Confined magnetic monopoles in dense QCD

A. Gorsky,[1,2] M. Shifman,[2,3] and A. Yung[2,4]

[1]*Theory Department, Institute for Theoretical and Experimental Physics, Moscow, Russia*
[2]*William I. Fine Theoretical Physics Institute, University of Minnesota, Minneapolis, Minnesota 55455, USA*
[3]*Center for Theoretical Physics, Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*
[4]*Petersburg Nuclear Physics Institute, Gatchina, St. Petersburg 188300, Russia*
(Received 11 January 2011; published 25 April 2011)

Non-Abelian strings exist in the color-flavor locked phase of dense QCD. We show that kinks appearing in the world-sheet theory on these strings, in the form of the kink-antikink bound pairs, are the magnetic monopoles—descendants of the 't Hooft–Polyakov monopoles surviving in such a special form in dense QCD. Our consideration is heavily based on analogies and inspiration coming from certain supersymmetric non-Abelian theories. This is the first ever analytic demonstration that objects unambiguously identifiable as the magnetic monopoles are native to non-Abelian Yang–Mills theories (albeit our analysis extends only to the phase of the monopole confinement and has nothing to say about their condensation). Technically, our demonstration becomes possible due to the fact that low-energy dynamics of the non-Abelian strings in dense QCD is that of the orientational zero modes. It is described by an effective two-dimensional $CP(2)$ model on the string world sheet. The kinks in this model representing confined magnetic monopoles are in a highly quantum regime.

DOI: 10.1103/PhysRevD.83.085027 PACS numbers: 21.65.Qr, 11.27.+d

I. INTRODUCTION

Despite a huge number of works attempting to treat the monopole condensation in Yang–Mills theories as the quark confinement mechanism, the very notion of the magnetic monopoles remains obscure in QCD: a clear-cut gauge invariant framework for their description and analysis is still absent. This is in contradistinction with a remarkable progress in supersymmetric Yang–Mills theories where, in 1994, Seiberg and Witten analytically proved, for the first time ever, that the dual Meissner effect does take place in a certain model, chromoelectric flux tubes do form, and quark confinement ensues [1]. Further explorations in this area led people to such finds as the non-Abelian flux tubes [2,3] and confined magnetic monopoles [4,5] in a well-defined and fully controllable setting.

In this paper we demonstrate that *confined* non-Abelian magnetic monopoles can be identified in a well-defined manner in high-density QCD. The very issue of flux tubes in high-density QCD [6–9] is the result of crossbreeding of two recent developments: the discovery of non-Abelian flux tubes in supersymmetric gauge theories [2–5] (for a review see [10–13]) and the color-flavor locked (CFL) phases in QCD with a nonvanishing chemical potential μ [14,15]. In principle, non-Abelian flux tubes in dense QCD can show up either in quark-gluon plasma or neutron stars. Leaving aside experimental identification issues, we argue that sufficiently long flux tubes (strings) of this type support kink-antikink pairs in stable "mesonic" states. The kinks, in their turn, can be shown to represent the magnetic monopoles in the confined phase. By the confined phase we mean that (i) the monopoles cannot be disassociated from the strings attached to them; and (ii) the monopole-antimonopole pairs are confined along the string, while the parent QCD *per se* is in the CFL phase. It is not ruled out that dualizing this picture á la [16] we could arrive at a description of *condensed* dyons or monopoles. This latter aspect is left for a separate study.

Large-μ QCD exhibits a rich phase structure of quark-gluon matter at weak coupling. With high enough chemical potential $\mu \gg \Lambda$, the theory supports color superconductivity due to the Cooper pair condensation of diquarks with the vanishing orbital momentum (for reviews see [14,15]). Several superconducting phases are known which differ by the residual symmetry as well as the structure of the condensates. For instance, in the so-called 2SC phase, in which only u and d quarks of two colors pair up [17], the strange quark density plays no role, while at larger values of μ the color-flavor locked (CFL) phase is realized in which all three quark flavors condense, implying the complete breaking of the non-Abelian global flavor group [18]. In addition, all eight gauge bosons are Higgsed. At the same time, a diagonal global SU(3) survives.

It is the CFL phase of dense QCD which is the subject of our studies. Magnetic monopoles in this environment have been already mentioned (in the negative sense) in [7]. However, our conclusion—the presence of well-defined states of magnetic monopoles in the CFL phase of dense QCD—is opposite to what was advocated in [7]. The crucial additional element of our analysis which was absent in [7] is the existence of kinks on the world sheet of the non-Abelian strings that form in dense QCD.

The paper is organized as follows. We briefly review the CFL phase of dense QCD (Sec. II) and the emergence of non-Abelian strings in this phase (Sec. IV), formulate an effective Ginzburg–Landau description (Sec. III), and then proceed to the demonstration that kinks supported by the Ginzburg–Landau model are in fact distorted magnetic monopoles—descendants of the 't Hooft–Polyakov monopoles surviving in such a special form in the environment of dense QCD (Secs. V, VI, and VII). Sec. VIII is devoted to perfecting the simplified Ginzburg–Landau model of Sec. III to make it more realistic. In particular, in this section we introduce a nonvanishing mass for the strange quark. Sec. IX describes possible effects due to the θ term. Finally, Sec. X briefly summarizes our results.

II. COLOR-FLAVOR-LOCKED PHASE

To begin with, we briefly outline the structure of the CFL phase in dense QCD. The one-gluon exchange interaction results in nontrivial diquark condensates of the Cooper type

$$\langle X_{kC}\rangle \propto \delta_{kC} \neq 0, \qquad \langle Y^{Ck}\rangle \propto \delta^{kC} \neq 0,$$
$$X_{kC} = q^{iA}_{\alpha} q^{jB\alpha} \varepsilon_{ijk}\varepsilon_{ABC}, \qquad Y^{Ck} = \tilde{q}_{iA\alpha}\tilde{q}^{\alpha}_{jB}\varepsilon^{ijk}\varepsilon^{ABC}, \tag{2.1}$$

where the small and capital Latin letters mark color and flavor, respectively; q_α and $\tilde{q}_\alpha$ are left-handed spinors; the first one is in the triplet representation of the color and flavor SU(3) groups, while the second one is in the antitriplet representation. The Dirac spinor of each flavor is composed of a pair $(q_\alpha, \bar{\tilde{q}}^{\dot\alpha})$. Needless to say, the complex-conjugate condensates $\langle X^\dagger\rangle$, $\langle Y^\dagger\rangle$ do not vanish as well. In the chiral limit, when $m_{u,d,s} = 0$, the QCD Lagrangian is globally invariant under two chiral SU(3) symmetries. In addition, it is invariant under $\mathrm{U}(1)_B$ and $U(1)_A$ where B marks the baryon number and A stands for axial. $\mathrm{U}(1)_A$ is anomalous; however, at large μ the impact of anomaly is small and can be treated as a correction.

The X condensate transforms under $\mathrm{SU}(3)_C \times \mathrm{SU}(3)_L$ while the $Y^\dagger$ condensate under $\mathrm{SU}(3)_C \times \mathrm{SU}(3)_R$ where $\mathrm{SU}(3)_{L,R}$ are the global chiral groups associated with flavor. The condensates (2.1) break all three groups; however, the diagonal vectorial $SU(3)_{C+F}$ obviously survives. In addition, the X and Y condensates spontaneously break $\mathrm{U}(1)_B$ and $\mathrm{U}(1)_A$. Altogether we have $8 + 8 + 2$ Goldstone bosons in the limit $m_{u,d,s} = 0$. The boson coupled to $\mathrm{U}(1)_A$ is, in fact, a quasi-Goldstone. Since the vectorial flavor SU(3) is unbroken, all Goldstones fall into representations of SU(3): two octets and two singlets. Dynamics in the CFL phase enjoy the properties of systems with superconductivity and superfluidity (due to the broken $\mathrm{U}(1)_B$ symmetry).

Out of 18 Goldstone bosons, 8 degrees of freedom corresponding to the vectorlike fluctuations are eaten by gluons via the Higgs mechanism. Hence we end up with two Goldstone mesons corresponding to the broken Abelian symmetries ($\mathrm{U}(1)_B$ and $\mathrm{U}(1)_A$) and eight pseudoscalar Goldstones coupled to the axial SU(3) currents.[1] It is useful to introduce a gauge invariant order parameter

$$\Sigma = YX, \tag{2.2}$$

which has the chiral transformation properties similar to those of the ordinary chiral condensate in QCD. The order parameter Σ transforms nontrivially under action the axial singlet $U(1)_A$ charge. The eight mesons parameterizing the "phases" of the matrix Σ are

$$\Sigma = |\Sigma| \exp\left(2i\frac{T^a\pi^a}{F_\pi}\right), \tag{2.3}$$

where T^a are the SU(3) generators and F_π is the "pion" constant. As for the absolute value of Σ,

$$|\Sigma| \propto \frac{\mu^4\Delta_0^2}{g^2}, \tag{2.4}$$

where Δ_0 is the superconducting gap at zero temperature

$$\Delta_0 \propto \mu(g(\mu))^{-5}\exp(c/g(\mu)), \tag{2.5}$$

and g is the QCD coupling constant. Numerically, μ is assumed to lie in the ballpark

$$\mu \sim 1\ \mathrm{GeV}$$

considered to be large in the scale

$$\Lambda_{\mathrm{QCD}} \sim 200\ \mathrm{MeV}.$$

The value of the gap parameter Δ_0 is

$$\Delta_0 \sim 10\ \mathrm{MeV}.$$

The CFL mesons have to be considered as four-quark states contrary to conventional two-quark Goldstone states in zero-density QCD. However, let us emphasize that the Goldstone mesons in the CFL phase have exactly the same quantum numbers as the Goldstone mesons in QCD at at $\mu = 0$. The coupling constant F_π in (2.3) is proportional [19] to μ, though.

The electromagnetic $\mathrm{U}(1)_Q$ is broken by the condensates (2.1). A closer look at these condensates shows, however, that a linear combination of the photon A_μ and A^3_μ, A^8_μ gluons

$$\tilde{A}_\mu \propto \left\{A_\mu - \frac{e}{g}\left(A^3_\mu + \frac{1}{\sqrt{3}}A^8_\mu\right)\right\} \tag{2.6}$$

remains massless (the corresponding charge is unbroken). Instead of $\mathrm{U}(1)_Q$, one can consider the charges with respect to $\mathrm{U}(1)_{\tilde{Q}}$. In the CFL phase the mixing angle θ between the photon and the gluon component is obviously small. Therefore, the massless gauge field is dominated by the original photon. Note that the off-diagonal gluons are charged with respect to the $\mathrm{U}(1)_{\tilde{Q}}$ charge.

[1] In supersymmetric theories *all* would-be Goldstones are eaten up by the Higgs mechanism [4,12].

When we switch on nonvanishing quark masses, strictly massless excitations disappear, of course. A number of observables develop a rather contrived mass hierarchy. Our initial consideration will refer to a model which is semirealistic, at best. However, it explains our basic observation in the most transparent way. We will change some details later to make our model more realistic.

In the spirit of the Ginzburg–Landau theory of superconductivity, we will represent the diquark order parameters by a 3×3 matrix of scalar fields Φ^{kA}, where k and A are the color and flavor indices, respectively.

III. GINZBURG–LANDAU EFFECTIVE DESCRIPTION

If the value of the chemical potential μ is large, QCD is in the CFL phase. The order parameter which develops a vacuum expectation value (VEV) is the diquark condensate

$$\Phi^{kC} \sim \varepsilon_{ijk}\varepsilon_{ABC}(q_\alpha^{iA} q^{jB\alpha} + \bar{\tilde{q}}^{iA\dot{\alpha}} \bar{\tilde{q}}_{\dot{\alpha}}^{jB}), \tag{3.1}$$

cf. Equation (2.1), where q_α and $\tilde{q}_\alpha$ are left-handed spinors; the first one is in the triplet representation of the color and flavor SU(3) groups, while the second one is in the antitriplet representation. The Dirac spinor of each flavor is composed from a pair $(q_\alpha, \bar{\tilde{q}}^{\dot{\alpha}})$.

At first, we consider dense QCD in the chiral limit, i.e. assume that quark masses all vanish,

$$m_u = m_d = m_s = 0. \tag{3.2}$$

Later (Sect. VIII) we will be able to relax this condition and switch on a nonvanishing strange quark mass. With the vanishing quark masses the symmetry group is

$$\mathrm{SU}(3)_C \times \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \times \mathrm{U}(1)_B \tag{3.3}$$

where $\mathrm{SU}(3)_{L,R}$ are the global chiral groups associated with three flavors, $\mathrm{U}(1)_B$ is the global rotation associated with the baryon number, while $\mathrm{SU}(3)_C$ is the gauge group.

At small temperatures, the gap (the diquark condensate) is large, while the light degrees of freedom are the Goldstone modes associated with the chiral rotations. These Goldstone modes are quite similar to ordinary pions of zero-μ QCD. The effective Lagrangian is a chiral Lagrangian for these pions; see [15] for a review.

At temperatures approaching (from below) the critical temperature T_c, the gap becomes small, and its fluctuations become exceedingly more important. It is assumed in what follows that the chiral fluctuations are small (and can be ignored) compared to the nonchiral gap fluctuations and those of the gauge fields.[2] This regime can be described in terms of a Ginzburg–Landau (GL) effective theory of a complex matrix scalar field Φ^{kA} defined in (3.1). The Ginzburg–Landau action has the form [7,20][3]

$$S = \int d^4x \left\{ \frac{1}{4g^2}(F_{\mu\nu}^a)^2 + 3\,\mathrm{Tr}(\mathcal{D}_0\Phi)^\dagger(\mathcal{D}_0\Phi) + \mathrm{Tr}(\mathcal{D}_i\Phi)^\dagger(\mathcal{D}_i\Phi) + V(\Phi) \right\} \tag{3.4}$$

with the potential

$$V(\Phi) = -m_0^2 \mathrm{Tr}(\Phi^\dagger\Phi) + \lambda([\mathrm{Tr}(\Phi^\dagger\Phi)]^2 + \mathrm{Tr}[(\Phi^\dagger\Phi)^2]), \tag{3.5}$$

where

$$\mathcal{D}_\mu\Phi \equiv (\partial_\mu - iA_\mu^a T^a)\Phi, \tag{3.6}$$

and T^a stands for the $\mathrm{SU}(3)_{\mathrm{gauge}}$ generator, while g^2 is the QCD coupling constant. The global flavor SU(3) transformations are similar, with T^a acting on Φ from the right. Various parameters in (3.5) are defined as follows:

$$m_0^2 = \frac{48\pi^2}{7\zeta(3)} T_c(T_c - T), \qquad \lambda = \frac{18\pi^2}{7\zeta(3)} \frac{T_c^2}{N(\mu)}, \tag{3.7}$$

while $N(\mu) = \mu^2/(2\pi^2)$ is the density of states on the Fermi surface. Note that our field Φ in (3.4) is rescaled as compared to that in [7,20]: its kinetic energy is canonically normalized.

The critical temperature T_c is much smaller than μ,

$$T_c \sim \frac{\mu}{(g(\mu))^5} \exp\left(-\frac{3\pi^2}{\sqrt{2}g(\mu)}\right) \ll \mu. \tag{3.8}$$

implying

$$m_0^2 \sim T_c(T_c - T), \qquad \lambda \sim \frac{T_c^2}{\mu^2} \ll 1, \tag{3.9}$$

Minimizing (3.5), we find the Φ field VEV,

$$\Phi_{\mathrm{vac}} = v\,\mathrm{diag}\{1, 1, 1\}, \tag{3.10}$$

where the value of the parameter v is given by[4]

$$v^2 = \frac{m_0^2}{8\lambda} = \frac{4\pi^2}{3} \frac{T_c - T}{T_c} \mu^2. \tag{3.11}$$

The diagonal form (3.10) of the vacuum Φ matrix, of the Bardakci–Halpern type [21], expresses the phenomenon of the color-flavor locking. The gauge symmetry of the Lagrangian is SU(3) while its flavor symmetry is $\mathrm{SU}(3) \times \mathrm{U}(1)$, see (3.3). The symmetry of the vacuum state is the diagonal $\mathrm{SU}(3)_{\mathrm{CF}}$,

[2]Strictly speaking, at zero quark masses chiral fluctuations (''pions'') are massless and should be included in the low-energy effective theory. We discuss their impact in Sec. VIII C.

[3]Here and below we use a formally Euclidean notation, e.g. $F_{\mu\nu}^2 = 2F_{0i}^2 + F_{ij}^2$, $(\partial_\mu a)^2 = (\partial_0 a)^2 + (\partial_i a)^2$, etc. This is appropriate, since we are going to study static (time-independent) field configurations, and $A_0 = 0$. Then the Euclidean action is nothing but the energy functional.

[4]Because of the Φ field rescaling in (3.4) our VEV $v \sim \mu$ is different from the standard definition of the gap which is $\Delta \sim T_c$.

$$\mathrm{SU}(3)_C \times \mathrm{SU}(3)_F \times \mathrm{U}(1)_B \to \mathrm{SU}(3)_{\mathrm{CF}}. \tag{3.12}$$

Nine symmetries are spontaneously broken. Out of nine Goldstone modes in this model, eight are eaten up by the gauge bosons, which are fully Higgsed, while one—a common phase of the matrix Φ—survives as a massless excitation. It is associated with broken global baryon symmetry $\mathrm{U}(1)_B$.

The spectrum of massive excitations around this vacuum can be read off [7] from the GL model Lagrangian (3.4). The Higgsed gluons acquire masses

$$m_g = gv \sim g\mu\sqrt{\frac{T_c - T}{T_c}}, \tag{3.13}$$

while nine remaining scalars of the complex matrix Φ^{kA} fill singlet and octet representations of the unbroken $\mathrm{SU}(3)_{\mathrm{CF}}$. Their masses are

$$m_1 = 2m_8 = \sqrt{2}m_0 \sim \sqrt{T_c(T_c - T)}. \tag{3.14}$$

Since $m_g \gg m_0$, as a consequence of the condition $T_c \ll \mu$, we deal here with the extreme type I superconductivity [22].

The GL model (3.4) is in the weak coupling regime if we assume that

$$m_g \gg \Lambda_{\mathrm{QCD}}. \tag{3.15}$$

IV. NON-ABELIAN STRINGS IN THE CFL PHASE

The model (3.4) we use for our analysis is similar to that in which the non-Abelian strings were first considered in the nonsupersymmetric setting [23]. Compared to the original version [23], we discard the U(1) gauge bosons, since in high-density QCD only the non-Abelian color SU(3) is gauged. The baryon current is not gauged, while the photon interaction with the electromagnetic current can be neglected for the time being due to its weakness compared to the quark-gluon interactions (see, however, Sec. VIII). This means that the vortices we will deal with are not fully local. In their U(1) part they are global. This would make their tension infinite if the perpendicular dimensions were infinite too. In the context of dense QCD, with finite-size samples, this is not a problem: the logarithmic divergence of the tension will be cut off by the sample size.

The existence of non-Abelian strings, with the tension 3 times smaller than that of the U(1) global string, is due to the fact that the Z_3 center elements of $\mathrm{SU}(3)_{\mathrm{gauge}}$ simultaneously belong to the global U(1). Everybody knows that $\pi_1(\mathrm{SU}(3))$ is trivial. In arranging a topologically nontrivial winding of the scalar fields on the large circle encompassing the Z_3 string axis, one can do the following. The needed 2π winding will be split in two parts (there are three possible options): $\pm 2\pi/3$ will come from U(1) while the remainder will come from rotations in the Cartan subgroup of SU(3) (in other words, rotations around the third and the eighth axes, with the generators $T^{3,8}$); for instance,

$$\begin{aligned}\Phi(r\to\infty,\alpha) &= v\,\mathrm{diag}(e^{i\alpha},1,1),\\ A_i(r\to\infty) &= \frac{\varepsilon_{ij}x^j}{r^2}\,\mathrm{diag}\left(-\frac{2}{3},\frac{1}{3},\frac{1}{3}\right),\end{aligned} \tag{4.1}$$

where i, $j = 1, 2$ are the directions perpendicular to the string axis and α is the polar angle in the 12 plane. The topological stability of the straight Z_3 strings is due to the fact that

$$\pi_1\left(\frac{\mathrm{SU}(3)_{\mathrm{CF}} \times \mathrm{U}(1)_{\mathrm{F}}}{(Z_3)_{\mathrm{CF}}}\right)$$

is nontrivial. There are three distinct Z_3 strings. Assembling all three such strings in one straight line, one obtains a string with triple tension topologically equivalent to the global U(1) string.

It is rather obvious that on each of the Z_3 strings the diagonal SU(3) symmetry is further broken down to $\mathrm{SU}(2) \times \mathrm{U}(1)$. Now, one can construct non-Abelian strings out of the Z_3 strings. To this end, one must rotate the given Z_3 solution inside the unbroken diagonal $\mathrm{SU}(3)_{\mathrm{CF}}$. This costs no energy; therefore orientational moduli associated with these rotations appear. Since the symmetry breaking pattern is

$$\mathrm{SU}(3)_{\mathrm{CF}} \to \mathrm{SU}(2) \times \mathrm{U}(1),$$

one has four moduli fields on the string world sheet, with the $CP(2)$ target space. We refer the reader to the reviews [10–13] for a detailed discussion.

The string solution involves the global U(1); hence, it contains a power tail from the uneaten Goldstone boson. This tail results in the logarithmic divergence of the string tension which is well familiar to the global strings explorers. We have already mentioned this circumstance above.

More specifically, following the general procedure (see [23] or the review paper [12]), we parametrize the solution for one of the Z_3 strings, say, that in (4.1), as follows:

$$\begin{aligned}\Phi(r,\alpha) &= \mathrm{diag}(e^{i\alpha}\phi_1,\phi_2,\phi_2),\\ A_i(r) &= \frac{\varepsilon_{ij}x^j}{r^2}(1-f)\mathrm{diag}\left(-\frac{2}{3},\frac{1}{3},\frac{1}{3}\right),\end{aligned} \tag{4.2}$$

where $\phi_1(r)$, $\phi_2(r)$, and $f(r)$ are scalar and gauge profile functions of the string, respectively. They satisfy the obvious boundary conditions

$$\phi_1(0) = 0, \qquad f(0) = 1, \tag{4.3}$$

at $r = 0$, and

$$\phi_1(\infty) = v, \qquad \phi_2(\infty) = v, \qquad f(\infty) = 0, \tag{4.4}$$

at $r = \infty$. Then the solution for the non-Abelian string can be written as [12,23]

$$\Phi = e^{i\alpha/3}\frac{1}{3}[2\phi_2 + \phi_1] + e^{i\alpha/3}(\phi_1 - \phi_2)\left(n\cdot\bar{n} - \frac{1}{3}\right),$$
$$A_i = \left(n\cdot\bar{n} - \frac{1}{3}\right)\varepsilon_{ij}\frac{x_j}{r^2}f(r), \tag{4.5}$$

where n^A ($A = 1, 2, 3$) are complex orientational moduli[5] of the string parameterizing the $CP(2)$ moduli space, $|n^A|^2 = 1$. The profile functions of the non-Abelian strings satisfy the second order equations of motion,

$$f'' - \frac{f'}{r} - \frac{g^2}{3}(1+2f)\phi_1^2 + \frac{g^2}{3}(1-f)\phi_2^2 = 0,$$
$$\phi_1'' + \frac{\phi_1'}{r} - \frac{1}{9}\frac{(1+2f)^2}{r^2}\phi_1 - \frac{1}{2}\frac{\partial V}{\partial\phi_1} = 0, \tag{4.6}$$
$$\phi_2'' + \frac{\phi_2'}{r} - \frac{1}{9}\frac{(1-f)^2}{r^2}\phi_2 - \frac{1}{4}\frac{\partial V}{\partial\phi_2} = 0,$$

where primes denote differentiations with respect to r. Here V is given by (3.5), and we note that $\partial V/\partial\phi_{1,2} \sim m_0^2(\phi_{1,2} - v)$ are rather small due to the smallness of m_0.

These equations were studied in [24]. A feature of the non-Abelian string in the CFL phase is the power falloff of the singlet scalar profile function at spatial infinity due to the presence of the corresponding massless Goldstone. From (4.6) we find that at $r\to\infty$ ($r \gg 1/m_0$) the profile functions behave as [24]

$$\phi_1 \sim \phi_2 \sim v\left(1 - \frac{1}{3m_1^2r^2} + \ldots\right),$$
$$(\phi_1 - \phi_2) \sim ve^{-m_0 r}, \tag{4.7}$$
$$f \sim e^{-m_0 r}.$$

In [24], solutions to Eq. (4.6) were found numerically. Here, to study these equations *analytically*, we will apply the method used for the Abelian strings in the extreme type I superconductors [25]. At distances $r \lesssim 1/m_0$, the string profile functions in fact have a two-scale structure due to smallness of the ratio of the scalar to gluon mass. At $1/m_g \ll r \ll 1/m_0$, the gluons can be considered as heavy, and we can neglect the gauge kinetic term. This boils down to dropping two first terms in the first equation in (4.6). Then, this equation becomes algebraic and yields

$$f \approx \frac{\phi_2^2 - \phi_1^2}{\phi_2^2 + 2\phi_1^2}. \tag{4.8}$$

Substituting this result in the two last equations in (4.6), we find the approximate solution in a simple form

$$\phi_1 \approx bv(m_0 r) + \ldots, \qquad \phi_2 \approx v[1 + O((m_0 r)^4)],$$
$$f \approx 1 - 3b^2(m_0 r)^2 + \ldots, \qquad 1/m_g \ll r \ll 1/m_0, \tag{4.9}$$

where the expansion goes in powers of $(m_0 r)^2$, while b is a number, $b \sim 1$. Here we use the fact that both the singlet and octet scalar masses are of the same order $\sim m_0$, much less then the gluon mass m_g; see (3.13) and (3.14).

Properties of the Z_3 strings in application to the CFL phase of dense QCD were also considered in [8,9,26–28].

V. ON THE STRING WORLD SHEET

The low-energy description of massless excitations on the string world sheet includes two decoupled sectors: the two translational moduli and four orientational. The translational moduli are described by the Nambu–Goto action with the constant T, which logarithmically diverges,

$$S_{\rm NG} = T_0\int d^4x\mathcal{L}_{\rm NG}, \qquad T_0 = 2\pi v^2\ln(Lm_0), \tag{5.1}$$

where L is a typical size of the color-flavor locked medium. This part is well known and will be of no concern to us here.

The orientational moduli's interaction is governed by $CP(N-1)$ (with $N = 3$ in the case at hand). In the non-supersymmetric setting, this model was shown to appear on the world sheet of a non-Abelian string in [23]. For non-Abelian strings in the model (3.4), the effective world-sheet theory was obtained in [29]. In the gauged formulation, the $CP(2)$ model takes the form

$$S_{CP(2)} = 2\beta\int dtdx_3\{3|\mathcal{D}_0n^A|^2 + |\mathcal{D}_3n^A|^2\}, \tag{5.2}$$

where β is the $CP(2)$ coupling constant, while the complex fields n^A ($A = 1, 2, 3$) are orientational moduli of the string promoted to world-sheet fields; see (4.5). They transform in the fundamental representation of SU(3). These fields are subject to the constraint

$$\bar{n}_An^A = 1. \tag{5.3}$$

The n fields have a U(1) charge which is gauged,

$$\mathcal{D}_\alpha n^A \equiv (\partial_\alpha - iA_\alpha)n^A. \tag{5.4}$$

The two-dimensional photon field A_μ has no kinetic term in the Lagrangian (5.2) and can be viewed as auxiliary,

$$A_\alpha = \frac{i}{2}(\bar{n}_A\overleftrightarrow{\partial}_\alpha n^A). \tag{5.5}$$

It does acquire a kinetic term in the solution of the model, however, which plays an important dynamical role.

The coupling constant β is determined by substituting the solution for the non-Abelian string (4.5) in the kinetic terms of the bulk action (3.4) and assuming that the moduli n^A has a slow adiabatic dependence on the the world-sheet coordinates t and x_3. We also use the following expressions for the A_0 and A_3 components of the gauge potential [12,23]:

$$A_\alpha = -i[\partial_\alpha n\cdot\bar{n} - n\cdot\partial_\alpha\bar{n} - 2n\cdot\bar{n}(\bar{n}\partial_\alpha n)]\rho(r), \qquad \alpha = 0, 3, \tag{5.6}$$

[5] A_i is a matrix; correspondingly, $n\cdot\bar{n}$ should be understood as $n^i\bar{n}_j$. The bar stands for the complex conjugation.

where we introduce a new profile function $\rho(r)$ with the boundary conditions

$$\rho(\infty) = 0, \qquad \rho(0) = 1. \tag{5.7}$$

The function $\rho(r)$ in Eq. (5.6) is determined through a minimization procedure [3,4,12,23] which generates ρ's own equation of motion.

This procedure leads us to the $CP(2)$ model (5.2), where coupling constant β is determined by the integral

$$\beta = \frac{2\pi}{g^2}\int_0^\infty rdr\bigg\{\bigg(\frac{d}{dr}\rho(r)\bigg)^2 + \frac{1}{r^2}f^2(1-\rho)^2 + g^2\bigg[\frac{\rho^2}{2}(\phi_1^2+\phi_2^2) + (1-\rho)(\phi_2-\phi_1)^2\bigg]\bigg\}. \tag{5.8}$$

Then the equation for the profile function ρ is

$$-\frac{d^2}{dr^2}\rho - \frac{1}{r}\frac{d}{dr}\rho - \frac{1}{r^2}f^2(1-\rho) + \frac{g^2}{2}(\phi_1^2+\phi_2^2)\rho - \frac{g^2}{2}(\phi_1-\phi_2)^2 = 0. \tag{5.9}$$

In Ref. [29] it was shown that, despite of the presence of the power falloff of the string solution, the coupling β is finite, i.e. the orientational modes of non-Abelian string are normalizable. The reason for this is that the only profile function with the power falloff is that of the singlet component of the scalar field [24], see also (4.7). On the other hand, the coupling β is associated with dynamics of the string orientational moduli and, therefore, is determined by the octet component of the scalar field and the gauge profile functions, which have the exponential falloff. In particular, at $r\to\infty$, the function ρ behaves as

$$\rho \sim e^{-2m_0 r}.$$

The main contribution to the integral in (5.8) comes from the region of intermediate r,

$$1/m_g \ll r \lesssim 1/m_0.$$

In this domain we can neglect, as previously, the kinetic term of the gauge field. This leaves us with only two last terms in Eq. (5.9). The equation becomes algebraic, and we can write

$$\rho \approx \frac{(\phi_2-\phi_1)^2}{\phi_1^2+\phi_2^2}. \tag{5.10}$$

Substituting here the expansions (4.7) and (4.9) we get an estimate for the coupling β,

$$\beta \approx \pi\int_0^\infty rdr\frac{(\phi_2^2-\phi_1^2)^2}{\phi_1^2+\phi_2^2} = c\frac{v^2}{m_0^2} \sim \frac{\mu^2}{T_c^2} \gg 1. \tag{5.11}$$

Unfortunately, we cannot calculate the constant c from the expansions (4.7) and (4.9); we only know that $c \sim 1$. Numerically, β was calculated in [29] for different values of the masses m_g, m_1 and m_8. On the other hand, our estimate has all virtues of the analytic expression.

We see that β is rather large. The reason for this is that the falloff of the string solution is controlled by the small scalar mass m_0.

In quantum theory, the coupling constant of the $CP(2)$ model runs. The $CP(N-1)$ models are asymptotically free and generate their own scale Λ_{CP}. The estimate (5.11) is classical and refers to the scale which determines the inverse thickness of the string [12] given by m_0. This is because the $CP(2)$ model (5.2) is an effective low-energy theory on the string world sheet. Its physical ultraviolet cutoff is given by the inverse thickness of the string. This implies

$$4\pi\beta(m_0) = N\ln\frac{m_0}{\Lambda_{CP}}, \qquad N=3, \tag{5.12}$$

an equation determining the scale Λ_{CP} of the effective world-sheet theory for the non-Abelian string. From (5.12) we get

$$\Lambda_{CP} = m_0\exp\bigg(-\frac{4\pi c}{N}\frac{v^2}{m_0^2}\bigg) \ll m_0, \qquad N=3. \tag{5.13}$$

We see that Λ_{CP} is exponentially small. Note that for the BPS-saturated strings in $\mathcal{N}=2$ supersymmetric QCD, the relevant parameter Λ_{CP} turns out to be equal to the scale Λ of the bulk theory. This feature is specific for $\mathcal{N}=2$ supersymmetry. In the CFL phase of dense QCD, such an equality does not hold. The reason is that we deal with the extreme type I superconductivity in the case at hand.

VI. KINK-ANTIKINK MESONS AT LARGE N

The $CP(N-1)$ model at large N was solved in [30,31] and the qualitative features of this solution are known to stay valid down to $N=3$ and even $N=2$. Below we will outline the features which are important for our purposes.

In the $CP(N-1)$ model, the genuine vacuum state is unique. However, there are of order N quasivacua [32] (local minima of the "potential"), which lie higher in energy than the genuine one. (Figure 1).

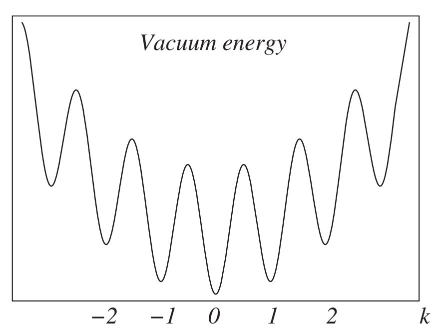

FIG. 1. The vacuum structure of the $CP(N-1)$ model with the vanishing vacuum angle. The genuine vacuum is labeled by $k=0$. All minima with $k\neq 0$ are quasivacua.

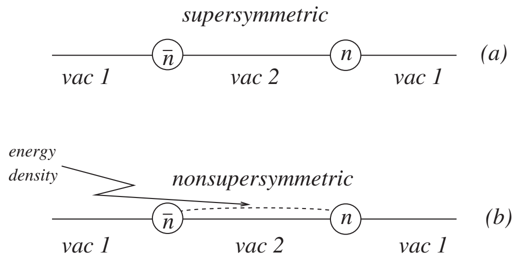

FIG. 2. A kink-antikink state in the $CP(N-1)$ model. In the nonsupersymmetric version, the vacuum 2 is a quasivacuum whose energy density is higher than that of the genuine vacuum 1 by $\sim\Lambda_{CP}/N$.

A family of quasivacua have energies

$$E_k \sim N\Lambda_{CP}^2\left\{1+\text{const}\left(\frac{2\pi k}{N}\right)^2\right\}, \qquad k=0\ldots,N-1. \tag{6.1}$$

The energy split between two neighboring (quasi)vacua is $O(1/N)$. In fact, the fields n and $\bar{n}$ represent kinks and antikinks interpolating between the genuine vacuum and its neighbors. Since their energies are nondegenerate, neither kinks nor antikinks can exist on the string in isolation. Only the kink-antikink pairs—mesons—are allowed.

A kink-antikink configuration on the flux tube is shown in Fig. 2. It is pretty obvious that the energy of this configuration linearly depends on the distance between n and $\bar{n}$, so that these kinks are confined along the string and form a meson.

We see that the splitting at large N is Λ_{CP}^2/N, while the mass of an individual kink is of the order of Λ_{CP},

$$m_{\text{kink}} \sim \Lambda_{CP}. \tag{6.2}$$

Thus, the distance between kinks in the meson is $\sim N/\Lambda_{CP}$, i.e. much larger than the size of the individual kink ($\sim 1/\Lambda_{CP}$). Hence, the kink and antikink are well separated inside the bound state, the meson. Since the n kinks are in the fundamental representation of the global $SU(N)_{CF}$ symmetry [31,33], the mesons can be of two types: $SU(N)_{CF}$ singlets or adjoints. Mesons with the quantum numbers of the adjoint representation cannot decay because the global $SU(N)_{CF}$ is unbroken and this global quantum number is conserved. Given that kink and antikink are well separated inside such a meson (provided the string is sufficiently long, which we assume, of course), it is clear that the kink-antikink mesons are in fact the lightest adjoint states and are *stable*. This conclusion is supported by the exact solution of the $CP(1)$ model [34] demonstrating stability of the kink-antikink mesons even at $N = 2$.

VII. CONFINED MONOPOLES IN THE CFL PHASE

What is the bulk interpretation of kinks interpolating between the neighboring (quasi)vacua of world-sheet $CP(N-1)$ model? This problem was studied in detail in supersymmetric gauge theories [2,4,5,12], and the answer is known.

The non-Abelian strings were first found in $\mathcal{N}=2$ supersymmetric QCD with the gauge group U(N) and $N_f = N$, where N_f is the number of the quark flavors. The scalar quarks (squarks) develop condensate of the color-flavor locked form, which leads to formation of the non-Abelian strings. $\mathcal{N}=2$ supersymmetric QCD have adjoint scalar fields along with the gauge fields form the $\mathcal{N}=2$ vector supermultiplet. These adjoint scalars develop VEVs as well, Higgsing the gauge U(N) group down to its maximal Abelian subgroup, which ensures existence of the conventional 't Hooft-Polyakov monopoles in the theory. The squark condensates then break the gauge $U(N)$ group completely, Higgsing all gauge bosons. Since the gauge group is fully Higgsed, the 't Hooft-Polyakov monopoles are *confined*. As we know, in the Higgsed U(N) gauge theories the magnetic monopoles show up only as junctions of two distinct elementary non-Abelian strings [4,5,35].

Now, let us verify that the confined magnetic monopole in the case at hand, CFL phase of dense QCD, is a junction of two strings seen in the two-dimensional $CP(N-1)$ model at $N=3$. Consider the junction of two Z_3 strings given by (4.5). Three distinct Z_3 strings mentioned above correspond to three choices of the orientation vector n^A,

$$n^A = (1,0,0), \qquad n^A = (0,1,0), \quad \text{and} \quad n^A = (0,0,1);$$

see (4.2). The magnetic flux of the junction of say, $n^A = (1,0,0)$ and $n^A = (0,1,0)$ strings is given by the difference of the fluxes of these two strings. Using (4.5), we get that the flux emanating from the junction is

$$4\pi \times \text{diag}\frac{1}{2}\{1,-1,0\}. \tag{7.1}$$

This is exactly the flux of the 't Hooft–Polyakov monopole with the magnetic charge

$$(n_3, n_8) = (1, 0), \tag{7.2}$$

where (n_3, n_8) are the magnetic charges with respect to T_3 and T_8 generators of $SU(3)_{\text{gauge}}$, respectively.

Similarly, two other string junctions, namely $n^A = (0,1,0)$, $n^A = (0,0,1)$ and $n^A = (0,0,1)$, $n^A = (1,0,0)$, have the magnetic fluxes equal to the fluxes of two other elementary monopoles in SU(3) (i.e. given by two other roots of SU(3) algebra), namely,

$$(n_3,n_8)=\left(-\frac{1}{2},\frac{\sqrt{3}}{2}\right), \quad (n_3,n_8)=\left(-\frac{1}{2},-\frac{\sqrt{3}}{2}\right). \tag{7.3}$$

How is this picture seen in the effective world-sheet theory on the non-Abelian string? For $\mathcal{N}=2$ supersymmetric bulk theory, the world sheet $CP(N-1)$ model is also supersymmetric and has N *degenerate* vacua. The elementary non-Abelian strings are in fact represented by N distinct vacuum states in the effective world-sheet

$CP(N-1)$ model, while the confined monopoles are the kinks interpolating between these distinct vacua [4,5,35].

Now, we can add a mass $m_{\rm adjoint}$ to the adjoint scalars, increase it eventually tending to infinity and decoupling these adjoint scalars from the bulk theory (this breaks supersymmetry in the bulk down to $\mathcal{N}=1$). What happens to the confined monopoles? The orientational moduli of the non-Abelian fluctuate, with their fluctuations becoming exceedingly stronger as $m_{\rm adjoint}$ grows. When $m_{\rm adjoint} \gg \Lambda$ strings' fluxes no longer have particular directions. Fluxes are smeared over the whole group space. Since the adjoint scalars are no longer present in the theory, naively it seems that there are no 't Hooft-Polyakov monopoles. At least they are not seen in the quasiclassical approximation.

In the world-sheet theory this corresponds to a highly quantum, strong coupling regime. However, we know that in this regime still there are N degenerate vacua in the world-sheet $CP(N-1)$ model [12,36,37]. Moreover, there are kinks interpolating between these vacua. They are stabilized by quantum effects and have a nonzero mass (of order Λ_{CP}) and finite-size (of the order of Λ_{CP}^{-1}). It is clear that these kinks correspond to confined monopoles of the bulk theory in the highly quantum regime. More exactly, we should say that they represent what *becomes* of the confined 't Hooft–Polyakov monopoles deep in the quantum non-Abelian regime when they cannot be seen in the quasiclassical approximation.

If we break supersymmetry in the bulk even further, down to nothing, the vacuum energies of the $CP(N-1)$ model split, as was explained in the previous section [23]. Kinks and antikinks are confined along the string and form kink-antikink mesons. The kink confinement in the two-dimensional CP model can be interpreted [23] as the following phenomenon: the non-Abelian monopoles, in addition to the four-dimensional confinement (which ensures that the monopoles are attached to the strings) acquire a two-dimensional confinement along the string: a monopole–antimonopole pair forms a mesonlike configuration, with necessity.

Moreover, as was shown in [31,33] for the $CP(N-1)$ model, the kinks belong to the fundamental representation of the global group $SU(N)_{CF}$. Therefore, the monopole-antimonopole mesons which belong to the adjoint representation with respect to this group are stable.

What lessons can we learn from this ''supersymmetry saga'' for dense QCD? We might think that there is a certain deformation of the GL model (3.4) which includes adjoint scalar fields. If these fields develop VEVs, the conventional 't Hooft-Polyakov monopoles are formed. If, in addition, the diquark condensate that develops in the color-flavor locked phase produces non-Abelian strings which confine these monopoles attaching them to the strings. These confined monopoles are seen as loosely bound kinks in the $CP(N-1)$ model on the string world sheet.

Now we give a mass to the adjoint scalars and eventually decouple them in the bulk theory, ending up with our GL model (3.4). No monopoles can now be found in the quasiclassical approximation. However, we are aware of their presence: they manifest themselves as confined monopoles seen as kinks on the non-Abelian strings.

Because of the quantum splitting of the string tensions (the vacuum energies in $CP(N-1)$ model split; see (6.1)), the magnetic monopoles cannot move freely along the string. Monopoles are bound with antimonopoles in the monopole-antimonopole mesons on the string. However, as long as the string is very long and the splitting is small, the monopole and antimonopole in the pair are well-separated inside the meson, and our conclusion of the presence of the confined monopoles in the CFL phase of QCD stays intact.

It is plausible to suggest that these monopoles become unconfined as we reduce μ and cross the phase transition line into the normal phase of QCD at smaller temperatures and chemical potentials. This is a pure speculation, of course, but, if so, they might condense in this phase, triggering the quark confinement.

VIII. TOWARDS A MORE REALISTIC SETTING

Now we will try to work out a more realistic setup, to make details of our analysis more closely related to actual dense QCD, with three flavors of unequal mass.

A. $N = 3$

In the real world, N is not so large, $N=3$. Therefore, strictly speaking, the large-N solution of the $CP(N-1)$ model [31] briefly reviewed in Sec. VI does not apply. However, as was mentioned in Sec. VI, the qualitative features of this solution are valid for not-so-large N such as $N=3$ and even $N=2$. Indeed, at $N=2$ the model was solved exactly [34]. The Zamolodchikovs found that the spectrum of the $O(3)$ model (equivalent to the $CP(1)$ model) consists of a triplet of degenerate states (with mass $\sim\Lambda_{CP}$). At $N=2$ the action (5.2) is built of doublets. In this sense, one can say that the Zamolodchikovs' solution exhibits confinement of doublets. This is in qualitative accord with the large-N solution [31].

In our case ($N=3$), we have two quasivacua, in addition to the lowest-energy state, the vacuum, and a triplet of kinks which, because of their linear attraction, are confined and form singlet and octet mesons on the string.

The question we address here is: given a state belonging to the adjoint representation of $SU(3)_{CF}$, can we say whether this state is a ''perturbative'' Higgsed gauge boson, or a kink-antikink meson? In other words, if the kink comes too close to antikink, can they annihilate into the perturbative state with the same SU(3) global charge?

Here the analogy with the supersymmetric setting is useful (although the string vacua are not split in supersymmetric $CP(N-1)$ models). In the quasiclassical regime, outside the so-called curves of the marginal stability (CMS), perturbative states are present in the spectrum of the $CP(N-1)$ model, while inside CMS, in the strong coupling domain, they just do not exist as stable states. They decay into the kink-antikink pairs [38–40]. In the nonsupersymmetric $CP(N-1)$ models we do not have that degree of control over the spectrum, while the curves of the marginal stability are replaced by the phase transition lines. However, qualitatively, the analogy with supersymmetric case is instructive. Because of string vacua splitting the kink and antikink cannot fly apart along the string. If N is large, we have well-separated monopole and antimonopole in such a meson, as was explained in Sec. VI. Even if N is not large, say, $N=2$, it is almost impossible to think that the Zamolodchikov triplet is anything other than a kink-antikink bound state. This is in line with the supersymmetric case which shows that there are no perturbative stable states inside CMS in the strong coupling domain.

Now, let us translate this two-dimensional picture in terms of strings and monopoles of the bulk theory. At large N, monopoles and antimonopoles are well-separated on the non-Abelian string at hand, and we can clearly identify these states in the theory. Moreover, even if N is not that large, the analogy with the supersymmetric set-up tells us that the adjoint string-attached mesons are most likely formed by monopole-antimonopole pairs. This is true as long as the splitting of non-Abelian strings is a quantum strong coupling effect determined by the $CP(N-1)$ scale Λ_{CP}, see (6.1).

B. Nonzero strange quark mass

Now we will move on towards reality in another direction, considering the effect of the nonvanishing strange quark mass m_s (we will continue to assume that the u and d-quarks are strictly massless),

$$m_u = m_d = 0, \qquad m_s \neq 0. \tag{8.1}$$

For small nonvanishing m_s, the GL potential (3.5) acquires the following correction [41]:

$$\delta V(\Phi) = \epsilon\{\Phi_u^\dagger \Phi^u + \Phi_d^\dagger \Phi^d\}, \tag{8.2}$$

which affects only quadratic terms in the Lagrangian, while the corrections to λ is negligible. Here the contraction of color indices is assumed, while the parameter ϵ is

$$\epsilon = \frac{48\pi^2}{7\zeta(3)}\frac{m_s^2}{4\mu^2}T_c^2 \ln\frac{\mu}{T_c}. \tag{8.3}$$

As a result, the VEVs of the diquark fields change, namely,

$$\langle\Phi\rangle = \mathrm{diag}(v_u, v_u, v_s), \tag{8.4}$$

where

$$v_u^2 = \frac{m_0^2 - 2\epsilon}{8\lambda}, \qquad v_s^2 = \frac{m_0^2 + 2\epsilon}{8\lambda}. \tag{8.5}$$

We see that the us and ds condensates are smaller than the ud condensate. The nonvanishing difference between v_s and $v_u = v_d$ breaks the residual global color-flavor symmetry,

$$\mathrm{SU}(3)_{\mathrm{CF}} \to \mathrm{SU}(2)_{\mathrm{CF}} \times \mathrm{U}(1). \tag{8.6}$$

Only the reduced color-flavor locked $SU(2)_{\mathrm{CF}}$ survived the perturbation.

It is worth mentioning that, when we switch on the strange quark mass perturbation in (8.2), we do not take into account the effect of the overall charge neutrality. The latter leads to shifts in the chemical potentials of u, d and s quarks proportional to the chemical potential of the electrons [41]. These shifts effectively further break $SU(2)_{\mathrm{CF}}$, because the electric charges of the u and d-quarks are different. For our purposes, it is all right to neglect this effect, since we do not address in this paper dynamical processes in the actual neutron stars.

Instead, we consider a *gedanken* QCD at large chemical potential μ, switching off electromagnetic interactions. No electrons are present in our system, just the quark-gluon matter. In this system, the nonvanishing mass of the strange quark does not break the $SU(2)_{\mathrm{CF}}$ global symmetry. This is an important difference between our results concerning the confined monopoles and those reported in [7];see below. The degree of relevance of this *gedanken* dense QCD, and consequences that ensue, to actual experiments on the quark-gluon plasma is to be investigated.

The split (8.5) breaks $\mathrm{SU}(3)_{\mathrm{CF}}$ and lifts some of the orientational modes of the non-Abelian string. This can be described by a small potential on the modular space of the string, i.e. a potential term in the model (5.2). In supersymmetric theories, the scalar potentials in the $CP(N-1)$ models on the string world sheet induced by the squark mass differences were calculated in [4,5]. At the same time, for dense QCD this problem was addressed in [7].

To calculate this potential to the leading order in the small parameter ϵ in (8.2), we can still use the solution (4.5), where the profile functions are unchanged. The only modification is that the common value of the scalar profile functions at $r \to \infty$ in (4.4) should be modified as

$$v \to \tilde{v}, \qquad \tilde{v}^2 = \frac{m_0^2 - \frac{2}{3}\epsilon}{8\lambda}, \tag{8.7}$$

which is the average value for the three VEVs in (8.4). The potential $(V + \delta V)$ gives the tension of the string $2\pi\tilde{v}^2 \ln(Lm_0)$ plus a finite (nonlogarithmic) contribution, which depends on the moduli fields n^A. It is given by

$$\frac{\epsilon}{3}\int d^4x\, \mathrm{Tr}[\Phi^\dagger \mathrm{diag}(1,1,-2)\Phi]. \tag{8.8}$$

Now, substituting here Eq. (4.5), we get the potential in the deformed $CP(2)$ model on the string world sheet, to the leading order in ϵ/m_0^2,

$$V_{CP} = \omega \int dt dx_3 (3n_3^2 - 1), \tag{8.9}$$

where

$$\omega = \frac{2\pi}{3}\epsilon \int_0^\infty r dr(\phi_2^2 - \phi_1^2) \sim \epsilon \frac{v^2}{m_0^2}. \tag{8.10}$$

Here we used the expansions (4.7) and (4.9) to make the last estimate.

From (8.9), it is clearly seen that, with $m_s \neq 0$, the (0, 0, 1) string has a significantly larger tension than the (1, 0, 0) and (0, 1, 0) strings and is, in fact, classically unstable. It is not even a local minimum of the potential (rather, it corresponds to a maximum). Note that the parameter ω is much larger than the quantum scale Λ_{CP} of the $CP(2)$ model, a crucial circumstance. Therefore, the classical splitting by far dominates over the quantum one in (6.1).

This instability means, in particular, that the monopole-antimonopole meson formed through the insertion of a piece of the (0, 0, 1) string in the (1, 0, 0) or (0, 1, 0) strings (see Fig. 2) is highly unstable and decays into a perturbative state with the same global (singlet or adjoint) quantum numbers with respect to the unbroken $SU(2)_{CF} \times U(1)$. This corresponds to the process in which monopoles with the magnetic charges given in (7.3) are annihilated with their antimonopole partners inside the monopole-antimonopole mesons. Correspondingly, the monopoles with these magnetic charges disappear from the string.

The potential in (8.9) shows that the n^3 field is heavy and can be integrated out from the $CP(2)$ model under consideration. Then we are left with the $CP(1)$ model (5.2) on the string world sheet, which includes now only the fields n^1 and n^2, and no potential on the target space. Its global group $SU(2)_{CF}$ remains unbroken.

In the quantum regime, two non-Abelian strings whose low-energy dynamics is described by this $CP(1)$ model (the $CP(1)$ vacua) are split, as was discussed in Sec. VI; see (6.1). There are mesons on the strings with the lowest tension which include pieces of the excited string. These are formed by monopoles and antimonopoles with magnetic charges classically given by (7.2). (Remember, in the quantum non-Abelian regime the magnetic monopole charge is averaged to zero.) Stable mesons are triplets with respect to unbroken $SU(2)_{CF}$. Thus, our conclusion on the presence of the confined non-Abelian monopoles attached to the non-Abelian strings in dense QCD stays *valid* even in a more realistic setting of dense QCD with $N = 3$ and nonvanishing strange quark mass.

To conclude this section, let us compare our results with those obtained in [7]. In [7], a realistic dense matter inside neutron stars was studied. In particular, the electromagnetic interactions and the presence of electrons were taken into account. As was mentioned above, this leads to the complete breaking of the non-Abelian color-flavor symmetry $SU(3)_{CF} \to U(1)^3$. All three strings are classically split by the strange quark mass. Two excited strings become classically unstable, and the monopoles effectively disappear from the string. They are annihilated by the would-be antimonopoles.

In our paper, we do not attempt to study realistic neutron stars. Instead, we focus just on the quark-gluon matter in dense QCD, and demonstrate that in the CFL phase there are confined magnetic monopoles attached to the non-Abelian strings. Whether or not the quark-gluon plasma can exist in terrestrial experiments sufficiently long allowing for the formation of long non-Abelian strings is a separate issue left for further studies.

C. Nonzero u and d-quark masses

Now let us introduce nonvanishing u and d quark masses,

$$m_u = m_d \ll m_s. \tag{8.11}$$

We stress that we assume that u and d quarks are strictly degenerate. This ensures that the color-flavor group (8.6) remains unbroken. The introduction of the common u and d quark mass just shifts the parameter m_0 (and λ) in the GL model (3.4), leaving our results intact.

Another effect due to $m_u = m_d \neq 0$ is that now "pions" become massive. Their masses were estimated (see e.g. the review [15]),

$$m_\pi \sim \sqrt{m_u m_s}\frac{T_c}{\mu}. \tag{8.12}$$

For reasonable values of the quark masses, the pion masses are rather small, $\sim m_{u,d}(6T_c/\mu)$. In particular, they are smaller than the Higgsed gluon mass constrained by the week coupling condition (3.15). Strictly speaking, this means that we cannot totally ignore pions in our GL effective description (3.4) of QCD in the CFL phase. They should be included in consideration of the low-energy theory.

The detailed study of the impact of these pions on the non-Abelian strings is left for a future work. Here we will make a few qualitative comments. First, they are "neutral" with respect to the gauge fields and, therefore, their presence in the bulk theory does not affect the classical solution (4.5) for the non-Abelian strings. However, they will show up in loops producing, generally speaking, long-range tails of string profile functions. Effectively, their presence forces the string to swell in the transverse dimension, acquiring the transverse size of the order of $1/m_\pi$. We assume, however, that

$$m_\pi \gg \Lambda_{CP}. \tag{8.13}$$

This constraint can be easily achieved since Λ_{CP} is quite small, see (5.13).

The condition (8.13) ensures that the $CP(1)$ model we arrived at still can be used for the low-energy description of dynamics of the orienational zero modes on the non-Abelian string. It means that the inverse transverse size of the string (although small) is still much larger than typical excitation energies on the world sheet, which are of order of Λ_{CP}. Higher-derivative corrections to the $CP(1)$ model (see (5.2) with $N = 2$) run in powers of the ratio of the typical excitation energies over m_π which can be considered small due to the condition (8.13). Of course, here we speak only about the rotational moduli fields which are responsible for nearly degenerate strings and kinks/monopoles.

To conclude this section, we stress again that, with $m_u = m_d$, the color-flavor locked SU(2) stays unbroken. Therefore, two non-Abelian strings described by $CP(1)$ model are in highly quantum regime which entails with necessity monopole-antimonopole pairs in the form of ‘‘mesons’’ attached to these strings.

IX. ON THE θ DEPENDENCE

In this section, we add the bulk θ term and trace its impact on the non-Abelian strings and monopoles of dense QCD.

A. θ term on the world sheet

In this section, we will make a few comments concerning possible effects due to the θ term and axions. In QCD all quarks are massive, hence the θ term effect cannot be eliminated by chiral rotations. We are interested in whether or not the θ term affects the world-sheet theory on the non-Abelian string in dense QCD. The nonsupersymmetric $CP(2)$ model allows one to introduce a θ term which, as usual, is coupled to the topological charge,

$$\mathcal{L}_\theta = \frac{\theta}{2\pi}\varepsilon_{\mu\nu}\partial^\mu A^\nu = \frac{\theta}{2\pi}\varepsilon_{\mu\nu}\partial^\mu(\bar{n}_i\partial^\nu n^i). \qquad (9.1)$$

Previously, in the simplest model [23], we demonstrated that the four-dimensional (bulk) θ term penetrates in the two-dimensional sigma model

$$\theta_{3+1} = \theta_{1+1}, \qquad (9.2)$$

with self-evident notation. The above equality between the four- and two-dimensional θ’s, however, is *not* a common property of all non-Abelian strings; see [42] for a counter-example. To find out what happens in the case at hand, we can substitute the gauge field from the string solution into the four-dimensional topological term and integrate over the transverse directions. As a result, we get that the equality (9.2) is fulfilled. There are nontrivial phenomena in the bulk theory at $\theta = \pi$ due to vacuum double degeneracy. On the world sheet as well, something remarkable occurs at $\theta = \pi$, namely, the *deconfinement* phenomenon. At this point the vacua on the world sheet become doubly degenerate too, and the single kink-monopole state becomes liberated and free to move along the string.

It is usually assumed that the θ dependence in dense QCD is negligible since the instanton-induced effects are exponentially suppressed due to Higgsing and large values of the diquark condensates. Our discussion implies that the non-Abelian string provides a nontrivial environment for the θ dependence to show up in full.

Care should be taken of the fermion modes on the string. If there were fermion zero modes in the world-sheet theory, the θ_{1+1} term could be eliminated from physics by chiral rotations, as happens in the supersymmetric version. If the bulk quarks are strictly massless, the fermion zero modes on the string do indeed exist [43]. In this limit, there is no θ dependence on the string. However, if nonzero quark masses are introduced, the fermion zero modes on the string are lifted and the fermions become irrelevant.

Let us comment on the interaction of the non-Abelian string in the CFL phase with the background fields. The simplest example of the bulk field coupled to the string is the axion. The string-axion interaction was considered in [44], where it was argued that a kind of axion halo emerges around the string, provided the string exists long enough for the halo to form. For shorter time intervals, the string with kinks on it acts as an antenna for the axion emission.

Note also that, due to the Witten effect, monopoles acquire the electric charge if the θ term is switched on. Hence the dyons emerge and the monopole-antimonole pair gets substituted by the dyon-antidyon state on the string world sheet.

B. A Holographic viewpoint

Remembering that a holographic representation in the (nonsupersymmetric) problem at hand can be exploited, if at all, only for a general guidance, let us have a closer look at the derivation of the world-sheet θ term via holography. Note that the very idea to look for such holographic realization of the string and confined monopoles is that it could help in interpretation of their hypothetical counterparts in different limits of the full QCD phase diagram.

To this end, we first comment on the known dual representation of the non-Abelian strings. In supersymmetric QCD with the Fayet–Iliopoulos term $\xi \neq 0$, the non-Abelian string is represented by a D2 brane stretched between two NS5 branes displaced by distance ξ in some internal coordinate [2]. Evidently, the tension of the string in the four-dimensional space-time is proportional to this distance. Similarly, the monopoles confined on the string are represented by the D2 branes with two internal world-volume coordinates [2].

Such non-Abelian strings as the D2 branes have a very clear-cut counterpart in holographic QCD with vanishing chemical potential [45,46]. At zero temperature, the dual geometry has the vanishing circle D2 brane that can wrap around, which yields a very small tension of the emerging magnetic string. Above the confinement-deconfinement phase transition, which corresponds to the Hawking–Page transition on the dual side, the magnetic string acquires a nonvanishing tension, since the relevant vanishing circle disappears.

From the consideration above, we have learned two well-established facts concerning non-Abelian strings in dense QCD: its tension is proportional to the diquark condensate squared and it carries a nontrivial θ term on the world sheet. These features have to be reproduced holographically. Since we deal with conventional (nonsupersymmetric) QCD, the best we can do is to work with the Sakai–Sugimoto model [47], which is a version of the black-hole background [48] and *qualitatively* seems to reproduce basic QCD phenomena. It involves, in the thermal case, two periodic coordinates x_5, t_E, radial coordinate U, and the internal S^4 manifold. Below the phase transition, the (x_5, U) coordinates provide the cigarlike geometry, while above T_c it is the (t_E, U) pair that yields this cigarlike geometry.

Let us try to argue that, within the Sakai–Sugimoto type models, the best candidate for the non-Abelian string is a wrapped D6 brane in the geometry of the charged black hole, much in the same spirit as in [42]. The black-hole charge corresponds to density in the holographic picture. The D2 realization cannot reproduce the correct string tension; hence a higher-dimensional D brane has to be involved. If there are no S^2 cycles in the background, there is no simple possibility to get the correct tension from the D4 brane. However, we cannot fully exclude the latter option with a more contrived background.

Consider the candidate D6 brane wrapped around S^4 and extended along x_5. The Chern–Simons term on its world-volume reads as

$$S_{\rm CS} = \int d^7 x C_0 \wedge F \wedge F \wedge F \tag{9.3}$$

and the integral over S^4 yields the factor N amounting to

$$S_{CS} = N \int dx_5 C_0 \int d^2 x F, \tag{9.4}$$

where $C_0 \propto \theta_{3+1}$

If we assume that the integration runs over the whole x_5 circle, we get a contradiction with the field-theoretical calculation because of the N factor. To avoid this contradiction, we assume that the integration runs over the segment of the x_5 circle between two flavor branes involved in the diquark condensate, which yields an additional $\frac{1}{N_f} = \frac{1}{N}$ factor. The tension of the string is given by the area of the corresponding disc, by the same token as in [42]. Hence we can qualitatively reproduce the required features. However, since there is no clear-cut holographic representation of the CFL phase of dense QCD in the Sakai–Sugimoto model yet, this D6 interpretation certainly calls for an additional study.

X. CONCLUSIONS

What has been achieved in this work? We started from the earlier observation of non-Abelian strings in the color-flavor locked phase of dense QCD below T_c. These strings develop orientational zero modes, which become dynamical fields of the $CP(2)$ model on the string world sheet. The above model supports kinks (antikinks) which are confined to the string, and, moreover, confined into kink-antikink bound states along the string, albeit the kink constituents are still identifiable.

The most nontrivial part of our further argument is as follows. We show that the kinks appearing in the world-sheet theory on these strings, in the form of the kink-antikink bound pairs, are, in fact, magnetic monopoles, as they manage to adapt and survive in such a peculiar form in dense QCD. The kinks of $CP(2)$ are the descendants of the 't Hooft–Polyakov monopoles—the latter appear in the quasiclassical regime while the former are the objects appearing in the highly quantum regime. Our considerations are heavily based on analogies and inspiration we abstracted from certain supersymmetric non-Abelian theories.

This is the first-ever analytic demonstration that the magnetic monopoles are native to non-Abelian Yang–Mills theories such as QCD (albeit our analysis extends only to the phase of the monopole confinement and has nothing to say about their condensation). Abundant speculations can be presented here, but we will refrain from them at this stage in the hope that a solid consideration allowing one to move towards the monopole condensation can be worked out later.

In conclusion, let us comment on possible signatures of the non-Abelian strings in the neutron stars. The baryon chemical potential in these stars depends on the location of the domain under consideration, and increases towards the star center. In other words, the CFL phase can be realized in a certain domain at a certain distance from the star center. The non-Abelian strings can be created via rotations of the neutron star [49].

The key question concerns specific detectable signals from the non-Abelian strings. Being created by some mechanism, the non-Abelian string could emit axions from inside the star. Another question which can be raised concerns the moving string. Assume that the string moves towards the boundary of the star from the domain inside the star with the CFL phase. At a certain distance from the center, the CFL phase becomes impossible, i.e. the non-Abelian string solution is no longer supported. This means that, at this point, the non-Abelian strings have to somehow join into the Abelian string excitations existing in the subsequent 2SC phase.

Finally, we would like to mention a very recent paper on non-Abelian strings in dense QCD [50] where interactions of string degrees of freedom with light bulk fields are studied.

ACKNOWLEDGMENTS

This work was completed during an extended visit of M. S. to the MIT Center for Theoretical Physics. M. S.

would like to thank his colleagues for hospitality. The work of M. S. was supported in part by DOE Grant No. DE-FG02-94ER408. The work of A. G. is supported in part by Grants No. RFBR-09-02-00308 and No. CRDF - RUP2-2961-MO-09. A. G. thanks IPhT at Saclay, where a part of the work was carried out, for hospitality and support. The work of A. Y. was supported by FTPI, University of Minnesota, by RFBR Grant No. 09-02-00457a and by Russian State Grant for Scientific Schools No. RSGSS-65751.2010.2.

---

[1] N. Seiberg and E. Witten, Nucl. Phys. **B426**, 19 (1994); **B430**, 485(E) (1994); **B431**, 484 (1994).
[2] A. Hanany and D. Tong, J. High Energy Phys. 07 (2003) 037.
[3] R. Auzzi, S. Bolognesi, J. Evslin, K. Konishi, and A. Yung, Nucl. Phys. **B673**, 187 (2003).
[4] M. Shifman and A. Yung, Phys. Rev. D **70**, 045004 (2004).
[5] A. Hanany and D. Tong, J. High Energy Phys. 04 (2004) 066.
[6] E. Nakano, M. Nitta, and T. Matsuura, Prog. Theor. Phys. Suppl. **174**, 254 (2008).
[7] M. Eto, M. Nitta, and N. Yamamoto, Phys. Rev. Lett. **104**, 161601 (2010).
[8] A. Gorsky and V. Mikhailov, Phys. Rev. D **76**, 105008 (2007).
[9] A. Yung (unpublished).
[10] D. Tong, arXiv:hep-th/0509216.
[11] M. Eto, Y. Isozumi, M. Nitta, K. Ohashi, and N. Sakai, J. Phys. A **39**, R315 (2006); K. Konishi, Lect. Notes Phys. **737**, 471 (2008).
[12] M. Shifman and A. Yung, Rev. Mod. Phys. **79**, 1139 (2007); an expanded version in *Supersymmetric Solitons* (Cambridge University Press, Cambridge, 2009).
[13] D. Tong, Ann. Phys. (N.Y.) **324**, 30 (2009).
[14] K. Rajagopal and F. Wilczek, in *At the Frontier of Particle Physics*, edited by M. Shifman (World Scientific, Singapore 2001), Vol. 3, p. 2061.
[15] M. G. Alford, A. Schmitt, K. Rajagopal, and T. Schäfer, Rev. Mod. Phys. **80**, 1455 (2008).
[16] M. Shifman and A. Yung, Phys. Rev. D **79**, 125012 (2009).
[17] M. G. Alford, K. Rajagopal and F. Wilczek, Phys. Lett. B **422**, 247 (1998); R. Rapp, T. Schäfer, E. V. Shuryak, and M. Velkovsky, Phys. Rev. Lett. **81**, 53 (1998); B. C. Barrois, Ph.D. thesis, CalTech UMI [Report No. 79-04847, 1979]; D. Bailin and A. Love, Phys. Rep. **107**, 325 (1984).
[18] M. G. Alford, K. Rajagopal, and F. Wilczek, Nucl. Phys. **B537**, 443 (1999).
[19] D. T. Son and M. A. Stephanov, Phys. Rev. D **61**, 074012 (2000); Phys. Rev. D **62**, 059902(E) (2000).
[20] I. Giannakis and H. c. Ren, Phys. Rev. D **65**, 054017 (2002); K. Iida and G. Baym, Phys. Rev. D **63**, 074018 (2001); **66**, 059903(E) (2002).
[21] K. Bardakci and M. B. Halpern, Phys. Rev. D **6**, 696 (1972).
[22] I. Giannakis and H.-c. Ren, Nucl. Phys. **B669**, 462 (2003).
[23] A. Gorsky, M. Shifman, and A. Yung, Phys. Rev. D **71**, 045010 (2005).
[24] M. Eto and M. Nitta, Phys. Rev. D **80**, 125007 (2009).
[25] A. Yung, Nucl. Phys. **B562**, 191 (1999).
[26] A. P. Balachandran, S. Digal, and T. Matsuura, Phys. Rev. D **73**, 074009 (2006).
[27] K. Iida and G. Baym, Phys. Rev. D **66**, 014015 (2002); K. Iida, *ibid.* **71**, 054011 (2005).
[28] M. M. Forbes and A. R. Zhitnitsky, Phys. Rev. D **65**, 085009 (2002).
[29] M. Eto, E. Nakano, and M. Nitta, Phys. Rev. D **80**, 125011 (2009).
[30] A. D'Adda, M. Lüscher, and P. Di Vecchia, Nucl. Phys. **B146**, 63 (1978); A. D'Adda, P. Di Vecchia, and M. Lüscher, Nucl. Phys. **B152**, 125 (1979).
[31] E. Witten, Nucl. Phys. **B149**, 285 (1979).
[32] E. Witten, Phys. Rev. Lett. **81**, 2862 (1998).
[33] K. Hori and C. Vafa, arXiv:hep-th/0002222.
[34] A. B. Zamolodchikov and A. B. Zamolodchikov, Ann. Phys. (N.Y.) **120**, 253 (1979).
[35] D. Tong, Phys. Rev. D **69**, 065003 (2004).
[36] A. Gorsky, M. Shifman, and A. Yung, Phys. Rev. D **75**, 065032 (2007).
[37] M. Shifman and A. Yung, Phys. Rev. D **77**, 125017 (2008).
[38] N. Dorey, J. High Energy Phys. 11 (1998) 005.
[39] A. Hanany and K. Hori, Nucl. Phys. **B513**, 119 (1998).
[40] M. Shifman and A. Yung, Phys. Rev. D **81**, 085009 (2010).
[41] K. Iida, T. Matsuura, M. Tachibana, and T. Hatsuda, Phys. Rev. Lett. **93**, 132001 (2004).
[42] R. Auzzi and S. Prem Kumar, J. High Energy Phys. 12 (2008) 077; Phys. Rev. Lett. **103**, 231601 (2009).
[43] S. Yasui, K. Itakura, and M. Nitta, Phys. Rev. D **81**, 105003 (2010).
[44] A. Gorsky, M. Shifman, and A. Yung, Phys. Rev. D **73**, 125011 (2006).
[45] A. Gorsky and V. Zakharov, Phys. Rev. D **77**, 045017 (2008).
[46] A. S. Gorsky, V. I. Zakharov, and A. R. Zhitnitsky, Phys. Rev. D **79**, 106003 (2009).
[47] T. Sakai and S. Sugimoto, Prog. Theor. Phys. **113**, 843 (2005).
[48] E. Witten, Adv. Theor. Math. Phys. **2**, 505 (1998).
[49] D. M. Sedrakian, D. Blaschke, K. M. Shahabasyan, and M. K. Shahabasyan, arXiv:0810.3003.
[50] Y. Hirono, T. Kanazawa, and M. Nitta, arXiv:1012.6042.