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Instanton effects on the twist-three light-cone distribution amplitudes of pion and light scalar mesons above 1 GeV

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By use of a single-instanton approximation the twist-three light-cone distribution amplitudes of the pion as well as the p-wave mesons, i.e., $f_0(1370)$, $K_0^*(1430)$, and $a_0(1450)$, are investigated within the framework of QCD moment sum rules with the inclusion of instanton effects based on the valence quark model. The results show that there is much change on light-cone distribution amplitudes due to the isospin- and chirality-dependent instanton contribution compared with the instanton-free ones. We find that the instanton-involved twist-three light-cone distribution amplitudes are non-positive-definite within some range of the momentum fraction and that there are rapid changes at two ends of the momentum fraction. To guarantee the convergence of moments using the method in this work, a low instanton density should be adopted; for instance, $n_c = \frac{1}{2}\ \text{fm}^{-4}$. Possible ingredients which might have an impact on the results are briefly discussed. These light-cone distribution amplitudes may be helpful to analyze exclusive heavy-flavored processes.

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I. INTRODUCTION

The hadronic light-cone distribution amplitude (LCDA or light-cone wave functions) which parametrize the nonperturbative effects play an important role in understanding exclusive hard processes in QCD [1–3]. The LCDA is also one of the key ingredients in the QCD factorization approach [4] for describing exclusive hadronic B decays which are helpful to ascertain the quark content of light scalars [5–7] and other observables of phenomenological interest. Therefore, if we have more information on LCDAs, there will be a more complete study on heavy-flavored processes and some important parameters in the standard model. To extract LCDA the nonperturbative method should be employed due to its nonperturbative nature. The QCD sum rules [8] have been proved to be very successful in obtaining useful information about the hadronic region [9]. Later it was found that, in order to produce reasonable results for the lowest-lying states of the pseudoscalar channel, the instanton effect [10–14] should be taken into account in QCD sum rules [15].[1] Then the role of the instanton in QCD sum rules can be extensively investigated [17,18]. Recently it was noticed that the instanton may be helpful in lifting the mass degeneration in light scalar mesons above 1 GeV [19]. The calculation of LCDAs within the framework of QCD sum rules was introduced in [20] by studying the correlation function of currents with derivatives; in this way, the desired Gegenbauer coefficients can be expressed in terms of the moments derived from QCD sum rules. Using this method Chernyak and Zhitnitsky (CZ) were able to describe many experimental data available at that time [21]. Using lattice simulation, the first two Gegenbauer coefficients of the pion and kaon were studied [22] under this method.

In analyzing B decays to p-wave mesons [23,24] (for a more complete review on B decays, see [25]), such as $f_0(1370)$, $K_0^*(1430)$, $a_0(1450)$, the LCDAs are important. Having noticed the significant implication of instantons in producing the realistic mass spectra of f_0, K_0^*, and a_0 above 1 GeV [19], in this paper we will make some effort to investigate the instanton effects on the twist-three LCDAs of these mesons using the CZ method. The work in this paper can be regarded as partly following Ref. [19], and the quark content is assigned as follows:

$$f_0 = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}), \qquad K_0^* = d\bar{s}, \qquad a_0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}). \tag{1}$$

The LCDAs of the pion, as a widely used quantity in exclusive processes, naturally attract attention based on various methods from the instanton model [26]. At this point we would like to stress that the instanton is crucial in reproducing reasonable results of the pion and its partners in the 0^- channel by QCD sum rules [15]. Perturbatively, the intuition is that, due to the chirality suppression, there should be a larger size of the $s\bar{s}$ component in ω than η. If this were the case, ω would be close to ϕ, but the fact is that ω nearly degenerates with ρ while there is a considerable mass gap between ω and ϕ. The empirical spectrum indicates that η is a nearly pure SU(3) octet, while there is a much smaller $s\bar{s}$ component in ω. Then the question is why there is a much larger splitting between the pion and η than that of ρ and ω, which is difficult to explain perturbatively. The puzzle can be solved if the instanton is

[1]For a thorough review on the instanton in QCD, one can refer to [16].

considered. Because of the definite chirality of the fermion zero mode, there is a direct instanton contribution to the pseudoscalar channel, but not in the vector and tensor ones. Additionally, this instanton-induced correction is isospin dependent; thus, the pion and η are lighter than ρ and ω, and there is a larger mass gap between the pion and η than ρ and ω. Hence, the pion provides a natural laboratory to study the instanton, and it is expected that there should be some effects of the instanton on its twist-three LCDAs. Therefore, it is meaningful to analyze the twist-three pion LCDAs by the CZ method with the inclusion of the instanton. In fact, the isospin dependence of the instanton effects is also important to reproduce a realistic mass gap between a_0 and f_0 in the 0^+ channel above 1 GeV by the QCD sum rules, assuming a small $s\bar{s}$ mixing into f_0 [19]. The quark content of the pion is, as usual,

$$\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}).$$

It is hoped that these twist-three LCDAs may shed some light on the exclusive hadronic B decays. We would like to emphasize that, in QCD language, a real hadron should be described by a set of Fock states, each of which has the same quantum number as the hadron. For example,

$$|K_0\rangle = \psi^K_{d\bar{s}}|d\bar{s}\rangle + \psi^K_{d\bar{s}g}|d\bar{s}g\rangle + \psi^K_{d\bar{s}q\bar{q}}|d\bar{s}q\bar{q}\rangle + \ldots. \quad (2)$$

It is no doubt that there are also twist-three LCDAs introduced by higher Fock states, but we will not consider them here. In other words, we deal with leading Fock states or the valence quarks of the hadrons. Additionally, there has been some effort in obtaining pion LCDAs from holographic QCD methods [27] and nonlocal condensates based on QCD sum rules [28], as well as some work on the shape of pion LCDAs [29]. This work gives us some insight on pion LCDAs.

The outline of this paper is as follows: in Sec. II we present the sum rules with the inclusion of instanton contributions to obtain the LCDAs of f_0, K_0^*, a_0, and the pion. In Sec. III the numerical results and discussions will be presented. Finally, we summarize our conclusions in Sec. IV. An appendix is given to show the vanishing of the instanton contribution to tensor moment sum rules within the method.

II. BASIC FORMULAS

A. Moments and light-cone distribution amplitudes

First, we define the decay constants of the scalar meson S and the pseudoscalar meson P,

$$\langle 0|\bar{q}_2 q_1|S(p)\rangle = m_S f_S,$$
$$\langle 0|\bar{q}_2 i\gamma_5 q_1|P(p)\rangle = \frac{f_P m_P^2}{m_1 + m_2},$$

where m_S, m_P, m_1, and m_2 are the masses of the scalar meson, the pseudoscalar meson, q_1, and q_2, respectively. The twist-three LCDAs $\phi_S^s(u)$ and $\phi_S^\sigma(u)$ for the scalar meson S with quark content $q_1\bar{q}_2$ are defined as [6]

$$\langle 0|\bar{q}_2(z_2)q_1(z_1)|S(p)\rangle = m_S f_S \int_0^1 du e^{i(up\cdot z_2 + \bar{u}p\cdot z_1)}\phi_S^s(u), \quad (3)$$

$$\langle 0|\bar{q}_2(z_2)\sigma_{\mu\nu}q_1(z_1)|S(p)\rangle = -m_S f_S(p_\mu z_\nu - p_\nu z_\mu)\int_0^1 du e^{i(up\cdot z_2 + \bar{u}p\cdot z_1)}\frac{\phi_S^\sigma(u)}{6}. \quad (4)$$

The two twist-three two-particle LCDAs $\phi_P^p(u)$ and $\phi_P^\sigma(u)$ of the pseudoscalar meson are defined as [30]

$$\langle 0|\bar{q}_2(z_2)i\gamma_5 q_1(z_1)|P(p)\rangle = \frac{f_P m_P^2}{m_1 + m_2}\int_0^1 du e^{i(up\cdot z_2 + \bar{u}p\cdot z_1)}\phi_P^p(u), \quad (5)$$

$$\langle 0|\bar{q}_2(z_2)\sigma_{\mu\nu}\gamma_5 q_1(z_1)|P(p)\rangle = -\frac{i}{3}\frac{f_P m_P^2}{m_1 + m_2}(p_\mu z_\nu - p_\nu z_\mu)\int_0^1 du e^{i(up\cdot z_2 + \bar{u}p\cdot z_1)}\phi_P^\sigma(u), \quad (6)$$

where u always refers to the momentum fraction carried by one quark and $\bar{u} = 1 - u$ is another quark momentum fraction; $z = z_2 - z_1$. Note that the gauge-invariant Wilson path-ordered integral

$$[z_2, z_1] = P\exp\left[ig\int_{z_1}^{z_2} d\sigma_\mu A^\mu(\sigma)\right]$$

has been suppressed. The normalization of these four twist-three LCDAs are

$$\int_0^1 du\phi_S^s(u) = \int_0^1 du\phi_S^\sigma(u) = 1,$$
$$\int_0^1 du\phi_P^p(u) = \int_0^1 du\phi_P^\sigma(u) = 1. \quad (7)$$

To proceed, first we remind the reader that there is no pure zero-mode contribution to tensor moment sum rules (see the Appendix) since the instanton effect is chirality dependent; thus, we do not consider $\phi_S^\sigma(u)$ and $\phi_P^\sigma(u)$ here. To obtain twist-three LCDAs of p-wave mesons above 1 GeV by using the instanton-free tensor moment sum rules, one can refer to Ref. [31]. For simplicity, in this paper we study $\phi_S^s(u)$ and $\phi_S^p(u)$. In fact, we can only concentrate on the scalar moment sum rules because the pseudoscalar one can be deduced from the scalar one by some appropriate substitutions; therefore, in the following we deal with the scalar sum rules only.

Generally, the twist-three LCDA $\phi_S^s(u)$ has the following form:

$$\phi_S^s(u, \mu) = 1 + \sum_{n=1}^{\infty} a_n(\mu)C_n^{1/2}(2u - 1), \quad (8)$$

where $C_n^{1/2}(x)$ is the Gegenbauer polynomial of order 1/2; the lowest ones are [32]

$$C_0^{1/2}(x) = 1, \quad C_1^{1/2}(x) = x, \quad C_2^{1/2}(x) = \frac{1}{2}(3x^2 - 1),$$
$$C_3^{1/2}(x) = \frac{5}{2}x^3 - \frac{3}{2}x, \quad C_4^{1/2}(x) = \frac{1}{8}(35x^4 - 30x^2 + 3), \tag{9}$$

and the orthogonality relation is

$$\int_{-1}^{1} C_n^{1/2}(x) C_m^{1/2}(x) dx = \frac{2}{2n+1}\delta_{nm}. \tag{10}$$

From Eq. (3) one can easily derive

$$\langle 0|\bar{q}_1(0)(iz\cdot \overleftrightarrow{D})^n q_2(0)|S(p)\rangle = m_S f_S (p\cdot z)^n \langle \xi_s^n\rangle, \tag{11}$$

where

$$\overleftrightarrow{D} = \overrightarrow{D}_\mu - \overleftarrow{D}_\mu, \quad \overrightarrow{D}_\mu = \overrightarrow{\partial}_\mu - igA_\mu^a t^a,$$
$$\langle \xi_s^n\rangle = \int_0^1 du(2u-1)^n \phi_S^s(u,\mu). \tag{12}$$

From the orthogonal relation, Eq. (10), the Gegenbauer moments a_n can be expressed in terms of $\langle \xi^n\rangle$; for our purpose,

$$a_2 = \frac{5}{2}(3\langle\xi^2\rangle - 1), \qquad a_4 = \frac{9}{8}(35\langle\xi^4\rangle - 30\langle\xi^2\rangle + 3). \tag{13}$$

The next step is to calculate the so-called moments appearing in Eq. (11); to this end, we consider the following two-point correlation function with derivatives,

$$I_n(z,q) = i\int d^4x e^{iqx}\langle 0|TO_n(x)O^\dagger(0)|0\rangle$$
$$= (z\cdot q)^n I_n^{\rm OPE}(q^2) \tag{14}$$

with

$$O_n(x) = \bar{q}_1(x)(iz\cdot\overleftrightarrow{D})^n q_2(x), \qquad O^\dagger(0) = \bar{q}_2(0)q_1(0).$$

The above correlation function can be expressed in terms of the operator product expansion. Up to leading order of α_s and dimension six, we get[2]

$$I_n(z,q) = (z\cdot q)^n I_n^{\rm OPE}(q^2)$$
$$= (z\cdot q)^n\Big[-\frac{3}{8\pi^2}\frac{1}{n+1}q^2\ln\frac{-q^2}{\mu^2} + \frac{3+n}{24}\Big\langle\frac{\alpha_s}{\pi}G^2\Big\rangle - \frac{1}{q^2}\Big(\frac{n+1}{2}m_1 + m_2\Big)\langle\bar{q}_1q_1\rangle - \frac{1}{q^2}\Big(m_1 + \frac{n+1}{2}m_2\Big)\langle\bar{q}_2q_2\rangle$$
$$- \frac{1}{2q^4}m_2\langle g_s\bar{q}_1\sigma Gq_1\rangle - \frac{1}{2q^4}m_1\langle g_s\bar{q}_2\sigma Gq_2\rangle + \frac{4\pi}{27}\frac{\alpha_s}{q^4}(n^2+3n-4)(\langle\bar{q}_1q_1\rangle^2 + \langle\bar{q}_2q_2\rangle^2) - \frac{48}{9}\frac{\alpha_s}{q^4}\langle\bar{q}_1q_1\rangle\langle\bar{q}_2q_2\rangle\Big], \tag{15}$$

where n is even; thus, only the scalar even moments exist. This is the theoretical side of the correlation function from the quark-gluon dynamics. On the other hand, Eq. (14) can also be derived phenomenologically based on the dispersion relation

$$I_n^{\rm OPE}(q^2) = \frac{1}{\pi}\int_0^\infty ds\frac{{\rm Im}I_n^{\rm ph}(s)}{s-q^2} + \text{subtr.const.} \tag{16}$$

The imaginary part ${\rm Im}I_n^{\rm ph}(s)$ is obtained by inserting a complete quantum set $\sum|n\rangle\langle n|$ into Eq. (14), which reads

$${\rm Im}I_n^{\rm ph}(q^2) = \pi m_S^2 f_S^2\langle\xi^n\rangle\delta(q^2 - m_S^2)$$
$$+ \frac{3}{8\pi^2}\frac{1}{n+1}\pi q^2\theta(q^2 - s_0). \tag{17}$$

By equating the theoretical and phenomenological sides of $I_n^{\rm OPE}(q^2)$, we get the sum rules

$$I_n^{\rm OPE}(q^2) = \frac{1}{\pi}\int_0^\infty ds\frac{{\rm Im}I_n^{\rm ph}(s)}{s-q^2} + \text{subtr.const.} \tag{18}$$

Substituting Eqs. (15) and (17) into Eq. (18), taking the Borel transformation, and subtracting the continuum contributions, we arrive at the desired scalar moment sum rules,

$$m_S^2 f_S^2\langle\xi_s^n\rangle\exp\Big[-\frac{m_S^2}{M^2}\Big]$$
$$= \frac{3}{8\pi^2}\frac{1}{n+1}\int_0^{s_0} ds\, s\exp\Big[-\frac{s}{M^2}\Big] + \frac{3+n}{24}\Big\langle\frac{\alpha_s}{\pi}G^2\Big\rangle$$
$$+ \Big(\frac{n+1}{2}m_1 + m_2\Big)\langle\bar{q}_1q_1\rangle + \Big(m_1 + \frac{n+1}{2}m_2\Big)\langle\bar{q}_2q_2\rangle$$
$$- \frac{1}{2M^2}m_2\langle g_s\bar{q}_1\sigma Gq_1\rangle - \frac{1}{2M^2}m_1\langle g_s\bar{q}_2\sigma Gq_2\rangle$$
$$+ \frac{4\pi}{27}\frac{\alpha_s}{M^2}(n^2+3n-4)[\langle\bar{q}_1q_1\rangle^2 + \langle\bar{q}_2q_2\rangle^2]$$
$$- \frac{48\pi}{9}\frac{\alpha_s}{M^2}\langle\bar{q}_1q_1\rangle\langle\bar{q}_2q_2\rangle. \tag{19}$$

Up to now our analysis is still confined in the conventional QCD sum rules; in the next subsection the instanton will be included.

[2]Notice the operator product expansion is different from Ref. [31] on the mass-dependent condensate terms, but there is little impact on the results since these terms are greatly suppressed by the quark mass. One will see in the following that we can get an explicit extremum from the sum rules which are derived from the operator product expansion in Eq. (15).

B. Inclusion of the instanton effects in moment sum rules

In this subsection our calculations are in *four-dimensional Euclidean space*, unless explicitly pointed out otherwise. The instanton is the nontrivial solution of the classical field equation in four-dimensional Euclidean gauge-field theories, which was first discovered by Belavin *et al.* [10]. Subsequently, 't Hooft [12] derived the instanton with topological charge $Q = 1$ in four-dimensional Euclidean space,

$$A^a_\mu(x) = \frac{2}{g}\eta_{a\mu\nu}\frac{(x-x_0)_\nu}{(x-x_0)^2+\rho^2}, \tag{20}$$

$$G^a_{\mu\nu}(x) = -\frac{4}{g}\eta_{a\mu\nu}\frac{\rho^2}{[(x-x_0)^2+\rho^2]^2}, \tag{21}$$

where ρ is the instanton size, $\eta_{a\mu\nu}$ is the 't Hooft η symbol, and x_0 is a point in four-dimensional Euclidean space called the instanton center. In this instanton background field there is a quark zero mode which represents the tunneling effects; for our purpose, we write the quark zero-mode propagator explicitly in *regular gauge*,[3]

$$S^{zm}(x,y;x_0) = \frac{\rho^2}{8\pi^2 m^*}\frac{1}{[(x-x_0)^2+\rho^2]^{3/2}} \times \frac{1}{[(y-x_0)^2+\rho^2]^{3/2}}\left[\gamma_\mu\gamma_\nu\frac{1}{2}(1-\gamma_5)\right] \otimes [\tau^+_\mu\tau^-_\nu], \tag{22}$$

where m^* is the effective mass and

$$\tau^\pm_\mu = (\boldsymbol{\tau}, \pm i), \qquad \boldsymbol{\tau} = \boldsymbol{\sigma}, \tag{23}$$

with the useful relations

$$\begin{aligned} \tau^a\tau^b &= \delta^{ab} + i\varepsilon^{abc}\tau^c, \\ \tau^+_\mu\tau^-_\nu &= \delta_{\mu\nu} + i\eta_{a\mu\nu}\tau^a, \\ \tau^-_\mu\tau^+_\nu &= \delta_{\mu\nu} + i\bar{\eta}_{a\mu\nu}\tau^a. \end{aligned} \tag{24}$$

The instanton contribution to the scalar moment sum rules is obtained by substituting the zero-mode propagator, Eq. (22), into the correlation function, Eq. (14),

$$\begin{aligned} &\int d^4x e^{iQx}\langle 0|TO_n(x)O^\dagger(0)|0\rangle \\ &= \frac{8}{\pi^4 m_1^* m_2^*}\int d^4x e^{iQx}\int d\rho n(\rho)\rho^4\int d^4x_0\frac{1}{(x_0^2+\rho^2)^3} \\ &\times\frac{1}{[(x-x_0)^2+\rho^2]^{3/2}}(iz\cdot\overleftrightarrow{D}_I)^n\frac{1}{[(x-x_0)^2+\rho^2]^{3/2}}, \end{aligned} \tag{25}$$

with

$$O_n(x) = \bar{q}_{10}(x)(iz\cdot\overleftrightarrow{D}_I)^n q_{20}(x), \qquad O^\dagger(0) = \bar{q}_{20}(0)q_{10},$$

where the integration over collective coordinates of the instanton is explicit, and the anti-instanton contribution as well as traces over γ and the SU(2) matrix are completed implicitly. The instanton density has the simple form proposed in [15],

$$n(\rho) = n_c\delta(\rho - \rho_c). \tag{26}$$

But now one should notice that the covariant derivative in Eq. (25) contains the instanton background field $A^{(I)}_\mu$ to guarantee gauge invariance,

$$D^I_\mu = \partial_\mu - igA^{(I)a}_\mu t^a, \tag{27}$$

where t^a is the SU(2) generator $t^a = \sigma^a/2$ with the normalization condition

$$\mathrm{tr}[t^a t^b] = \tfrac{1}{2}\delta^{ab}. \tag{28}$$

First we take a close look at the covariant derivative

$$\begin{aligned} (iz\cdot\overleftrightarrow{D}_I)^n &= [iz\cdot(\overrightarrow{D}_I - \overleftarrow{D}_I)]^n \\ &= \{iz_\mu[(\overrightarrow{\partial}_\mu - igA^{(I)a}_\mu t^a) - (\overleftarrow{\partial}_\mu + igA^{(I)a}_\mu t^a)]\}^n. \end{aligned} \tag{29}$$

We can sort the terms in the expansion of Eq. (29) into two kinds. The first one only contains the differential operator which acts on the zero-mode propagator from the left and right; the second one is the remaining part which includes all the instanton field involved terms. In the following, we will elucidate the complete contribution from these two kinds of terms at $n = 2$ and $n = 4$ by an explicit calculation.

Contribution to the correlation function of the first kind is easy to calculate, such that we can derive a general formula for n as follows[4]:

$$\begin{aligned} \Pi^{\rm first}_n(Q^2) &= \int d^4x e^{iQx}\langle 0|TO_n(x)O^\dagger(0)|0\rangle \\ &= \frac{8\rho^4}{\pi^4 m_1^* m_2^*}\int d^4x_0 e^{iQx_0}\frac{1}{(x_0^2+\rho^2)^3} \\ &\times A(2i)^n\left(-iz\cdot\frac{\partial}{\partial Q}\right)^n\int d^4x e^{iQx}\frac{1}{(x^2+\rho^2)^{n+3}}, \end{aligned} \tag{30}$$

where A is constant,

$$A = [1+(-1)^n]\cdot\frac{3}{2}\cdot\frac{5}{2}\cdot\ldots\cdot\frac{2n+1}{2} + \sum_{k=1}^{n-1}C^k_n(-1)^{n+k} \cdot\frac{3}{2}\cdot\frac{5}{2}\cdot\ldots\cdot\frac{2(n-k)+1}{2}\times\frac{3}{2}\cdot\frac{5}{2}\cdot\ldots\cdot\frac{2k+1}{2}.$$

[3] For the propagator in the singular gauge, one can refer to [33].

[4] For simplicity, we take $\rho_c \to \rho$.

It is worth mentioning that in deriving Eq. (30) from Eq. (25) the light-cone constraint $z^2 = 0$ is crucial; otherwise, there will be large masses. Now the remaining work is trivial, with the help of the following formulas [32,34]:

$$\int d^4x \frac{e^{iQx}}{(x^2+\rho^2)^\nu} = \frac{2\pi^2}{\Gamma(\nu)}\left(\frac{Q\rho}{2}\right)^{\nu-2}\frac{K_{2-\nu}(Q\rho)}{\rho^{2\nu-4}},$$
$$\left(\frac{d}{zdz}\right)^m [z^\nu K_\nu(z)] = (-)^m z^{\nu-m} K_{\nu-m}(z), \tag{31}$$
$$K_{-\nu}(z) = K_\nu(z),$$

where $Q^2 = -q^2$ and $K_\nu(z)$ is the MacDonald function. When the smoke clears, we get the desired results for the first kind of contribution,

$$\Pi_n^{\text{first}}(Q^2) = \int d^4x e^{iQx}\langle 0|TO_n(x)O^\dagger(0)|0\rangle = \frac{n_c\rho^2}{\pi m_1^* m_2^*}\frac{2^{n+1}[1+(-1)^n](1+n)}{\Gamma(n+3)} \times \left[\Gamma\left(\frac{n+1}{2}\right)\right]^2 (z\cdot Q)^n Q^2 K_1^2(Q\rho). \tag{32}$$

We find that the nonvanishing contribution of the second kind for $n = 2$ is

$$\Pi_2^{\text{second}}(Q^2) = i^2 \int d^4x e^{iQx} S_{10}(0,x;x_0)(-2igz\cdot A^{(I)})^2 S_{20}(x,0;x_0). \tag{33}$$

Substituting the zero-mode propagator, Eq. (22), and instanton field, Eq. (20), into Eq. (33), for definiteness we write the explicit form of Eq. (33) as follows (one should notice that both the propagator and the instanton field are in regular gauge):

$$\begin{aligned}\Pi_2^{\text{second}}(Q^2) = &-\frac{4g^2}{64\pi^4 m_1^* m_2^*}\int d^4x e^{iQ\cdot x}\int d\rho n(\rho)\rho^4 \int d^4x_0 \frac{1}{(x_0^2+\rho^2)^{3/2}}\frac{1}{[(x-x_0)^2+\rho^2]^{3/2}} \\ &\times \gamma_\mu\gamma_\nu\frac{1}{2}(1-\gamma_5)\tau_\mu^+\tau_\nu^- z_\sigma \frac{2}{g}\eta_{a\sigma\delta}\frac{(x-x_0)_\delta}{(x-x_0)^2+\rho^2} t^a z_\rho \frac{2}{g}\eta_{b\rho\gamma}\frac{(x-x_0)_\gamma}{(x-x_0)^2+\rho^2} t^b \frac{1}{(x_0^2+\rho^2)^{3/2}} \\ &\times \frac{1}{[(x-x_0)^2+\rho^2]^{3/2}}\gamma_\alpha\gamma_\beta\frac{1}{2}(1-\gamma_5)\tau_\alpha^+\tau_\beta^- \\ = &-\frac{n_c\rho_c^4}{64\pi^4 m_1^* m_2^*}\int d^4x e^{iQ\cdot x}\int d^4x_0 \frac{1}{(x_0^2+\rho_c^2)^3}\frac{1}{[(x-x_0)^2+\rho_c^2]^5} z_\sigma \eta_{a\sigma\delta}(x-x_0)_\delta z_\rho \eta_{b\rho\gamma}(x-x_0)_\gamma \\ &\times \text{tr}\left[\gamma_\mu\gamma_\nu\frac{1}{2}(1-\gamma_5)\gamma_\alpha\gamma_\beta\frac{1}{2}(1-\gamma_5)\right]\text{tr}[\tau_\mu^+\tau_\nu^-\tau^a\tau^b\tau_\alpha^+\tau_\beta^-].\end{aligned} \tag{34}$$

After a lengthy calculation we obtain the following result from Eq. (34),[5]

$$\Pi_2^{\text{second}}(Q^2) = \frac{n_c\rho^2}{6m_1^* m_2^*}(z\cdot Q)^2 Q^2 K_1^2(Q\rho), \tag{35}$$

where the isospin dependence has been included. One can find that this part is comparable to the first kind of contribution for $n = 2$.

For $n = 4$ the situation is more complicated when we expand Eq. (29) order by order, since the differential operator ∂_μ can also act on the instanton field and give a nonvanishing contribution. In considering this effect on the instanton field, we find the following terms for the second kind of contribution to the correlator,

$$\begin{aligned}\Pi_4^{\text{second}}(Q^2) &= i^4 \int d^4x e^{iQx}\{S_{10}(0,x;x_0)4[z\cdot(\overrightarrow{\partial}-\overleftarrow{\partial})]^2 S_{20}(x,0;x_0)\} \\ &\times(-2igz\cdot A^{(I)})^2 + i^4\int d^4x e^{iQx} S_{10}(0,x;x_0) \\ &\times[4z_\mu z_\nu z_\alpha z_\beta A_\alpha^{(I)}(\partial_\mu\partial_\nu A_\beta^{(I)})(-2ig)^2 \\ &+ (-2igz\cdot A^{(I)})^4] S_{20}(x,0;x_0),\end{aligned} \tag{36}$$

where, for brevity, the SU(2) generator index is suppressed. After some effort, we obtain

$$\Pi_4^{\text{second}}(Q^2) = \frac{3n_c\rho^2}{20m_1^* m_2^*}(z\cdot Q)^4 Q^2 K_1^2(Q\rho). \tag{37}$$

Notice that we have included the anti-instanton effects in both Eqs. (35) and (37). Combining Eqs. (32), (35), and (37) we get the complete zero-mode contribution to the correlation function,

[5]For simplicity, we replace ρ_c by ρ.

$$\Pi_2^{\rm zm}(Q^2) = \Pi_2^{\rm first}(Q^2) + \frac{n_c\rho^2}{6m_1^* m_2^*}(z\cdot Q)^2 Q^2 K_1^2(Q\rho) \quad (38)$$

for $n = 2$, and

$$\Pi_4^{\rm zm}(Q^2) = \Pi_4^{\rm first}(Q^2) + \frac{3n_c\rho^2}{20m_1^* m_2^*}(z\cdot Q)^4 Q^2 K_1^2(Q\rho) \quad (39)$$

for $n = 4$. From the two equations above one can find that the two kinds of contributions are comparable; thus, we cannot simply omit one of them.

After dealing with the quark zero mode in the instanton background field, we now calculate the nonzero-mode contribution to the correlation function. For $n = 2$ this part is

$$\Pi_2^{\rm nzm}(Q^2) = i^2 \int d^4x e^{iQx} S_1^{\rm nzm}(0,x) \times (-2igz\cdot A^{(I)})^2 S_2^{\rm nzm}(x,0), \quad (40)$$

where $S_1^{\rm nzm}(0,x)$ and $S_2^{\rm nzm}(x,0)$ are the nonzero-mode propagators. In the instanton background field the complete form of the quark propagator consists of zero-mode and nonzero-mode parts,

$$S_I(x,y) = S_I^{\rm zm}(x,y) + S_I^{\rm nzm}(x,y). \quad (41)$$

In the previous paragraphs we have completed all the contributions to the correlation function from the zero mode in the single-instanton approximation. To obtain the nonzero-mode contribution we need $S_I^{\rm nzm}(x,y)$, which is a quite complicated object. It was shown [35–37] that it is reliable to take a massless free propagator approximation for $S_I^{\rm nzm}(x,y)$ if the zero-mode contribution to the Green function is maximal. Fortunately, in the present calculation this requirement can be met since there is a direct instanton contribution to the correlation function we considered; thus, it is convenient to take the massless free propagator approximation

$$S_I^{\rm nzm}(x,y) = \frac{1}{2\pi^2}\frac{\gamma_\mu(x_\mu - y_\mu)}{(x-y)^4}, \quad (42)$$

for the nonzero mode. At this point it is necessary to stress that in the vector channel this approximation is no longer valid; otherwise, the vector current is not always conserved [38]. In this case the propagator of the nonzero mode is very involved; one can refer to Ref. [16] for details.

Combining Eqs. (40) and (42) we arrive at

$$\Pi_2^{\rm nzm}(Q^2) = \frac{8n_c}{\pi^2}\int d^4x e^{iQx}\frac{1}{x^6}\int d^4x_0 \frac{[z\cdot(x-x_0)]^2}{[(x-x_0)^2+\rho^2]^2}. \quad (43)$$

According to translational invariance, $(x - x_0) \to u$, we can factorize out the integral with respect to the instanton center x_0; thus, there are two separate integrals given by the nonzero mode and the instanton field. Since $z^2 = 0$ it is easy to see that the instanton integral is vanishing; therefore, there is no contribution of the nonzero mode to the correlation function. In other words, the nonzero mode and the instanton field do not "entangle" each other. The physical meaning is that, in this case, the instanton field does not transfer momentum from one quark to another. A similar analysis also holds for $n = 4$.

To be consistent with Eq. (19) we should reformulate the total instanton-induced contributions in terms of a dispersion relation. For this purpose, we note that the properties of the MacDonald function under analytical continuation are [18,39]

$$K_\nu(z) = \begin{cases} \frac{i\pi}{2}e^{i\pi\nu/2}H_\nu^{(1)}(ze^{i\pi/2}) & -\pi < \arg z \le \frac{\pi}{2} \\ -\frac{i\pi}{2}e^{-i\pi\nu/2}H_\nu^{(1)}(ze^{-i\pi/2}) & \frac{\pi}{2} < \arg z \le \pi. \end{cases} \quad (44)$$

In the above expression $H_\nu^{(1)}(z)$ is the Hankel function of the first kind,

$$H_\nu^{(1)}(z) = J_\nu(z) + iY_\nu(z), \quad (45)$$

where $J_\nu(z)$ and $Y_\nu(z)$ are the Bessel functions and Neumann functions, respectively. The last step is to improve Eq. (32) so that it is consistent with Eq. (19). To this end, noticing the cut structure of the Hankel functions, Eq. (45), one can find

$$\mathrm{Im}\, K_1^2(-i\rho\sqrt{s}) = \frac{\pi^2}{2}J_1(\rho\sqrt{s})Y_1(\rho\sqrt{s}) + \text{singular term}. \quad (46)$$

As usual, in terms of the dispersion relation we get the final results, which are improved by the Borel transformation with the continuum contribution subtracted. Finally we collect the whole result as follows,

$$m_S^2 f_S^2\langle\xi_s^2\rangle \exp\left[-\frac{m_S^2}{M^2}\right] = \Pi_2^{\rm OPE}(s_0,M^2) + (-1)^I\frac{\pi n_c\rho^2}{3m_1^* m_2^*} \times \int_0^{s_0} ds s J_1(\rho\sqrt{s})Y_1(\rho\sqrt{s})\exp\left[-\frac{s}{M^2}\right] \quad (47)$$

for $\langle\xi_s^2\rangle$, and

$$m_S^2 f_S^2\langle\xi_s^4\rangle \exp\left[-\frac{m_S^2}{M^2}\right] = \Pi_4^{\rm OPE}(s_0,M^2) + (-1)^I\frac{8\pi n_c\rho^2}{45m_1^* m_2^*} \times \int_0^{s_0} ds s J_1(\rho\sqrt{s})Y_1(\rho\sqrt{s})\exp\left[-\frac{s}{M^2}\right] \quad (48)$$

for $\langle\xi_s^4\rangle$, where the isospin dependence has been considered and $\Pi_n^{\rm OPE}$ is the right-hand side of Eq. (19). It is easy to derive the pseudoscalar moment sum rules from the scalar ones if we consider the effects of $i\gamma_5$ carefully in the calculations,

$$\frac{m_P^4 f_P^2 \langle \xi_p^2 \rangle}{(m_1+m_2)^2} \exp\left[-\frac{m_P^2}{M^2}\right] = \Pi_2^{ps,\mathrm{OPE}}(s_0, M^2) - (-1)^I \frac{\pi n_c \rho^2}{3 m_1^* m_2^*} \times \int_0^{s_0} ds s J_1(\rho\sqrt{s}) Y_1(\rho\sqrt{s}) \exp\left[-\frac{s}{M^2}\right] \quad (49)$$

for $\langle \xi_p^2 \rangle$, and

$$\frac{m_P^4 f_P^2 \langle \xi_p^4 \rangle}{(m_1+m_2)^2} \exp\left[-\frac{m_P^2}{M^2}\right] = \Pi_4^{ps,\mathrm{OPE}}(s_0, M^2) - (-1)^I \frac{8\pi n_c \rho^2}{45 m_1^* m_2^*} \times \int_0^{s_0} ds s J_1(\rho\sqrt{s}) Y_1(\rho\sqrt{s}) \exp\left[-\frac{s}{M^2}\right] \quad (50)$$

for $\langle \xi_p^4 \rangle$, where $\Pi_n^{ps,\mathrm{OPE}}$ represents the following instanton-free sum rules:

$$\begin{aligned}\Pi_n^{ps,\mathrm{OPE}}(s_0, M^2) &= \frac{3}{8\pi^2}\frac{1}{n+1}\int_0^{s_0} ds s \exp\left[-\frac{s}{M^2}\right] + \frac{n-3}{24}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle \\ &+ \left(\frac{n+1}{2}m_1 - m_2\right)\langle\bar{q}_1 q_1\rangle + \left(-m_1 + \frac{n+1}{2}m_2\right)\langle\bar{q}_2 q_2\rangle \\ &+ \frac{1}{2M^2} m_2 \langle g_s \bar{q}_1 \sigma G q_1\rangle + \frac{1}{2M^2} m_1 \langle g_s \bar{q}_2 \sigma G q_2\rangle \\ &+ \frac{4\pi}{27}\frac{\alpha_s}{M^2}(n^2+3n-4)[\langle\bar{q}_1 q_1\rangle^2 + \langle\bar{q}_2 q_2\rangle^2] \\ &+ \frac{48\pi}{9}\frac{\alpha_s}{M^2}\langle\bar{q}_1 q_1\rangle\langle\bar{q}_2 q_2\rangle.\end{aligned}$$

Obviously the instanton contribution in the pseudoscalar channel is opposite from the scalar one, which reflects the chirality dependence of the instanton effects.

Now all the formulas needed have been fixed. The parameters which will be adopted in our numerical analysis are as follows [6,40,41]:

$$\begin{aligned}&\alpha_s = 0.517, \qquad \left\langle\frac{\alpha_s}{\pi}G^2\right\rangle = 0.012 \pm 0.006\ \mathrm{GeV}^4, \\ &\langle\bar{u}u\rangle = \langle\bar{d}d\rangle = -(0.225 \pm 0.15)^3\ \mathrm{GeV}^3, \\ &\langle\bar{s}s\rangle = (0.8 \pm 0.2)\langle\bar{u}u\rangle, \qquad m_u = 0.004\ \mathrm{GeV}, \\ &m_d = 0.006\ \mathrm{GeV}, \qquad m_s = 0.12\ \mathrm{GeV}, \\ &\langle g_s\bar{u}\sigma G u\rangle = \langle g_s\bar{d}\sigma G d\rangle = 0.8\ \mathrm{GeV}^2\langle\bar{u}u\rangle, \\ &\langle g_s\bar{s}\sigma G s\rangle = 0.8\langle g_s\bar{u}\sigma G u\rangle.\end{aligned} \quad (51)$$

We read the masses and decay constants of f_0, K_0^*, and a_0 from [19]

$$\begin{aligned}&m_{f_0} = 1380\ \mathrm{MeV}, \qquad f_{f_0} = 375\ \mathrm{MeV}, \\ &m_{K_0^*} = 1450\ \mathrm{MeV}, \qquad f_{K_0^*} = 370\ \mathrm{MeV}, \\ &m_{a_0} = 1480\ \mathrm{MeV}, \qquad f_{a_0} = 370\ \mathrm{MeV}.\end{aligned} \quad (52)$$

The mass and decay constant pions are [41]

$$m_\pi = 140\ \mathrm{MeV}, \qquad f_\pi = 130\ \mathrm{MeV}. \quad (53)$$

All the parameters in Eq. (51) as well as in Eqs. (52) and (53) are taken at $\mu = 1$ GeV.

The remaining important parameters are the instanton-related ones. For the effective masses of the quarks and the instanton size, the following values work well [19,33]:

$$\begin{aligned}&m_u^* = m_d^* = 86\ \mathrm{MeV}, \\ &m_s^* = 114 \pm 28\ \mathrm{MeV}, \\ &\rho = \frac{1}{3}\ \mathrm{fm} = \frac{1}{0.6}\ \mathrm{GeV}^{-1}.\end{aligned}$$

The last parameter is the instanton density which still needs improvement. The original value $n_c = \frac{1}{2}\ \mathrm{fm}^{-4}$ is widely used [15,42,43], while the lattice calculation suggested $n_c \sim 1\ \mathrm{fm}^{-4}$ [44]. The work of Cristoforetti *et al.* [45], based on the interacting instanton liquid model, showed that an even larger one was needed, i.e. $n_c = 3\ \mathrm{fm}^{-4}$, to reproduce the nucleon mass and the low-energy constants in chiral perturbation theory. For this reason we will investigate the sensitivity of the moments and, consequently, the LCDAs for different instanton densities.

III. RESULTS AND DISCUSSIONS

First we present the selection rule of the threshold and Borel window. The continuum (and exited states) as well as the dimension-six condensates contribution should be controllable; as usual [8,31,46,47], we demand that in the instanton-free sum rules, the continuum contribution, the part in the dispersive integral from s_0 to ∞, should be less than 30% of the total dispersion integration; this gives us an upper limit of the Borel momentum M^2. In addition, the dimension-six condensates' contribution should be less than 15%; this gives us a lower limit of the Borel momentum. If there is an extremum within the Borel window, we take it as our calculated value; otherwise, the midvalue within the window will be adopted. Then we turn on the instanton contribution under the same threshold and Borel window since, in this way, there will be a good comparison for the two cases. Of course one can analyze the moment by a new threshold and Borel window when the instanton contribution is turned on; however, in general, the sum rule is sensitive to the threshold and Borel window, so the instanton effect may be smeared by the change which is induced by adopting a new threshold and Borel window. In the following discussions all the Borel windows satisfy the selection rule unless explicitly stated otherwise.

With the selection rule in mind, we find that for the pion, the moment $\langle \xi_p^2 \rangle$ at threshold $s_0 = 4.0 \pm 0.2\ \mathrm{GeV}^2$ and

Borel window $M^2 \in [1.35, 1.65]$ GeV^2 as well as $\langle \xi_p^4 \rangle$ at $s_0 = 4.4 \pm 0.2$ GeV^2 and $M^2 \in [1.2, 1.5]$ GeV^2 are stable from sum rules, Eqs. (49) and (50), at $n_c = 0$, respectively. The midvalues are $\langle \xi_p^2 \rangle = 0.34$ and $\langle \xi_p^4 \rangle = 0.21$, which are shown in Fig. 1. When turning on the instanton effects we find $\langle \xi_p^2 \rangle = 0.52$ and $\langle \xi_p^4 \rangle = 0.36$ at $n_c = \frac{1}{2}$ fm^{-4}. If the instanton density is increasing, for instance, at $n_c = 1$ fm^{-4}, we find $\langle \xi_p^2 \rangle = 0.71$ and $\langle \xi_p^4 \rangle = 0.50$. Obviously the moments increase as the instanton density increases. Hence it is expected that at some larger n_c the second moment will be more than 1; for instance, if we take $n_c = 2$ fm^{-4} as proposed in [45], we find $\langle \xi_p^2 \rangle = 1.10$ and $\langle \xi_p^4 \rangle = 0.80$. It is astonishing that the second moment is more than 1. This result is unnatural since it is obvious from Eq. (12), if assuming positive-definite LCDAs, that we have

$$\langle \xi^m \rangle = \int_0^1 du(2u-1)^m \phi(u, \mu) < \langle \xi^n \rangle = \int_0^1 du(2u-1)^n \phi(u, \mu) < 1, \tag{54}$$

where

$$n, m = 2, 4, \ldots\ldots, m > n.$$

But the LCDAs themselves are not measurable; to observe their exact role we should convolute them with the hard scattering amplitudes T in exclusive processes. In fact, the twist-two LCDAs of $f_0(980)$ given in [5] from the CZ method, the twist-three ones of the pion [30], as well as the work in Refs. [48–52] show non-positive-definite behavior within some range of the momentum fraction. In considering this, it seems that our results may indicate non-positive-definite LCDAs. Indeed the instanton-involved LCDAs of the pion show this property, as presented in Fig. 2, although the moments are still very convergent. The exact forms of LCDAs and the instanton density are not well known nowadays, so the impact of the high instanton density on LCDAs by the CZ method seems to need further study. Our results seem to show that the sum rules in Eq. (49) and (50) do not allow too large of an instanton density in order to get convergent moments. The obtained LCDAs of the pion for different instanton densities are shown in the left panel of Fig. 2.

Next we analyze $f_0(1370)$ with the quark content assigned in Eq. (1). The sum rules of this member are very similar to the pion except for some different condensate terms induced by chirality. We can see that the instanton contribution of $f_0(1370)$ is the same as for the pion since there are simultaneous changes in its isospin and chirality relative to the pion. At $n_c = 0$ we get the threshold and Borel window for $\langle \xi_{f_0}^2 \rangle$ and $\langle \xi_{f_0}^4 \rangle$ as $s_0 = 4.7 \pm 0.2$ GeV^2, $M^2 \in [1.3, 1.7]$ GeV^2 and $s_0 = 4.8 \pm 0.2$ GeV^2, $M^2 \in [1.8, 2.2]$ GeV^2, respectively. Under the threshold and Borel window we can obtain an explicit extremum both for $\langle \xi_{f_0}^2 \rangle$ and $\langle \xi_{f_0}^4 \rangle$, which is shown in Fig. 3. The extrema within the range of threshold are very stable, and we obtain $\langle \xi_{f_0}^2 \rangle = 0.35$, $\langle \xi_{f_0}^4 \rangle = 0.24$. When the instanton effects are turned on, we find that the midvalues are $\langle \xi_{f_0}^2 \rangle = 0.55$ and $\langle \xi_{f_0}^4 \rangle = 0.34$ for $n_c = \frac{1}{2}$ fm^{-4}, as well as $\langle \xi_{f_0}^2 \rangle = 0.76$ and $\langle \xi_{f_0}^4 \rangle = 0.44$ for $n_c = 1$ fm^{-4}. Similar to the case of π at $n_c = 2$ fm^{-4} we find $\langle \xi_{f_0}^2 \rangle = 1.17$, which is more than 1. The moments also increase as the instanton density takes a larger value, which is understandable since the instanton contributions to the pion and $f_0(1370)$ are equivalent due to the combined effects of isospin and chirality. Thus we conclude that the instanton contribution to the moment

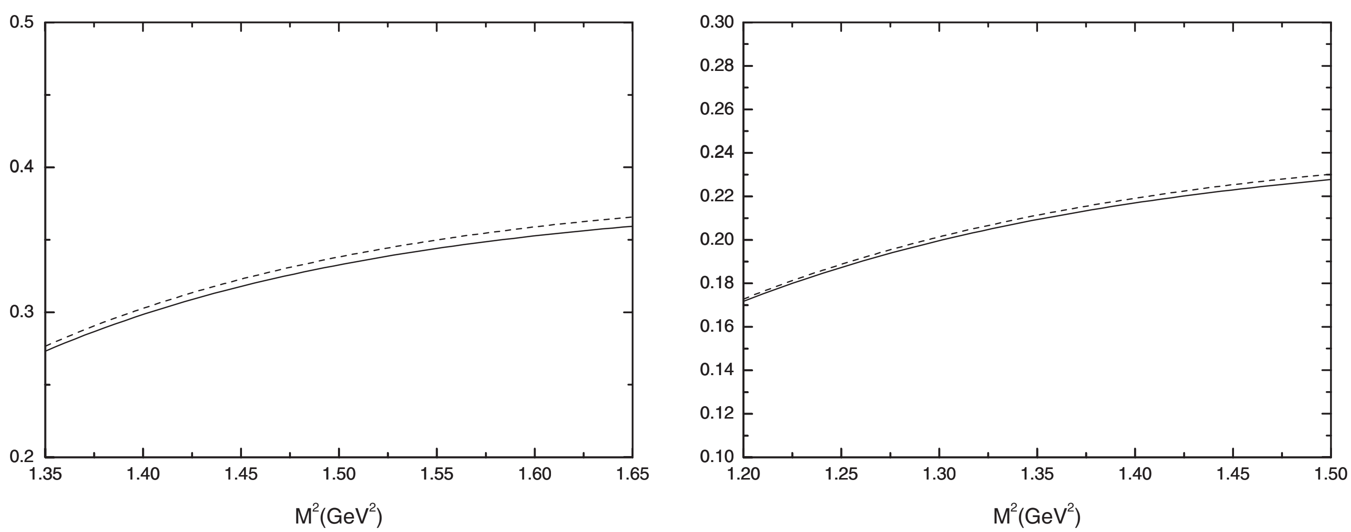

FIG. 1. Moments $\langle \xi_p^2 \rangle$ (left panel) and $\langle \xi_p^4 \rangle$ (right panel) of the pion from instanton-free sum rules. Solid lines correspond to the central value of the threshold, while the dashed lines represent the threshold increasing by 0.1 GeV^2 relative to the central value (the line descriptions are the same for Figs. 3–5).

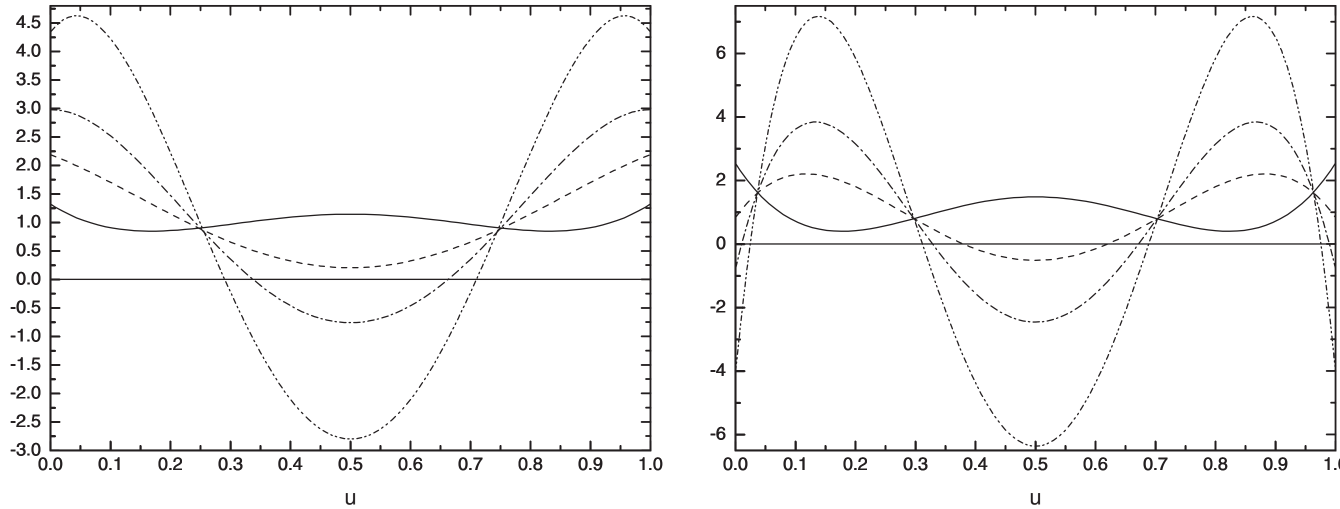

FIG. 2. Twist-three light-cone distribution amplitudes of the pion (left panel) and $f_0(1370)$ (right panel) as a function of the momentum fraction u at $\mu = 1$ GeV for different instanton densities: the solid lines show $n_c = 0$, the dashed line $n_c = \frac{1}{2}$ fm^{-4}, the dashed-dot line $n_c = 1$ fm^{-4}, and the dash-dot-dot line $n_c = 2$ fm^{-4} (the line descriptions are the same for Fig. 6).

sum rules of the pion and $f_0(1370)$ is positive. The LCDAs of $f_0(1370)$ are plotted in Fig. 2.

It is easy to understand the similar impact of the instanton on the LCDAs of the pion and $f_0(980)$ since the dominant parts, i.e., the perturbative dispersive integral and the instanton contribution of the moment sum rules for the pion and $f_0(1370)$, are equivalent except for the chirality-dependent condensates. In fact, this similarity is also reflected in the LCDAs, which can be observed clearly from Fig. 2. One more important thing is that the LCDAs are positive-definite when there are no instanton effects, while when the instanton is involved there is a strong impact on the profile of LCDAs. Because of the chirality-dependent parts at the two ends of the momentum fraction, the LCDAs of $f_0(1370)$ change more rapidly than in the pion.

The sum rules of $K_0^*(1430)$ and $a_0(1450)$ are nearly the same since they share the same isospin and chirality, in addition to the difference introduced by flavor symmetry breaking. The adopted threshold and Borel window of $\langle \xi^2_{K_0^*} \rangle$ and $\langle \xi^4_{K_0^*} \rangle$ at $n_c = 0$ are $s_0 = 4.7 \pm 0.2$ GeV^2, $M^2 \in [1.3, 1.7]$ GeV^2 and $s_0 = 5.5 \pm 0.2$ GeV^2, $M^2 \in [1.35, 1.75]$ GeV^2, respectively. There is little change of the extremum corresponding to a different threshold for the two moments; we get $\langle \xi^2_{K_0^*} \rangle = 0.34$ and $\langle \xi^4_{K_0^*} \rangle = 0.23$. When the instanton effects are involved, the moments change significantly, even at low density, $n_c = \frac{1}{2}$ fm^{-4}. Both moments decrease compared with the case $n_c = 0$; we obtain $\langle \xi^2_{K_0^*} \rangle = 0.17$, $\langle \xi^4_{K_0^*} \rangle = 0.13$. If the instanton density increases further, for instance, $n_c = 1$ fm^{-4}, we find there is flipping of the second and fourth moments,

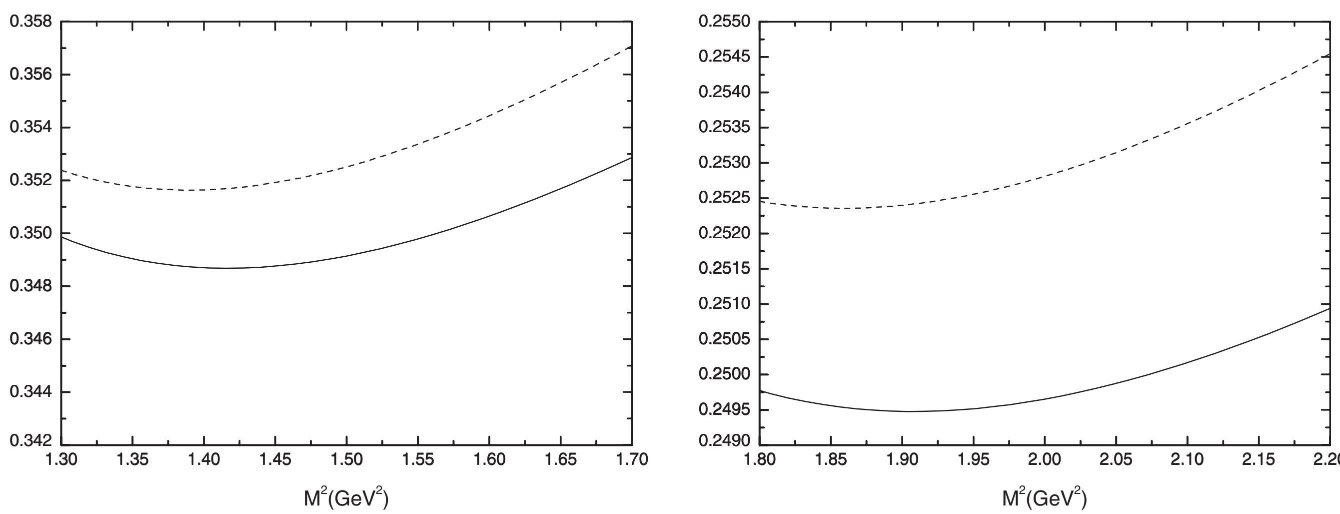

FIG. 3. Moments $\langle \xi_s^2 \rangle$ (left panel) and $\langle \xi_s^4 \rangle$ (right panel) of $f_0(1370)$ at $\mu = 1$ GeV from instanton-free sum rules.

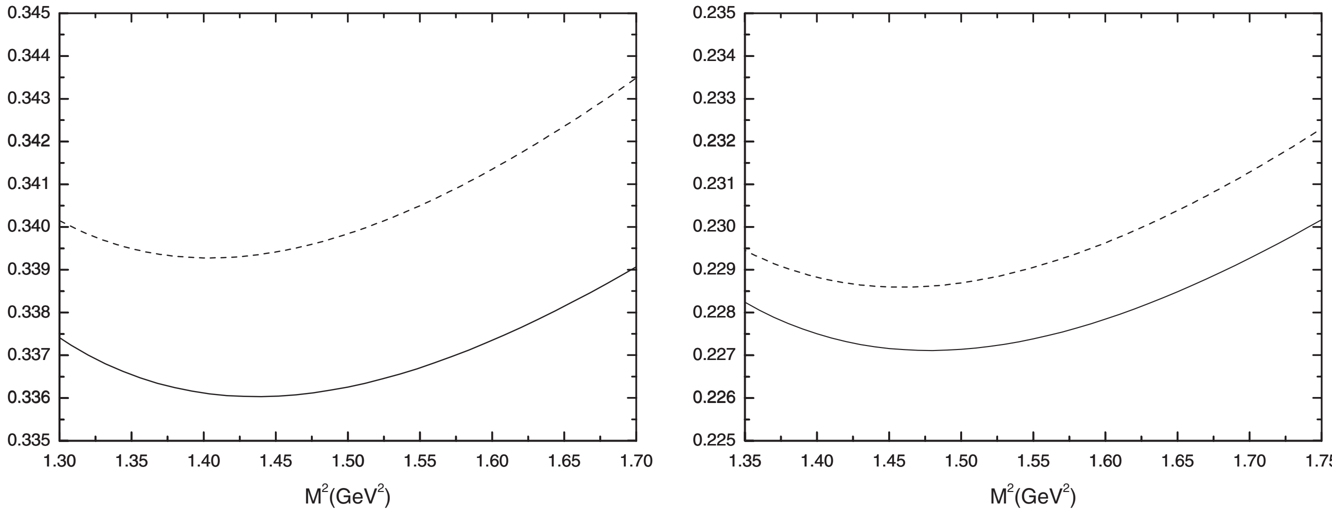

FIG. 4. Moments $\langle \xi_s^2 \rangle$ (left panel) and $\langle \xi_s^4 \rangle$ (right panel) of $K_0^*(1430)$ from instanton-free sum rules.

$\langle \xi^2_{K_0^*} \rangle = 0.01$ and $\langle \xi^4_{K_0^*} \rangle = 0.04$, which is unsatisfactory since it breaks the convergence. However, at higher density, $n_c = 2\ \mathrm{fm}^{-4}$, the convergence can recover, while it develops a negative value, $-\langle \xi^2_{K_0^*} \rangle = -0.32$ and $\langle \xi^4_{K_0^*} \rangle = -0.16$. The LCDAs of $K_0^*(1430)$ for different instanton densities are plotted in Fig. 6. It is clear that the profile of LCDAs of $K_0^*(1430)$ with $n_c \neq 0$ just reverse to that of the pion and $f_0(1370)$, which indicates the conspiracy of the chirality and isospin dependence of instanton effects.

The case of $a_0(1450)$ runs parallel to $K_0^*(1430)$. The threshold and working window of $\langle \xi^2_{a_0} \rangle$ and $\langle \xi^4_{a_0} \rangle$ determined from instanton-free sum rules are $s_0 = 4.9 \pm 0.2\ \mathrm{GeV}^2$, $M^2 \in [1.5, 1.9]\ \mathrm{GeV}^2$ and $s_0 = 5.6 \pm 0.2\ \mathrm{GeV}^2$, $M^2 \in [1.8, 2.2]\ \mathrm{GeV}^2$, respectively. We obtain the extrema $\langle \xi^2_{a_0} \rangle = 0.39$ and $\langle \xi^4_{a_0} \rangle = 0.28$ within the threshold range. When $n_c = \frac{1}{2}\ \mathrm{fm}^{-4}$, both moments are still convergent and decrease to lower values: $\langle \xi^2_{a_0} \rangle = 0.20$, $\langle \xi^4_{a_0} \rangle = 0.17$. Similar to the case of $\xi_{K_0^*}$, at $n_c = 1\ \mathrm{fm}^{-4}$ we find there is also flipping of the second and fourth moments—$\langle \xi^2_{a_0} \rangle = 0.01$ and $\langle \xi^4_{a_0} \rangle = 0.07$—which shows the breakdown of the convergence of the moment. At $n_c = 2\ \mathrm{fm}^{-4}$ we can get $\langle \xi^2_{a_0} \rangle = -0.37$ and $\langle \xi^4_{a_0} \rangle = -0.13$, which shows a negative value, but the convergence recovers. The moments and LCDAs of a_0 are plotted in Fig. 5 and in the right panel of Fig. 6, respectively.

The instanton-involved twist-three LCDAs calculated in this work present nontrivial properties; to some extent, this

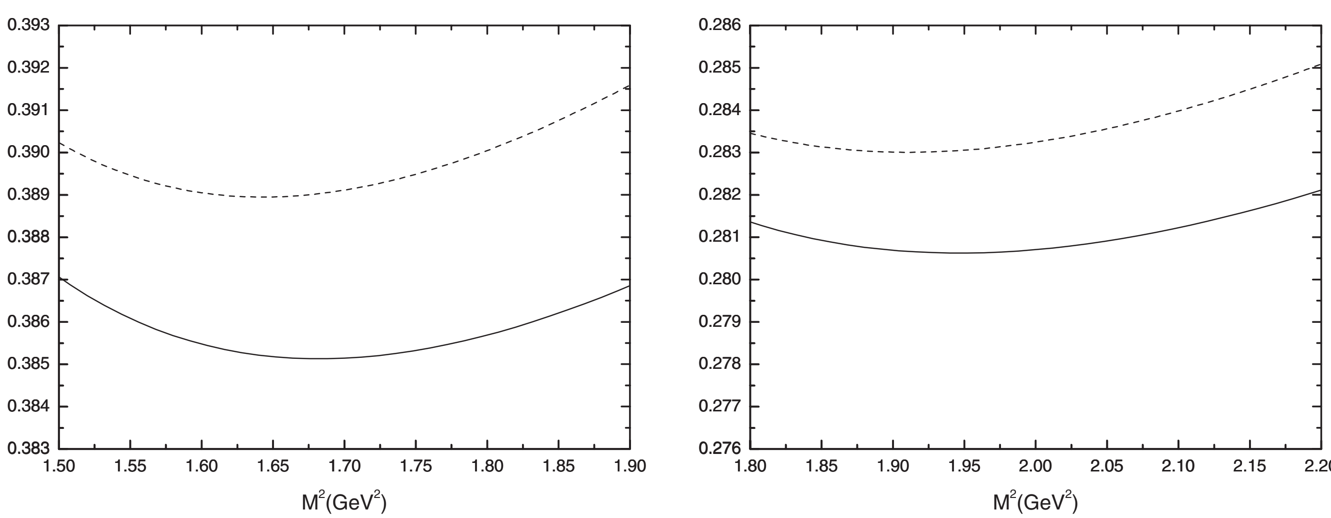

FIG. 5. Moments $\langle \xi_s^2 \rangle$ (left panel) and $\langle \xi_s^4 \rangle$ (right panel) of $a_0(1450)$ from instanton-free sum rules at $\mu = 1$ GeV.

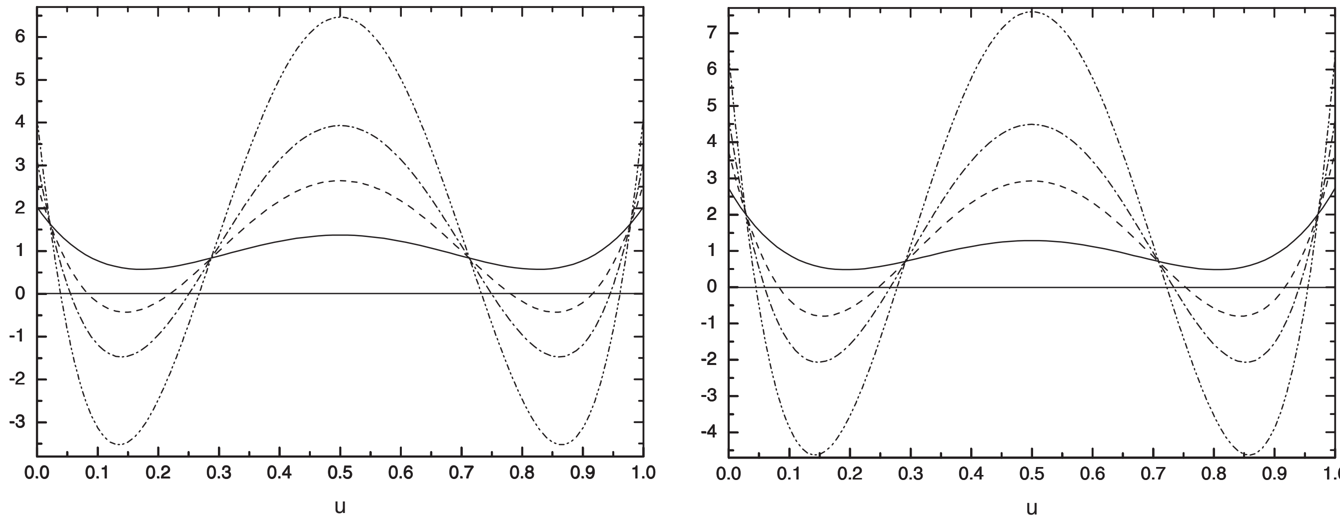

FIG. 6. Twist-three light-cone distribution amplitudes of $K_0^*(1430)$ (left panel) and $a_0(1450)$ (right panel) as a function of the momentum fraction u at $\mu = 1$ GeV for different instanton densities.

is unexpected. Although, in principle, non-positive-definite LCDAs are allowed, since here we lack a direct test of these LCDAs in exclusive processes, we would like to give a tentative discussion on the possible ingredients which are not mentioned above and may have some impact on our results.

(i) *Multi-instantons*.—For simplicity, we adopt the single-instanton approximation in our calculation; in fact, there might be multi-instanton contributions to the correlation function. But the results in Ref. [43] indicate that in singular gauge the multi-instanton contribution to the pion correlator is coincident with that given by the single-instanton approximation. While physical results should be gauge independent, we conjecture that it might be reasonable to utilize the single-instanton approximation in our calculation. On the other hand, one can easily see that it is very difficult to work out the instanton contribution in singular gauge for $n \neq 0$.

(ii) *Instanton density*.—It is obvious that the instanton density is crucial to obtain convergent results. Low instanton density is welcomed in our calculation. The density $\frac{1}{2}$ fm^{-4} is widely used under the single-instanton approximation, giving many reasonable results. In a way this indicates that low instanton density is consistent with a single approximation. In other words, it seems that high instanton density is questionable at the single-instanton approximation. Maybe this is the main reason that at high density the convergence is lost.

(iii) *Nonleading Fock states*.—From Eq. (2) one can see that we use the valence model to investigate twist-three LCDAs. The nonleading Fock states also may contribute a twist-three component via the mixing with other wave functions [30]. This correction can be added by using the renormalization group, so it is less relative to this work.

IV. CONCLUSIONS

In the present work we have investigated the instanton effect using the single-instanton approximation on the twist-three LCDAs of the pion, $f_0(1370)$, $K_0^*(1430)$, and $a_0(1450)$ from the valence quark model within the framework of QCD moment sum rules. Our results illustrate that the instanton-free twist-three LCDAs are always positive-definite, while the instanton-involved LCDAs show some nontrivial properties. We find that low instanton density is consistent with the method adopted in this work. Possible ingredients which might have an impact on the results are briefly discussed. Nonetheless, we hope these LCDAs will be helpful in some heavy-flavored exclusive processes since we conjecture that the instanton density may play some role as a tuning parameter in deriving experimentally favored results.

ACKNOWLEDGMENTS

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APPENDIX: VANISHING OF THE PURE ZERO-MODE CONTRIBUTION TO TENSOR MOMENT SUM RULES

We still work in four-dimensional Euclidean space. We consider the following two-point correlation function,

$$\int d^4x e^{iqx}\langle 0|TO_n(x)O^\dagger(0)|0\rangle$$
$$= \frac{1}{256\pi^4 m_1^* m_2^*}\int d^4x e^{iqx}\int d\rho n(\rho)\rho^4$$
$$\times \int d^4x_0 \frac{1}{(x_0^2+\rho^2)^3}\left\{\frac{1}{[(x-x_0)^2+\rho^2]^{3/2}}\right.$$
$$\left.\times (iz\cdot \overleftrightarrow{D}_I)^{n+1}\frac{1}{[(x-x_0)^2+\rho^2]^{3/2}}\right\}$$
$$\times \mathrm{tr}[\gamma_\mu\gamma_\nu(1-\gamma_5)\sigma_{\alpha\beta}\gamma_\sigma\gamma_\rho(1-\gamma_5)]\mathrm{tr}[\tau_\mu^+\tau_\nu^-\tau_\sigma^+\tau_\rho^-] \tag{A1}$$

with

$$O_n(x) = \bar{q}_{10}(x)\sigma_{\alpha\beta}(iz\cdot \overleftrightarrow{D}_I)^{n+1}q_{20}(x),$$
$$O^\dagger(0) = \bar{q}_{20}(0)q_{10},$$

where the integrations over the instanton collective coordinates are explicit. In fact, it is enough to concentrate on the trace part in Eq. (A1),

$$\mathrm{tr}[\tau_\mu^+\tau_\nu^-\tau_\sigma^+\tau_\rho^-] = \mathrm{tr}[(\delta_{\mu\nu}+i\eta_{a\mu\nu}\tau^a)(\delta_{\sigma\rho}+i\eta_{b\sigma\rho}\tau^b)]$$
$$= 2\delta_{\mu\nu}\delta_{\sigma\rho} - 2\eta_{a\mu\nu}\eta_{a\sigma\rho}$$
$$= 2(\delta_{\mu\nu}\delta_{\sigma\rho} - \delta_{\mu\sigma}\delta_{\nu\rho} + \delta_{\mu\rho}\delta_{\nu\sigma})$$
$$- 2\varepsilon_{\mu\nu\sigma\rho}, \tag{A2}$$

where we use Eqs. (23) and (24). Then, combining the above result with the trace over the γ matrix, we have

$$[2(\delta_{\mu\nu}\delta_{\sigma\rho} - \delta_{\mu\sigma}\delta_{\nu\rho} + \delta_{\mu\rho}\delta_{\nu\sigma}) - 2\varepsilon_{\mu\nu\sigma\rho}]$$
$$\times 2\,\mathrm{tr}[\gamma_\mu\gamma_\nu(1-\gamma_5)\sigma_{\alpha\beta}\gamma_\sigma\gamma_\rho]$$
$$= -2\varepsilon_{\mu\nu\sigma\rho}\times 2\,\mathrm{tr}[\gamma_\mu\gamma_\nu(1-\gamma_5)\sigma_{\alpha\beta}\gamma_\sigma\gamma_\rho], \tag{A3}$$

while in four-dimensional Euclidean space

$$\gamma_5 = \frac{1}{4!}\varepsilon_{\mu\nu\sigma\rho}\gamma_\mu\gamma_\nu\gamma_\sigma\gamma_\rho.$$

Now one can see the whole trace part in Eq. (A1) vanishes; thus, there is no pure zero-mode contribution to tensor moment sum rules, and consequently, there is no instanton effect on the tensor LCDA.

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