Observation concerning the process dependence of the Sivers functions

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The $k_\perp$ moment of a quark's Sivers function is known to be related to the corresponding twist-three quark-gluon correlation function $T_{q,F}(x,x)$. The two functions have been extracted from data for single-spin asymmetries in semi-inclusive deep-inelastic scattering and in single-inclusive hadron production in pp collisions, respectively. Performing a consistent comparison of the extracted functions, we find that they show a "sign mismatch": while the magnitude of the functions is roughly consistent, the $k_\perp$ moment of the Sivers function has opposite sign from that of $T_{q,F}(x,x)$, both for up and for down quarks. Barring any inconsistencies in our theoretical understanding of the Sivers functions and their process dependence, the implication of this mismatch is that either the Sivers effect is not dominantly responsible for the observed single-spin asymmetries in pp collisions or the current semi-inclusive lepton scattering data do not sufficiently constrain the $k_\perp$ moment of the quark Sivers functions. Both possibilities strengthen the case for further experimental investigations of single-spin asymmetries in high-energy pp and ep scattering.

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I. INTRODUCTION

Since the observation of surprisingly large single transverse spin asymmetries (SSAs) in $p^\uparrow p \to \pi X$ at Femilab in the 1980s [1], the exploration of the physics behind the observed SSAs has become a very active research branch in hadron physics, and has played an important role in our efforts to understand QCD and nucleon structure [2,3]. Defined as $A_N = (\sigma(s_\perp) - \sigma(-s_\perp))/(\sigma(s_\perp) + \sigma(-s_\perp))$, the ratio of the difference and the sum of the cross sections when the hadron's spin vector $s_\perp$ is flipped, significant SSAs have by now been consistently observed in various experiments at different collision energies. These include semi-inclusive hadron production at low transverse momentum $P_{h\perp}$ in deep-inelastic scattering, $\ell N^\uparrow \to \ell' hX$, by the HERMES Collaboration at DESY [4], COMPASS at CERN [5], and CLAS at Jefferson Lab [6], as well as inclusive single-hadron production at high $P_{h\perp}$ in hadron-hadron collisions, $p^\uparrow p \to hX$, by the STAR, PHENIX, and BRAHMS collaborations at RHIC [7]. The observed large size of SSAs in hadronic scattering initially presented a challenge for QCD theorists [8]. Later two complementary mechanisms were proposed to describe the measured SSAs, and both of them have been quite successful phenomenologically [9–17].

One mechanism relies on the so-called transverse-momentum dependent (TMD) factorization [18–25], and describes the SSAs in terms of the spin-dependent part of TMD parton distribution functions (PDFs), known as the Sivers functions [26], or TMD fragmentation functions (FFs), known as the Collins functions [27]. This TMD factorization approach is suitable for evaluating the SSAs of scattering processes with two very different momentum scales, $Q_1 \gg Q_2 \gtrsim \Lambda_{\rm QCD}$. The larger scale Q_1 is necessary for using perturbative QCD, while the lower scale Q_2 makes the observable sensitive to the parton's transverse motion. For example, the SSAs of hadron production at low $P_{h\perp}$ in lepton-hadron semi-inclusive deep-inelastic scattering (SIDIS) have this characteristic property: $Q \gg P_{h\perp} \sim \Lambda_{\rm QCD}$, and can be studied within the TMD factorization approach.

The other mechanism generalizes the successful leading-power QCD collinear factorization formalism to the next-to-leading power in the expansion in $1/Q$, where Q is the large momentum transfer of the collision, and describes the SSAs in terms of twist-3 transverse-spin-dependent three-parton correlation functions [28–31], or a combination of the transversity distribution and three-parton fragmentation functions [30–32]. This so-called twist-3 collinear factorization approach is more relevant to the SSAs for processes in which all observed momentum transfers Q are much larger than $\Lambda_{\rm QCD}$. This applies, for example, to the SSAs of inclusive single-hadron production at high $P_{h\perp}$ in $p^\uparrow p$ collisions. Although the two mechanisms describe the SSAs in two very different

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kinematic domains, they were shown to be equivalent in the overlap region where they both apply, and they thus provide a unified QCD description for the SSAs [33].

One of the potentially important contributions to the SSAs is the Sivers effect, which is generated by the initial- and final-state interactions between the struck parton and the spectators or the remnant of the polarized hadron [21]. The interactions provide the necessary phase that leads to the nonvanishing SSAs. In the TMD factorization approach, the role of these interactions is accounted for by including the appropriate color gauge links into the definition of the TMD parton distributions, whose spin-dependent part defines the Sivers functions [23,34,35]. Since the details of the initial- and final-state interactions depend on the color flow of the scattering process, the form of the gauge links including the phase of the interactions is process dependent. Since the gauge links are included in the definition of the TMD parton distributions, the Sivers functions, too, are found to be process dependent [24]. Because of parity and time-reversal invariance of the strong interactions, the process dependence of the Sivers functions is effectively reduced to a sign change between their definitions in SIDIS and in Drell-Yan lepton-pair production in $p^\uparrow p$ collisions [23,35]. The predictive power of the TMD factorization approach relies on this modified universality of the Sivers functions.

On the other hand, in the twist-3 collinear factorization approach, the process dependence of the initial- and final-state interactions is absorbed into the short-distance perturbative hard-part functions, while keeping the relevant twist-3 three-parton correlation functions universal or process independent. The necessary phase for generating the nonvanishing SSAs arises from the quantum interference between a scattering amplitude with one active collinear parton and an amplitude with two active collinear partons. The SSAs are therefore proportional to the nonprobabilistic three-parton correlation functions. Unlike the TMD parton distributions, which at given transverse momentum provide direct information on a parton's transverse motion, the twist-3 three-parton correlation functions provide a net asymmetry of the parton's transverse motion, after integration over all values of the parton's transverse momentum. As a result, the twist-3 three-parton correlation functions have a close connection with the transverse momentum $k_\perp$ moment of TMD parton distributions. More precisely, the twist-3 quark-gluon correlation function, $T_{q,F}(x, x)$, often referred to as Efremov-Teryaev-Qiu-Sterman (ETQS) function, is equal to the first $k_\perp$ moment of the quark Sivers function $f_{1T}^{\perp q}(x, k_\perp^2)$ probed in SIDIS (or Drell-Yan) processes [23,33,36].

Following the tremendous progress in experimental measurements of SSAs in recent years, the quark Sivers functions and the ETQS functions have been extracted for various quark flavors from the single-spin asymmetries in SIDIS and in pp scattering, respectively. In this paper, we examine the existing parametrizations of these two functions to see whether the first $k_\perp$ moments of Sivers functions are consistent with the existing twist-3 ETQS functions. Taking the quark Sivers functions $f_{1T}^{\perp q}(x, k_\perp^2)$ extracted from SIDIS [10,11], we evaluate their first $k_\perp$ moments, and derive the ETQS functions $T_{q,F}(x, x)$ with the help of the operator relation between the two functions [23,33,36]. We then compare the resulting "indirectly" obtained quark-gluon correlation functions with those "directly" extracted from the global fit [14] to the SSAs for inclusive single-hadron production in $p^\uparrow p$ collisions. In doing so, we first observe that the sign convention adopted for the SSA in $p^\uparrow p \to hX$ in the previous literature [14,29,30] is in fact not consistent with that used for the experimental data. As a result, the signs of the $T_{q,F}(x, x)$ functions extracted in [14] need to be reversed. After this adjustment, we find that the twist-3 correlation functions $T_{q,F}(x, x)$ obtained in the two different ways have conflicting signs.

The rest of our paper is organized as follows. In the next section, we briefly review the definitions of the quark Sivers functions and the twist-3 quark-gluon correlation functions (or ETQS functions). We recall the operator relation between the $k_\perp$ moment of the Sivers functions and the ETQS functions, and discuss its limitations and the corrections to it. In Sec. III, we present our findings regarding the sign "mismatch" between the existing parametrizations of quark Sivers functions and twist-3 quark-gluon correlation functions. We discuss the possible origins of this mismatch, and potential remedies. We also address the implications for phenomenology and propose further measurements to test the mechanisms for SSAs in hadronic processes. Finally, we give our conclusions and summary in Sec. IV. An Appendix describes the derivation of the correct signs of the ETQS functions in single-inclusive hadron production in pp scattering.

II. THE SIVERS FUNCTIONS AND THE ETQS FUNCTIONS

In this section, we recall the definitions and relations between the quark Sivers functions and the twist-3 quark-gluon correlation functions, or ETQS functions. We use light-cone coordinates with the two lightlike vectors,

$$\bar{n}^\mu = [1^+, 0^-, 0_\perp], \qquad n^\mu = [0^+, 1^-, 0_\perp], \qquad (1)$$

to project out the light-cone components: $v^+ = v^\mu g_{\mu\nu} n^\nu$ and $v^- = v^\mu g_{\mu\nu} \bar{n}^\nu$ of any four-vector v^μ. For the fully antisymmetric tensor $\epsilon^{\mu\nu\rho\sigma}$, we adopt the convention $\epsilon^{0123} = 1$. We choose a frame in which the momentum of the transversely polarized hadron, p, is in the "$+z$" direction, with no transverse components: $p^\mu = p^+ \bar{n}^\mu$.

The quark Sivers functions for SIDIS kinematics with the transversely polarized proton moving in the $+z$ direction is defined through the following quark-field correlator [37]:

$$\mathcal{M}(x, k_\perp) = \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{ixp^+\xi^-} e^{-i\vec{k}_\perp\cdot\vec{\xi}_\perp} \langle p, s_\perp | \bar{\psi}(0) W_{[0,\xi]} \psi(\xi) | p, s_\perp\rangle|_{\xi^+=0}, \tag{2}$$

where

$$W_{[0,\xi]} = \mathcal{P}\exp\left[ig\int_0^\infty d\eta^- A^+(\eta^-, 0_\perp)\right]\mathcal{P}\exp\left[ig\int_{0_\perp}^{\xi_\perp} d\eta_\perp A_\perp(\infty^-, \eta_\perp)\right]\mathcal{P}\exp\left[ig\int_\infty^{\xi^-} d\eta^- A^+(\eta^-, \xi_\perp)\right] \tag{3}$$

is the gauge link consistent with the SIDIS process and $\mathcal{P}$ indicates path ordering [34]. We note that the gauge link depends on the transverse separation $\xi_\perp$ of the two field operators, which is responsible for the process dependence of TMD parton distribution functions [35]. Here it is worth pointing out that a different sign convention for the strong coupling constant g (for the interaction between the quark and the gluon) would lead to a different sign in the exponent of the gauge link in Eq. (3) (i.e., from ig to $-ig$). For deriving Eq. (3) we adopted the convention of the covariant derivative D_μ as

$$D_\mu = \partial_\mu + igA_\mu, \tag{4}$$

for the relevant part of the QCD Lagrangian density $\mathcal{L} = \bar{\psi} i\gamma^\mu D_\mu \psi$. Different conventions for D_μ (such as $D_\mu = \partial_\mu - igA_\mu$) exist in the literature and in textbooks. Different conventions usually do not introduce any difference for a cross section, which has an even power in g. However, the single transverse spin asymmetry is proportional to the difference of two cross sections with the spin flipped, and the asymmetry is a consequence of an interference between scattering amplitudes of different phases, which is linearly proportional to ig. Therefore, one has to use the convention consistently in the theoretical definition and calculation of the single transverse spin asymmetry.

Following the so-called Trento convention [38], the correlator $\mathcal{M}(x, k_\perp)$ can be expanded as

$$\mathcal{M}(x,k_\perp) = \frac{1}{2}\left[f_1(x,k_\perp^2)\not{\bar{n}} + \frac{1}{M} f_{1T}^\perp(x,k_\perp^2)\epsilon^{\mu\nu\rho\sigma}\gamma_\mu \bar{n}_\nu k_{\perp\rho} s_{\perp\sigma}\right], \tag{5}$$

where M is the nucleon mass, $f_1(x, k_\perp^2)$ is the spin-averaged TMD PDF, and $f_{1T}^\perp(x, k_\perp^2)$ is the quark Sivers function. We note that a different convention for the Sivers function is commonly adopted in the phenomenological studies by the Torino group [10,11]. Here a function $\Delta^N f_{q/A^\uparrow}(x, k_\perp)$ is introduced which is defined from

$$\begin{aligned} f_{q/A^\uparrow}(x, k_\perp) &\equiv \mathrm{Tr}[\tfrac{1}{2}\not{n}\mathcal{M}(x, k_\perp)] \\ &= f_1(x, k_\perp^2) + \tfrac{1}{2}\Delta^N f_{q/A^\uparrow}(x, k_\perp)\mathbf{s}_\perp \cdot (\hat{\mathbf{p}} \times \hat{\mathbf{k}}_\perp). \end{aligned} \tag{6}$$

The relation between $\Delta^N f_{q/A^\uparrow}$ and the Sivers function in the Trento convention is

$$\Delta^N f_{q/A^\uparrow}(x, k_\perp) = -\frac{2k_\perp}{M} f_{1T}^{\perp q}(x, k_\perp^2). \tag{7}$$

In the twist-3 collinear factorization approach, the ETQS function $T_{q,F}(x, x)$ is defined as [14,16]

$$\begin{aligned} T_{q,F}(x,x) &= \int \frac{d\xi^- d\zeta^-}{4\pi} e^{ixp^+\xi^-} \\ &\times \langle p, s | \bar{\psi}(0) V_{[0,\zeta]} \gamma^+ [\epsilon^{s_\perp\sigma n\bar{n}} F_\sigma{}^+(\zeta^-)] V_{[\zeta,\xi]} \psi(\xi^-) | p, s\rangle, \end{aligned} \tag{8}$$

where $V_{[0,\zeta]}$ and $V_{[\zeta,\xi]}$ are the gauge links along the "$-$" light-cone direction and are given by

$$V_{[\zeta,\xi]} = \mathcal{P}\exp\left[ig\int_{\zeta^-}^{\xi^-} d\eta^- A^+(\eta^-)\right]. \tag{9}$$

Here the sign convention for coupling constant g is the same as that in Eq. (4). Choice of, for example, the convention with $D_\mu = \partial_\mu - igA_\mu$ would change the sign of the exponent. Within the collinear factorization approach, it is assumed that the typical transverse momentum of all active partons is much smaller than the hard scale of the scattering process, Q. Up to power corrections in $1/Q$, the transverse momenta of all active partons are completely integrated into nonperturbative PDFs, FFs, or the correlation functions. Consequently, unlike for the TMD distributions, all field operators defining the nonperturbative functions in the collinear factorization approach are evaluated at the same light-cone separation with zero "$+$" and "$\perp$" components, as shown, for example, in Eq. (8).

Since the quark-gluon correlation functions in the collinear factorization approach have all their active partons' transverse momenta integrated, these correlation functions can be related to $k_\perp$ moments of the TMD parton distribution functions. It was shown at the operator level [23,33,36] that the ETQS function $T_{q,F}(x, x)$ is closely related to the $k_\perp$ moment of Sivers function:

$$gT_{q,F}(x, x) = -\int d^2k_\perp \frac{|k_\perp|^2}{M} f_{1T}^{\perp q}(x, k_\perp^2)|_{\mathrm{SIDIS}}, \tag{10}$$

where the subscript "SIDIS" emphasizes that the Sivers functions here are probed in the SIDIS process. We stress again the importance of the sign convention for the coupling constant g in the definition of the gauge link. If the sign convention used to define $T_{q,F}(x, x)$ is different from that in the definition of $f_{1T}^{\perp q}(x, k_\perp^2)$, the difference will introduce an extra factor "-1" in the relation between these two functions, so that there will be no minus sign on the right-hand side of Eq. (10).

We emphasize that the operator definition in Eq. (8) does not completely fix the quark-gluon correlation function $T_{q,F}(x,x)$, unless the renormalization scheme is specified. As is well known from the case of ordinary PDFs, the matrix element in Eq. (8) is ultraviolet (UV) divergent [39]. Like in the case of PDFs, the quark-gluon correlation function is really defined in terms of the QCD factorization formalism. The leading UV divergent (the large $k_\perp$) region of the matrix element on the right-hand side of Eq. (8) corresponds to the region of phase space with large parton virtuality, and is required by factorization to be moved from the matrix element into the perturbatively calculated short-distance functions. The removal or subtraction of the UV divergence is not unique, which leads to the factorization scheme and scale (μ) dependence of the correlation functions $T_{q,F}(x,x,\mu)$ [40]. In this way, also the relation in Eq. (10) is subject to the UV subtractions and the adopted factorization scheme, and hence not a unique identity. That said, the relation (10) provides a natural "zeroth-order" connection between the Sivers and the ETQS functions. It plays an important role in establishing the consistency between the TMD factorization approach and the collinear twist-three quark-gluon correlation approach in the descriptions of the SSAs in SIDIS and the Drell-Yan process [33]. It also is a useful starting point for phenomenological studies and is of much help in testing the various constraints on the quark Sivers and quark-gluon correlation functions. In the following, we will therefore make use of relation (10), keeping in mind, however, the caveats we have made regarding UV renormalization.

III. THE "SIGN MISMATCH"

The quark Sivers functions $f_{1T}^{\perp q}(x,k_\perp^2)$ [or equivalently, $\Delta^N f_{q/A^\uparrow}(x,k_\perp)$] and the twist-3 quark-gluon correlation functions $T_{q,F}(x,x)$ have been extracted from experimental data on SSAs for single-hadron production in SIDIS and in hadron-hadron scattering, respectively. In this section, we compare the existing parametrizations of these two functions and present our findings concerning the "sign mismatch." We also introduce and discuss various loopholes that might resolve the apparent inconsistency.

So far, the quark Sivers functions have been extracted from the $A_{UT}^{\sin(\phi_h-\phi_s)}$ azimuthal asymmetries in SIDIS. We consider two such parametrizations here. One is from Ref. [10] (we refer to it as "old Sivers"), the other one ("new Sivers") from Ref. [11]. They both parametrize the spin-averaged TMD PDFs $f_1^q(x,k_\perp^2)$ and Sivers functions $\Delta^N f_{q/h^\uparrow}(x,k_\perp)$ for each quark flavor q in the form

$$f_1^q(x,k_\perp^2) = f_1^q(x)g(k_\perp), \tag{11}$$

$$\Delta^N f_{q/h^\uparrow}(x,k_\perp) = 2\mathcal{N}_q(x)f_1^q(x)h(k_\perp)g(k_\perp), \tag{12}$$

where $f_1^q(x)$ is the quark's spin-averaged collinear PDF, $\mathcal{N}_q(x)$ is a fitted function whose functional form is not relevant for our discussion below, and $g(k_\perp)$ is assumed to have a Gaussian form,

$$g(k_\perp) = \frac{1}{\pi\langle k_\perp^2\rangle}e^{-k_\perp^2/\langle k_\perp^2\rangle}, \tag{13}$$

with a fitting parameter $\langle k_\perp^2\rangle$ for the width. However, the two parametrizations adopt different functional forms for the $k_\perp$ dependence of the Sivers function:

$$\text{old Sivers: } h(k_\perp) = \frac{2k_\perp M_0}{k_\perp^2 + M_0^2}, \tag{14}$$

$$\text{new Sivers: } h(k_\perp) = \sqrt{2e}\frac{k_\perp}{M_1}e^{-k_\perp^2/M_1^2}, \tag{15}$$

where M_0 and M_1 are fitted parameters.

Since for both parametrizations the $k_\perp$ dependence is assumed to be decoupled from the x dependence, we can derive the x dependence of the associated twist-3 quark-gluon correlation $T_{q,F}(x,x)$ analytically, using the relation in Eq. (10). By substituting the parametrization of the Sivers function in Eq. (12) into the right-hand side of Eq. (10), and using the fitting parameters extracted in Refs. [10,11], we obtain the following two parametrizations for the correlation function $T_{q,F}(x,x)$:

$$gT_{q,F}(x,x)|_{\text{old Sivers}} = 0.40f_1^q(x)\mathcal{N}_q(x)|_{\text{old}}, \tag{16}$$

$$gT_{q,F}(x,x)|_{\text{new Sivers}} = 0.33f_1^q(x)\mathcal{N}_q(x)|_{\text{new}}. \tag{17}$$

From the existing data, the best constrained Sivers functions are those of u and d quarks. Using the fitted functions $\mathcal{N}_q(x)|_{\text{old}}$ and $\mathcal{N}_q(x)|_{\text{new}}$ from Refs. [10,11], respectively, we plot the derived quark-gluon correlation functions $xgT_{u,F}(x,x)$ (left) and $xgT_{d,F}(x,x)$ (right) in Fig. 1. The dashed lines are for the quark-gluon correlation functions obtained by using the new Sivers parametrization, while the dotted lines are for the old Sivers parametrization. We find that for these indirectly obtained quark-gluon correlation functions, $T_{u,F}(x,x)$ is positive, while $T_{d,F}(x,x)$ is negative.

On the other hand, the ETQS function $T_{q,F}(x,x)$ can be directly extracted from data on SSAs for inclusive single-hadron production in hadronic collisions, $p^\uparrow p \to h(P_{h\perp},y) + X$, assuming these asymmetries are predominantly generated by the Sivers effect (or rather, its twist-3 counterpart). Such SSAs have been measured at sufficiently large transverse momentum $P_{h\perp}$ by the E704 Collaboration at Fermilab [1], and the STAR, PHENIX, and BRAHMS collaborations at RHIC [7]. Since they depend only on one large momentum scale $P_{h\perp}$, these SSAs are better studied in the collinear factorization approach, where they may be generated by three possible mechanisms: (1) the twist-3 quark-gluon and trigluon correlation functions of the polarized hadron, (2) the transversity distribution of the polarized hadron combined with

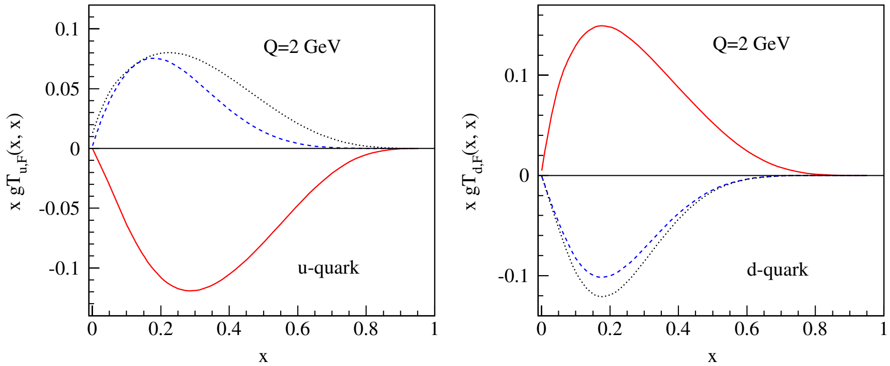

FIG. 1 (color online). The quark-gluon correlation function $gT_{q,F}(x, x)$ as a function of momentum fraction x for u quarks (left) and d quarks (right). The dashed (dotted) lines are $gT_{q,F}(x, x)|_{\text{new Sivers}}$ $[gT_{q,F}(x, x)|_{\text{old Sivers}}]$ obtained by taking the $k_\perp$ moments of the corresponding quark Sivers functions according to the right-hand side of Eq. (10). The solid lines represent the correlation functions extracted directly from data on SSAs for inclusive pion production in proton-proton collisions, $p^\uparrow p \to \pi + X$ [14], after correcting for the sign convention (see text).

the twist-3 quark-gluon fragmentation functions to the observed hadron, and (3) the transversity distribution combined with possible twist-3 unpolarized quark-gluon correlation functions [30]. It was found that the third mechanism only makes a small contribution [31]. By assuming that the observed SSAs are mainly generated by the ETQS functions $T_{q,F}(x, x)$, a set of $T_{q,F}(x, x)$ was extracted by a global fitting procedure [14]. In the course of our investigations, we have revisited the sign convention adopted in [14], which was based on the earlier work [14,29,30]. We have discovered that the convention was at odds with that chosen in the experimental studies. The inconsistency can be traced to the value of the contracted Levi-Cività tensor appropriate for the spin asymmetry. We provide a detailed discussion of this issue in the Appendix. Correcting the sign convention of [14,29,30] means that one needs to change the signs of the $T_{q,F}(x, x)$ functions extracted in [14]. We plot the resulting directly extracted $T_{q,F}(x, x)$ functions as solid lines in Fig. 1, along with the previous ones derived indirectly from the $k_\perp$ moment of the quark Sivers functions. Surprisingly, we find that the two sets of functions have *opposite signs*, both for up and for down quarks.

At first sight, it may seem that we have created a problem where none used to be. After all, the sign mismatch we find becomes apparent only after we have changed the signs of the $T_{q,F}(x, x)$ functions of [14]. However, the basic problem is easy to see: as we discussed in the Introduction, the Sivers contributions to the single-spin asymmetries depend on initial or final-state interactions in the scattering processes. The SSA in SIDIS comes from a final-state interaction. A negative up-quark Sivers function is known to generate a positive SIDIS spin asymmetry for π^+ production. In $p^\uparrow p \to \pi^+ X$ at forward rapidities, however, the main partonic channel is $ug \to ug$, for which initial-state interactions play the dominant role, resulting in negative partonic hard-scattering functions. Therefore, if the Sivers mechanism [or its twist-3 variant via Eq. (10)] is primarily responsible for the SSA in this process, one would expect a negative asymmetry for π^+, contrary to what is observed. Thus, the $T_{q,F}(x, x)$ functions needed to describe the RHIC single-spin asymmetries cannot have the signs suggested by Eq. (10). We note that in these considerations, one has to carefully take into account the experimental definitions of the SSAs; see the Appendix for some details.

There are two main caveats regarding the sign mismatch. The first one is that the integral over $k_\perp$ in Eq. (10) might produce a different sign from that of $f_{1T}^{\perp q}(x, k_\perp^2)$ itself in the region of $k_\perp$ where it is constrained by data. The HERMES SIDIS data that are mostly relevant for the extraction of $f_{1T}^{\perp q}(x, k_\perp^2)$ are at a relatively modest $Q^2 \sim 2.4\ \text{GeV}^2$. Since the TMD factorization formalism is valid only for $k_\perp \ll Q$, the data constrain the function and its sign only at very low $k_\perp \sim \Lambda_{\text{QCD}}$. The existing parametrizations of the quark Sivers functions [10,11] assume a purely Gaussian form of the $k_\perp$ dependence and hence would not allow a sign change of the function at some $k_\perp$. This leads to significant uncertainties in the determination of the twist-3 quark-gluon correlation functions via Eq. (10), because taking the $k_\perp$ moment enhances the contribution from the unknown larger-$k_\perp$ region. Also, the issue of UV renormalization discussed in the previous section becomes relevant for the $k_\perp$ moment. We note that future SIDIS experiments at an electron ion collider [41] would have the kinematic reach to precisely map out the $k_\perp$ dependence, and to allow measurements of the transverse-momentum weighted asymmetries, providing direct access to the twist-3 quark-gluon correlation functions. In this way, reliable comparisons with the correlation functions extracted from pp collisions would become possible.

It is worth keeping in mind that the SIDIS and pp single-spin asymmetry data also probe slightly different values of x. The former reach up to $x \sim 0.4$, while the latter mostly access yet larger values, $x \sim 0.6$. While it is in principle possible that a rapid sign change could occur towards large x which would explain the mismatch, there is nothing in the SIDIS data or the pp data with a sufficiently large x_F coverage that would indicate such a behavior, and we do not consider this to be a likely scenario.

The second possibility is that there are other significant contributions to the SSAs for single-hadron production in $p^\uparrow p$ collisions, besides the Sivers mechanism. In addition to the asymmetry due to the spin-dependent twist-3 quark-gluon correlation functions of the polarized hadron, the SSAs in hadronic collisions may also be generated at the hadronization stage by a combination of the transversity distribution of the polarized hadron and the twist-3 quark-gluon fragmentation functions [32], which is effectively a representation of the Collins effect in the collinear factorization approach. If this mechanism makes a large contribution to the hadronic SSAs, with sign opposite to that by the $T_{q,F}(x,x)$, it might explain the observed features. Unlike the measurement of SSAs in SIDIS, where the Sivers effect and the Collins effect can be separated by using different azimuthal angle weighting, the two effects cannot be separated in single-hadron inclusive production in hadronic collisions. Nonetheless, other measurements are available in pp scattering that would allow to disentangle them. The prime example is the Drell-Yan process, which allows direct access to the Sivers or $T_{q,F}(x,x)$ functions [23,35]. A similar role could be played by photon pair production [42]. Here we will briefly consider two further processes that have the advantage of being somewhat more copious at RHIC.

In order to get clean access to the quark-gluon correlation functions $T_{q,F}(x,x)$, we need observables that are not sensitive to the details of the hadronization stage. At RHIC, for example, direct photon production [25] at large transverse momentum $P_{h\perp}$ and inclusive single jet production at large transverse jet energy are two promising observables of this kind. Since the fragmentation contribution to prompt photon production at large p_T is much smaller than the direct contribution at RHIC energies, in particular if photon isolation cuts are imposed, the SSAs of these two observables could provide direct information on the twist-3 quark-gluon correlation functions, thus allowing one to see if their signs are consistent with those derived from Eq. (10).

In Fig. 2 we present our estimates for the SSAs for direct photon (left) and inclusive single jet production (right) in $p^\uparrow p$ collisions at $\sqrt{S} = 200$ GeV. We consider production in the forward region of the polarized proton, so that to a good approximation we only need to include the valence quark contribution for the polarized beam. For the relevant unpolarized PDFs, we use those specified in Refs. [10,11,14] correspondingly. The solid curves represent the SSAs calculated by using the *directly* extracted $T_{q,F}(x,x)$ [with $T_{u,F}(x,x) < 0$ and $T_{d,F}(x,x) > 0$], which were shown as the solid lines in Fig. 1. The dashed and dotted curves show the SSAs calculated by using the *indirectly* derived $T_{q,F}(x,x)$ from Eqs. (16) and (17), respectively, which again were shown by the same line patterns in Fig. 1 and have $T_{u,F}(x,x) > 0$ and $T_{d,F}(x,x) < 0$. The results in Fig. 2 demonstrate that positive A_N for direct photon and inclusive single jet production should be expected at RHIC if the directly extracted $T_{q,F}(x,x)$ are correct. If, on the other hand, the signs of $T_{q,F}(x,x)$ follow Eqs. (16) and (17), negative values for the A_N for the two processes are predicted. We note that direct photon and inclusive single jet production both receive contributions from the u and d quark ETQS functions. Since these have opposite signs and rather similar magnitude, their effects cancel to some degree for jet production. For photons, the situation is more favorable thanks to the weighting by the quark's charge squared, which explains why here the spin asymmetries are overall larger.

IV. SUMMARY

We have computed the $k_\perp$ moments for two parametrizations of up and down quark Sivers functions determined from semi-inclusive lepton scattering data given in [10,11]. These are related to the quark-gluon correlation functions

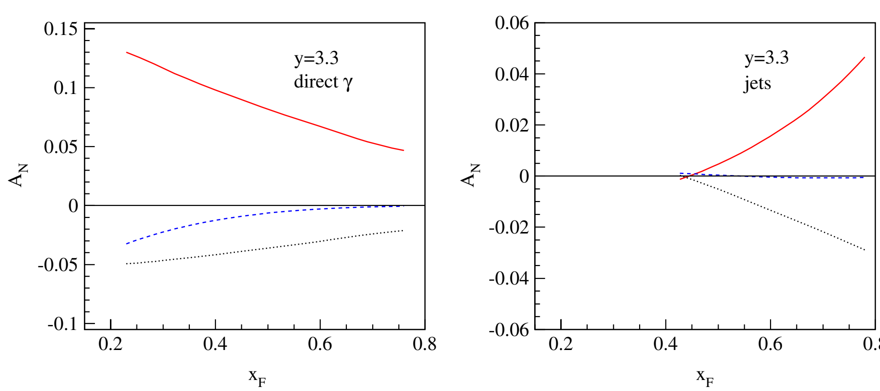

FIG. 2 (color online). The SSAs for direct photon (left) and single-inclusive jet (right) production in $p^\uparrow p$ collisions at $\sqrt{S} = 200$ GeV, as functions of x_F for rapidity $y = 3.3$. The various curves correspond to the $T_{q,F}(x,x)$ shown in Fig. 1.

$T_{q,F}(x,x)$ relevant for the description of single-spin asymmetries in single-hadron production in pp scattering. The latter have in the past been extracted from RHIC data [14]. Correcting an inconsistency in previous theoretical treatments of the spin asymmetries in pp scattering, we have found that the resulting $T_{q,F}(x,x)$ functions have signs opposite to those predicted from the analysis of the $k_\perp$ moments of the Sivers functions. We have discussed various possible explanations for this apparent discrepancy.

Our finding highlights the importance of additional measurements of single-spin asymmetries. Measurements of the $k_\perp$ dependence of the Sivers functions with wide kinematic reach would be feasible at an electron ion collider and should shed light on the contributions from various $k_\perp$ regions to the moment of the Sivers functions. We have also shown that A_N measurements for jet and direct photon production in pp collisions at RHIC should be valuable tools for a cleaner determination of the quark-gluon correlation functions $T_{q,F}(x,x)$.

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APPENDIX: THE SIGN OF $T_{q,F}(x,x)$ IN INCLUSIVE HADRON PRODUCTION

In this Appendix, we demonstrate why the SSA data for $p^\uparrow p \to hX$ require $T_{u,F}(x,x) < 0$ and $T_{d,F}(x,x) > 0$, if the ETQS functions are the dominant sources of the observed asymmetries.

We start with the QCD factorization formalism for the spin-averaged cross section for inclusive single particle production in hadronic collisions, $A^\uparrow(S_\perp) + B \to h(P_{h\perp}) + X$:

$$E_h \frac{d\sigma}{d^3P_h} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int \frac{dz}{z^2} D_{c\to h}(z) \int \frac{dx'}{x'} f_{b/B}(x') \times \int \frac{dx}{x} f_{a/A}(x) H^U_{ab\to c}(\hat{s},\hat{t},\hat{u})\delta(\hat{s}+\hat{t}+\hat{u}), \tag{A1}$$

where $f_{a/A}(x)$ and $f_{b/B}(x')$ are the PDFs, $D_{c\to h}(z)$ are the FFs, and $H^U_{ab\to c}$ are the partonic hard-scattering functions, with $\hat{s}$, $\hat{t}$, and $\hat{u}$ the Mandelstam variables at the parton level. Including only the contributions by the twist-3 quark-gluon correlation functions, the spin-dependent cross section $d\Delta\sigma(s_\perp) \equiv [d\sigma(s_\perp) - d\sigma(-s_\perp)]/2$ is given by

$$E_h \frac{d\Delta\sigma(s_\perp)}{d^3P_h} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int \frac{dz}{z^2} D_{c\to h}(z) \int \frac{dx'}{x'} f_{b/B}(x') \int \frac{dx}{x} \sqrt{4\pi\alpha_s} \left(\frac{\epsilon^{P_{h\perp}s_\perp n\bar{n}}}{z\hat{u}}\right)\left[T_{a,F}(x,x) - x\frac{d}{dx}T_{a,F}(x,x)\right] H_{ab\to c}(\hat{s},\hat{t},\hat{u})\delta(\hat{s}+\hat{t}+\hat{u}), \tag{A2}$$

where the relevant hard-scattering functions $H_{ab\to c}(\hat{s},\hat{t},\hat{u})$ can be written as

$$H_{ab\to c}(\hat{s},\hat{t},\hat{u}) = H^I_{ab\to c}(\hat{s},\hat{t},\hat{u}) + H^F_{ab\to c}(\hat{s},\hat{t},\hat{u})\left(1+\frac{\hat{u}}{\hat{t}}\right), \tag{A3}$$

with $H^I_{ab\to c}$ and $H^F_{ab\to c}$ representing the contributions from initial- and final-state interactions, respectively. The explicit forms of $H^U_{ab\to c}$, $H^I_{ab\to c}$, and $H^F_{ab\to c}$ are given in [14]. It is important to point out that the spin-dependent cross section in Eq. (A2) is calculated from an interference between two partonic amplitudes. It thus depends on the sign convention for the coupling constant g; the form given in Eq. (A2) is based on the convention in Eq. (4). If one uses the other sign convention for the covariant derivative, there will be an extra minus sign appearing on the right-hand side of Eq. (A2), which would be compensated by an extra sign in Eq. (10).

The SSA, A_N, is given by the ratio of spin-dependent and spin-averaged cross sections:

$$E_h \frac{d\Delta\sigma(s_\perp)}{d^3P_h} \bigg/ E_h \frac{d\sigma}{d^3P_h} \equiv A_N \sin(\phi_s - \phi_h), \tag{A4}$$

where ϕ_h and ϕ_s are the azimuthal angles of the hadron transverse momentum $P_{h\perp}$ and the spin vector $s_\perp$, respectively. The absolute sign of A_N depends on the

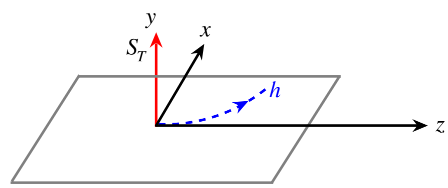

FIG. 3 (color online). Illustration of the sign convention for A_N: positive A_N means that more hadrons are produced to the *left* of the beam direction when the beam's spin is vertically *upward*.

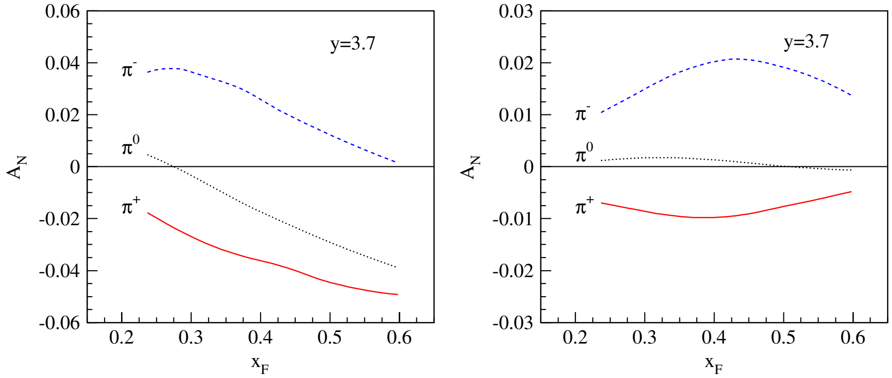

FIG. 4 (color online). The SSA, A_N, for inclusive single pion production in $p^\uparrow p \to \pi + X$ at $\sqrt{s} = 200$ GeV, as a function of x_F and at rapidity $y = 3.7$, evaluated by using the old Sivers functions in Eq. (16) (left), and the new Sivers functions in Eq. (17) (right).

choice of frame and the coordinate system. In the experiment the following convention is used: positive values of A_N correspond to a larger cross section for hadron production to the beam's *left* when the beam's proton spin is vertically *upward* [30], as sketched in Fig. 3. In the center-of-mass frame of A and B, a convenient coordinate system (consistent with the experimental convention) is given by choosing the polarized nucleon A to move along $+z$, the unpolarized B along $-z$, the spin vector $s_\perp$ along y, and the produced hadron's transverse momentum $P_{h\perp}$ along the x direction. In this frame, $\phi_h = 0$, $\phi_s = \pi/2$, and

$$\epsilon^{P_{h\perp} s_\perp n \bar{n}} = -|P_{h\perp}||s_\perp|. \quad \text{(A5)}$$

We note at this point that there is an overall sign error in [30] and consequently in [14], because in these papers the choice $\epsilon^{P_{h\perp} s_\perp n \bar{n}} > 0$ was made [see Eq. (73) of [30], in contrast to Eq. (A5) above].

In the forward direction, $qg \to qg$ is the dominant partonic scattering channel for inclusive single-hadron production. The corresponding hard-scattering functions are given by [14]

$$H^U_{qg\to qg} = \frac{N_c^2 - 1}{2N_c^2}\left[-\frac{\hat{s}}{\hat{u}} - \frac{\hat{u}}{\hat{s}}\right]\left[1 - \frac{2N_c^2}{N_c^2 - 1}\frac{\hat{s}\,\hat{u}}{\hat{t}^2}\right] \xrightarrow{|\hat{t}| \ll \hat{s} \sim |\hat{u}|} \left[\frac{2\hat{s}^2}{\hat{t}^2}\right], \quad \text{(A6)}$$

$$H^I_{qg\to qg} = \frac{1}{2(N_c^2 - 1)}\left[-\frac{\hat{s}}{\hat{u}} - \frac{\hat{u}}{\hat{s}}\right]\left[1 - N_c^2\frac{\hat{u}^2}{\hat{t}^2}\right] \xrightarrow{|\hat{t}| \ll \hat{s} \sim |\hat{u}|} \left[-\frac{N_c^2}{2(N_c^2 - 1)}\right]\left[\frac{2\hat{s}^2}{\hat{t}^2}\right], \quad \text{(A7)}$$

$$H^F_{qg\to qg} = \frac{1}{2N_c^2(N_c^2 - 1)}\left[-\frac{\hat{s}}{\hat{u}} - \frac{\hat{u}}{\hat{s}}\right]\left[1 + 2N_c^2\frac{\hat{s}\,\hat{u}}{\hat{t}^2}\right] \xrightarrow{|\hat{t}| \ll \hat{s} \sim |\hat{u}|} \left[-\frac{1}{N_c^2 - 1}\right]\left[\frac{2\hat{s}^2}{\hat{t}^2}\right]. \quad \text{(A8)}$$

This shows that both $H^I_{qg\to qg}$ and $H^F_{qg\to qg}$ have opposite sign to that of the spin-averaged hard-scattering function $H^U_{qg\to qg}$. Furthermore it is clear that the SSA in π^+ production is mainly sensitive to $T_{u,F}(x,x)$, while the one for π^- production probes $T_{d,F}(x,x)$. Since

$$\frac{\epsilon^{P_{h\perp} s_\perp n \bar{n}}}{\hat{u}} > 0, \quad \text{(A9)}$$

we conclude from Eq. (A2) that the observed positive SSAs for π^+ production indicates a *negative* $T_{u,F}(x,x)$, while the observed negative asymmetry for π^- production indicates a *positive* $T_{d,F}(x,x)$, as shown by the solid curves in Fig. 1.

To conclude this Appendix, we demonstrate the apparent "sign mismatch" again numerically, by evaluating the SSAs for inclusive single-hadron production using the ETQS functions indirectly derived via Eq. (10) from the quark Sivers functions in Eqs. (16) and (17). The results are shown in Fig. 4. As expected, the signs of the calculated SSAs are opposite to those observed experimentally.

[1] G. Bunce *et al.*, Phys. Rev. Lett. **36**, 1113 (1976); D. L. Adams *et al.* (E581 and E704 Collaborations), Phys. Lett. B **261**, 201 (1991); D. L. Adams *et al.* (FNAL-E704 Collaboration), Phys. Lett. B **264**, 462 (1991); K. Krueger *et al.*, Phys. Lett. B **459**, 412 (1999).

[2] U. D'Alesio and F. Murgia, Prog. Part. Nucl. Phys. **61**, 394 (2008).

[3] V. Barone, F. Bradamante, and A. Martin, Prog. Part. Nucl. Phys. **65**, 267 (2010).

[4] A. Airapetian *et al.* (HERMES Collaboration), Phys. Rev. Lett. **94**, 012002 (2005); **103**, 152002 (2009).

[5] V. Y. Alexakhin *et al.* (COMPASS Collaboration), Phys. Rev. Lett. **94**, 202002 (2005); A. Martin (COMPASS Collaboration), Czech. J. Phys. **56**, F33 (2006); M. Alekseev *et al.* (COMPASS Collaboration), Phys. Lett. B **673**, 127 (2009).

[6] H. Avakian, P. E. Bosted, V. Burkert, and L. Elouadrhiri (CLAS Collaboration), AIP Conf. Proc. **792**, 945 (2005).

[7] J. Adams *et al.* (STAR Collaboration), Phys. Rev. Lett. **92**, 171801 (2004); B. I. Abelev *et al.* (STAR Collaboration), Phys. Rev. Lett. **99**, 142003 (2007); **101**, 222001 (2008); S. S. Adler *et al.* (PHENIX Collaboration), Phys. Rev. Lett. **95**, 202001 (2005); I. Arsene *et al.* (BRAHMS Collaboration), Phys. Rev. Lett. **101**, 042001 (2008).

[8] G. L. Kane, J. Pumplin, and W. Repko, Phys. Rev. Lett. **41**, 1689 (1978).

[9] J. C. Collins, A. V. Efremov, K. Goeke, S. Menzel, A. Metz, and P. Schweitzer, Phys. Rev. D **73**, 014021 (2006); S. Arnold, A. V. Efremov, K. Goeke, M. Schlegel, and P. Schweitzer, arXiv:0805.2137.

[10] M. Anselmino, M. Boglione, U. D'Alesio, A. Kotzinian, F. Murgia, and A. Prokudin, Phys. Rev. D **72**, 094007 (2005); **72**, 099903(E) (2005).

[11] M. Anselmino *et al.*, Eur. Phys. J. A **39**, 89 (2008).

[12] M. Anselmino, M. Boglione, U. D'Alesio, S. Melis, F. Murgia, and A. Prokudin, Phys. Rev. D **79**, 054010 (2009); Z. B. Kang and J. W. Qiu, Phys. Rev. Lett. **103**, 172001 (2009); Phys. Rev. D **81**, 054020 (2010).

[13] M. Anselmino, M. Boglione, U. D'Alesio, A. Kotzinian, F. Murgia, A. Prokudin, and C. Turk, Phys. Rev. D **75**, 054032 (2007).

[14] C. Kouvaris, J. W. Qiu, W. Vogelsang, and F. Yuan, Phys. Rev. D **74**, 114013 (2006).

[15] K. Kanazawa and Y. Koike, Phys. Rev. D **82**, 034009 (2010); Y. Koike and T. Tomita, Phys. Lett. B **675**, 181 (2009).

[16] Z. B. Kang and J. W. Qiu, Phys. Rev. D **78**, 034005 (2008); Z. B. Kang, J. W. Qiu, W. Vogelsang, and F. Yuan, Phys. Rev. D **78**, 114013 (2008).

[17] M. Anselmino, M. Boglione, and F. Murgia, Phys. Lett. B **362**, 164 (1995); U. D'Alesio and F. Murgia, Phys. Rev. D **70**, 074009 (2004); M. Anselmino, M. Boglione, U. D'Alesio, E. Leader, S. Melis, and F. Murgia, Phys. Rev. D **73**, 014020 (2006); M. Boglione, U. D'Alesio, and F. Murgia, Phys. Rev. D **77**, 051502 (2008); L. Gamberg and Z. B. Kang, Phys. Lett. B **696**, 109 (2011); Z. B. Kang and F. Yuan, Phys. Rev. D **81**, 054007 (2010).

[18] J. C. Collins and D. E. Soper, Nucl. Phys. **B193**, 381 (1981); **B213**, 545 (1983).

[19] X. d. Ji, J. p. Ma, and F. Yuan, Phys. Rev. D **71**, 034005 (2005); Phys. Lett. B **597**, 299 (2004).

[20] J. C. Collins and A. Metz, Phys. Rev. Lett. **93**, 252001 (2004).

[21] S. J. Brodsky, D. S. Hwang, and I. Schmidt, Phys. Lett. B **530**, 99 (2002); Nucl. Phys. **B642**, 344 (2002).

[22] P. J. Mulders and R. D. Tangerman, Nucl. Phys. **B461**, 197 (1996); **B484**, 538(E) (1997); D. Boer and P. J. Mulders, Phys. Rev. D **57**, 5780 (1998).

[23] D. Boer, P. J. Mulders, and F. Pijlman, Nucl. Phys. **B667**, 201 (2003).

[24] A. Bacchetta, C. J. Bomhof, P. J. Mulders, and F. Pijlman, Phys. Rev. D **72**, 034030 (2005); C. J. Bomhof, P. J. Mulders, and F. Pijlman, Eur. Phys. J. C **47**, 147 (2006); T. C. Rogers and P. J. Mulders, Phys. Rev. D **81**, 094006 (2010).

[25] A. Bacchetta, C. Bomhof, U. D'Alesio, P. J. Mulders, and F. Murgia, Phys. Rev. Lett. **99**, 212002 (2007).

[26] D. W. Sivers, Phys. Rev. D **41**, 83 (1990); **43**, 261 (1991).

[27] J. C. Collins, Nucl. Phys. **B396**, 161 (1993).

[28] A. V. Efremov and O. V. Teryaev, Yad. Fiz. **36**, 242 (1982) [Sov. J. Nucl. Phys. **36**, 140 (1982)]; A. V. Efremov and O. V. Teryaev, Phys. Lett. **150B**, 383 (1985).

[29] J. W. Qiu and G. Sterman, Phys. Rev. Lett. **67**, 2264 (1991); Nucl. Phys. **B378**, 52 (1992).

[30] J. W. Qiu and G. F. Sterman, Phys. Rev. D **59**, 014004 (1998).

[31] H. Eguchi, Y. Koike, and K. Tanaka, Nucl. Phys. **B763**, 198 (2007); Y. Koike and K. Tanaka, Phys. Lett. B **646**, 232 (2007); **668**, 458(E) (2008); Phys. Rev. D **76**, 011502 (2007).

[32] Z. B. Kang, F. Yuan, and J. Zhou, Phys. Lett. B **691**, 243 (2010).

[33] X. Ji, J. W. Qiu, W. Vogelsang, and F. Yuan, Phys. Rev. Lett. **97**, 082002 (2006); Phys. Rev. D **73**, 094017 (2006); Phys. Lett. B **638**, 178 (2006); Y. Koike, W. Vogelsang, and F. Yuan, Phys. Lett. B **659**, 878 (2008); A. Bacchetta, D. Boer, M. Diehl, and P. J. Mulders, J. High Energy Phys. 08 (2008) 023.

[34] X. d. Ji and F. Yuan, Phys. Lett. B **543**, 66 (2002); A. V. Belitsky, X. Ji, and F. Yuan, Nucl. Phys. **B656**, 165 (2003).

[35] J. C. Collins, Phys. Lett. B **536**, 43 (2002).

[36] J. P. Ma and Q. Wang, Eur. Phys. J. C **37**, 293 (2004).

[37] A. Bacchetta, M. Diehl, K. Goeke, A. Metz, P. J. Mulders, and M. Schlegel, J. High Energy Phys. 02 (2007) 093.

[38] A. Bacchetta, U. D'Alesio, M. Diehl, and C. A. Miller, Phys. Rev. D **70**, 117504 (2004).

[39] J. C. Collins, Acta Phys. Pol. B **34**, 3103 (2003).

[40] Z. B. Kang and J. W. Qiu, Phys. Rev. D **79**, 016003 (2009); J. Zhou, F. Yuan, and Z. T. Liang, Phys. Rev. D **79**, 114022 (2009); W. Vogelsang and F. Yuan, Phys. Rev. D **79**, 094010 (2009); V. M. Braun, A. N. Manashov, and B. Pirnay, Phys. Rev. D **80**, 114002 (2009); Z. B. Kang, Phys. Rev. D **83**, 036006 (2011).

[41] EIC wiki page, https://wiki.bnl.gov/eic/index.php/Main_Page.

[42] J. Qiu, M. Schlegel, and W. Vogelsang, arXiv:1103.3861.