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Cosmological ultraviolet/infrared divergences and de Sitter spacetime

Wei Xue,* Keshav Dasgupta,† and Robert Brandenberger‡

Physics Department, McGill University, 3600 University Street, Montréal QC, Canada H3A 2T8

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We consider one-loop graviton corrections to Green's scalar field functions in the de Sitter phase of an inflationary space-time, a topic relevant to the computation of cosmological observables beyond linear order. By embedding de Sitter space into an ultraviolet complete theory, such as M theory, we argue that the ultraviolet (UV) cutoff of the effective field theory should be taken to be fixed in physical coordinates, whereas the infrared (IR) cutoff is expanding as space expands. In this context, we demonstrate how to implement three different regularization schemes—the brute-force cutoff regularization, dimensional regularization and Pauli-Villars regularization—obtaining the same result for the scalar propagator if we use any of the three regularization schemes.

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I. INTRODUCTION

The study of quantum fields in an expanding universe has become a cornerstone of modern cosmology. It is believed that the fluctuations of a scalar quantum field induce the gravitational fluctuations, which in an inflationary universe scenario evolve into the inhomogeneities in the distribution of galaxies and into the anisotropies of the cosmic microwave background which we observe today. The computation of the generation and evolution of cosmological perturbations is typically performed at linear order (see e.g. [1] for an overview of the theory of cosmological perturbations), i.e. at tree level. Recently, however, there has been a great deal of interest in studying perturbations at higher order (see e.g. [2] for recent reviews). There are both phenomenological and purely theoretical reasons for this interest. On the phenomenological side, an important issue has been the study of non-Gaussianities induced by the nonlinearities in the underlying field equations (see e.g. [3] for some recent reviews and [4] for two early papers on this subject). On the purely theoretical side, there are various questions. Foremost, there is the question of divergences which arise in computing perturbatively to higher orders. There will clearly be ultraviolet (UV) divergences as there are in any quantum field theory. Furthermore, in a theory which involves massless modes like gravity there is the possibility of infrared (IR) divergences. In inflationary cosmology, these divergences were noticed in early work [5] and lead to a linear growth of the coincident point of the two-point function of a scalar field ϕ in the de Sitter phase

$$\langle \phi^2(x) \rangle \sim H^3 t, \tag{1.1}$$

where H is the Hubble constant and t is physical time. The role of these divergences for cosmological perturbations was first discussed in [6]. They could have both interesting and also dangerous effects. For one, they could invalidate the entire perturbative approach (see e.g. [7]). On the other hand, there have been speculations that IR divergences in the gravity wave sector [8] or in the sector of scalar metric fluctuations [9] could lead to a dynamical relaxation mechanism for the cosmological constant, a mechanism which would leave behind a remnant cosmological constant of exactly the right magnitude to explain dark energy [10].

The interest has focused on studying quantum fields in a de Sitter background since such a background is a good approximation for the phase of inflationary expansion in the very early Universe. Free quantum fields in nontrivial gravitational backgrounds have been studied extensively (see e.g. [11] for a textbook treatment), and the study of such fields has evolved into a mature subject.

In comparison, the study of interacting fields in cosmological backgrounds is a field still in its infancy. A good understanding of this topic, however, is vital if we are to compute correlation functions of cosmological fluctuation variables to higher than tree order. In particular, we are interested in a correlation function of the variable commonly denoted by either ζ or $\mathcal{R}$ [1,12] which represents the curvature perturbation in a comoving gauge. Since General Relativity is intrinsically nonlinear, nonlinearities will be important for the evolution of ζ even if the matter field is a free field. It is particularly important to consider the interactions of scalar fields (such as ζ) with gravitons. For early work on the quantum theory of gravitons in de Sitter space the reader is referred to [8]. Early work on interacting quantum fields in de Sitter-like backgrounds see e.g. [13–16]. For a mathematical physics approach to interacting quantum field theory in cosmological backgrounds see e.g. [17].

A few years ago, Weinberg [18] wrote a seminal paper studying quantum contributions to correlation functions of curvature perturbations in de Sitter space.[1] Using

*xuewei@hep.physics.mcgill.ca
†keshav@hep.physics.mcgill.ca
‡rhb@hep.physics.mcgill.ca

[1] See [19] for fully dimensionally regulated and renormalized computations involving gravitons in de Sitter.

dimensional regularization, and working in the "in-in formalism" [20] he found the one-loop result[2]

$$\langle \zeta_k^2 \rangle \sim \frac{1}{k^3} \frac{H^4}{M_{\rm pl}^2} \log\left(\frac{k}{\mu}\right), \tag{1.2}$$

where k is a comoving wave number, μ is a physical ultraviolet renormalization scale, and H is the Hubble constant during the de Sitter phase. As expected, there are both ultraviolet and infrared divergences.[3] The former are removed via renormalization, but the latter persist and could have an important effect.

Similar calculations to those of Weinberg were then performed using a "brute-force" cutoff regularization [24–27] yielding consistent results. However, in [28] the result of Weinberg [18] was questioned on the basis of the fact that k is a comoving scale whereas μ is physical. The authors of [28] found the result

$$\langle \zeta_k^2 \rangle \sim \frac{1}{k^3} \frac{H^4}{M_{\rm pl}^2} \log\left(\frac{H}{\mu}\right). \tag{1.3}$$

The difference between the results of [18,28] can be traced to different assumptions made about the nature of the UV and IR cutoff scales (see e.g. [24,29] for other recent work). Recently, Weinberg [30] has reconsidered the problem and now advocates that one should use Pauli-Villars regularization instead of dimensional regularization. However, one should be able to obtain the same result using different renormalization schemes.[4]

The purpose of this paper is to address the two issues mentioned in the previous paragraph. First, by putting the scalar quantum field theory model in the context of an ultraviolet complete theory, we will be able to argue that the correct way to impose the cutoffs is to use a fixed physical ultraviolet cutoff scale but a comoving infrared cutoff. Second, we will show in terms of a simple example that all three regularization schemes commonly used—-brute-force cutoff, dimensional regulations, and Pauli-Villars regularization—all give the same result.

In the following section we will discuss what we can learn about the renormalization issue of quantum fields in de Sitter space by embedding de Sitter into an ultraviolet complete theory such as string or M theory. From this discussion, we can draw conclusions on how an effective quantum field theory of one low-mass scalar field is embedded in the ultraviolet complete theory. In particular, this teaches us how to impose the ultraviolet cutoff. In Sec. III we review the quantization of fields in de Sitter space-time. We then turn to the computation of Green's function for a massless scalar field in de Sitter space. In Sec. IV, V, and VI we perform the calculation of the one-loop corrections to Green's function using, in turn, brute-force cutoff, dimensional regularization, and Pauli-Villars regularization. We obtain the same results. In the final section we discuss our results.

[2]Here, ζ is the curvature fluctuation in comoving coordinates—see [1].

[3]There has been a lot of recent work focusing on whether the infrared divergences are real or not (see e.g. [21] for a review and [22] for a selection of other references). This is an issue which we will not touch here. Note that there is a close connection between the IR divergences and stochastic inflation [23].

[4]There have also been some attempts to go beyond a pure loop expansion [31].

II. INDUCED EFFECTIVE FIELD THEORY IN FOUR SPACE-TIME DIMENSIONS FROM STRING THEORY

General Relativity is not renormalizable, and its quantization is an outstanding challenge. Cosmological perturbations (both scalar metric fluctuations associated with matter perturbations, and gravitational waves) can be quantized at the linear level [32,33], but in the absence of an ultraviolet complete theory of quantum gravity which reduces to General Relativity in the low-energy limit questions of consistency of the quantization scheme remain.

By embedding the de Sitter phase of an inflationary universe into an ultraviolet complete quantum theory of gravity, it is possible to study how the conventional theory of fields on de Sitter space arises as a low-energy effective field theory. This, in turn, will help us justify the cutoff scales which need to be introduced in order to eliminate UV and IR divergences.

In spite of "no-go" theorems [34] derived in the context of supergravity, it has been possible in the past years to construct inflationary solutions of string theory, using input from string theory which goes beyond simple supergravity (see [35] for some reviews).[5]

String theory (M theory) is defined in 10 (11) space-time dimensions. In order to make contact with four-dimensional physics at low energies, it is crucial to compactify the extra spatial dimensions and to stabilize their

[5]One example of an inflationary model arising from type IIB reduction of M theory [36–38] is obtained by considering a D3 brane moving in the presence of a stack of D7 branes. The D7 branes wrap the extra dimensions which are taken to have the form of a particular Calabi-Yau manifold. In the four-dimensional effective field theory, a hybrid inflationary model of a D-term type results: the separation between the D3 brane and the stack of D7 branes yields the inflaton field, and the scalar field χ whose condensation ends inflation can be identified with a D3-D7 string mode that becomes tachyonic at a critical value of the D3/D7 interbrane distance. It has been established [39–41] that fluxes about the internal dimensions can stabilize all complex structure moduli at tree level, and nonperturbative effects can stabilize the radial moduli [42,43]. Alternatively, in a heterotic string theory, string gases winding the extra dimensions [44,45] can stabilize all geometric moduli [46–49], and gaugino condensation can be used to stabilize the dilaton [50]. In a separate paper we will revisit the geometry of this D3-D7 system, and show that from an M theory starting point the geometry of our four-dimensional world becomes de Sitter space [51].

sizes and shapes (which are the geometrical moduli). String theory also admits branes and fluxes and they lead to more moduli, i.e. more degrees of freedom which from the four-dimensional point of view act as fields. There are Kaluza-Klein modes of the higher dimensional fields which once again yield four-dimensional fields. The upshot of this is that string or M theory give rise to a vast number of scalar fields in the four-dimensional effective field theory.

If all of the moduli are successfully stabilized, then only a very small number of fields remain light, including the graviton and the scalar field which plays the role of the inflaton. All other fields obtain a mass which is characterized by a combination of the compactification scale and the string scale. We will denote this scale as M.

Since there are an infinite number of scalar fields in four-dimensional fields, all interacting with each other, the UV theory is highly nontrivial at energy scales larger than M. This scale is the natural cutoff scale of the low-energy effective field theory. This cutoff scale must be fixed (at least at late cosmological times) in physical coordinates to avoid time-dependent coupling constants at low energies. Thus, we learn that the ultraviolet cutoff energy scale is fixed in physical coordinates and hence increasing in comoving coordinates (as used in most works, in particular, in [28]).

If we follow a mode with an initial frequency smaller than M forward in time as space expands, its physical frequency decreases while the comoving frequency stays the same. If we imagine setting initial conditions for all modes at the beginning of the inflationary period, or at some fixed time in de Sitter space (using the cosmological slicing of de Sitter space), then there is a natural infrared cutoff, as can be seen by the following argument: Physics inside the initial Hubble radius H^{-1} cannot determine the initial conditions on scales larger than H^{-1}. Hence, the initial conditions on the initial super-Hubble scales depend on the prehistory of the cosmological model, the history before the onset of the inflationary phase. If we are interested in effects due to the de Sitter phase, we must cut out the modes which are initially super-Hubble. Hence, the initial Hubble radius is the natural infrared wavelength cutoff. The wavelengths of modes are stretched as space expands, and hence the IR cutoff length will expand as well. The IR energy cutoff is fixed in comoving coordinates but decreases in physical coordinates.

There is a geometrical picture which represents the setup we are considering here and which is illustrated in Fig. 1. The horizontal plane represents our four space-time dimensional universe; the vertical axis is the energy scale. Each mode corresponds to a wave in the horizontal plane at a particular height. Modes which lie between the UV and IR cutoff scales are within the domain of the effective field theory. As the Universe expands, the height of the wave decreases, and new waves enter the region of the effective field theory from the UV sea of modes. The IR cutoff scale decreases as t increases.[6]

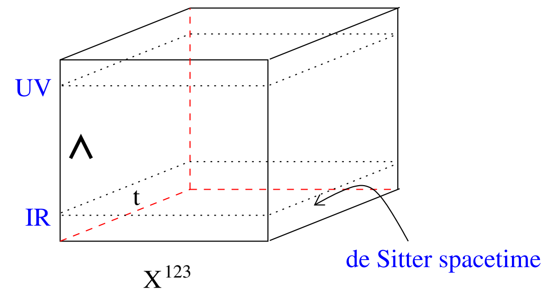

FIG. 1 (color online). Fields in a de Sitter space can be arranged succinctly in a five-dimensional space. The horizontal plane represents our four space-time dimensional expanding universe (the spatial dimensions in the plane of the paper, the time direction into the paper), and the fifth dimension is the energy scale $\mathbf{\Lambda} \equiv (\mathbf{\Lambda}_{\rm UV}, \mathbf{\Lambda}_{\rm IR})$ of the field. As the Universe expands, the energy of a mode decreases. Thus, as t increases, the mode moves downwards in the cube. The modes tracked in the effective field theory lie between the UV and IR cutoffs. If the z-axis represents physical energy, then the UV cutoff is at a height which is fixed in time, whereas the IR cutoff scale is decreasing as t increases. New modes continually enter the region of the effective field theory from the UV sea as t increases.

The low-energy effective action can be written completely in terms of a small number of low-mass scalar fields σ_i interacting with graviton fluctuations h_{ij}. The masses m_i of all stringy modes and other moduli fields must be larger than the UV cutoff scale. Otherwise, the effective action would not make any sense beyond the scale m_i since these modes would have to be inserted in the loop diagrams of correlation functions of the light modes, e.g. of σ, which would change the behavior of the $\langle\sigma\sigma\rangle$ correlator.

The massive modes contain the KK reductions of all the ten-dimensional light states in four dimensions over any compactifying manifold as well as the massive stringy modes and their KK reductions over the same six-dimensional internal manifold. All the massive states do couple to gravity, and correspondingly every low-energy state has an infinite number of interaction terms. Thus, with all the massive modes and the infinite series expansion of the metric, the loop integrals for correlation functions of low-mass fields at energies above the scale M are completely out of control. However as is well known, the new stringy states that enter the Wilsonian action at the scale where Einstein gravity breaks down change the UV picture completely and in fact control the UV divergences. The final UV behavior is finite and shows no divergences.

[6]Note that we are not the first to use these IR and UV cutoff prescriptions—see e.g. [52].

Thus, it makes sense to study the UV behavior of a single light state coupled to gravitational fluctuations up to a specified UV cutoff using the four-dimensional effective field theory of the low-energy modes. Beyond the UV cutoff we expect the behavior that we discussed above will kick in.

But for an effective theory to make sense below $\Lambda_{\rm UV}$ we should be able to regularize the loop integrals unambiguously without resorting to extra massive degrees of freedom. We must therefore study the effects of the choice of regulators and of the regularization schemes. These questions make sense within the effective four-dimensional action and will concern the rest of this paper.

The simplified Lagrangian of the low-energy effective theory, expressed using the Arnowittt-Deser-Misner (ADM) decomposition of the four-dimensional spacetime metric into spatial metric g_{ij} (the Latin indices run over the three spatial coordinates), shift vector N_i and lapse function N, is

$$\mathcal{L} = \frac{1}{2}\int \sqrt{\det g_{ij}}[NR^{(3)} + N^{-1}(E_{ij}E^{ij} - E^2) + N^{-1}(\dot{\sigma} - N^i\partial_i\sigma) - Ng^{ij}\partial_i\sigma\partial_j\sigma], \tag{2.1}$$

where E_{ij} represents the extrinsic curvature tensor, and R^3 denotes the Ricci scalar of the spatial metric. We can even insert a potential $-2NV$ for the scalar field σ. The above Lagrangian implies that there are at least two interaction vertices:

$$\mathcal{L}_1 \equiv \frac{a}{2}h_{ij}\partial_i\sigma\partial_j\sigma, \qquad \mathcal{L}_2 \equiv -\frac{a}{4}h_{il}h_{lj}\partial_i\sigma\partial_j\sigma \tag{2.2}$$

which enter into the computation of one-loop corrections of gravitons to $\langle\sigma\sigma\rangle$, i.e. the two-point correlation function.

In our five-dimensional picture, (2.2) will give rise to two distinct nonbifurcating surfaces. An alternative, but equivalent, viewpoint will be to take (2.1) as the Wilsonian action at the scale $\Lambda_{\rm UV}$, and as we go down the scale we will only consider modes with momenta up to that scale. Clearly, the existence of any extra massive states will spoil this simple Wilsonian picture.

III. QUANTIZATION AND MODES IN DE SITTER SPACE-TIME

To study the modes in de Sitter space it is convenient to go to the comoving frame in which the four-dimensional background metric becomes

$$ds^2 = a^2(\eta)\left(-d\eta^2 + \sum_{i=1}^{3} dz_i.dz_i\right). \tag{3.1}$$

For the sake of simplicity, we consider the spatial sections to be flat. Note that η is conformal time.

In the presence of scalar field matter, the metric contains 10 degrees of freedom for fluctuations, four of them scalar, four vector and two tensor (classifying the fluctuations according to how they transform under spatial rotations, the usual procedure in cosmology). Two scalar and two vector modes are gauge. We fix the gauge by setting four metric fluctuation variables to zero. Note that in this gauge there are no propagating ghosts[7] or interactions, unlike what would happen if one were to use a covariant gauge-fixing procedure (see e.g. [53] for studies using covariant gauges). The Hamiltonian and momentum constraints eliminate four further variables, leaving us with one[8] scalar mode (a combination of the scalar matter field and the scalar metric fluctuation) and the two tensor modes which are the two graviton polarization fields. The scalar field and the graviton obey the following differential equations (with a prime denoting the derivative with respect to conformal time):

$$\begin{aligned} \sigma'' + 2aH\sigma' - \nabla^2\sigma &= 0 \\ h_{ij}'' + 2aHh_{ij}' - \nabla^2 h_{ij} &= 0. \end{aligned} \tag{3.2}$$

We can use the above equations to describe the modes of the scalar field and the metric. The mode expansion for the scalar field is the standard one. For the graviton we will use a basis of polarization tensors $e_{ij}(k, s)$ ($s = \pm$) to describe the mode expansion as

$$h_{ij}(\mathbf{z}, \eta) = \int d^3k \sum_{s=\pm}[a(\mathbf{k})e_{ij}(k, s)h_{s,k}(\eta)e^{i\mathbf{k}\cdot\mathbf{z}} + a^\dagger(\mathbf{k})e^*_{ij}(k, s)h^*_{s,k}(t)e^{-i\mathbf{k}\cdot\mathbf{z}}], \tag{3.3}$$

where $h_{\pm,k}$ denote the mode functions. The polarization tensors satisfy the identity:

$$\begin{aligned}\sum_s e_{ij}(\hat{p}, s)e^*_{kl}(\hat{p}, s) = {}& \delta_{ik}\delta_{jl} + \delta_{il}\delta_{kl} - \delta_{ij}\delta_{kl} + \delta_{kl}\hat{p}_i\hat{p}_j \\ & - \delta_{ik}\hat{p}_j\hat{p}_l - \delta_{il}\hat{p}_j\hat{p}_k - \delta_{jk}\hat{p}_i\hat{p}_l \\ & - \delta_{jl}\hat{p}_i\hat{p}_k + \hat{p}_i\hat{p}_j\hat{p}_k\hat{p}_l,\end{aligned} \tag{3.4}$$

and the subscript s of the graviton is the helicity. We have used $\hat{p}$ to denote the unit momenta and k to denote the absolute value.

The mode functions in de Sitter space are easy to obtain. We assume that the modes start out on sub-Hubble scales in their vacuum state, the usual assumption made in inflationary cosmology. Let us denote the scalar field modes that appear in the standard mode expansion by u_k. The final result for these modes is

[7] We thank Alex Maloney and Guy Moore for asking probing questions which stimulated us to focus on this issue.

[8] For n scalar fields there would be n scalar modes.

$$u_k = \frac{H}{(2\pi)^{3/2}\sqrt{2k^3}}(1 + ik\eta)e^{-ik\eta}$$
$$h_{\pm,k} = \frac{H}{(2\pi)^{3/2}\sqrt{k^3}}(1 + ik\eta)e^{-ik\eta}, \tag{3.5}$$

where we note that both the modes $h_{+,k}$ and $h_{-,k}$ are given by the same expression.

Now that we have identified the physical modes of the system, we can move on to study the behavior of these modes in a de Sitter phase of an inflationary universe. Specifically, we need to understand what it means to impose IR and UV cutoffs in an expanding universe. The mode decomposition of the fields is defined in comoving coordinates. As we have argued in the previous section, the UV cutoff of the effective field theory should be at a fixed physical wavelength, and hence at a comoving wavelength which is decreasing as $a(t)^{-1}$. The comoving high frequency cutoff is hence increasing. In physical coordinates, it is the UV cutoff which remains the same whereas the physical IR wavelength cutoff is increasing. Since the Hubble radius $H^{-1}(t)$, the length which separates the high frequency region where the modes are oscillating from the low frequency region where the modes are frozen out, is at a fixed value in physical coordinates, we conclude that the volume of phase space of short wavelength modes is constant, whereas that of the long wavelength modes is increasing. As short wavelength modes exit the Hubble radius during the inflationary expansion of space, new UV modes enter the region of validity of the effective field theory to replenish the phase space.

Figure 2 gives a sketch of the evolution of various scales in de Sitter space-time. The horizontal axis represents physical distance, the vertical distance time. The wavelength of a fixed mode increases as time proceeds. If t_0 is the beginning of inflation, then the mode whose

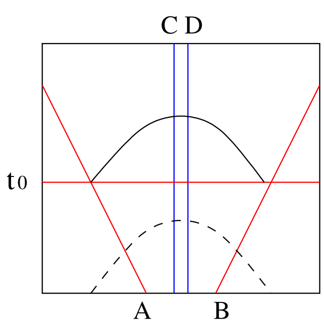

FIG. 2 (color online). A representation of how modes evolve in de Sitter space. The vertical axis denotes time, the horizontal axis physical space. The grey dashed half wave and the solid grey half wave at time t_0 denotes waves of Hubble length. The red slanted solid lines delineate the wavelength of a wave which exits the Hubble radius at time t_0 (whose initial value is given by the length between A and B). The solid blue vertical lines indicate the ultraviolet cutoff wavelength (the distance between C and D) which is constant in physical coordinates.

wavelength is indicated by the distance between the two red solid slanted lines indicates the IR cutoff scale whose wavelength is growing with time. In contrast, the UV cutoff wavelength is fixed in physical coordinates—by the distance between C and D in the sketch. The Hubble radius has a constant physical distance.

In the following, we will consider the one-loop graviton corrections to the two-point function of σ. We perform the computation using three different regularization schemes:

(i) Brute cutoff regularization,
(ii) Dimensional regularization,
(iii) Pauli-Villars regularization.

and show that all the three regularization schemes yield identical results.[9]

In an upcoming work [54] other interactions of σ will be studied, and an extension of our method to higher n-correlation functions will be worked out. However, we want to point out that the steps presented in this paper to regularize the one-loop two-point function extend in a straightforward way to higher n-point functions, including nontrivial fermionic and gauge interactions.

IV. BRUTE CUTOFF REGULARIZATION

A. Preliminaries

The "Brute Cutoff" scheme consists of eliminating the contributions of modes with wave numbers above a certain ultraviolet cutoff scale and below a certain infrared cutoff scale from the loop integrals. The specification of these cutoffs is the first place where all the subtleties that we mentioned above regarding UV and IR cutoffs will show up.

In scattering experiments one is interested in computing transition rates between prescribed in and out states. The questions in cosmology are very different. Here, one is interested in the evolution for a finite duration of time of a certain initial state. Thus, we are not interested in calculating S-matrix elements, but rather expectation values of operators at some later time: t evaluated in a state set Ω up at some initial time t_i. The formalism appropriate for performing this computation is the Schwinger-Keldysh [20] or "in-in" formalism.

We are interested in computing the expectation value

$$\langle\Omega|\mathcal{O}(t)|\Omega\rangle \tag{4.1}$$

[9]One might worry about preserving the Ward identity using the above regulators. The Pauli-Villars regulator (as like the dimensional regulator) *does* preserve the Ward identity. However, to preserve the Ward identity using the brute cutoff regulator one might need to add extra terms to the Lagrangian. The final answer is that any violation of the Ward identity is cancelled by these extra terms *without* affecting the two-point correlation functions. Therefore in this work we will ignore these subtleties.

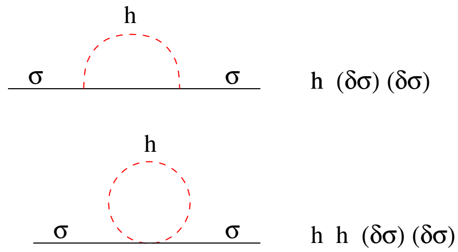

FIG. 3 (color online). The two one-loop diagrams which we study here. They are related to the two amplitudes given in the text. The first diagram gives amplitude I and the second one amplitude II.

of some operator $\mathcal{O}$ evaluated at time $t \gg t_i$ in some state Ω prescribed at some early time t_i, which we consider to tend to $-\infty$. We take the state Ω to be the initial vacuum state. Working in the interaction representation, the formula for this correlation function (4.1) becomes

$$\langle\Omega|\mathcal{O}(t)|\Omega\rangle = \langle 0|\bar{T}e^{i\int_{-\infty}^{t}H_I(t')dt'}\mathcal{O}_I(t)Te^{-i\int_{-\infty}^{t}H_I(t')dt'}|0\rangle \tag{4.2}$$

where $|\Omega\rangle$ is the vacuum in the interacting theory and $|0\rangle$ is the free field vacuum. The right-hand side of the equation is calculated in the interaction picture. T and $\bar{T}$ denote time-ordered and anti-time-ordered products.

Considering a free scalar field theory for matter,[10] there are two diagrams which contribute to the above expectation value at the one-loop level. They are shown in Fig. 3. We will evaluate these diagrams in the following subsection.

B. In-in calculation of loop corrections to the two-point function

The contribution $G_p^I(\eta)$ of the first diagram of Fig. 3 to the one-loop corrections of the scalar correlation is due to the three field interaction (2.2) is given by the following expression:

$$\begin{aligned} G_p^I(\eta) = &-4(2\pi)^6\,\mathbf{Re}\int_{-\infty}^{\eta}d\eta_1 a^2(\eta_1)\int_{-\infty}^{\eta}d\eta_2 a^2(\eta_2)\\ &\times\int d^3q p^4\sin^4\theta\Big[\theta(\eta_1-\eta_2)u_p^2(\eta)u_p^*(\eta_1)u_p^*(\eta_2)\\ &\times u_{p'}(\eta_1)u_{p'}^*(\eta_2)h_q(\eta_1)h_q^*(\eta_2)\\ &-\frac{1}{2}|u_p(\eta)|^2u_p(\eta_1)u_p^*(\eta_2)u_{p'}(\eta_1)\\ &\times u_{p'}^*(\eta_2)h_q(\eta_1)h_q^*(\eta_2)\Big], \end{aligned} \tag{4.3}$$

[10]To be complete, we should consider the self-coupling of σ which is unavoidable if the coupling of matter to scalar gravitational fluctuations is taken into account.

where p is the momentum of the external line, q is the momentum of the graviton and $p' \equiv p - q$ is the momentum of the internal scalar field. The factor $p^4\sin^4\theta$ comes from summing over the graviton polarization states:

$$\sum_s e_{ij}(\hat{p},s)e_{kl}^*(\hat{p},s)p_ip_jp_kp_l = p^4\sin^4\theta. \tag{4.4}$$

Note that in Eq. (4.3), the second part in the bracket is exponentially small in the UV because of the exponential oscillation of the modes. Therefore, when we calculate the UV divergence of Diagram I, we can just integrate the first part without effecting the result. In the UV limit, the correlation can then be simplified as

$$\begin{aligned} G_p^{I,UV}(\eta) &= -4\,\mathbf{Re}\int_{-\infty}^{\eta}d\eta_1\int_{-\infty}^{\eta_1}d\eta_2 u_p^2(\eta)u_p^*(\eta_1)u_p^*(\eta_2)\\ &\quad\times\int\frac{d^3q}{2q^2}p^4\sin^4\theta e^{-2iq(\eta_1-\eta_2)}\\ &= -\lim_{\eta_1\to\eta_2}\int_{-\infty}^{\eta}d\eta_1\,\mathbf{Im}[u_p^2(\eta)u_p^{*2}(\eta_1)]\\ &\quad\times\int\frac{d^3q}{q^3}p^4\sin^4\theta. \end{aligned} \tag{4.5}$$

On the other hand, if we want to keep both the IR and UV effects, we can take (4.3), do the time integral first and then integrate over the loop momentum. The final result (writing only the divergent part) can be expressed in the following way:

$$\begin{aligned} &G_p^I(\eta=0)\\ &\quad=\frac{H^2}{(2\pi)^3 2p^3}\frac{H^2}{(2\pi)^2}\Big[2\log(p/\Lambda_{\rm IR})+\frac{2}{3}\log(\Lambda_{\rm UV}/p)\Big] \end{aligned} \tag{4.6}$$

where H is the Hubble constant and p is the incoming momentum of the scalar field.

Once we have the result for Diagram I of Fig. 3, we can evaluate Diagram II. This follows arguments more or less similar to those made when considering (4.3). The one-loop correction to the scalar correlation function from the four field interaction in (2.2) then takes the form

$$\begin{aligned} G_p^{II}(\eta) = &(2\pi)^3\int_{-\infty}^{\eta}d\eta_1 a^2(\eta_1)\,\mathbf{Im}[u_p^2(\eta)u_p^{*2}(\eta_1)]\\ &\times\int d^3q 2p^2\sin^2\theta|h_q(\eta_1)|^2 \end{aligned} \tag{4.7}$$

where now the quantity $2p^2\sin^2\theta$ comes from the summation of the following graviton polarization:

$$\sum_s e_{il}(\hat{p},s)e_{jl}^*(\hat{p},s)p_ip_j = 2p^2\sin^2\theta. \tag{4.8}$$

To evaluate this one should once again first do the time integral and then integrate the loop momentum. The final result (once again writing only the divergent terms) is given by

$$G_p^{II}(\eta=0)=\frac{H^2}{(2\pi)^3 2p^3}\frac{H^2}{(2\pi)^2}\times\left[2\log(\Lambda_{\rm IR}/\Lambda_{\rm UV})-\frac{5}{6}\Lambda_{\rm UV}^2\right], \qquad (4.9)$$

where all the terms appearing have been defined above. So far, the two results (4.6) and (4.9) agree with those given in [52] (see also [28]).

Finally, we can rewrite (4.6) and (4.9) using the renormalization scales $\mu_{\rm IR}$ and $\mu_{\rm UV}$. In this language the two amplitudes are reexpressed as

$$G_p^{I}(\eta=0)=\frac{H^2}{(2\pi)^3 2p^3}\frac{H^2}{(2\pi)^2}\times\left[2\log(p/\mu_{\rm IR})+\frac{2}{3}\log(\mu_{\rm UV}/p)\right]$$
$$G_p^{II}(\eta=0)=\frac{H^2}{(2\pi)^3 2p^3}\frac{H^2}{(2\pi)^2}\times\left[2\log(\mu_{\rm IR}/\mu_{\rm UV})-\frac{5}{6}\mu_{\rm UV}^2\right]. \qquad (4.10)$$

However, this is *not* the final answer because there is a subtlety associated with applying cutoffs in the comoving and in the physical coordinate systems. To understand this and analyze the results correctly, we turn to the implementation of the cutoffs.

C. Physical UV and comoving IR cutoffs

In the previous subsection we have not carefully discussed the UV and IR cutoffs for our case. The subtlety about this has been discussed earlier in Sec. II. It is now time to apply what we learned in that section to our two results (4.6) and (4.9) or equivalently to (4.10).

Note that the calculations in the above subsection are based on the assumption that the UV and IR cutoffs are both comoving. During inflation, we know that the UV modes are generated and IR modes are stretched outside of the Hubble radius. Thus, the phase space is increasing in de Sitter space. Therefore (using for the moment a superscript "c" to designate a quantity in comoving coordinates), in the comoving frame the IR cutoff $\Lambda_{\rm IR}^c$ will remain unchanged, whereas the UV cutoff $\Lambda_{\rm UV}^c$ will change because new high energy modes will enter the system. Therefore we can rewrite $\Lambda_{\rm UV}^c$ as

$$\Lambda_{\rm UV}^c=\lambda_{\rm UV}a(\eta) \qquad (4.11)$$

such that $\lambda_{\rm UV}$ is the physical UV cutoff (i.e. which remains unchanged in the physical (x^i,t) frame). Thus, we can reexpress Eq. (4.7) as

$$G_p^{II}(\eta)=(2\pi)^3\int_{-\infty}^{\eta}d\eta_1 a^2(\eta_1)\,\mathbf{Im}[u_p^2(\eta)u_p^{*2}(\eta_1)]\times\int_{\Lambda_{\rm IR}}^{\lambda_{\rm UV}a(\eta_1)}d^3q\,2p^2\sin^2\theta|h_q(\eta_1)|^2 \qquad (4.12)$$

where both $\lambda_{\rm UV}$ and $\Lambda_{\rm IR}$ are constant. Thus, completing the above integral and replacing the cutoffs by the renormalization scales $\tilde{\mu}_{\rm UV}$ and $\mu_{\rm IR}$, we get

$$G_p^{II}(\eta=0)=\frac{H^2}{(2\pi)^3 2p^3}\frac{H^2}{(2\pi)^2}\times\left[2\log(\mu_{\rm IR}/p)+2\log(H/\tilde{\mu}_{\rm UV})-\frac{\tilde{\mu}_{\rm UV}^2}{H^2}\right]. \qquad (4.13)$$

For the other one-loop diagram in Fig. 3 i.e. G_p^I, we can use (4.3) and (4.5) to easily obtain the following result:

$$G_p^{I}(\eta=0)=\frac{H^2}{(2\pi)^3 2p^3}\frac{H^2}{(2\pi)^2}\times\left[2\log(p/\mu_{\rm IR})+\frac{2}{3}\log(\tilde{\mu}_{\rm UV}/H)\right] \qquad (4.14)$$

where one may note the appearance of H inside the logarithm as above. Note also that $\tilde{\mu}_{\rm UV}$ is the physical renormalization scale, whereas $\mu_{\rm IR}$ is the comoving renormalization scale.

V. DIMENSIONAL REGULARIZATION

Our next step is to analyze the dimensional regularization of the two one-loop diagrams of Fig. 3. Dimensional regularization is rather subtle here because we need to first find the modes in de Sitter space in $d=4+\delta$ dimensions where $\delta\to 0$. This is related to one of the issues associated with dimensional regularization: absence of the usual analytic form of the action integrand as a function of the wave number. Of course a way out of this problem is to analyze the system in a slow roll inflationary scenario or an equivalent de Sitter space-time.

In the following we will start by finding the modes in d space-time dimensions for the de Sitter background.

A. Free field quantization

In d dimensional space-time, the Lagrangian of the massless scalar field in the conformal FRW background metric is written as

$$\begin{aligned}\mathcal{L}&=\sqrt{-\det g}\left[-\frac{1}{2}a^{-2}\eta^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma\right]\\&=-\frac{1}{2}a^{d-2}\eta^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma.\end{aligned} \qquad (5.1)$$

Canonical quantization requires that the field should be redefined. Considering the power of the scale factor a in Eq. (5.1), we are led to defining canonical variables v and $\tilde{h}$ as

$$v\equiv\sigma a^{-1-\delta/2} \qquad \tilde{h}\equiv\frac{h}{\sqrt{2}}a^{-1-\delta/2}, \qquad (5.2)$$

where $\delta=d-4$. In the limit $\eta\to-\infty$, we get

$$v_k = \frac{1}{(2\pi)^{3/2}\sqrt{2k}}e^{-ik\eta} \qquad \tilde{h}_{\pm,k} = \frac{1}{(2\pi)^{3/2}\sqrt{2k}}e^{-ik\eta}. \tag{5.3}$$

Once we know the behavior for the redefined components at $\eta \to -\infty$, one may use them to determine the expressions for the modes u_k and $h_{\pm,k}$. With the usual vacuum initial conditions (5.3), the d-dimensional equation of motion of the free fields can be solved to give us the following mode expansions:

$$\begin{aligned} u_k &= \frac{e^{i\pi(1+(\delta/4))}H^{1+(\delta/2)}(-k\eta)^{(3+\delta)/2}}{4\pi\sqrt{2}k^{(3+\delta)/2}}\mathcal{H}_{(3+\delta)/2}(-k\eta) \\ h_{\pm,k} &= \frac{e^{i\pi(1+(\delta/4))}H^{1+(\delta/2)}(-k\eta)^{(3+\delta)/2}}{4\pi k^{(3+\delta)/2}}\mathcal{H}_{(3+\delta)/2}(-k\eta), \end{aligned} \tag{5.4}$$

where $\mathcal{H}_{(3+\delta)/2}$ is a Hankel function. The d-dimensional modes can now be Taylor expanded around $\delta = 0$ to give us the following mode components:

$$u_k = \frac{H}{(2\pi)^{3/2}\sqrt{2k^3}}(1+ik\eta)e^{-ik\eta}\left(1+\frac{\delta}{2}\log(-H\eta)+\cdots\right). \tag{5.5}$$

Note that there is an extra term $\delta\log(-H\eta)$ in the mode expansion. It is important to consider this term in dimensional regularization. As was pointed out in [28], this correction term will change the one-loop final result from $\log(k/\mu)$ to $\log(H/\mu)$. In the following analysis we will carefully consider the implication of this term in the regularization process.

B. Zeroth order regularization

Let us start by ignoring the $\delta\log(-H\eta)$ term in the mode expansion. We will call this the zeroth order calculation and in the next section we will introduce the correction term. For dimensional regularization, it is useful to introduce the RG scale μ_D in the interaction Lagrangians that modifies our earlier interactions (2.2) in the following way:

$$\begin{aligned} \mathcal{L}_1 &= \frac{a\mu_D^{-(\delta/2)}}{2}h_{ij}\partial_i\sigma\partial_j\sigma \\ \mathcal{L}_2 &= \frac{a\mu_D^{-\delta}}{2}h_{il}h_{lj}\partial_i\sigma\partial_j\sigma. \end{aligned} \tag{5.6}$$

Using these, the value of the first diagram in Fig. 3 can now be worked out in the following way:

$$\begin{aligned} G_p^I(\eta) = &-4(2\pi)^{6+2\delta}\mathbf{Re}\int_{-\infty}^{\eta}d\eta_1 a^{2+\delta}(\eta_1) \\ &\times\int_{-\infty}^{\eta}d\eta_2 a^{2+\delta}(\eta_2)\int d^{3+\delta}q\,p^4\sin^4\theta\Big[\theta(\eta_1-\eta_2) \\ &\times u_p^2(\eta)u_p^*(\eta_1)u_p^*(\eta_2)u_{p'}(\eta_1)u_{p'}^*(\eta_2)h_q(\eta_1)h_q^*(\eta_2) \\ &-\frac{1}{2}|u_p^2(\eta)|^2u_p(\eta_1)u_p^*(\eta_2)u_{p'}(\eta_1) \\ &\times u_{p'}^*(\eta_2)h_q(\eta_1)h_q^*(\eta_2)\Big]. \end{aligned} \tag{5.7}$$

The analysis of the above integral will be a little easier compared to the integrals arising in the brute cutoff scheme because in the current scheme the cutoff δ does not depend on time. This means that we can do the time integral first, yielding the following expression:

$$\begin{aligned} G_p^I(\eta) = &\frac{H^4}{(2\pi)^6p^4}\int d^{3+\delta}q \\ &\times\int d^{3+\delta}p'\delta^3(\mathbf{p}+\mathbf{p}'+\mathbf{q})f(p,p',q), \end{aligned} \tag{5.8}$$

where we have kept all the momentum dependences from (5.7) in the function $f(p, p', q)$. An immediate advantage in writing (5.7) in the form (5.8) is that we can use dimensional analysis to predict

$$\int d^{3+\delta}q\int d^{3+\delta}p'\delta^3(\mathbf{p}+\mathbf{p}'+\mathbf{q})f(p,p',q) \propto p^{1+\delta}F, \tag{5.9}$$

where the constant of proportionality will be determined below, and F represents the following functional form in either the UV or the IR:

$$F = \frac{F_0}{\delta} + F_1 \tag{5.10}$$

where F_0 and F_1 are both independent of δ as expected. In the above Eq. (5.10) one might wonder why F just has a single pole. This is because in flat space-time one-loop dimensional regularization has only a single pole. The curved space-time cannot yield more poles.

In the limit of $\delta \to 0$, the δ appearing in the denominator cancels out in the standard way to give us the following result:

$$\begin{aligned} &\int d^{3+\delta}q\int d^{3+\delta}p'\delta^3(\mathbf{p}+\mathbf{p}'+\mathbf{q})f(p,p',q) \\ &\quad = p(F_0\log p + \text{constant}), \end{aligned} \tag{5.11}$$

where the constant term in independent of p but allows a δ^{-1} singular term. This divergence is not a problem as it will be renormalized by the UV completion of our theory.

Let us now choose a momentum scale p such that we can divide the d^3q momentum integral into two parts:[11] $0 \le q \le p$ and $p \le q \le \infty$. We can equivalently divide F_0 into two parts: $F_0^{(\mathrm{IR})}$ and $F_0^{(\mathrm{UV})}$ whose values can be read off from (5.7). This gives us[12]:

$$\frac{2\pi F_0^{(\mathrm{UV})}}{3}\int_p^\infty \frac{dq}{q}\left(\frac{q}{\mu_{D,\mathrm{UV}}}\right)^\delta = p\left[\frac{2\pi F_0^{(\mathrm{UV})}}{3}\log(\mu_{D,\mathrm{UV}}/p) + \text{constant}\right]$$

$$2\pi p F_0^{(\mathrm{IR})}\int_0^p \frac{dq}{q}\left(\frac{q^2}{\mu_{D,\mathrm{IR}}(\eta_1)\mu_{D,\mathrm{IR}}(\eta_2)}\right)^{\delta/2} = [a(\eta_1)a(\eta_2)]^{\delta/2} p[2\pi F_0^{(\mathrm{IR})}\log(a\mu_{D,\mathrm{IR}}/p) + \text{constant}], \tag{5.12}$$

whereas before, the constant parts in the above equations are divergent but independent of p. Note also that, since in dimensional regularization we always choose physical renormalization group (RG) scales, they are related to the RG scales used in the brute cutoff scheme of the previous section in the following way:

$$\mu_{D,\mathrm{UV}} = \tilde{\mu}_{\mathrm{UV}}, \qquad \mu_{D,\mathrm{IR}} = \frac{\mu_{\mathrm{IR}}}{a}. \tag{5.13}$$

Combining (5.12) with (5.7), we get our final result for the zeroth order analysis:

$$G_p^{I,0}(\eta = 0) = \frac{H^2}{(2\pi)^3 2p^3}\frac{H^2}{(2\pi)^2} \times \left[2\log(p/\mu_{\mathrm{IR}}) + \frac{2}{3}\log(\tilde{\mu}_{\mathrm{UV}}/p)\right], \tag{5.14}$$

where the subscript in $G_p^{I,0}$ denotes the one-loop term without the $\delta\log(-H\eta)$ correction. Note also that we have chosen the RG scales μ_{IR} and μ_{UV} as before.[13]

For the second diagram in Fig. 3 we can basically follow the same set of ideas to get the UV and IR divergence. The amplitude for the second diagram is given by

$$G_p^{II}(\eta) = (2\pi)^{3+\delta}\int_{-\infty}^{\eta} d\eta_1 a^{2+\delta}(\eta_1)\,\mathbf{Im}[u_p^2(\eta)u_p^{*2}(\eta_1)] \times \int d^{3+\delta}q\, 2p^2\sin^2\theta |h_q(\eta_1)|^2. \tag{5.15}$$

If we ignore the $\delta\log(-H\eta)$ correction to the modes, we can rewrite the amplitude as

$$G_p^{II,0}(\eta = 0) \simeq \frac{H^4}{(2\pi)^6 2p^3}\int d^{3+\delta}q\,\frac{(6p^2 + 5q^2)2p^2\sin^2\theta}{8p^4q^3} = \frac{H^2}{(2\pi)^3 2p^3}\frac{H^2}{(2\pi)^2}[2\log(\mu_{\mathrm{IR}}/p) + 2\log(p/\tilde{\mu}_{\mathrm{UV}})]. \tag{5.16}$$

C. First order regularization

Let us now add back the $\delta\log(-H\eta)$ correction term. In the following we will compute the effect of this addition to our loop regularization. Before we go about doing this, note that every mode has this kind of correction. In the loop integrals for the two diagrams, i.e. (5.7) and (5.15), the $\delta\log(-H\eta)$ in the loop can be cancelled by the scale factor in the equation. This is because the correction disappears if the modes are multiplied with $a^{\delta/2}$. When the loop diagram is considered, there is a factor proportional to $a^{2+\delta}$ coming from the time integral. However the correction from $u_p^2(\eta)$ can be absorbed in the redefinition of the renormalization scale, as we mentioned earlier.

Taking all the above into account, the correction to G_p^{II} is as follows:

$$G_p^{II,1}(\eta = 0) \simeq \frac{H^4\delta}{(2\pi)^6 2p^3}\int d^{3+\delta}q\log(H/p) \times \frac{(6p^2 + 5q^2)2p^2\sin^2\theta}{8p^4q^3} + \delta\times\text{constant}. \tag{5.17}$$

The above integral can be computed in exactly the way that we did in the previous subsection, i.e. by dividing into two parts dealing with IR and UV divergences, respectively. In fact there are no IR divergences because the prefactor $[a(\eta_1)a(\eta_2)]^{\delta/2}$ of (5.12) in the IR calculations will cancel the first order corrections from $u_p(\eta_1)u_p(\eta_2)$ in (5.7) and

[11] This is because p is the only allowed scale for us here. Therefore this gives a natural demarcation between UV and IR physics.

[12] It is easy to see that the UV and IR divergence are determined by the sign of δ. First consider the momentum integral, which contains UV and IR divergences simultaneously:

$$\int_{\Lambda_{\mathrm{IR}}}^{\Lambda_{\mathrm{UV}}}\frac{dp}{p} = \log\left(\frac{\Lambda_{\mathrm{UV}}}{\Lambda_{\mathrm{IR}}}\right).$$

This integral, which is defined in four dimensions, can be rewritten in $d = 4 + \delta$ dimensions by making the following change:

$$\int\frac{dp}{p^{1-\delta}} = \frac{p^\delta}{\delta}.$$

The δ dependence of the above integral tells us that when we use dimensional regularization to deal with UV and IR divergences, different choices of δ should be made. For example, when $p\to\infty$, and $\delta < 0$ can keep the UV divergence under control. Whereas when $p\to 0$ then $\delta > 0$ in order to keep the IR divergence under control.

[13] Note that the additional $1 + \frac{\delta}{2}\log(-H\eta)$ correction term coming from $u_p^2(\eta)$ in (5.7) only adds a correction term to (5.14) that can be absorbed in the definition of the renormalization scale.

(5.15). Therefore combining $G_p^{II,1}$ and $G_p^{II,0}$, we get our final result:

$$G_p^{II}(\eta=0) = G_p^{II,0} + G_p^{II,1} = \frac{H^2}{(2\pi)^3 2p^3}\frac{H^2}{(2\pi)^2}[2\log(\mu_{\rm IR}/p) + 2\log(H/\tilde{\mu}_{\rm UV})]. \tag{5.18}$$

Similarly one may also compute $G_p^{I,1}$. For this, if we are considering the UV divergence, we can use the approximation $\eta_2 \to \eta_1$. Since there are no IR divergences, this gives us

$$G_p^{I,1}(\eta=0) = \frac{H^2}{(2\pi)^3 2p^3}\frac{H^2}{(2\pi)^2}\left[\frac{2}{3}\log(p/H)\right] \tag{5.19}$$

which, when combined with $G_p^{I,0}$ in (5.14), gives us the following result:

$$G_p^{I}(\eta=0) = \frac{H^2}{(2\pi)^3 2p^3}\frac{H^2}{(2\pi)^2} \times\left[2\log(p/\mu_{\rm IR}) + \frac{2}{3}\log(\tilde{\mu}_{\rm UV}/H)\right]. \tag{5.20}$$

One may now compare (5.18) and (5.20) with (4.13) and (4.14), respectively, that we got using the brute cutoff. They match precisely for the logarithmic terms. One might however wonder about the additional quadratic piece in (4.13):

$$-\frac{H^2}{2p^3(2\pi)^5}\cdot\tilde{\mu}_{\rm UV}^2 \tag{5.21}$$

This term cannot be seen from the dimensional regularization because this method is optimized to capture the logarithmic divergences. Any divergences higher than logarithmic require a more involved regularization scheme.

VI. PAULI-VILLARS REGULARIZATION

Now that we have seen how the results from the brute cutoff and dimension regularization match up precisely, it is time to analyze the UV/IR divergences using the Pauli-Villars regularization scheme. Our starting point is to take the interactions (2.2) along with a potential $V(\sigma)$ that could in principle appear from moduli stabilization in M theory.

A. Pauli-Villars regulators

Pauli-Villars regularization is a scheme to cancel the divergences in loops by introducing one or several massive fields, sacrificing general covariance or gauge symmetry of the underlying theory. In this regularization process, the propagator of a field becomes proportional to

$$\frac{1}{k^2} - \frac{1}{k^2+M_0^2} = \frac{1}{k^2 + k^4/M_0^2}, \tag{6.1}$$

where M is a cutoff mass. In the high energy $E \gg M_0$ limit the propagator vanishes, cancelling the UV divergence by the heavy field. Similarly in the low-energy $E \ll M_0$ limit the propagator regains its massless limit.

The above is the standard story behind Pauli-Villars regularization. To extend this to more complicated scenarios we need to add more than one type of Pauli-Villars fields. This would, for example, change (6.1) to the following propagator:

$$\frac{1}{k^2} + \sum_n \frac{Z_n^{-1}}{k^2+M_n^2} = \frac{1}{k^2\prod_n(k^2+M_n^2)/\prod_n M_n^2}, \tag{6.2}$$

where Z_n^{-1} are the typical coefficents for the free parts of the Pauli-Villars Lagrangian and M_n are the masses of the regulator fields. These coefficients satisfy

$$\sum_n Z_n^{-1} = -1, \qquad \sum_n Z_n^{-1}M_n^2 = 0. \tag{6.3}$$

To apply the Pauli-Villars regularization scheme to our case, we need two sets of Pauli-Villars fields: one set for the scalar field σ and the other set for the graviton field h_{ij}. We will call these fields as χ_n and γ_{ij}^n, respectively. The typical Lagrangian for our scalar field σ in the presence of gravitational interaction is given by

$$\mathcal{L} = \sqrt{-g}\left[\frac{R}{2} - \frac{a^2}{2}\partial_\mu\sigma\partial^\mu\sigma - V_1(h_{ij},\sigma) - V_2(\sigma)\right], \tag{6.4}$$

where $V_2(\sigma)$ is a potential term for σ whose form follows from the ultraviolet theory being considered, and $V_1(h_{ij},\sigma)$ is the minimal coupling of the metric h_{ij} with σ that generates the two interactions (2.2).

Once we switch on the two sets of Pauli-Villars fields the Lagrangian with the scalar field σ takes the following form:

$$\mathcal{L}_s = \sqrt{-g}\left[-\frac{a^2}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}\sum_n Z_n^{-1}(g^{\mu\nu}\partial_\mu\chi_n\partial_\nu\chi_n + M_n^2\chi_n^2) - V_1\left(h_{ij}+\sum_n\gamma_{ij}^n, \sigma+\sum_n\chi_n\right) - V_2\left(\sigma+\sum_n\gamma_n\right)\right], \tag{6.5}$$

where we have shifted the fields (h_{ij},σ) by their corresponding regulator fields in the potential to generate the required couplings between them. The gravitational part of the Lagrangian can also be adjusted to take into account the action of the regulator fields:

$$\mathcal{L}_g = \sqrt{-g}\left[\frac{1}{2}R_0 - \frac{1}{8}g^{\mu\nu}\partial_\mu h_{ij}\partial_\nu h_{ij} - \frac{1}{8}\sum_n\tilde{Z}_n^{-1}(g^{\mu\nu}\partial_\mu\gamma_{ij}^n\partial_\nu\gamma_{ij}^n + \tilde{M}_n^2\gamma_{ij}^{n2})\right], \tag{6.6}$$

where R_0 is the curvature associated with the background de Sitter space-time and $(\tilde{Z}_n, \tilde{M}_n)$ satisfy relations similar to (6.3). The above action for the graviton h_{ij} and the regulator fields γ^n_{ij} make sense because the free graviton field and free scalar field are the same up to some constant. Thus, adding the regulator fields should be identical for both cases.

Having obtained the full action for our case, let us investigate the interaction terms. The linear shift in the potential $V_1(h_{ij}, \sigma)$ immediately gives us the following relevant interactions:

$$\mathcal{L}_3 = \frac{a}{2}\left(h_{ij} + \sum_n \gamma^n_{ij}\right)\partial_i\sigma\partial_j\sigma\mathcal{L}_4 = -\frac{a}{4}\left(h + \sum_n \gamma^n\right)_{il}\left(h + \sum_m \gamma^m\right)_{lj}\partial_i\sigma\partial_j\sigma. \quad (6.7)$$

The above are the two kinds of interactions that we will consider for our case. However there are additional interactions of the form:

$$\frac{a}{2}\sum_{n,m,l}(h_{ij} + \gamma^n_{ij})(\partial_i\sigma\partial_j\chi_m + \partial_i\chi_l\partial_j\chi_m) - V_2\left(\sigma + \sum_k \chi_k\right) - \frac{a}{4}\sum_{n,m,k,p}(h + \gamma^n)_{il}(h + \gamma^m)_{lj}(\partial_i\sigma\partial_j\chi_k + \partial_i\chi_p\partial_j\chi_k). \quad (6.8)$$

Before we analyze the above interactions, it is easy to see from (6.5) that the interaction $V_2(\sigma + \sum_k\chi_k)$ leads to diagrams that are unambiguously regularized in [30]. So all we need to consider are the other interactions. Most of the interactions in (6.8) are not one-loop. The only one-loop diagrams are given in Fig. 4. Since both diagrams have the same χ_m fields propagating in the loop, the regularization for h_{ij} proceeds in exactly the same way as in the case of the first interaction in (6.7). This is evident when we incorporate the diagrams in Fig. 4 in our computations.

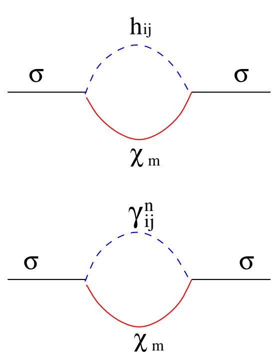

FIG. 4 (color online). The other one-loop interactions in the theory.

B. Quantization of the Pauli-Villars fields

As we see from (6.7), the only relevant Pauli-Villars fields that we need to consider in our case are the fields γ^n_{ij}, since the χ_n fields appear in diagrams that are not relevant in our case. Therefore, let us start by defining $\gamma^0_{ij} \equiv h_{ij}$ with $\tilde{Z}_0 \equiv 1$, $\tilde{M}_0 = 0$, and γ^n_{ij} for $n \geq 1$ being the set of Pauli-Villars fields. We can then write down the following mode expansion:

$$\gamma^n_{ij}(\mathbf{z}, \eta) = \int d^3q \sum_{s=\pm}[a(\mathbf{q})e_{ij}(q, s)\gamma^n_{s,q}(\eta)e^{i\mathbf{q}\cdot\mathbf{z}} + a^\dagger(\mathbf{q})e^*_{ij}(q, s)\gamma^{n*}_{s,q}(\eta)e^{-i\mathbf{q}\cdot\mathbf{z}}] \quad (6.9)$$

where e_{ij} satisfies the identity (3.4) and $\gamma^n_{s,q} \equiv \gamma^n_q$ denote the modes of γ^n_{ij} for a given momentum q. The creation and annihilation operators then satisfy

$$[a_n(\mathbf{k}_1), a^\dagger_m(\mathbf{k}_2)] = \delta^3(\mathbf{k}_1 - \mathbf{k}_2)\delta_{mn}. \quad (6.10)$$

One may similarly quantize the other set of Pauli-Villars fields χ_n, but will not do so here. The mode expansion of the fields χ_n is used to compute the two diagrams of Fig. 4.

C. The regularization process

We are now ready to perform the actual regularization process using the two interactions (6.7). Our method will be very similar to the one recently developed in [30], which the reader may consult for additional information. Note that the analysis of [30] only dealt with the scalar field σ and its Pauli-Villars partners χ_n. We therefore extend the technique of [30] to apply to graviton fluctuation.

As before, our aim is to regularize the two-point functions using a Pauli-Villars field. To do this, note that the internal momentum integral may be separated into two parts, exactly as in [30]. As one part we choose an interval where the internal momenta are much smaller than the masses of the regulator fields, in the other part we take the interval where the internal momenta are of the same order as the masses or larger. We will call the separation scale Q (following [30]). It plays no physical role. Therefore, the final two-point function should not depend on the scale Q.

The two-point function for the second interaction in (6.7) is then given (using comoving coordinates) by

$$G^{II}_p = (2\pi)^3\int_{-\infty}^{\eta} d\eta_1 a^2(\eta_1)\,\mathbf{Im}[u^2_p(\eta)u^{*2}_p(\eta_1)]p_ip_j \times \sum_s\sum_{KMN}\tilde{Z}^{-1}_K\int d^3q\, e_{il}(\hat{q}, s)e^*_{lj}(\hat{q}, s)\gamma^K_{Mq}(\eta_1)\gamma^{K*}_{Nq}(\eta_1). \quad (6.11)$$

The above integral may now be divided into two intervals as discussed above by introducing the momentum scale Q. If we also choose the masses $\tilde{M}_n = M_n$ ($n \geq 1$, $\tilde{M}_0 = 0$) for simplicity, then we can express (6.11) for $q > Q$ in the following way:

$$\begin{aligned}
G_p^{II,>Q} &= \frac{4}{3}\pi\int_{-\infty}^{\eta} d\eta_1 a^2(\eta_1)\,\mathbf{Im}[u_p^2(\eta)u_p^{*2}(\eta_1)]p^2\Big[\sum_n \tilde{Z}_m^{-1}M_n^2\ln M_n + (\dot{H}(\eta_1)+2H^2(\eta_1))\Big(\ln 2 - \sum_n \tilde{Z}_n^{-1}\ln M_n\Big) \\
&\quad - \frac{Q^2}{a(\eta_1)^2} - (\dot{H}(\eta_1)+2H^2(\eta_1))\ln\Big(\frac{Q}{a(\eta_1)}\Big)\Big] \\
&= \frac{4}{3}\pi\int_{-\infty}^{\eta} d\eta_1 a^2(\eta_1)\,\mathbf{Im}[u_p^2(\eta)u_p^{*2}(\eta_1)]p^2\Big[\mu_A^2 + (\dot{H}(\eta_1)+2H^2(\eta_1))\log\Big(\frac{\mu_B a(\eta_1)}{Q}\Big) - \frac{Q^2}{a(\eta_1)^2},\Big]
\end{aligned} \tag{6.12}$$

where in the second equality we have chosen the renormalization scales μ_A and μ_B. These two scales will be related to each other, but we will make the identification after we add the contributions from amplitudes with $q < Q$. For $q < Q$ our result is

$$G_p^{II,<Q} = (2\pi)^3\int_{-\infty}^{\eta} d\eta_1 a^2(\eta_1)\,\mathbf{Im}[u_p^2(\eta)u_p^{*2}(\eta_1)]2p^2\int_{\Lambda_{\rm IR}}^{Q} d^3q\sin^2\theta|h_q(\eta_1)|^2. \tag{6.13}$$

Once we restrict our result to the de Sitter space-time, the final amplitude will be given by the sum of the above two, i.e. the sum of (6.12) and (6.13). This gives

$$\begin{aligned}
G_p^{II} &\equiv G_p^{II,>Q} + G_p^{II,<Q} \\
&= \frac{8}{3}\pi(2\pi)^3\int_{-\infty}^{\eta} d\eta_1 a^2(\eta_1)\,\mathbf{Im}[u_p^2(\eta)u_p^{*2}(\eta_1)]p^2\Big[\mu_A^2 + 2H^2(\eta_1)\log\Big(\frac{\mu_B a(\eta_1)}{\Lambda_{\rm IR}}\Big)\Big] \\
&= \frac{H^2}{(2\pi)^3 2p^3}\frac{H^2}{(2\pi)^2}\Big[2\log(\mu_{\rm IR}/p) + 2\log(H/\tilde{\mu}_{\rm UV}) - \frac{\tilde{\mu}_{\rm UV}^2}{H^2}\Big],
\end{aligned} \tag{6.14}$$

where to go from the second step to the final one, we have identified $\mu_A = \mu_B = \tilde{\mu}_{\rm UV}$ and $\Lambda_{\rm IR} \equiv \mu_{\rm IR} a$. This matches precisely with the other two regularization schemes defined with physical UV cutoff and comoving IR cutoff.

Finally, for the first interaction of (6.7) the amplitude is given by the following expression (again using comoving coordinates):

$$\begin{aligned}
G_p^I &= -4(2\pi)^6\,\mathbf{Re}\int_{-\infty}^{\eta} d\eta_1 a^2(\eta_1)\int_{-\infty}^{\eta} d\eta_2 a^2(\eta_2)\sum_s\sum_{KLMNM'N'}\tilde{Z}_K^{-1}Z_L^{-1}\int d^3q\,p_ip'_jp_kp'_le_{ij}(\hat{q},s)e^*_{kl}(\hat{q},s) \\
&\quad\times\Big[\theta(\eta_1-\eta_2)u_p^2(\eta)u_p^*(\eta_1)u_p^*(\eta_2)\gamma^K_{Mq}(\eta_1)\gamma^{K*}_{M'q}(\eta_2)u^L_{Np'}(\eta_1)u^{L*}_{N'p'}(\eta_2) \\
&\quad - \frac{1}{2}|u_p(\eta)|^2u_p^*(\eta_1)u_p(\eta_2)\gamma^{K*}_{Mq}(\eta_1)\gamma^K_{M'q}(\eta_2)u^{L*}_{Np'}(\eta_1)u^L_{N'p'}(\eta_2)\Big].
\end{aligned} \tag{6.15}$$

The above integral can again be restricted to the two intervals exactly as before. For $q > Q$, (6.15) takes the following form:

$$\begin{aligned}
G_p^{I,>Q} &= \frac{16}{15}\pi\int_{-\infty}^{\eta} d\eta_1 a^2(\eta_1)\,\mathbf{Im}[u_p^2(\eta)u_p^{*2}(\eta_1)]p^4\Big[\sum_{mn}\tilde{Z}_m^{-1}Z_n^{-1}\frac{M_n^2\ln M_n - M_m^2\ln M_m}{M_n^2 - M_m^2} \\
&\quad + \sum_n \tilde{Z}_n^{-1}\ln M_n + \sum_n Z_n^{-1}\ln M_n + \ln\Big(\frac{Q}{a(\eta_1)}\Big)\Big] \\
&= \frac{16}{15}\pi\int_{-\infty}^{\eta} d\eta_1 a(\eta_1)\,\mathbf{Im}[u_p^2(\eta)u_p^{*2}(\eta_1)]p^4\log\frac{Q}{a(\eta_1)\tilde{\mu}_{\rm UV}},
\end{aligned} \tag{6.16}$$

where $\tilde{\mu}_{\rm UV}$ is the required physical UV renormalization scale. On the other hand, for $q < Q$ we have the following expression which is a slight variation of (4.3):

$$\begin{aligned}
G_p^{I,<Q} &= -4(2\pi)^6\,\mathbf{Re}\int_{-\infty}^{\eta} d\eta_1 a^2(\eta_1)\int_{-\infty}^{\eta} d\eta_2 a^2(\eta_2)\int_{\Lambda_{\rm IR}}^{Q} d^3q\,p^4\sin^4\theta \\
&\quad\times\Big[\theta(\eta_1-\eta_2)u_p^2(\eta)u_p^*(\eta_1)u_p^*(\eta_2)u_{p'}(\eta_1)u^*_{p'}(\eta_2)h_q(\eta_1)h_q^*(\eta_2) \\
&\quad - \frac{1}{2}|u_p(\eta)|^2u_p(\eta_1)u_p^*(\eta_2)u_{p'}(\eta_1)u^*_{p'}(\eta_2)h_q(\eta_1)h_q^*(\eta_2)\Big].
\end{aligned} \tag{6.17}$$

In de Sitter space we can add up (6.17) and (6.16) to give us the following final expression for the one-loop interaction:

$$G_p^I \equiv G_p^{I,>Q} + G_p^{I,<Q} = \frac{H^2}{(2\pi)^3 2p^3} \frac{H^2}{(2\pi)^2} \left[2 \log(p/\mu_{\rm IR}) + \frac{2}{3} \log(\tilde{\mu}_{\rm UV}/H) \right], \tag{6.18}$$

which again matches precisely with the results that we got from the other two regularization schemes.

VII. CONCLUSIONS

In this paper we have computed one-loop graviton corrections to Green's function of a scalar matter field in de Sitter space using three different regularization schemes: brute-force cutoff, dimensional regularization and the Pauli-Villars prescription. We have shown that careful evaluation in the three cases leads to the identical result.

There are both infrared and ultraviolet divergences which appear in the one-loop computation. By embedding the de Sitter phase into an ultraviolet complete theory we were able to justify the use of an ultraviolet cutoff at a fixed physical scale. This leads to the conclusion that the Hilbert space of modes of the effective low-energy field theory is growing as space is expanding, and this in turn leads to the presence of a growing contribution of infrared modes to correlation functions.

The embedding of an inflationary model in the context of the string theory allows us to study the trans-Planckian "problem" [55] for cosmological fluctuations in inflationary cosmology. In toy models of inflation, it is unclear what state to choose for the fluctuation modes when they arise from the "trans-Planckian sea". The answer, however, will be well defined once the inflationary model is embedded in an ultraviolet complete theory. We are currently investigating this issue [56]. We are also planning to study further consequences of the growth of the phase space of infrared modes which contribute to correlation functions.

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