmatrix}I & 0\\ 0 & -I\end{pmatrix}, \qquad \gamma^i = \begin{pmatrix}0 & \sigma_i\\ -\sigma_i & 0\end{pmatrix}, \qquad i = 1, 2, 3. \quad \text{(A21)}$$

Now we list the relation between the square of the helicity amplitudes and square of the coupling constants from our amplitudes for $B^*(\frac{1}{2}^\pm, \frac{3}{2}^\pm, \frac{5}{2}^\pm, \frac{7}{2}^\pm) \to B\gamma$ as follows:

$$M = g_\gamma^a M_1 + g_\gamma^b M_2, \quad \text{(A22)}$$

$$B^*\left(\frac{1^+}{2}\right): \quad M1 = i\Phi_\mu^{(1)}\epsilon^{\mu\nu\lambda\sigma}\epsilon_\nu^*\tilde{t}_\lambda^{(1)}\hat{p}_{B^*\sigma}, \quad \text{(A23)}$$

$$|A_{1/2}|^2 = \frac{E_B + m_B}{2m_B}4k|g_\gamma^a|^2. \quad \text{(A24)}$$

$$B^*\left(\frac{1^-}{2}\right):\ M1 = -\frac{2}{3}(\tilde{r}\cdot\tilde{r})\Phi^{(1)}_{\mu}\epsilon^{*\mu} + \Phi^{(1)}_{\mu}\epsilon^*_{\nu}\tilde{t}^{(2)\mu\nu}, \qquad (A25)$$

$$|A_{1/2}|^2 = \frac{E_B + m_B}{m_B} 16k|g^a_{\gamma}|^2. \qquad (A26)$$

$$B^*\left(\frac{3^+}{2}\right):\quad M_1 = i\phi^{(1)}_{\mu}\epsilon^{\mu\nu\lambda\sigma}\tilde{t}^{(1)}_{\lambda}\hat{p}_{B^*\sigma}\epsilon^*_{\nu}, \qquad (A27)$$

$$M_2 = -\frac{3}{5}(\tilde{r}\cdot\tilde{r})\Phi^{(2)\mu\nu}\epsilon^*_{\mu}\tilde{t}^{(1)}_{\nu} + \Phi^{(2)}_{\mu\lambda}\tilde{t}^{(3)\mu\lambda\nu}\epsilon^*_{\nu}, \qquad (A28)$$

$$|A_{1/2}|^2 = \frac{E_B + m_B}{2m_B}\frac{2}{3}k|g^a_{\gamma} - 12g^b_{\gamma}k^2|^2, \qquad (A29)$$

$$|A_{3/2}|^2 = \frac{E_B + m_B}{2m_B}2k|g^a_{\gamma} + 4g^b_{\gamma}k^2|^2. \qquad (A30)$$

$$B^*\left(\frac{3^-}{2}\right):\quad M_1 = -\frac{2}{3}(\tilde{r}\cdot\tilde{r})\phi^{(1)\mu}\epsilon^*_{\mu} + \phi^{(1)}_{\mu}\tilde{t}^{(2)\mu\nu}\epsilon^*_{\nu}, \qquad (A31)$$

$$M_2 = i\Phi^{(2)}_{\mu\alpha}\epsilon^{\mu\nu\lambda\sigma}\epsilon^*_{\nu}\tilde{t}^{(1)\alpha}_{\lambda}\hat{p}_{B^*\sigma}, \qquad (A32)$$

$$|A_{1/2}|^2 = \frac{E_B + m_B}{2m_B}\frac{8}{3}k^3| - g^a_{\gamma} + 3g^b_{\gamma}|^2, \qquad (A33)$$

$$|A_{3/2}|^2 = \frac{E_B + m_B}{2m_B}8k^3| - g^a_{\gamma} - g^b_{\gamma}|^2. \qquad (A34)$$

$$B^*\left(\frac{5^+}{2}\right):\quad M_1 = -\frac{3}{5}(\tilde{r}\cdot\tilde{r})\phi^{(2)\mu\nu}\epsilon^*_{\mu}\tilde{t}_{(1)\nu} + \phi^{(2)}_{\mu\nu}\tilde{t}^{(3)\mu\nu\lambda}\epsilon^*_{\lambda}, \qquad (A35)$$

$$M_2 = i\Phi^{(3)}_{\mu\alpha\beta}\epsilon^{\mu\nu\lambda\sigma}\epsilon^*_{\nu}\tilde{t}^{(3)\alpha\beta}_{\lambda}\hat{p}_{B^*\sigma}, \qquad (A36)$$

$$|A_{1/2}|^2 = \frac{E_B + m_B}{2m_B}\frac{32}{5}k^5| - g^a_{\gamma} + 4g^b_{\gamma}|^2, \qquad (A37)$$

$$|A_{3/2}|^2 = \frac{E_B + m_B}{2m_B}\frac{64}{5}k^5| - g^a_{\gamma} - 2g^b_{\gamma}|^2. \qquad (A38)$$

$$B^*\left(\frac{5^-}{2}\right):\quad M_1 = i\phi^{(2)}_{\mu\alpha}\epsilon^{\mu\nu\lambda\sigma}\epsilon^*_{\nu}\tilde{t}^{(2)\alpha}_{\lambda}\hat{p}_{B^*\sigma}, \qquad (A39)$$

$$M_2 = -\frac{4}{7}(\tilde{r}\cdot\tilde{r})\Phi^{(3)\mu\nu\lambda}\epsilon^*_{\mu}\tilde{t}^{(2)}_{\nu\lambda} + \Phi^{(3)}_{\mu\nu\lambda}\tilde{t}^{(4)\mu\nu\lambda\sigma}\epsilon^*_{\sigma}, \qquad (A40)$$

$$|A_{1/2}|^2 = \frac{E_B + m_B}{2m_B}\frac{8}{5}k^3|g^a_{\gamma} - 16g^b_{\gamma}k^2|^2, \qquad (A41)$$

$$|A_{3/2}|^2 = \frac{E_B + m_B}{2m_B}\frac{16}{5}k^3| - g^a_{\gamma} + 4g^b_{\gamma}k^2|^2. \qquad (A42)$$

$$B^*\left(\frac{7^+}{2}\right):\quad M_1 = i\phi^{(3)}_{\mu\alpha\beta}\epsilon^{\mu\nu\lambda\sigma}\epsilon^*_{\nu}\tilde{t}^{(3)\alpha\beta}_{\lambda}\hat{p}_{B^*\sigma}, \qquad (A43)$$

$$M_2 = -\frac{5}{9}(\tilde{r}\cdot\tilde{r})\Phi^{(4)\mu\nu\lambda\sigma}\epsilon^*_{\mu}\tilde{t}^{(3)}_{\nu\lambda\sigma} + \Phi^{(4)}_{\mu\nu\lambda\sigma}\tilde{t}^{(5)\mu\nu\lambda\sigma\delta}\epsilon^*_{\delta}, \qquad (A44)$$

$$|A_{1/2}|^2 = \frac{E_B + m_B}{2m_B}\frac{128}{35}k^5|g^a_{\gamma} - 20g^b_{\gamma}k^2|^2, \qquad (A45)$$

$$|A_{3/2}|^2 = \frac{E_B + m_B}{2m_B}\frac{128}{21}k^5|g^a_{\gamma} + 12g^b_{\gamma}k^2|^2. \qquad (A46)$$

$$B^*\left(\frac{7^-}{2}\right):$$

$$M_1 = -\frac{4}{7}(\tilde{r}\cdot\tilde{r})\phi^{(3)\mu\nu\lambda}\epsilon^*_{\mu}\tilde{t}_{(1)\nu\lambda} + \phi^{(3)}_{\mu\nu\lambda}\tilde{t}^{(4)\mu\nu\lambda\sigma}\epsilon^*_{\sigma}, \qquad (A47)$$

$$M_2 = i\Phi^{(4)}_{\mu\alpha\beta\gamma}\epsilon^{\mu\nu\lambda\sigma}\epsilon^*_{\nu}\tilde{t}^{(4)\alpha\beta\gamma}_{\lambda}\hat{p}_{B^*\sigma}, \qquad (A478)$$

$$|A_{1/2}|^2 = \frac{E_B + m_B}{2m_B}\frac{512}{35}k^7| - g^a_{\gamma} + 5g^b_{\gamma}|^2, \qquad (A49)$$

$$|A_{3/2}|^2 = \frac{E_B + m_B}{2m_B}\frac{512}{21}k^7| - g^a_{\gamma} - 3g^b_{\gamma}|^2. \qquad (A50)$$

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