PHYSICAL REVIEW D **83,** 086002 (2011)

Cosmology in p-brane systems

Masato Minamitsuji*

Department of Physics, Graduate School of Science and Technology, Kwansei Gakuin University, Sanda 669-1337, Japan

Kunihito Uzawa

Department of Physics, Kinki University, Higashi-Osaka, Osaka 577-8502, Japan

(Received 10 November 2010; published 7 April 2011)

We present time-dependent solutions in the higher-dimensional gravity which are related to supergravity in the particular cases. Here, we consider p-branes with a cosmological constant and the intersections of two and more branes. The dynamical description of p-branes can be naturally obtained as the extension of static solutions. In the presence of a cosmological constant, we find accelerating solutions if the dilaton is not dynamical. In the case of intersecting branes, the field equations normally indicate that time-dependent solutions in supergravity can be found if only one harmonic function in the metric depends on time. However, if the special relation between dilaton couplings to antisymmetric tensor field strengths is satisfied, one can find a new class of solutions where all harmonic functions depend on time. We then apply our new solutions to study cosmology, with and without performing compactifications.

DOI: 10.1103/PhysRevD.83.086002 PACS numbers: 11.25.−w, 11.27.+d, 98.80.Cq

I. INTRODUCTION

The dynamical brane systems in supergravity (and more general intersecting brane systems) have attracted growing interests in recent years since they can be used to construct the cosmological model under the compactifications in string theory. The simplest dynamical solution to Einstein equations in supergravity is a p-brane in an asymptotically time-dependent background, obtained in a system composed of gravity, a scalar field and an antisymmetric form field strength. In such a solution, a naked singularity is formed at the places where the warp factor vanishes. Such a solution can be naturally constructed as an extension of a static p-brane solution. This construction has a natural interpretation in terms of D-branes and has served as an important example in string theory. In the absence of the time dependence, a brane system is supersymmetric. Solutions can also be constructed by lifting the Maki-Shiraishi solutions [1] to higher dimensions. These models have interesting effects that can spoil asymptotic flatness and supersymmetry even if they hold in static solutions; much attention has been paid on determining conditions to obtain a supersymmetric solution (for example, see [2,3]). A close cousin of the above solution is a p-brane with a cosmological constant. For a single 2-form field strength, this is an asymptotically Milne universe. The examples relevant for us are the multicentered Kastor-Traschen solutions [4]. Some of these developments have been motivated by de Sitter compactifications in the four-dimensional effective theory. Borrowing these results, we acquire a few novel insights about the physics.

*masato.minamitsuji@kwansei.ac.jp

In a p-brane model, the dynamics can be characterized by the warp factor which is given in terms of the linear combination of the linear functions of time and the harmonic function in the space transverse to the brane. This function contains information about the dynamics of the underlying model, but this has not been fully exploited yet. See [5–8] for an example of determination of such a function. Since a warp factor arises from a field strength, the dynamics of a system composed of $n$branes can be characterized by n warp factors arising from n field strengths [9–16]. In such a system, some of branes can naturally intersect. However, for M-branes and D-branes, among these warp factors only one function can depend on time. These harmonic functions of D-brane model are related to the string coupling constant in string theory. They have been studied from many points of view; for recent discussion, see [17,18].

The purpose of the present paper is to make this result more transparent and to generalize it. We will consider solutions with more general couplings of dilaton to the field strengths. In the classical solution of a p-brane in a D-dimensional theory, the coupling to dilaton for field strength includes the parameter N. Though there are classical solutions for particular values of N, the solutions of $N \neq 4$ models are no longer related to D-branes and M-branes. The dynamical solutions for $N = 4$ were also developed independently in [5,6,8]; the property of cosmological evolution had in essence been introduced earlier [7,19]. For any number of dimension, we will show that the time-dependent solutions can be obtained for $N \neq 4$ by extending the ansatz. The case of $N \neq 4$ gives new intersecting brane solutions that all warp factors arising from field strengths can depend on time if the number N has the appropriate values. There are also the dynamical intersecting solutions that one has $N = 4$, the others have $N \neq 4$. As a simple example, we will study the dynamical intersecting solution in a class of the six-dimensional Romans supergravity [20,21] with a vanishing cosmological constant. We will also see that the effect of a cosmological constant often changes the picture radically, in particular, triggering the accelerating expansion of the Universe. This can only happen when the scalar field vanishes, since a nonzero scalar field is an obstruction to accelerating expansion. Our results will also be interesting for cosmological applications of string theories.

The dynamical solutions in the six-dimensional Nishino-Salam-Sezgin (NSS) supergravity [22–26] have been investigated in [27–31], including applications to brane world models. A particular construction of dynamical solutions was discussed recently in this context and then applied to brane world models in [27] or 1-brane collision [32]. In the present paper, the dynamical 0-brane solution in the NSS model will be derived as a special case and used to study the possibility of brane collisions, which in the special case of p-branes has been originally discussed in [33].

If one drops the requirement of $N = 4$ in the coupling to dilaton for field strengths, the solutions obtained in an Einstein-Maxwell model are a special case of a larger class of dynamical solutions that lead to de Sitter spacetime. In Sec. II, we describe this larger class and apply it to construct brane world models in the five-dimensional theory. In Sec. III, we characterize the intersecting brane system that arises two kinds of form fields without the condition of $N = 4$. In Sec. IV, we perform explicit calculations illustrating how the dynamical solutions of n kinds of intersecting brane system arise from the condition of $N \neq 4$. These examples are inspired by and generalize an example considered in Sec. 2 of [6] as well as the detailed analysis of cosmological models in [27]. Section V is devoted to concluding remarks.

II. DYNAMICAL SOLUTIONS WITH A COSMOLOGICAL CONSTANT

A. Theory

We will start from the D-dimensional theory, for which the action in the Einstein frame contains the metric g_{MN}, the dilaton ϕ, the cosmological constant Λ, and the antisymmetric tensor field of rank $(p+2)$, $F_{(p+2)}$

$$S = \frac{1}{2\kappa^2}\int\left[(R - 2e^{\alpha\phi}\Lambda)*\mathbf{1}_D - \frac{1}{2}d\phi\wedge *d\phi - \frac{1}{2\cdot(p+2)!}e^{\epsilon c\phi}F_{(p+2)}\wedge *F_{(p+2)}\right], \tag{1}$$

where α is constant, κ^2 is the D-dimensional gravitational constant, $*$ is the Hodge operator in the D-dimensional spacetime, $F_{(p+2)}$ is the $(p+2)$-form field strength, and c, ϵ are constants given by

$$c^2 = N - \frac{2(p+1)(D-p-3)}{D-2}, \tag{2a}$$

$$\epsilon = \begin{cases} + & \text{if } p\text{-brane is electric} \\ - & \text{if } p\text{-brane is magnetic.} \end{cases} \tag{2b}$$

Here, N is a constant. The field strength $F_{(p+2)}$ is given by the $(p+1)$-form gauge potential $A_{(p+1)}$

$$F_{(p+2)} = dA_{(p+1)}. \tag{3}$$

In this section, we focus on dimensions of $D > 2$. In $D = 10$ and $D = 11$, the cases of $\Lambda = 0$ and $N = 4$ of the theory (1) correspond to supergravities. The bosonic part of the action of $D = 11$ supergravity includes only 4-form $(p = 2)$ without the dilaton, since $c = 0$ automatically. For $D = 10$ and $N = 4$, the constant c is precisely the dilaton coupling for the Ramond-Ramond $(p+2)$-form in the type II supergravities. The dynamical solutions for the case of $N = 4$ have been already discussed in [32]. The bosonic part of the six-dimensional NSS model [22–24] is given by the expression (1) with $\Lambda > 0$. In this section, we will discuss the dynamical solution for $N \neq 4$.

After varying the action with respect to the metric, the dilaton, and the $(p+1)$-form gauge field, we obtain the field equations

$$R_{MN} = \frac{2}{D-2}e^{\alpha\phi}\Lambda g_{MN} + \frac{1}{2}\partial_M\phi\partial_N\phi + \frac{1}{2\cdot(p+2)!}e^{\epsilon c\phi}\times\left[(p+2)F_{MA_2\cdots A_{p+2}}F_N{}^{A_2\cdots A_{p+2}} - \frac{p+1}{D-2}g_{MN}F^2_{(p+2)}\right], \tag{4a}$$

$$d*d\phi - \frac{\epsilon c}{2\cdot(p+2)!}e^{\epsilon c\phi}F_{(p+2)}\wedge *F_{(p+2)} - 2\alpha e^{\alpha\phi}\Lambda *\mathbf{1}_D = 0, \tag{4b}$$

$$d[e^{\epsilon c\phi}*F_{(p+2)}] = 0. \tag{4c}$$

To solve the field equations, we assume that the D-dimensional metric takes the form

$$ds^2 = h^a(x,z)q_{\mu\nu}(X)dx^\mu dx^\nu + h^b(x,z)u_{ab}(Z)dz^a dz^b, \tag{5}$$

where $q_{\mu\nu}(X)$ is a $(p+1)$-dimensional metric which depends only on the $(p+1)$-dimensional coordinates x^μ, and $u_{ab}(Z)$ is the $(D-p-1)$-dimensional metric which depends only on the $(D-p-1)$-dimensional coordinates z^a. Here, X space represents the world volume directions, while Z space does the space transverse to the p-brane. The parameters a and b are given by

$$a = -\frac{4(D-p-3)}{N(D-2)}, \qquad b = \frac{4(p+1)}{N(D-2)}. \tag{6}$$

The form of the metric (5) is a straightforward generalization of the case of a static p-brane system with a dilaton coupling [5,34]. Furthermore, we assume that the scalar field ϕ and the gauge field strength $F_{(p+2)}$ are given by

$$e^{\phi} = h^{2\epsilon c/N}, \tag{7a}$$

$$F_{(p+2)} = \frac{2}{\sqrt{N}} d(h^{-1}) \wedge \Omega(X), \tag{7b}$$

where $\Omega(X)$ denotes the volume $(p+1)$-form,

$$\Omega(X) = \sqrt{-q} dx^0 \wedge dx^1 \wedge \cdots \wedge dx^p. \tag{8}$$

q is the determinant of the metric $q_{\mu\nu}$.

B. Asymptotically milne solution

Firstly, we consider the Einstein Eqs. (4a) with $c \neq 0$. We assume that the parameter α is given by

$$\alpha = \left[-N + \frac{2(D-p-3)}{D-2} \right] (\epsilon c)^{-1}. \tag{9}$$

Using the assumptions (5) and (7), the Einstein equations are given by

$$\begin{aligned} &R_{\mu\nu}(X) - \frac{4}{N} h^{-1} D_\mu D_\nu h + \frac{2}{N}\left(1 - \frac{4}{N}\right) \partial_\mu \ln h \partial_\nu \ln h \\ &\quad - \frac{2}{D-2} \Lambda q_{\mu\nu} h^{-2} \\ &\quad - \frac{a}{2} q_{\mu\nu} \left[h^{-1} \triangle_X h - \left(1 - \frac{4}{N}\right) q^{\rho\sigma} \partial_\rho \ln h \partial_\sigma \ln h \right] \\ &\quad - \frac{a}{2} q_{\mu\nu} h^{-4/N-1} \triangle_Z h = 0, \end{aligned} \tag{10a}$$

$$h^{-1} \partial_\mu \partial_a h = 0, \tag{10b}$$

$$\begin{aligned} &R_{ab}(Z) - \frac{b}{2} h^{4/N} u_{ab} \left[h^{-1} \triangle_X h - \left(1 - \frac{4}{N}\right) q^{\rho\sigma} \partial_\rho \ln h \partial_\sigma \ln h \right] \\ &\quad - \frac{b}{2} u_{ab} h^{-1} \triangle_Z h - \frac{2}{D-2} \Lambda u_{ab} h^{-2+4/N} = 0, \end{aligned} \tag{10c}$$

where D_μ is the covariant derivative with respect to the metric $q_{\mu\nu}$, $\triangle_X$ and $\triangle_Z$ are the Laplace operators on X and Z, respectively. Similarly, $R_{\mu\nu}(\mathrm{X})$ and $R_{ab}(\mathrm{Z})$ are the Ricci tensors associated with the metrics $q_{\mu\nu}$ and u_{ab}, respectively. From Eq. (10b), we see that the function h must be in the form

$$h(x, z) = h_0(x) + h_1(z). \tag{11}$$

With this form of h, the other components of the Einstein Eqs. (10a) and (10c) are rewritten as

$$\begin{aligned} &R_{\mu\nu}(\mathrm{X}) - \frac{4}{N} h^{-1} D_\mu D_\nu h_0 + \frac{2}{N}\left(1 - \frac{4}{N}\right) h^{-2} \partial_\mu h_0 \partial_\nu h_0 \\ &\quad - \frac{2}{D-2} \Lambda q_{\mu\nu} h^{-2} - \frac{a}{2} q_{\mu\nu} \\ &\quad \times \left[h^{-1} \triangle_X h_0 - \left(1 - \frac{4}{N}\right) h^{-2} q^{\rho\sigma} \partial_\rho h_0 \partial_\sigma h_0 \right] \\ &\quad - \frac{a}{2} q_{\mu\nu} h^{-4/N-1} \triangle_Z h_1 = 0, \end{aligned} \tag{12a}$$

$$\begin{aligned} &R_{ab}(Z) - \frac{b}{2} h^{4/N} u_{ab} \\ &\quad \times \left[h^{-1} \triangle_X h_0 - \left(1 - \frac{4}{N}\right) h^{-2} q^{\rho\sigma} \partial_\rho h_0 \partial_\sigma h_0 \right] \\ &\quad - \frac{b}{2} u_{ab} h^{-1} \triangle_Z h_1 - \frac{2}{D-2} \Lambda u_{ab} h^{-2+4/N} = 0. \end{aligned} \tag{12b}$$

Under the assumption (7b), the Bianchi identity is automatically satisfied. The equation of motion for the gauge field (4c) becomes

$$\triangle_Z h_1 \Omega(\mathrm{Z}) = 0, \tag{13}$$

where we have used (11), and $\Omega(Z)$ is defined by

$$\Omega(\mathrm{Z}) = \sqrt{u} dz^1 \wedge \cdots \wedge dz^{D-p-1}. \tag{14}$$

Hence, the gauge field equation gives

$$\triangle_Z h_1 = 0. \tag{15}$$

Let us next consider the scalar field equation. Substituting Eqs. (7) and (11) into Eq. (4b), we obtain

$$\begin{aligned} &\frac{2}{N} \epsilon c h^{4/N-b} \left[h^{-1} \triangle_X h_0 - \left(1 - \frac{4}{N}\right) h^{-2} q^{\rho\sigma} \partial_\rho h_0 \partial_\sigma h_0 \right. \\ &\quad \left. + h^{-1} \triangle_Z h_1 \right] - 2\alpha h^{-2-a} \Lambda = 0. \end{aligned} \tag{16}$$

Because of Eq. (15), we are left with

$$\begin{aligned} &\triangle_X h_0 = 0, \\ &\frac{1}{N}\left(1 - \frac{4}{N}\right) q^{\rho\sigma} \partial_\rho h_0 \partial_\sigma h_0 + (\epsilon c)^{-1} \alpha \Lambda = 0. \end{aligned} \tag{17}$$

Let us go back to the Einstein Eqs. (12). If $F_{(p+2)} = 0$, the function h_1 becomes trivial. On the other hand, for $F_{(p+2)} \neq 0$, the first term in Eq. (12a) depends on only x whereas the rest on both x and y. Thus Eqs. (12) together with (15) and (17) give

$$R_{\mu\nu}(\mathrm{X}) = 0, \qquad D_\mu D_\nu h_0 = 0, \tag{18a}$$

$$\frac{2}{N}\left(1-\frac{4}{N}\right)\partial_\mu h_0 \partial_\nu h_0 + \frac{a}{2}\left(1-\frac{4}{N}\right)q_{\mu\nu}q^{\rho\sigma}\partial_\rho h_0 \partial_\sigma h_0 - \frac{2}{D-2}\Lambda q_{\mu\nu} = 0, \tag{18b}$$

$$R_{ab}(\mathrm{Z}) + \frac{b}{2}\left(1-\frac{4}{N}\right)h^{-2+4/N}u_{ab}q^{\rho\sigma}\partial_\rho h_0 \partial_\sigma h_0 - \frac{2}{D-2}\Lambda u_{ab}h^{-2+4/N} = 0. \tag{18c}$$

The Eqs. (17) and (18b) give $N = 2$. Then, the Eqs. (18b) and (18c) are written by

$$\partial_\mu h_0 \partial_\nu h_0 = -\frac{4}{c^2(D-2)}\Lambda q_{\mu\nu}, \tag{19a}$$

$$R_{ab}(\mathrm{Z}) - \frac{4p}{c^2(D-2)}\Lambda u_{ab} = 0, \tag{19b}$$

respectively. If one solves these Eqs. (18) with Eq. (15), the solution of the present system is given by Eqs. (5) and (7) with (11).

For a nonvanishing cosmological constant, Eq. (18c) implies that the $(D-p-1)$-dimensional space Z is an Einstein manifold. The $(D-p-1)$-dimensional flat space is allowed only for $p = 0$. Equation (19a) implies that the $(p+1)$-dimensional metric $q_{\mu\nu}(\mathrm{X})$ is expressed as a product of two vectors. Hence, for $p \neq 0$, the determinant of the metric $q_{\mu\nu}(\mathrm{X})$ becomes zero, which is not permissible. In the following, we will discuss the solution of $p = 0$ case. We find that Eqs. (15), (18), and (19b) reduce to

$$h(t, z) = h_0(t) + h_1(z), \qquad h_0 = At + B, \qquad \triangle_{\mathrm{Z}} h_1 = 0, \tag{20a}$$

$$R_{ab}(\mathrm{Z}) = 0, \tag{20b}$$

where A is defined by $A \equiv \pm\sqrt{2\Lambda}$ and B is constant parameter. Thus, there is no solution for $\Lambda < 0$. For the special case

$$u_{ab} = \delta_{ab}, \tag{21}$$

where δ_{ij} is the $(D-1)$-dimensional Euclidean space metric, the solution for h is obtained explicitly as

$$h(t, z) = At + B + h_1(z), \tag{22}$$

where the harmonic function h_1 is found to be

$$h_1(z) = \sum_{\ell=1}^{L} \frac{M_\ell}{|z - z_\ell|^{D-3}} \quad \text{for } D \neq 3, \tag{23a}$$

$$h_1(z) = \sum_{\ell=1}^{L} M_\ell \ln|z - z_\ell| \quad \text{for } D = 3. \tag{23b}$$

Here, $|z - z_\ell| = \sqrt{(z^1 - z^1_\ell)^2 + (z^2 - z^2_\ell)^2 + \cdots + (z^{D-1} - z^{D-1}_\ell)^2}$, and $M_\ell (\ell = 1 \cdots L)$ are mass constants of 1-branes located at z_ℓ. The behavior of the harmonic function h_1 is classified into two classes depending on the dimensions D, i.e. $D > 3$, and $D = 3$.

For $D = 3$, the harmonic function h_1 diverges both at infinity and near 0-branes. In particular, because $h_1 \to -\infty$, there is no regular spacetime region near branes. Hence, such solutions are not physically relevant. The original theory is ill-defined. In the following, we will focus on the case $D > 3$.

Assuming $\Lambda > 0$, and introducing a new time coordinate τ by

$$\frac{\tau}{\tau_0} = (At + B)^{1/(D-2)}, \qquad \tau_0 = \frac{(D-2)}{A}, \tag{24}$$

we find the D-dimensional metric (62) as

$$ds^2 = \left[1 + \left(\frac{\tau}{\tau_0}\right)^{-(D-2)} h_1\right]^{-((2(D-3))/(D-2))} \times \left[-d\tau^2 + \left\{1 + \left(\frac{\tau}{\tau_0}\right)^{-(D-2)} h_1\right\}^2 \left(\frac{\tau}{\tau_0}\right)^2 u_{ab}dz^a dz^b\right]. \tag{25}$$

For $h_1 \to 0$, the spacetime approaches an isotropic and homogeneous universe, whose scale factor is proportional to τ, i.e., the D-dimensional Milne universe. This is realized in the limit $\tau \to \infty$, which is guaranteed by a scalar field with the exponential potential. The D-dimensional spacetime becomes inhomogeneous for $h_1 \neq 0$. The power exponent of the scale factor is always larger than that in the matter or radiation-dominated era. It is interesting to note that in the case of $D = 6$, $p = 0$ with $\Lambda > 0$, Eq. (25) describes the cosmological solution in the NSS model with the vanishing 3-form field strength. The late time evolution has a scaling behavior. Note that the scaling solution in the NSS model obtained in Ref. [30] has the similar time dependence, although in this case the 2-form field strength is magnetic.

The D-dimensional spacetime is regular in the region of $h > 0$, but has curvature singularities where $h = 0$, since ϕ diverges there. The physical spacetime exists only inside the domain restricted by

$$h(t, z) \equiv At + B + h_1(z) > 0. \tag{26}$$

To see the detailed dynamics of spacetime, let us illustrate the case of two 0-branes, which are sharing the same charge M and located at $z = (\pm L, 0, \cdots, 0)$. Here, we focus on the period of $t > 0$. For the period of $t < 0$, the spacetime dynamics is obtained simply by reversing evolution of the case of $t > 0$.

In the case of $A > 0$, for $t \geq 0$ the metric is always regular. The metric (5) implies that the transverse dimensions expand asymptotically as $\tilde{\tau}$, where $\tilde{\tau}$ is the proper time of the coordinate observer. However, it is observer-dependent. As we mentioned before, it is static near branes, and the spacetime approaches a Milne universe in the far region ($|z| \to \infty$), which expands in all directions isotropically. Defining

$$z_\perp = \sqrt{(z^2)^2 + \cdots + (z^{D-1})^2}, \tag{27}$$

the proper distance at $z_\perp = 0$ between two branes is given by

$$d(t) = \int_{-L}^{L} dz^1 \left[At + \frac{M}{|z^1 + L|^{D-3}} + \frac{M}{|z^1 - L|^{D-3}} \right]^{((1)/(D-2))} = (ML^2)^{((1)/(D-2))} \int_{-1}^{1} d\eta \left[\left(\frac{AL^{D-3}}{M} \right) t + \frac{1}{|\eta + 1|^{D-3}} + \frac{1}{|\eta - 1|^{D-3}} \right]^{((1)/(D-2))}, \tag{28}$$

which is a monotonically increasing function of t.

Next, we discuss the case of $A < 0$. Initially ($t = 0$), all of the region of $(D-1)$-dimensional space is regular except at $z \to \infty$. As time evolves, the singular hypersurface erodes the z-coordinate region. As a result, only the region near 0-branes remains regular. When we watch this process on the $(z^1, z_\perp)$ plane, the singular circle appears at infinity. It eventually approaches 0-branes and finally the regular spatial region splits into two isolated throats surrounding each 0-brane. The proper distance d between two branes, given by Eq. (28), is now a monotonically decreasing function of t. At a glance, it could realize brane collisions. However, since a singularity appears between two branes before the distance vanishes, a regular brane collision cannot be realized.

C. Asymptotically de Sitter solution

Next, we consider the solution with a trivial dilaton which is the case of $c = 0$ and hence $\alpha = 0$. The scalar field becomes constant because of the ansatz (4b), and the scalar field Eq. (17) is automatically satisfied. In terms of $c = 0$, Eq. (2a) give

$$N = \frac{2(D-p-3)(p+1)}{D-2}. \tag{29}$$

In this case, the field equations are reduced to

$$R_{\mu\nu}(\mathrm{X}) = 0, \quad R_{ab}(\mathrm{Z}) = 0, \tag{30a}$$

$$h(x,z) = h_0(x) + h_1(z), \tag{30b}$$

$$D_\mu D_\nu h_0 = 0,$$

$$\partial_\mu h_0 \partial_\nu h_0 + \frac{2(p+1)(D-p-3)^2}{(D-2)((1-p)D + p^2 + 4p - 1)} \Lambda q_{\mu\nu} = 0, \tag{30c}$$

$$\triangle_Z h_1 = 0. \tag{30d}$$

We will focus on the solution of $p = 0$, since from Eq. (30c) it turns out that the solution for $p \neq 0$ is not permissible. Then Eq. (30c) gives

$$h_0 = c_1 t + c_2, \tag{31}$$

where c_2 is an integration constant and c_1 is given by

$$c_1 = \pm(D-3)\sqrt{\frac{2\Lambda}{(D-1)(D-2)}}. \tag{32}$$

Thus, there is no solution for $\Lambda < 0$. If the metric $u_{ab}(Z)$ is assumed to be Eq. (21), the function h_1 is given by Eq. (23). Now we introduce a new time coordinate τ by

$$c_1 \tau = \ln t, \tag{33}$$

where we have taken $c_1 > 0$ for simplicity. The D-dimensional metric (5) is then rewritten as

$$ds^2 = -(1 + c_1^{-1} e^{-c_1\tau} h_1)^{-2} d\tau^2 + (1 + c_1^{-1} e^{-c_1\tau} h_1)^{2/(D-3)} \times (c_1 e^{c_1\tau})^{2/(D-3)} u_{ab}(\mathrm{Z}) dz^a dz^b. \tag{34}$$

Equation (34) implies that the spacetime describes an isotropic and homogeneous universe if $h_1 = 0$. In the limit when the terms with h_1 are negligible, which is realized in the limit $\tau \to \infty$ and for $c_1 > 0$, we find a D-dimensional de Sitter universe. The solution (34) has been discussed by [1]. Furthermore, for $D = 4$, the solution is found by Kastor and Traschen [4].

D. Application to the brane world

The asymptotically de Sitter solution in the case of $D = 5$ is now applied to construct a cosmological brane world. We start from the general metric

$$ds^2 = -d(T, \xi)^2 dT^2 + f(T, \xi)^2 d\xi^2 + a(T, \xi)^2 d\Omega_{(3)}^2, \tag{35}$$

where $d\Omega_{(3)}^2$ denotes a unit 3-sphere. ξ and T denote the dimensionless coordinates of the extra space and time. For a given background spacetime, applying the standard copy and paste method, it is possible to construct a cosmological 3-brane world embedded into a five-dimensional bulk as in the Randall-Sundrum model [35]. For simplicity, we impose the Z_2-symmetry across the brane world volume.

A cosmological brane world evolves along a trajectory specified by an affine parameter τ, $(T, \xi) = (T(\tau), \xi(\tau))$. The induced metric on the brane world is then given by the closed Friedmann-Robertson-Walker metric with the scale factor a

$$ds_{(\mathrm{ind})}^2 = -d\tau^2 + a(T(\tau), \xi(\tau))^2 d\Omega_{(3)}^2, \tag{36}$$

where we imposed

$$-d(T(\tau), \xi(\tau))^2 \dot{T}^2 + f(T(\tau), \xi(\tau))^2 \dot{\xi}^2 = -1. \tag{37}$$

A dot denotes a derivative with respect to τ, which is interpreted as the cosmic proper time.

The trajectory of the brane world is through the junction conditions. Here, we focus on the Israel conditions given by

$$\frac{1}{2}\kappa^2\bar{\rho} = -3\epsilon\left\{\frac{1}{f}\frac{a_{,\xi}}{a}\sqrt{1+f^2\dot{\xi}^2} + \frac{f}{d}\frac{a_{,T}}{a}\dot{\xi}\right\}, \tag{38a}$$

$$\frac{1}{2}\kappa^2\bar{p} = \epsilon\left\{\left(2\frac{a_{,\xi}}{a} + \frac{d_{,\xi}}{d}\right)\frac{1}{f}\sqrt{1+f^2\dot{\xi}^2} + \frac{f_{,\xi}\dot{\xi}^2 + \ddot{\xi}f}{\sqrt{1+f^2\dot{\xi}^2}} + \frac{f}{d}\left(2\frac{a_{,T}}{a} + \frac{2f_{,T}}{f}\right)\dot{\xi}\right\}, \tag{38b}$$

where $\bar{\rho}$ and $\bar{p}$ represent the total energy density and pressure, obtained by varying the brane world action. From now on, we focus on the energy density equation, Eq. (38a). Note that $\epsilon = +1$ denotes the normal vector pointing the direction of increasing ξ (and $\epsilon = -1$ vice versa).

The derivative of the scale factor with respect to the cosmic proper time is given by

$$\dot{a} = \alpha_\xi f\dot{\xi} + \alpha_T\sqrt{1+f^2\dot{\xi}^2}, \tag{39}$$

where we defined

$$\alpha_\xi := \frac{a_{,\xi}}{f}, \qquad \alpha_T := \frac{a_{,T}}{d}. \tag{40}$$

Replacing $\dot{\xi}$ with $\dot{a}$ through Eq. (39) and squaring the energy density component of (38a), we obtain the generalized Friedmann equation for a. In the $|\alpha_T| \gg |\alpha_\xi|$ limit, where the time dependence rules the spatial one, the cosmological equation reduces to

$$\frac{\dot{a}^2}{a^2} + \frac{1}{a^2} \approx \frac{\kappa^4\bar{\rho}^2}{36} + \frac{\alpha_T^2+1}{a^2}. \tag{41}$$

Similarly, in the $|\alpha_T| \ll |\alpha_\xi|$ limit, where the spacetime is approximately static, the cosmological equation reduces to

$$\frac{\dot{a}^2}{a^2} + \frac{1}{a^2} \approx \frac{\kappa^4\bar{\rho}^2}{36} - \frac{\alpha_\xi^2-1}{a^2}. \tag{42}$$

1. Brane world supported by the tension

One possibility is to support the brane world by tension. Decomposing $\bar{\rho} = \sigma + \rho$, where σ and ρ denote the tension and matter energy density localized on the brane, respectively, we obtain

$$\frac{\dot{a}^2}{a^2} + \frac{1}{a^2} \approx \frac{1}{3}\Lambda_{\rm eff} + \frac{\kappa_4^2}{3}\rho + O(\rho^2). \tag{43}$$

Here, we assumed $\rho \ll \sigma$. The four-dimensional effective cosmological constant (not exactly constant, of course) and the gravitational constant are given by

$$\Lambda_{\rm eff} := \frac{1}{12}\kappa^4\sigma^2 + \frac{3(\alpha_t^2+1)}{a^2}, \qquad \kappa_4^2 := \frac{1}{6}\kappa^4\sigma, \tag{44}$$

for $|\alpha_T| \gg |\alpha_\xi|$, and

$$\Lambda_{\rm eff} := \frac{1}{12}\kappa^4\sigma^2 - \frac{3(\alpha_\xi^2-1)}{a^2}, \qquad \kappa_4^2 := \frac{1}{6}\kappa^4\sigma, \tag{45}$$

for $|\alpha_T| \ll |\alpha_\xi|$. $\Lambda_{\rm eff}$ is composed of the tension part and the bulk part. For $|\alpha_T| \gg |\alpha_\xi|$, from Eq. (38a) assuming $\alpha_T > 0$, to obtain a positive gravitational constant, namely, a positive tension, we have to impose $\dot{\xi} > 0$ for $\epsilon = -1$ and $\dot{\xi} < 0$ for $\epsilon = +1$. For $|\alpha_T| \ll |\alpha_\xi|$, similarly from Eq. (38a) assuming $\alpha_\xi > 0$, to obtain a positive gravitational constant, we have to impose $\epsilon = -1$.

2. Brane world supported by the induced gravity

The other possibility is to support the brane world induced gravity term [36]

$$\bar{\rho} = \rho + \mu^2 G_{({\rm ind})}{}^0{}_0 = \rho - 3\mu^2\left(\frac{\dot{a}^2}{a^2} + \frac{1}{a^2}\right), \tag{46}$$

where $G_{({\rm ind})\mu\nu}$ is the Einstein tensor associated with the brane world metric Eq. (36). We will see that the parameter μ plays the role of the four-dimensional Planck scale in the high density region. We then obtain for $|\alpha_T| \gg |\alpha_\xi|$,

$$\left(\frac{\dot{a}^2}{a^2} + \frac{1}{a^2}\right)_\pm = \frac{2}{\kappa^4\mu^4}\left[1 + \frac{\kappa^4\mu^2}{6}\rho \pm \sqrt{1 + \frac{\kappa^4\mu^2}{3}\rho - \frac{\kappa^4\mu^4(\alpha_T^2+1)}{a^2}}\right], \tag{47}$$

and for $|\alpha_T| \ll |\alpha_\xi|$,

$$\left(\frac{\dot{a}^2}{a^2} + \frac{1}{a^2}\right)_\pm = \frac{2}{\kappa^4\mu^4}\left[1 + \frac{\kappa^4\mu^2}{6}\rho \pm \sqrt{1 + \frac{\kappa^4\mu^2}{3}\rho + \frac{\kappa^4\mu^4(\alpha_\xi^2-1)}{a^2}}\right], \tag{48}$$

where in both cases $(+)$ and $(-)$ denote two independent branches. Here, in the first limit, if $\dot{\xi} > 0$, we take $\epsilon = +1$ for the $(+)$-branch and $\epsilon = -1$ for the $(-)$-branch (for $\dot{\xi} < 0$ vice versa). In the second limit, we take $\epsilon = +1$ for the $(+)$-branch and $\epsilon = -1$ for the $(-)$-branch.

Let us discuss the cosmological behaviors in high and low energy density limits, respectively. In both limits, we recover the ordinary cosmological equation in the high density region, where the term linear in ρ dominates others

$$\left(\frac{\dot{a}^2}{a^2} + \frac{1}{a^2}\right)_\pm \approx \frac{1}{3\mu^2}\rho, \tag{49}$$

where clearly the four-dimensional gravitational constant is given by μ^{-1}. Thus, in this region, the standard Friedmann equation is recovered due to the induced gravity term. On the other hand, in the low density region, if $|\alpha_T| \gg |\alpha_\xi|$,

$$\left(\frac{\dot{a}^2}{a^2}+\frac{1}{a^2}\right)_\pm \simeq \frac{2}{\kappa^4\mu^4}\left(1\pm\sqrt{1-\frac{\kappa^4\mu^4(\alpha_T^2+1)}{a^2}}\right), \quad (50)$$

and if $|\alpha_T| \ll |\alpha_\xi|$,

$$\left(\frac{\dot{a}^2}{a^2}+\frac{1}{a^2}\right)_\pm \simeq \frac{2}{\kappa^4\mu^4}\left(1\pm\sqrt{1+\frac{\kappa^4\mu^4(\alpha_\xi^2-1)}{a^2}}\right). \quad (51)$$

In the first limit, for $\frac{\sqrt{\alpha_T^2+1}}{a} \ll \frac{1}{\kappa^2\mu^2}$, $(\frac{\dot{a}}{a})_+ \approx \frac{2}{\kappa^2\mu^2}$ (ignoring the $\frac{1}{a^2}$ term compared to the constant part) and $(\frac{\dot{a}}{a})_- \approx |\frac{\alpha_T}{a}|$. In the second limit, for $\frac{|1-\alpha_\xi^2|^{(1/2)}}{a} \ll \frac{1}{\kappa^2\mu^2}$, $(\frac{\dot{a}}{a})_+ \approx \frac{2}{\kappa^2\mu^2}$ and there is no regular behavior in the $(-)$-branch. The $(+)$-branch has the expansion rate of the self-accelerating solution given by Dvali, Gabadadze, and Porrati (DGP) [36]. However, this branch is known to suffers a ghost instability [37]. On the other hand, the $(-)$-branch does not contain any pathology.

3. Brane world in the asymptotically de Sitter spacetime

We apply our formulation to the case of the asymptotically de Sitter solution (34) in $D = 5$. Here, we have to assume that the harmonic function h_1 found in (23a) is given by the contribution of a single brane with a mass M. Then, in $D = 5$, the asymptotically de Sitter solution reduces to

$$ds^2 = M\left(1+\frac{e^{-T}}{\xi^2}\right)e^T d\xi^2 - c_1^{-2}\left(1+\frac{e^{-T}}{\xi^2}\right)^{-2}dT^2 + M\xi^2\left(1+\frac{e^{-T}}{\xi^2}\right)e^T d\Omega_{(3)}^2, \quad (52)$$

where $c_1 = \sqrt{\frac{2}{3}\Lambda}$ is given by Eq. (32) (we assume $c_1 > 0$). The dimensionless coordinates run $-\infty < T < \infty$ and $0 < \xi < \infty$. Since the combination in the round bracket is always positive, no curvature singularity appears. Comparing with (35), d, f and a read

$$d := c_1^{-1}\left(1+\frac{e^{-T}}{\xi^2}\right)^{-1},$$
$$f := M^{(1/2)}\sqrt{1+\frac{e^{-T}}{\xi^2}}e^{(T/2)}, \quad (53)$$
$$a := M^{(1/2)}\xi\sqrt{1+\frac{e^{-T}}{\xi^2}}e^{(T/2)},$$

and we find

$$\alpha_T = \frac{\sqrt{c_1^2M}\zeta(T,\xi)}{2}\sqrt{1+\frac{1}{\zeta(T,\xi)^2}},$$
$$\alpha_\xi = \frac{\zeta(T,\xi)^2}{1+\zeta(T,\xi)^2} < 1, \quad (54)$$

where $\zeta(T,\xi) := e^{(T/2)}\xi$. We then define

$$F(\zeta) := \frac{\alpha_T}{\alpha_\xi} = (c_1^2M)^{(1/2)}\nu(\zeta), \qquad \nu(\zeta) := \frac{(1+\zeta^2)^{3/2}}{2\zeta^2}. \quad (55)$$

Here, $\nu(\zeta)$ takes minimum at $\zeta = \sqrt{2}$, where $\nu(\sqrt{2}) = \frac{3^{(3/2)}}{4} \simeq 1.299$. Therefore, as long as $c_1^2M > \frac{16}{27}$, we always obtain $\alpha_T > \alpha_\xi$. In particular, for both limits of $\zeta \gg 1$ and $0 < \zeta \ll 1$, $\alpha_T \gg \alpha_\xi$, irrespective of c_1^2M. Thus, the cosmological equation can be described by Eq. (41) with $\frac{\alpha_T}{a} = \frac{c_1}{2}$.

If the brane world is supported by the tension, from (41) the effective cosmological and gravitational constants read

$$\Lambda_{\rm eff} \approx \frac{1}{12}\kappa^4\sigma^2 + \frac{3c_1^2}{4}, \qquad \kappa_4^2 := \frac{1}{6}\kappa^4\sigma. \quad (56)$$

To obtain a positive gravitational constant, we impose $\epsilon = -1$ for $\dot{\xi} > 0$ and $\epsilon = +1$ for $\dot{\xi} < 0$. In addition, the bulk volume is not finite in the direction of increasing ξ, as seen from $\sqrt{-g} = c_1M^2\xi^3e^{2T}(1+\frac{e^{-T}}{\xi^2})$. Thus, to obtain the localized graviton on the brane world at low energy, ξ must have an upper bound $0 < \xi < \xi_0$, where ξ_0 is the position of the brane world. Thus, we also impose $\dot{\xi} > 0$. Then, the effective cosmology is the ΛCDMtype one.

If the brane world is supported by the induced gravity, we obtain

$$\left(\frac{\dot{a}^2}{a^2}+\frac{1}{a^2}\right)_\pm = \frac{2}{\kappa^4\mu^4}\left[1+\frac{\kappa^4\mu^2}{6}\rho \pm\sqrt{1+\frac{\kappa^4\mu^2}{3}\rho-\kappa^4\mu^4\left(\frac{c_1^2}{4}+\frac{1}{a^2}\right)}\right]. \quad (57)$$

To ensure the regular cosmological behavior at the low energy density, here we impose $c_1 < \frac{2}{\kappa^2\mu^2}$. In the high density region, as shown in Eq. (49), the four-dimensional cosmological equation is recovered. In the low density region,

$$\left(\frac{\dot{a}^2}{a^2}+\frac{1}{a^2}\right)_\pm \simeq \frac{2}{\kappa^4\mu^4}\left(1\pm\sqrt{1-\kappa^4\mu^4\left(\frac{c_1^2}{4}+\frac{1}{a^2}\right)}\right). \quad (58)$$

If $c_1 \ll \frac{2}{\kappa^2\mu^2}$, $(\frac{\dot{a}}{a})_+ \approx \frac{2}{\kappa^2\mu^2}$ and $(\frac{\dot{a}}{a})_- \approx \frac{c_1}{2}$. The latter shows that the healthy $(-)$ branch, as well as the $(+)$-branch can give accelerating solutions in the later times. The result is similar to a higher-dimensional extension of DGP [38]. Although this property looks fascinating, in order to explain the cosmic acceleration of today, we have to require that the bulk cosmological constant becomes very tiny as $\Lambda^{(1/2)} \simeq 10^{-42}$ GeV. Therefore, for any reasonable choice of the five-dimensional Planck scale, as TeV scale, the huge fine-tuning for Λ cannot be avoided.

III. THE INTERSECTION OF TWO BRANES IN *D*-DIMENSIONAL THEORY

A. Theory

In this section, we consider a D-dimensional theory composed of the metric g_{MN}, dilaton ϕ, and two antisymmetric tensor fields of rank $(p_r + 2)$ and $(p_s + 2)$:

$$S = \frac{1}{2\kappa^2}\int\Big[R * \mathbf{1} - \frac{1}{2}d\phi \wedge *d\phi - \frac{1}{2}\frac{1}{(p_r + 2)!}e^{\epsilon_r c_r \phi}F_{(p_r+2)} \wedge *F_{(p_r+2)} - \frac{1}{2}\frac{1}{(p_s + 2)!}e^{\epsilon_s c_s \phi}F_{(p_s+2)} \wedge *F_{(p_s+2)}\Big], \quad (59)$$

where κ^2 is the D-dimensional gravitational constant, $*$ is the Hodge operator in the D-dimensional spacetime, $F_{(p_r+2)}$ and $F_{(p_s+2)}$ are $(p_r + 2)$-form, $(p_s + 2)$-form field strengths, respectively. And c_I, $\epsilon_I (I = r, s)$ are constants given by

$$c_I^2 = N_I - \frac{2(p_I + 1)(D - p_I - 3)}{D - 2}, \quad (60a)$$

$$\epsilon_I = \begin{cases} + & \text{if } p_I\text{- brane is electric} \\ - & \text{if } p_I\text{- brane is magnetic.} \end{cases} \quad (60b)$$

Here, N_I is constant. After varying the action with respect to the metric, the dilaton, and the $(p_r + 1)$-form and $(p_s + 1)$-form gauge fields, we obtain the field equations,

$$R_{MN} = \frac{1}{2}\partial_M\phi\partial_N\phi + \frac{1}{2}\frac{e^{\epsilon_r c_r \phi}}{(p_r + 2)!}\Big[(p_r + 2)F_{MA_2\cdots A_{(p_r+2)}}F_N{}^{A_2\cdots A_{(p_r+2)}} - \frac{p_r + 1}{D - 2}g_{MN}F^2_{(p_r+2)}\Big] + \frac{1}{2}\frac{e^{\epsilon_s c_s \phi}}{(p_s + 2)!}\Big[(p_s + 2)F_{MA_2\cdots A_{(p_s+2)}}F_N{}^{A_2\cdots A_{(p_s+2)}} - \frac{p_s + 1}{D - 2}g_{MN}F^2_{(p_s+2)}\Big], \quad (61a)$$

$$d * d\phi - \frac{1}{2}\frac{\epsilon_r c_r}{(p_r + 2)!}e^{\epsilon_r c_r \phi}F_{(p_r+2)} \wedge *F_{(p_r+2)} - \frac{1}{2}\frac{\epsilon_s c_s}{(p_s + 2)!}e^{\epsilon_s c_s \phi}F_{(p_s+2)} \wedge *F_{(p_s+2)} = 0, \quad (61b)$$

$$d[e^{\epsilon_r c_r \phi} * F_{(p_r+2)}] = 0, \quad (61c)$$

$$d[e^{\epsilon_s c_s \phi} * F_{(p_s+2)}] = 0. \quad (61d)$$

To solve the field equations, we assume that the D-dimensional metric takes the form

$$ds^2 = h_r^{a_r}h_s^{a_s}q_{\mu\nu}(\mathrm{X})dx^\mu dx^\nu + h_r^{b_r}h_s^{a_s}\gamma_{ij}(\mathrm{Y}_1)dy^i dy^j + h_r^{a_r}h_s^{b_s}w_{mn}(\mathrm{Y}_2)dv^m dv^n + h_r^{b_r}h_s^{b_s}u_{ab}(\mathrm{Z})dz^a dz^b, \quad (62)$$

where $q_{\mu\nu}$ is a $(p + 1)$-dimensional metric which depends only on the $(p + 1)$-dimensional coordinates x^μ, γ_{ij} is the $(p_s - p)$-dimensional metric which depends only on the $(p_s - p)$-dimensional coordinates y^i, w_{mn} is the $(p_r - p)$-dimensional metric which depends only on the $(p_r - p)$-dimensional coordinates v^m and finally u_{ab} is the $(D + p - p_r - p_s - 1)$-dimensional metric which depends only on the $(D + p - p_r - p_s - 1)$-dimensional coordinates z^a. The parameters $a_I (I = r, s)$ and $b_I (I = r, s)$ in the metric (62) are given by

$$a_I = -\frac{4(D - p_I - 3)}{N_I(D - 2)}, \qquad b_I = \frac{4(p_I + 1)}{N_I(D - 2)}. \quad (63)$$

The D-dimensional metric (62) implies that the solutions are characterized by two functions, h_r and h_s, which depend on the coordinates transverse to the brane as well as the world volume coordinate. For the configurations of two branes, the powers of harmonic functions have to obey the intersection rule, and then split the coordinates in three parts. One is the overall world-volume coordinates, $\{x\}$, which are common to the two branes. The others are overall transverse coordinates, $\{z\}$, and the relative transverse coordinates, $\{y\}$ and $\{v\}$, which are transverse to only one of the two branes. The field equations of intersecting branes allow for the following three kinds of possibilities on p_r- and p_s-branes in D dimensions [8,39,40].

(I) Both h_r and h_s depend on the overall transverse coordinates: $h_r = h_r(x, z)$, $h_s = h_s(x, z)$.

(II) Only h_s depends on the overall transverse coordinates, but the other h_r does on the corresponding relative coordinates: $h_r = h_r(x, y)$, $h_s = h_s(x, z)$.

(III) Each of h_r and h_s depends on the corresponding relative coordinates: $h_r = h_r(x, y)$, $h_s = h_s(x, v)$.

In the following, we consider intersections where each participating brane corresponds to an independent harmonic function in the solution and derive the dynamical intersecting brane solution in D dimensions satisfying the above conditions.

B. Case (I)

We first consider the case (I). Under our classification, the D-dimensional metric ansatz Eq. (62) now explicitly becomes

$$ds^2 = h_r^{a_r}(x,z)h_s^{a_s}(x,z)q_{\mu\nu}(\mathrm{X})dx^\mu dx^\nu + h_r^{b_r}(x,z)h_s^{a_s}(x,z)\gamma_{ij}(\mathrm{Y}_1)dy^i dy^j + h_r^{a_r}(x,z)h_s^{b_s}(x,z)w_{mn}(\mathrm{Y}_2)dv^m dv^n + h_r^{b_r}(x,z)h_s^{b_s}(x,z)u_{ab}(\mathrm{Z})dz^a dz^b. \tag{64}$$

We also assume that the scalar field ϕ and the gauge field strengths $F_{(p_r+2)}$, $F_{(p_s+2)}$ are given by

$$e^\phi = h_r^{2\epsilon_r c_r/N_r} h_s^{2\epsilon_s c_s/N_s}, \tag{65a}$$

$$F_{(p_r+2)} = \frac{2}{\sqrt{N_r}} d[h_r^{-1}(x,z)] \wedge \Omega(\mathrm{X}) \wedge \Omega(\mathrm{Y}_2), \tag{65b}$$

$$F_{(p_s+2)} = \frac{2}{\sqrt{N_s}} d[h_s^{-1}(x,z)] \wedge \Omega(\mathrm{X}) \wedge \Omega(\mathrm{Y}_1), \tag{65c}$$

where $\Omega(\mathrm{X})$, $\Omega(\mathrm{Y}_1)$, and $\Omega(\mathrm{Y}_2)$ denote the volume $(p+1)$-form, $(p_s - p)$-form, $(p_r - p)$-form respectively

$$\Omega(\mathrm{X}) = \sqrt{-q}dx^0 \wedge dx^1 \wedge \cdots \wedge dx^p, \tag{66a}$$

$$\Omega(\mathrm{Y}_1) = \sqrt{\gamma}dy^1 \wedge dy^2 \wedge \cdots \wedge dy^{p_s-p}, \tag{66b}$$

$$\Omega(\mathrm{Y}_2) = \sqrt{w}dv^1 \wedge dv^2 \wedge \cdots \wedge dv^{p_r-p}. \tag{66c}$$

Here, q, γ, w are the determinant of the metric $q_{\mu\nu}$, γ_{ij}, w_{mn}, respectively. Let us first consider the gauge field Eqs. (61c) and (61d). Under the assumptions (65b) and (65c), we find

$$d[h_s^{4\chi/N_s}\partial_a h_r(*_Z dz^a) \wedge \Omega(\mathrm{Y}_1)] = 0, \tag{67a}$$

$$d[h_r^{4\chi/N_r}\partial_a h_s(*_Z dz^a) \wedge \Omega(\mathrm{Y}_2)] = 0, \tag{67b}$$

where $*_{Y_1}$, $*_{Y_2}$ denote the Hodge operator on Y_1, Y_2 respectively, and χ is defined by

$$\chi = p + 1 - \frac{(p_r+1)(p_s+1)}{D-2} + \frac{1}{2}\epsilon_r\epsilon_s c_r c_s. \tag{68}$$

Then, the Eq. (67a) leads to

$$h_s^{4\chi/N_s} \triangle_Z h_r = 0, \tag{69a}$$

$$\partial_\mu h_s^{4\chi/N_s}\partial_a h_r + h_s^{4\chi/N_s}\partial_\mu\partial_a h_r = 0, \tag{69b}$$

where $\triangle_Z$ is the Laplace operators on the space of Z. On the other hand, it follows from (67b) that

$$h_r^{4\chi/N_r} \triangle_Z h_s = 0, \tag{70a}$$

$$\partial_\mu h_r^{4\chi/N_r}\partial_a h_s + h_r^{4\chi/N_r}\partial_\mu\partial_a h_s = 0. \tag{70b}$$

For $\chi = 0$, the Eq. (69) gives

$$\triangle_Z h_r = 0, \qquad \partial_\mu\partial_a h_r = 0, \tag{71a}$$

and the Eq. (70) reduces to

$$\triangle_Z h_s = 0, \qquad \partial_\mu\partial_a h_s = 0. \tag{72a}$$

The relation $\chi = 0$ is consistent with the intersection rule [6,8,41–44].

Next, we consider the Einstein Eq. (61a). Using the assumptions (62) and (65), the Einstein equations are given by

$$R_{\mu\nu}(\mathrm{X}) - \frac{4}{N_r}h_r^{-1}D_\mu D_\nu h_r - \frac{4}{N_s}h_s^{-1}D_\mu D_\nu h_s + \frac{2}{N_r}\partial_\mu \ln h_r\left[\left(1-\frac{4}{N_r}\right)\partial_\nu \ln h_r - \frac{4}{N_s}\partial_\nu \ln h_s\right] + \frac{2}{N_s}\partial_\mu \ln h_s\left[\left(1-\frac{4}{N_s}\right)\partial_\nu \ln h_s - \frac{4}{N_r}\partial_\nu \ln h_r\right] - \frac{1}{2}q_{\mu\nu}h_r^{-4/N_r}h_s^{-4/N_s}(a_r h_r^{-1}\triangle_Z h_r + a_s h_s^{-1}\triangle_Z h_s) - \frac{1}{2}q_{\mu\nu}\left[a_r h_r^{-1}\triangle_X h_r - a_r q^{\rho\sigma}\partial_\rho \ln h_r\left\{\left(1-\frac{4}{N_r}\right)\partial_\sigma \ln h_r - \frac{4}{N_s}\partial_\sigma \ln h_s\right\} + a_s h_s^{-1}\triangle_X h_s - a_s q^{\rho\sigma}\partial_\rho \ln h_s\left\{\left(1-\frac{4}{N_s}\right)\partial_\sigma \ln h_s - \frac{4}{N_r}\partial_\sigma \ln h_r\right\}\right] = 0, \quad (73a)$$

$$h_r^{-1}\partial_\mu \partial_a h_r = 0, \quad (73b)$$

$$h_s^{-1}\partial_\mu \partial_a h_s = 0, \quad (73c)$$

$$R_{ij}(\mathrm{Y}_1) - \frac{1}{2}h_r^{4/N_r}\gamma_{ij}\left[b_r h_r^{-1}\triangle_X h_r - b_r q^{\rho\sigma}\partial_\rho \ln h_r\left\{\left(1-\frac{4}{N_r}\right)\partial_\sigma \ln h_r - \frac{4}{N_s}\partial_\sigma \ln h_s\right\} + a_s h_s^{-1}\triangle_X h_s - a_s q^{\rho\sigma}\partial_\rho \ln h_s\left\{\left(1-\frac{4}{N_s}\right)\partial_\sigma \ln h_s - \frac{4}{N_r}\partial_\sigma \ln h_r\right\}\right] - \frac{1}{2}\gamma_{ij}h_s^{-4/N_s}(b_r h_r^{-1}\triangle_Z h_r + a_s h_s^{-1}\triangle_Z h_s) = 0, \quad (73d)$$

$$R_{mn}(\mathrm{Y}_2) - \frac{1}{2}h_s^{4/N_s}w_{mn}\left[a_r h_r^{-1}\triangle_X h_r - a_r q^{\rho\sigma}\partial_\rho \ln h_r\left\{\left(1-\frac{4}{N_r}\right)\partial_\sigma \ln h_r - \frac{4}{N_s}\partial_\sigma \ln h_s\right\} + b_s h_s^{-1}\triangle_X h_s - b_s q^{\rho\sigma}\partial_\rho \ln h_s\left\{\left(1-\frac{4}{N_s}\right)\partial_\sigma \ln h_s - \frac{4}{N_r}\partial_\sigma \ln h_r\right\}\right] - \frac{1}{2}w_{mn}h_r^{-4/N_r}(a_r h_r^{-1}\triangle_Z h_r + b_s h_s^{-1}\triangle_Z h_s) = 0, \quad (73e)$$

$$R_{ab}(\mathrm{Z}) - \frac{1}{2}h_r^{4/N_r}h_s^{4/N_s}u_{ab}\left[b_r h_r^{-1}\triangle_X h_r - b_r q^{\rho\sigma}\partial_\rho \ln h_r\left\{\left(1-\frac{4}{N_r}\right)\partial_\sigma \ln h_r - \frac{4}{N_s}\partial_\sigma \ln h_s\right\} + b_s h_s^{-1}\triangle_X h_s - b_s q^{\rho\sigma}\partial_\rho \ln h_s\left\{\left(1-\frac{4}{N_s}\right)\partial_\sigma \ln h_s - \frac{4}{N_r}\partial_\sigma \ln h_r\right\}\right] - \frac{1}{2}u_{ab}(b_r h_r^{-1}\triangle_Z h_r + b_s h_s^{-1}\triangle_Z h_s) = 0, \quad (73f)$$

where we have used the intersection rule $\chi = 0$, and D_μ is the covariant derivative with respect to the metric $q_{\mu\nu}$, $\triangle_X$ is the Laplace operators on X space, and $R_{\mu\nu}(X)$, $R_{ij}(Y_1)$, $R_{mn}(Y_2)$, and $R_{ab}(Z)$ are the Ricci tensors associated with the metrics $q_{\mu\nu}(X)$, $\gamma_{ij}(Y_1)$, $w_{mn}(Y_2)$ and $u_{ab}(Z)$, respectively.

We see from Eqs. (73b) and (73c) that the warp factors h_r and h_s must take the form

$$h_r(x,z) = h_0(x) + h_1(z), \qquad h_s(x,z) = k_0(x) + k_1(z). \quad (74)$$

With this form of h_r and h_s, the other components of the Einstein Eqs. (73) are rewritten as

$$R_{\mu\nu}(\mathrm{X}) - \frac{4}{N_r}h_r^{-1}D_\mu D_\nu h_0 - \frac{4}{N_s}h_s^{-1}D_\mu D_\nu k_0 + \frac{2}{N_r}\partial_\mu \ln h_r\left[\left(1-\frac{4}{N_r}\right)\partial_\nu \ln h_r - \frac{4}{N_s}\partial_\nu \ln h_s\right] + \frac{2}{N_s}\partial_\mu \ln h_s\left[\left(1-\frac{4}{N_s}\right)\partial_\nu \ln h_s - \frac{4}{N_r}\partial_\nu \ln h_r\right] - \frac{1}{2}q_{\mu\nu}(a_r h_r^{-1}\triangle_Z h_1 + a_s h_s^{-1}\triangle_Z k_1) - \frac{1}{2}q_{\mu\nu}\left[a_r h_r^{-1}\triangle_X h_0 - a_r q^{\rho\sigma}\partial_\rho \ln h_r\left\{\left(1-\frac{4}{N_r}\right)\partial_\sigma \ln h_r - \frac{4}{N_s}\partial_\sigma \ln h_s\right\} + a_s h_s^{-1}\triangle_X k_0 - a_s q^{\rho\sigma}\partial_\rho \ln h_s\left\{\left(1-\frac{4}{N_s}\right)\partial_\sigma \ln h_s - \frac{4}{N_r}\partial_\sigma \ln h_r\right\}\right] = 0, \quad (75a)$$

$$R_{ij}(\mathrm{Y}_1) - \frac{1}{2}h_r^{4/N_r}\gamma_{ij}\left[b_r h_r^{-1}\triangle_X h_0 - b_r q^{\rho\sigma}\partial_\rho \ln h_r\left\{\left(1-\frac{4}{N_r}\right)\partial_\sigma \ln h_r - \frac{4}{N_s}\partial_\sigma \ln h_s\right\} + a_s h_s^{-1}\triangle_X k_0 - a_s q^{\rho\sigma}\partial_\rho \ln h_s\left\{\left(1-\frac{4}{N_s}\right)\partial_\sigma \ln h_s - \frac{4}{N_r}\partial_\sigma \ln h_r\right\}\right] - \frac{1}{2}\gamma_{ij}h_s^{-4/N_s}(b_r h_r^{-1}\triangle_Z h_1 + a_s h_s^{-1}\triangle_Z k_1) = 0, \quad (75b)$$

$$R_{mn}(\mathrm{Y}_2) - \frac{1}{2}h_s^{4/N_s}w_{mn}\left[a_r h_r^{-1}\triangle_X h_0 - a_r q^{\rho\sigma}\partial_\rho \ln h_r\left\{\left(1-\frac{4}{N_r}\right)\partial_\sigma \ln h_r - \frac{4}{N_s}\partial_\sigma \ln h_s\right\} + b_s h_s^{-1}\triangle_X k_0 - b_s q^{\rho\sigma}\partial_\rho \ln h_s\left\{\left(1-\frac{4}{N_s}\right)\partial_\sigma \ln h_s - \frac{4}{N_r}\partial_\sigma \ln h_r\right\}\right] - \frac{1}{2}w_{mn}h_r^{-4/N_r}(a_r h_r^{-1}\triangle_Z h_1 + b_s h_s^{-1}\triangle_Z k_1) = 0, \quad (75c)$$

$$R_{ab}(\mathrm{Z}) - \frac{1}{2}h_r^{4/N_r}h_s^{4/N_s}u_{ab}\left[b_r h_r^{-1}\triangle_X h_0 - b_r q^{\rho\sigma}\partial_\rho \ln h_r\left\{\left(1-\frac{4}{N_r}\right)\partial_\sigma \ln h_r - \frac{4}{N_s}\partial_\sigma \ln h_s\right\} + b_s h_s^{-1}\triangle_X k_0 - b_s q^{\rho\sigma}\partial_\rho \ln h_s\left\{\left(1-\frac{4}{N_s}\right)\partial_\sigma \ln h_s - \frac{4}{N_r}\partial_\sigma \ln h_r\right\}\right] - \frac{1}{2}u_{ab}(b_r h_r^{-1}\triangle_Z h_1 + b_s h_s^{-1}\triangle_Z k_1) = 0. \quad (75d)$$

Finally we should consider the scalar field equation. Substituting Eqs. (65) and (74) and the intersection rule $\chi = 0$ into Eq. (61b), we obtain

$$\begin{aligned}
&\epsilon_r c_r h_r^{4/N_r} h_s^{4/N_s}\Big[h_r^{-1}\triangle_X h_0 - q^{\rho\sigma}\partial_\rho \ln h_r \\
&\quad\times\Big\{\Big(1-\frac{4}{N_r}\Big)\partial_\sigma \ln h_r - \frac{4}{N_s}\partial_\sigma \ln h_s\Big\}\Big] \\
&\quad+\epsilon_s c_s h_r^{4/N_r} h_s^{4/N_s}\Big[h_r^{-1}\triangle_X k_0 - q^{\rho\sigma}\partial_\rho \ln h_s \\
&\quad\times\Big\{\Big(1-\frac{4}{N_s}\Big)\partial_\sigma \ln h_s - \frac{4}{N_r}\partial_\sigma \ln h_r\Big\}\Big] \\
&\quad+\epsilon_r c_r h_r^{-1}\triangle_Z h_1 + \epsilon_s c_s h_s^{-1}\triangle_Z k_1 = 0. \qquad (76)
\end{aligned}$$

Thus, the warp factors h_r and h_s should satisfy the equations

$$\triangle_X h_0 + q^{\rho\sigma}\partial_\rho h_0\Big[\Big(1-\frac{4}{N_r}\Big)\partial_\sigma \ln h_r - \frac{4}{N_s}\partial_\sigma \ln h_s\Big] = 0, \quad \triangle_Z h_1 = 0, \qquad (77a)$$

$$\triangle_X k_0 + q^{\rho\sigma}\partial_\rho k_0\Big[\Big(1-\frac{4}{N_s}\Big)\partial_\sigma \ln h_s - \frac{4}{N_r}\partial_\sigma \ln h_r\Big] = 0, \quad \triangle_Z k_1 = 0. \qquad (77b)$$

Combining these, we find that these field equations lead to

$$R_{\mu\nu}(\mathrm{X}) = 0, \qquad R_{ij}(\mathrm{Y}_1) = 0, \qquad R_{mn}(\mathrm{Y}_2) = 0, \qquad R_{ab}(\mathrm{Z}) = 0, \qquad (78a)$$

$$h_r = h_0(x) + h_1(z), \qquad h_s = k_0(x) + k_1(z), \qquad (78b)$$

$$D_\mu D_\nu h_0 = 0, \qquad \Big(1-\frac{4}{N_r}\Big)\partial_\mu \ln h_r - \frac{4}{N_s}\partial_\mu \ln h_s = 0, \qquad \triangle_Z h_1 = 0, \qquad (78c)$$

$$D_\mu D_\nu k_0 = 0, \qquad \Big(1-\frac{4}{N_s}\Big)\partial_\mu \ln h_s - \frac{4}{N_r}\partial_\mu \ln h_r = 0, \qquad \triangle_Z k_1 = 0. \qquad (78d)$$

If $F_{(p_r+2)} = 0$ and $F_{(p_s+2)} = 0$, the functions h_1 and k_1 become trivial, and the D-dimensional spacetime is no longer warped [7,19].

1. The case of $\frac{1}{N_r} + \frac{1}{N_s} = \frac{1}{4}$

As a special example, let us consider the case

$$q_{\mu\nu} = \eta_{\mu\nu}, \qquad \gamma_{ij} = \delta_{ij}, \qquad w_{mn} = \delta_{mn}, \qquad u_{ab} = \delta_{ab}, \qquad \frac{1}{N_r} + \frac{1}{N_s} = \frac{1}{4}, \qquad h_r = h_s, \qquad (79)$$

where $\eta_{\mu\nu}$ is the $(p+1)$-dimensional Minkowski metric and δ_{ij}, δ_{mn}, δ_{ab} are the $(p_s - p)$-, $(p_r - p)$- and $(D + p - p_r - p_s - 1)$-dimensional Euclidean metrics, respectively. This physically means that both branes have the same total amount of charge. The solution for h_r and h_s can be obtained explicitly as

$$h_r(x,z) = A_\mu x^\mu + B + \sum_\ell \frac{M_\ell}{|z - z_\ell|^{D+p-p_r-p_s-3}}, \qquad (80a)$$

$$h_s(x,z) = h_r(x,z), \qquad (80b)$$

where A_μ, B, C, M_ℓ and M_c are constant parameters, and z_ℓ and z_c are constant vectors representing the positions of the branes. Since the functions coincide, the locations of the brane will also coincide.

Let us consider the intersection rule in the D-dimensional theory. For $p_r = p_s$ and $N_r = N_s = 8$, the intersection rule $\chi = 0$ leads to

$$p = p_r - 4. \qquad (81)$$

Then, we get the intersection involving two p_r-brane

$$p_r \cap p_r = p_r - 4. \qquad (82)$$

Equation (82) tells us that the numbers of intersection for $p_r < 4$ are negative, which means that there is no intersecting solution of these brane systems. Since p is positive or zero, the number of the total dimension must be $D \geq 10$.

2. The case of $\frac{1}{N_r} + \frac{1}{N_s} \neq \frac{1}{4}$

If we consider the case $\frac{1}{N_r} + \frac{1}{N_s} \neq \frac{1}{4}$, the field equations can be satisfied only if there is only one function $h_I (I = r \,\mathrm{or}\, s)$ depending on both z^a and x^μ, and other functions are either dependent on z^a or constant.

In the case of $\frac{1}{N_r} + \frac{1}{N_s} \neq \frac{1}{4}$, if $\partial_\mu k_0 = 0$, from Eq. (77) we obtain $(1 - \frac{4}{N_r}) \times q^{\rho\sigma}\partial_\rho h_0 \partial_\sigma h_0 = 0$ with $\Delta_X h_0 = 0$. Thus, in this case it is clear that there is no solution for $h_0(x)$ such as $\partial_\mu h_0 \neq 0$ unless $N_r = 4$. Then we consider the case

$$q_{\mu\nu} = \eta_{\mu\nu}, \qquad \gamma_{ij} = \delta_{ij}, \qquad w_{mn} = \delta_{mn}, \qquad u_{ab} = \delta_{ab}, \qquad N_r = 4, \qquad (83)$$

where $\eta_{\mu\nu}$ is the $(p+1)$-dimensional Minkowski metric and δ_{ij}, δ_{mn}, δ_{ab} are the $(p_s - p)$-, $(p_r - p)$- and $(D + p - p_r - p_s - 1)$-dimensional Euclidean metrics, respectively. For $\partial_\mu h_s = 0$, the solution for h_r and h_s can be obtained explicitly as

$$h_r(x,z) = A_\mu x^\mu + B + \sum_\ell \frac{M_\ell}{|z - z_\ell|^{D+p-p_r-p_s-3}}, \qquad (84a)$$

$$h_s(z) = C + \sum_c \frac{M_c}{|z - z_c|^{D+p-p_r-p_s-3}}, \qquad (84b)$$

where A_μ, B, C, M_ℓ and M_c are constant parameters, and z_ℓ and z_c are constant vectors representing the positions of the branes. Thus, in our time-dependent generalization of intersecting brane solutions, only D-, NS-branes as well as M-branes can be time dependent in the $D = 11$ and $D = 10$ supergravities.

Note that the conditions on the intersection $\chi = 0$ (see Eq. (68)) have now potentially more solutions, since p can now take also the value $p = -1$ (thus defining an intersection on a point in Euclidean space). There is also a single brane solution which only exists in the Euclidean formulation is the (-1)-brane, or D-instanton for $\partial_\mu k_0 = 0$. The intersections for the (-1)-brane (D-instantons) in $N_I = 4$ are given by [8,42,44]

$$\mathrm{D}(-1) \cap \mathrm{D}p = \frac{1}{2}(p-5), \tag{85a}$$

$$\mathrm{D}(-1) \cap \mathrm{F1} = 0. \tag{85b}$$

We then consider the NSS model among the theories of $D = 6$. The couplings of the 3-form ($p_r = 1$) and the 2-form ($p_s = 0$) field strengths to the dilaton are given by $\epsilon_r c_r = -\sqrt{2}$, $\epsilon_s c_s = -\frac{1}{\sqrt{2}}$, respectively. From Eq. (60a), this case is realized by choosing $N_r = 4$ and $N_s = 2$. However, the number of the intersections dimensions is -1, according to the intersection rule $\chi = 0$. Though meaningless in ordinary spacetime, these configurations are relevant in the Euclidean space, for instance representing instantons.

Similarly, for the $\mathcal{N} = 4^g$ class of the six-dimensional Romans theory [20], following the classification in Ref. [21], the coupling of the 3-form and of the 2-form field strengths to the dilaton are given by $\epsilon_r c_r = -\sqrt{2}$, $\epsilon_s c_s = 1/\sqrt{2}$, respectively. The number of the intersecting dimensions is zero from the intersection rule. On the other hand, for the $\mathcal{N} = \tilde{4}^g$ class, the number of the intersecting dimension becomes -1 as for the NSS model and therefore the solution is classically meaningless.

C. Case (II)

We next consider the case (II). For this class, the D-dimensional metric ansatz (62) gives

$$\begin{aligned} ds^2 &= h_r^{a_r}(x,y) h_s^{a_s}(x,z) q_{\mu\nu}(\mathrm{X}) dx^\mu dx^\nu \\ &\quad + h_r^{b_r}(x,y) h_s^{a_s}(x,z) \gamma_{ij}(\mathrm{Y}_1) dy^i dy^j \\ &\quad + h_r^{a_r}(x,y) h_s^{b_s}(x,z) w_{mn}(\mathrm{Y}_2) dv^m dv^n \\ &\quad + h_r^{b_r}(x,y) h_s^{b_s}(x,z) u_{ab}(\mathrm{Z}) dz^a dz^b. \end{aligned} \tag{86}$$

We also take the following ansatz for the scalar field ϕ and the gauge field strengths:

$$e^\phi = h_r^{2\epsilon_r c_r/N_r} h_s^{2\epsilon_s c_s/N_s}, \tag{87a}$$

$$F_{(p_r+2)} = \frac{2}{\sqrt{N_r}} d[h_r^{-1}(x,y)] \wedge \Omega(\mathrm{X}) \wedge \Omega(\mathrm{Y}_2), \tag{87b}$$

$$F_{(p_s+2)} = \frac{2}{\sqrt{N_s}} d[h_s^{-1}(x,z)] \wedge \Omega(\mathrm{X}) \wedge \Omega(\mathrm{Y}_2), \tag{87c}$$

where $\Omega(X)$, $\Omega(Y_1)$, and $\Omega(Y_2)$ are defined in (66). Since we use the same procedure as in Sec. III B, we can derive the intersection rule $\chi = 0$ from the field equations. For $\chi = 0$, it is easy to show that the field equations reduce to

$$R_{\mu\nu}(\mathrm{X}) = 0, \qquad R_{ij}(\mathrm{Y}_1) = 0, \qquad R_{mn}(\mathrm{Y}_2) = 0, \qquad R_{ab}(\mathrm{Z}) = 0, \tag{88a}$$

$$h_r = h_0(x) + h_1(y), \qquad h_s = k_0(x) + k_1(z), \tag{88b}$$

$$D_\mu D_\nu h_0 = 0, \qquad \left(1 - \frac{4}{N_r}\right)\partial_\mu \ln h_r - \frac{4}{N_s}\partial_\nu \ln h_s = 0, \qquad \triangle_{\mathrm{Y}_1} h_1 = 0, \tag{88c}$$

$$D_\mu D_\nu k_0 = 0, \qquad \left(1 - \frac{4}{N_s}\right)\partial_\mu \ln h_s - \frac{4}{N_r}\partial_\nu \ln h_r = 0, \qquad \triangle_{\mathrm{Z}} k_1 = 0, \tag{88d}$$

where $\triangle_{\mathrm{Y}_1}$ is the Laplace operators on the space of Y_1. If $F_{(p_r+2)} \neq 0$ and $F_{(p_s+2)} \neq 0$, the functions h_1 and k_1 are nontrivial. There is no dynamical solution for $\frac{1}{N_r} + \frac{1}{N_s} = \frac{1}{4}$ because we can not take both functions to be equal.

In the case of $\frac{1}{N_r} + \frac{1}{N_s} \neq \frac{1}{4}$, as in the case (I), if $\partial_\mu k_0 = 0$ there is no solution for $h_0(x)$ such as $\partial_\mu h_0 \neq 0$ unless $N_r = 4$. Let us consider the following case in more detail:

$$q_{\mu\nu} = \eta_{\mu\nu}, \qquad \gamma_{ij} = \delta_{ij}, \qquad w_{mn} = \delta_{mn}, \qquad u_{ab} = \delta_{ab}, \qquad N_r = 4, \tag{89}$$

where $\eta_{\mu\nu}$ is the $(p+1)$-dimensional Minkowski metric and δ_{ij}, δ_{mn}, δ_{ab} are the $(p_s - p)$-, $(p_r - p)$-, and $(D + p - p_r - p_s - 1)$-dimensional Euclidean metrics, respectively. For $\partial_\mu h_s = 0$, the solution for h_r and h_s can be obtained explicitly as

$$h_r(x,y) = A_\mu x^\mu + B + \sum_\ell \frac{M_\ell}{|\boldsymbol{y} - \boldsymbol{y}_\ell|^{p_s - p - 2}}, \tag{90a}$$

$$h_s(z) = C + \sum_c \frac{M_c}{|\boldsymbol{z} - \boldsymbol{z}_c|^{D+p-p_r-p_s-3}}, \tag{90b}$$

where A_μ, B, C, $\boldsymbol{y}_\ell$, $\boldsymbol{z}_c$, M_ℓ and M_c are constant parameters. Thus, in our time-dependent generalization of intersecting brane solutions, only D-, NS-branes as well as M-branes can be time dependent in the $D = 11$ and $D = 10$ supergravities.

D. Case (III)

Finally we consider the case (III). For this class, the D-dimensional metric ansatz (62) reduces to

$$\begin{aligned} ds^2 &= h_r^{a_r}(x,y) h_s^{a_s}(x,v) q_{\mu\nu}(\mathrm{X}) dx^\mu dx^\nu \\ &\quad + h_r^{b_r}(x,y) h_s^{a_s}(x,v) \gamma_{ij}(\mathrm{Y}_1) dy^i dy^j \\ &\quad + h_r^{a_r}(x,y) h_s^{b_s}(x,v) w_{mn}(\mathrm{Y}_2) dv^m dv^n \\ &\quad + h_r^{b_r}(x,y) h_s^{b_s}(x,v) u_{ab}(\mathrm{Z}) dz^a dz^b. \end{aligned} \tag{91}$$

We also assume that the scalar field ϕ and the gauge field strengths are given as

$$e^{\phi} = h_r^{2\epsilon_r c_r/N_r} h_s^{2\epsilon_s c_s/N_s}, \tag{92a}$$

$$F_{(p_r+2)} = \frac{2}{\sqrt{N_r}} h_s^{4/N_s} d[h_r^{-1}(x,y)] \wedge \Omega(\mathrm{X}) \wedge \Omega(\mathrm{Y}_2), \tag{92b}$$

$$F_{(p_s+2)} = \frac{2}{\sqrt{N_s}} h_r^{4/N_r} d[h_s^{-1}(x,v)] \wedge \Omega(\mathrm{X}) \wedge \Omega(\mathrm{Y}_1), \tag{92c}$$

where $\Omega(\mathrm{X})$, $\Omega(\mathrm{Y}_1)$, and $\Omega(\mathrm{Y}_2)$ denote the volume $(p+1)$-, (p_s-p)-, and (p_r-p)-forms, respectively.

Under the assumption, the field equations give the intersection rule $\chi = -2$ [8,45]. This is different from the usual rule applicable to the cases (I) and (II). Upon using the intersection rule $\chi = -2$, it is easy to show that the field equations reduce to

$$R_{\mu\nu}(\mathrm{X}) = 0, \qquad R_{ij}(\mathrm{Y}_1) = 0, \qquad R_{mn}(\mathrm{Y}_2) = 0, \qquad R_{ab}(\mathrm{Z}) = 0, \tag{93a}$$

$$h_r = h_0(x) + h_1(y), \qquad h_s = k_0(x) + k_1(v), \tag{93b}$$

$$D_\mu D_\nu h_0 = 0, \qquad \left(1 - \frac{4}{N_r}\right)\partial_\mu \ln h_r - \frac{4}{N_s}\partial_\nu \ln h_s = 0, \qquad \triangle_{\mathrm{Y}_1} h_1 = 0, \tag{93c}$$

$$D_\mu D_\nu k_0 = 0, \qquad \left(1 - \frac{4}{N_s}\right)\partial_\mu \ln h_s - \frac{4}{N_r}\partial_\nu \ln h_r = 0, \qquad \triangle_{\mathrm{Y}_2} k_1 = 0, \tag{93d}$$

where $\triangle_{\mathrm{Y}_1}$ and $\triangle_{\mathrm{Y}_2}$ are the Laplace operators on the spaces of Y_1 and Y_2, respectively. The functions h_1 and k_1 are nontrivial for $F_{(p_r+2)} \neq 0$ and $F_{(p_s+2)} \neq 0$. There is no dynamical solution for $\frac{1}{N_r} + \frac{1}{N_s} = \frac{1}{4}$ because of $h_r \neq h_s$.

In the case of $\frac{1}{N_r} + \frac{1}{N_s} \neq \frac{1}{4}$, as in the cases (I) and (II), if $\partial_\mu k_0 = 0$ there is no solution for $h_0(x)$ such as $\partial_\mu h_0 \neq 0$ unless $N_r = 4$. Now we consider the case

$$q_{\mu\nu} = \eta_{\mu\nu}, \qquad \gamma_{ij} = \delta_{ij}, \qquad w_{mn} = \delta_{mn}, \qquad u_{ab} = \delta_{ab}, \qquad N_r = 4, \tag{94}$$

where $\eta_{\mu\nu}$ is the $(p+1)$-dimensional Minkowski metric and δ_{ij}, δ_{mn}, δ_{ab} are the (p_s-p)-, (p_r-p)-, and $(D+p-p_r-p_s-1)$-dimensional Euclidean metrics, respectively. For $\partial_\mu h_s = 0$, the solution for h_r and h_s can be obtained explicitly as

$$h_r(x,y) = A_\mu x^\mu + B + \sum_\ell \frac{M_\ell}{|\boldsymbol{y} - \boldsymbol{y}_\ell|^{p_s-p-2}}, \tag{95a}$$

$$h_s(v) = C + \sum_c \frac{M_c}{|\boldsymbol{v} - \boldsymbol{v}_c|^{p_r-p-2}}, \tag{95b}$$

where A_μ, B, C, $\boldsymbol{y}_\ell$, $\boldsymbol{v}_c$, M_ℓ and M_c are constant parameters. Thus, in our time-dependent generalization of intersecting brane solutions, only D-, NS-branes as well as M-branes can be time dependent in $D = 11$ and $D = 10$ supergravities. There is also a single brane solution which only exists in the Euclidean formulation is the (-1)-brane, or D-instanton. The intersections for the RR-charged D-instantons are thus given by

$$\mathrm{D}(-1) \cap \mathrm{D}p = \frac{1}{2}(p-9). \tag{96}$$

For $D = 6$, we can construct the dynamical intersecting brane solutions in the NSS and the Romans theories with a vanishing cosmological constant. However, the number of intersections involving the 1-brane and 0-brane are $p = -3$ for the NSS theory and $p = -2$ for the $\mathcal{N} = 4^g$ class of the Romans theory, respectively. Therefore, both of them are classically meaningless.

E. Cosmology

Let us consider the dynamical solutions for the p_r- and p_s-brane system which appears in the D-dimensional theory. In this section, we apply the above solutions to the four-dimensional cosmology. We assume an isotropic and homogeneous three-space in the four-dimensional spacetime. We assume that the $(p+1)$-dimensional spacetime is the Minkowski spacetime with $q_{\mu\nu}(\mathrm{X}) = \eta_{\mu\nu}(\mathrm{X})$, and drop the coordinate dependence on X space except for the time.

The D-dimensional metric (64) can be expressed as

$$ds^2 = -h dt^2 + ds^2(\tilde{\mathrm{X}}) + ds^2(\mathrm{Y}_1) + ds^2(\mathrm{Y}_2) + ds^2(\mathrm{Z}), \tag{97}$$

where we have defined

$$ds^2(\tilde{\mathrm{X}}) \equiv h\delta_{PQ}(\tilde{\mathrm{X}}) d\theta^P d\theta^Q, \tag{98a}$$

$$ds^2(\mathrm{Y}_1) \equiv h_r^{b_r} h_s^{a_s} \gamma_{ij}(\mathrm{Y}_1) dy^i dy^j, \tag{98b}$$

$$ds^2(\mathrm{Y}_2) \equiv h_r^{a_r} h_s^{b_s} w_{mn}(\mathrm{Y}_2) dv^m dv^n, \tag{98c}$$

$$ds^2(\mathrm{Z}) \equiv h_r^{b_r} h_s^{b_s} u_{ab}(\mathrm{Z}) dz^a dz^b, \tag{98d}$$

$$h \equiv h_r^{a_r} h_s^{a_s}. \tag{98e}$$

Here, $\delta_{PQ}(\tilde{\mathrm{X}})$ is the p-dimensional Euclidean metric, and θ^P denotes the coordinate of the p-dimensional Euclid space $\tilde{\mathrm{X}}$.

We focus on the case $h_r = h_s$ and $\frac{1}{N_r} + \frac{1}{N_s} = \frac{1}{4}$ and set $h_r = At + h_1$. The D-dimensional metric (97) can be written as

$$
\begin{aligned}
ds^2 = \left[1 + \left(\frac{\tau}{\tau_0}\right)^{-2/(a_r+a_s+2)} h_1\right]^{a_r+a_s} \Bigg[& -d\tau^2 + \left(\frac{\tau}{\tau_0}\right)^{2(a_r+a_s)/(a_r+a_s+2)} \delta_{PQ}(\tilde{\mathrm{X}}) d\theta^P d\theta^Q \\
& + \left\{1 + \left(\frac{\tau}{\tau_0}\right)^{-2/(a_r+a_s+2)} h_1\right\}^{4/N_r} \left(\frac{\tau}{\tau_0}\right)^{2(a_r+a_s+4/N_r)/(a_r+a_s+2)} \gamma_{ij}(\mathrm{Y}_1) dy^i dy^j \\
& + \left\{1 + \left(\frac{\tau}{\tau_0}\right)^{-2/(a_r+a_s+2)} h_1\right\}^{1-4/N_r} \left(\frac{\tau}{\tau_0}\right)^{2(a_r+a_s+1-4/N_r)/(a_r+a_s+2)} w_{mn}(\mathrm{Y}_2) dv^m dv^n \\
& + \left\{1 + \left(\frac{\tau}{\tau_0}\right)^{-2/(a_r+a_s+2)} h_1\right\} \left(\frac{\tau}{\tau_0}\right)^{2(a_r+a_s+1)/(a_r+a_s+2)} u_{ab}(\mathrm{Z}) dz^a dz^b \Bigg], \qquad (99)
\end{aligned}
$$

where we have introduced the cosmic time τ defined by

$$\frac{\tau}{\tau_0} = (At)^{(a_r+a_s+2)/2}, \qquad \tau_0 = \frac{2}{(a_r + a_s + 2)A}. \qquad (100)$$

The D-dimensional metric (99) implies that the power of the scale factor in the fastest expanding case is

$$\frac{a_r + a_s + 1}{a_r + a_s + 2} = \frac{p_r + 1}{D + p_r - 1} < 1, \quad \text{for } D > 2, \qquad (101)$$

for $p_r = p_s$ and $N_r = N_s = 8$. Then, it is impossible to find the cosmological model that our Universe exhibits an accelerating expansion.

We compactify $d(\equiv d_1 + d_2 + d_3 + d_4)$ dimensions to fit our Universe, where d_1, d_2, d_3 and d_4 denotes the compactified dimensions with respect to the $\tilde{\mathrm{X}}$, Y_1, Y_2 and Z spaces. The metric (97) is then described by

$$ds^2 = ds^2(\mathrm{M}) + ds^2(\mathrm{N}), \qquad (102)$$

where $ds^2(\mathrm{M})$ is the $(D - d)$-dimensional metric and $ds^2(\mathrm{N})$ is the metric of compactified dimensions.

By the conformal transformation

$$ds^2(\mathrm{M}) = h_r^B ds^2(\bar{\mathrm{M}}), \qquad (103)$$

we can rewrite the $(D - d)$-dimensional metric in the Einstein frame. Here, B and C are

$$B = -\frac{(a_r + a_s)d + d_3 + d_4 + \frac{4(d_2-d_3)}{N_r}}{D - d - 2}. \qquad (104)$$

Hence the $(D - d)$-dimensional metric in the Einstein frame is

$$
\begin{aligned}
ds^2(\bar{\mathrm{M}}) = h_r^{B'}[& -d\tau^2 + \delta_{P'Q'}(\tilde{\mathrm{X}}') d\theta^{P'} d\theta^{Q'} \\
& + h_r^{4/N_r} \gamma_{k'l'}(\mathrm{Y}_1{}') dy^{k'} dy^{l'} \\
& + h_r^{1-4/N_r} w_{m'n'}(\mathrm{Y}_2{}') dv^{m'} dv^{n'} \\
& + h_r u_{a'b'}(\mathrm{Z}') dz^{a'} dz^{b'}], \qquad (105)
\end{aligned}
$$

where B' is defined by $B' = -B + a_r + a_s$, and $\tilde{\mathrm{X}}'$, $\mathrm{Y}_1{}'$, $\mathrm{Y}_2{}'$ and Z' denote the $(p - d_1)$-, $(p_s - p - d_2)$-, $(p_r - p - d_3)$-, and $(D + p - p_r - p_s - 1 - d_4)$-dimensional spaces, respectively.

For $h_r = At + h_1$, the metric (105) is thus rewritten as

$$
\begin{aligned}
ds^2(\bar{\mathrm{M}}) = \left[1 + \left(\frac{\tau}{\tau_0}\right)^{-2/(B'+2)} h_1\right]^{B'} \Bigg[& -d\tau^2 + \left(\frac{\tau}{\tau_0}\right)^{2B'/(B'+2)} \delta_{P'Q'}(\tilde{\mathrm{X}}') d\theta^{P'} d\theta^{Q'} \\
& + \left\{1 + \left(\frac{\tau}{\tau_0}\right)^{-2/(B'+2)} h_1\right\}^{4/N_r} \left(\frac{\tau}{\tau_0}\right)^{2(B'+4/N_r)/(B'+2)} \gamma_{k'l'}(\mathrm{Y}_1{}') dy^{k'} dy^{l'} \\
& + \left\{1 + \left(\frac{\tau}{\tau_0}\right)^{-2/(B'+2)} h_1\right\}^{1-4/N_r} \left(\frac{\tau}{\tau_0}\right)^{2(B'+1-4/N_r)/(B'+2)} w_{m'n'}(\mathrm{Y}_2{}') dv^{m'} dv^{n'} \\
& + \left\{1 + \left(\frac{\tau}{\tau_0}\right)^{-2/(B'+2)} h_1\right\} \left(\frac{\tau}{\tau_0}\right)^{2(B'+1)/(B'+2)} u_{a'b'}(\mathrm{Z}') dz^{a'} dz^{b'} \Bigg], \qquad (106)
\end{aligned}
$$

where the cosmic time τ is defined by

$$\frac{\tau}{\tau_0} = (At)^{(B'+2)/2}, \qquad \tau_0 = \frac{2}{(B' + 2)A}. \qquad (107)$$

For the Einstein frame, the power of the scale factor in the fastest expanding case is also given by

$$\frac{B' + 1}{B' + 2} < 1, \quad \text{for } D > d + 2, \qquad d > 0. \qquad (108)$$

Therefore, we cannot find the solution which exhibits an accelerating expansion of our Universe.

We list the Friedmann-Lemaitre-Robertson-Walker (FLRW) cosmological solutions with an isotropic and homogeneous three-space for the solutions (106) in Table I for $p0$-$p1$ brane system in the six-dimensional Romans theory. The power exponents of the scale factor of possible four-dimensional cosmological models are given by $a(\tilde{\mathrm{M}}) \propto \tau^{\lambda(\tilde{\mathrm{M}})}$, where τ is the cosmic time, and $a(\tilde{\mathrm{M}})$ and $a_E(\tilde{\mathrm{M}})$ denote the scale factors of the

TABLE I. Intersections of 0-branes and 1-brane of the six-dimensional Romans theory with $N_r = 4$ for 1-brane and $N_s = 2$ for 0-brane in the case (I) and (II) are shown. Time dependence appears only in 1-brane.

Branes		0	1	2	3	4	5	$\tilde{M}$	$\lambda(\tilde{M})$	$\lambda_E(\tilde{M})$
$p0$-$p1$	$p1$	∘	∘					Z	$\lambda(Z) = 1/3$	$\lambda_E(Z) = \frac{2-d_3}{6-2d_3-d_4}$
	$p0$	∘								
	x^N	t	v	z^1	z^2	z^3	z^4			

TABLE II. The power exponent of the fastest expansion in the Einstein frame for $p0$-$p1$ brane of the six-dimensional Romans theory is shown. "TD" in the table represents which brane is time dependent.

Branes	TD	dim(M)	$\tilde{M}$	(d_1, d_2, d_3, d_4)	$\lambda_E(\tilde{M})$	Case
$p0$-$p1$	$p1$	5	Z	(0, 0, 1, 0)	1/4	I & II

space $\tilde{\mathrm{M}}$ in Jordan and Einstein frames with the exponents carrying the same suffices, respectively. Here, $\tilde{\mathrm{M}}$ denotes the spatial part of the spacetime M.

Since the time dependence in the metric comes from only one brane in the intersections, the obtained expansion law is simple. In order to find an expanding universe, one may have to compactify the vacuum bulk space as well as the brane world volume. Unfortunately we find that the fastest expanding case in the Jordan frame has the power $\lambda(\tilde{\mathrm{M}}) < 1/2$, which is too small to give a realistic expansion law like that in the matter-dominated era ($a \propto \tau^{2/3}$) or that in the radiation-dominated era ($a \propto \tau^{1/2}$).

When we compactify the extra dimensions and go to the four-dimensional Einstein frame, the power exponents are different depending on how we compactify the extra dimensions even within one solution. For $p0$-$p1$brane in the six-dimensional Romans theory, we give the power exponent of the fastest expansion of our four-dimensional Universe in the Einstein frame in Table II. We again see that the expansion is too small. Hence, we have to conclude that in order to find a realistic expansion of the Universe in this type of models, one has to include additional "matter" fields on the brane.

IV. THE INTERSECTION OF nBRANES IN D-DIMENSIONAL THEORY

A. Theory

Let us consider a gravitational theory with the metric g_{MN}, dilaton ϕ, and antisymmetric tensor fields of rank $(p_I + 2)$, where I denotes the type of the corresponding branes. The most general action for the intersecting-brane system is written as

$$S = \frac{1}{2\kappa^2} \int \Bigg[R * \mathbf{1}_D - \frac{1}{2} d\phi \wedge * d\phi - \sum_I \frac{1}{2(p_I + 2)!} e^{\epsilon_I c_I \phi} F_{(p_I+2)} \wedge * F_{(p_I+2)} \Bigg], \quad (109)$$

where κ^2 is the D-dimensional gravitational constant, $*$ is the Hodge dual operator in the D-dimensional spacetime, c_I, ϵ_I are constants given by

$$c_I^2 = N_I - \frac{2(p_I + 1)(D - p_I - 3)}{D - 2}, \quad (110)$$

$$\epsilon_I = \begin{cases} + & \text{for the electric brane} \\ - & \text{for the magnetic brane.} \end{cases} \quad (111)$$

The expectation values of fermionic fields are assumed to be zero.

The field equations are given by

$$R_{MN} = \frac{1}{2}\partial_M \phi \partial_N \phi + \frac{1}{2}\sum_I \frac{1}{(p_I+2)!} e^{\epsilon_I c_I \phi} \times \Bigg[(p_I+2) F_{MA_2 \cdots A_{p_I+2}} F_N{}^{A_2 \cdots A_{p_I+2}} - \frac{p_I+1}{D-2} g_{MN} F^2_{(p_I+2)} \Bigg], \quad (112a)$$

$$d * d\phi = \frac{1}{2}\sum_I \frac{\epsilon_I c_I}{(p_I+2)!} e^{\epsilon_I c_I \phi} F_{(p_I+2)} \wedge * F_{(p_I+2)}, \quad (112b)$$

$$d[e^{\epsilon_I c_I \phi} * F_{(p_I+2)}] = 0. \quad (112c)$$

B. Solutions

To solve the field equations, we assume the D-dimensional metric of the form

$$ds^2 = -\mathcal{A}(t, z) dt^2 + \sum_{\alpha=1}^{p} \mathcal{B}^{(\alpha)}(t, z)(dx^\alpha)^2 + \mathcal{C}(t, z) u_{ij}(\mathrm{Z}) dz^i dz^j, \quad (113)$$

where $u_{ij}(\mathrm{Z})$ is the metric of the $(D - p - 1)$-dimensional Z space which depends only on the $(D - p - 1)$-dimensional coordinates z^i. $\mathcal{A}$, $\mathcal{B}^{(\alpha)}$ and $\mathcal{C}$ are given by

$$\mathcal{A} = \prod_I [h_I(t, z)]^{a_I}, \quad \mathcal{B}^{(\alpha)} = \prod_I [h_I(t, z)]^{\delta_I^{(\alpha)}}, \quad \mathcal{C} = \prod_I [h_I(t, z)]^{b_I}, \quad (114)$$

where the parameters a_I, b_I and $\delta_I^{(\alpha)}$ are defined by

$$a_I = -\frac{4(D-p_I-3)}{N_I(D-2)}, \qquad b_I = \frac{4(p_I+1)}{N_I(D-2)}, \qquad \delta_I^{(\alpha)} = \begin{cases} a_I & \text{for } \alpha \in I \\ b_I & \text{for } \alpha \notin I \end{cases}, \tag{115}$$

and $h_I(t, z)$, which depends on t and z^i, is a straightforward generalization of the harmonic function associated with the brane I in a static brane system [42].

We also assume that the scalar field ϕ and the gauge field strength $F_{(p+2)}$ are given by

$$e^{\phi} = \prod_I h_I^{2\epsilon_I c_I/N_I}, \qquad F_{(p_I+2)} = \frac{2}{\sqrt{N_I}} d(h_I^{-1}) \wedge \Omega(\mathrm{X}_I), \tag{116}$$

where X_I is the space associated with the brane I, and $\Omega(X_I) = dt \wedge dx^{p_1} \wedge \cdots \wedge dx^{p_I}$ is the volume (p_I+1)-form.

Let us assume [42]

$$\mathcal{A}^{\,(D-p-3)} \prod_{\alpha=1}^{p} \mathcal{B}^{(\alpha)} \mathcal{C} = 1, \qquad \mathcal{A}^{-1} \prod_{\alpha \in I} (\mathcal{B}^{(\alpha)})^{-1} e^{\epsilon_I c_I \phi} = h_I^2. \tag{117}$$

The Einstein equations Eq. (112a) then reduce to

$$\sum_{I,I'} \frac{2}{N_I}\left(\delta_{II'} - \frac{2}{N_{I'}} M_{II'}\right)\partial_t \ln h_I \partial_t \ln h_{I'} + \frac{1}{2}\sum_I b_I\left[\left(1-\frac{4}{N_I}\right)\partial_t \ln h_I - \sum_{I'\neq I}\frac{4}{N_{I'}}\partial_t \ln h_{I'}\right]\partial_t \ln h_I - \frac{1}{2}\sum_I\left(\frac{4}{N_I} + b_I\right)h_I^{-1}\partial_t^2 h_I - \frac{1}{2}\prod_{I'} h_{I'}^{-4/N_{I'}} \sum_I a_I h_I^{-1} \triangle_{\mathrm{Z}} h_I = 0, \tag{118a}$$

$$\sum_I h_I^{-1}\partial_t \partial_i h_I + \sum_{I,I'}\left(\frac{2}{N_{I'}} M_{II'} - \delta_{II'}\right)\partial_t \ln h_I \partial_i \ln h_{I'} = 0, \tag{118b}$$

$$\prod_{J'} h_{J'}^{-a_{J'}} \sum_\gamma \prod_J h_J^{\delta_J^{(\gamma)}} \sum_I \delta_I^{(\gamma)}\left[h_I^{-1}\partial_t^2 h_I - \left\{\left(1-\frac{4}{N_I}\right)\partial_t \ln h_I - \sum_{I'\neq I}\frac{4}{N_{I'}}\partial_t \ln h_{I'}\right\}\partial_t \ln h_I\right] - \prod_{J'} h_{J'}^{-b_{J'}} \sum_\gamma \prod_J h_J^{\delta_J^{(\gamma)}} \sum_I \delta_I^{(\gamma)} h_I^{-1} \triangle_{\mathrm{Z}} h_I = 0, \tag{118c}$$

$$R_{ij}(\mathrm{Z}) + \frac{1}{2}u_{ij}\prod_J h_J^{4/N_J}\sum_I b_I\left[h_I^{-1}\partial_t^2 h_I - \left\{\left(1-\frac{4}{N_I}\right)\partial_t \ln h_I - \sum_{I'\neq I}\frac{4}{N_{I'}}\partial_t \ln h_{I'}\right\}\partial_t \ln h_I\right] - \frac{1}{2}u_{ij}\sum_I b_I h_I^{-1}\triangle_{\mathrm{Z}} h_I - \sum_{I,I'}\frac{2}{N_I}\left(\frac{2}{N_{I'}}M_{II'} - \delta_{II'}\right)\partial_i \ln h_I \partial_j \ln h_{I'} = 0, \tag{118d}$$

where $R_{ij}(\mathrm{Z})$ is the Ricci tensor of the metric u_{ij}, and $M_{II'}$ is given by

$$M_{II'} \equiv \frac{N_I N_{I'}}{16}\left[a_I a_{I'} + \sum_\alpha \delta_I^{(\alpha)}\delta_{I'}^{(\alpha)} + (D-p-3)b_I b_{I'}\right] + \frac{1}{2}\epsilon_I \epsilon_{I'} c_I c_{I'}. \tag{119}$$

Let us consider Eq. (118b). We can rewrite this as

$$\sum_{I,I'}\left[\frac{2}{N_{I'}}M_{II'} + \delta_{II'}\frac{\partial_t\partial_i \ln h_I}{\partial_t \ln h_I \partial_i \ln h_I}\right]\partial_t \ln h_I \partial_i \ln h_{I'} = 0. \tag{120}$$

In order to satisfy this equation for arbitrary coordinate values and independent functions h_I, the second term in the square bracket must be constant:

$$\frac{\partial_t\partial_i \ln h_I}{\partial_t \ln h_I \partial_i \ln h_I} = k_I. \tag{121}$$

Then in order for Eq. (120) to be satisfied identically, we must have

$$\frac{2}{N_{I'}}M_{II'} + k_I \delta_{II'} = 0. \tag{122}$$

Using Eqs. (110), (115), and (119), we get

$$M_{II} = \left(\frac{N_I}{4}\right)^2[(p_I+1)a_I^2 + (p-p_I)b_I^2 + (D-p-3)b_I^2] + \frac{1}{2}c_I^2 = \frac{N_I}{2}. \tag{123}$$

This means that the constant k_I in Eq. (122) is $k_I = -1$, namely

$$M_{II'} = \frac{N_{I'}}{2}\delta_{II'}. \tag{124}$$

It then follows from Eq. (121) that

$$\partial_t \partial_i [h_I(t, z)] = 0. \tag{125}$$

As a result, the warp factor h_I must be separable as

$$h_I(t, z) = K_I(t) + H_I(z). \tag{126}$$

For $I \neq I'$, Eq. (124) gives the intersection rule on the dimension $\bar{p}$ of the intersection for each pair of branes I and I' ($\bar{p} \leq p_I, p_{I'}$) [42,44–46]:

$$\bar{p} = \frac{(p_I + 1)(p_{I'} + 1)}{D - 2} - 1 - \frac{1}{2}\epsilon_I c_I \epsilon_{I'} c_{I'}. \tag{127}$$

Let us next consider the gauge field. Under the ansatz Eq. (116) for electric background, we find

$$dF_{(p_I+2)} = h_I^{-1}(2\partial_i \ln h_I \partial_j \ln h_I + h_I^{-1}\partial_i\partial_j h_I)dz^i \wedge dz^j \wedge \Omega(\mathrm{X}_I) = 0. \tag{128}$$

Thus, the Bianchi identity is automatically satisfied. Also the equation of motion for the gauge field becomes

$$d[\partial_i H_I(*_{\mathrm{Z}} dz^i) \wedge *_{\mathrm{X}}\Omega(\mathrm{X}_I)] = 0, \tag{129}$$

where we used Eq. (126), and $*_{\mathrm{X}}$, $*_{\mathrm{Z}}$ denotes the Hodge dual operator on $\mathrm{X}(\equiv \cup_I \mathrm{X}_I)$ and Z, respectively, and we have used Eq. (117). Hence, we again find the condition Eq. (126) and

$$\triangle_{\mathrm{Z}} H_I = 0. \tag{130}$$

We note that the roles of the Bianchi identity and field equations are interchanged for magnetic ansatz [42,44], but the net result is the same.

Let us finally consider the scalar field equation. Substituting the scalar field and the gauge field in Eq. (116), and the warp factor Eq. (126) into the equation of motion for the scalar field Eq. (112b), we obtain

$$-\prod_{I''} h_{I''}^{-a_{I''}} \sum_I \frac{1}{N_I}\epsilon_I c_I\left[h_I^{-1}\partial_t^2 K_I - \left\{\left(1 - \frac{4}{N_I}\right)\partial_t \ln h_I - \sum_{I' \neq I}\frac{4}{N_{I'}}\partial_t \ln h_{I'}\right\}\partial_t \ln h_I\right] + \prod_{I''} h_{I''}^{-b_{I''}} \sum_I \frac{1}{N_I} h_I^{-1}\epsilon_I c_I \triangle_{\mathrm{Z}} H_I = 0. \tag{131}$$

This equation is satisfied if

$$\partial_t^2 K_I = 0, \tag{132a}$$

$$\triangle_{\mathrm{Z}} H_I = 0, \tag{132b}$$

$$\sum_I \frac{1}{N_I}\epsilon_I c_I\left[\left(1 - \frac{4}{N_I}\right)\partial_t \ln h_I - \sum_{I' \neq I}\frac{4}{N_{I'}}\partial_t \ln h_{I'}\right] = 0. \tag{132c}$$

Equation (132a) gives $K_I = A_I t + B_I$, where A_I and B_I are integration constants.

(A): Let us first consider the case that we take all functions to be equal:

$$h_I(t, z) = h(t, z) \equiv K(t) + H(z), \qquad N_I = N_{I'} = N. \tag{133}$$

We can find the solutions if the function h and N satisfy

$$K(t) = At + B, \qquad N = 4\ell, \tag{134}$$

where ℓ denotes the number of the functions h_I. Then the remaining Einstein equations Eq. (118) are

$$R_{ij}(\mathrm{Z}) = 0. \tag{135}$$

Now we assume

$$u_{ij} = \delta_{ij}, \tag{136}$$

where δ_{ij} is the $(D - p - 1)$-dimensional Euclidean metric. In this case, the solution for h can be obtained explicitly as

$$h(t, z) = At + B + \sum_k \frac{Q_k}{|z - z_k|^{D-p-3}}, \tag{137}$$

where Q_k's are constant parameters and z_k represents the positions of the branes in Z space, z_K is constant vector representing the positions of the branes. Since the functions coincide, the locations of the brane will also coincide. This physically means that all branes have the same total amount of charge at same position.

Let us consider the intersection rule in the D-dimensional theory. If we choose $p_I = \tilde{p}$ for all p_I, the intersection rule Eq. (127) leads to

$$\bar{p} = \tilde{p} - 2\ell. \tag{138}$$

Then, we get the intersection involving two $\tilde{p}$-brane

$$\tilde{p} \cap \tilde{p} = \tilde{p} - 2\ell. \tag{139}$$

Equation (139) tells us that the number of intersection for $\tilde{p} < 2\ell$ is negative, which means that there is no intersecting solution of these brane systems.

For $K = 0$ ($A = B = 0$), the metric describes the known static and extremal multi-black hole solution with black hole charges Q_k [42–44].

(B): Next, we consider the case that there is only one function h_I depending on both z^i and t, which we denote with the subscript $\tilde{I}$, and other functions are either dependent on z^i or constant. We also assume $N_{\tilde{I}} = 4$. Then, we have

$$K_{\tilde{I}}(t) = At + B_{\tilde{I}}, \qquad N_{\tilde{I}} = 4, \tag{140a}$$

$$K_I = B_I, \qquad (I \neq I_{\tilde{I}}). \tag{140b}$$

If we assume $u_{ij} = \delta_{ij}$, the solution for H_I can be obtained explicitly as

$$H_I(z) = 1 + \sum_k \frac{Q_{I,k}}{|z - z_k|^{D-p-3}}, \tag{141}$$

where $Q_{I,k}$'s are constant parameters and z_k represents the positions of the branes in Z space. We can find the solution

(140) for any N_I. If we choose $N_I = 4$, the solutions have already discussed in [6].

C. Cosmology

Let us consider the dynamical solutions for the $p_I$brane system which appears in the D-dimensional theory. In this section, we apply the above solutions to the four-dimensional cosmology. We assume that the four-dimensional spacetime is an isotropic and homogeneous three-space, and either the world volume space or the transverse space can be (a part of) our four-dimensional Universe.

In what follows, we concentrate on the $(D-p-1)$-dimensional Euclidean space with $u_{ij}(\mathrm{Z}) = \delta_{ij}(\mathrm{Z})$, and consider the case that all functions h_I are equal to h and all parameter of N_I has the same value $N_I = N = 4\ell$, where ℓ is the number of p_I-brane. In this case, the D-dimensional metric (113) can be expressed as

$$ds^2 = -h^a dt^2 + \sum_\alpha h^{\delta^{(\alpha)}}(dx^\alpha)^2 + h^b \delta_{ij}(\mathrm{Z}) dz^i dz^j, \quad (142)$$

where h is defined by (133) and the parameters a, b, $\delta^{(\alpha)}$ are given by

$$a = -\sum_I \frac{D - p_I - 3}{\ell(D-2)},$$
$$b = \sum_I \frac{p_I + 1}{\ell(D-2)}, \quad (143)$$
$$\delta^{(\alpha)} = \sum_I \delta_I^{(\alpha)}.$$

For $K = At$, the metric (142) is thus rewritten as

$$\begin{aligned} ds^2 = &-\left[1 + \left(\frac{\tau}{\tau_0}\right)^{-((2)/(a+2))} H\right]^a d\tau^2 \\ &+ \sum_\alpha \left[1 + \left(\frac{\tau}{\tau_0}\right)^{-((2)/(a+2))} H\right]^{\delta^{(\alpha)}} \\ &\times \left(\frac{\tau}{\tau_0}\right)^{((2\delta^{(\alpha)})/(a+2))} (dx^\alpha)^2 + \left[1 + \left(\frac{\tau}{\tau_0}\right)^{-((2)/(a+2))} H\right]^b \\ &\times \left(\frac{\tau}{\tau_0}\right)^{((2b)/(a+2))} \delta_{ij}(\mathrm{Z}) dz^i dz^j, \end{aligned} \quad (144)$$

where we have introduced the cosmic time τ defined by

$$\frac{\tau}{\tau_0} = (At)^{(a+2)/2}, \qquad \tau_0 = \frac{2}{(a+2)A}. \quad (145)$$

Here, the H is defined by

$$H(z) = 1 + \sum_k \frac{Q_k}{|z - z_k|^{D-p-3}}, \quad (146)$$

where Q_k's are constant parameters and z_k represents the positions of the branes in Z space.

The D-dimensional metric (144) implies that the power of the scale factor in the fastest expanding case is

$$\frac{b}{a+2} = \frac{\sum_I (p_I + 1)}{\sum_{I'} (p_{I'} + 1) + \ell(D-2)} < 1, \quad \text{for } D > 2. \quad (147)$$

Then, it is impossible to find the cosmological model that our Universe exhibits an accelerating expansion.

We compactify $d (\equiv \sum_\alpha d_\alpha + d_z)$ dimensions to fit our Universe, where d_α and d_z denotes the compactified dimensions with respect to the relative transverse space, Z spaces. The metric (113) is then described by

$$ds^2 = ds^2(\mathrm{M}) + ds^2(\mathrm{N}), \quad (148)$$

where $ds^2(\mathrm{M})$ is the $(D-d)$-dimensional metric and $ds^2(\mathrm{N})$ is the metric of compactified dimensions.

By the conformal transformation

$$ds^2(\mathrm{M}) = h^B ds^2(\bar{\mathrm{M}}), \quad (149)$$

we can rewrite the $(D-d)$-dimensional metric in the Einstein frame. Here, B is

$$B = -\frac{\sum_\alpha d_\alpha \delta^{(\alpha)} + d_z b}{D - d - 2}. \quad (150)$$

Hence, the $(D-d)$-dimensional metric in the Einstein frame is

$$\begin{aligned} ds^2(\bar{\mathrm{M}}) = h^{-B}\Big[&-h^a dt^2 + \sum_{\alpha'} h^{\delta^{(\alpha')}} (dx^{\alpha'})^2 \\ &+ h^b \delta_{i'j'}(\mathrm{Z}') dz^{i'} dz^{j'}\Big], \end{aligned} \quad (151)$$

where $x^{\alpha'}$ is the coordinate of $(p - d_\alpha)$-dimensional relative transverse space, and Z' denote $(D - p - 1 - d_z)$-dimensional spaces, respectively.

For $K = At$, the metric (151) is thus rewritten as

$$\begin{aligned} ds^2(\bar{\mathrm{M}}) = &-\left[1 + \left(\frac{\tau}{\tau_0}\right)^{-((2)/(B'+2))} H\right]^{B'} d\tau^2 \\ &+ \sum_{\alpha'} \left[1 + \left(\frac{\tau}{\tau_0}\right)^{-((2)/(B'+2))} H\right]^{-B+\delta^{(\alpha')}} \\ &\times \left(\frac{\tau}{\tau_0}\right)^{((2(-B+\delta^{(\alpha')}))/(B'+2))} (dx^{\alpha'})^2 \\ &+ \left[1 + \left(\frac{\tau}{\tau_0}\right)^{-((2)/(B'+2))} H\right]^{B'+1} \\ &\times \left(\frac{\tau}{\tau_0}\right)^{((2(B'+1))/(B'+2))} \delta_{i'j'}(\mathrm{Z}') dz^{i'} dz^{j'}, \end{aligned} \quad (152)$$

where B' is defined by $B' = -B + a$, and the cosmic time τ is defined by

$$\frac{\tau}{\tau_0} = (At)^{(B'+2)/2}, \qquad \tau_0 = \frac{2}{(B'+2)A}. \quad (153)$$

For the Einstein frame, the power of the scale factor in the fastest expanding case is also given by

$$\frac{B' + 1}{B' + 2} < 1, \quad \text{for } D > d + 2, \qquad d > 0. \tag{154}$$

Then, we cannot find the solution which exhibits an accelerating expansion of our Universe.

Next we consider the case that there is only one function h_I depending on both z^i and t, which we denote with the subscript $\tilde{I}$, and other functions are either dependent on z^i or constant. We also assume $N_{\tilde{I}} = 4$. Then we have

$$\begin{aligned} ds^2 = &-\prod_{I\neq\tilde{I}} h_I^{a_I}\left[1 + \left(\frac{\tau}{\tau_0}\right)^{-((2)/(a_{\tilde{I}}+2))} H_{\tilde{I}}\right]^{a_{\tilde{I}}} d\tau^2 \\ &+ \sum_\alpha \prod_{I\neq\tilde{I}} h_I^{\delta_I^{(\alpha)}}\left[1 + \left(\frac{\tau}{\tau_0}\right)^{-((2)/(a_{\tilde{I}}+2))} H_{\tilde{I}}\right]^{\delta_{\tilde{I}}^{(\alpha)}} \\ &\times \left(\frac{\tau}{\tau_0}\right)^{((2\delta_{\tilde{I}}^{(\alpha)})/(a_{\tilde{I}}+2))} (dx^\alpha)^2 + \prod_{I\neq\tilde{I}} h_I^{b_I}\Bigg[1 \\ &+ \left(\frac{\tau}{\tau_0}\right)^{-((2)/(a_{\tilde{I}}+2))} H_{\tilde{I}}\Bigg]^{b_{\tilde{I}}} \\ &\times \left(\frac{\tau}{\tau_0}\right)^{((2b_{\tilde{I}})/(a_{\tilde{I}}+2))} \delta_{ij}(\mathrm{Z}) dz^i dz^j, \end{aligned} \tag{155}$$

where the function $H_{\tilde{I}}$ is defined by (141), and we have introduced the cosmic time τ defined by

$$\frac{\tau}{\tau_0} = (At)^{(a_{\tilde{I}}+2)/2}, \qquad \tau_0 = \frac{2}{(a_{\tilde{I}} + 2)A}. \tag{156}$$

The D-dimensional metric (155) implies that the power of the scale factor in the fastest expanding case is

$$\frac{b_{\tilde{I}}}{a_{\tilde{I}} + 2} = \frac{p + 1}{D + p - 1} < 1, \quad \text{for } D > 2. \tag{157}$$

Then, it is impossible to find the cosmological model that our Universe exhibits an accelerating expansion.

We compactify $d(\equiv \sum_\alpha d_\alpha + d_z)$ dimensions to fit our Universe, where d_α and d_z denotes the compactified dimensions with respect to the relative transverse space, Z spaces. The metric (113) is then described by (148). By the conformal transformation

$$ds^2(\mathrm{M}) = h_{\tilde{I}}^{B_{\tilde{I}}} \prod_{I\neq\tilde{I}} h_I^{C_I} ds^2(\bar{\mathrm{M}}), \tag{158}$$

we can rewrite the $(D - d)$-dimensional metric in the Einstein frame. Here, $B_{\tilde{I}}$ and C_I are

$$B_{\tilde{I}} = -\frac{\sum_\alpha d_\alpha \delta_{\tilde{I}}{}^{(\alpha)} + d_z b_{\tilde{I}}}{D - d - 2}, \qquad C_I = -\frac{\sum_\alpha d_\alpha \delta_I^{(\alpha)} + d_z b_I}{D - d - 2}. \tag{159}$$

Hence, the $(D - d)$-dimensional metric in the Einstein frame is

$$\begin{aligned} ds^2(\bar{\mathrm{M}}) = h_{\tilde{I}}^{-B_{\tilde{I}}} \prod_{J\neq\tilde{I}} h_J{}^{-C_J}\Bigg[&-h^{a_{\tilde{I}}} \prod_{I\neq\tilde{I}} h_I^{a_I'} dt^2 \\ &+ \sum_{\alpha'} h_{\tilde{I}}^{\delta_{\tilde{I}}^{(\alpha')}} \prod_{I\neq\tilde{I}} h_I^{\delta_I^{(\alpha')}} (dx^{\alpha'})^2 \\ &+ h_{\tilde{I}}^{b_{\tilde{I}}} \prod_{I\neq\tilde{I}} h_I^{b_I} \delta_{i'j'}(\mathrm{Z}') dz^{i'} dz^{j'}\Bigg], \end{aligned} \tag{160}$$

where $x^{\alpha'}$ is the coordinate of $(p - d_\alpha)$-dimensional relative transverse space, and Z$\prime$ denote $(D - p - 1 - d_z)$-dimensional spaces, respectively. For $K_{\tilde{I}} = At$, the $(D - d)$-dimensional metric in the Einstein frame is thus written as

$$\begin{aligned} ds^2(\bar{\mathrm{M}}) = \prod_{I\neq\tilde{I}} h_I^{-C_I}\Bigg[&-\prod_{I\neq\tilde{I}} h_I^{a_I}\left\{1 + \left(\frac{\tau}{\tau_0}\right)^{-((2)/(B'_{\tilde{I}}+2))} H_{\tilde{I}}\right\}^{B'_{\tilde{I}}} d\tau^2 + \sum_{\alpha'} \prod_{I\neq\tilde{I}} h_I^{\delta_I^{(\alpha')}}\left\{1 + \left(\frac{\tau}{\tau_0}\right)^{-((2)/(B'_{\tilde{I}}+2))} H_{\tilde{I}}\right\}^{-B_{\tilde{I}}+\delta_{\tilde{I}}^{(\alpha')}} \\ &\times \left(\frac{\tau}{\tau_0}\right)^{((2(-B_{\tilde{I}}+\delta_{\tilde{I}}^{(\alpha')}))/(B'_{\tilde{I}}+2))} (dx^{\alpha'})^2 + \prod_{I\neq\tilde{I}} h_I^{b_I}\left\{1 + \left(\frac{\tau}{\tau_0}\right)^{-((2)/(B'_{\tilde{I}}+2))} H_{\tilde{I}}\right\}^{B'_{\tilde{I}}+1} \left(\frac{\tau}{\tau_0}\right)^{((2(B'_{\tilde{I}}+1))/(B'_{\tilde{I}}+2))} \delta_{i'j'}(\mathrm{Z}') dz^{i'} dz^{j'}\Bigg], \end{aligned} \tag{161}$$

where $B'_{\tilde{I}}$ is defined by $B'_{\tilde{I}} = -B_{\tilde{I}} + a_{\tilde{I}}$, and the cosmic time τ is defined by

$$\frac{\tau}{\tau_0} = (At)^{(B'_{\tilde{I}}+2)/2}, \qquad \tau_0 = \frac{2}{(B'_{\tilde{I}} + 2)A}. \tag{162}$$

For the Einstein frame, the power of the scale factor in the fastest expanding case is also given by

$$\frac{B'_{\tilde{I}} + 1}{B'_{\tilde{I}} + 2} < 1, \quad \text{for } D > d + 2, \qquad d > 0. \tag{163}$$

Hence, we can not find the solution which exhibits an accelerating expansion of our Universe.

V. DISCUSSIONS

In the first part of the paper, we have seen that dynamical solutions of p-brane have several remarkable properties. If the scalar and gauge fields are related to the functions h_I like (7), then by counting solutions of the Einstein equations, one would construct only the cosmological model of decelerating expansion of our Universe. We recall that the cosmological constant leads to the accelerating expansion which was described somewhat abstractly in Sec. II.

It appears that the exact forms of the field strengths are given by the ansatz (7b), which depends on the dilaton coupling parameter N. The $N = 4$ case is apparently related to the classical solutions of string theory. We observed that the dynamical solutions with $N \neq 4$ certainly have many attractive properties. Firstly, these solutions were obtained by replacing the time-independent warp factor of the static solution with the time-dependent function. The warp factor for $N \neq 4$ is the same form as that for $N = 4$. Secondly, we could not obtain any analytic solution of a single p-brane with time dependence of the warp factor, if there is no cosmological constant because of the ansatz of the gauge field. Since the field strength has the component along the time coordinate, the time derivative of the warp factor is not permissible in the field strength. Hence, in the Einstein equations, the term of the time derivative of the warp factor arises only from the Ricci tensor, and cannot be compensated by the scalar and gauge fields, except for $N = 4$.

In the case of $N \neq 4$ with a flat transverse space to the brane and a positive cosmological constant $\Lambda > 0$, the Einstein equations give an asymptotically de Sitter solution for a single 2-form field strength. To find the solutions to the Einstein equations in this way, we need a D-dimensional theory with vanishing dilaton in which the cosmological constant is related to a field strength. This is a generalization of Kastor-Traschen solution in four-dimensional Einstein-Maxwell theory. We have simply started with D-dimensional gravitational theory and introduced the cosmological constant with a scalar field that preserves time dependence. For the 0-brane in the NSS model of $D = 6$, an asymptotically Milne solution is obtained. However, it cannot provide an accelerating universe. We have also applied the asymptotically de Sitter solution of five dimensions to construct the brane world model. We have employed the standard copy and paste method to construct a cosmological 3-brane world, supported by either the tension or induced gravity, and embedded into a five-dimensional bulk. We have derived the effective gravitational equations via the junction condition, and shown that the solution gives an accelerating expansion on the 3-brane. However, in our model there is no natural way to explain why the bulk cosmological constant is so small.

In the second part of the paper, we have discussed the time-dependent intersecting brane solutions. For $N_I = 4$, which are the parameters in the coupling of the field strengths to the dilaton, there is only one function h_I depending on both the time and coordinates of transverse space. All the field strengths in the $D = 11$ and $D = 10$ supergravities have $N_I = 4$ couplings.

If all the branes have equal number of world volume dimensions and the same charge, it is possible to get a solution in which all functions h_I depend on both the time and the coordinates of overall transverse space. This turns out to be the only situation where the parameters N_I have proper values within the framework of the intersecting p-brane systems.

If at least one of branes has $N_{\tilde{I}} = 4$, we can construct the time-dependent solutions even if all other $N_I \neq 4$. In this case, only one time-dependent h_I is obtained from the brane of $N_{\tilde{I}} = 4$. For instance, in the case of $D = 6$ without a cosmological constant, we have obtained a dynamical $p0$-$p1$ brane solution in a class of the Romans theory. A dynamical intersecting brane system in this class of the Romans theory was allowed only for the 1-brane.

Supposing that our four-dimensional spacetime is located at a particular place of the extra spatial dimensions, we have obtained expanding FLRW universes. The power of the scale factor in these solutions, however, is too small to give a realistic expansion law even in the case that all functions h_I depend on both the time and coordinates of overall transverse space. This means that we have to consider additional matter on the brane in order to get a realistic expanding universe.

As we have observed, there is a serious difficulty in obtaining an accelerating expansion from the dynamical intersecting solutions. We have discussed the possible solutions of field equations for given scalar and gauge fields in Secs. III and IV. For a given choice of ansatz of fields in the D-dimensional spacetime for the dilaton coupling parameter c_I, the functions h_I in the metric have a condition corresponding to the relation between the warp factors associated to the parameter N_I in the coupling constant c_I. In terms of the field equations, the functions h_I have a structure of the linear combination of the functions $h_0(t)$ and $h_1(z)$. The condition for the form of h_I to be harmonic function to the transverse space is not relevant to the choice of N_I. Though this result is really natural in the viewpoint of the extension of the static solution, it prevents us from obtaining an accelerating expansion because the field equations lead to the function $h_0(t)$ depending on the linear function of time.

Of course, whether this makes sense depends on the ansatz of fields associated with D-dimensional symmetry. A more precise statement with respect to an accelerating expansion in the p-brane system will be presented in the near future.

ACKNOWLEDGMENTS

M. M. is grateful for fruitful discussions during the JGRG 20 and the COSMO/COSPA 2010 held in Japan. K. U. would like to thank H. Kodama, M. Sasaki, N. Ohta and T. Okamura for continuing encouragement. K. U. is supported by Grant-in-Aid for Young Scientists (B) of JSPS Research, under Contract No. 20740147.

[1] T. Maki and K. Shiraishi, Classical Quantum Gravity **10**, 2171 (1993).
[2] K. i. Maeda, N. Ohta, M. Tanabe, and R. Wakebe, J. High Energy Phys. 06 (2009) 036.
[3] K. i. Maeda, N. Ohta, M. Tanabe, and R. Wakebe, J. High Energy Phys. 04 (2010) 013.
[4] D. Kastor and J. H. Traschen, Phys. Rev. D **47**, 5370 (1993).
[5] P. Binetruy, M. Sasaki, and K. Uzawa, Phys. Rev. D **80**, 026001 (2009).
[6] K. i. Maeda, N. Ohta, and K. Uzawa, J. High Energy Phys. 06 (2009) 051.
[7] H. Kodama and K. Uzawa, J. High Energy Phys. 07 (2005) 061.
[8] M. Minamitsuji, N. Ohta, and K. Uzawa, Phys. Rev. D **82**, 086002 (2010).
[9] K. i. Maeda and M. Nozawa, Phys. Rev. D **81**, 044017 (2010).
[10] K. i. Maeda and M. Nozawa, Phys. Rev. D **81**, 124038 (2010).
[11] N. Ohta and T. Shimizu, Int. J. Mod. Phys. A **13**, 1305 (1998).
[12] N. Ohta and J. G. Zhou, Int. J. Mod. Phys. A **13**, 2013 (1998).
[13] N. Ohta, K. L. Panigrahi, and S. Siwach, Nucl. Phys. **B674**, 306 (2003); **B748**, 333 (2006).
[14] Y. G. Miao and N. Ohta, Phys. Lett. B **594**, 218 (2004).
[15] C. M. Chen, D. V. Gal'tsov, and N. Ohta, Phys. Rev. D **72**, 044029 (2005).
[16] N. Ohta and K. L. Panigrahi, Phys. Rev. D **74**, 126003 (2006).
[17] G. W. Gibbons and K. i. Maeda, Phys. Rev. Lett. **104**, 131101 (2010).
[18] M. Nozawa and K. i. Maeda, Phys. Rev. D **83**, 024018 (2011).
[19] H. Kodama and K. Uzawa, J. High Energy Phys. 03 (2006) 053.
[20] L. J. Romans, Nucl. Phys. **B269**, 691 (1986).
[21] C. Nunez, I. Y. Park, M. Schvellinger, and T. A. Tran, J. High Energy Phys. 04 (2001) 025.
[22] H. Nishino and E. Sezgin, Phys. Lett. B **144**, 187 (1984).
[23] A. Salam and E. Sezgin, Phys. Lett. B **147**, 47 (1984).
[24] H. Nishino and E. Sezgin, Nucl. Phys. **B278**, 353 (1986).
[25] G. W. Gibbons, R. Gueven, and C. N. Pope, Phys. Lett. B **595**, 498 (2004).
[26] Y. Aghababaie *et al.*, J. High Energy Phys. 09 (2003) 037.
[27] M. Minamitsuji, N. Ohta, and K. Uzawa, Phys. Rev. D **81**, 126005 (2010).
[28] K. Maeda and H. Nishino, Phys. Lett. B **154**, 358 (1985).
[29] K. Maeda and H. Nishino, Phys. Lett. B **158**, 381 (1985).
[30] A. J. Tolley, C. P. Burgess, C. de Rham, and D. Hoover, New J. Phys. **8**, 324 (2006).
[31] A. J. Tolley, C. P. Burgess, C. de Rham, and D. Hoover, J. High Energy Phys. 07 (2008) 075.
[32] K. i. Maeda, M. Minamitsuji, N. Ohta, and K. Uzawa, Phys. Rev. D **82**, 046007 (2010).
[33] G. W. Gibbons, H. Lu, and C. N. Pope, Phys. Rev. Lett. **94**, 131602 (2005).
[34] H. Lu, C. N. Pope, E. Sezgin, and K. S. Stelle, Nucl. Phys. **B456**, 669 (1995).
[35] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999).
[36] G. R. Dvali, G. Gabadadze, and M. Porrati, Phys. Lett. B **485**, 208 (2000).
[37] K. Koyama, Classical Quantum Gravity **24**, R231 (2007).
[38] M. Minamitsuji, Phys. Lett. B **684**, 92 (2010).
[39] K. Behrndt, E. Bergshoeff, and B. Janssen, Phys. Rev. D **55**, 3785 (1997).
[40] E. Bergshoeff, M. de Roo, E. Eyras, B. Janssen, and J. P. van der Schaar, Nucl. Phys. **B494**, 119 (1997).
[41] A. A. Tseytlin, Nucl. Phys. **B475**, 149 (1996).
[42] R. Argurio, F. Englert, and L. Houart, Phys. Lett. B **398**, 61 (1997).
[43] R. Argurio, arXiv:hep-th/9807171.
[44] N. Ohta, Phys. Lett. B **403**, 218 (1997).
[45] J. P. Gauntlett, D. A. Kastor, and J. H. Traschen, Nucl. Phys. **B478**, 544 (1996).
[46] A. A. Tseytlin, Nucl. Phys. **B487**, 141 (1997).