Baryogenesis via leptogenesis from quark-lepton symmetry and a compact heavy N_R spectrum

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By demanding a compact spectrum for the right-handed neutrinos and an approximate quark-lepton symmetry inspired from $SO(10)$ gauge unification (assuming a Dirac neutrino mass matrix close to the up quark mass matrix), we construct a *fine-tuning* scenario for baryogenesis via leptogenesis. We find two solutions with a normal hierarchy, with the lightest neutrino mass m_1 different from zero, providing an absolute scale for the spectrum. In the approximations of the model, there are three independent CP phases: δ_L (that we take of the order of the quark Kobayashi-Maskawa phase) and the two light neutrino Majorana phases α and β. A main conclusion is that, although this general scheme is rather flexible, in some regions of parameter space we find that the necessary baryogenesis with its sign is given in terms of the δ_L phase alone. The light Majorana phases can also be computed, and they turn out to be close to $\pi/2$ or very small. Moreover, $SO(10)$ breaks down to the Pati-Salam group $SU(4) \times SU(2) \times SU(2)$ at the expected natural intermediate scale of about 10^{10}–10^{11} GeV. A prediction is made for the effective mass in $(\beta\beta)_{0\nu}$ decay, the ν_e mass, and the sum of all light neutrino masses.

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I. INTRODUCTION AND QUALITATIVE REMARKS

The discovery of oscillations, advocated so many years ago by Pontecorvo [1] in solar and atmospheric neutrinos, is one of the most important experimental discoveries of the last century, the most relevant after the proposal of the standard model and its precision tests. The discovery of neutrino oscillations is also a milestone in the search of new physics.

Up to now four quantities related to the Pontecorvo, Maki, Nagakawa and Sakata matrix [2,3] have been experimentally measured:

$$\Delta m_s^2 \simeq 8 \times 10^{-5}\ \text{eV}^2, \tag{1}$$

$$\tan^2\theta_s \simeq 0.4, \tag{2}$$

$$\Delta m_a^2 \simeq 2.5 \times 10^{-3}\ \text{eV}^2, \tag{3}$$

$$\tan^2\theta_a \simeq 1, \tag{4}$$

where the subindices s and a mean, respectively, solar and atmospheric neutrinos.

An upper bound has been found for the component of ν_{eL} along the heaviest ν_L mass eigenstate

$$\sin^2\theta_{13} < 0.05 \tag{5}$$

and the limits

$$m_{\nu_e} < 2.2\ \text{eV} \tag{6}$$

$$|\langle m_{ee}\rangle| < 0.4\ \text{eV} \tag{7}$$

$$\sum_i m_{\nu_i} < 1\ \text{eV} \tag{8}$$

from the high energy spectrum of the electrons in nuclear beta decay, from the upper limit on the rate in neutrinoless double beta decay (for Majorana neutrinos) and from astrophysics.

Interestingly, a more restrictive bound combining all cosmological data has been obtained recently by G. Fogli *et al.* [4]:

$$\sum_i m_{\nu_i} < 0.2\ \text{eV}, \tag{9}$$

to which we will refer in Sec. VIII, comparing it to our results.

But for the moment, in this qualitative introduction, we will rely on the generally accepted looser bound (8).

The most natural framework to account for the order of magnitude of neutrino masses is the seesaw model [5], where the 6×6 neutrino mass matrix has the form

$$\begin{pmatrix} 0 & m_D^t \\ m_D & M_R \end{pmatrix}, \tag{10}$$

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where the 3×3 Dirac neutrino mass matrix m_D has elements of the order of the masses of charged fermions and M_R is the Majorana mass matrix of the right-handed neutrinos, which are singlets of the standard model gauge group, with elements of the order of the scale of breaking the lepton quantum number.

The information on oscillations gives us only four of the nine parameters of the light neutrino mass matrix. Within the simplifying assumption of neglecting θ_{13} and consequently the neutrino Dirac CP-violating phase, we will be able to strongly constrain the value of its smallest eigenvalue, and fix the values of the two higher ones, as well as the two Majorana phases, simply by demanding that these parameters have a soft dependence on the values of the matrix elements of M_R. We will obtain these results despite the fact that we expect a rather hierarchical spectrum for the eigenvalues of m_D, as it happens for the other fermions and as is natural in a $SO(10)$ framework. The mathematical principle is quite simple: the inverse of a function with a critical dependence on a variable is a very slowly varying function; the product of the derivatives is of $O(1)$. The demand of having matrix elements and eigenvalues of M_R of the same order, given a mixing matrix of leptons similar to the one for quarks, will fix m_1 and the Majorana phases of light neutrinos. As a result of this requirement, we shall get a compact spectrum for the N_R masses, which will make the leptogenesis scenario for baryogenesis natural, as well as predictions for the electron neutrino mass bounded from tritium β decay and the matrix element $|\langle m_{ee}\rangle|$ appearing in neutrinoless double beta decay. By compact spectrum for the heavy right-handed neutrinos we simply mean to have eigenvalues of the same order of magnitude.

From the seesaw formula

$$m_L = -m_D M_R^{-1} m_D^t, \tag{11}$$

one gets

$$\det M_R = -\frac{(\det m_D)^2}{\det m_L}. \tag{12}$$

From Eq. (8) one obtains the upper limit

$$|\det m_L| < \frac{1}{27}\ \mathrm{eV}^3, \tag{13}$$

while in principle there is no lower limit for the left-hand side (lhs) of the inequality (13). Notice that we write the absolute value in the lhs of (13) because neutrino masses, being Majorana masses, can differ in sign for neutrinos with opposite CP.

Moreover, from Eqs. (1) and (3) we get:

$$\Delta m_s^2 = |m_2|^2 - |m_1|^2 \simeq 8 \times 10^{-5}\ \mathrm{eV}^2, \tag{14}$$

$$\begin{aligned}\Delta m_a^2 &= |m_3|^2 - \cos^2\theta_s |m_2|^2 - \sin^2\theta_s |m_1|^2 \\ &\simeq 2.5 \times 10^{-3}\ \mathrm{eV}^2,\end{aligned} \tag{15}$$

where the unfamiliar formula (15) for Δm_a^2, proposed in [6], is demonstrated in the Appendix. This formula is an improvement over the usual ones found in the literature, $\Delta m_a^2 = |m_3|^2 - |m_2|^2$ or $\Delta m_a^2 = |m_3|^2 - |m_1|^2$. Of course, in the limit $|m_2| \simeq |m_1|$, all these formulas coincide. However, we must underline that the results of this paper are not really sensitive to adopting formula (15) or the usual ones.

From the preceding formula one gets a lower limit for the ratio:

$$\left|\frac{m_2}{m_3}\right| > 0.18 \tag{16}$$

A tentative lower bound for $|\det m_L|$ may be found in the $SO(10)$ framework by taking, as in [7],

$$|\det m_D| = 4 \times 10^{-2}\ \mathrm{GeV}^3 \tag{17}$$

and for $|\det M_R|$ the upper limit

$$|\det M_R| \leq 2.7 \times 10^{34}\ \mathrm{GeV}^3, \tag{18}$$

which comes by assuming that the three right-handed neutrinos take a mass at the scale of B–L spontaneous symmetry breaking in the $SO(10)$ model, with breaking to the $SU(4) \times SU(2) \times SU(2)$ Pati-Salam group [8] at the intermediate scale 3×10^{11} [9,10].

We then get, from the seesaw formula (12),

$$|\det m_L| \geq 6 \times 10^{-11}\ \mathrm{eV}^3. \tag{19}$$

Assuming a normal hierarchy for the light neutrinos,

$$|m_2| \sim \sqrt{\Delta m_s^2} \simeq 8.9 \times 10^{-3}\ \mathrm{eV}, \tag{20}$$

$$|m_3| \sim \sqrt{\Delta m_a^2} \simeq 5.0 \times 10^{-2}\ \mathrm{eV}, \tag{21}$$

Eq. (19) will then imply the following lower bound for $|m_1|$:

$$|m_1| \geq 1.3 \times 10^{-7}\ \mathrm{eV} \tag{22}$$

i.e., *a nonvanishing value for the lightest neutrino mass* m_1, an absolute scale for the light neutrino spectrum.

As we will see below, a rather sharp prediction for m_1 and relevant predictions for the lhs of Eqs. (6)–(8) will be achieved by our demand of a compact M_R spectrum and successful leptogenesis.

The measurements of the cosmic microwave background anisotropies [11] and the abundance of light nuclei produced in primordial nucleosynthesis [12] give a consistent value for the baryon asymmetry:

$$Y_B = \frac{n_B - n_{\bar{B}}}{s} \simeq \frac{1}{7.04}\frac{n_B - n_{\bar{B}}}{n_\gamma} \simeq 9 \times 10^{-11}. \tag{23}$$

This baryonic asymmetry may arise from the leptogenesis scenario [13], with a leptonic asymmetry produced at a high scale, which gives rise by the B–L conserving sphaleron processes [14] at the electroweak scale to a baryon asymmetry below that scale.

Within the leptogenesis scenario, the baryon asymmetry, baryon to entropy fraction, is given by

$$Y_B \simeq -\frac{1}{2} Y_L \tag{24}$$

that should be compared with the experimental value given by (23).

Concerning grand unification, the $SU(5)$ minimal model is disfavored, since it generates a small baryon asymmetry at the high scale, washed out at the electroweak scale, since in that model B–L is conserved. Thus, $SO(10)$, with its B–L generator spontaneously broken, that we will adopt in its nonsupersymmetric version, should be preferred to $SU(5)$ to realize the leptogenesis scenario.

The paper is organized as follows. In Sec. II we give the relevant formulas for the inverse seesaw, mass matrices and mixings. In Sec. III we formulate our $SO(10)$ ansatz. In Sec. IV we give the formulas needed for CP violation and the baryon asymmetry. Section V is devoted to a simple mathematical procedure to obtain a quasidegenerate right-handed neutrino spectrum (that presents a level crossing) and a realistic light neutrino spectrum. We underline an illuminating limit of considering, for the matrix diagonalizing m_D, a pure Cabibbo matrix that we then extend to a general matrix of the Cabibbo-Kobayashi-Maskawa (CKM) form, therefore introducing CP violation. We find two possible solutions. In Sec. VI we expose a simple procedure to slowly lift the degeneracy of the heavy right-handed neutrinos, and give the corresponding evolution of Δm_s^2 and Δm_a^2. In Sec. VII we exhibit the results for CP violation and baryon asymmetry, in the one-flavor approximation, and in Sec. VIII we give the predictions for m_{ν_e} and the effective neutrino mass in $(\beta\beta)_{0\nu}$. In Sec. IX we relax a reality assumption used in Secs. VI and VII. In Sec. X we comment on the compact heavy neutrino spectrum and on the level crossing region. Finally in Secs. XI and XII we underline open problems within the present approach and we conclude.

II. INVERSE SEESAW, MASS MATRICES AND MIXINGS

From Eq. (11), we can deduce the inverse seesaw formula,

$$M_R = -m_D^t m_L^{-1} m_D, \tag{25}$$

and diagonalizing the neutrino Dirac mass matrix m_D by

$$m_D = V^{L+} m_D^{\mathrm{diag}} V^R, \tag{26}$$

one gets the formula [6]

$$\begin{aligned} M_R &= -m_D^t m_L^{-1} m_D = -V^{Rt} m_D^{\mathrm{diag}} V^{L*} m_L^{-1} V^{L+} m_D^{\mathrm{diag}} V^R \\ &= -V^{Rt} m_D^{\mathrm{diag}} A^L m_D^{\mathrm{diag}} V^R, \end{aligned} \tag{27}$$

where the last equality follows from the definition [6] of the matrix A^L:

$$A^L = V^{L*} m_L^{-1} V^{L+}. \tag{28}$$

The neutrino mass matrix m_L is diagonalized by the Pontecorvo, Maki, Nagakawa and Sakata matrix U:

$$m_L = U^* m_L^{\mathrm{diag}} U^+, \tag{29}$$

where

$$m_L^{\mathrm{diag}} = \mathrm{diag}(m_1, m_2, m_3), \tag{30}$$

and U writes:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} .\mathrm{diag}(1, e^{i\alpha}, e^{i\beta}), \tag{31}$$

where δ is the Dirac phase and α and β are the Majorana phases. For the latter we adopt the convention of Davidson *et al.* [13].

Let us make a remark on the counting of phases. One has, in all generality, 6 independent phases in the Type I seesaw scheme, as established in [15,16] and as exposed in the review [13], last reference, Sec. 2.1. In the model that we develop below, the number of independent phases will be reduced according to the hypotheses adopted.

Taking into account the data on solar and atmospheric neutrinos and the fact that s_{13} is bounded to be small, we will take

$$s_{13} \simeq 0 \tag{32}$$

and approximate, from now on, the matrix U as follows:

$$U \simeq \begin{pmatrix} c_s & s_s & 0 \\ -\frac{s_s}{\sqrt{2}} & \frac{c_s}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s_s}{\sqrt{2}} & -\frac{c_s}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} .\mathrm{diag}(1, e^{i\alpha}, e^{i\beta}), \tag{33}$$

and CP violation originating in the Dirac phase δ drops out.

To simplify the expressions in what follows, we change the notation for the diagonal matrix in (29)–(31):

$$\mathrm{diag}\,(m_1, e^{-2i\alpha} m_2, e^{-2i\beta} m_3) \rightarrow \mathrm{diag}(m_1, m_2, m_3), \tag{34}$$

where, from now on, m_2 and m_3 are assumed to be *complex parameters*.

Eq. (29) now writes, in the approximation (33), and with the notation convention of the rhs of (34),

$$m_L \simeq \begin{pmatrix} c_s & s_s & 0 \\ -\frac{s_s}{\sqrt{2}} & \frac{c_s}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s_s}{\sqrt{2}} & -\frac{c_s}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \mathrm{diag}(m_1, m_2, m_3) \begin{pmatrix} c_s & -\frac{s_s}{\sqrt{2}} & \frac{s_s}{\sqrt{2}} \\ s_s & \frac{c_s}{\sqrt{2}} & -\frac{c_s}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{35}$$

Let us strongly underline again that in (35), and in what follows, the parameters m_2, m_3 are assumed to be complex, containing, according to (34), the Majorana phases defined by (31).

These phases will be computed at different stages. Their calculation will depend on some hypotheses to be made explicit below, and on the successive parametrizations assumed for the matrix m_D (26).

From the previous hypotheses we obtain the following complex symmetric matrix:

$$m_L^{-1} = \begin{pmatrix} \frac{c_s^2}{m_1} + \frac{s_s^2}{m_2} & -\frac{c_s s_s}{\sqrt{2}}\left(\frac{1}{m_1} - \frac{1}{m_2}\right) & \frac{c_s s_s}{\sqrt{2}}\left(\frac{1}{m_1} - \frac{1}{m_2}\right) \\ -\frac{c_s s_s}{\sqrt{2}}\left(\frac{1}{m_1} - \frac{1}{m_2}\right) & \frac{1}{2}\left(\frac{s_s^2}{m_1} + \frac{c_s^2}{m_2} + \frac{1}{m_3}\right) & -\frac{1}{2}\left(\frac{s_s^2}{m_1} + \frac{c_s^2}{m_2} - \frac{1}{m_3}\right) \\ \frac{c_s s_s}{\sqrt{2}}\left(\frac{1}{m_1} - \frac{1}{m_2}\right) & -\frac{1}{2}\left(\frac{s_s^2}{m_1} + \frac{c_s^2}{m_2} - \frac{1}{m_3}\right) & \frac{1}{2}\left(\frac{s_s^2}{m_1} + \frac{c_s^2}{m_2} + \frac{1}{m_3}\right) \end{pmatrix}, \tag{36}$$

and the matrix A_L (28) is also complex symmetric:

$$A^{Lt} = A^L. \tag{37}$$

In a previous work [6], to comply with the lower bound for the mass of the lightest right-handed neutrino claimed by [17], we did set upper limits on the coefficients of contributions proportional to the products of the Dirac matrix eigenvalues $(m_{D_3})^2$ and $m_{D_2}m_{D_3}$ in the M_R matrix, related to m_L by the inverse seesaw formula.

From formula (27) we see that to get a quasidegenerate heavy Majorana neutrino spectrum we need that the terms proportional to $(m_{D_3})^2$ and $m_{D_2}m_{D_3}$ have to be small, which means that, for a hierarchical Dirac mass spectrum, the matrix elements of (28) A^L_{33} and $A^L_{23} = A^L_{32}$ have to be small. From (28), we get the following expression for these matrix elements of interest:

$$\begin{aligned} A^L_{23} = {} & V^{L*}_{31}\left[\left(\frac{c_s^2}{m_1} + \frac{s_s^2}{m_2}\right)V^{L*}_{21} - \frac{c_s s_s}{\sqrt{2}}\left(\frac{1}{m_1} - \frac{1}{m_2}\right)V^{L*}_{22} + \frac{c_s s_s}{\sqrt{2}}\left(\frac{1}{m_1} - \frac{1}{m_2}\right)V^{L*}_{23}\right] \\ & + V^{L*}_{32}\left[-\frac{c_s s_s}{\sqrt{2}}\left(\frac{1}{m_1} - \frac{1}{m_2}\right)V^{L*}_{21} + \frac{1}{2}\left(\frac{s_s^2}{m_1} + \frac{c_s^2}{m_2} + \frac{1}{m_3}\right)V^{L*}_{22} - \frac{1}{2}\left(\frac{s_s^2}{m_1} + \frac{c_s^2}{m_2} - \frac{1}{m_3}\right)V^{L*}_{23}\right] \\ & + V^{L*}_{33}\left[\frac{c_s s_s}{\sqrt{2}}\left(\frac{1}{m_1} - \frac{1}{m_2}\right)V^{L*}_{21} - \frac{1}{2}\left(\frac{s_s^2}{m_1} + \frac{c_s^2}{m_2} - \frac{1}{m_3}\right)V^{L*}_{22} + \frac{1}{2}\left(\frac{s_s^2}{m_1} + \frac{c_s^2}{m_2} + \frac{1}{m_3}\right)V^{L*}_{23}\right], \end{aligned} \tag{38}$$

$$\begin{aligned} A^L_{33} = {} & V^{L*}_{31}\left[\left(\frac{c_s^2}{m_1} + \frac{s_s^2}{m_2}\right)V^{L*}_{31} - \frac{c_s s_s}{\sqrt{2}}\left(\frac{1}{m_1} - \frac{1}{m_2}\right)V^{L*}_{32} + \frac{c_s s_s}{\sqrt{2}}\left(\frac{1}{m_1} - \frac{1}{m_2}\right)V^{L*}_{33}\right] \\ & + V^{L*}_{32}\left[-\frac{c_s s_s}{\sqrt{2}}\left(\frac{1}{m_1} - \frac{1}{m_2}\right)V^{L*}_{31} + \frac{1}{2}\left(\frac{s_s^2}{m_1} + \frac{c_s^2}{m_2} + \frac{1}{m_3}\right)V^{L*}_{32} - \frac{1}{2}\left(\frac{s_s^2}{m_1} + \frac{c_s^2}{m_2} - \frac{1}{m_3}\right)V^{L*}_{33}\right] \\ & + V^{L*}_{33}\left[\frac{c_s s_s}{\sqrt{2}}\left(\frac{1}{m_1} - \frac{1}{m_2}\right)V^{L*}_{31} - \frac{1}{2}\left(\frac{s_s^2}{m_1} + \frac{c_s^2}{m_2} - \frac{1}{m_3}\right)V^{L*}_{32} + \frac{1}{2}\left(\frac{s_s^2}{m_1} + \frac{c_s^2}{m_2} + \frac{1}{m_3}\right)V^{L*}_{33}\right]. \end{aligned} \tag{39}$$

III. OUR $SO(10)$ ANSATZ

The seesaw model is realized in the framework of $SO(10)$ unified gauge theories [18], where B–L is a generator which has to be spontaneously broken. Long before we saw the firm evidence for neutrino oscillations, this phenomenon had been claimed [19] as the most promising experimental signal for $SO(10)$ unification.

A systematic study of the spontaneous symmetry breaking in $SO(10)$ unified theories has lead researchers to propose [9] the model with $SU(4) \times SU(2) \times SU(2)$ [8] intermediate gauge group, broken at the scale of order 3×10^{11} GeV [10].

A general analysis has been done in [7] on the possibility to construct a realistic leptogenesis scenario within the seesaw model with neutrino Dirac masses in a hierarchical ratio, as is the case for u-type quarks. The most promising case has been found with $M_3 \sim 10^{14}$ GeV and nearby values for the masses of the two lightest right-handed neutrinos.

Although in the present paper we follow the general idea [7] of leptogenesis generated by quasidegenerate right-handed neutrinos, we look for a more compact spectrum for N_R, with the heaviest right-handed neutrino at the intermediate scale, of the order 10^{11} GeV.

In $SO(10)$ the hypothesis that the electroweak Higgs transforms as a combination of **10** representations implies at the unification scale the equalities among mass matrices

$$m_e = m_d, \tag{40}$$

$$m_D = m_u. \tag{41}$$

For b and τ masses, relation (40) at the intermediate scale is in reasonable agreement with experiment but, as Georgi and Jarlskog [20] have shown in the $SU(5)$ case, one also needs higher dimensional representations. The generalization of this argument to $SO(10)$ was given by Harvey *et al.* [21]. For an overview on fermion masses and mixings in gauge theories, see the review article [22].

Within $SO(10)$, with the electroweak Higgs boson belonging to the **10** and/or **126** representations, and *no component along the* **120** *representation*, the mass matrices are symmetric. As a consequence, the unitary matrices V^R and V^L that diagonalize the Dirac neutrino matrix (26) are related:

$$V^R = V^{L*}, \tag{42}$$

and the matrix M_R (27) becomes

$$\begin{aligned} M_R &= -m_D^t m_L^{-1} m_D = -V^{L+} m_D^{\mathrm{diag}} V^{L*} m_L^{-1} V^{L+} m_D^{\mathrm{diag}} V^{L*} \\ &= -V^{L+} m_D^{\mathrm{diag}} A^L m_D^{\mathrm{diag}} V^{L*}. \end{aligned} \tag{43}$$

Let us now go back to the question of the phase counting, quoted for the seesaw scheme in Sec. III, in the particular case of $SO(10)$ with a symmetric Dirac neutrino matrix. Since V^L has only one phase, and m_L, through the mixing matrix U (31), has three phases, we have reduced the number of independent phases, from 6 in the general case to 4 independent phases. In the approximation (32) $s_{13} \simeq 0$ that we have adopted, this means that we have 3 independent phases, namely, a phase from V^L, that we will call δ_L, and the two Majorana phases α and β from (33).

Below, in Secs. V and VI, we will impose two other conditions that further reduce the number of independent phases, from 3 to a single one.

For the diagonalized Dirac neutrino matrix

$$m_D^{\mathrm{diag}} = \begin{pmatrix} m_{D_1} & 0 & 0 \\ 0 & m_{D_2} & 0 \\ 0 & 0 & m_{D_3} \end{pmatrix}, \tag{44}$$

we will adopt the numerical values proposed in [7], inspired from the up quark mass matrix:

$$m_{D_1} = 10^{-3}\ \mathrm{GeV}, \qquad m_{D_2} = 0.4\ \mathrm{GeV}, \qquad m_{D_3} = 100\ \mathrm{GeV}. \tag{45}$$

The matrix $m_D^{\mathrm{diag}} A^L m_D^{\mathrm{diag}}$ appearing in (27) has the form

$$m_D^{\mathrm{diag}} A^L m_D^{\mathrm{diag}} = \begin{pmatrix} m_{D_1}^2 A_{11}^L & m_{D_1} m_{D_2} A_{12}^L & m_{D_1} m_{D_3} A_{13}^L \\ m_{D_1} m_{D_2} A_{12}^L & m_{D_2}^2 A_{22}^L & m_{D_2} m_{D_3} A_{23}^L \\ m_{D_1} m_{D_3} A_{13}^L & m_{D_2} m_{D_3} A_{23}^L & m_{D_3}^2 A_{33}^L \end{pmatrix}, \tag{46}$$

which clearly shows that in order to have a compact N_R spectrum from (27) one needs small values for the matrix elements A_{23}^L and A_{33}^L.

For V^L we will assume a form qualitatively similar to the Cabibbo-Kobayashi-Maskawa (CKM) quark matrix, that reads, in the standard convention (except for the phase δ_L, we take the same notation as for the light neutrino mixing matrix (31), but in what follows there is no ambiguity):

$$V^L = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_L} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_L} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_L} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_L} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_L} & c_{23}c_{13} \end{pmatrix}, \tag{47}$$

where δ_L is the CP-violating phase. This form is exact in the limit of considering only **10** Higgs representations.

We define, as usual, in terms of Wolfenstein parameters:

$$s_{12} = \lambda, \quad s_{23} = A\lambda^2, \quad s_{13}e^{i\delta_L} = A\lambda^3(\rho + i\eta). \tag{48}$$

Of course, in our problem the parameters λ, A, ρ, η do not necessarily have the same precise values as in the quark sector: we are interested only in an order-of-magnitude estimate.

Let us say some words concerning the diagonalization of the right-handed neutrino matrix. Since in our $SO(10)$ ansatz M_R (43) is complex and symmetric, we can diagonalize it by using a single unitary matrix:

$$M_R = W_R M_R^{\mathrm{diag}} W_R^t. \tag{49}$$

The matrix W_R is such that all eigenvalues are real and positive. The effect of phases will appear in the matrix W_R. These phases will of course have consequences for baryogenesis and for neutrinoless double beta decay.

As we will see below, our demand of suppressed values for A^L_{33} and A^L_{23} generates a compact form for the M_R spectrum, which helps in getting the desired lepton asymmetry in a natural way.

IV. LEPTOGENESIS AND BARYON ASYMMETRY

In this section we recall the basic formulas concerning the CP-violating asymmetry ϵ_1 and the corresponding baryogenesis asymmetry Y_{B_1}. We work in the basis in which the mass matrices of charged leptons and of right-handed neutrinos are diagonal, i.e. from (25) and (49):

$$M_R^{\rm diag} = -W_R^+ m_D^t m_L^{-1} m_D W_R^*. \tag{50}$$

Therefore, in the computation of the CP-violating asymmetry ϵ_1, we define

$$\hat{m}_D = m_D W_R^* \tag{51}$$

such that

$$M_R^{\rm diag} = -\hat{m}_D^t m_L^{-1} \hat{m}_D. \tag{52}$$

By convention we label the masses of the heavy neutrinos N_{R_i} $(i = 1, 2, 3)$:

$$0 \leq M_1 \leq M_2 \leq M_3. \tag{53}$$

In terms of $\hat{m}_D$, the CP asymmetry writes, for the lightest heavy neutrino N_{R_1}:

$$\epsilon_1 = \frac{1}{8\pi v^2} \sum_{k\neq 1} f\left(\frac{M_k^2}{M_1^2}\right) \frac{{\rm Im}[(\hat{m}_D^+ \hat{m}_D)^2_{1k}]}{(\hat{m}_D^+ \hat{m}_D)_{11}}, \tag{54}$$

where $v = 174$ GeV is the scale of electroweak symmetry breaking, and the function $f(x)$ is given by [23]

$$f(x) = \sqrt{x}\left[\frac{1}{1-x} + 1 - (1+x)\log\left(\frac{1+x}{x}\right)\right], \tag{55}$$

that in the limit $x \gg 1$ becomes

$$f(x) \simeq -\frac{3}{2\sqrt{x}}, \tag{56}$$

and the effective neutrino mass, that controls the amount of washout, writes:

$$\tilde{m}_1 = \frac{(\hat{m}_D^+ \hat{m}_D)_{11}}{M_1}. \tag{57}$$

The cases that we encounter in our calculations below satisfy the strong washout condition

$$\tilde{m}_1 \gg 3 \times 10^{-3}\ {\rm eV}, \tag{58}$$

and the corresponding baryon asymmetry writes, *in the one-flavor approximation* that we will adopt in the following [24]

$$Y_{B_1} = -\frac{1}{2} 0.3 \frac{\epsilon_1}{g_*}\left(\frac{0.55\times 10^{-3}\ {\rm eV}}{\tilde{m}_1}\right)^{1.16}, \tag{59}$$

where $g_* \simeq 107$ in the standard model, in the nonsupersymmetric case.

V. QUASIDEGENERATE HEAVY RIGHT-HANDED NEUTRINOS AND A REALISTIC LIGHT NEUTRINO SPECTRUM

In order to get a compact N_R spectrum, a sufficient condition is to impose that the matrix elements A^L_{33} and A^L_{23} are suppressed, because we are dealing with the matrix (46) and the m_D eigenvalues (45). As a first exercise, we thus consider the solutions of the equations, linear and homogeneous in the inverse of the neutrino masses $\frac{1}{m_i}$ $(i = 1, 2, 3)$,

$$A^L_{23}(m_1, m_2, m_3) = A^L_{33}(m_1, m_2, m_3) = 0. \tag{60}$$

We are aware that this is a very drastic assumption, but it will help to guide our research of a compact right-handed neutrino spectrum and also to look for its consequences on the light neutrino masses and the amount of baryogenesis that one can get. We must emphasize that, in this section and in the following ones, we are dealing with a *fine-tuning* scheme. We cannot content ourselves with just order-of-magnitude estimates; instead, we need precise numerical calculations.

Notice a new important point in the phase counting of Eq. (43) with the hypothesis (32). Under the two reality conditions (60) that we now impose, the 3 phases (see Sec. III) are now reduced to a single phase, either δ_L or one of the two Majorana phases α or β.

Since we do not have experimental information on the Majorana phases, we will, from now on, compute α and β, and later the CP asymmetry ϵ_1 and baryon asymmetry Y_{B_1} in terms of δ_L. Of course, in principle, one could also compute the pair (δ_L, α) in terms of β or (δ_L, β) in terms of α. But in the present $SO(10)$ approach the natural thing to do is to take δ_L as input, since we can take it to be of the order of Kobayashi-Maskawa (KM) phase $\delta_{\rm KM}$, on which we have information.

A. V^L in the limit of a pure Cabibbo matrix

For our purpose, it is a good illustration to study the consequences of this hypothesis, considering it within the very simplified approximation of a 2×2 Cabibbo matrix:

$$V^L = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{61}$$

Since from (61) δ_L drops out, we are left with only two phases, namely, the Majorana phases α and β. Imposing the two reality conditions (60), these phases will be fixed, as we see below.

From (60) and (61), we find that the light and heavy neutrino spectra turn out to be reasonable. From (38) and (39), the matrix elements of A^L we are interested in are

$$A^L_{23} = A^L_{32} = -\frac{1}{2}\left(\frac{s_s^2}{m_1} + \frac{c_s^2}{m_2} - \frac{1}{m_3}\right)c_{12} - \frac{c_s s_s}{\sqrt{2}}\left(\frac{1}{m_1} - \frac{1}{m_2}\right)s_{12},$$
$$A^L_{33} = \frac{1}{2}\left(\frac{s_s^2}{m_1} + \frac{c_s^2}{m_2} + \frac{1}{m_3}\right). \tag{62}$$

If we impose the very strong assumption (60), we have the two equations

$$\frac{s_s^2}{m_1} + \frac{c_s^2}{m_2} - \frac{1}{m_3} + \sqrt{2}c_s s_s\left(\frac{1}{m_1} - \frac{1}{m_2}\right)\tan\theta_{12} = 0,$$
$$\frac{s_s^2}{m_1} + \frac{c_s^2}{m_2} + \frac{1}{m_3} = 0, \tag{63}$$

and solving for m_2 and m_3 in terms of m_1, one gets:

$$m_2 = -\frac{\sqrt{2} - \tan\theta_s \tan\theta_{12}}{\sqrt{2}\tan^2\theta_s + \tan\theta_s \tan\theta_{12}} m_1,$$
$$m_3 = \frac{\sqrt{2} - \tan\theta_s \tan\theta_{12}}{\tan\theta_s \tan\theta_{12}} m_1. \tag{64}$$

From the data (1)–(4) and formulas (14) and (15) one gets, from (64), a number of solutions for m_1 and θ_{12}.

However, if one looks for solutions with θ_{12} *in the neighborhood of the Cabibbo angle* θ_C, one finds the following two solutions, according to the sign of $\tan\theta_s$, since only its square (2) is measured:

(1) For $\tan\theta_s \simeq -\sqrt{0.4}$, one gets

$$\tan\theta_{12} = 0.140, \tag{65}$$

$$|m_1| = 0.0030 \text{ eV}, \qquad m_2 = -3.1522 m_1,$$
$$m_3 = -16.9273 m_1. \tag{66}$$

We have taken several digits for the m_i values to get a compact spectrum for the N_{R_i} and, as we will see below, for later to obtain the nondegeneracy of the two higher states. The reason is that we are dealing with a fine-tuning problem. Of course, one could take the first digits and say the degree of approximation at each stage, but we believe that our way of presenting the results, although harder to read, corresponds better to the reality of the calculation.

In consistency with (34), we assume the convention $m_1 > 0$, and one gets the following spectrum:

$$m_1 = 0.0030 \text{ eV}, \qquad m_2 = -0.0094 \text{ eV},$$
$$m_3 = -0.0507 \text{ eV}, \tag{67}$$

where two heavier neutrinos have opposite *CP* from the lighter one. This means that the Majorana phases are

$$\alpha = \frac{\pi}{2}, \qquad \beta = \frac{\pi}{2}. \tag{68}$$

We find, from (26) and (42) and the value $\tan\theta_{12}$ (65), for the symmetric Dirac mass matrix:

$$m_D = \begin{pmatrix} 0.0087 & -0.0549 & 0 \\ -0.0549 & 0.3923 & 0 \\ 0 & 0 & 100 \end{pmatrix} \text{ GeV}, \tag{69}$$

and

$$M_1 = 5.5504 \times 10^9 \text{ GeV},$$
$$M_2 = 1.429\,91 \times 10^{10} \text{ GeV},$$
$$M_3 = 1.429\,92 \times 10^{10} \text{ GeV}. \tag{70}$$

(2) For $\tan\theta_s \simeq +\sqrt{0.4}$, one gets

$$\tan\theta_{12} = 0.243, \tag{71}$$

$$|m_1| = 0.0062 \text{ eV}, \qquad m_2 = -1.752 m_1,$$
$$m_3 = 8.191 m_1. \tag{72}$$

Assuming again the convention $m_1 > 0$, one gets the following hierarchical spectrum,

$$m_1 = 0.0062 \text{ eV}, \qquad m_2 = -0.0109 \text{ eV},$$
$$m_3 = 0.0509 \text{ eV}, \tag{73}$$

where two neutrinos (the lightest and the heaviest) have opposite *CP* from the third one. This means that the Majorana phases are

$$\alpha = -\frac{\pi}{2}, \qquad \beta = 0. \tag{74}$$

We obtain for this solution, from the value $\tan\theta_{12}$ (71), the Dirac neutrino matrix:

$$m_D = \begin{pmatrix} 0.0233 & -0.0916 & 0 \\ -0.0916 & 0.3777 & 0 \\ 0 & 0 & 100 \end{pmatrix} \text{ GeV} \tag{75}$$

and the quasidegenerate right-handed heavy neutrino spectrum:

$$M_1 = 6.72168 \times 10^9 \text{ GeV},$$
$$M_2 = 8.30366 \times 10^9 \text{ GeV},$$
$$M_3 = 8.30409 \times 10^9 \text{ GeV}. \tag{76}$$

The results for these two solutions seem encouraging because we get in both cases a value of the angle θ_{12} that is rather close to the Cabibbo angle θ_C. It seems highly nontrivial and amazing that such a simplified form of the V^L matrix could already give these results consistent with quark-lepton symmetry.

B. V^L with approximate CKM form

We now switch on the other V^L parameters and consider the full matrix (47). To perform the calculations we adopt for m_1 and $\tan\theta_{12}$ the values obtained in the pure Cabibbo limit, namely, $m_1 = 0.0030$ eV, $\tan\theta_{12} = 0.140$ for solution (1) and $m_1 = 0.0062$ eV, $\tan\theta_{12} = 0.243$ for solution (2).

For the numerical calculations we thus proceed in the following way:

(i) From either solution (1) or solution (2) of Sec. VA, we fix the parameters m_1 and $\tan\theta_{12}$:

(1)

$$\tan\theta_s = -\sqrt{0.4}, \qquad m_1 = 0.0030 \text{ eV}, \qquad \tan\theta_{12} = 0.140, \tag{77}$$

(2)

$$\tan\theta_s = +\sqrt{0.4}, \qquad m_1 = 0.0062 \text{ eV}, \qquad \tan\theta_{12} = 0.243. \tag{78}$$

(ii) For the rest of the V^L matrix elements we deduce the Wolfenstein parameter λ in (48) from (77) and (78) and take, just a guess, the parameters A, ρ and η from the quark sector CKM matrix, i.e., for example,

$$A = 0.8, \qquad \rho = 0.13, \qquad \eta = 0.35, \tag{79}$$

and, using these parameters, we fix s_{23}, s_{13} following (48):

(1)

$$\tan\theta_{12} = 0.140, \qquad s_{23} = 0.0154, \qquad s_{13} = 0.0008, \qquad \delta_L = 1.2152 \tag{80}$$

gives

$$V^L \simeq \begin{pmatrix} 0.990 & 0.139 & (2.8 - 7.5i) \times 10^{-4} \\ -0.139 & 0.990 & 0.015 \\ (18.6 - 7.4i) \times 10^{-4} & -0.015 & 1 \end{pmatrix}, \tag{81}$$

while from

(2)

$$\tan\theta_{12} = 0.243, \qquad s_{23} = 0.0446, \qquad s_{13} = 0.0039, \qquad \delta_L = 1.2152 \tag{82}$$

one obtains

$$V^L \simeq \begin{pmatrix} 0.972 & 0.236 & (1.37 - 3.68i) \times 10^{-3} \\ -0.236 & 0.971 & 0.045 \\ (9.20 - 3.58i) \times 10^{-3} & -0.045 & 1 \end{pmatrix}. \tag{83}$$

(iii) Then, we solve Eqs. (60) for m_2 and m_3 and compare with experiment for Δm_s^2 and Δm_a^2.
Notice that this numerical procedure is less rigid that the one adopted in the simpler case of a pure Cabibbo matrix of the preceding subsection, where Δm_s^2 and Δm_a^2 were fixed to the experimental central values (1) and (3) and we did solve for $\tan\theta_{12}$, m_1, m_2 and m_3. We prefer to change our numerical approach here due to the extreme fine tuning of the problem. It seems to us sensible enough if we get results for Δm_s^2 and Δm_a^2 that are roughly consistent with experiment.
We find the following results.

(1) For $\tan\theta_s \simeq -\sqrt{0.4}$, $m_1 = 0.0030$ eV and $\tan\theta_{12} = 0.140$, one gets

$$m_2 = -0.0095 e^{0.0036i} \text{ eV}, \qquad m_3 = -0.0495 e^{0.0075i} \text{ eV} \tag{84}$$

that correspond to the Majorana phases

$$\alpha = \frac{\pi}{2} - 0.0018, \qquad \beta = \frac{\pi}{2} - 0.0038. \tag{85}$$

Let us notice an important point. We obtain the CP-violating part of the Majorana phases for the light neutrinos [i.e. their departure relative to $\frac{\pi}{2}$ in (85)] from the δ_L phase, that we take close to the KM phase δ_{KM}. This can seem paradoxical, because δ_L concerns the Dirac neutrino mass. However, because of δ_L, the matrix A^L is complex. This implies that, setting m_1 real as we have done above, the solutions from Eqs. (60) for m_2 and m_3 [with the notation (34)] must be complex. The departure of these Majorana phases relative to the ones obtained in the real Cabibbo limit (68) and (74) turn out to be numerically small.

We obtain, from (14), (15), and (84),

$$\Delta m_s^2 = 8.1 \times 10^{-5}\ \mathrm{eV}^2, \qquad \Delta m_a^2 = 2.4 \times 10^{-3}\ \mathrm{eV}^2. \tag{86}$$

The agreement with the data is good.

Let us now give the complex *symmetric* Dirac neutrino mass and the right-handed heavy neutrino spectrum for this solution. We get

$$m_D = \begin{pmatrix} 0.0090 + 0.0003i & -0.0576 - 0.0011i & 0.1849 + 0.0739i \\ -0.0576 - 0.0011i & 0.4155 - 0.0003i & -1.5206 + 0.0103i \\ 0.1849 + 0.0739i & -1.5206 + 0.0103i & 99.9764 \end{pmatrix} \mathrm{GeV} \tag{87}$$

and the quasidegenerate right-handed heavy neutrino spectrum:

$$M_1 = 5.53144 \times 10^9\ \mathrm{GeV}, \qquad M_2 = 1.43230 \times 10^{10}\ \mathrm{GeV}, \qquad M_3 = 1.43232 \times 10^{10}\ \mathrm{GeV}. \tag{88}$$

(2) For $\tan\theta_s \simeq +\sqrt{0.4}$, $m_1 = 0.0062$ eV and $\tan\theta_{12} = 0.243$ one gets:

$$m_2 = -0.0106 e^{-0.016i}\ \mathrm{eV}, \qquad m_3 = 0.0455 e^{0.0078i}\ \mathrm{eV}, \tag{89}$$

that correspond to the Majorana phases

$$\alpha = -\frac{\pi}{2} + 0.0080, \qquad \beta = -0.0039. \tag{90}$$

We obtain, from (14), (15), and (89):

$$\Delta m_s^2 = 7.4 \times 10^{-5}\ \mathrm{eV}^2, \qquad \Delta m_a^2 = 2.0 \times 10^{-3}\ \mathrm{eV}^2. \tag{91}$$

The agreement with the data is not as good as for solution (1). We could change the initial conditions for m_1 and $\tan\theta_{12}$ and get a better agreement. However, it is not our intention to make a fit but to get a qualitative agreement with the data.

We get the Dirac matrix for this solution:

$$m_D = \begin{pmatrix} 0.0304 + 0.0066i & -0.1319 - 0.0148i & 0.9152 + 0.3575i \\ -0.1319 - 0.0148i & 0.5676 - 0.0076i & -4.3450 + 0.0869i \\ 0.9152 + 0.3575i & -4.3450 + 0.0869i & 99.8003 \end{pmatrix} \mathrm{GeV}, \tag{92}$$

and the quasidegenerate right-handed heavy neutrino spectrum:

$$M_1 = 6.846\,78 \times 10^9\ \mathrm{GeV}, \qquad M_2 = 8.848\,78 \times 10^9\ \mathrm{GeV}, \qquad M_3 = 8.849\,09 \times 10^9\ \mathrm{GeV}. \tag{93}$$

The signs and phases of the results (84) and (89) will have quantitative consequences for the effective neutrino mass in neutrinoless double beta decay, as we will see below.

C. *CP* violation and baryon asymmetry

The results of Sec. V B show that, imposing the very drastic conditions (60), one gets quasidegenerate right-handed neutrino spectra.

To have a feeling on how to proceed, let us make an exercise in the case (1), where the quasidegeneracy (88) is less pronounced. Let us compute the Dirac matrix (51) in the basis in which the heavy right-handed neutrino mass matrix is diagonal, ϵ_1, $\tilde{m}_1$ and finally Y_{B_1}. We will assume that the lightest neutrino decays out of equilibrium and that one can apply the one-flavor approximation.

We find the following result:

$$\hat{m}_D \simeq \begin{pmatrix} -0.055i & -0.052 + 0.132i & -0.131 - 0.052i \\ -0.001 + 0.391i & -0.006 - 1.081i & 1.080 - 0.006i \\ -0.002 + 0.347i & -0.064 + 70.702i & -70.702 - 0.064i \end{pmatrix} \mathrm{GeV}. \tag{94}$$

Of course, unlike expression (87), this matrix is no longer symmetric. Using it in Eqs. (54), (57), and (59), we obtain

$$\epsilon_1 \simeq -3.805 \times 10^{-10}, \qquad \tilde{m}_1 \simeq 0.050 \text{ eV}, \qquad Y_{B_1} \simeq 3.05 \times 10^{-15}, \tag{95}$$

where we have used the exact formula (55) for the function $f(x)$ since the three heavy neutrinos are rather close in mass, although N_{R_1} is lighter, and formula (59) is applied because, according to the value of $\tilde{m}_1$ (95), we are in the strong washout regime (58).

The obtained baryon asymmetry $Y_{B_1} \cong 3 \times 10^{-15}$ is much too small, by about four to 5 orders of magnitude, although of the right sign. The reason is the smallness of ϵ_1, that follows from the quasidegeneracy of M_2 and M_3 and the opposite CP asymmetry contribution from both heavy neutrinos. Indeed, one finds, for the two terms in (54):

$$f\left(\frac{M_2^2}{M_1^2}\right)\frac{\text{Im}[(\hat{m}_D\hat{m}_D^+)_{12}^2]}{8\pi v^2(\hat{m}_D\hat{m}_D^+)_{11}} \simeq -f\left(\frac{M_3^2}{M_1^2}\right)\frac{\text{Im}[(\hat{m}_D\hat{m}_D^+)_{13}^2]}{8\pi v^2(\hat{m}_D\hat{m}_D^+)_{11}} \simeq -3.634 \times 10^{-6} \tag{96}$$

that shows a strong cancellation giving a very small CP violation ϵ_1.

Although for the moment we do not get good phenomenological results, we should however emphasize an interesting limit of the present scheme, namely:

$$\begin{aligned} &\delta_L \to 0 \quad \text{implies} \quad \epsilon_1 \to 0, \qquad Y_{B_1} \to 0, \\ &\alpha = \beta \to \frac{\pi}{2} \quad [\text{solution (1)}], \\ &\alpha \to -\frac{\pi}{2}, \qquad \beta \to 0 \quad [\text{solution (2)}]. \end{aligned} \tag{97}$$

Notice that we have an intuitive argument to understand the quasidegeneracy between M_2 and M_3 (88) and the smallness of the CP violation (95). Indeed, in the limit $A_{23}^L = A_{33}^L = 0$ (60), and neglecting terms of order $m_{D_1}^2$ and $m_{D_1}m_{D_2}$, since we take V^L to be close to a diagonal matrix, the M_R matrix has only $M_{R_{13}}$, $M_{R_{31}}$ and $M_{R_{22}}$ sizeable matrix elements with $M_{R_{13}} \simeq M_{R_{31}}$, and the matrix M_R is close to real. Therefore, one has $M_2 \simeq M_3$ and $\epsilon_1 \simeq 0$.

We can try to modify our very simplified scheme by lifting the degeneracy of N_{R_2} and N_{R_3}. We will thus relax somewhat the strong condition (60), but keep the general physical idea of a compact heavy N_R spectrum. We will allow for nonvanishing values for the matrix elements $|A_{23}^L|$ and $|A_{33}^L|$, keeping them "small," i.e. values much smaller than each of their individual contributions that, due to the smallness of the light neutrino masses, are naturally of the order $\sim 10^{11}$ GeV^{-1}, as can be seen in Eqs. (38) and (39).

As we will further examine, one can thus obtain a rather compact N_R spectrum, and also reasonable values for the baryon asymmetry consistent with the data without spoiling the good properties of the light neutrino spectrum.

VI. LIFTING THE QUASIDEGENERACY OF HEAVY NEUTRINOS

We will proceed now, in terms of some parameters, to a continuous and slow lifting of the quasidegeneracy of the heavy right-handed neutrino masses obtained in the previous section within the strong hypothesis (60).

In this section we do the calculation considering nonvanishing values for the rhs of Eqs. (60). Moreover, since we have seen that even the drastic assumption of taking $A_{33}^L = A_{23}^L = 0$ gives reasonable neutrino spectra (84) or (89), we will allow $|A_{23}^L|$ and $|A_{33}^L|$ to vary within a very wide range, keeping small values ($\ll 10^{11}$ GeV^{-1}), and observe how the heavy neutrino and the light neutrino spectra evolve, as well as the consequences for the CP-violation asymmetry ϵ_1, the effective neutrino mass $\hat{m}_1$ and baryon asymmetry Y_{B_1}. We will perform the calculations for both solutions (1) ($\tan\theta_s \simeq -\sqrt{0.4}$) and (2) ($\tan\theta_s \simeq +\sqrt{0.4}$).

Notice that the strong conditions (60) $A_{33}^L = A_{23}^L = 0$ are linear homogeneous equations in $\frac{1}{m_i}$ $(i = 1, 2, 3)$. We now allow for nonvanishing inhomogeneous terms:

$$A_{23}^L(m_1, m_2, m_3) = C_{23}^L, \tag{98}$$

$$A_{33}^L(m_1, m_2, m_3) = C_{33}^L. \tag{99}$$

To lift the very close degeneracy between M_2 and M_3 in the case examined before, $A_{33}^L = A_{23}^L = 0$, we just need to have nonvanishing, in general complex parameters C_{23}^L and C_{33}^L in the rhs of (98) and (99). However, to have an overall compact heavy neutrino spectrum, we need inhomogeneous terms that should be small in modulus relative to each individual term in A_{33}^L and A_{23}^L (38) and (39). This means that we will take nonvanishing values for C_{23}^L and C_{33}^L with the condition

$$|C_{23}^L|, |C_{23}^L| \ll 10^{11} \text{ GeV}^{-1}. \tag{100}$$

In principle, one should scan the general two *complex* numbers C_{23}^L and C_{33}^L and see how the heavy neutrino spectrum evolves, as well as the light neutrino masses, the light neutrino Majorana phases α and β, the CP asymmetry ϵ_1 and final baryon asymmetry Y_{B_1}.

As pointed out at the beginning of Sec. V, the Majorana phases (85) and (90), that we found for $C_{23}^L = C_{33}^L = 0$ have their origin in the approximation adopted for the matrix V^L, that we take close to the CKM matrix. In order to preserve this interesting feature, we will assume that the nonvanishing values of the inhomogeneous terms C_{23}^L and C_{33}^L are *real* and satisfy (100). Later, in Sec. IX we will relax this reality assumption and will see that we have a wide domain of values for complex C_{23}^L and C_{33}^L that can give reasonable results.

The first important observation to be made is that the degeneracy between N_{R_2} and N_{R_3}, that we have found solving Eqs. (60), is lifted considerably if $|C_{33}^L|$ is nonvanishing in some region with $|C_{33}^L| \ll 10^{11}$ GeV^{-1}, and we have realized that the mass difference $M_3 - M_2$ is rather insensitive to the precise value of $|C_{23}^L|$, provided that its value is not "too large." Importantly, the amount of CP violation, and therefore of baryon asymmetry, also depends on the value adopted for $|C_{23}^L|$.

To reduce the number of parameters, we assume that C_{23}^L and C_{33}^L are real and of equal modulus $|C_{23}^L| = |C_{33}^L|$. As we vary C_{23}^L and C_{33}^L we find essentially the same heavy neutrino spectrum and roughly the same values for Δm_s^2 and Δm_a^2, independent of their relative sign. We find that what is dependent on this relative sign is the amount of CP violation and baryon asymmetry. If $C_{23}^L = C_{33}^L$ (independent of its sign), the baryon asymmetry can be at most of $O(10^{-12})$, but if C_{23}^L and C_{33}^L are of opposite sign, one can get a correct amount of baryon asymmetry.

In conclusion, after some trial and error guesses, we, respectively, adopt real numbers for C_{23}^L and C_{33}^L for both solutions (1) ($\tan\theta_s < 0$) and (2) ($\tan\theta_s > 0$):

(1)
$$-C_{23}^L = C_{33}^L > 0, \tag{101}$$

(2)
$$-C_{23}^L = C_{33}^L < 0, \tag{102}$$

with $|C_{33}^L| \ll 10^{11}$ GeV. As will become clear below, the adopted sign for each of the solutions corresponds to the experimental sign $Y_B > 0$ in some region for the parameters C_{23}^L, C_{33}^L. We assume that $Y_{B_1} \simeq Y_B$, a hypothesis that will be justified in Sec. X.

We will now show how the heavy neutrino and the light neutrino spectra evolve under the conditions (101) and

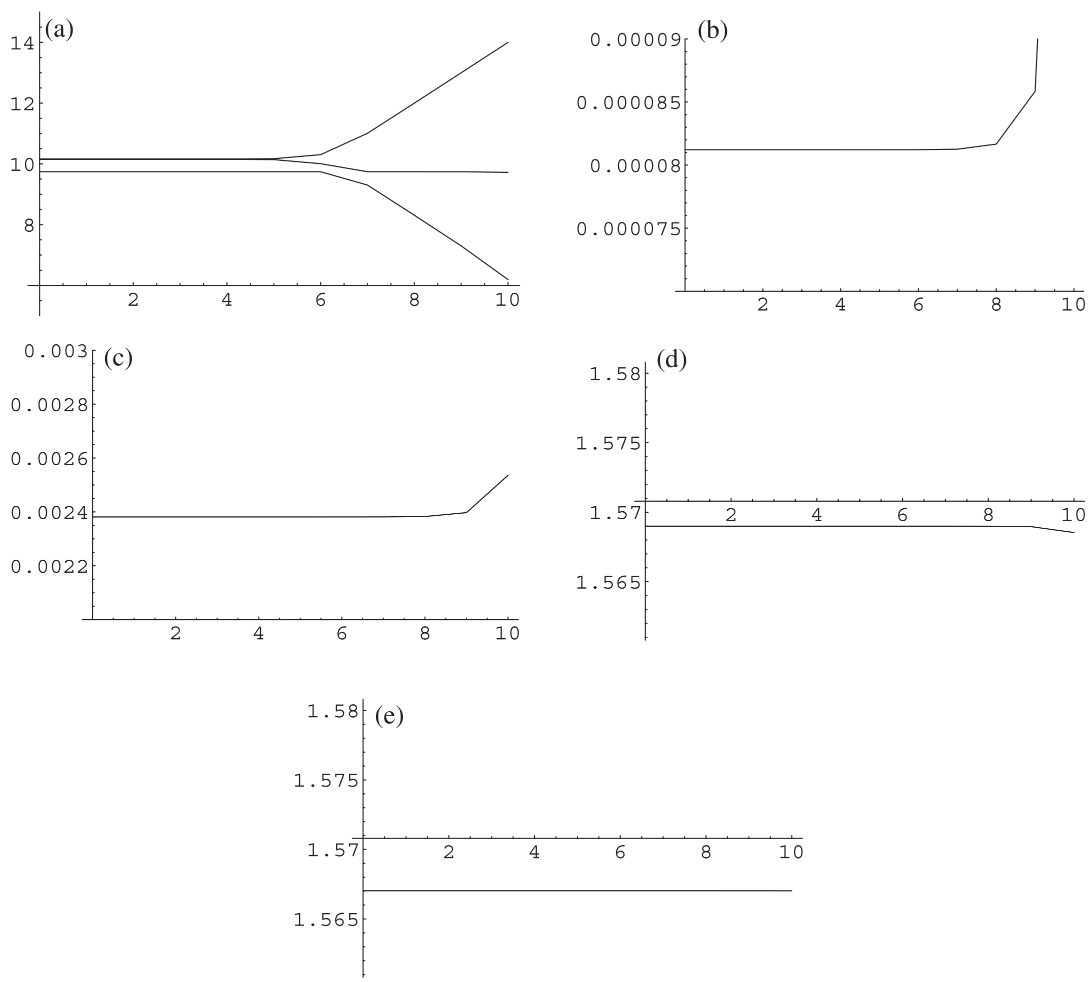

FIG. 1. (a) Log-log plot of the right-handed heavy neutrino spectrum (masses in GeV units) as a function of $-C_{23}^L = C_{33}^L > 0$ in units of GeV^{-1}, for fixed $m_1 = 0.0030$ eV and $\tan\theta_{12} = 0.140$. Within the range $-C_{23}^L = C_{33}^L = 10^6$–$10^7$ GeV^{-1}, there is a level crossing. The angular points come from the fact that the curves are obtained from interpolation of a finite number of points. The same applies to the other figures. (b) Δm_s^2 in eV^2 units as a function of $-C_{23}^L = C_{33}^L > 0$ in units of GeV^{-1}, for fixed $m_1 = 0.0030$ eV and $\tan\theta_{12} = 0.140$, in a log scale for C_{33}^L. (c) Δm_a^2 in eV^2 units as a function of $-C_{23}^L = C_{33}^L > 0$ in units of GeV^{-1}, for fixed $m_1 = 0.0030$ eV and $\tan\theta_{12} = 0.140$, in a log scale for C_{33}^L. (d) The Majorana phase α as a function of $-C_{23}^L = C_{33}^L > 0$ in units of GeV^{-1}, in a log scale for C_{33}^L, for fixed $m_1 = 0.0030$ eV and $\tan\theta_{12} = 0.140$. The x-axis is centered at $\pi/2$. (e) The Majorana phase β as a function of $-C_{23}^L = C_{33}^L > 0$ in units of GeV^{-1}, in a log scale for C_{33}^L, for fixed $m_1 = 0.0030$ eV and $\tan\theta_{12} = 0.140$. The x-axis is centered at $\pi/2$.

(102). In the next section we will show how ϵ_1, $\hat{m}_1$ and Y_{B_1} behave.

Of course, our ansatz for C^L_{23}, C^L_{33} is just a guess. We do not intend to make a fit to the overall data, light neutrino spectrum and baryon asymmetry. We just want to see if the description of these data is possible within this scheme of a compact heavy neutrino spectrum and approximate quark-lepton symmetry. Other equations of the type (98) and (99), with *complex* rhs values for the parameters C^L_{23}, C^L_{33} also give acceptable results, as we will see in Sec. IX.

A. Heavy and light neutrino spectra for case (1) $\tan\theta_s < 0$

To perform the calculations we again adopt the values of the pure Cabibbo limit, namely, $m_1 = 0.0030$ eV, $\tan\theta_{12} = 0.140$, although one could change these values slightly to get a better fit.

In Fig. 1(a) the right-handed heavy neutrino spectrum is plotted. In Figs. 1(b) and 1(c) we show, respectively, the solar and atmospheric quantities Δm^2_s, Δm^2_a.

Let us comment on these figures. The angular points that appear in the figures are an artifact of the representation of the curves, obtained from an interpolation of a finite number of points. Notice that *for each point* we must perform the singular value decomposition of the matrix M_R in order to compute the quantities necessary to obtain the baryon asymmetry.

The first striking point is that, as we have learned from Eqs. (98) and (99), the N_R spectrum [Fig. 1(a)] is very compact for C^L_{33} not "too large," $C^L_{33} < 10^7$ GeV^{-1}. As explained in the Introduction, there is an expected correlation between the stability of the light neutrino spectrum and the compact heavy right-handed neutrino one. The fine tuning for the close heavy neutrino masses ensures the stability of the light neutrino ones.

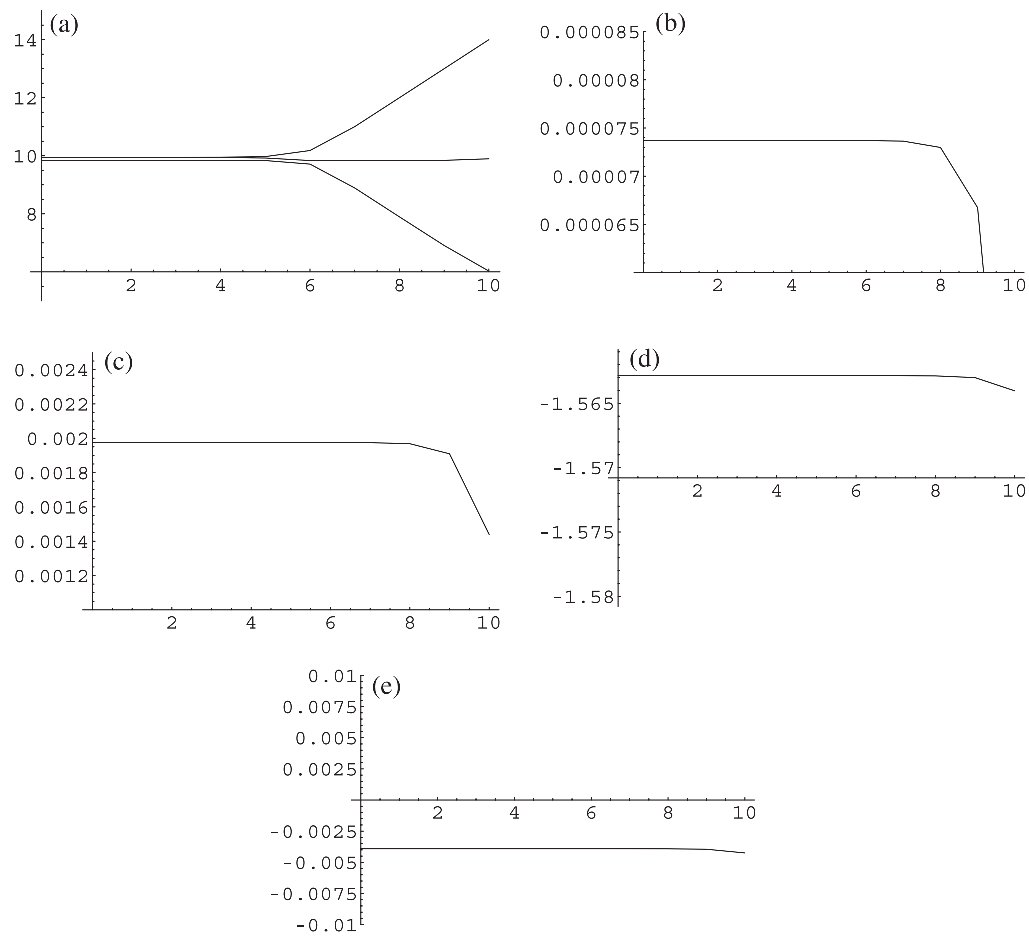

FIG. 2. (a) Log-log plot of the right-handed heavy neutrino spectrum (masses in GeV units) as a function of $C^L_{23} = -C^L_{33} > 0$ in units of GeV^{-1}, for fixed $m_1 = 0.0062$ eV and $\tan\theta_{12} = 0.243$. Within the range $-C^L_{33} = 10^5$–10^6 GeV^{-1}, there is a level crossing. (b) Δm^2_s in eV^2 units as a function of $C^L_{23} = -C^L_{33} > 0$ in units of GeV^{-1}, in a log scale for $-C^L_{33}$, for fixed $m_1 = 0.0062$ eV and $\tan\theta_{12} = 0.243$. (c) Δm^2_a in eV^2 units as a function of $C^L_{23} = -C^L_{33} > 0$ in units of GeV^{-1}, in a log scale for $-C^L_{33}$, for fixed $m_1 = 0.0062$ eV and $\tan\theta_{12} = 0.243$. (d) The Majorana phase α as a function of $C^L_{23} = -C^L_{33} > 0$ in units of GeV^{-1}, in a log scale for $-C^L_{33}$, for fixed $m_1 = 0.0062$ eV and $\tan\theta_{12} = 0.243$. The x-axis is centered at $-\pi/2$. (e) The Majorana phase β as a function of $C^L_{23} = -C^L_{33} > 0$ in units of GeV^{-1}, in a log scale for $-C^L_{33}$, for fixed $m_1 = 0.0062$ eV and $\tan\theta_{12} = 0.243$.

For $-C_{23}^L = C_{33}^L > 10^7$ GeV^{-1}, the right-handed neutrino spectrum evolves into a hierarchical spectrum. The values obtained for Δm_s^2 [Fig. 1(b)] and Δm_a^2 [Fig. 1(c)] are very stable and consistent with experiment for a wide range of values of $-C_{23}^L = C_{33}^L$, of about 8 orders of magnitude.

We observe two other important things in Fig. 1(a): the degeneracy between N_{R_2} and N_{R_3} is lifted, and there is a level crossing around $-C_{23}^L \simeq C_{33}^L = 3 \times 10^6$ GeV^{-1}.

An important point also to be underlined is that one of the levels (N_{R_1} before the crossing) remains practically constant in the whole studied range, while the mass of N_{R_2} decreases. After the crossing we will call N_{R_1} this right-handed neutrino, being the lightest, according to convention (53).

As we can see from Figs. 1(b) and 1(c), the values obtained for Δm_s^2 and Δm_a^2 are in good agreement with the data for a very wide range of the parameters.

For the Majorana phases α and β, shown in Figs. 1(d) and 1(e), we find rather constant values (in a logarithmic scale) that are very close but a little smaller than $\frac{\pi}{2}$, as shown in the figures.

B. Heavy and light neutrino spectra for case (2) $\tan\theta_s > 0$

To perform the calculations, we again adopt the values obtained in the pure Cabibbo limit, namely, $m_1 = 0.0062$ eV, $\tan\theta_{12} = 0.243$.

In Fig. 2(a) the right-handed heavy neutrino spectrum is plotted. In Figs. 2(b) and 2(c) we show, respectively, the solar and atmospheric quantities Δm_s^2, Δm_a^2. Figures 2(d) and 2(e) display the results for the Majorana phases α and β.

As shown in the figures, in a logarithmic scale, we find for the Majorana phase α a rather constant value that is very close to but a little larger than $-\frac{\pi}{2}$ and for β a small negative, almost constant value.

The first striking point in case (2) is that, as imposed from Eqs. (98) and (99), the N_R spectrum [Fig. 2(a)] has the same features as for the precedent solution, although it is very compact for $-C_{33}^L < 10^6$ GeV^{-1}, much more than in case (1). Within the range $C_{23}^L = -C_{33}^L = 10^5$–$10^6$ GeV^{-1}, there is also a level crossing, on which we will comment below. For $C_{23}^L = -C_{33}^L > 10^6$ GeV^{-1} the right-handed neutrino spectrum evolves also into a hierarchical spectrum. Second, Δm_s^2 [Fig. 2(b)] and Δm_a^2 [Fig. 2(c)] are very stable for a wide range of values of $C_{23}^L = -C_{33}^L$, of about 7 orders of magnitude. However, the agreement with experiment is not as good as it was for solution (1), although it is acceptable within a 3σ range. Of course, we could somewhat change the initial conditions $m_1 = 0.0062$ eV, $\tan\theta_{12} = 0.243$ and get results closer to the data. This could be done, but we will not do it because our purpose is only a qualitative one within our (fine-tuning) scheme.

VII. *CP* VIOLATION AND BARYON ASYMMETRY IN THE REGION APPROACHING THE LEVEL CROSSING

Let us turn now to the quantities that are important for baryogenesis via leptogenesis. Labeling the lightest heavy right-handed neutrino N_{R_1}, our calculations show that the quantities $-\epsilon_1$, $\tilde{m}_1$ and Y_{B_1} have a strong discontinuity across the level crossing region. We always call N_{R_1} the lightest heavy neutrino, even after the crossing, according to the convention (53).

We will justify and characterize this term of level crossing, and discuss its implications before and after this region in Sec. X.

To simplify the presentation of the results, we will restrict ourselves to the region *before the crossing*, where M_2 and M_1 become relatively close, i.e. to the following regions, slightly different in both cases:

(1) $$10^5\ \mathrm{GeV}^{-1} \le -C_{23}^L = C_{33}^L \le 10^{6.4}\ \mathrm{GeV},^{-1}$$
$$10^9\ \mathrm{GeV} \le M_2 - M_1 \le 8.3 \times 10^9\ \mathrm{GeV}, \quad (103)$$

(2) $$10^4\ \mathrm{GeV}^{-1} \le C_{23}^L = -C_{33}^L \le 10^{5.6}\ \mathrm{GeV},^{-1}$$
$$10^9\ \mathrm{GeV} \le M_2 - M_1 \le 2. \times 10^9\ \mathrm{GeV}. \quad (104)$$

To avoid the delicate situation related to the quasidegeneracy of two heavy neutrinos, extensively studied by A. Pilaftsis *et al.* [25], we need the condition (see also [7])

$$\Gamma_1 \ll M_2 - M_1, \quad (105)$$

where Γ_1 is the width of the lightest heavy right-handed neutrino, that has an upper bound qualitatively given by [7]:

$$\Gamma_1 \le \frac{m_t^2}{16\pi v^2} M_1. \quad (106)$$

Before the level crossing region one gets, from the parameters quoted above ($m_t \simeq m_{D_3} \simeq 100$ GeV, $v = 174$ GeV and $M_1 \simeq 5. \times 10^9$ GeV),

$$\Gamma_1 \le 3. \times 10^7\ \mathrm{GeV}. \quad (107)$$

Therefore, before the level crossing region, taking into account the inequalities (103) and (104), we see that, in both cases (1) and (2), the condition (105) is satisfied. We are far away from resonant leptogenesis and we do not have to face complications related to quasidegenerate heavy neutrinos, for which $M_2 - M_1 \le \Gamma_1$.

Let us now show the quantities $-\epsilon_1$, $\tilde{m}_1$ and Y_{B_1} for both solutions within the interesting ranges (103) and (104).

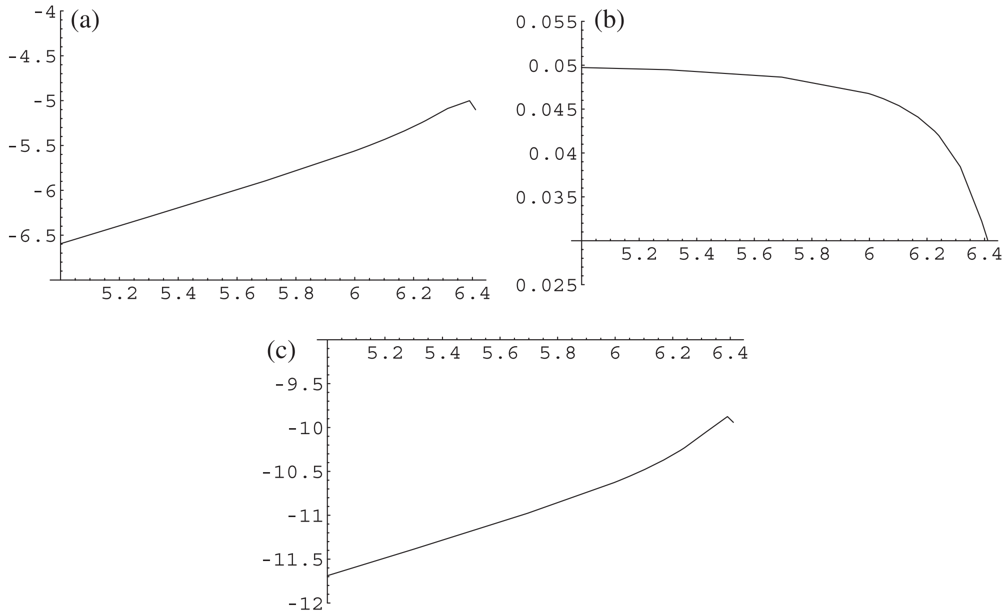

FIG. 3. (a) Log-log plot of $-\epsilon_1$ as a function of $-C^L_{23} = C^L_{33}$ in units of GeV^{-1}, for fixed $m_1 = 0.0030$ eV and $\tan\theta_{12} = 0.140$. (b) $\tilde{m}_1$ in eV units as a function of $-C^L_{23} = C^L_{33}$ in units of GeV^{-1}, in a log scale for C^L_{33}, for fixed $m_1 = 0.0030$ eV and $\tan\theta_{12} = 0.140$. (c) Log-log plot of Y_{B_1} as a function of $-C^L_{23} = C^L_{33}$ in units of GeV^{-1}, for fixed $m_1 = 0.0030$ eV and $\tan\theta_{12} = 0.140$.

A. Case (1) $\tan\theta_s < 0$

Figure 3(a) displays the CP violation parameter $-\epsilon_1$, Fig. 3(b) plots the washout parameter $\tilde{m}_1$, and Fig. 3(c) shows the baryon asymmetry Y_{B_1} in the one-flavor approximation.

We observe that ϵ_1 is negative and becomes large enough in absolute magnitude to give a large positive Y_{B_1} with rather stable values of the parameter $\tilde{m}_1$ that imply strong washout in the whole region. Notice that $-\epsilon_1$ as well as Y_{B_1} grow as the mass difference $M_3 - M_2$ slowly grows and the mass difference $M_2 - M_1$ becomes smaller, a phenomenon already underlined by Akhmedov *et al.* [7].

The main conclusion that we can draw from Fig. 3(c) is that there is no problem to getting a baryon asymmetry Y_{B_1} of the right order of magnitude $(Y_B)_{\mathrm{exp}} \simeq 9 \times 10^{-11}$.

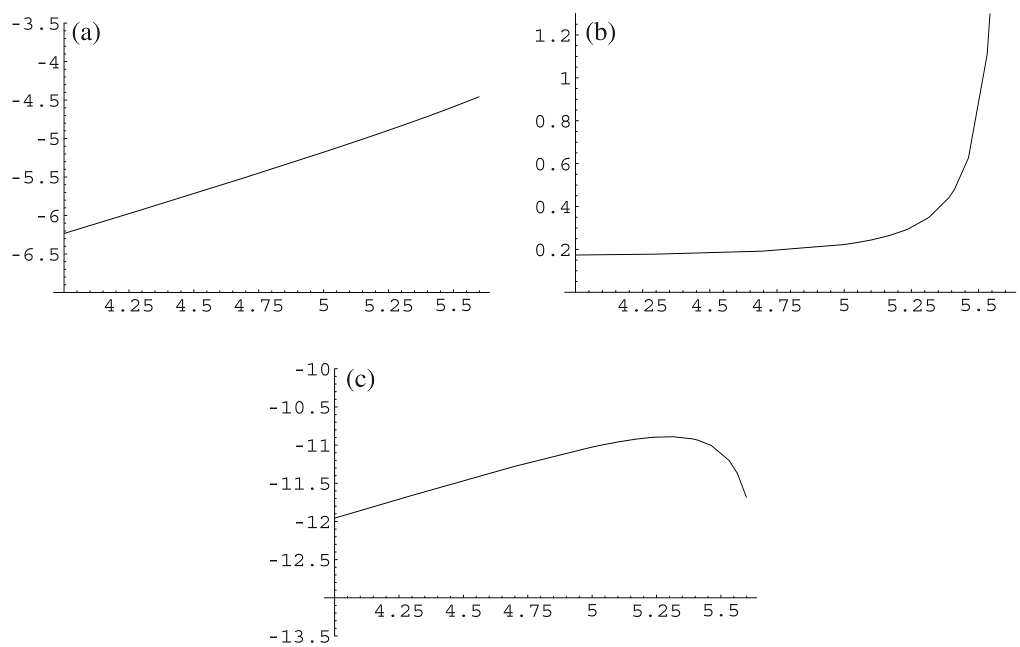

FIG. 4. (a) Log-log plot of $-\epsilon_1$ as a function of $C^L_{23} = -C^L_{33}$ in units of GeV^{-1}, for fixed $m_1 = 0.0062$ eV and $\tan\theta_{12} = 0.243$. (b) $\tilde{m}_1$ in eV units as a function of $C^L_{23} = -C^L_{33}$ in units of GeV^{-1}, in a log scale for $-C^L_{33}$, for fixed $m_1 = 0.0062$ eV and $\tan\theta_{12} = 0.243$. (c) Log-log plot of Y_{B_1} as a function of $C^L_{23} = -C^L_{33}$ in units of GeV^{-1}, for fixed $m_1 = 0.0062$ eV and $\tan\theta_{12} = 0.243$.

We must underline that if we had taken the opposite sign in (101), we would have obtained the opposite sign for ϵ_1 and therefore also for Y_{B_1}. Therefore, our scheme *does not predict* the sign of Y_B, since it depends on the chosen sign of the inhomogeneous terms.

B. Case (2) $\tan\theta_s > 0$

Figure 4(a) displays the CP violation parameter $-\epsilon_1$, Fig. 4(b) plots the washout parameter $\tilde{m}_1$, and Fig. 4(c) shows the baryon asymmetry Y_{B_1} in the one-flavor approximation.

We observe that ϵ_1 is negative and becomes large in absolute magnitude to give a rather large positive Y_{B_1} with values of the parameter $\tilde{m}_1$ within the strong washout regime.

We must again point out that if we had taken the opposite sign in the rhs of (102), we would have obtained the opposite sign for ϵ_1 and therefore also for Y_{B_1}.

The main conclusion that we can draw from Fig. 4(c) is that in this case we are somewhat short of having a baryon asymmetry Y_{B_1} of the right order of magnitude $(Y_B)_{\rm exp} \simeq 9 \times 10^{-11}$. However, as pointed out above, one could modify the initial conditions (the values $m_1 = 0.0062$ and $\tan\theta_{12} = 0.243$) and get results in better agreement with the data. But it is not our purpose to make a detailed fit for Δm_s^2, Δm_a^2 and Y_{B_1}; we only want to give a qualitative trend.

Let us emphasize again that in the present scheme developed in Secs. VI and VII, due to the reality conditions on C_{23}^L and C_{33}^L, we again have the interesting limit (97):

$$\begin{aligned} &\delta_L \to 0 \quad \text{implies} \quad \epsilon_1 \to 0 \qquad Y_{B_1} \to 0 \\ &\alpha = \beta \to \frac{\pi}{2} \quad [\text{solution (1)}], \\ &\alpha \to -\frac{\pi}{2}, \qquad \beta \to 0 \quad [\text{solution (2)}]. \end{aligned} \tag{108}$$

VIII. RESULTS FOR m_{ν_e}, $(\beta\beta)_{0\nu}$ AND THE SUM OF THE NEUTRINO MASSES

We give here the predictions for the electron neutrino mass, on which one has limits from tritium β decay,

$$m_{\nu_e} = \cos^2\theta_s |m_1| + \sin^2\theta_s |m_2| \tag{109}$$

and for the effective mass relevant for neutrinoless double beta decay $|\langle m_{ee}\rangle|$ that writes, within the approximation (33),

$$|\langle m_{ee}\rangle| = |\cos^2\theta_s m_1 + \sin^2\theta_s m_2|. \tag{110}$$

The neutrino masses and their phases for both solutions (84) and (89) are very close to those obtained in the region that give an acceptable value for Y_B. Taking thus the values for both solutions, and using the notation (34),

$$(1)\quad m_1 = 0.0030\ \text{eV}, \qquad m_2 = -0.0095 e^{0.0036i}\ \text{eV}, \qquad m_3 = -0.0495 e^{0.0075i}\ \text{eV}, \tag{111}$$

$$(2)\quad m_1 = 0.0062\ \text{eV}, \qquad m_2 = -0.0106 e^{-0.016i}\ \text{eV}, \qquad m_3 = 0.0455 e^{0.0078i}\ \text{eV}, \tag{112}$$

we obtain, respectively:

$$(1)\qquad m_{\nu_e} \simeq 4.9 \times 10^{-3}\ \text{eV}, \qquad |\langle m_{ee}\rangle| \simeq 5.7 \times 10^{-4}\ \text{eV}, \tag{113}$$

$$(2)\qquad m_{\nu_e} \simeq 7.5 \times 10^{-3}\ \text{eV}, \qquad |\langle m_{ee}\rangle| \simeq 1.4 \times 10^{-3}\ \text{eV}. \tag{114}$$

For both solutions, due to the relative signs among the m_i ($i = 1, 2, 3$) (i.e. due to the Majorana phases), there is a strong cancellation between the two terms in (110), a phenomenon already exhibited in [6] in another context. The cancellation is stronger for solution (1).

For the sum of the absolute magnitude of all neutrino masses, we obtain:

$$(1)\qquad \sum_i |m_{\nu_i}| = 0.0620\ \text{eV}, \tag{115}$$

$$(2)\qquad \sum_i |m_{\nu_i}| = 0.0623\ \text{eV}. \tag{116}$$

One gets very close results for both solutions that comply with the cosmological bounds (8) and (9) [4].

Let us make a last qualitative remark comparing the different possible future experiments on neutrino masses and stress the importance of cosmological limits.

If one takes $m_1 \simeq 0$, one finds, from the data, $|m_2| \simeq \sqrt{\Delta m_s^2} \simeq 9 \times 10^{-3}$ eV and $|m_3| \simeq \sqrt{\Delta m_a^2 + \cos^2\theta_s \Delta m_s^2} \simeq 5 \times 10^{-2}$ eV, which correspond to a value for the lhs in Eqs. (8) and (9) of the order 6×10^{-2} eV, very near the value that we have found and only a factor 3.3 below the bound (9) [4]. So, according to the present scenario, the most promising search for effects of neutrino masses, apart from oscillation experiments, is the analysis of cosmological data, while for beta decay and neutrinoless double beta decay one would need an improvement of more than 2 orders of magnitude.

IX. RELAXING THE ADDITIONAL REALITY CONSTRAINTS OF THE MODEL

We now relax the conditions of the particular model that we have studied quantitatively, namely, those given by

Eqs. (98) and (99) with real C^L_{23} and C^L_{33} satisfying (101) and (102), and allow complex numbers for these parameters, keeping, however, "small" values for the moduli, as stated in (100).

Notice the important point that now we have two new sources of CP violation besides a single independent phase δ_L (or α or β), and we recover the situation in which there are three independent phases: δ_L, α and β. However, as we will see below, in the range of interest for Y_{B_1}, the magnitude of the Majorana phases is not very different than in the real case studied in detail in Secs. V, VI, and VII. However, their new contributions have important implications for the baryon asymmetry.

Just to have a feeling of what can happen, we take an extreme case and adopt relations (101) and (102) with the condition (100), but now taking C^L_{23} and C^L_{33} *purely imaginary*. Interestingly, the results are phenomenologically good and show that our general scheme of a compact N_R spectrum is flexible enough.

We do not give the corresponding curves of Secs. VI and VII, and give values for representative points with acceptable phenomenological results.

For case (1), i.e. $\tan\theta_s \simeq -\sqrt{0.4}$ with $m_1 = 0.0030$ eV and $\tan\theta_{12} = 0.140$, taking

$$-C^L_{23} = C^L_{33} = i10^{-5}, \tag{117}$$

we find the following results:

$$\begin{aligned}
&\Delta m_s^2 = 8.1\times 10^{-5}\,\mathrm{eV}^2, \quad \Delta m_a^2 = 2.4\times 10^{-3}\,\mathrm{eV}^2,\\
&\quad \alpha = \frac{\pi}{2} - 0.0018, \quad \beta = \frac{\pi}{2} - 0.0038,\\
&\quad M_1 = 5.531\times 10^{9}\,\mathrm{GeV}, \quad M_2 = 1.383\times 10^{10}\,\mathrm{GeV},\\
&\quad M_3 = 1.483\times 10^{10}\,\mathrm{GeV}, \quad \tilde m_1 = 0.050\,\mathrm{eV},\\
&\quad \epsilon_1 = -2.755\times 10^{-5}, \quad Y_{B_1} = 2.211\times 10^{-10},
\end{aligned} \tag{118}$$

while for case (2), i.e. $\tan\theta_s \simeq +\sqrt{0.4}$ with $m_1 = 0.0062$ eV and $\tan\theta_{12} = 0.243$, taking

$$-C^L_{23} = C^L_{33} = -i10^{-3}, \tag{119}$$

we find:

$$\begin{aligned}
&\Delta m_s^2 = 7.4\times 10^{-5}\,\mathrm{eV}^2, \quad \Delta m_a^2 = 2.0\times 10^{-3}\,\mathrm{eV}^2,\\
&\quad \alpha = -\frac{\pi}{2} + 0.0079, \quad \beta = -0.0039,\\
&\quad M_1 = 6.847\times 10^{9}\,\mathrm{GeV}, \quad M_2 = 8.844\times 10^{9}\,\mathrm{GeV},\\
&\quad M_3 = 8.954\times 10^{9}\,\mathrm{GeV}, \quad \tilde m_1 = 0.170\,\mathrm{eV},\\
&\quad \epsilon_1 = -3.622\times 10^{-5}, \quad Y_{B_1} = 7.035\times 10^{-11},
\end{aligned} \tag{120}$$

These results are phenomenologically reasonable, and we find a whole region in their neighborhood that also gives good results.

Notice that in the numbers of case (2) we are not far away from saturating the bound (107), and therefore we are approaching the regime of resonant leptogenesis.

Let us emphasize again that in the case examined here we have two different sources of CP violation: δ_L and the Majorana phases α, β.

To illustrate how these new contributions to Majorana phases occur, it is useful to recall again how we perform our calculations. Proceeding like in Sec. VI, using Eqs. (117) and (119), we compute, from (38) and (39), m_2 and m_3 [with the convention (34)] in terms of the given values for m_1 and $\tan\theta_{12}$. Then, m_2 and m_3 get by construction new CP-violating contributions to the Majorana phases because, according to (117) and (119), the inhomogeneous terms must be pure imaginary.

Of course, since we now have new sources of CP violation in the Majorana phases, in the limit $\delta_L \to 0$ we do not recover the simple limit (108) that we got for real values of C^L_{23} and C^L_{33} or, equivalently, for CP violation in the Majorana phases fixed exclusively from the phase δ_L.

We had CP violation in Majorana phases that were induced by their calculation for a given δ_L in the case of vanishing C^L_{23} and C^L_{33} (Sec. V B). But, from the imaginary inhomogeneous terms of the present section, we now have new sources of CP violation in these phases.

These new sources of CP violation in the Majorana phases, although small, are very efficient in producing a baryon asymmetry, as we realize from the results (118) and (120).

It can easily be understood that the constraints (117) and (119) imply new contributions to the baryon asymmetry. These equations mean that the entries $A^L_{23} = A^L_{32}$ and A^L_{33} are purely imaginary. This in turn implies, from (43), new CP violation contributions to the mass matrix M_R, providing, after its diagonalization, new contributions to ϵ_1 and Y_{B_1}. This important phenomenon certainly deserves further investigation for general complex inhomogeneous terms, keeping however a compact N_R spectrum.

The important conclusion of the calculations of the present section is that the results presented in Secs. VI and VII remain much more general than the very particular model that, for the sake of simplicity, was exposed there. Our scheme, although fine-tuned because we look for a compact heavy neutrino spectrum, allows for a wide range of parameters giving good results.

X. COMMENTS ON THE COMPACT HEAVY NEUTRINO SPECTRUM AND ON THE LEVEL CROSSING REGION

We now go back to the case that we have studied in quantitative detail, namely, Eqs. (98) and (99) with the conditions (101) and (102).

Before and around the level crossing region, we have a rather or very compact heavy neutrino spectrum. For simplicity, we have assumed that the lightest heavy neutrino N_{R_1} decays out of equilibrium and gives the main contribution to the important quantities relevant for baryogenesis: ϵ, $\tilde m$ and Y_B.

In such a fine-tuned situation, this can seem rather artificial. Actually, one should consider the contributions of all three heavy neutrinos, and therefore the contributions of all the CP-violation parameters ϵ_1, ϵ_2 and ϵ_3, and the corresponding washout factors. Notice that there are studies in the literature that consider all these contributions. See, for example, the paper by E. Bertuzzo *et al.* [26] and also, in a qualitative way, the work by Akhmedov *et al.* [7]. However, to take into account the contributions of all three heavy neutrinos can present some subtleties. We have not dared, for the moment, to roughly add Y_{B_1}, Y_{B_2} and Y_{B_3}. We simply expect that the possible contributions of all three heavy neutrinos will not strongly affect the results that we have found from N_{R_1}. An argument given below supports this hypothesis.

Let us now comment on the crossing region. As pointed out above, there is a level crossing in both cases: (1) $\tan\theta_s < 0$ and (2) $\tan\theta_s > 0$. This happens around $-C^L_{23} = C^L_{33} \simeq 3 \times 10^6\ \mathrm{GeV}^{-1}$ for solution (1) and $C^L_{23} = -C^L_{33} \simeq 5 \times 10^5\ \mathrm{GeV}^{-1}$ for solution (2). At some point in this region, the heavy neutrinos N_{R_1} and N_{R_2} become degenerate.

But we want to make more precisely explicit what we understand by level crossing. What we mean is that the properties of the N_{R_1} neutrino before the level crossing become (up to signs) those of the N_{R_2} neutrino after the level crossing, and vice versa, exchanging their effect on the absolute magnitude of the final quantities ϵ, $\tilde{m}$ and Y_B. This can easily be seen by writing the effective Dirac neutrino mass that enters in formulas (51) and (54), before and after the crossing region.

To illustrate what happens, we will take as an example solution (1). Similar features appear for solution (2). To be definite, we consider N_{R_1} (the lightest neutrino) and N_{R_2} (the next-to-lightest neutrino) before the crossing region, as we have computed in Sec. VII. Naïvely applying for N_{R_2} the formulas (54), (57), and (59) and making just the exchange $M_1 \leftrightarrow M_2$, let us give for solution (1) the quantities $\tilde{m}_1$, ϵ_1, Y_{B_1} and $\tilde{m}_2$, ϵ_2, Y_{B_2} at one point before the crossing region, for example, for the value $C^L_{33} = 10^6\ \mathrm{GeV}^{-1}$, and at one point after the crossing, for example, for $C^L_{33} = 10^7\ \mathrm{GeV}^{-1}$. Let us recall that the Dirac matrix (87) is completely fixed, but the redefined matrix (51) changes from point to point because it depends on the diagonalization of the matrix M_R by (49).

One finds, *before the crossing*, for $C^L_{33} = 10^6\ \mathrm{GeV}^{-1}$, these values for the heavy neutrino masses:

$$M_1 = 5.531 \times 10^9\ \mathrm{GeV}, \qquad M_2 = 1.017 \times 10^{10}\ \mathrm{GeV}, \qquad M_3 = 2.017 \times 10^{10}\ \mathrm{GeV}, \tag{121}$$

and the Hermitian matrix that enters in (57) and (54):

$$\hat{m}_D^+ \hat{m}_D \simeq \begin{pmatrix} 0.259 & 18.176 - 0.089i & -0.125 - 25.597i \\ 18.176 + 0.089i & 3352.06 & -0.000\,06 - 4720.59i \\ -0.125 + 25.597i & -0.000\,06 + 4720.59i & 6647.84 \end{pmatrix} \mathrm{GeV},^2 \tag{122}$$

and one gets therefore:

$$\tilde{m}_1 = 0.047\ \mathrm{eV}, \quad \epsilon_1 = -2.749 \times 10^{-6}, \quad Y_{B_1} = 2.383 \times 10^{-11}, \quad \tilde{m}_2 = 329.586\ \mathrm{eV}, \quad \epsilon_2 = -1.425 \times 10^{-10}, \quad Y_{B_2} = 4.245 \times 10^{-20}. \tag{123}$$

Notice one point here. The numbers obtained in (123) are very interesting in relation to the calculations done in Sec. VII for the region before the level crossing. We have assumed there that Y_B is dominated by the contribution of the lightest neutrino Y_{B_1}. We see indeed that, at least within these naïve estimates, this is true as far as the consideration of the next-to-lightest neutrino is concerned.

Remember that we have adopted the level ordering convention (53) that also applies after the crossing: N_{R_1} is the lightest neutrino and N_{R_2} is the next-to-lightest neutrino.

One finds, *after the crossing*, for example, for $C^L_{33} = 10^7\ \mathrm{GeV}^{-1}$, the heavy neutrino masses:

$$M_1 = 2.011 \times 10^9\ \mathrm{GeV}, \qquad M_2 = 5.531 \times 10^9\ \mathrm{GeV}, \qquad M_3 = 1.020 \times 10^{11}\ \mathrm{GeV}, \tag{124}$$

and the Hermitian matrix that enters in (57) and (54):

$$\hat{m}_D^+ \hat{m}_D \simeq \begin{pmatrix} 193.27 & -5.937 - 0.020i & -0.000\,29 + 1376.69i \\ -5.937 + 0.020i & 0.342 & -0.143 - 42.293i \\ -0.000\,29 - 1376.69i & -0.143 + 42.293i & 9806.55 \end{pmatrix} \mathrm{GeV}^2, \tag{125}$$

and one gets therefore:

$$\tilde{m}_1 = 96.125\ \text{eV}, \qquad \epsilon_1 = 8.023 \times 10^{-10}, \\ Y_{B_1} = -9.981 \times 10^{-19}, \qquad \tilde{m}_2 = 0.062\ \text{eV}, \\ \epsilon_2 = 3.694 \times 10^{-6}, \qquad Y_{B_2} = -2.312 \times 10^{-11}. \tag{126}$$

The shifts in order of magnitude among the elements of the matrices before and after the crossing, (122) or (125), explain the strong differences (in magnitude and even in sign) of the relevant quantities in these two regions, (123) or (126). We observe a strong discontinuity for the lightest neutrino properties (and for the next-to-lightest ones) that happens going through the crossing region. Up to the sign of ϵ and therefore of Y_B, we see that after the crossing the lightest neutrino has very strong washout and very small $|\epsilon_1|$ and therefore $|Y_{B_1}|$, and that the opposite is true for the next-to-lightest heavy neutrino N_{R_2}. It is easy to examine this for the parameter $\tilde{m}_i$ ($i = 1, 2$), just by inspection of the matrix elements $(\hat{m}_D^+ \hat{m}_D)_{ii}$ in (122) and (125). For ϵ_i it is a little more involved, but it can also be seen by looking at the squares of the matrix elements of $\hat{m}_D^+ \hat{m}_D$.

Let us now comment on the change of sign of ϵ_i and Y_{B_i} before and after the level crossing region (123) and (126). For solution (1), that we discuss here, the sign of the inhomogeneous term (101) $-C_{23}^L = C_{33}^L > 0$ gives $\epsilon_i < 0$ and $Y_{B_i} > 0$ before the crossing, and $\epsilon_i > 0$ and $Y_{B_i} < 0$ after the crossing. We have realized that, if one changes the sign of (101), i.e. $-C_{23}^L = C_{33}^L < 0$, then one has the opposite: $\epsilon_i > 0$ and $Y_{B_i} < 0$ before the crossing, and $\epsilon_i < 0$ and $Y_{B_i} > 0$ after the crossing. Adopting this latter sign for C_{23}^L, C_{33}^L, nothing essential changes for the heavy neutrino spectrum and for Δm_s^2 and Δm_a^2. The same considerations apply to solution (2) using (102) and changing its sign.

Our conclusion is that, provided N_{R_1} and N_{R_2} are close enough in mass, one can have the right order of magnitude and sign for Y_B before and after the crossing. This happens only at the price of changing the sign of our single free real parameter $-C_{23}^L = C_{33}^L$.

An interesting conclusion is that, after the level crossing, the second-to-lightest heavy neutrino N_{R_2} dominates. The possibility of next-to-lightest neutrino dominance has been extensively studied recently by S. Antusch *et al.* [27].

XI. OPEN PROBLEMS WITHIN THE PRESENT APPROACH

There are a number of problems to face and study within the present approach. Let us make an incomplete list:

(i) There is the possibility that more than one heavy right-handed neutrino decays out of equilibrium, contributing to the leptogenesis, a point that, in particular, has been suggested rather clearly in Ref. [7]. If we guess a temperature $T \simeq 10^{11}$ GeV below which all heavy neutrinos decay out of equilibrium, then not only the lightest N_{R_1} decays out of equilibrium after the level crossing, but also N_{R_2} is in the same situation. On the other hand, the heavy neutrino spectrum being rather compact, the natural thing to do would be to consider the contributions to Y_B of all three heavy neutrinos N_{R_1}, N_{R_2} and N_{R_3}, i.e. to compute ϵ_1, ϵ_2 and ϵ_3 and the relevant washout factors.
For the moment we just expect that the consideration of the three neutrinos will not spoil the good features of the calculations of the present paper, which take into account only the lightest neutrino N_{R_1} before the crossing.
We have given an argument in this sense in Sec. X where we have seen that, before the level crossing, the contribution of the next-to-lightest neutrino N_{R_2} is negligible compared to the one of the lightest one N_{R_1}.

(ii) One should also take into account the level crossing region and therefore the finite width of the right-handed neutrinos, as well as the delicate question of their interference. These problems have been treated in great detail by A. Pilaftsis and collaborators [25] and, to be complete, need to be adopted within our approach.

(iii) The flavor effects, thoroughly studied by A. Abada *et al.* [28], are also a delicate question to study in this region of compact right-handed neutrino spectrum, and this should be performed.

(iv) It would be worth studying the more general case for CP violation outlined in Sec. IX, and making a detailed scan of the results in the case of general complex inhomogeneous terms—or equivalently general light neutrino Majorana phases—with the constraint of having a compact heavy neutrino spectrum.

(v) It could be that the homogeneous Eqs. (60) correspond to some symmetry, the inhomogeneous terms (that we have introduced to get a large enough CP violation ϵ_1) being a breaking of this symmetry, a possibility that would also be interesting to study.

XII. CONCLUSIONS

Our demand of a compact N_R spectrum, and of an approximate quark-lepton symmetry implying a hierarchical spectrum for the Dirac neutrino masses with a similar structure between V^L and the CKM mixing matrix, brings us to a scenario where the lepton asymmetry comes out naturally, producing the required order of magnitude for the baryon asymmetry $Y_B \sim O(10^{-10})$. We have assumed and justified that Y_B is dominated by the contribution of the lightest neutrino N_{R_1}.

In this way, not only can one get a good magnitude for Y_B, but as a natural consequence there are also a number of other strong points in this approach.

We get two possible solutions with a normal hierarchical light neutrino mass spectrum and an absolute scale, i.e. the lightest neutrino mass m_1 must be nonvanishing.

The light neutrino squared mass differences Δm_s^2 and Δm_a^2 are very stable and consistent with the data.

There are three CP-violating phases in the whole approach, the phase δ_L of the V^L unitary matrix, and the light neutrino Majorana phases α and β. We take δ_L to be of the order of the Kobayashi-Maskawa phase $\delta_{\rm KM}$.

We have thoroughly studied in a quantitative way a particular case in which all CP-violating effects are computed in terms of δ_L, in particular, ϵ_1 and Y_{B_1}. It is interesting that one can get a baryon asymmetry of the right order of magnitude taking $\delta_L \simeq \delta_{\rm KM}$. Of course, this result is not obtained in the standard model, but in a new physics scheme under particular assumptions: baryogenesis via leptogenesis, $SO(10)$ grand unification, approximate quark-lepton symmetry and the compact heavy N_R spectrum. In the limit $\delta_L \to 0$, one indeed gets $\epsilon_1 \to 0$ and $Y_{B_1} \to 0$.

The ν_e mass, bounded by tritium β decay, is of the order of a few times 10^{-3} eV.

The sum $\sum_i m_{\nu_i}$ satisfies the cosmological bounds, with a value rather close to the present upper limits.

Let us emphasize that δ_L also induces small CP-violating corrections to the light neutrino Majorana phases that turn out to be naturally close to $\alpha = \frac{\pi}{2}$, $\beta = \frac{\pi}{2}$ or 0. The effective neutrino mass, relevant for neutrinoless double beta decay, comes out to be rather small, of the order of 10^{-3} eV, because of strong cancellations due to the Majorana phases.

In the region of quasidegeneracy, the heaviest N_R has a mass of the order 1.5×10^{10} GeV, roughly consistent with the expected scale of B–L symmetry breaking, so that $SO(10)$ breaks down to the Pati-Salam group $SU(4) \times SU(2) \times SU(2)$ at the expected natural intermediate scale.

We also expose an example in which the phase of the Dirac neutrino mass matrix δ_L and the Majorana phases α, β are independent, providing an efficient generation of baryon asymmetry.

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APPENDIX

In this Appendix we demonstrate the approximate formula (15) for Δm_a^2:

$$\Delta m_a^2 = |m_3|^2 - \cos^2\theta_s |m_2|^2 - \sin^2\theta_s |m_1|^2. \qquad \text{(A1)}$$

The mass eigenstates read:

$$\begin{aligned} |\nu_1\rangle &= c_s|\nu_e\rangle - \frac{s_s}{\sqrt{2}}|\nu_\mu\rangle + \frac{s_s}{\sqrt{2}}|\nu_\tau\rangle \\ |\nu_2\rangle &= s_s|\nu_e\rangle + \frac{c_s}{\sqrt{2}}|\nu_\mu\rangle - \frac{c_s}{\sqrt{2}}|\nu_\tau\rangle, \\ |\nu_3\rangle &= \frac{1}{\sqrt{2}}|\nu_\mu\rangle + \frac{1}{\sqrt{2}}|\nu_\tau\rangle, \end{aligned} \qquad \text{(A2)}$$

and therefore the μ-neutrino state is given, in terms of the mass eigenstates:

$$|\nu_\mu\rangle = -\frac{s_s}{\sqrt{2}}|\nu_1\rangle + \frac{c_s}{\sqrt{2}}|\nu_2\rangle + \frac{1}{\sqrt{2}}|\nu_3\rangle \qquad \text{(A3)}$$

that evolves in time according to:

$$\begin{aligned} |\nu_\mu(t)\rangle &= -\frac{s_s}{\sqrt{2}}e^{(-ip-i(m_1^2/2p))t}|\nu_1\rangle + \frac{c_s}{\sqrt{2}}e^{(-ip-i(m_2^2/2p))t}|\nu_2\rangle \\ &\quad + \frac{1}{\sqrt{2}}e^{(-ip-i(m_3^2/2p))t}|\nu_3\rangle \\ &= e^{(-ip-i(m_3^2/2p))t}\Big[-\frac{s_s}{\sqrt{2}}e^{i((m_3^2-m_1^2)/2p)t}|\nu_1\rangle \\ &\quad + \frac{c_s}{\sqrt{2}}e^{i((m_3^2-m_2^2)/2p)t}|\nu_2\rangle + \frac{1}{\sqrt{2}}|\nu_3\rangle\Big], \end{aligned} \qquad \text{(A4)}$$

and, from (A2), we can write the scalar products:

$$\begin{aligned} &e^{(ip+i(m_3^2/2p))t}\langle\nu_e|\nu_\mu(t)\rangle \\ &\quad = \Big[-\frac{s_s}{\sqrt{2}}e^{i((m_3^2-m_1^2)/2p)t}\langle\nu_e|\nu_1\rangle \\ &\qquad + \frac{c_s}{\sqrt{2}}e^{i((m_3^2-m_2^2)/2p)t}\langle\nu_e|\nu_2\rangle + \frac{1}{\sqrt{2}}\langle\nu_e|\nu_3\rangle\Big] \\ &\quad = \frac{s_s c_s}{\sqrt{2}}[-e^{i((m_3^2-m_1^2)/2p)t} + e^{i((m_3^2-m_2^2)/2p)t}] \\ &\quad = \frac{s_s c_s}{\sqrt{2}}e^{i((m_3^2-m_2^2)/2p)t}[-1 + e^{i((m_2^2-m_1^2)/2p)t}], \end{aligned} \qquad \text{(A5)}$$

$$\begin{aligned} &e^{(ip+i(m_3^2/2p))t}\langle\nu_\mu|\nu_\mu(t)\rangle \\ &\quad = \Big[-\frac{s_s}{\sqrt{2}}e^{i((m_3^2-m_1^2)/2p)t}\langle\nu_\mu|\nu_1\rangle \\ &\qquad + \frac{c_s}{\sqrt{2}}e^{i((m_3^2-m_2^2)/2p)t}\langle\nu_\mu|\nu_2\rangle + \frac{1}{\sqrt{2}}\langle\nu_\mu|\nu_3\rangle\Big] \\ &\quad = \frac{1}{2}[s_s^2 e^{i((m_3^2-m_1^2)/2p)t} + c_s^2 e^{i((m_3^2-m_2^2)/2p)t} + 1], \end{aligned} \qquad \text{(A6)}$$

$$\begin{aligned} &e^{(ip+i(m_3^2/2p))t}\langle\nu_\tau|\nu_\mu(t)\rangle \\ &\quad = \Big[-\frac{s_s}{\sqrt{2}}e^{i((m_3^2-m_1^2)/2p)t}\langle\nu_\tau|\nu_1\rangle \\ &\qquad + \frac{c_s}{\sqrt{2}}e^{i((m_3^2-m_2^2)/2p)t}\langle\nu_\tau|\nu_2\rangle + \frac{1}{\sqrt{2}}\langle\nu_\tau|\nu_3\rangle\Big] \\ &\quad = \frac{1}{2}[-s_s^2 e^{i((m_3^2-m_1^2)/2p)t} - c_s^2 e^{i((m_3^2-m_2^2)/2p)t} + 1]. \end{aligned} \qquad \text{(A7)}$$

Denoting

$$\alpha_{ji} = \frac{(m_j^2 - m_i^2)t}{2p}, \tag{A8}$$

one obtains, from (A7) and (A8) and the relation

$$\alpha_{31} - \alpha_{32} = \alpha_{21}, \tag{A9}$$

the following probabilities:

$$|\langle\nu_e|\nu_\mu(t)\rangle|^2 = s_s^2 c_s^2[1 - \cos(\alpha_{21})], \tag{A10}$$

$$|\langle\nu_\mu|\nu_\mu(t)\rangle|^2 = \frac{1}{4}[s_s^4 + c_s^4 + 1 + 2s_s^2 c_s^2 \cos(\alpha_{21}) + 2s_s^2 \cos(\alpha_{31}) + 2c_s^2 \cos(\alpha_{32})], \tag{A11}$$

$$|\langle\nu_\tau|\nu_\mu(t)\rangle|^2 = \frac{1}{4}[s_s^4 + c_s^4 + 1 + 2s_s^2 c_s^2 \cos(\alpha_{21}) - 2s_s^2 \cos(\alpha_{31}) - 2c_s^2 \cos(\alpha_{32})], \tag{A12}$$

and one obtains, as expected:

$$|\langle\nu_e|\nu_\mu(t)\rangle|^2 + |\langle\nu_\mu|\nu_\mu(t)\rangle|^2 + |\langle\nu_\tau|\nu_\mu(t)\rangle|^2 = 1. \tag{A13}$$

Performing an expansion in powers of $\alpha_{ji} = \frac{(m_j^2 - m_i^2)t}{2p}$, one finds

$$|\langle\nu_e|\nu_\mu(t)\rangle|^2 \simeq \frac{s_s^2 c_s^2}{2}\left[\frac{(m_2^2 - m_1^2)t}{2p}\right]^2, \tag{A14}$$

$$|\langle\nu_\mu|\nu_\mu(t)\rangle|^2 \simeq 1 - \frac{s_s^2 c_s^2}{4}\left[\frac{(m_2^2 - m_1^2)t}{2p}\right]^2 - \frac{s_s^2}{4}\left[\frac{(m_2^2 - m_1^2)t}{2p}\right]^2 - \frac{c_s^2}{4}\left[\frac{(m_3^2 - m_2^2)t}{2p}\right]^2, \tag{A15}$$

$$|\langle\nu_\tau|\nu_\mu(t)\rangle|^2 \simeq -\frac{s_s^2 c_s^2}{4}\left[\frac{(m_2^2 - m_1^2)t}{2p}\right]^2 + \frac{s_s^2}{4}\left[\frac{(m_2^2 - m_1^2)t}{2p}\right]^2 + \frac{c_s^2}{4}\left[\frac{(m_3^2 - m_2^2)t}{2p}\right]^2, \tag{A16}$$

with:

$$|\langle\nu_e|\nu_\mu(t)\rangle|^2 + |\langle\nu_\mu|\nu_\mu(t)\rangle|^2 + |\langle\nu_\tau|\nu_\mu(t)\rangle|^2 \simeq 1. \tag{A17}$$

The last terms in the rhs of (A15) and (A16) read:

$$\frac{s_s^2}{4}\left[\frac{(m_2^2 - m_1^2)t}{2p}\right]^2 + \frac{c_s^2}{4}\left[\frac{(m_3^2 - m_2^2)t}{2p}\right]^2 = \frac{1}{4}[s_s^2(m_3^2 - m_1^2)^2 + c_s^2(m_3^2 - m_2^2)^2]\left(\frac{t}{2p}\right)^2, \tag{A18}$$

and, for m_1^2, $m_2^2 \ll m_3^2$, the bracket in (A18) becomes

$$[s_s^2(m_3^2 - m_1^2)^2 + c_s^2(m_3^2 - m_2^2)^2] \simeq [m_3^2 - (s_s^2 m_1^2 + c_s^2 m_2^2)]^2, \tag{A19}$$

and therefore, formulas (A14)–(A16) can be approximated by

$$|\langle\nu_e|\nu_\mu(t)\rangle|^2 \simeq \frac{s_s^2 c_s^2}{2}\left[\frac{(m_2^2 - m_1^2)t}{2p}\right]^2, \tag{A20}$$

$$|\langle\nu_\mu|\nu_\mu(t)\rangle|^2 \simeq 1 - \frac{s_s^2 c_s^2}{4}\left[\frac{(m_2^2 - m_1^2)t}{2p}\right]^2 - \frac{1}{4}\left[\frac{(m_3^2 - m_x^2)t}{2p}\right]^2, \tag{A21}$$

$$|\langle\nu_\tau|\nu_\mu(t)\rangle|^2 \simeq -\frac{s_s^2 c_s^2}{4}\left[\frac{(m_2^2 - m_1^2)t}{2p}\right]^2 + \frac{1}{4}\left[\frac{(m_3^2 - m_x^2)t}{2p}\right]^2, \tag{A22}$$

that satisfies (A17) and where

$$m_x^2 = s_s^2 m_1^2 + c_s^2 m_2^2. \tag{A23}$$

Therefore the formula (A1) or (15) follows.

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