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Non-Gaussianities of single field inflation with nonminimal coupling

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We investigate the non-Gaussianities of inflation driven by a single scalar field coupling nonminimally to the Einstein Gravity. We assume that the form of the scalar field is very general with an arbitrary sound speed. For convenience, we take the subclass that the nonminimal coupling term is linear to the Ricci scalar R. We define a parameter $\mu \equiv \epsilon_h/\epsilon_\theta$, where ϵ_h and ϵ_θ are two kinds of slow-roll parameters, and obtain the dependence of the shape of the 3-point correlation function on μ. We also show the estimator $F_{\rm NL}$ in the equilateral limit. Finally, based on numerical calculations, we present the non-Gaussianities of nonminimal coupling chaotic inflation as an explicit example.

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I. INTRODUCTION

The inflation theory is one of the most successful theories of modern cosmology. Having a period of very rapidly accelerating expansion, it can not only solve many theoretical problems in cosmology, such as flatness, horizon, monopole, and so on, but also gives the right amount of primordial fluctuations with nearly scale-invariant power spectrum, which fits the data very well in structure formation [1–3].

There are many ways to construct inflation models, one of which is to introduce a scalar field called "inflaton" ϕ (see [2,3]). Moreover, one may expect that inflaton could have nonminimal coupling to the Ricci scalar R. The most usual coupling form is $R\phi^2$, which was initially studied for the new inflation scenario [4] and chaotic inflation scenario [5]. Later, various models were taken on and deeply investigated. With a nonminimal coupling term, inflation can be easily obtained and an attractor solution is also available [5]. Perturbations based on nonminimal coupling inflation are discussed in [6], where the coupling term may give rise to corrections on power spectrum which can be used to fit the data or constrain the parameters. Nonminimal couplings can be extended to multifields, see [7], or kinetic term coupling [8]. The constraints from observational data were also performed, e.g. in [9], where the authors claimed that for nonminimal coupling chaotic inflation models, a tiny tensor to scalar ratio will be obtained. Other applications of nonminimal coupling inflation include the realization of warm inflation [10] and the avoidance of the so-called "η" problem [11] in the framework of string theory [12]. One can also see Refs. [13,14] for comprehensive reviews of nonminimal coupling theories.

The non-Gaussianity of the primordial perturbation has been widely acknowledged to be an important probe in the early Universe [15–21]. Experimentally, more and more accurate data allow us to study the nonlinear properties of the fluctuation in cosmic microwave background and large scale structure [22–24]; Theoretically, the redundancy of inflation models requires more information than those of linear perturbations only to have them distinguished. The non-Gaussianity of the fluctuations was first considered in [25], and it was further shown in [26] that the canonical single field slow-roll inflation can only give rise to a negligible amount of non-Gaussianity. To get large non-Gaussianity new inflation models must be found. A *partial* list of references includes multifield models [27], k inflation [28], Dirac-Born-Infeld-type inflation [29], curvaton scenario [30], ghost inflation [31], warm inflation [32], non-Bunch-Davies vacuum scenario [33], bounce scenario [34], island cosmology [35], loop correction [36], noncommutativity [37], string gas scenario [38], cosmic string [39], "end-in-inflation" scenario [40], Ekpyrotic scenario [41], vector field [42], Hořava theories [43], and so on and so forth [44].

In this paper, we investigate the non-Gaussianity of inflation driven by a general single field $P(X, \phi)$ coupling nonminimally to the Einstein gravity. Some specific examples of non-Gaussianities of the nonminimal coupled field has been studied in, e.g. [46], and non-Gaussianity generated by modified gravity is expected to have effects that can be tested by cosmic microwave background anisotropies [47]. By taking a subclass of linear coupling, we calculated various shapes depending on the ratio between two slow-roll parameters, ϵ_h and ϵ_θ, which describe the evolution of cosmic expansion and the nonminimal correction, respectively. The power spectrum will deviate from scale invariance due to the existence of nonminimal coupling [14], and the shape of the 3-point correlation function is correspondingly affected. In this paper we find that for different (red or blue) tilts of the power spectrum, the shape will include different parts which will obtain different amplitudes of non-Gaussianities. However, since we have only calculated up to leading order in the slow-roll parameter, this conclusion has not been very clear as of yet. Nevertheless, if it can be verified after completely considering all the orders, one can find the relations between the 2- and 3-point correlation functions, which can be used to

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constrain nonminimal coupling models. This will be one of our future works.

This paper is organized as follows: Section II briefly reviews the preliminaries and basic equations of the general nonminimal coupling single field inflation. We study the non-Gaussianities of the general nonscalar field with linear coupling in Sec. III, which is the main part of the paper. We first study the perturbed action of the system up to third order, and obtain the mode solution at the quadratic level. After that, we calculate various shapes of the 3-point correlation functions using the mode solution. We also study their equilateral limit and relation with slow-roll parameters at their leading order. In the last part of this section, we present the non-Gaussianities of nonminimal coupling chaotic inflation as an explicit example using numerical calculations. Sec. IV is the conclusion and discussions.

II. PRELIMINARY

To begin with, let us consider the most general action of a single scalar field with nonminimal coupling

$$S = \frac{1}{2}\int dtd^3x\sqrt{-g}[f(R,\phi) + 2P(X,\phi)], \qquad (1)$$

where $X \equiv -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ is the kinetic term and the metric $g_{\mu\nu} = \mathrm{diag}[-1, a^2(t), a^2(t), a^2(t)]$ with $a(t)$ the scale factor of the Universe. For the background evolution, one can vary the action (1) with respect to the field ϕ and the metric $g_{\mu\nu}$ to get the equation of motion for ϕ:

$$f_\phi + 2P_\phi + 2(P_{XX}\nabla^\mu X + P_{X\phi}\nabla^\mu\phi)\nabla_\mu\phi + 2P_X\Box\phi = 0, \qquad (2)$$

and the Einstein equations

$$\Sigma_{\mu\nu} = T^{(\phi)}_{\mu\nu}, \qquad (3)$$

where

$$\Sigma_{\mu\nu} \equiv \Box f_R g_{\mu\nu} - \nabla_\mu\nabla_\nu f_R + f_R R_{\mu\nu} - \tfrac{1}{2}fg_{\mu\nu}, \qquad (4)$$

$$T^{(\phi)}_{\mu\nu} \equiv P_X\nabla_\mu\phi\nabla_\nu\phi + Pg_{\mu\nu}, \qquad (5)$$

with ∇_μ being the covariant derivative with respect to the metric $g_{\mu\nu}$ and $\Box \equiv \nabla_\mu\nabla^\mu$.

The evolution of the Universe can be described by the slow-roll parameter

$$\epsilon_h \equiv -\frac{\dot{H}}{H^2}, \qquad (6)$$

where $H = \dot{a}/a$ is the Hubble parameter and the dot denotes the time derivative. In the inflation case, we require that ϵ_h be small. Moreover, we can define two more parameters following Ref. [26]:

$$\Sigma \equiv XP_X + 2X^2P_{XX}, \qquad (7)$$

$$\lambda \equiv X^2P_{XX} + \tfrac{2}{3}X^3P_{XXX}, \qquad (8)$$

which will be used in later parts of the paper.

To study non-Gaussianities, we adopt the usual convention of using the Arnowitt-Deser-Misner metric [48] as follows:

$$ds^2 = -N^2dt^2 + h_{ij}(dx^i + N^idt)(dx^j + N^jdt), \qquad (9)$$

where $N(x)$ and $N_i(x)$ are the lapse function and the shift vector, respectively. It is useful to decompose the action (1) into the 3 + 1 form, saying

$$S = \frac{1}{2}\int dtd^3x\sqrt{h}N\Big\{f\Big[R^{(3)} + K_{ij}K^{ij} - K^2 + \frac{2}{N\sqrt{h}}\partial_t(\sqrt{h}K) - \frac{2}{N\sqrt{h}}\partial_i(\sqrt{h}KN^i + \sqrt{h}h^{ij}\partial_jN), \phi\Big] + 2P(X,\phi)\Big\}. \qquad (10)$$

The extrinsic tensor K_{ij} is defined as

$$K_{ij} \equiv \frac{1}{2N}(\dot{h}_{ij} - \nabla_iN_j - \nabla_jN_i), \qquad (11)$$

where ∇_i is the covariant derivative with respect to the metric h_{ij} and its indices can be raised and lowered by h_{ij}. The contraction $K \equiv K^i_i$. The three-dimensional Ricci scalar $R^{(3)}$ is computed from the metric h_{ij}. From this action, we are able to obtain the equations of motion for N and N_i (constraint equations) as

$$\begin{aligned} f - 2f_R\Big[&K_{ij}K^{ij} - K^2 + \frac{1}{\sqrt{h}N}\partial_t(\sqrt{h}K) - \frac{1}{\sqrt{h}N}\partial_i(\sqrt{h}KN^i + \sqrt{h}h^{ij}\partial_jN)\Big] \\ &+ \frac{2K}{N}(\partial_tf_R - N^i\partial_if_R) - \frac{2}{\sqrt{h}}\partial_j(\sqrt{h}h^{ij}\partial_if_R) \\ &+ 2P - 2P_Xv^2N^{-2} = 0, \end{aligned} \qquad (12)$$

$$\begin{aligned} \nabla_j(f_RK^{ij}) &- h^{ij}\nabla_j(f_RK) - h^{ij}\nabla_j(N^{-1}\partial_tf_R) \\ &+ h^{ij}\nabla_j(N^{-1}N^l\partial_lf_R) + h^{ij}\partial_jf_RK \\ &- a^{-2}N^{-1}P_Xv\partial^i\phi = 0, \end{aligned} \qquad (13)$$

respectively, where $v \equiv \dot{\phi} - N^i\partial_i\phi$. Moreover, a gauge choice is needed to eliminate the redundant degrees of freedom. Here we choose the uniform density (comoving) gauge, where the perturbations of the scalar field and metric take the following form:

$$\delta\phi = 0, \qquad h_{ij} = a^2e^{2\zeta}\delta_{ij}. \qquad (14)$$

We also need to expand the constraining variables N and N_i. It is only needed to expand them up to the $(n-2)$-th order when we calculate n-th order perturbation [26]. So here we expand them to first order as follows:

$$N = 1 + \alpha, \qquad N_i = \tilde{N}_i + \partial_i \psi, \qquad \partial^i \tilde{N}_i = 0, \quad (15)$$

where α, $\tilde{N}_i$ and ψ are in the first order of ϵ.

III. THE NON-GAUSSIANITY CALCULATION

Since it is very complicated to consider the full types of nonminimal coupling theory, in this paper we will only pick up a subclass where the Ricci scalar is coupled linearly to the scalar field, saying $f(R, \phi) = R f_R(\phi)$. This type of nonminimal coupling is often referred to as scalar-tensor theory [49] or linear coupling [50]. For the nonlinear coupling case, the constraint equations are listed in Appendix A and the solutions will be postponed to future studies.

A. Up to third-order action

Considering all the equations from (12) to (15) in the linear coupling case, one can get

$$-\frac{1}{2} a^{-2} f_{R0} \partial^2 \tilde{N}^i + a^{-2}(2 f_{R0} H + \dot{f}_{R0}) \partial^i \alpha - 2a^{-2} f_{R0} \partial^i \dot{\zeta} = 0, \quad (16)$$

$$4\alpha\Sigma + 12 H f_{R0} \dot{\zeta} - 12 H^2 f_{R0} \alpha - 4a^{-2} f_{R0} H \partial^2 \psi - 4a^{-2} f_{R0} \partial^2 \zeta - (12\alpha H - 6\dot{\zeta} + 2a^{-2}\partial^2\psi) \dot{f}_{R0} = 0, \quad (17)$$

where f_{R0} is the background value of f_R and for the current case $f_{R0} = f_R$. The equations above give the specific solution at first order in ζ:

$$\tilde{N}^i = 0,$$
$$\alpha = \frac{2 f_{R0} \dot{\zeta}}{2 f_{R0} H + \dot{f}_{R0}}, \quad (18)$$
$$\psi = \frac{-2 f_{R0}}{2 f_{R0} H + 2\dot{f}_{R0}} \zeta + a^2 \frac{3\dot{f}_{R0}^2 + 4 f_{R0}\Sigma}{(2 f_{R0} H + \dot{f}_{R0})^2} \partial^{-2} \dot{\zeta}.$$

We define $\theta \equiv \frac{1}{2}\ln(f_{R0} a^2)$, so it will be followed that $e^\theta = f_{R0}^{1/2} a$ and $\dot{\theta} = H + \frac{\dot{f}_{R0}}{2 f_{R0}}$. Note that when $f_{R0} \to 1$, the system will return to the minimal coupling case and $\dot{\theta}$ coincides with the Hubble parameter. It is also convenient to rewrite α and ψ in terms of θ as

$$\alpha = \frac{\dot{\zeta}}{\dot{\theta}},$$
$$\psi = -\frac{\zeta}{\dot{\theta}} + \chi, \quad (19)$$
$$\chi = \left[3a^2\left(1 - \frac{H}{\dot{\theta}}\right)^2 + a^4 \dot{\theta}^{-2} e^{-2\theta}\Sigma\right]\partial^{-2}\dot{\zeta}.$$

Hereafter, we will use this parameter for convenience throughout the paper.

Expanding the action (10) to third order of ζ and substituting Eq. (19) into the action, one can get the expanded action in each order:

$$S_0 = \frac{1}{2}\int dt d^3x a^3 (f_0 + 2P_0), \quad (20)$$

$$S_1 = 0, \quad (21)$$

$$S_2 = \int dt d^3x a e^{2\theta}\left\{3\left(\frac{H^2}{\dot{\theta}^2} - 2\frac{H}{\dot{\theta}} + 1\right)\dot{\zeta}^2 + a^2 e^{-2\theta}\Sigma\frac{\dot{\zeta}^2}{\dot{\theta}^2} + a^{-2}\left(\frac{H}{\dot{\theta}} - 1 + \frac{\ddot{\theta}}{\dot{\theta}^2}\right)(\partial\zeta)^2\right\}, \quad (22)$$

$$\begin{aligned} S_3 = \frac{1}{2}\int dt d^3x a e^{2\theta}\Big\{&+6\left(+2\frac{H}{\dot{\theta}} - 1 - \frac{H^2}{\dot{\theta}^2}\right)\frac{\dot{\zeta}^3}{\dot{\theta}} + 18\left(-2\frac{H}{\dot{\theta}} + 1 + \frac{H^2}{\dot{\theta}^2}\right)\zeta\dot{\zeta}^2 + 2a^{-2}\left(\frac{H}{\dot{\theta}} - 1\right)\zeta(\partial\zeta)^2 \\ &+ 2a^{-2}\frac{\ddot{\theta}\zeta}{\dot{\theta}^2}(\partial\zeta)^2 - 4a^{-4}(\partial\zeta\cdot\partial\psi)\partial^2\psi \\ &+ 3a^{-4}\zeta\partial_i\partial_j\psi\partial^j\partial^i\psi - a^{-4}\frac{\dot{\zeta}}{\dot{\theta}}\partial_i\partial_j\psi\partial^i\partial^j\psi \\ &- 3a^{-4}\zeta(\partial^2\psi)^2 + a^{-4}\frac{\dot{\zeta}}{\dot{\theta}}(\partial^2\psi)^2\Big\} \\ &+ \frac{1}{2}\int dt d^3x a^3\left\{6\Sigma\frac{\dot{\zeta}^2}{\dot{\theta}^2}\zeta - 2(\Sigma + 2\lambda)\frac{\dot{\zeta}^3}{\dot{\theta}^3}\right\}. \end{aligned} \quad (23)$$

This process is very straightforward, but rather tedious. The physical meaning of each equation is easy to understand: The zeroth-order expansion (20) is just the background part of the original action (1) where subscript "0" denotes the background value; the first-order expansion (21) is just the background equation of motion. The second- and third-order expansions only deviate from the general relativity case due to the difference in the parameter $\dot{\theta}$ from H, which will coincide when $f_{R0} \to 1$. However, as will be seen below, this deviation causes very different results of non-Gaussianity in our case from that of GR.

B. Quadratic part: Mode solution

First of all, let us consider the solution of the second-order action (22). This is the most important and kernel step in the calculation of the bispectrum, which will be performed later. The second-order action can be written as

$$\begin{aligned} S_2 &= \int dt d^3x a e^{2\theta}\left\{3\left(\frac{H^2}{\dot{\theta}^2} - 2\frac{H}{\dot{\theta}} + 1\right)\dot{\zeta}^2 + a^2 e^{-2\theta}\Sigma\frac{\dot{\zeta}^2}{\dot{\theta}^2} + a^{-2}\left(\frac{H}{\dot{\theta}} - 1 + \frac{\ddot{\theta}}{\dot{\theta}^2}\right)(\partial\zeta)^2\right\} \\ &= \int d\tau d^3x e^{2\theta}\left\{3\left(\frac{H^2}{\dot{\theta}^2} - 2\frac{H}{\dot{\theta}} + 1\right)u_{\vec{k}}'^2(\tau) + a^2 e^{-2\theta}\frac{\Sigma}{\dot{\theta}^2}u_{\vec{k}}'^2(\tau) - \left(\frac{H}{\dot{\theta}} - 1 + \frac{\ddot{\theta}}{\dot{\theta}^2}\right)k^2 u_{\vec{k}}^2(\tau)\right\}, \end{aligned} \quad (24)$$

where in the second step we have used conformal time $\tau \equiv \int \frac{dt}{a(t)}$ and transformed the variable $\zeta(\tau, \vec{x})$ into its Fourier form, namely,

$$\zeta(\tau, \vec{x}) = \int \frac{d\vec{k}}{\sqrt{2E}}[u_{\vec{k}}(\tau)a_{\vec{k}}e^{-i\vec{k}\cdot\vec{x}} + u^*_{\vec{k}}(\tau)a^\dagger_{\vec{k}}e^{i\vec{k}\cdot\vec{x}}]. \quad (25)$$

It is convenient to define another variable v_k as $v_k \equiv zu_{\vec{k}}(\tau)$, where

$$z = \sqrt{\left|3e^{2\theta}\left(\frac{H}{\dot{\theta}} - 1\right)^2 + \frac{a^2\Sigma}{\dot{\theta}^2}\right|} \quad (26)$$

to let the equation of motion for v_k be in a canonical form, which is

$$v_k'' + \left(c_s^2k^2 - \frac{z''}{z}\right)v_k = 0. \quad (27)$$

Here the effective sound speed squared c_s^2 is defined as $c_s^2 = (\dot{\theta}^2 - H\dot{\theta} - \ddot{\theta})/|3(H - \dot{\theta})^2 + f_{R0}^{-1}\Sigma|$, and thus one can have $z = \frac{e^\theta}{c_s}\sqrt{\epsilon_\theta + 1 - \frac{H}{\dot{\theta}}}$.

In the inflation period, the Hubble parameter changes slowly so as to have fast enough expansion. Here in order to solve the equation above, we need to introduce another parameter:

$$\epsilon_\theta \equiv -\frac{\ddot{\theta}}{\dot{\theta}^2}, \quad (28)$$

which describes the variation of f_{R0} with respect to time. In the case where ϵ_h [see Eq. (6)] and ϵ_θ are both small and $\dot{\epsilon}_i \ll \epsilon_i (i = h, \theta)$, we can take the leading order of ϵ_h and ϵ_θ so that Eq. (27) becomes

$$v_{\vec{k}}'' + \left(c_s^2k^2 - \frac{\mu^2 + \mu}{\tau^2}\right)v_{\vec{k}} = 0, \quad (29)$$

where $\mu \equiv \epsilon_h/\epsilon_\theta$ is the ratio of the two slow-roll parameters. The solution of Eq. (29) can be presented in form of the Hankal function

$$v_{\vec{k}}(\tau) = C\sqrt{c_sk|\tau|}H_{\pm(\mu+(1/2))}(c_sk|\tau|), \quad (30)$$

where C is an undetermined constant denoting the amplitude of the solution. In deriving this, the approximation of slow-varying sound speed $\dot{c}_s \ll 1$ is also taken.

The solution (30) can be split into two limits corresponding to the sub-Hubble and super-Hubble regions, respectively. In the super-Hubble region, we can take the limit to be

$$\begin{aligned} v_{\vec{k}} \to \sqrt{c_sk|\tau|}\Big(&\frac{C}{\Gamma(\mu + \frac{3}{2})}(c_sk|\tau|)^{\mu+(1/2)} \\ &+ \frac{C}{\Gamma(-\mu + \frac{1}{2})}(c_sk|\tau|)^{-(\mu+(1/2))}\Big) \\ = &\frac{C}{\Gamma(\mu + \frac{3}{2})}(c_sk|\tau|)^{\mu+1} + \frac{C}{\Gamma(-\mu + \frac{1}{2})}(c_sk|\tau|)^{-\mu}, \end{aligned} \quad (31)$$

and from the relation $v_k = zu_{\vec{k}}(\tau)$ one has

$$\begin{aligned} u_{\vec{k}} = &\frac{e^{-\theta}c_s}{\sqrt{|\epsilon_\theta + 1 - \frac{H}{\dot{\theta}}|}}\Big\{\frac{C}{\Gamma(\mu + \frac{3}{2})}(c_sk|\tau|)^{\mu+1} \\ &+ \frac{C}{\Gamma(-\mu + \frac{1}{2})}(c_sk|\tau|)^{-\mu}\Big\} \\ = &\frac{\mu^{1/2}H}{f_{R0}^{1/2}(-1)c_s^{\mu-1}k^\mu|\epsilon_h + \mu - 1|^{1/2}} \\ &\times\Big\{\frac{C}{\Gamma(\mu + \frac{3}{2})}(c_sk|\tau|)^{2\mu+1} + \frac{C}{\Gamma(-\mu + \frac{1}{2})}\Big\} \end{aligned} \quad (32)$$

where $f_{R0}(-1)$ denotes the value of f_{R0} at $\tau = 1$ and f_{R0} can be parameterized as $f_{R0} \simeq f_{R0}(-1)|\tau|^{2(1-\mu)}$. In deriving these equations we also used the approximation $\tau = -1/(aH) + \mathcal{O}(\epsilon_h)$. The solution contains a constant mode and a decaying mode, the latter of which is irrelevant and should be discarded. In the sub-Hubble region, we can take the limit as

$$v_{\vec{k}} \to C\sqrt{\frac{2}{\pi}}e^{ic_sk|\tau|}e^{i(\mu\pi/2)}, \quad (33)$$

where we also discarded the $+(\mu + \frac{1}{2})$ branch. On the other hand, one can use WKB method to calculate the sub-Hubble solution of Eq. (29), which is

$$v_{\vec{k}} \simeq \frac{i\mathcal{H}}{\sqrt{8c_s^3k^3}}e^{-ic_sk\tau}[\mu(1 + \mu) + 2ic_sk\tau]. \quad (34)$$

Comparing Eqs. (33) and (34) at $\tau \to -\infty$, one can determine the coefficient C as

$$C = \sqrt{\frac{\pi}{4c_sk}}e^{-i(\mu\pi/2)}. \quad (35)$$

With this in hand, we can have the exact solution of $u_{\vec{k}}$ in both super-Hubble and sub-Hubble limits. Substituting this back to (32), we get the final form of the super-Hubble solution:

$$\begin{aligned} u_{\vec{k}} = &\frac{\mu^{1/2}H}{2f_{R0}^{1/2}(-1)c_s^{\mu-1}k^\mu|\epsilon_h + \mu - 1|^{1/2}\Gamma(-\mu + \frac{1}{2})} \\ &\times\sqrt{\frac{\pi}{c_sk}}e^{-i(\mu\pi/2)}. \end{aligned} \quad (36)$$

It is a time-independent mode and thus can be applied to far future where $\tau = 0$. From this we can also obtain the power spectrum of ζ, which is

$$\mathcal{P}_k^\zeta \equiv \frac{k^3}{2\pi^2}|u_{\vec{k}}|^2 = \frac{\mu H^2}{8\pi f_{R0}(-1)c_s^{2\mu-1}k^{2\mu-2}|\epsilon_h + \mu - 1|\Gamma^2(-\mu+\frac{1}{2})}, \tag{37}$$

and the spectrum index is $n_\zeta \equiv \frac{d\ln\mathcal{P}_k^\zeta}{d\ln k} + 1 = 2(1-\mu)+1$. One can see from this that the power gets a red spectrum ($n_\zeta < 1$) when $\mu > 1$ while a blue spectrum ($n_\zeta > 1$) will be obtained at $\mu < 1$. Moreover, the constraints that the primordial spectrum must be nearly scale invariant requires that $|\mu - 1| \sim \mathcal{O}(\epsilon)$.

Furthermore, using Eq. (33), the sub-Hubble solution can be solved as

$$\begin{aligned} u_{\vec{k}} &\simeq \frac{i\mathcal{H}e^{-\theta}\mu^{1/2}}{2\sqrt{2}c_s^{1/2}|\vec{k}|^{3/2}|\epsilon_h+\mu-1|^{1/2}} \times e^{-ic_s|\vec{k}|\tau}[\mu(1+\mu)+2ic_s|\vec{k}|\tau] \\ &= \frac{iH\mu^{1/2}}{2\sqrt{2}c_s^{1/2}k^{3/2}f_{R0}^{1/2}|\epsilon_h+\mu-1|^{1/2}} \times e^{-ic_sk\tau}[\mu(1+\mu)+2ic_sk\tau] \\ &= \frac{iH\mu^{1/2}}{2\sqrt{2}c_s^{1/2}k^{3/2}f_{R0}^{1/2}(-1)|\epsilon_h+\mu-1|^{1/2}} \times e^{-ic_sk\tau}[\mu(1+\mu)|\tau|^{\mu-1} - 2ic_sk|\tau|^{\mu}], \end{aligned} \tag{38}$$

and

$$\frac{d}{d\tau}u_{\vec{k}}^*(\tau) = \frac{iH\mu^{1/2}c_s^{3/2}k^{1/2}e^{ic_s|\vec{k}|\tau}}{\sqrt{2}f_{R0}^{1/2}(-1)|\epsilon_h+\mu-1|^{1/2}}|\tau|^{\mu}, \tag{39}$$

where we keep only the leading order terms in terms of ϵ. The results above will be useful for our analysis of non-Gaussianity in the next paragraph (i.e., Sec. III C).

C. Cubic part: Non-Gaussianities

According to "in-in" formalism [51], the 3-point correlation function is characterized in the interaction picture as

$$\langle|\zeta(\tau,\vec{k}_1)\zeta(\tau,\vec{k}_2)\zeta(\tau,\vec{k}_3)|\rangle = -i\mathcal{T}\int_{t_0}^{t}dt'\langle|[\zeta(t,\vec{k}_1)\zeta(t,\vec{k}_2)\zeta(t,\vec{k}_3), H_{\rm int}^{p}(t')]|\rangle, \tag{40}$$

where $H_{\rm int}^p$ is the third-order interaction Hamiltonian and $\mathcal{T}$ is the time-ordering operator. From the third-order action (23), the third-order Hamiltonian can be written as

$$\begin{aligned} H_{\rm int} = \int dtd^3xae^{2\theta}\Big\{&[a^2e^{-2\theta}(\Sigma+2\lambda)+3(\dot\theta-H)^2]\frac{\dot\zeta^3}{\dot\theta^3} - 3[a^2e^{-2\theta}\Sigma+3(\dot\theta-H)^2]\frac{\zeta\dot\zeta^2}{\dot\theta^2} \\ &- a^{-2}\Big(\frac{H}{\dot\theta}-1+\frac{\ddot\theta}{\dot\theta^2}\Big)\zeta(\partial\zeta)^2 - a^{-4}\dot\theta^{-1}\Big[3a^2\Big(1-\frac{H}{\dot\theta}\Big)^2 + a^4\dot\theta^{-2}e^{-2\theta}\Sigma\Big]\dot\zeta(\partial\zeta)^2 \\ &+ 2a^{-4}\Big[3a^2\Big(1-\frac{H}{\dot\theta}\Big)^2 + a^4\dot\theta^{-2}e^{-2\theta}\Sigma\Big]\dot\zeta\partial\zeta\partial\chi\Big\}, \end{aligned} \tag{41}$$

or, if changed to momentum space,

$$\begin{aligned} H_{\rm int}^p = \int &\frac{d^3p_1d^3p_2d^3p_3}{(2\pi)^9}(2\pi)^3\delta^3(\vec{p}_1+\vec{p}_2+\vec{p}_3)ae^{2\theta}\Big\{[a^2e^{-2\theta}(\Sigma+2\lambda)+3(\dot\theta-H)^2]\frac{\dot\zeta(t,\vec{p}_1)\dot\zeta(t,\vec{p}_2)\dot\zeta(t,\vec{p}_3)}{\dot\theta^3} \\ &- 3[a^2e^{-2\theta}\Sigma+3(\dot\theta-H)^2]\frac{\zeta(t,\vec{p}_1)\dot\zeta(t,\vec{p}_2)\dot\zeta(t,\vec{p}_3)}{\dot\theta^2} - a^{-2}\Big(\frac{H}{\dot\theta}-1+\frac{\ddot\theta}{\dot\theta^2}\Big)(\vec{p}_2\cdot\vec{p}_3)\zeta(t,\vec{p}_1)\zeta(t,\vec{p}_2)\zeta(t,\vec{p}_3) \\ &- a^{-4}\dot\theta^{-1}\Big[3a^2\Big(1-\frac{H}{\dot\theta}\Big)^2 + a^4\dot\theta^{-2}e^{-2\theta}\Sigma\Big](\vec{p}_2\cdot\vec{p}_3)\dot\zeta(t,\vec{p}_1)\zeta(t,\vec{p}_2)\zeta(t,\vec{p}_3) \\ &+ 2a^{-4}\Big[3a^2\Big(1-\frac{H}{\dot\theta}\Big)^2 + a^4\dot\theta^{-2}e^{-2\theta}\Sigma](\vec{p}_2\cdot\vec{p}_3)\dot\zeta(t,\vec{p}_1)\zeta(t,\vec{p}_2)\chi(t,\vec{p}_3)\Big\}. \end{aligned} \tag{42}$$

There are five terms of third order, each containing a long prefactor. Using Eq. (40), we can calculate their contributions to non-Gaussianities. Neglecting the detailed calculation process, we only give the final results of the contributions to non-Gaussianities from each term as follows:

The contribution from $\dot\zeta^3$:

$$\begin{aligned} &-6i\frac{\Sigma+2\lambda}{\mu^3H^4}u_{\vec{k}_1}(0)u_{\vec{k}_2}(0)u_{\vec{k}_3}(0)\int_{-\infty}^{0}\frac{-1}{\tau}d\tau(2\pi)^3\delta^3\Big(\sum_i\vec{k}_i\Big)\frac{d}{d\tau}u^*_{-\vec{k}_1}(\tau)\frac{d}{d\tau}u^*_{-\vec{k}_2}(\tau)\frac{d}{d\tau}u^*_{-\vec{k}_3}(\tau) \\ &\quad -18i\frac{(\mu-1)^2f_{R0}(-1)}{\mu^3H^2}u_{\vec{k}_1}(0)u_{\vec{k}_2}(0)u_{\vec{k}_3}(0)\int_{-\infty}^{0}|\tau|^{1-2\mu}d\tau(2\pi)^3\delta^3\Big(\sum_i\vec{k}_i\Big)\frac{d}{d\tau}u^*_{-\vec{k}_1}(\tau)\frac{d}{d\tau}u^*_{-\vec{k}_2}(\tau)\frac{d}{d\tau}u^*_{-\vec{k}_3}(\tau) + c.c., \end{aligned} \tag{43}$$

The contribution from $\zeta\dot\zeta^2$:

$$6i\frac{\Sigma}{\mu^2H^4}u_{\vec{k}_1}(0)u_{\vec{k}_2}(0)u_{\vec{k}_3}(0)\int_{-\infty}^{0}\frac{1}{\tau^2}d\tau(2\pi)^3\delta^3\left(\sum_i\vec{k}_i\right)[u^*_{-\vec{k}_1}(\tau)\frac{d}{d\tau}u^*_{-\vec{k}_2}(\tau)\frac{d}{d\tau}u^*_{-\vec{k}_3}(\tau)+2\,\text{perms}]$$
$$18i\frac{(\mu-1)^2f_{R0}(-1)}{\mu^2H^2}u_{\vec{k}_1}(0)u_{\vec{k}_2}(0)u_{\vec{k}_3}(0)\int_{-\infty}^{0}|\tau|^{-2\mu}d\tau(2\pi)^3\delta^3\left(\sum_i\vec{k}_i\right)\left[u^*_{-\vec{k}_1}(\tau)\frac{d}{d\tau}u^*_{-\vec{k}_2}(\tau)\frac{d}{d\tau}u^*_{-\vec{k}_3}(\tau)+2\,\text{perms}\right]+c.c., \tag{44}$$

The contribution from $\zeta(\partial\zeta)^2$:

$$2i\frac{(\mu-1)f_{R0}(-1)}{\mu H^2}u_{\vec{k}_1}(0)u_{\vec{k}_2}(0)u_{\vec{k}_3}(0)\int_{-\infty}^{0}|\tau|^{-2\mu}d\tau(2\pi)^3\delta^3\left(\sum_i\vec{k}_i\right)[(\vec{k}_2\cdot\vec{k}_3)u^*_{-\vec{k}_1}(\tau)u^*_{-\vec{k}_2}(\tau)u^*_{-\vec{k}_3}(\tau)+2\,\text{perms}]+c.c., \tag{45}$$

The contribution from $\dot{\zeta}(\partial\zeta)^2$:

$$-6i\frac{(\mu-1)^2f_{R0}(-1)}{\mu^3H^2}u_{\vec{k}_1}(0)u_{\vec{k}_2}(0)u_{\vec{k}_3}(0)\int_{-\infty}^{0}|\tau|^{1-2\mu}d\tau(2\pi)^3\delta^3\left(\sum_i\vec{k}_i\right)[(\vec{k}_2\cdot\vec{k}_3)\frac{d}{d\tau}u^*_{-\vec{k}_1}(\tau)u^*_{-\vec{k}_2}(\tau)u^*_{-\vec{k}_3}(\tau)+2\,\text{perms}]$$
$$-2i\frac{\Sigma}{\mu^3H^4}u_{\vec{k}_1}(0)u_{\vec{k}_2}(0)u_{\vec{k}_3}(0)\int_{-\infty}^{0}\frac{-1}{\tau}d\tau(2\pi)^3\delta^3\left(\sum_i\vec{k}_i\right)[(\vec{k}_2\cdot\vec{k}_3)\frac{d}{d\tau}u^*_{-\vec{k}_1}(\tau)u^*_{-\vec{k}_2}(\tau)u^*_{-\vec{k}_3}(\tau)+2\,\text{perms}]+c.c., \tag{46}$$

The contribution from $\dot{\zeta}\partial\zeta\partial\chi$:

$$-18i\frac{(\mu-1)^4f_{R0}(-1)}{\mu^4H^2}u_{\vec{k}_1}(0)u_{\vec{k}_2}(0)u_{\vec{k}_3}(0)\int_{-\infty}^{0}|\tau|^{-2\mu}d\tau(2\pi)^3\delta^3\left(\sum_i\vec{k}_i\right)\left[\frac{\vec{k}_2\cdot\vec{k}_3}{k_3^2}\frac{d}{d\tau}u^*_{-\vec{k}_1}(\tau)u^*_{-\vec{k}_2}(\tau)\frac{d}{d\tau}u^*_{-\vec{k}_3}(\tau)+5\,\text{perms}\right]$$
$$-2i\frac{\Sigma^2}{\mu^4H^6f_{R0}(-1)}u_{\vec{k}_1}(0)u_{\vec{k}_2}(0)u_{\vec{k}_3}(0)\int_{-\infty}^{0}|\tau|^{2\mu-4}d\tau(2\pi)^3\delta^3\left(\sum_i\vec{k}_i\right)\left[\frac{\vec{k}_2\cdot\vec{k}_3}{k_3^2}\frac{d}{d\tau}u^*_{-\vec{k}_1}(\tau)u^*_{-\vec{k}_2}(\tau)\frac{d}{d\tau}u^*_{-\vec{k}_3}(\tau)+5\,\text{perms}\right]$$
$$-12i\frac{(\mu-1)^2\Sigma}{\mu^4H^4}u_{\vec{k}_1}(0)u_{\vec{k}_2}(0)u_{\vec{k}_3}(0)\int_{-\infty}^{0}\frac{1}{\tau^2}d\tau(2\pi)^3\delta^3\left(\sum_i\vec{k}_i\right)\left[\frac{\vec{k}_2\cdot\vec{k}_3}{k_3^2}\frac{d}{d\tau}u^*_{-\vec{k}_1}(\tau)u^*_{-\vec{k}_2}(\tau)\frac{d}{d\tau}u^*_{-\vec{k}_3}(\tau)+5\,\text{perms}\right]+c.c. \tag{47}$$

In all of the contributions above, we can substitute the explicit forms of $\frac{d}{d\tau}u^*_{\vec{k}}$ into the equation above to get a lot of integrals with τ. It is straightforward, but the result is rather boring and page-wasting, so we have listed them in Appendix B. Actually, it has nine terms differing from each an order of τ sequentially, plus permutations and complex conjugates. At the end we will compile all these terms coming from contributions of all the terms in the Hamiltonian according to their indices in a clearer form in order to make our study more readable.

D. The shapes of bispectrum

From Eqs. (43)–(47) and also (B1)–(B5), we can integrate them out to have different shapes. Since the power-law indices of τ in each integration depend on the value of μ, one may worry that for indices less than -1, the infrared (IR) divergence will occur. However, that is not the case. As has already been shown in Sec. III B, the mode solution is frozen outside of horizon and there will be no more IR evolution. Actually, the IR divergences will all be canceled with each other, and we do not see any real singularity, though it is tedious to check analytically and even difficult numerically. The same argument can be found in Chen and Wang (J. Cosmol. Astropart. Phys., 2010) as cited in Ref. [28] of this paper. Furthermore, we can replace the value of $u_{\vec{k}}$ at far future $\tau=0$ to that at a horizon-crossing time. Taking these into consideration, we first write down all the possible shapes of the bispectrum. Here we define

$$\langle|\zeta(\tau,\vec{k}_1)\zeta(\tau,\vec{k}_2)\zeta(\tau,\vec{k}_3)|\rangle=(2\pi)^3\delta^3\left(\sum_i\vec{k}_i\right)\mathcal{B}(k_1,k_2,k_3), \tag{48}$$

where $\mathcal{B}(k_1,k_2,k_3)$ are the shapes of non-Gaussianity. There are a total of 10 shapes at the leading order:

$$\mathcal{B}_{3\mu-3} = \frac{(2\pi)^{3/2}\Sigma H^2\cos(3\mu\pi)\Gamma(3\mu-2)}{8^2 f_{R0}^3(-1)c_s^{6\mu-6}(k_1k_2k_3)^{\mu+2}K^{3\mu-2}|\epsilon_h+\mu-1|^3\Gamma^3(-\mu+\frac{1}{2})} \times\left((1+\mu)(3\mu^2+\frac{\mu^2(1+\mu)}{2c_s^2}-12(\mu-1)^2)\sum_{i>j}k_i^2k_j^2-(1+\mu)\left(\frac{\mu^2(1+\mu)}{4c_s^2}-6(\mu-1)^2\right)\sum_i k_i^4\right), \tag{49}$$

$$\mathcal{B}_{3\mu-2} = \frac{(2\pi)^{3/2}\Sigma H^2\cos(3\mu\pi)\Gamma(3\mu-1)}{8^2 f_{R0}^3(-1)c_s^{6\mu-6}(k_1k_2k_3)^{\mu+1}K^{3\mu-1}|\epsilon_h+\mu-1|^3\Gamma^3(-\mu+\frac{1}{2})} \times\left(\frac{1}{\mu}\left(\frac{\mu^2(1+\mu)}{2c_s^2}-6(\mu-1)^2\right)\left(\prod_i\frac{1}{k_i}\right)\sum_{i\neq j}(k_i^2k_j^3-k_ik_j^4)+\frac{1}{\mu}\left(6\mu^2+\frac{\mu^2(1+\mu)}{c_s^2}-12(\mu-1)^2\right)\sum_{i>j}k_ik_j\right), \tag{50}$$

$$\mathcal{B}_{3\mu-1} = \frac{2(2\pi)^{3/2}H^2\cos(3\mu\pi)\Gamma(3\mu)}{8^2 f_{R0}^3(-1)c_s^{6\mu-6}(k_1k_2k_3)^{\mu}K^{3\mu}|\epsilon_h+\mu-1|^3\Gamma^3(-\mu+\frac{1}{2})}\left(\frac{\Sigma}{2c_s^2}\left(\prod_i\frac{1}{k_i}\right)\left(\sum_{i\neq j}k_ik_j^2-\sum_i k_i^3\right)-3(\Sigma+2\lambda)\right), \tag{51}$$

$$\mathcal{B}_{\mu-3} = \frac{(2\pi)^{3/2}\mu^5(\mu-1)(1+\mu)^3H^4\cos(2\mu\pi)\Gamma(\mu-2)}{8^3c_s^{4\mu-2}f_{R0}^2(-1)(\prod_i k_i^{\mu+2})K^{\mu-2}|\epsilon_h+\mu-1|^3\Gamma^3(-\mu+\frac{1}{2})}\sum_i k_i^2, \tag{52}$$

$$\mathcal{B}_{\mu-2} = -\frac{2(2\pi)^{3/2}\mu^4(\mu-1)(1+\mu)^2H^4\cos(2\mu\pi)\Gamma(\mu-1)}{8^3c_s^{4\mu-2}f_{R0}^2(-1)(\prod_i k_i^{\mu+2})K^{\mu-2}|\epsilon_h+\mu-1|^3\Gamma^3(-\mu+\frac{1}{2})}\sum_i k_i^2, \tag{53}$$

$$\mathcal{B}_{\mu-1} = \frac{(2\pi)^{3/2}(\mu^2-1)H^4\cos(2\mu\pi)\Gamma(\mu)}{8^2 f_{R0}^2(-1)c_s^{4\mu-4}(k_1k_2k_3)^{\mu+2}K^{\mu}|\epsilon_h+\mu-1|^3\Gamma^3(-\mu+\frac{1}{2})} \times\left(9(\mu-1)(2(\mu-1)^2-\mu^2)\sum_{i>j}k_i^2k_j^2-9(\mu-1)^3\sum_i k_i^4+\frac{\mu^3}{2c_s^2}\left(\sum_{i>j}k_ik_j\right)\sum_i k_i^2\right), \tag{54}$$

$$\mathcal{B}_{\mu} = \frac{2(2\pi)^{3/2}H^4\cos(2\mu\pi)\Gamma(\mu+1)}{8^2 f_{R0}^2(-1)c_s^{4\mu-4}(k_1k_2k_3)^{\mu+2}K^{\mu+1}|\epsilon_h+\mu-1|^3\Gamma^3(-\mu+\frac{1}{2})} \times\left((\mu-1)\left(\prod_i k_i\right)\left(9\left(\frac{1}{\mu}-1\right)(2\mu-1)\sum_{i>j}k_ik_j+\frac{\mu^2}{2}\sum_i k_i^2\right)+\frac{9(\mu-1)^4}{2\mu}\sum_{i\neq j}(k_i^2k_j^3-k_ik_j^4)\right), \tag{55}$$

$$\mathcal{B}_{\mu+1} = \frac{18(2\pi)^{3/2}(\mu-1)^2H^4\cos(2\mu\pi)\Gamma(\mu+2)}{8^3 f_{R0}^2(-1)c_s^{4\mu-4}(k_1k_2k_3)^{\mu}K^{\mu+2}|\epsilon_h+\mu-1|^3\Gamma^3(-\mu+\frac{1}{2})}, \tag{56}$$

$$\mathcal{B}_{5\mu-5} = \frac{(2\pi)^{3/2}\Sigma^2(1+\mu)\cos(4\mu\pi)\Gamma(5\mu-4)}{8^2 f_{R0}^4(-1)c_s^{8\mu-8}(k_1k_2k_3)^{\mu+2}K^{5\mu-4}|\epsilon_h+\mu-1|^3\Gamma^3(-\mu+\frac{1}{2})}\left(2\sum_{i>j}k_i^2k_j^2-\sum_i k_i^4\right), \tag{57}$$

$$\mathcal{B}_{5\mu-4} = \frac{2(2\pi)^{3/2}\Sigma^2\cos(4\mu\pi)\Gamma(5\mu-3)}{8^2 f_{R0}^4(-1)c_s^{8\mu-8}(k_1k_2k_3)^{\mu+2}K^{5\mu-3}\mu|\epsilon_h+\mu-1|^3\Gamma^3(-\mu+\frac{1}{2})}\left(\frac{1}{2}\sum_{i\neq j}k_i^2k_j^3+\left(\prod_i k_i\right)\sum_{i>j}k_ik_j-\frac{1}{2}\sum_{i\neq j}k_ik_j^4\right). \tag{58}$$

For the next step, we have to discard the terms that will be divergent as the indices in the original integral becomes less than -1, and sum up all the convergent terms to give the total shape. It depends on the value of μ, obviously. In the following we give the total shape for different values of μ. The final result is

$$\mathcal{B}_{\text{total}} = \sum_y \Theta(y+1)\mathcal{B}_y, \tag{59}$$

where Θ is the Heaviside step function defined as

$$\Theta(x) = \begin{cases} 0, & \text{for } x<0, \\ 1 & \text{for } x>0, \end{cases} \tag{60}$$

and y for all the subscripts of $\mathcal{B}$ above. From this result, we can also divide all the shapes to four classes: (i) $\mathcal{B}_{3\mu-1}$, $\mathcal{B}_{\mu-1}$, $\mathcal{B}_{\mu}$, $\mathcal{B}_{\mu+1}$: Since we assume that the slow-roll parameters ϵ_h and ϵ_θ larger than 0, thus the parameter $\mu>0$, and the indices of these shapes are larger than -1.

So they will definitely contribute to $\mathcal{B}_{\rm total}$; (ii) $\mathcal{B}_{3\mu-2}$, $\mathcal{B}_{3\mu-3}$, $\mathcal{B}_{5\mu-4}$, $\mathcal{B}_{5\mu-5}$: If $\mu > 1$, i.e., the power spectrum gets a red index, the indices of these shapes are larger than -1 and they will contribute to $\mathcal{B}_{\rm total}$; (iii) $\mathcal{B}_{\mu-3}$: If $\mu < 1$, i.e., the power spectrum gets a blue index, the indices of these shapes are smaller than -1 and they will not contribute to $\mathcal{B}_{\rm total}$; and (iv) $\mathcal{B}_{\mu-2}$: The indices of these shapes are at "divide" values. So when $\mu > 1$, they will contribute to $\mathcal{B}_{\rm total}$ while when $\mu < 1$, they will not.

We can also define the estimator through

$$\mathcal{B}(k_1k_2k_3) = \frac{6}{5}F_{NL}\left\{\frac{2\pi^2}{k_1^3}\frac{2\pi^2}{k_2^3}\mathcal{P}^{\zeta}_{\vec{k}_1}\mathcal{P}^{\zeta}_{\vec{k}_2} + 2\,\text{perms}\right\}, \tag{61}$$

so,

$$F_{\rm NL} = \frac{5}{6}\frac{\mathcal{B}(k_1k_2k_3)}{\{\frac{2\pi^2}{k_1^3}\frac{2\pi^2}{k_2^3}\mathcal{P}^{\zeta}_{\vec{k}_1}\mathcal{P}^{\zeta}_{\vec{k}_2} + 2\,\text{perms}\}} = \frac{40f_{R0}^2(-1)c_s^{4\mu-2}|\epsilon_h+\mu-1|^2\Gamma^4(-\mu+\frac{1}{2})}{3\pi^2\mu^2H^4\sum_{i>j}(k_ik_j)^{-(2\mu+1)}}\mathcal{B}(k_1,k_2,k_3) \tag{62}$$

for each shape listed above. The result is rather obvious by just substituting each shape into Eq. (6), so we will not list them here for simplicity. We will only show the equilateral limits of these $F_{\rm NL}$'s, of which $k_1 = k_2 = k_3 = k$, in the next paragraph (i.e., Sec. III E).

E. The equilateral limit ($k_1 = k_2 = k_3 = k$)

In this section, we can take the equilateral limit, namely, $k_1 = k_2 = k_3 = k$, which gives simpler form of $F_{\rm NL}$. Following Sec. III D above, we have

$$(F_{\rm NL})^{\rm equil}_{3\mu-3} = k^{2-2\mu}\frac{5(2\pi)^{3/2}\Sigma\cos(3\mu\pi)\Gamma(3\mu-2)\Gamma(-\mu+\frac{1}{2})}{24\times 3^{3\mu-2}\pi^2f_{R0}(-1)\mu^2c_s^{2\mu-4}H^2|\epsilon_h+\mu-1|}(1+\mu)\left(3\mu^2+\frac{\mu^2(1+\mu)}{4c_s^2}-6(\mu-1)^2\right), \tag{63}$$

$$(F_{\rm NL})^{\rm equil}_{3\mu-2} = k^{2-2\mu}\frac{5(2\pi)^{3/2}\Sigma\cos(3\mu\pi)\Gamma(3\mu-1)\Gamma(-\mu+\frac{1}{2})}{24\times 3^{3\mu-1}\pi^2f_{R0}(-1)\mu^3c_s^{2\mu-4}H^2|\epsilon_h+\mu-1|}\left(6\mu^2+\frac{\mu^2(1+\mu)}{c_s^2}-12(\mu-1)^2\right), \tag{64}$$

$$(F_{\rm NL})^{\rm equil}_{3\mu-1} = k^{2-2\mu}\frac{5(2\pi)^{3/2}\cos(3\mu\pi)\Gamma(3\mu)\Gamma(-\mu+\frac{1}{2})}{12\times 3^{3\mu}\pi^2f_{R0}(-1)\mu^2H^2c_s^{2\mu-4}|\epsilon_h+\mu-1|}\left(\Sigma\left(\frac{1}{2c_s^2}-1\right)-2\lambda\right), \tag{65}$$

$$(F_{\rm NL})^{\rm equil}_{\mu-3} = \frac{5(2\pi)^{3/2}\mu^3(\mu-1)(1+\mu)^3\cos(2\mu\pi)\Gamma(\mu-2)\Gamma(-\mu+\frac{1}{2})}{192\times 3^{\mu-2}\pi^2|\epsilon_h+\mu-1|}, \tag{66}$$

$$(F_{\rm NL})^{\rm equil}_{\mu-2} = -\frac{5(2\pi)^{3/2}\mu^2(\mu-1)(1+\mu)^2\cos(2\mu\pi)\Gamma(\mu-1)\Gamma(-\mu+\frac{1}{2})}{96\times 3^{\mu-2}\pi^2|\epsilon_h+\mu-1|}, \tag{67}$$

$$(F_{\rm NL})^{\rm equil}_{\mu-1} = \frac{5(2\pi)^{3/2}(\mu^2-1)c_s^2\cos(2\mu\pi)\Gamma(\mu)\Gamma(-\mu+\frac{1}{2})}{8\times 3^{\mu}\pi^2\mu^2|\epsilon_h+\mu-1|}\left(3(\mu-1)^3-3(\mu-1)\mu^2+\frac{\mu^3}{2c_s^2}\right), \tag{68}$$

$$(F_{\rm NL})^{\rm equil}_{\mu} = \frac{5(2\pi)^{3/2}c_s^2\cos(2\mu\pi)\Gamma(\mu+1)\Gamma(-\mu+\frac{1}{2})}{24\times 3^{\mu+1}\pi^2\mu^3|\epsilon_h+\mu-1|}(\mu-1)(\mu^3-18(\mu-1)(2\mu-1)), \tag{69}$$

$$(F_{\rm NL})^{\rm equil}_{\mu+1} = \frac{5(2\pi)^{3/2}(\mu-1)^2c_s^2\cos(2\mu\pi)\Gamma(\mu+2)\Gamma(-\mu+\frac{1}{2})}{32\times 3^{\mu+2}\pi^2\mu^2|\epsilon_h+\mu-1|}, \tag{70}$$

$$(F_{\rm NL})^{\rm equil}_{5\mu-5} = k^{4-4\mu} \frac{5(2\pi)^{3/2}\Sigma^2(1+\mu)\cos(4\mu\pi)\Gamma(5\mu-4)\Gamma(-\mu+\frac{1}{2})}{24\times 3^{5\mu-4}\pi^2 f_{R0}^2(-1)\mu^2 H^4 c_s^{4\mu-6}|\epsilon_h+\mu-1|}, \tag{71}$$

$$(F_{\rm NL})^{\rm equil}_{5\mu-4} = k^{4-4\mu} \frac{5(2\pi)^{3/2}\Sigma^2\cos(4\mu\pi)\Gamma(5\mu-3)\Gamma(-\mu+\frac{1}{2})}{12\times 3^{5\mu-3}\pi^2 f_{R0}^2(-1)\mu^3 H^4 c_s^{4\mu-6}|\epsilon_h+\mu-1|}, \tag{72}$$

and as same as the shape, we have for the total nonlinear parameter

$$(F_{\rm NL})^{\rm equil}_{\rm total} = \sum_y \Theta(y+1)(F_{\rm NL})^{\rm equil}_y. \tag{73}$$

From the result above we can see that $(F_{\rm NL})^{\rm equil}$ has a slight running behavior due to the deviation of μ from 1, that is, due to the nonminimal coupling behavior. This is different from the usual minimal coupling case which was studied in [26] where the equilateral limit of $F_{\rm NL}$ is independent of k. This is another result in this paper and will later be confirmed with numerical calculations. One can also obtain $F_{\rm NL}$ in the local limit ($k_1 \approx k_2 \gg k_3$) and folded limit ($k_1 = 2k_2 = 2k_3$). Whichever limits they are in, all the values of $F_{\rm NL}$ with different indices can also be divided into four classes by the same criteria used for $\mathcal{B}(k_1, k_2, k_3)$.

F. An explicit example: Nonminimal coupled chaotic inflation

In order to support our long analytical derivation, in this section we focus on an explicit model of nonminimal coupling inflation. For simplicity but without losing generality, we consider the chaotic inflation, of which the potential has a quadratic form as $V(\phi) = \lambda\phi^4/4$, where λ is the coupling coefficients. Furthermore, we set the nonminimal coupling term $f(R,\phi) = \frac{R}{8\pi G} + \xi R\phi^2$, where G is the Newtonian gravitational constant and ξ is the nonminimal coupling coefficient. This model is indeed very interesting since with the presence of the nonminimal coupling term, the coefficient λ does not need to go to an incredibly small value (Fakir and Unruh, [6]) to meet the observational constraint, and this property has been used to construct Higgs inflation models, which connects inflation theory to particle physics in the standard model [52]. In Higgs inflation, the Higgs potential asymptotically coincides with the chaotic potential in inflation period where the scalar field is in a high energy region with some large value, and a too small value of λ will make the models inconsistent with the constraints from the standard model [53]. Putting aside its physical motivation, in this paper we study whether it can give rise to large non-Gaussianities.

From the original action (1), one can obtain the equation of motion for the fields

$$\ddot{\phi} + 3H\dot{\phi} - 6\xi(\dot{H}+2H^2)\phi + \frac{\partial V(\phi)}{\partial\phi} = 0, \tag{74}$$

and the Friedmann equation

$$3H^2\left(\frac{1}{8\pi G} + \xi\phi^2\right) = \frac{1}{2}\dot{\phi}^2 + V(\phi) - 6\xi H\phi\dot{\phi}, \tag{75}$$

and the above two equations can be combined to give another equation:

$$\dot{H} = \left(\frac{1}{8\pi G} + \xi\phi^2\right) = -\frac{1}{2}\dot{\phi}^2 + \xi H\phi\dot{\phi} - \xi\dot{\phi}^2 - \xi\phi\ddot{\phi}. \tag{76}$$

First, we draw the background evolution of the system in Figs. 1 and 2. From the plots we can see that with the natural choice of initial conditions and parameters, a period of inflation can be easily constructed with a sufficient number of e-folds. Setting $\xi = 1000$, the parameter λ could be raised up to $\mathcal{O}(10^{-3})$, compared to the unnatural choice of $\lambda \sim 10^{-14}$ in the case with $\xi = 0$ (Fakir and Unruh, [6]). Figures 3 and 4 show the amplitude and the k dependence of its quadratic perturbation spectrum, whose analytical form has already been given in Eq. (37). For the given initial conditions and parameters, we can see that the spectrum behaves nearly k independent, with a slight tilt caused by the deviation of μ from 1, which is mildly favored by the WMAP-7 data [54]. The amplitude of the spectrum is also consistent with the observations.

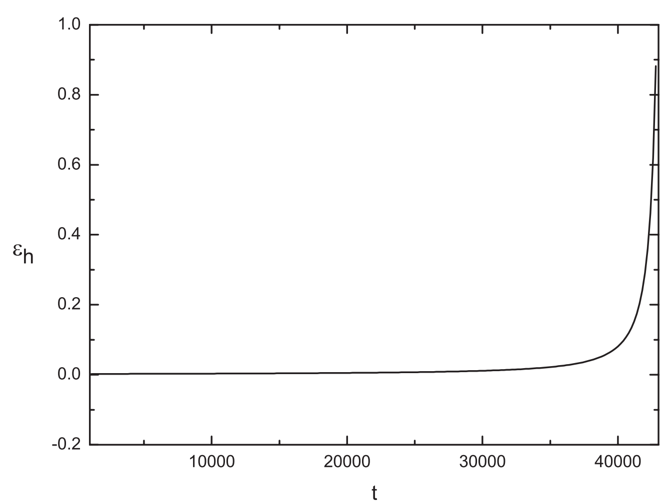

FIG. 1. The evolution of slow-roll parameter ϵ w.r.t. cosmic time t. The arrival of ϵ at 1 stops the inflation. Parameters and initial values: $\xi = 1000$, $\lambda = 10^{-3}$, $\phi_i = 4.9$, $\dot{\phi}_i = 0.063$. The normalization is $8\pi G = 1$.

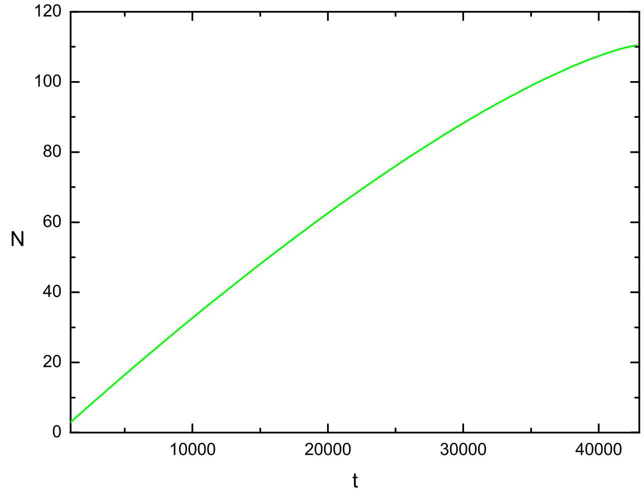

FIG. 2 (color online). The e-folding number N w.r.t. cosmic time t. Parameters and initial values are the same as in Fig. 1.

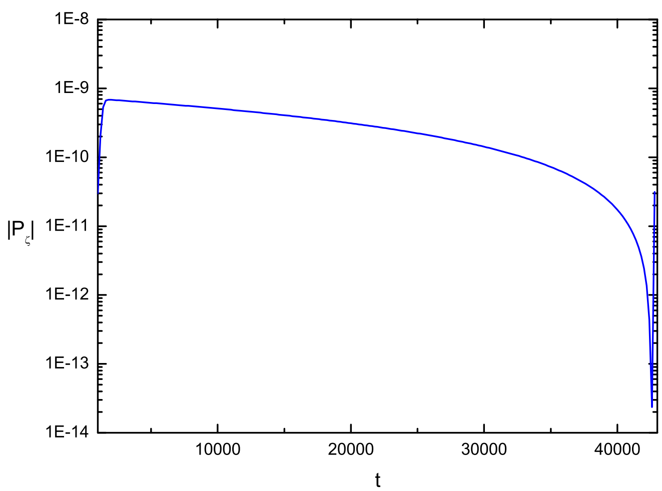

FIG. 3 (color online). The amplitude of power spectrum w.r.t. cosmic time t.

Next let us move on to the non-Gaussianities that this model can give rise to. Since the power spectrum of this model has a blue tilt, the first two classes of the total four in the shape $\mathcal{B}(k_1, k_2, k_3)$ as well as the estimator F_{NL} which was shown in the last paragraphs will be applied. From our numerical calculations, we can obtain the values of every parameter that appears in Eqs. (49)–(58) as well as (63)–(72). With this in hand, we can easily obtain the numerical results of the non-Gaussianities generated in this model. The total shape (of leading order) of the non-Gaussianities and the estimator in the equilateral limit $(F_{NL})^{equil}$ are shown in Figs. 5 and 6. From the plots we can see that the shape of the non-Gaussianities are well within the constraints of the observational data. Note that the estimator $(F_{NL})^{equil}$ shows a running behavior with respect to $k(= k_1 = k_2 = k_3)$, with a positive sign. This is because of the effect of nonminimal coupling which

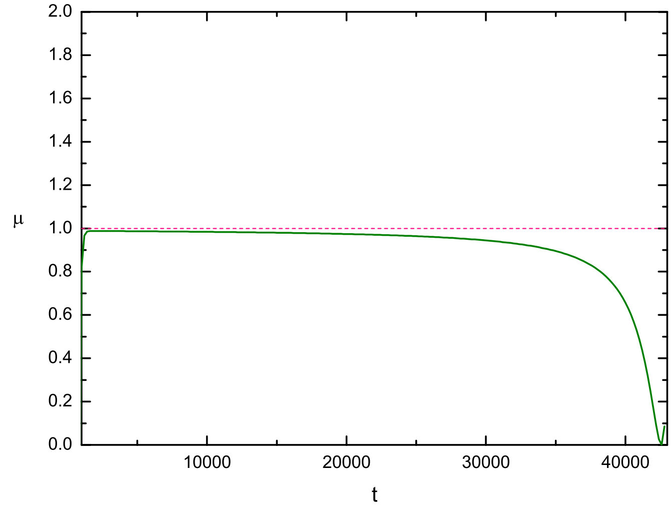

FIG. 4 (color online). μ w.r.t. cosmic time t. A slight deviation from 1 is obtained due to the nonminimal coupling.

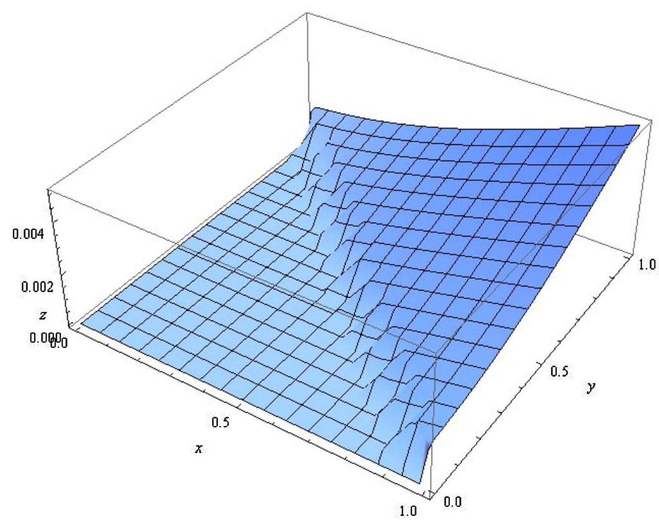

FIG. 5 (color online). The shape of non-Gaussianities $\mathcal{B}(k_1, k_2, k_3)$. Here we renormalize $x \equiv k_1/k_3$, $y \equiv k_2/k_3$, and set $k_3 = 1$.

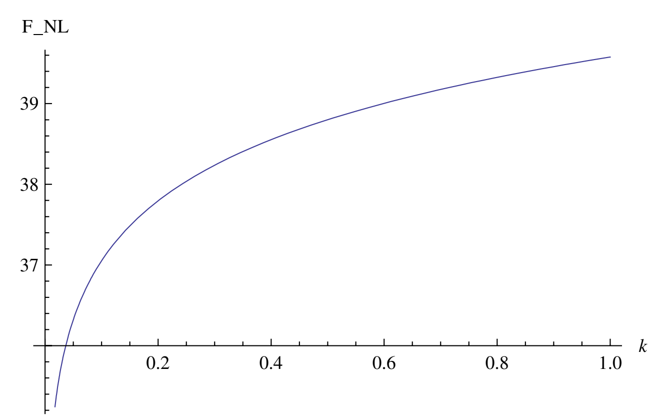

FIG. 6 (color online). The estimator of non-Gaussianities in the equilateral limit: $(F_{NL})^{equil}$. Running behavior is obtained due to the nonminimal coupling effect.

makes the parameter μ deviate from 1 and thus $(F_{\rm NL})^{\rm equil}$ will be dependent on k. In the GR limit $\mu \to 1$, $(F_{\rm NL})^{\rm equil}$ will have a constant value, as shown in [26].

IV. CONCLUSION AND DISCUSSION

In this paper, we performed the non-Gaussianities of a general single scalar field which linearly couples to gravity. Our result shows that due to the nonminimal coupling, the power spectrum will deviate from scale invariance, which in order lead to the complicated non-Gaussianities in the third order. We obtained all the possible shapes of the 3-point correlation functions and for different tilts of power spectrum, we showed that different shapes will be involved in giving rise to non-Gaussianities. Our calculation presents the description in general nonminimal coupling inflation and this result, if verified to all the orders, can provide the relation between 2- and 3-point correlation functions and can be used to constrain nonminimal coupling models.

Another result that was presented in this paper is that there is some running behavior of the estimator $F_{\rm NL}$ in the equilateral limit with respect to k which is different from the normal minimal coupling case. This behavior is due to the nonminimal coupling, and are expected to have signature on observations in order to distinguish the minimal and nonminimal cases.

Besides the analytical calculations, we also performed numerical computations on a specific example of a nonminimal coupling chaotic inflation model. This model was extensively studied with the application of Higgs inflation. We obtained the behavior of background, 2-point power spectrum as well as the shape and estimator of its non-Gaussianity. We showed that the non-Gaussianities are well within the observational constraint, with the running behavior of $F_{\rm NL}^{\rm equil}$ with respect to (w.r.t.) k.

Other than inflation, such a nonminimal coupling system can also be applied to other aspects in cosmology. For example, nonminimal coupling theory can act as dark energy [55] or give rise to a bouncing/cyclic universe [56]. Moreover, nonminimal coupling can be used to make up an open/closed universe [57], while dualities of Einstein's gravity in the presence of a nonminimal coupling was taken on in [58]. The stabilities and singularities in superacceleration phases were discussed in [59], and the removal of singularities in loop quantum gravity with nonminimal coupling was studied in [60]. Our calculation of non-Gaussianities is also expected to be applied to these interesting fields.

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APPENDIX A: EXTENSION TO NONLINEAR COUPLING

Here, we only list the constraint equations for the general form of the nonminimal coupling single scalar field, including the nonlinear coupling case. The constraints from N and N_i are

$$
\begin{aligned}
&4\alpha\Sigma + 12Hf_{R0}\dot\zeta - 12H^2 f_{R0}\alpha - 4a^{-2}f_{R0}H\partial^2\psi - 4a^{-2}f_{R0}\partial^2\zeta - (12\alpha H - 6\dot\zeta + 2a^{-2}\partial^2\psi)\partial_t f_{R0}\\
&\quad + 72(\dot H - H^2)(\dot H + 2H^2)f_{RR0}\alpha - 36(\dot H - H^2)f_{RR0}\ddot\zeta - 36(\dot H - H^2)Hf_{RR0}(4\dot\zeta - \dot\alpha)\\
&\quad + 12a^{-2}(\dot H - H^2)f_{RR0}(\partial^2\dot\psi + 2H\partial^2\psi + \partial^2\alpha + 2\partial^2\zeta) + 6H\partial_t[-12(\dot H + 2H^2)f_{RR0}\alpha + 6Hf_{RR0}(4\dot\zeta - \dot\alpha)\\
&\quad + 6f_{RR0}\ddot\zeta - 2a^{-2}f_{RR0}(\partial^2\dot\psi + 2H\partial^2\psi + \partial^2\alpha + 2\partial^2\zeta)] + 24a^{-2}(\dot H + 2H^2)f_{RR0}\partial^2\alpha\\
&\quad - 12a^{-2}Hf_{RR0}\partial^2(4\dot\zeta - \dot\alpha) - 12a^{-2}f_{RR0}\partial^2\ddot\zeta + 4a^{-4}f_{RR0}\partial^2(\partial^2\dot\psi + 2H\partial^2\psi + \partial^2\alpha + 2\partial^2\zeta) = 0
\end{aligned}
\tag{A1}
$$

and

$$
\begin{aligned}
&-\frac{1}{2}a^{-2}f_{R0}\partial^2\tilde N^i - 12a^{-2}H(\dot H + 2H^2)f_{RR0}\partial^i\alpha + 6a^{-2}H^2 f_{RR0}\partial^i(4\dot\zeta - \dot\alpha) + 6a^{-2}Hf_{RR0}\partial^i\ddot\zeta\\
&\quad - 2a^{-4}Hf_{RR0}\partial^i(\partial^2\dot\psi + 2H\partial^2\psi + \partial^2\alpha + 2\partial^2\zeta) + a^{-2}\partial_t[12(\dot H + 2H^2)f_{RR0}\partial^i\alpha - 6Hf_{RR0}\partial^i(4\dot\zeta - \dot\alpha)\\
&\quad - 6f_{RR0}\partial^i\ddot\zeta + 2a^{-2}f_{RR0}\partial^i(\partial^2\dot\psi + 2H\partial^2\psi + \partial^2\alpha + 2\partial^2\zeta)] + 2a^{-2}f_{R0}H\partial^i\alpha + a^{-2}\partial_t f_{R0}\partial^i\alpha - 2a^{-2}f_{R0}\partial^i\dot\zeta = 0
\end{aligned}
\tag{A2}
$$

respectively, where f_{R0} and f_{RR0} denotes the background value of the first and second derivatives of $f(R, \phi)$ with respect to R.

APPENDIX B: CONTRIBUTIONS FROM TERMS IN $H_{\rm int}^p$ W.R.T. τ

Here, we list the contributions from terms in $H_{\rm int}^p$, i.e. Eq. (43)–(47), in terms of τ by substituting $\frac{d}{d\tau}u_k^*$ in. One can see that there contains integrals of different power laws of τ, each differing one order from the other in every term. One can combine the integrals of the same order power law to have neater forms, as in Sec. III D.

The contribution from $\dot\zeta^3$:

$$\langle\dot\zeta^3\rangle \supset -6\frac{(2\pi)^3\delta^3(\sum_i\vec{k}_i)(\Sigma+2\lambda)\pi^{3/2}H^2e^{-3i(\mu\pi/2)}}{8^{3/2}f_{R0}^3(-1)c_s^{3\mu-6}(k_1k_2k_3)^{\mu}|\epsilon_h+\mu-1|^3\Gamma^3(-\mu+\frac{1}{2})}\int_{-\infty}^0 d\tau e^{ic_sK\tau}|\tau|^{3\mu-1} - 18\frac{(2\pi)^3\delta^3(\sum_i\vec{k}_i)(\mu-1)^2\pi^{3/2}H^4e^{-3i(\mu\pi/2)}}{8^{3/2}f_{R0}^2(-1)c_s^{3\mu-6}(k_1k_2k_3)^{\mu}|\epsilon_h+\mu-1|^3\Gamma^3(-\mu+\frac{1}{2})}\int_{-\infty}^0 d\tau e^{ic_sK\tau}|\tau|^{\mu+1} + c.c. \tag{B1}$$

The contribution from $\zeta\dot\zeta^2$:

$$\langle\zeta\dot\zeta^2\rangle \supset -6\frac{(2\pi)^3\delta^3(\sum_i\vec{k}_i)\Sigma\pi^{3/2}\mu H^2e^{-3i(\mu\pi/2)}}{8^{5/2}f_{R0}^3(-1)c_s^{3\mu}(k_1k_2k_3)^{\mu+2}|\epsilon_h+\mu-1|^3\Gamma^3(-\mu+\frac{1}{2})}\int_{-\infty}^0 d\tau e^{ic_sK\tau}[\{+4c_s^4\mu(1+\mu)k_2^2k_3^2|\tau|^{3\mu-3} + 8ic_s^5k_1k_2^2k_3^2|\tau|^{3\mu-2}\} + 2\,\text{perms}] - 18\frac{(2\pi)^3\delta^3(\sum_i\vec{k}_i)\pi^{3/2}\mu(\mu-1)^2H^4e^{-3i(\mu\pi/2)}}{8^{5/2}f_{R0}^2(-1)c_s^{3\mu}(k_1k_2k_3)^{\mu+2}|\epsilon_h+\mu-1|^3\Gamma^3(-\mu+\frac{1}{2})} \times\int_{-\infty}^0 d\tau e^{ic_sK\tau}[\{+4c_s^4\mu(1+\mu)k_2^2k_3^2|\tau|^{\mu-1} + 8ic_s^5k_1k_2^2k_3^2|\tau|^{\mu}\} + 2\,\text{perms}] + c.c. \tag{B2}$$

The contribution from $\zeta(\partial\zeta)^2$:

$$\langle\zeta(\partial\zeta)^2\rangle \supset -\frac{2(2\pi)^{9/2}\delta^3(\sum_i\vec{k}_i)(\mu-1)\mu^2H^4e^{-3i(\mu\pi/2)}}{8^3c_s^{3\mu}f_{R0}^2(-1)(\prod_i k_i^{\mu+2})|\epsilon_h+\mu-1|^3\Gamma^3(-\mu+\frac{1}{2})}\int_{-\infty}^0 d\tau[(\vec{k}_2\cdot\vec{k}_3)e^{ic_sK\tau}\{\mu^3(1+\mu)^3|\tau|^{\mu-3} + 2ic_s\mu^2(1+\mu)^2K|\tau|^{\mu-2} - 4c_s^2\mu(1+\mu)\Big(\sum_{i>j}k_ik_j\Big)|\tau|^{\mu-1} - 8ic_s^3(k_1k_2k_3)|\tau|^{\mu}\} + 2\,\text{perms.}] + c.c. \tag{B3}$$

The contribution from $\dot\zeta(\partial\zeta)^2$:

$$\langle\dot\zeta(\partial\zeta)^2\rangle \supset -\frac{12(2\pi)^3\delta^3(\sum_i\vec{k}_i)\pi^{3/2}(\mu-1)^2H^4e^{-3i(\mu\pi/2)}}{8^{5/2}f_{R0}^2(-1)c_s^{3\mu-2}(k_1k_2k_3)^{\mu+2}|\epsilon_h+\mu-1|^3\Gamma^3(-\mu+\frac{1}{2})}\int_{-\infty}^0 d\tau[(\vec{k}_2\cdot\vec{k}_3)k_1^2e^{ic_sK\tau}[\mu^2(1+\mu)^2|\tau|^{\mu-1} + 2ic_s\mu(1+\mu)(k_2+k_3)|\tau|^{\mu} - 4c_s^2k_2k_3|\tau|^{\mu+1}] + 2\,\text{perms}] - \frac{4(2\pi)^3\delta^3(\sum_i\vec{k}_i)\pi^{3/2}\Sigma H^2e^{-3i(\mu\pi/2)}}{8^{5/2}f_{R0}^3(-1)c_s^{3\mu-2}(k_1k_2k_3)^{\mu+2}|\epsilon_h+\mu-1|^3\Gamma^3(-\mu+\frac{1}{2})}\int_{-\infty}^0 d\tau[(\vec{k}_2\cdot\vec{k}_3)k_1^2e^{ic_sK\tau}[\mu^2(1+\mu)^2|\tau|^{3\mu-3} + 2ic_s\mu(1+\mu)(k_2+k_3)|\tau|^{3\mu-2} - 4c_s^2k_2k_3|\tau|^{3\mu-1}] + 2\,\text{perms}] + c.c. \tag{B4}$$

The contribution from $\dot\zeta\partial\zeta\partial\chi$:

$$\langle\dot\zeta\partial\zeta\partial\chi\rangle \supset \frac{18(2\pi)^3\delta^3(\sum_i\vec{k}_1)\pi^{3/2}(\mu-1)^4H^4e^{-3i(\mu\pi/2)}}{8^{5/2}f_{R0}^2(-1)c_s^{3\mu}(k_1k_2k_3)^{\mu+2}\mu|\epsilon_h+\mu-1|^3\Gamma^3(-\mu+\frac{1}{2})}\int_{-\infty}^0 d\tau e^{ic_sK\tau}$$
$$\times\left[\frac{\vec{k}_2\cdot\vec{k}_3}{k_3^2}\{+4c_s^4\mu(1+\mu)k_1^2k_3^2|\tau|^{\mu-1}+8ic_s^5k_2k_1^2k_3^2|\tau|^{\mu}\}+5\,\text{perms}\right]$$
$$+\frac{2(2\pi)^3\delta^3(\sum_i\vec{k}_i)\pi^{3/2}\Sigma^2e^{-3i(\mu\pi/2)}}{8^{5/2}f_{R0}^4(-1)c_s^{3\mu}(k_1k_2k_3)^{\mu+2}\mu|\epsilon_h+\mu-1|^3\Gamma^3(-\mu+\frac{1}{2})}$$
$$\times\int_{-\infty}^0 d\tau e^{ic_sK\tau}\left[\frac{\vec{k}_2\cdot\vec{k}_3}{k_3^2}\{+4c_s^4\mu(1+\mu)k_1^2k_3^2|\tau|^{5\mu-5}+8ic_s^5k_2k_1^2k_3^2|\tau|^{5\mu-4}\}+5\,\text{perms}\right]$$
$$+\frac{12(2\pi)^3\delta^3(\sum_i\vec{k}_i)\pi^{3/2}(\mu-1)^2\Sigma H^2e^{-3i(\mu\pi/2)}}{8^{5/2}f_{R0}^3(-1)c_s^{3\mu}(k_1k_2k_3)^{\mu+2}\mu|\epsilon_h+\mu-1|^3\Gamma^3(-\mu+\frac{1}{2})}$$
$$\times\int_{-\infty}^0 d\tau e^{ic_sK\tau}\left[\frac{\vec{k}_2\cdot\vec{k}_3}{k_3^2}\{+4c_s^4\mu(1+\mu)k_1^2k_3^2|\tau|^{3\mu-3}+8ic_s^5k_2k_1^2k_3^2|\tau|^{3\mu-2}\}+5\text{perms}\right]+c.c. \qquad \text{(B5)}$$

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