Dual instantons in antimembranes theory

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We introduce two ansatzs for the 3-form potential of Euclidean $11d$ supergravity on skew-whiffed $\mathrm{AdS}_4 \times S^7$ background which results in two scalar modes with $m^2 = -2$ on AdS_4. Being conformally coupled with a quartic interaction, it is possible to find the exact solutions of the scalar equation on this background. These modes turn out to be invariant under the $SU(4)$ subgroup of the $SO(8)$ isometry group, whereas there are no corresponding $SU(4)$ singlet Bogomol'nyi-Prasad-Sommerfeld operators of dimensions one or two on the boundary theory constructed by Aharony, Bergman, Jafferis, and Maldacena. Noticing the interchange of $\mathbf{8}_s$ and $\mathbf{8}_c$ representations under skew-whiffing in the bulk, we propose the theory of antimembranes should similarly be obtained from Aharony, Bergman, Jafferis, and Maldacena's theory by swapping these representations. In particular, this enables us to identify the dual boundary operators of the two scalar modes. We deform the boundary theory by the dual operators and examine the fermionic field equations, and compare the solutions of the deformed theory with those of the bulk.

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I. INTRODUCTION

Aharony, Bergman, Jafferis, and Maldacena (ABJM) have recently succeeded in constructing a Chern-Simons-matter theory, which describes the low energy dynamics of N $M2$-branes at the tip of the orbifold $\mathbf{C}_4/\mathbf{Z}_k$ [1]. This theory is conjectured to be dual to M-theory on $\mathrm{AdS}_4 \times S^7/\mathbf{Z}_k$, where k is the level of Chern-Simons on the gauge theory side. For large k ($k^5 \gg N$), the dual theory is better described in terms of type IIA string theory on $\mathrm{AdS}_4 \times CP^3$. In this note, though, we are interested in the $k = 1$ case where the boundary theory is conjectured to have an enhanced $\mathcal{N} = 8$ supersymmetry together with a global $SO(8)$ symmetry.

For the supergravity solution of $\mathrm{AdS}_4 \times S^7$, one can flip the sign of F_4 flux (skew-whiffed) and still get a solution which is maximally supersymmetric. On this background, we obtain two supergravity modes with a mass squared $m^2 = -2$ which could couple to operators of dimensions 1 or 2. Further, with our particular ansatzs these modes turn out to be singlet under $SU(4)$ subgroup of $SO(8)$ isometry group of S^7, and hence, according to the AdS/CFT duality, the dual Bogomol'nyi-Prasad-Sommerfeld (BPS) operators must also be invariant under $SU(4)$. However, in ABJM boundary theory there are no such BPS operators of dimension 1 or 2 which are $SU(4)$ invariant. A look at the supergravity side, though, provides a hint: in the skew-whiffed solution of the bulk, which corresponds to antimembranes, one needs to interchange 8_s and 8_c representations to get the right amount of supersymmetries upon compactification on the S^1 fiber [2]. Therefore, we propose the boundary theory of antimembranes should similarly be related to that of ABJM by swapping these representations. This change of representations, however, is only possible when $k = 1$ or $k = 2$, where the ABJM Lagrangian has an enhanced $\mathcal{N} = 8$ supersymmetry with $SO(8)$ global symmetry. The triality of $SO(8)$ then allows one to permute the three 8-dimensional representations, i.e., $\mathbf{8}_v$, $\mathbf{8}_c$, and $\mathbf{8}_s$, into one another and get three inequivalent Lagrangians. Note that from the standpoint of $SO(8)$, these three Lagrangians are completely equivalent, but they look different under $SU(4)$ decomposition.

In ABJM, scalars, fermions and supersymmetry charges are decomposed under $SU(4)$ as

$$\mathbf{8}_v = \mathbf{4} \oplus \bar{\mathbf{4}}, \qquad \mathbf{8}_c = \mathbf{4} \oplus \bar{\mathbf{4}}, \qquad \mathbf{8}_s = \mathbf{6} \oplus \mathbf{1} \oplus \mathbf{1}, \tag{1}$$

respectively. Hence, for antimembranes to conform with the supergravity side, fermions should decompose as

$$\mathbf{8}_s = \mathbf{6} \oplus \mathbf{1} \oplus \mathbf{1}; \tag{2}$$

then, triality of $SO(8)$ implies that for supercharges we should have

$$\mathbf{8}_c = \mathbf{4} \oplus \bar{\mathbf{4}} \tag{3}$$

while scalar decomposition is not changed.

Now, we can see how $SU(4)$ singlet BPS operators of dimension 2 can arise. First, note that rank two symmetric traceless operators of dimension one sit in **35** representation of $SO(8)$, which under $SU(4)$ decomposes as

$$\mathbf{35}_v = \mathbf{15} \oplus \mathbf{10} \oplus \mathbf{10}. \tag{4}$$

Let Y^A denote four scalar fields in $\mathbf{4}$ of $SU(4)$; then, the above decomposition corresponds to having the following BPS operators of dimension 1:

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$$O^A{}_B = \mathrm{tr}\Big(Y^A Y_B^\dagger - \frac{\delta^A{}_B}{4}(Y^C Y_C^\dagger)\Big),$$
$$O^{AB} = \mathrm{tr}(Y^A T^\dagger Y^B),$$
$$\bar{O}_{AB} = \mathrm{tr}(Y_A^\dagger T Y_B^\dagger),$$

where T's indicate monopole operators (for $k = 1$ and $k = 2$), which are needed to make gauge invariant operators out of two Y^A's [3]. Since for antimembranes, supercharges Q_A are in $\bar{\mathbf{4}}$ of $SU(4)$, we can get singlet scalar operators of dimension 2 by acting with supercharges twice on the above operators:

$$\mathcal{O}_1 = \{Q_A, [\bar{Q}^B, O^A{}_B]\},$$
$$\mathcal{O}_2 = \{Q_A, [Q_B, O^{AB}]\}, \tag{5}$$
$$\bar{\mathcal{O}}_2 = \{\bar{Q}^A, [\bar{Q}^B, \bar{O}_{AB}]\}.$$

These are, in fact, the three singlets in the decomposition of $\mathbf{35}_s$:

$$\mathbf{35}_s = \mathbf{20} \oplus \mathbf{6} \oplus \mathbf{6} \oplus \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}. \tag{6}$$

These three operators have the right symmetry properties to be identified with the three linearized supergravity scalar modes that we find in the bulk (note that $SU(4)$ singlet BPS operators of dimension one are still missing, and therefore, in the following, we are only considering operators of dimension 2). Moreover, neglecting the backreaction, we are able to solve the field equation of F_4 exactly. Examining the behavior of the solution near the boundary, we find that it satisfies a mixed boundary condition. A scalar of $m^2 = -2$ could couple to operators of dimensions 1 or 2. For a boundary operator with dimension 2, we have to choose the leading term as a source while the subleading term would correspond to the expectation value of that operator. The opposite is true for an operator of dimension one [4]. Following [5], we perturb the boundary theory by a dual operator corresponding to $\mathcal{O}_1$, which has no $U(1)_b$ charge. We observe that the new Lagrangian admits exact solutions with no monopole charge, and hence they are identified with bulk solutions which are invariant under $SU(4) \times U(1)$.

Another possibility is to decompose eight scalars as $\mathbf{6} \oplus \mathbf{1} \oplus \mathbf{1}$ with fermions and supercharges in $\mathbf{4} \oplus \bar{\mathbf{4}}$. This time, we obtain three scalar BPS operators of dimension one, which are invariant under $SU(4)$. In contrast to operators in (4), these operators will be primary. However, note that in the $11d$ supergravity the scalars in $\mathbf{8}_v$ must decompose as $\mathbf{4} \oplus \bar{\mathbf{4}}$ to get the 10-dimensional scalars on right representations upon compactification [2]. Further, the scalar modes that we find in the bulk are all coming from the 3-form potential and hence are pseudoscalars, whereas the above three scalar operators are real scalars. So, we conclude that this pattern of decomposition cannot be realized on antimembranes.

In Sec. II, we begin with 11 dimensional skew-whiffed background of $\mathrm{AdS}_4 \times S^7$. On this background, we provide two different ansatzs for the 3-form potential, reducing the 4-form field equation to a 4d scalar equation on AdS_4. As the scalar is conformally coupled with a quartic self-interaction, we are able to find its exact solutions. In Sec. III, we examine the behavior of this solution near the boundary. Taking the leading term as a source, we see that the dual operator has to have a dimension 2. We discuss how triality of $SO(8)$ allows us to rearrange the field representations in ABJM theory in order to get the antimembranes theory. We deform the Lagrangian with a multitrace operator and examine the fermionic field equations. We then obtain exact solutions when a fermionic field and the $U(1)$ gauge fields are turned on. Conclusions and outlook are brought in Sec. IV.

II. 11*d* SUPERGRAVITY IN SKEW-WHIFFED BACKGROUND

Let us start with the 11 dimensional supergravity action

$$S = \frac{-1}{2\kappa_{11}^2}\int d^{11}x\sqrt{g}R + \frac{1}{4\kappa_{11}^2}\int (F_4 \wedge *F_4) + \frac{i}{12\kappa_{11}^2}\int A_3 \wedge F_4 \wedge F_4, \tag{7}$$

where we have included a factor of i in the Chern-Simons term as we work in Euclidean space. For the equation of motion of A_3, we have

$$d * F_4 = -\frac{i}{2} F_4 \wedge F_4. \tag{8}$$

Let us take the background to be $\mathrm{AdS}_4 \times S^7/\mathbf{Z}_k$, with the metric

$$ds^2 = \frac{R^2}{4} ds^2_{\mathrm{AdS}_4} + R^2 ds^2_{S^7/\mathbf{Z_k}}, \tag{9}$$

where

$$ds^2_{S^7/\mathbf{Z_k}} = \frac{1}{k^2}(d\varphi + k\omega)^2 + ds^2_{CP^3}, \tag{10}$$

and ω is related to the Kähler form J of CP^3 through

$$J = d\omega. \tag{11}$$

This metric describes S^7 as a $U(1)$ bundle over CP^3. φ parametrizes the $U(1)$ circle with radius R/k, so for large k the radius of the circle is small and the effective description will be in terms of $10d$ type IIA supergravity. For the background 4-form flux we have

$$F_4 = \frac{3i}{8} R^3 \epsilon_4, \tag{12}$$

with an i factor in Euclidean space. The sign of 4-form background flux is important in getting conformally coupled scalars in 4 dimensions. In fact, the sign in (12) corresponds to skew-whiffed solutions in Minkowskian signature. To solve the field equations in this background, in the following, we consider two different ansatzs which reduce the 4-form field equation to a single scalar equation in four dimensions.[1] Being conformally invariant, we are able to solve the effective $4d$ equation exactly by neglecting the backreaction.

Amusingly, the solution will, in fact, have a zero $4d$ modified energy momentum tensor, and so it will not backreact on AdS_4 background [7,8]. However, it cannot be uplifted to an exact solution in 11 dimensions as the energy momentum tensor does have nonzero components along S^7. On the other hand, as long as one is only interested in the behavior of the solution near the boundary and the correlation functions of the dual operators, one can ignore the backreaction on the metric [9].

A. The first ansatz

To write our first ansatz, we note that there are gauge (with respect to the $U(1)$ gauge factor of CP^3) covariantly constant spinors θ, which could be used to construct complex charged 3-forms on CP^3 [10]:

$$K_{ijk} = \bar{\theta}\Gamma_{ijk}\theta \tag{13}$$

such that

$$dK = 4i\omega \wedge K. \tag{14}$$

Now, if we define

$$L = e^{4i\varphi/k}K \tag{15}$$

we have

$$dL = 4i\left(\frac{d\varphi}{k} + \omega\right) \wedge L = 4ie^7 \wedge L. \tag{16}$$

Further, the Hodge dual of L is

$$*_7 L = ie^7 \wedge L \tag{17}$$

so that, together with Eq. (16), this implies that

$$dL = 4 *_7 L. \tag{18}$$

The above properties of L allow for the appearance of identical terms on both sides of the equation of motion (8) so that we can reduce it to a 4-dimensional equation.

So to proceed, we take the following ansatz for F_4:

$$F_4 = N\epsilon_4 + R^9 d(f\Omega) = N\epsilon_4 + R^9 df \wedge \Omega + R^9 f d\Omega, \tag{19}$$

with

$$\Omega = L + \bar{L}, \tag{20}$$

N and f are scalar functions on AdS_4, so that $dF_4 = 0$. We have included an R^9 factor to account for the dimension 5 of L. For the Hodge dual, we obtain

$$*F_4 = R^3\left(\frac{8}{3}NJ^3 \wedge e^7 - \frac{R^9}{4} *_4 df \wedge *_7\Omega + \frac{R^9}{16} f\epsilon_4 \wedge *_7 d\Omega\right), \tag{21}$$

where $\epsilon_{12345\cdots 11} = \epsilon_{1234}\epsilon_{5\cdots 11}$, and, hence, the minus sign in the second term. Note that

$$*_7\,\Omega = ie^7 \wedge (L - \bar{L}), \qquad *_7 d\Omega = *(dL + d\bar{L}) = 4\Omega,$$

therefore, the equation of motion (8) reads

$$\begin{aligned}
&\frac{8}{3}dN \wedge J^3 \wedge e^7 - \frac{iR^9}{4} d *_4 df \wedge e^7 \wedge (L - \bar{L}) \\
&\quad + iR^9 f\epsilon_4 \wedge e^7 \wedge (L - \bar{L}) \\
&= -iR^6 N f\epsilon_4 \wedge d\Omega - iR^{15} f df \wedge \Omega \wedge d\Omega \\
&= 4R^6 N f\epsilon_4 \wedge e^7 \wedge (L - \bar{L}) + 8R^{15} f df \wedge e^7 \wedge L \wedge \bar{L},
\end{aligned} \tag{22}$$

where use has been made of

$$d *_7 \Omega = 0, \qquad d\Omega \wedge d\Omega = 0.$$

Let us normalize L so that

$$L \wedge \bar{L} = -\frac{i\lambda}{48R^{10}} J^3 \tag{23}$$

for some real dimensionless parameter λ. Recalling that L is a (3,0) form and $J_{\alpha\bar{\beta}} = ig_{\alpha\bar{\beta}}$, we can see that λ must be positive

$$\lambda > 0. \tag{24}$$

Using (23), the equation of motion splits to:

$$*_4\, d *_4 df - 4f = 16i\frac{N}{R^3} f, \qquad dN = -i\frac{\lambda}{32} R^5 df^2. \tag{25}$$

The last equation implies

$$N = -i\frac{\lambda}{32}R^5 f^2 + \frac{3i}{8}R^3; \tag{26}$$

here, we have chosen the constant of integration equal to the background field. Plugging this back into the first equation of (25), we have

$$* d * df + 2f - \frac{\lambda R^2}{2} f^3 = 0, \tag{27}$$

with $*$ indicating the Hodge dual on AdS_4 with a unit radius. Going back to an AdS_4 metric with radius $R/2$, we have

$$\frac{1}{\sqrt{g}}\partial_\mu(\sqrt{g}\partial^\mu f) + \frac{8}{R^2} f - 2\lambda f^3 = 0, \tag{28}$$

which we solve at the end of this section.

[1] Similar ansatzs have also been independently proposed in [6].

B. The second ansatz

For the second ansatz, let

$$F_4 = N\epsilon_4 + R^4 d(f\Omega) = N\epsilon_4 + R^4 df \wedge \Omega + R^4 f d\Omega, \quad (29)$$

where now

$$\Omega = e^7 \wedge J, \quad (30)$$

with

$$d\Omega = 2 *_7 \Omega. \quad (31)$$

Repeating steps (21) and (26), we get

$$* d * df + 2f - \frac{3R^2}{2} f^3 = 0, \quad (32)$$

or

$$\frac{1}{\sqrt{g}} \partial_\mu (\sqrt{g} \partial^\mu f) + \frac{8}{R^2} f - 6f^3 = 0, \quad (33)$$

for an AdS_4 metric with radius $R/2$. Note that in contrast with our previous ansatz, where we have an arbitrary "coupling constant" λ, here its value gets fixed to $\lambda = 3$. Note that, however, instead of (26), we have

$$N = \frac{-3i}{16} R^5 f^2 + \frac{3i}{8} R^3. \quad (34)$$

Although, the mass squared $m^2R^2 = -2$ is negative but it is above the lower bound, $-9/4$, for the stability. Further, it falls in the range

$$-9/4 < m^2R^2 < -5/4, \quad (35)$$

permitting a quantization with Dirichlet or Neumann boundary conditions, and hence coupling to operators of dimension 2 or 1, respectively [4,11].

Note that if we had chosen

$$N_0 = -\frac{3i}{8} R^3, \quad (36)$$

in (34) or (26) for the background field (corresponding to the Euclidean version of ABJM background), we would have obtained scalars of $m^2R^2 = 10$, which are not conformally coupled and hence, the $4d$ equations could not be solved exactly. We will comment on the dual operators of these modes in the next section.

To solve Eq. (27) or (32), we note that the mass term in this equation is such that it permits a conformal transformation to the flat space. To see this, let us write the metric of AdS_4 in Poincare coordinate

$$ds^2 = \frac{R^2}{4z^2}(dz^2 + \delta_{ij} dx^i dx^j), \qquad i, j = 1, 2, 3 \quad (37)$$

so that Eq. (28) reads

$$\frac{4z^4}{R^2} \partial_z \left(\frac{\partial_z f}{z^2} \right) + \frac{4z^2}{R^2} \partial_i \partial^i f + \frac{8}{R^2} f - 2\lambda f^3 = 0. \quad (38)$$

Now, let us make the following scaling transformation on f

$$f = \frac{2z}{R} g. \quad (39)$$

In terms of g, Eq. (38) simply reduces to the equation of a massless scalar (with the same cubic term) on flat space:

$$\partial_z \partial_z g + \delta_{ij} \partial^i \partial^j g - 2\lambda g^3 = 0, \quad (40)$$

which is easily solved for

$$g = \frac{2}{\sqrt{\lambda}} \left(\frac{b}{-b^2 + (z + a)^2 + (\vec{x} - \vec{x}_0)^2} \right), \quad (41)$$

with b a free parameter of the solution. We need to choose $a^2 > b^2$ with $a > 0$ to have a smooth solution. So, for f we have

$$f = \frac{4}{R\sqrt{\lambda}} \left(\frac{bz}{-b^2 + (z + a)^2 + (\vec{x} - \vec{x}_0)^2} \right). \quad (42)$$

Similar equations to (27) and (32) have appeared in [7,8,12]. As we noticed above, these 4-dimensional equations are *conformally coupled* in the sense that the action from which the equations are derived is conformally invariant.

It is also possible to make a comparison between our ansatzs and the more general ansatzs that have been independently proposed in [6]. First, note that as we have ignored the backreaction on the metric, we should set $U = V = 0$ in Eq. (2.4) of that paper. Further, if we set $H_3 = H_2 = A_1 = h = 0$ in Eq. (2.5) of that paper and define $f \equiv \chi + \bar{\chi}$, we get our first ansatz (19). On the other hand, our second ansatz in (29) is obtained by setting $H_3 = H_2 = A_1 = \chi = 0$ with $f \equiv h$ in Eq. (2.5). For the skew-whiffed solution we see that Eq. (B.14) (with $U = V = \chi = 0$) of that paper reduces to

$$f = 6(-1 + h^2); \quad (43)$$

plugging this back into Eq. (B.11) (with $H_3 = H_2 = A_1 = 0$), we get

$$d * dh + 4h(f + 4)\mathrm{vol}_4 = 0, \quad (44)$$

or

$$* d * dh + 8h - 24h^3 = 0, \quad (45)$$

which upon a scaling gives the conformally coupled scalar equation in (33).

III. DUAL OPERATORS

We would now like to look at the behavior of the solution near the boundary. As we will be discussing the second ansatz, let us set $\lambda = 3$, and $R = 1$ for convenience. When $z \to 0$ for solution (42), we have

$$f(z, x) \to \alpha(x) z + \beta(x) z^2, \quad (46)$$

where

$$\alpha(x) = \frac{4}{\sqrt{3}}\left(\frac{b}{-b^2 + a^2 + (\vec{x} - \vec{x}_0)^2}\right) \tag{47}$$

and

$$\beta(x) = \frac{-16ab}{\sqrt{3}(-b^2 + a^2 + (\vec{x} - \vec{x}_0)^2)^2} \tag{48}$$

are taken to be the source and the expectation value of the dual boundary operator corresponding to the solution (42) in the bulk. As mentioned in the Introduction, since we are taking the leading term in (46) as a source, the dual operator will have dimension 2. On the other hand, for a bulk mode with $m^2 = -2$, it is also possible to take the second term as a source, and thus the dual operator would have dimension one. In the following, we consider the first possibility, as there is no $SU(4)$ invariant BPS operator of dimension one in antimembranes boundary theory.

We will assume $b < 0$, and note that α and β are related through

$$\alpha(x) = -\eta\beta(x)^{1/2}, \tag{49}$$

where we have defined

$$\eta = \sqrt{\frac{-b}{a\sqrt{3}}}; \tag{50}$$

therefore, the solution satisfies a mixed boundary condition. Following the prescription in [5], the boundary condition (49) corresponds to deforming the boundary theory by

$$W = -\frac{2\eta}{3}\int d^3x \mathcal{O}_1(x)^{3/2}. \tag{51}$$

This is so because $\langle\mathcal{O}_1(x)\rangle = \beta(x)$; hence, (49) can formally be written as

$$\alpha(x) = \frac{\delta W}{\delta\beta}. \tag{52}$$

From (51) we see that η is the deformation parameter of the boundary theory, so that different values of η define different boundary deformations. Hence, for a fix value of η, we are left with four moduli parameters for solution (42). As we will see next, this matches the moduli parameters of the solution in the field theory side.

What are the corresponding dual boundary operators to the solution (42)? To answer this question, let us consider S^7 as a $U(1)$ bundle over CP^3. Now we note that in the first ansatz, since spinors θ are covariantly constant, they are invariant under $SU(4)$ isometry group of CP^3 but are charged under $U(1)$. This property is inherited by L [10], and thus from our ansatz for A_3 we conclude that the dual operator should be invariant under $SU(4)$ global symmetry of the field theory. However, as the $U(1)$ isometry of the bulk is identified with the baryonic number symmetry $U(1)_b$, the dual operator is charged under $U(1)_b$ and so it comes with a monopole operator. For the second ansatz, we note that J, the Kähler form, is invariant under $SU(4)$. It can also be shown that e^7 is invariant under this group. Further, J and e^7 carry no charge under $U(1)$ so the whole ansatz is invariant under $SU(4) \times U(1)$. Therefore, the dual operator on the boundary must be a singlet under $SU(4) \times U(1)_b$. As already noted, the scaling behavior of the bulk solution near the boundary indicates that these operators must have dimension one or two.

Let us see if we can identify such operators in the ABJM boundary theory. In this model, a Chern-Simons-matter theory describes the boundary dynamics, where scalars $X^I = (Y^A, Y_A^\dagger)$ transform as

$$\mathbf{8}_v = \mathbf{4} \oplus \bar{\mathbf{4}} \tag{53}$$

under $SU(4)$ R-symmetry. However, the only singlet scalar operator of dimension 1 that one can construct is the Konishi operator, i.e. $\mathrm{tr}(Y^A Y_A^\dagger)$, which as we know is not a BPS operator. Moreover, the Konishi operator is invariant under the whole $SO(8)$ symmetry group, whereas our ansatz in the bulk is only invariant under $SU(4)$ subgroup. All these indicate that the dual operator cannot be the Konishi operator. For dual operators of dimension two, we note that the fermionic fields are in $\mathbf{8}_c = \mathbf{4} \oplus \bar{\mathbf{4}}$, hence the second descendant of $\mathrm{tr}(X^{\{I}X^{J\}})$ (of dimension 2) contains $\mathbf{35}_c = \mathbf{15} \oplus \mathbf{10} \oplus \mathbf{10}$ with no $SU(4)$ singlet.

Note that had we chosen the background field as in ABJM, i.e., (36), with the above ansatz, we obtain scalars of $m^2 = 10$ (instead of getting $m^2 = -2$ for skew-whiffed), which are singlets under $SU(4)$. These modes, however, can be recognized as the sixth descendant of $\mathrm{tr}(X^{\{I}X^J X^K X^{L\}})$. They sit in $\mathbf{35}_s$ as indicated in Table 1 of [13], and this representation—in the sixth descendant—contains three $SU(4)$ singlets of $\Delta = 5$, which we identify with the three scalars with $m^2 = 10$ in the ABJM background (36). However, if we assume the low energy dynamics of antimembranes is also given by the ABJM (Chern-Simons-matter) theory, we run into problems in identifying the dual operators.

Recall that our solution had a flipped sign of F_4, and hence we should really discuss the theory of antimembranes on the boundary. It has been observed that the spectrum of the Kaluza-Klein modes of the bulk supergravity of the two theories are related by interchanging the two representations $\mathbf{8}_s$ and $\mathbf{8}_c$ of $SO(8)$ [2,14]. In ABJM theory, supercharges and fermionic matter fields sit in $\mathbf{8}_s$ and $\mathbf{8}_c$, respectively. Therefore, for antimembranes boundary theory we propose fermions to sit in $\mathbf{8}_s$, which under $SU(4)$ decompose as

$$\mathbf{8}_s = \mathbf{6} \oplus \mathbf{1} \oplus \mathbf{1}. \tag{54}$$

Triality of $SO(8)$ then implies that the supercharges and scalars should decompose as

$$\mathbf{8} = \mathbf{4} \oplus \bar{\mathbf{4}}. \tag{55}$$

What is the field theory describing the low energy dynamics of antimembranes? First, note that the skew-whiffed bulk solution preserves supersymmetry only when $k = 1$, i.e., when we have an $AdS_4 \times S^7$ [14] for which the boundary theory should have an $\mathcal{N} = 8$ supersymmetry. On the other hand, ABJM theory has been conjectured to have an enhanced $\mathcal{N} = 8$ supersymmetry for $k = 1$ and $k = 2$ [1]. This happens because of the existence of appropriate monopole operators, which allow to impose reality conditions on dynamical fields. The enhanced $\mathcal{N} = 8$ supersymmetry of ABJM theory for $k = 1$ and $k = 2$ has further been investigated in [15–17]. In particular, it has been shown that in these cases the action has a manifest $SO(8)$ symmetry. One can thus arrange the scalars, fermions, and the supercharges in three $SO(8)$ representations of $\mathbf{8}_v$, $\mathbf{8}_c$, and $\mathbf{8}_s$, respectively.

The antimembranes action (for $k = 1$) can therefore be obtained from ABJM action by a parity transformation and by swapping $\mathbf{8}_s$ and $\mathbf{8}_c$ representations of $SO(8)$ by triality. The explicit form of the action can be worked out as in [15], though we do not need it as we will turn on only one $SU(4)$ singlet spinor and the gauge fields. Setting Y^A's to zero, all the interaction terms (including the fermionic ones) disappear so that we are left with the kinetic term of the spinor field and the Chern-Simons term. On the other hand, since we have chosen α in (46) to act as a source for the operator $\mathcal{O}_1$ (of dimension 2), turning on the scalar mode in the bulk corresponds to adding W term (51) to the boundary action:

$$S \to S + W. \tag{56}$$

Let us call ψ the singlet spinor field in (54) and see if the new action admits nontrivial classical solutions when we turn on only the ψ field. By looking at the field equations, however, it is seen that we need also turn on the gauge fields. In this case, the $SU(4)$ singlet operator is

$$\mathcal{O}_1 = \mathrm{tr}(\bar{\psi}\psi), \tag{57}$$

and hence the deformed action, with only ψ and the gauge fields turned on, becomes

$$S = \int d^3x \mathrm{tr}\Big(i\bar{\psi}\not{D}\psi - \frac{ik}{4\pi}\epsilon^{\mu\nu\rho}\Big(A_\mu\partial_\nu A_\rho + \frac{2i}{3}A_\mu A_\nu A_\rho - \hat{A}_\mu\partial_\nu\hat{A}_\rho - \frac{2i}{3}\hat{A}_\mu\hat{A}_\nu\hat{A}_\rho\Big)\Big) - \frac{2\eta}{3}\int d^3x(\mathrm{tr}(\bar{\psi}\psi))^{3/2}, \tag{58}$$

where we have included an extra i factor in front of the Chern-Simons term because of Euclidean signature. The field equations then read

$$\begin{aligned} &i\not{D}\psi - \eta(\mathrm{tr}(\bar{\psi}\psi))^{1/2}\psi = 0,\\ &\frac{ik}{4\pi}\epsilon^{\mu\nu\rho}F_{\nu\rho} + \bar{\psi}\gamma^\mu\psi = 0,\\ &\frac{ik}{4\pi}\epsilon^{\mu\nu\rho}\hat{F}_{\nu\rho} + \bar{\psi}\gamma^\mu\psi = 0. \end{aligned} \tag{59}$$

To find a solution, we simply set

$$\psi^a_{\hat{a}} = \frac{\delta^a_{\hat{a}}}{N}\psi, \tag{60}$$

which is then equivalent to looking at the two $U(1)$ sectors of the gauge group. Now we can set the $SU(N)$ gauge fields to zero. For the $U(1)$ part, let $A^\pm_\mu = A_\mu \pm \hat{A}_\mu$; then we have

$$\frac{ik}{4\pi}\epsilon^{\mu\nu\rho}F^+_{\nu\rho} = -2\bar{\psi}\gamma^\mu\psi, \qquad F^-_{\nu\rho} = 0. \tag{61}$$

So, we can further set $A^-_\mu = 0$. However, note that the matter field only couples to A^-_μ, so setting $A^- = 0$, we are left with a self-interacting spinor field in the first equation of (59). As the solutions we found in the bulk are nonsingular and spherically symmetric near the boundary, to solve the ψ equation we make the following ansatz, which is similarly nonsingular and rotationally symmetric:

$$\psi = \frac{(c + i(x - x_0)^\mu\sigma_\mu)}{(c^2 + (\vec{x} - \vec{x}_0)^2)^\gamma}\xi, \tag{62}$$

with c a free constant and ξ an arbitrary constant spinor. This ansatz has been proposed earlier in solving the Seiberg-Witten equations on $\mathbf{R}^3$ [18], and, interestingly, the solution we obtain will be identical to theirs up to a constant. Plugging this ansatz into the field equation of ψ, the normalization constant and γ get fixed:

$$\psi = \frac{3c\sqrt{N}}{\eta}\frac{(c + i(x - x_0)^\mu\sigma_\mu)}{(c^2 + (\vec{x} - \vec{x}_0)^2)^{3/2}}\begin{pmatrix}1\\0\end{pmatrix}. \tag{63}$$

Further, let us compute the action of the above solution (with $A^- = 0$):

$$S = \int d^3x\Big[\mathrm{tr}(i\bar{\psi}\not{\partial}\psi) - \frac{2\eta}{3}(\mathrm{tr}(\bar{\psi}\psi))^{3/2}\Big]. \tag{64}$$

Using the field equations and plugging (63) into the action, we obtain

$$S = \frac{\eta}{3}\int d^3x(\mathrm{tr}(\bar{\psi}\psi))^{3/2} = \frac{9c^3}{\eta^2}\int\frac{d^3x}{(c^2 + (\vec{x} - \vec{x}_0)^2)^3} = \frac{9\pi^2}{4\eta^2}. \tag{65}$$

Having a finite action, solution (63) thus represents an instanton of the deformed boundary theory. For the gauge field A^+_μ, we plug solution (63) into Eq. (61) and take the integral

$$\oint_s F^+ = \oint_s \epsilon^{\mu\nu\rho}F^+_{\nu\rho}ds_\mu = 0, \tag{66}$$

with s a round sphere at infinity. Therefore, this solution has no net magnetic charge, and we can consistently identify it with the solution in the bulk which is invariant under $U(1)$ isometry group ($SU(4) \times U(1) \subset SO(8)$).

One can also examine the correlation functions of ψ's in instanton background (63). In particular, we can obtain the dominant contribution to the expectation value of $\mathcal{O} = \mathrm{tr}(\bar{\psi}\psi)$ by evaluating it in this background

$$\mathrm{tr}\,(\bar{\psi}\psi) = \frac{9c^2}{\eta^2(c^2 + (\vec{x} - \vec{x}_0)^2)^2}. \quad (67)$$

Moreover, if we set $c^2 = a^2 - b^2$, this will be proportional to (48), the expectation value we obtained in the bulk by analyzing the behavior of solution (42) near the boundary. So, as expected, the field theory analysis is consistent with the bulk computations. Also, note that the moduli parameters of the bulk and boundary solutions match.

IV. CONCLUSIONS AND OUTLOOK

In this paper, we provided two ansatzs which reduced the $11d$ supergravity field equations on the background of skew-whiffed $\mathrm{AdS}_4 \times S^7$ to a 4d conformally coupled scalar equation. We found the exact solutions and examined their behavior near the boundary. The scalar modes turned out to be singlets under $SU(4)$ subgroup of $SO(8)$ with $m^2 = -2$. Our main task was to find the dual operators to these modes. We argued that there are no BPS operators in the boundary ABJM theory whose quantum numbers could match with those of bulk scalars. Therefore, we inferred that the boundary theory of antimembranes cannot be identical with the theory of membranes. A crucial hint came from the skew-whiffed bulk theory, where one has to swap the s and c representations for consistency. Hence, we proposed that the theory of antimembranes should analogously be obtained from ABJM by interchanging the s and c representations. Doing so, we were able to identify the BPS operators corresponding to the scalar modes that we found in the bulk. On the field theory side, we deformed the action by the dual operator and found an exact classical solution which we identified with the bulk solution invariant under $U(1) \times SU(4)$. Apart from this, there are two more operators (being complex conjugate of each other) that are invariant under $SU(4)$ but carry $U(1)$ charge, and so contain monopole operators. These correspond to bulk solutions that we found in our first ansatz.

Our analysis of antimembranes theory provides a realization of the boundary toy model discussed in [7,8]. Therefore, it is interesting to study the instability of the bulk vacuum through the instantons discussed above. Similarly, the instabilities of the supergravity solutions that have recently been studied in [19] should be understandable in this framework.

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