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Stückelberg mechanisms for tensor multiplets and compactification on $AdS_3 \times S^3$

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We present Stückelberg mechanisms for tensor multiplets coupled to supergravity in four dimensions (4D), six dimensions (6D), and three dimensions (3D). For $N = 1$ supergravity in 4D, our field content is $(e_\mu{}^m, \psi_\mu)$, $(B_{\mu\nu}, \chi, \varphi)$ and (A_μ, λ), respectively, for the supergravity, tensor, and vector multiplets. In our Stückelberg mechanism, the Abelian vector field A_μ is absorbed into the longitudinal component of the tensor $B_{\mu\nu}$, which becomes massive. The field strength $F = dA$ of A is replaced by $\mathcal{F} \equiv F + mB$, where m is a coupling constant with the dimension of mass. In 6D, we utilize the so-called *dual* version for $N = 2$ supergravity, in order to avoid the obstruction caused by the Chern-Simons term $F \wedge A$ in the B-field strength G. Instead of the $F \wedge A$-term in G, the 6D Lagrangian has a peculiar topological and cubic interaction term proportional to $m^{-1}\mathcal{F} \wedge \mathcal{F} \wedge \mathcal{F}$. In 3D, we also show that a similar mechanism works for $N = 1$ supergravity. Interestingly, the basic structure is parallel to the 4D case, except that the originally nonpropagating field B starts propagating, after absorbing the A-field. We also show a possible compactification of 6D theory on $AdS_3 \times S^3$.

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I. INTRODUCTION

In supergravity theories associated with superstring [1], supermembrane [2], M-theory [3], or D-branes [4], the so-called axion field $B_{\mu\nu}$ and dilaton field φ arise as massless fields in the Neveu-Schwarz sector, or in compactifications of higher-rank flux fields from higher dimensions, such as ten (10D) or 11 (11D) dimensions. These fields are also called moduli, and they are problematic for realistic phenomenological model building in 4D.

In our previous papers [5,6], we have presented supersymmetric Stückelberg mechanisms [7,8] that can absorb the problematic massless fields $B_{\mu\nu}$ and φ, respectively, into the tensor field $C_{\mu\nu\rho}$ and the vector A_μ, making the tensor field and vector massive. Upon this mechanism, $B_{\mu\nu}$ and φ are absorbed into the longitudinal components of $C_{\mu\nu\rho}$ and A_μ, and they completely disappear from the low-energy spectrum, as desired for low-energy models.

In the present paper, we present *alternative* or *reversed* Stückelberg mechanisms, in which a tensor $B_{\mu\nu}$ absorbs a vector field A_μ, and becomes massive. Its mass m can be as heavy as desired, so that it can again disappear from the low-energy particle spectrum. The original Abelian field strength $F = dA$ of A is replaced by $\mathcal{F} \equiv F + mB$, where m is the coupling constant with the dimension of mass. In other words, the field strength F will be completely gauged away by the tensorial transformation $\delta_\Lambda B = d\Lambda$ of the B-field. Effectively, we can replace $\mathcal{F}$ by mB everywhere in the system. For example, the kinetic term $-(1/4)(\mathcal{F}_{\mu\nu})^2$ of A will play the role of the mass term of B. Because of the consistency under supersymmetry, the partner fermionic field χ will be also massive, forming a Dirac field with the gaugino λ.

Actually, such a supersymmetric Stückelberg mechanism is not new at all, and it has been practiced in supergravity in higher dimensions. The most typical example is the massive type IIA theory in 10D [9], where the original field strength $F = dA$ of A is replaced by $\mathcal{F} \equiv F + mB$, and the vector field A_μ is absorbed into $B_{\mu\nu}$.

It then seems straightforward to practice similar mechanisms in supergravity in lower dimensions. However, there are obstructions against such an idea. The most serious one is the common feature that the field strength G of the tensor B contains the Chern-Simons (CS) term as $G = dB + F \wedge A$. This is closely related to the universal exponential couplings of the dilaton field φ to bosonic fields, yielding the kinetic terms such as $e^{c\varphi}(F_{\mu\nu})^2$. The reason is that this term generates the variation of the type $\bar\epsilon\chi F^2$, which in turn necessitates the Lagrangian term $\bar\chi\lambda F$. Such a term then generates a variation $\bar\epsilon\lambda F G$, which necessitates the CS term $F \wedge A$ in G in the B-kinetic term.

The obstruction caused by the $F \wedge A$-term in G is that the *bare* field A is involved in the Lagrangian, so that the Stückelberg mechanism does *not* work here. This is because the simple replacement of F by $\mathcal{F}$ does not work for the *bare* A-term in the $F \wedge A$-term. In a system with G *without* the $F \wedge A$-term, there arises *no* such obstruction. For example, in the case of type IIA supergravity [9], the original field strength $G \equiv dB$ had *no* CS term $F \wedge A$ with *no* obstruction against the Stückelberg mechanism for *massive* B. Our desirable supergravity system, therefore, should have the field strength G *without* any $F \wedge A$-type CS term.

Fortunately, such desirable systems do exist in 4D, 6D, and also 3D. In 4D, we presented in our previous paper [6] a Lagrangian for supergravity $(e_\mu{}^m, \psi_\mu)$, tensor

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$(B_{\mu\nu}, \chi, \varphi)$ [10],[1] and vector (A_μ, λ) multiplets, such that the field strength G has *no* $F \wedge A$ term.

In 6D, one way to avoid the $F \wedge A$-term in G is to use the *dual* version [11] for Lagrangian formulation. We use the supergravity multiplet $(e_\mu{}^m, \psi_\mu{}^A, B_{\mu\nu}^{(+)})$,[2] a tensor multiplet $(B_{\mu\nu}^{(-)}, \chi^A, \varphi)$ [12], and a vector multiplet (A_μ, λ^A). The tensors $B_{\mu\nu}^{(+)}$ and $B_{\mu\nu}^{(-)}$ are, respectively, self-dual and anti–self-dual tensors [12]. The former two multiplets are combined to form a long (reducible) multiplet $(e_\mu{}^m, \psi_\mu{}^A, B_{\mu\nu}, \chi^A, \varphi)$, so that the field content is essentially the same as in the 4D case. In the dual formulation [11], the original field strength G is replaced by its Hodge dual $N = *G$.[3] Accordingly, the field strength $N \equiv dM$ involves *no* CS term which is desirable for our objective. The price we have to pay is the presence of the new CS term in the Lagrangian of the form $M \wedge F \wedge F$. However, as will be explained shortly, this term has more advantage than drawback, because this term can be easily replaced by $\mathcal{F} \wedge \mathcal{F} \wedge \mathcal{F}$ in the Stückelberg mechanism.

In this paper, we start with the 4D case in the next section, due to its direct relevance to low-energy physics. We study $N = 1$ supergravity in 4D with the multiplets $(e_\mu{}^m, \psi_\mu)$, $(B_{\mu\nu}, \chi, \varphi)$, and (A_μ, λ), using the Lagrangian terms in [6]. In Sec. III, we study the 6D case with more relevance to compactifications of higher-dimensional theories [1,2,4]. We use the long supergravity multiplet $(e_\mu{}^m, \psi_\mu{}^A, B_{\mu\nu}, \chi^A, \varphi)$, and (A_μ, λ^A) for $N = 2$ supergravity in 6D in terms of the dual formulation [11]. The 3D case is studied in Sec. IV, due to its relevance to the base space for supermembrane [2]. We show that a similar mechanism works also for $N = 1$ supergravity in 3D. In Sec. V, we show a possible compactification of our 6D theory on $\mathrm{AdS}_3 \times \mathrm{S}^3$, by assigning a nontrivial solution to the field strength G. Concluding remarks are given in Sec. VI.

II. LAGRANGIAN IN 4D

We start with 4D with the multiplet of supergravity $(e_\mu{}^m, \psi_\mu)$, the tensor multiplet $(B_{\mu\nu}, \chi, \varphi)$ [10], and the Abelian vector multiplet (A_μ, λ),[4] adopting the essentially same interaction terms given in [6]. The vector A_μ is absorbed into the longitudinal components of $B_{\mu\nu}$ as the Stückelberg mechanism.

As we also mentioned in [6], there are certain ambiguities for the couplings involving the tensor multiplet. The Lagrangian presented in [6] is desirable because of the absence of the $F \wedge A$-term in G.

A drawback, however, is the presence of the 4th rank field strength $H \equiv dC$ for the 3rd rank potential C. For our purpose, the potential C or H is not playing a crucial role. As a close invariance confirmation reveals, there arises *no* problem, even if we consistently truncate both C and H everywhere in the Lagrangian (Eq. 2.1) and the transformation rule (Eq. 2.4) in [6].

Another drawback of the Lagrangian (2.1) in [6] is the absence of exponential couplings for the tensor B_μ. It seems that the potential term of the form $m^2 e^{-4\varphi}$ should be present, for the $m\lambda\partial_\mu\varphi$-type variation in $\delta_Q \mathcal{L}_{4D}$ to be cancelled, arising from the $m(\bar{\lambda}\chi)$-term. Another trouble is that the G-field strength has the $Bd\varphi$-type term with the *bare* B-field. The trouble is that everywhere the B-field appears, there should be the field strength F, so that the latter is absorbed into B, as the Stückelberg mechanism. Actually, these two problems seem related to each other. In order to avoid this problem, we rescale the B-field such that the new B-field scales under the dilaton shift $\varphi \to \varphi + c$.

Based on these guiding principles, our action $I_{4D} \equiv \int d^4x \mathcal{L}_{4D}$ has the Lagrangian[5]

$$
\begin{aligned}
e^{-1}\mathcal{L}_{4D} = & -\frac{1}{4}R(\omega) - (\bar{\psi}_\mu \gamma^{\mu\nu\rho} D_\nu(\omega)\psi_\rho) \\
& - \frac{1}{12}e^{4\varphi}(G_{\mu\nu\rho})^2 - \frac{1}{2}(\partial_\mu\varphi)^2 - \frac{1}{4}(\mathcal{F}_{\mu\nu})^2 \\
& + \frac{1}{2}(\bar{\chi}\gamma^\mu D_\mu(\omega)\chi) + \frac{1}{2}(\bar{\lambda}\gamma^\mu D_\mu(\omega)\lambda) \\
& + (\bar{\psi}_\mu\gamma^\nu\gamma^\mu\chi)\partial_\nu\varphi - \frac{1}{2}(\bar{\psi}_\mu\gamma^{\rho\sigma}\gamma^\mu\lambda)\mathcal{F}_{\rho\sigma} \\
& + e^{2\varphi}\bigg[+ \frac{1}{6}(\bar{\psi}_\mu\gamma^{\rho\sigma\tau}\gamma^\mu\chi) - \frac{1}{8}(\bar{\chi}\gamma^{\rho\sigma\tau}\chi) \\
& - \frac{1}{24}(\bar{\lambda}\gamma^{\rho\sigma\tau}\lambda)\bigg] G_{\rho\sigma\tau} - me^{-2\varphi}(\bar{\lambda}\chi) \\
& + \frac{1}{2}me^{-2\varphi}(\bar{\psi}_\mu\gamma^\mu\lambda) - \frac{1}{8}m^2 e^{-4\varphi},
\end{aligned} \tag{2.1}
$$

up to quartic fermion terms. As has been mentioned, we have

$$
\mathcal{F}_{\mu\nu} \equiv F_{\mu\nu} + mB_{\mu\nu} \equiv +2\partial_{[\mu}A_{\nu]} + mB_{\mu\nu}, \tag{2.2a}
$$

$$
G_{\mu\nu\rho} \equiv +3\partial_{[\mu}B_{\nu\rho]}. \tag{2.2b}
$$

Our action I_{4D} is invariant under $N = 1$ local supersymmetry:

[1]It may be more common to call it *linear multiplet* in 4D. However, in order to be consistent with 6D, we also call it "tensor multiplet" in 4D.

[2]We use the index A for the **2** of $Sp(1)$.

[3]For the 3rd-rank field strength in Sec. III, we use the symbol G instead of N for the *dual* formulation [11]. We believe that this is not confusing, as long as it is clear from the context.

[4]For a reason to be mentioned shortly, we truncate the 3rd rank field $C_{\mu\nu\rho}$ in [6].

[5]Our metric is $(\eta^{mn}) \equiv \mathrm{diag.}(-, +, +, +)$. We also use the "plus-favored" metrics in 6D and 3D.

$$
\begin{aligned}
\delta_Q e_\mu{}^m &= -2(\bar{\epsilon}\gamma^m\psi_\mu),\\
\delta_Q\psi_\mu &= +D_\mu(\hat{\omega})\epsilon - \frac{1}{12}e^{2\varphi}(\gamma_\mu{}^{\rho\sigma\tau}\epsilon)\hat{G}_{\rho\sigma\tau},\\
\delta_Q B_{\mu\nu} &= +e^{-2\varphi}(\bar{\epsilon}\gamma_{\mu\nu}\chi) + 2e^{-2\varphi}(\bar{\epsilon}\gamma_{[\mu}\psi_{\nu]}),\\
\delta_Q\chi &= -(\gamma^\mu\epsilon)\hat{D}_\mu\varphi + \frac{1}{6}e^{2\varphi}(\gamma^{\rho\sigma\tau}\epsilon)\hat{G}_{\rho\sigma\tau},\\
\delta_Q\varphi &= +(\bar{\epsilon}\chi),\\
\delta_Q A_\mu &= +(\bar{\epsilon}\gamma_\mu\lambda),\\
\delta_Q\lambda &= +\frac{1}{2}(\gamma^{\mu\nu}\epsilon)\hat{\mathcal{F}}_{\mu\nu} + \frac{1}{2}me^{-2\varphi}\epsilon.
\end{aligned}
\tag{2.3}
$$

As usual in supergravity [13], all the *hatted* quantities are supercovariant, such as $\hat{D}_\mu\varphi$, $\hat{\mathcal{F}}_{\mu\nu}$, and $\hat{G}_{\mu\nu\rho}$.

Since the field strength $G_{\mu\nu\rho}$ has no CS-term, there is *no* additional term with $\delta_Q A_\mu$ in $\delta_Q B_{\mu\nu}$, either. Note also the absence of the $\bar{\chi}\lambda\mathcal{F}$-term that is present in the 6D case, as will be seen in the next section. The absence of this term is related to the absence of the exponential dilaton factor in the A_μ-kinetic term. The m-linear term in $\delta_Q\lambda$ is required for the invariance $\delta_Q I_{4D}$.

As has been mentioned, our action has also the global invariance of the dilaton shift:

$$
\varphi \to \varphi + c, \qquad B_{\mu\nu} \to e^{-2c}B_{\mu\nu}, \qquad m \to e^{+2c}m,
\tag{2.4}
$$

while A_μ is invariant. This is also consistent with the mB-term in $\mathcal{F}$.

Note that the potential term $V(\varphi) = +(1/8)m^2e^{-4\varphi}$ is positive definite. This feature is shared with the gauged supergravity in 6D [14], in which the dilaton potential is also positive definite: $\tilde{V} = +(1/8)e^{-\sqrt{2}\varphi}(C^{i\hat{I}})^2$, containing a similar positive-definite dilaton potential part: $+(3/8)\times(g')^2e^{-\sqrt{2}\varphi}$ [14].

The confirmation of the invariance $\delta_Q I_{4D} = 0$ up to quartic terms works as follows: For all the m-independent terms generated in $\delta_Q I_{4D}$, the cancellation mechanism is just parallel to [6]. There are two subtleties to be mentioned. The first one is related to the truncation of the C-field, while the second one is associated with the rescaling of the B-field. For the former subtlety, fortunately, their truncation does not affect all other cancellation structures in the Lagrangian. For the second subtlety, the rescaling deletes the $Bd\varphi$-term in G as in (2.2b), which makes the computation easier.

All the m-dependent terms in $\delta_Q I_{4D}$ are categorized into the seven sectors: (i) $mD\lambda$, (ii) $m\chi\mathcal{F}$, (iii) $m\psi\mathcal{F}^2$, (iv) $m\lambda G$, (v) $m\lambda\varphi$, (vi) $m^2\chi$, and (vii) $m^2\psi$. The subtle sector is (iv), to which the variation of the Noether term $\bar{\psi}\lambda\mathcal{F}$ contributes via $\partial_{[\mu}\mathcal{F}_{\rho\sigma]} = (1/3)mG_{\mu\rho\sigma}$. Another subtlety is with the sector (vi), which necessitates the presence of the exponential dilaton factor in the potential. This is why we have to go to the special frame in which the B-field and the mass m are transforming under (2.4).

As in the usual Stückelberg mechanism [7], the original vector field A_μ appears only in the $\mathcal{F}_{\mu\nu}$-term with $mB_{\mu\nu}$, so it is completely gauged away by the Λ-transformation $\delta_\Lambda B_{\mu\nu} = 2\partial_{[\mu}\Lambda_{\nu]}$ of $B_{\mu\nu}$. Eventually, we can replace all the $\mathcal{F}_{\mu\nu}$-field strength by $mB_{\mu\nu}$ everywhere in the Lagrangian (2.1), as well as in the transformation rules (2.3). In particular, the kinetic term for A_μ-field becomes the mass term of $B_{\mu\nu}$:

$$
\begin{aligned}
&-\frac{1}{12}e^{4\varphi}(G_{\rho\sigma\tau})^2 - \frac{1}{4}(\mathcal{F}_{\mu\nu})^2\\
&\to -\frac{1}{12}e^{4\varphi}(G_{\rho\sigma\tau})^2 - \frac{1}{4}m^2(B_{\mu\nu})^2.
\end{aligned}
\tag{2.5}
$$

Accordingly, the original Majorana fields λ and χ are combined into a massive Dirac field, because of the mixture term $m(\bar{\lambda}\chi)$. This prescription is equivalent to adopting the gauge in which $A_\mu = 0$, so that the new supersymmetry transformation rule of A_μ is $\delta'_Q A_\mu = 0$, while the new $\delta'_Q B_{\mu\nu}$ has an additional term $2m^{-1}\partial_{[\mu}(\bar{\epsilon}\gamma_{\nu]}\lambda)$.

III. LAGRANGIAN IN 6D

As has been mentioned for 6D, we need to use the dual version [11] of $N = 2$ supergravity [12,14]. We use the reducible supergravity multiplet $(e_\mu{}^m, \psi_\mu{}^A, B_{\mu\nu}, \chi^A, \varphi)$, and the Abelian vector multiplet (A_μ, λ^A). The former is the combination of the supergravity multiplet $(e_\mu{}^m, \psi_\mu{}^A, B^{(+)}_{\mu\nu})$ and the tensor multiplet $(B^{(-)}_{\mu\nu}, \chi^A, \varphi)$ [12]. Each of these two multiplets do not have Lagrangian formulation due to the (anti) self-duality of $B^{(\pm)}_{\mu\nu}$, so that we need to combine them to have a Lagrangian [14]. Even though we use the notation $B_{\mu\nu}$ for the tensor, the original symbol for the tensor is $M_{\mu\nu}$ [11], which is dual to the usual tensor $B_{\mu\nu}$ [12,14].

As has been also mentioned, instead of the $F\wedge A$ term in the field strength G, the Lagrangian of the dual version [11] has the $B\wedge F\wedge F$-type CS term, as desirable for our purpose of a Stückelberg mechanism.

As in the 4D case, the basic prescription is to replace the field strength $F = dA$ by $\mathcal{F} \equiv F + mB$. The only subtlety is about the above-mentioned $B\wedge F\wedge F$-term. However, it turns out to be rather simple, because the leading part $F\wedge F\wedge F$ itself is a total divergence, while if these F's are replaced by the $\mathcal{F}$'s, it becomes covariant under the Λ-transformation $\delta_\Lambda B_{\mu\nu} = 2\partial_{[\mu}\Lambda_{\nu]}$. Accordingly, its supersymmetry transformation becomes also covariantized:

$$\delta_Q\Big(-\frac{1}{24}m^{-1}\epsilon^{\mu\nu\rho\sigma\tau\lambda}\mathcal{F}_{\mu\nu}\mathcal{F}_{\rho\sigma}\mathcal{F}_{\tau\lambda}\Big) = -\frac{1}{6}\epsilon^{\mu\nu\rho\sigma\tau\lambda}(\delta_Q A_\mu)G_{\nu\rho\sigma}\mathcal{F}_{\tau\lambda} - \frac{1}{8}\epsilon^{\mu\nu\rho\sigma\tau\lambda}(\delta_Q B_{\mu\nu})\mathcal{F}_{\rho\sigma}\mathcal{F}_{\tau\lambda}. \tag{3.1}$$

With this caveat in mind, our action $I_{6D} \equiv \int d^6x \mathcal{L}_{6D}$ has the Lagrangian

$$\begin{aligned} e^{-1}\mathcal{L}_{6D} = &+\frac{1}{4}R(\omega) - \frac{1}{2}(\bar\psi_\mu\gamma^{\mu\rho\sigma}D_\nu(\omega)\psi_\rho) - \frac{1}{12}e^{-2\sqrt{2}\varphi}(G_{\mu\nu\rho})^2 - \frac{1}{2}(\bar\chi\gamma^\mu D_\mu(\omega)\chi) - \frac{1}{2}(\partial_\mu\varphi)^2 - \frac{1}{4}e^{\sqrt{2}\varphi}(\mathcal{F}_{\mu\nu})^2 \\ &- \frac{1}{2}(\bar\lambda\gamma^\mu D_\mu(\omega)\lambda) + \frac{1}{\sqrt{2}}(\bar\psi_\mu\gamma^\nu\gamma^\mu\chi)\partial_\nu\varphi - \frac{1}{24}e^{-\sqrt{2}\varphi}[(\bar\psi^\mu\gamma_{[\mu}\gamma^{\rho\sigma\tau}\gamma_{\nu]}\psi^\nu) \\ &+ 2(\bar\psi_\mu\gamma^{\rho\sigma\tau}\gamma^\mu\chi) - (\bar\chi\gamma^{\rho\sigma\tau}\chi) - (\bar\lambda\gamma^{\rho\sigma\tau}\lambda)]G_{\rho\sigma\tau} + \frac{1}{2\sqrt{2}}e^{\varphi/\sqrt{2}}[(\bar\psi_\mu\gamma^{\rho\sigma}\gamma^\mu\lambda) - (\bar\chi\gamma^{\rho\sigma}\lambda)]\mathcal{F}_{\rho\sigma} \\ &- \frac{1}{24}m^{-1}e^{-1}\epsilon^{\mu\nu\rho\sigma\tau\lambda}\mathcal{F}_{\mu\nu}\mathcal{F}_{\rho\sigma}\mathcal{F}_{\tau\lambda} + \frac{3}{2\sqrt{2}}me^{3\varphi/\sqrt{2}}(\bar\lambda\chi) - \frac{1}{2\sqrt{2}}me^{3\varphi/\sqrt{2}}(\bar\psi_\mu\gamma^\mu\lambda) - \frac{1}{8}m^2e^{3\sqrt{2}\varphi}, \end{aligned} \tag{3.2}$$

up to quartic terms. We omit contracted A-indices, e.g., $(\bar\chi\gamma^{\rho\sigma}\lambda) \equiv (\bar\chi^A\gamma^{\rho\sigma}\lambda_A)$, etc. As in the 4D case, we have

$$\begin{aligned} \mathcal{F}_{\mu\nu} &\equiv F_{\mu\nu} + mB_{\mu\nu} \equiv +2\partial_{[\mu}A_{\nu]} + mB_{\mu\nu}, \\ G_{\mu\nu\rho} &\equiv +3\partial_{[\mu}B_{\nu\rho]}. \end{aligned} \tag{3.3}$$

Our action I_{6D} is invariant under $N = 2$ local supersymmetry:

$$\begin{aligned} \delta_Q e_\mu{}^m &= +(\bar\epsilon\gamma^m\psi_\mu), \\ \delta_Q\psi_\mu{}^A &= +D_\mu(\hat\omega)\epsilon^A + \frac{1}{24}e^{-\sqrt{2}\varphi}(\gamma^{\rho\sigma\tau}\gamma_\mu\epsilon^A)\hat G_{\rho\sigma\tau}, \\ \delta_Q B_{\mu\nu} &= -\frac{1}{2}e^{\sqrt{2}\varphi}(\bar\epsilon\gamma_{\mu\nu}\chi) - e^{\sqrt{2}\varphi}(\bar\epsilon\gamma_{[\mu}\psi_{\nu]}), \\ \delta_Q\chi^A &= +\frac{1}{\sqrt{2}}(\gamma^\mu\epsilon^A)\hat D_\mu\varphi + \frac{1}{12}e^{-\sqrt{2}\varphi}(\gamma^{\rho\sigma\tau}\epsilon^A)\hat G_{\rho\sigma\tau}, \\ \delta_Q\varphi &= +\frac{1}{\sqrt{2}}(\bar\epsilon\chi), \\ \delta_Q A_\mu &= -\frac{1}{\sqrt{2}}e^{-\varphi/\sqrt{2}}(\bar\epsilon\gamma_\mu\lambda), \\ \delta_Q\lambda^A &= +\frac{1}{2\sqrt{2}}e^{\varphi/\sqrt{2}}(\gamma^{\mu\nu}\epsilon^A)\hat{\mathcal{F}}_{\mu\nu} + \frac{1}{2\sqrt{2}}me^{3\varphi\sqrt{2}}\epsilon^A. \end{aligned} \tag{3.4}$$

As in the 4D case, the m^2 potential term is positive definite, implying 6D de Sitter space-time (dS_6). There is global scale invariance of the Lagrangian, dictated by

$$\begin{aligned} &\varphi \to \varphi + c, \qquad B_{\mu\nu} \to e^{\sqrt{2}c}B_{\mu\nu}, \\ &A_\mu \to e^{-c/\sqrt{2}}A_\mu, \qquad m \to e^{-3c/\sqrt{2}}m, \end{aligned} \tag{3.5}$$

so that each term in our Lagrangian is invariant under this global transformation. In particular, the $m^{-1}\mathcal{F}\wedge\mathcal{F}\wedge\mathcal{F}$-term is invariant.

Note the existence of other "symmetry" of our Lagrangian. Under this symmetry our Lagrangian is *not* invariant, but is *covariant* instead [14]:

$$\begin{aligned} &\varphi \to \varphi + c, \qquad e_\mu{}^m \to e^{-c/\sqrt{2}}e_\mu{}^m, \\ &e_m{}^\mu \to e^{c/\sqrt{2}}e_m{}^\mu, \qquad \psi_\mu \to e^{-c/2\sqrt{2}}\psi_\mu, \\ &(\chi, \lambda) \to e^{c/2\sqrt{2}}(\chi, \lambda), \qquad m \to e^{-\sqrt{2}c}m, \\ &\mathcal{L}_{6D} \to e^{-2\sqrt{2}c}\mathcal{L}_{6D}. \end{aligned} \tag{3.6}$$

To our knowledge, the peculiar $m^{-1}\mathcal{F}\wedge\mathcal{F}\wedge\mathcal{F}$-term has not been presented in the past in the context of 6D supergravity, or, at least, not in the context of a Stückelberg mechanism consistent with supergravity.

The confirmation of $\delta_Q I_{6D} = 0$ up to quartic terms goes as follows. First, for the m-independent terms, all the cancellations work as in [11]. The only subtlety is related to (3.1), but fortunately their structures are eventually the same as the original $m = 0$ case without any disturbance.

Second, for the m-dependent terms, there are seven sectors: (i) $mD\lambda$, (ii) $m\chi\mathcal{F}$, (iii) $m\psi\mathcal{F}^2$, (iv) $m\lambda G$, (v) $m\lambda\varphi$, (vi) $m^2\chi$, and (vii) $m^2\psi$, that are parallel to the previous 4D case. The only differences are as follows. To the sector (ii), there is a new contribution from the $\bar\chi\lambda\mathcal{F}$-term in the Lagrangian which was absent in the previous 4D case [6]. To the sector (iv), the contribution from $\delta_Q\psi_\mu$ is zero in the 6D case due to the identity $\gamma_\mu\gamma^{\rho\sigma\tau}\gamma^\mu \equiv 0$, and therefore there is no contribution from the $m(\bar\psi_\mu\gamma^\mu\lambda)$-term.

As in the previous 4D case, the vector field A_μ in $\mathcal{F}_{\mu\nu} \equiv F_{\mu\nu} + mB_{\mu\nu}$ is completely gauged away by the tensorial transformation $\delta_\Lambda B_{\mu\nu} = 2\partial_{[\mu}\Lambda_{\nu]}$. Accordingly, the original A_μ-kinetic term $(-1/4)(F_{\mu\nu})^2$ is replaced by the mass term $-(1/4)m^2(B_{\mu\nu})^2$. This is also equivalent to adopting the gauge in which $A_\mu = 0$, so that the new transformation rule for A_μ is $\delta'_Q A_\mu = 0$, while the new $\delta'_Q B_{\mu\nu}$ has an additional term $-\sqrt{2}m^{-1}\partial_{[\mu|}(e^{\varphi/\sqrt{2}}\bar\epsilon\gamma_{|\nu]}\lambda)$.

The importance of our result in 6D is multifold. First, we have shown the system with a positive-definite potential, without the usual *gauging* technique [14]. Even though this mechanism is similar to the massive type IIA case [9], it does not seem to have been pointed out in the past in 6D. Second, we have the peculiar $m^{-1}\mathcal{F}\wedge\mathcal{F}\wedge\mathcal{F}$-term which is equivalent to the cubic self-interaction $m^2 B\wedge B\wedge B$ after the A-field is gauged away. This does not seem to have been presented in the past, either. Third, we can regard the Stückelberg mechanism for $B_{\mu\nu}$ as an important application of the dual version [11] of $N=2$ supergravity in 6D [12,14]. In other words, the Stückelberg mechanism for the tensor $B_{\mu\nu}$ differentiates the dual version [11] from the usual version [14]. Relevantly, a duality transformation becomes impossible once the *bare* $B_{\mu\nu}$-field is involved in the field strength $\mathcal{F}$.

IV. LAGRANGIAN IN 3D

We now show that a similar Stückelberg mechanism also works for $N=1$ supergravity in 3D. Even though 3D is not useful for dimensional reductions to phenomenologically interesting models in 4D, it still has some significance in the context of supermembrane [2] or M-theory [3]. The field content is the sum of the multiplet of supergravity $(e_\mu{}^m, \psi_\mu)$, the tensor multiplet $(B_{\mu\nu}, \chi, \varphi)$, and the vector multiplet (A_μ, λ). Actually, the second multiplet is a scalar multiplet, where $B_{\mu\nu}$ is originally an auxiliary field in 3D. However, to be consistent with 4D and 6D, we call it temporarily "tensor multiplet."

Since most of the notations used are the same as in the 4D case with the same metric $(\eta^{mn})\equiv\text{diag.}(-,+,+)$, we directly show the Lagrangian and transformation rule. Our total action $I_{3D}\equiv\int d^3x\mathcal{L}_{3D}$ has the Lagrangian

$$\begin{aligned}e^{-1}\mathcal{L}_{3D}=&-\frac{1}{4}R(\omega)-(\bar{\psi}_\mu\gamma^{\mu\nu\rho}D_\nu(\omega)\psi_\rho)-\frac{1}{12}e^{4\varphi}(G_{\mu\nu\rho})^2\\&-\frac{1}{2}(\partial_\mu\varphi)^2-\frac{1}{4}(\mathcal{F}_{\mu\nu})^2+\frac{1}{2}(\bar{\chi}\gamma^\mu D_\mu(\omega)\chi)\\&+\frac{1}{2}(\bar{\lambda}\gamma^\mu D_\mu(\omega)\lambda)+(\bar{\psi}_\mu\gamma^\nu\gamma^\mu\chi)\partial_\nu\varphi\\&-\frac{1}{2}(\bar{\psi}_\mu\gamma^{\rho\sigma}\gamma^\mu\lambda)\mathcal{F}_{\rho\sigma}+e^{2\varphi}\Big[+\frac{1}{6}(\bar{\psi}_\mu\gamma^{\rho\sigma\tau}\gamma^\mu\chi)\\&-\frac{1}{6}(\bar{\chi}\gamma^{\rho\sigma\tau}\chi)\Big]G_{\rho\sigma\tau}-me^{-2\varphi}(\bar{\lambda}\chi)\\&+\frac{1}{2}me^{-2\varphi}(\bar{\psi}_\mu\gamma^\mu\lambda)-\frac{1}{8}m^2e^{-4\varphi},\end{aligned}\tag{4.1}$$

up to quartic terms. As in the 4D case, we have

$$\begin{aligned}\mathcal{F}_{\mu\nu}&\equiv F_{\mu\nu}+mB_{\mu\nu}\equiv+2\partial_{[\mu}A_{\nu]}+mB_{\mu\nu},\\G_{\mu\nu\rho}&\equiv+3\partial_{[\mu}B_{\nu\rho]}.\end{aligned}\tag{4.2}$$

Our action I_{3D} is invariant up to quartic terms under $N=1$ local supersymmetry:

$$\begin{aligned}\delta_Q e_\mu{}^m&=-2(\bar{\epsilon}\gamma^m\psi_\mu),\\\delta_Q\psi_\mu&=+D_\mu(\hat{\omega})\epsilon,\\\delta_Q B_{\mu\nu}&=+e^{-2\varphi}(\bar{\epsilon}\gamma_{\mu\nu}\chi)+2e^{-2\varphi}(\bar{\epsilon}\gamma_{[\mu}\psi_{\nu]}),\\\delta_Q\chi&=-(\gamma^\mu\epsilon)\hat{D}_\mu\varphi+\frac{1}{6}e^{2\varphi}(\gamma^{\rho\sigma\tau}\epsilon)\hat{G}_{\rho\sigma\tau},\\\delta_Q\varphi&=+(\bar{\epsilon}\chi),\\\delta_Q A_\mu&=+(\bar{\epsilon}\gamma_\mu\lambda),\\\delta_Q\lambda&=+\frac{1}{2}(\gamma^{\mu\nu}\epsilon)\hat{\mathcal{F}}_{\mu\nu}+\frac{1}{2}me^{-2\varphi}\epsilon.\end{aligned}\tag{4.3}$$

As in the 4D case, our action I_{3D} also has the global invariance of the dilaton shift:

$$\varphi\to\varphi+c,\qquad B_{\mu\nu}\to e^{-2c}B_{\mu\nu},\qquad m\to e^{+2c}m.\tag{4.4}$$

Compared with the 4D case (2.1) through (2.3), there are three major differences: (i) The absence of the $\bar{\lambda}\lambda G$-term in the Lagrangian, (ii) The absence of the G-linear term in $\delta_Q\psi_\mu$, and (iii) The original field $B_{\mu\nu}$ was auxiliary without physical degree of freedom, but it starts propagating after absorbing the vector A_μ. In particular, the first two features are related to each other, via the contribution to the λFG-sector. All other terms, including the m-linear terms, are exactly the same, and even their coefficients and dilaton exponential factors are the same. The cancellation structure for the invariance $\delta_Q I_{3D}=0$ is essentially parallel to the 4D case, so we do not elaborate the details.

As in the 4D and 6D cases, the A_μ-field is completely absorbed into $B_{\mu\nu}$. Thus the kinetic term of the former becomes the mass term for the latter. This is also equivalent to adopting the gauge, in which $A_\mu=0$, so that $\delta'_Q A_\mu=0$, while the new $\delta'_Q B_{\mu\nu}$ has an additional term $2m^{-1}\partial_{[\mu}(\bar{\epsilon}\gamma_{\nu]}\lambda)$.

V. COMPACTIFICATION ON $\text{AdS}_3\times\text{S}^3$

As an application of our 6D theory, we investigate possible compactifications from 6D to lower dimensions. It turns out that a compactification of 6D on $\text{AdS}_3\times\text{S}^3$ is indeed possible. In a sense, this is similar to other compactification patterns of higher-dimensional supergravity on anti–de Sitter space-time [15].

Our bosonic field equations for g^{MN},[6] A_M, B_{MN}, and φ from our $\mathcal{L}_{6D}$ (3.2) are

[6] Only in this section, we are using the capital alphabetic indices $M, N, \cdots = 0, 1, \cdots, 5$ for 6D space-time.

$$R_{MN} \doteq +e^{-2\sqrt{2}\hat{\varphi}} G_{MRS} G_N{}^{RS} + 2e^{\sqrt{2}\varphi} \mathcal{F}_{MR} \mathcal{F}_N{}^R + 2(\partial_M \varphi)(\partial_N \varphi) - g_{MN}\left[+\frac{1}{6} e^{-2\sqrt{2}\varphi}(G_{RST})^2 + \frac{1}{4} e^{\sqrt{2}\varphi}(\mathcal{F}_{RS})^2 - \frac{1}{8} m^2 e^{3\sqrt{2}\varphi}\right], \tag{5.1a}$$

$$\partial_N(e e^{\sqrt{2}\varphi} \mathcal{F}^{MN}) + \frac{1}{6} \epsilon^{MNRSTU} G_{NRS} \mathcal{F}_{TU} \doteq 0, \tag{5.1b}$$

$$\partial_R(e e^{-2\sqrt{2}\varphi} G^{MNR}) - m e e^{\sqrt{2}\varphi} \mathcal{F}^{MN} - \frac{1}{4} \epsilon^{MNRSTU} \mathcal{F}_{RS} \mathcal{F}_{TU} \doteq 0, \tag{5.1c}$$

$$D_M^2 \varphi + \frac{1}{3\sqrt{2}} e^{-2\sqrt{2}\varphi}(G_{MNR})^2 - \frac{1}{2\sqrt{2}} e^{\sqrt{2}\varphi}(\mathcal{F}_{MN})^2 - \frac{3}{4\sqrt{2}} m^2 e^{3\sqrt{2}\varphi} \doteq 0. \tag{5.1d}$$

From now on, we use the symbol $\doteq$ for a field equation or a solution.

Note that the relative sign between the $\mathcal{F}^2$ and m^2-terms in the φ-field equation (5.1d) is *positive*. Because of this relative sign, and due to our metric diag.$(-, +, +, +, +, +)$, there seems *no* direct way to compactify this 6D theory on (Minkowski)$_4 \times S^2$ via a monopole solution for $\mathcal{F}_{MN}$ in the extra 2D [16].

In conventional $N = 2$ gauged supergravity in 6D [14], the last g_{MN}-terms in (5.1a) could be expressed only in terms of $D_M^2 \varphi$ using the φ-field equation. This is because of Lagrangian "covariance" analogous to (3.6) *without* the gauge coupling g transforming. In our 6D theory, however, the m^2-term in (5.1a) has a different coefficient from the corresponding term in the φ-field equation (5.1d). This is related to the fact that the constant m is transforming under (3.6). Compared with the G^2 and F^2-terms, the last m^2-term in the last line of (5.1a) has different relative factors, and therefore it is *not* absorbed into $D_M^2\varphi$. This feature in turn becomes an obstruction against the compactification on (Minkowski)$_4 \times S^2$, as opposed to the gauged $N = 2$ supergravity in 6D [16] including dual version [11].

Despite this obstruction, there is a different compactification scheme. Since the relative sign between the G^2 and m^2-terms is negative, we can assign certain nontrivial value to G_{MNR}, compactifying from 6D into 3D. This is similar to the work [17] about the compactification of 6D supergravity on $AdS_3 \times$ [SU(2) Group Manifold]. Of course, the difference is that we use the Stückelberg mechanism in 6D, that generates the positive-definite potential, that in turn makes the compactification from 6D on $AdS_3 \times S^3$ possible.

For our compactification into 3D, our Ansätze with the constants a, b, g, and φ_0 are

$$R_{MN} = \begin{cases} R_{\alpha\beta} \doteq + a^{-2}\delta_{\alpha\beta} & (\text{for } M = \alpha, N = \beta), \\ R_{\mu\nu} \doteq - b^{-2}\eta_{\mu\nu} & (\text{for } M = \mu, N = \nu), \\ 0 & (\text{otherwise}), \end{cases}$$

$$G_{MNR} \doteq \begin{cases} g\epsilon_{\alpha\beta\gamma} & (\text{for } M = \alpha, N = \beta, R = \gamma), \\ 0 & (\text{otherwise}), \end{cases}$$

$$\mathcal{F}_{MN} \doteq 0, \qquad \varphi \doteq \varphi_0 = \text{const}, \tag{5.2}$$

where $\alpha, \beta, \cdots = 3, 4, 5$ (or $\mu, \nu, \cdots = 0, 1, 2$) are for S^3 (or AdS_3).

By substituting these Ansätze into the field equations (5.1), we get the conditions on g, a, b, φ_0 in terms of m as

$$g^2 \doteq +\frac{3}{8} m^2 e^{5\sqrt{2}\varphi_0}, \quad a^2 \doteq +2m^{-2} e^{-3\sqrt{2}\varphi_0}, \quad b^2 \doteq +4m^{-2} e^{-3\sqrt{2}\varphi_0}. \tag{5.3}$$

The important aspect here is that these solutions are consistent with the positive definiteness of g^2, a^2, and b^2. In fact, the signs of the scalar curvatures in each 3D are

$$R_\alpha{}^\alpha \doteq +3a^{-2} > 0 \quad (\text{for } S^3), \qquad R_\mu{}^\mu \doteq -3b^{-2} < 0 \quad (\text{for } AdS_3). \tag{5.4}$$

We have thus seen that our Stückelberg mechanism in 6D resulted in the interesting compactification on $AdS_3 \times S^3$.

VI. CONCLUDING REMARKS

In this paper we have presented locally supersymmetric Stückelberg mechanisms [7,8] for the massive tensor field $B_{\mu\nu}$ in 4D, 6D, and 3D. In the 4D case, we have used special couplings between the tensor multiplet and supergravity in [6]. In particular, the absence of the $F \wedge A$-term in the field strength G was crucial. In the 6D case, we have used the *dual* formulation [11] of $N = 2$ supergravity, which has no $F \wedge A$-term in G, either. The 3D case is just parallel to the 4D case: The field content is parallel; the Lagrangian and transformation rule are almost exactly the same.

In these dimensions, the inclusions of the mB-terms in $\mathcal{F} = dA + mB$ resulted in several new explicitly m-dependent terms, such as the dilaton potential terms at $\mathcal{O}(m^2)$, $m(\bar{\lambda}\chi)$, $m(\bar{\psi}_\mu \gamma^\mu \lambda)$, and the linear m-terms in $\delta_Q \lambda$. Interestingly, all these m-dependent Lagrangian terms have exactly the same structures in dimensions 4D, 6D, and 3D.

As an important application of our Stückelberg mechanism in 6D, we have presented a nontrivial compactification on $AdS_3 \times S^3$. Thanks to the positive-definite potential generated, the 6D theory can compactify into 3D by a nontrivial solution for the field strength $G_{\mu\nu\rho}$. We have seen that the extra 3D become S^3, while the final 3D become AdS_3.

There are differences as well as similarities in these dimensions 4D, 6D, and 3D. The most fundamental similarity is about the field contents which are essentially the same in these dimensions. Another similarity is the presence of the positive-definite potential at $\mathcal{O}(m^2)$, implying de Sitter space-time. The structure of the 3D case is just parallel to the 4D case, including the dilaton exponential factors. A difference in the 6D case is the usage of the dual version [11], while in 4D we used the special Lagrangian terms in [6] with *no* $F \wedge A$-term in G. Another difference in 6D is the presence of the peculiar term $\mathcal{F} \wedge \mathcal{F} \wedge \mathcal{F}$, containing the topological surface term $F \wedge F \wedge F$.

We emphasize that the result in this paper is highly nontrivial, and not obtained by straightforward computations. The special choice of frames, in particular, the dilaton exponential factors, play very crucial roles. The usage of the dual version [11] instead of the usual version [14] is also one of such nontrivial features. In all the dimensions 4D, 6D, and 3D, we have used the special frame of couplings with *no* $F \wedge A$-term in G.

Note that the Stückelberg mechanism [7] has problems at the quantum level for a non-Abelian gauge group, such as unitarity [8,18]. However, since we are dealing only with Abelian symmetries, our formulation does not seem to pose such a problem. Note also that the Abelian-type Stückelberg mechanism has been established in 10D as massive type IIA theory [9]. Moreover, the presence of local supersymmetry provides a better chance for the consistency also at the quantum level.

We can think of other space-time dimensions for similar mechanisms. However, a simple consideration immediately reveals that there are certain restrictions. For example, in $N = 2$ supergravity in 5D, the 2nd-rank tensor B needs the 3rd-rank field strength $G = dB + F \wedge A$ with the CS term [19,20], which is an obstruction against our mechanism. As opposed to the 6D case, we cannot use a duality transformation, either, because the original tensor $B_{\mu\nu}$ will be dualized into a vector which we do not want. In a sense, the series of 3D, 4D, and 6D is analogous to the space-time dimensions, in which the so-called Green-Schwarz formulations with fermionic κ-symmetries are possible [21]. These dimensions have also certain similarity with respect to fermions [22].

Despite such restrictions, our new mechanism has opened a new avenue for massive tensor multiplets. It provides not only a massive tensor field $B_{\mu\nu}$, but also nontrivial dilaton potentials and new interaction terms that are similar to but slightly different from conventional gauging techniques. Obviously, there is a considerable number of potential applications of our methods and results to supergravity theories in diverse space-time dimensions.

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