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Constraining asymmetric dark matter through observations of compact stars

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We put constraints on asymmetric dark matter candidates with spin-dependent interactions based on the simple existence of white dwarfs and neutron stars in globular clusters. For a wide range of the parameters (WIMP mass and WIMP-nucleon cross section), weakly interacting massive particles (WIMPs) can be trapped in progenitors in large numbers and once the original star collapses to a white dwarf or a neutron star, these WIMPs might self-gravitate and eventually collapse forming a mini–black hole that eventually destroys the star. We impose constraints competitive to direct dark matter search experiments, for WIMPs with masses down to the TeV scale.

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I. INTRODUCTION

Observations of clusters of galaxies, rotations curves of individual galaxies, cosmic microwave background anisotropies, and many other methods suggest the existence of dark matter. A possible realization of dark matter might be in the form of weakly interacting massive particles (WIMPs). A huge effort is being undertaken by experimentalists to directly detect WIMPs in underground or space experiments, as well as by theorists to incorporate them into viable theories beyond the standard model.

The situation experimentally is still not clear, as the majority of the experiments have not detected WIMPs so far. Direct search experiments with Earth based detectors like the Cryogenic Dark Matter Search (CDMS) [1] and XENON [2] have imposed constraints on the WIMP-nuclei cross sections, assuming the local dark matter density around the Earth as inferred from the cosmological and other data (see, e.g., Ref. [3] for the determination of the amount of dark matter from the WMAP data). On the other hand, DAMA experiment [4] claims dark matter detection with parameters that contradict other experiments if taken at face value.

Given the still unclear picture regarding the nature of dark matter, it is of crucial importance to constrain as much as possible the WIMP candidates, including their mass and interactions. Several such candidates exist in the market depending on what theory beyond the standard model one chooses, ranging from supersymmetry [5,6] and hidden sectors [7,8], to technicolor [9–15]. The WIMPs can be classified according to their properties, i.e., if they are produced thermally, if they are asymmetric [9,16,17], if they have spin-dependent or spin-independent cross section with the nuclei, if their collisions with the nuclei are elastic or inelastic [18–22], and/or whether they are self-interacting [23–26].

Apart from direct searches, constraints on the properties of the WIMPs might arise from astrophysical observations as, for example, in [27]. Concentration of the WIMPs within stars can affect, under certain circumstances, the evolution of the latter, and/or products of WIMP annihilation within the stars could be directly or indirectly detected. The capture of WIMPs in the Sun and the Earth [28–30] has been used to predict a possible signature for an indirect detection of dark matter based on neutrino production due to WIMP coannihilation [31,32]. Constraints on the dark matter properties and the dark matter profile can also be imposed due to the effect of dark matter on the evolution of low mass stars [33,34], and main sequence stars [35], on possible gravitational collapse of neutron stars [36], and on the cooling process of compact objects such as neutron stars and white dwarfs [37–42]. In particular, the authors of [36] have investigated under what conditions a neutron star can collapse gravitationally due to accretion of WIMPs, providing an upper bound for the WIMP masses given the local dark matter density and the time of accretion. Although the bound for the mass of a bosonic WIMP was low ~10 MeV, the upper bound for fermionic WIMPs was quite high $\sim 10^5$ TeV (and therefore not relevant for physics at the TeV scale).

In this paper we investigate possible constraints that can arise from stars that accrete WIMPs during their lifetime and then collapse into a more compact object, white dwarf or a neutron star, inheriting the accumulated dark matter. Depending on the location of the star and the WIMP-nuclei cross section, it might be possible to impose constraints on the mass of the WIMP, excluding in some cases candidates

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that are lighter than TeV. Such constraints improve significantly the existing ones, and may become relevant for LHC physics.

More specifically, we consider two different cases. In the first case, we examine the accretion of WIMPs with spin-dependent interactions with protons onto a Sun-like star. We deduce under what circumstances the accumulated WIMPs can trigger a gravitational collapse once the star has turned into a white dwarf. In the second case, a supermassive star accretes WIMPs which have a spin-dependent cross section with nucleons (protons and neutrons). A typical supermassive star of 15 solar masses lives about 10^7 yr and then explodes forming a neutron star. Under certain assumptions, the WIMPs inherited by the neutron star from its progenitor will thermalize, sink to the center, and, for some range of parameters, collapse further into a black hole. Thus, the mere existence of neutron stars might impose constraints on the mass or the cross section of the WIMPs. In all cases we will assume cross sections that are compatible with experimental constraints from the direct dark matter searches.

II. ACCRETION OF WIMPS ONTO STARS

The crucial parameter that determines the capture of WIMPs by a star is the WIMP-nucleon cross section. In this paper we will consider dark matter candidates that exhibit predominantly a spin-dependent cross section with nucleons. Spin-independent cross section is highly constrained from direct dark matter search experiments, whereas the spin-dependent one is less constrained. The present constraint on the spin-independent cross section (normalized to a single proton) is roughly 3×10^{-44} cm^2 for a WIMP of a mass 100 GeV [1]. However, the rate of events is proportional to the local WIMP number density, and because for a fixed dark matter energy density the number density decreases inversely proportional to the WIMP mass, the present constraint can be written (for masses higher than TeV) as

$$\sigma_{\rm SI} < 3 \times 10^{-43}\ {\rm cm}^2 \left(\frac{m}{\rm TeV}\right).$$

As one can see, the constraint becomes weaker at higher masses.

The constraints on spin-dependent cross section are not so strict. The best upper limit on the spin-dependent cross section of WIMP with neutron is $\sim 10^{-38}$ cm^2 [43], whereas the analysis for spin-dependent cross section between WIMP and proton gives a minimum upper limit by 1 order of magnitude higher [44]. The constraint for a spin-dependent cross section between WIMP-nucleon (proton or neutron) can be written as

$$\sigma_{\rm SD} < 7 \times 10^{-38}\ {\rm cm}^2 \left(\frac{m}{\rm TeV}\right).$$

This constraint also weakens linearly in m at high masses.

The difference between the spin-independent and spin-dependent constraints is due to several reasons. The first one is that WIMPs with the spin-independent cross section scatter coherently with the whole nucleus if their De Broglie wavelength is larger than the size of the nucleus. This condition is easily met on Earth-based detectors. The coherence increases the cross section between WIMP and nucleus compared to the one of WIMP-nucleon roughly by a factor of N^2 where N is the number of nucleons composing the nucleus. For example, this gives an enhancement by a factor of $\sim 73^2$ in the case of the Ge detectors of CDMS. In addition, form factors suppress further the spin-dependent cross section, making the resulting constraints even weaker.

Let us now briefly summarize the accretion of WIMPs onto a star. Following [28,37,41], one can write the accretion rate as follows:

$$F = \frac{8}{3}\pi^2 \frac{\rho_{\rm dm}}{m}\left(\frac{3}{2\pi\bar{v}^2}\right)^{3/2} GMR\bar{v}^2(1 - e^{-3E_0/\bar{v}^2})f, \quad (1)$$

where $\rho_{\rm dm}$ is the local dark matter density, $\bar{v}$ is the average WIMP velocity, M and R are the mass and the radius of the star, E_0 is the maximum energy of the WIMP per WIMP mass that can lead to a capture, and f denotes the probability for at least one WIMP-proton scattering to take place within the star. In this expression we have neglected relativistic corrections (which are very small for regular stars) and possible motion of the star with respect to the dark matter halo. The latter can reduce slightly the accretion rate, but not more than an order of magnitude. Since we are not targeting particular stars like the Sun, we are going to present results for stars that do not move relatively to the dark matter halo. However, one can easily get the correct accretion rate in the case of a moving star by multiplying the accretion rate by $(\sqrt{\pi}/2){\rm erf}(\eta)$, where $\eta = \sqrt{3/2}v_\odot/\bar{v}$, with $v_\odot$ being the velocity of the star.

Let us first estimate E_0. The recoil energy produced by a WIMP-proton scattering is within $0 < T < 4m_p m/(m + m_p)^2 E$ where E is the WIMP energy before the collision. Upon assuming that the distribution over the recoil energies is not very different from a uniform one, a typical scattering will produce a recoil of order $\sim 2m_p/m$ (for $m \gg m_p$). In order for a WIMP to be captured (i.e., to become gravitationally bound) after one collision, it must lose at least the initial kinetic energy it had far out from the star. This leads to

$$E_0 \simeq 2\frac{m_p}{m}\frac{GM}{R}.$$

This is a conservative estimate because we have implicitly assumed that the collision will take place at the outskirts of the star (at the radius R), and not somewhere deep inside where the kinetic energy (and therefore the recoil energy) would be larger.

For small cross sections, the probability f of at least one scattering of WIMP in a Sun-like star was estimated in Ref. [28] to be

$$f \simeq 0.89 \frac{\sigma}{\sigma_{\rm crit}}, \tag{2}$$

where

$$\sigma_{\rm crit} = \frac{m_p R^2}{M} \simeq 4 \times 10^{-36}\ {\rm cm}^2 \left(\frac{R}{R_\odot}\right)^2 \left(\frac{M}{M_\odot}\right)^{-1},$$

$M_\odot$ and $R_\odot$ being the mass and the radius of the Sun, respectively. Note that for another case of interest in what follows, a supermassive star of a mass $M = 15 M_\odot$ and radius $R = 6.75 R_\odot$ [45], the critical cross section is $\sigma_{\rm crit} = 1.25 \times 10^{-35}\ {\rm cm}^2$. The probability f saturates to 1 for $\sigma > \sigma_{\rm crit}$.

We should also mention that in principle, the probability f may change as a function of time. As the star burns its hydrogen to helium, WIMPs interacting via spin-dependent cross section passing through the star meet fewer protons (hydrogen) scatterers to interact. He^4 cannot interact through spin-dependent interactions with WIMPs. Therefore, if we make the simple assumption that the star has converted all the hydrogen to helium at the end of its hydrogen stage, the probability f would drop to zero as WIMPs passing through the star do not find protons to scatter off anymore and be captured. It is natural to include this effect in the definition of the critical cross section. In this way one gets for the spin-dependent case

$$\sigma_{\rm crit} \simeq 5.47 \times 10^{-36} \frac{1}{1 - t/t_0}\ {\rm cm}^2 \left(\frac{R}{R_\odot}\right)^2 \left(\frac{M}{M_\odot}\right)^{-1}, \tag{3}$$

where t_0 is the star's lifetime and we have assumed that the initial composition of the star is 75% hydrogen and 25% helium, i.e., the one of Big Bang nucleosynthesis. The critical cross section grows with time (the probability f decreases).

The WIMP capture rate, Eq. (1), can be written in convenient units as follows:

$$F = 1.1 \times 10^{27}\ {\rm s}^{-1} \left(\frac{\rho_{\rm dm}}{0.3\ {\rm GeV/cm}^3}\right) \left(\frac{220\ {\rm km/s}}{\bar{v}}\right) \left(\frac{\rm TeV}{m}\right) \times \left(\frac{M}{M_\odot}\right) \left(\frac{R}{R_\odot}\right) \left(1 - e^{((-3E_0)/(\bar{v}^2))}\right) f, \tag{4}$$

where f is defined by Eqs. (2) and (3). Making use of Eq. (4) one can estimate the amount of accreted WIMPs during the lifetime of the star. A typical Sun-like star will burn hydrogen for 10×10^9 years before it becomes a red giant and later a white dwarf. A typical 15-$M_\odot$ supermassive star burns first hydrogen to helium for 11.1×10^6 years and then helium to carbon, carbon to oxygen, etc. in a much smaller timescale (the last stage before the supernova explosion is the silicon burning lasting about 20 days).

III. GRAVITATIONAL COLLAPSE OF THE WIMP-SPHERE

Once the WIMPs are captured by the star, they start to thermalize through successive collisions with the nuclei inside the star and after sufficient time are described by the Maxwell-Boltzmann distribution in the velocity and distance from the center of the star. The majority of WIMPs then is concentrated within the radius

$$r_{\rm th} = \left(\frac{9T}{8\pi G \rho_c m}\right)^{1/2} \simeq 2 \times 10^8\ {\rm cm} \left(\frac{m}{\rm TeV}\right)^{-1/2}, \tag{5}$$

where T is the temperature of the star, ρ_c is the core density, and m is the mass of the WIMP. In the last equality we used typical values for a Sun-like star $T = 1.5 \times 10^7$ K, and $\rho_c = 150\ {\rm g/cm}^3$. For a supermassive star of mass $M = 15 M_\odot$, the thermal radius is roughly an order of magnitude larger (using typical values $T = 3.53 \times 10^7$ K and $\rho_c = 5.81\ {\rm g/cm}^3$ [45]).

Depending on the mass and the cross section between WIMP and nucleon, WIMPs might or might not thermalize during the lifetime of the star. Since our constraints will depend on the WIMP thermalization, we estimate here the thermalization time. The thermalization of captured WIMPs can be divided into two stages: at the first stage the WIMPs oscillate in the star's gravitational potential, crossing it twice per period. This lasts until the WIMP's orbit decreases to the size of the star. At the second stage, the WIMP moves completely inside the star on the orbit which shrinks to the thermal radius.

Consider the first stage. Each time the WIMP crosses the star it has a chance to collide and lose some energy. The time between collisions Δt is given by half a period of WIMPs oscillation divided by the ratio of the WIMP cross section to the critical cross section. At each collision the WIMP typically loses a fraction $2m_p/m$ of its energy. Averaging over the WIMP trajectory inside the star (assuming for simplicity that the latter passes through the center) the typical energy loss is

$$\Delta E = 2GMm_p \left(\frac{4}{3R} - \frac{1}{r}\right),$$

where M and R are the mass and the radius of the star, and r is the size of the WIMP's orbit. Dividing this energy change by Δt and expressing r in terms of energy gives the differential equation for the WIMP energy as a function of time,

$$\frac{dE}{dt} = -\frac{2\sqrt{2} m_p \sigma}{\pi G M m^{5/2}} \left(\frac{4}{3} E_* + E\right) |E|^{3/2},$$

where $E_* = GMm/R$ is the binding energy of the WIMP at the star surface. From this equation the duration of the first stage is

$$t_1 = \frac{\pi m R^{3/2}\sigma_{\rm crit}}{2m_p\sqrt{2GM}\sigma}\int_{\epsilon_0}^{1}\frac{d\epsilon}{(4/3-\epsilon)\epsilon^{3/2}} \sim \frac{3\pi m R^{3/2}\sigma_{\rm crit}}{4m_p\sqrt{2GM}\sigma}\sqrt{\frac{E_*}{|E_0|}}, \tag{6}$$

where $\epsilon_0 = |E_0|/E_*$ is the ratio of the WIMP initial energy E_0 to its binding energy at the star surface. Numerically, $\epsilon_0 \sim m_p/m$, and for a solar mass star and spin-dependent cross section we have

$$t_1 = 3\ {\rm yr}\left(\frac{m}{\rm TeV}\right)^{3/2}\left(\frac{\sigma}{10^{-35}\ {\rm cm}^2}\right)^{-1}. \tag{7}$$

The time becomes longer for larger masses and smaller cross sections.

At the second stage, the average time between two successive collisions is $\Delta t = 1/(n\sigma v)$, where n is the number density of the nucleons, σ is the WIMP-proton cross section, and $v = \sqrt{2E/m}$ is the average velocity of the WIMP. Therefore, the energy as a function of time is determined by the following equation:

$$\frac{dE}{dt} = -n\sigma v\Delta E = -2\sqrt{2}\rho\sigma\left(\frac{E}{m}\right)^{3/2}, \tag{8}$$

where ρ is the matter density of the star. Solving this equation gives the time t_2 needed to decrease the energy from $E_{\rm in}$ to E_f,

$$t_2 = \frac{m^{3/2}}{\sqrt{2}\rho\sigma}\left(\frac{1}{\sqrt{E_f}} - \frac{1}{\sqrt{E_{\rm in}}}\right). \tag{9}$$

Here the initial energy is of order $E_{\rm in} \simeq GM/R$, while the final energy is determined by the final size of the WIMP cloud. In case of cooling to the thermal radius this energy is $E_f = (3/2)kT$, where k is the Boltzmann constant and T is the temperature in the core of the star. One can safely neglect the term depending on $E_{\rm in}$ in Eq. (9) since $E_{\rm in} \gg E_f$.

In case of cooling to some fixed radius (e.g., to the radius of a future white dwarf ~4000 km, as will be of interest in what follows) the final energy is determined by this radius and the density of the star. Assuming typical parameters of a solar mass star, one gets

$$t_2 = 0.15\ {\rm yr}\left(\frac{m}{\rm TeV}\right)\left(\frac{\sigma}{10^{-35}\ {\rm cm}^2}\right)^{-1}. \tag{10}$$

Once inside either a white dwarf or a neutron star, WIMPs start to thermalize once again toward the much smaller thermal radius of a white dwarf (typically a few kilometers for a TeV WIMP) or of a neutron star (typically a few centimeters for a TeV WIMP). This process described by the same Eq. (9), but with different parameters. If a sufficiently large number of WIMPs have been accumulated, WIMPs may start self-gravitating and collapse gravitationally (in the absence, obviously, of a repulsive force between them). In this way the formation of a compact star (either a white dwarf or a neutron star) may trigger the collapse of the WIMP-sphere into a black hole.

The onset of the self-gravitating regime happens when the total mass of WIMPs inside the thermal radius becomes comparable to the total mass of the ordinary matter in the same region. This leads to the condition

$$N \gtrsim \left(\frac{T^3}{G^3 m^5 \rho_c}\right)^{1/2} \sim 10^{45}\left(\frac{m}{\rm TeV}\right)^{-5/2},$$

where we have substituted typical parameters of a white dwarf. For a neutron star the required number of dark matter particles is 5 orders of magnitude smaller.

The self-gravitating WIMP-sphere may collapse into a black hole if the Fermi pressure of the WIMPs cannot counterbalance the gravitational attraction. The onset of the gravitational collapse occurs when the potential energy of a WIMP exceeds the Fermi momentum, and therefore Pauli blocking cannot prevent the collapse anymore. This happens when

$$\frac{GNm^2}{r} > k_F = \left(\frac{3\pi^2 N}{V}\right) = \left(\frac{9\pi}{4}\right)^{1/3}\frac{N^{1/3}}{r}. \tag{11}$$

In the derivation of the above limit, we have considered that WIMPs are (semi)relativistic, which is justified since once WIMPs self-gravitate themselves, they get closer and closer, building up a Fermi momentum that eventually corresponds to relativistic velocities. From the above equation we can deduce the number of WIMPs needed for the collapse to take place,

$$N = \left(\frac{9\pi}{4}\right)^{1/2}\left(\frac{m_{\rm Pl}}{m}\right)^3 \simeq 5\times 10^{48}\left(\frac{m}{\rm TeV}\right)^{-3}, \tag{12}$$

where m is the WIMP mass and $m_{\rm Pl}$ is the Planck mass.

IV. CONSTRAINTS ON DARK MATTER

Having derived the accretion rate formula for a generic star and the amount of dark matter needed in order to form a mini-–black hole, we can proceed to the constraints that arise from the requirement that such mini–black holes are not created inside newly formed white dwarfs and neutron stars. We consider two different cases, i.e., constraints on spin-dependent cross sections from white dwarfs, and from neutron stars.

A. Constraints on spin-dependent cross section from white dwarfs

Dark matter WIMPs can have purely spin-independent or spin-dependent interactions with nuclei, or even both types at the same time. Because of the coherence effect, the spin-independent interactions are usually stronger than the spin-dependent ones. However, there are cases where the spin-independent cross section is either suppressed or absent. Such cases arise naturally in models where dark matter candidates have an axial coupling to gauge bosons. One characteristic example is Majorana particles. Majorana fermions scatter off nuclei without the N^2 enhancement mentioned earlier because the amplitudes of scattering on different nucleons add up incoherently. Since most of the nucleons within the nuclei come in pairs of opposite spin, the WIMP interacts effectively only with the unpaired nucleons. However, a Majorana particle is its own antiparticle, allowing therefore for WIMP coannihilation. The coannihilation invalidates our constraints because it destroys the WIMPs before they collapse into a black hole, unless the annihilation cross section is extremely small. However, in such a case the spin-dependent elastic cross section should be parametrically equally small.

The constraints we present in this subsection are valid for dark matter candidates which predominantly have a spin-dependent cross section without being Majorana particles. We constrain models of asymmetric dark matter with WIMPs that have axial couplings to gauge bosons like Z. Such candidates have been identified and studied in [46]. For example, Dirac fermions with predominant axial coupling to the Z boson have a suppressed spin-independent cross section and a dominant spin-dependent one. In asymmetric dark matter models of this type, although the annihilation cross section is not suppressed, annihilations are rare due to the asymmetry. In such cases we can impose constraints competitive to the direct dark matter search experiments.

In this subsection we look at potential candidates that have a spin-dependent cross section with protons, which is larger than the spin-independent cross section. We consider both types of cross sections compatible with the experimental constraints. In such a case, WIMPs will be captured by a solar mass star primarily due to their spin-dependent interactions. When the star turns into a white dwarf, most of the WIMPs which are inside the white dwarf radius will be inherited by the white dwarf.

In order to form a black hole, the accumulated WIMPs have to cool further. The spin-dependent interactions inside the white dwarf are suppressed because the latter is composed predominantly of spin-zero nuclei like He^4, C^{12}, and O^{16}. However, since the white dwarfs are much denser than the ordinary stars, much smaller interactions are sufficient for the successful WIMP thermalization. As we will show below, these interactions may be provided by a small spin-independent component in the interaction and/or by a small admixture of nuclei with nonzero spin such as, for example, C^{13}.

Consider as an example the case where there are spin-independent interactions in addition to the spin-dependent ones. In this case the dark matter particles are accumulated in a solar mass star mostly due to their spin-dependent interactions, but once the star collapses to a white dwarf made of spin-zero nuclei, they thermalize due to collisions of spin-independent nature. We do not have to assume that WIMPs interact also with neutrons. It is sufficient to have spin-dependent interactions with protons only. In fact, the experimental constraints on spin-dependent cross section of WIMPs with protons is even less stringent, the strongest bounds coming from CDMS [43], PICASSO [47], and KIMS [48]. The constraint is approximately given by

$$\sigma_{\mathrm{SD},p} < 10^{-36}\ \mathrm{cm}^2\left(\frac{m}{\mathrm{TeV}}\right)$$

at WIMP masses above 1 TeV. Note that stricter constraints on the spin-dependent cross section between WIMP-protons from the ICECUBE Collaboration [49] do not apply in our case since the constraints are based on annihilation of WIMPs, and we constrain candidates with spin-dependent cross section but of asymmetric nature, which effectively have no annihilations because the antiparticles are simply not present anymore.

In Fig. 1 we present the resulting constraints on the spin-dependent cross section, as a function of the WIMP mass. As one can see, these constraints are competitive with the direct ones at WIMPs masses of 1 TeV or less, depending on the dark matter density at the location of the observed white dwarf. The constraints cannot be extended to lower masses because, no matter what the cross section is, not enough WIMPs can be accumulated to trigger the collapse.

Let us now estimate the cross section required for the thermalization of the WIMPs inside the white dwarf. To begin, assume that WIMPs have a small spin-independent cross section with nucleons (this assumption

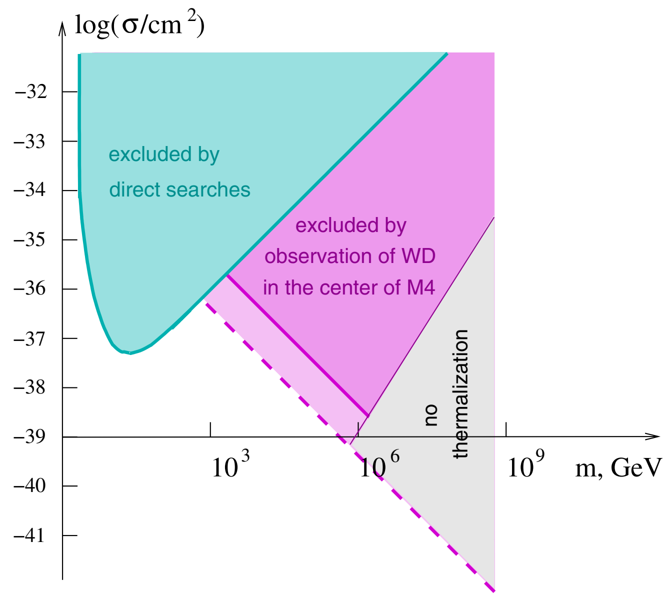

FIG. 1 (color online). Constraints on the spin-dependent WIMP-proton cross section. The constraints follow from the existence of old white dwarfs in globular clusters [50] with a core dark matter density of 10^3 GeV/cm^3 (straight solid line) and 10^4 GeV/cm^3 (straight dashed line). We also show the constraints from direct searches (solid curve).

will be relaxed later). Substituting typical white dwarf parameter in Eq. (9) one finds

$$t_2 = 4\ \mathrm{yr}\left(\frac{m}{\mathrm{TeV}}\right)^{3/2}\left(\frac{\rho}{10^8\ \mathrm{g/cm^3}}\right)^{-1}\left(\frac{\sigma}{10^{-43}\ \mathrm{cm^2}}\right)^{-1} \times\left(\frac{T}{10^7\ \mathrm{K}}\right)^{-1/2}. \quad (13)$$

Given that cold and old white dwarfs (more than several billion years) have been observed in the inner parts of globular clusters [50], the thermalization time may safely be as long as 1 Gyr. Assuming for simplicity that the white dwarf is made exclusively of C^{12}, this gives

$$\sigma_{\mathrm{SI}}^{C} > 4\times 10^{-52}\left(\frac{m}{\mathrm{TeV}}\right)^{3/2}\left(\frac{10^7\ \mathrm{K}}{T}\right)^{1/2}\left(\frac{t_0}{\mathrm{Gyr}}\right)^{-1}\ \mathrm{cm^2}, \quad (14)$$

where t_0 is the maximum time we allow for thermalization (e.g., 1×10^9 years) and σ_{SI}^{C} is the spin-independent WIMP-carbon cross section. The constraint can be rewritten as a constraint for the WIMP-proton cross section as

$$\sigma_{\mathrm{SI}}^{p} > 8\times 10^{-56}\left(\frac{m}{\mathrm{TeV}}\right)^{3/2}\left(\frac{T}{10^7\ \mathrm{K}}\right)^{-1/2}\left(\frac{t_0}{\mathrm{Gyr}}\right)^{-1}\ \mathrm{cm^2}, \quad (15)$$

taking into account that the spin-independent cross section WIMP-carbon is related to that of WIMP-proton as $\sigma_{\mathrm{SI}}^{C}/\sigma_{\mathrm{SI}}^{p} \simeq (\mu_C/\mu_H)^2 6^2$, where μ are the reduced masses of the respective WIMP-nucleus and 6^2 is the coherence enhancement due to the 6 protons of the carbon.

As we have already mentioned, a spin-independent interaction of WIMPs with nucleons or a small fraction of isotopes of carbon or oxygen with nonzero spin can play the same role in the thermalization of WIMPs. By inspection of Eq. (13), it is obvious that one can always trade the cross section for the number density. This means that, assuming a TeV WIMP, if thermalization can be achieved for $\sigma_{\mathrm{SI}}^{C} \sim 10^{-52}\ \mathrm{cm^2}$, it can equally be achieved by a spin-dependent cross section $\sigma_{\mathrm{SD}}^{C} = 10^{-41}\ \mathrm{cm^2}$ where C now represents C^{13} with a relative abundance C^{12}:C^{13} = 1:10^{-11}. Several red giants have been seen with a ratio C^{12}:C^{13} = 4:1 predicted by the equilibrium processes. This is because C^{12} from triple-alpha production can meet with hydrogen in outer shells of the red giant leading to C^{13}. In practice, this means that even with a 10^9 GeV WIMP an abundance of C^{13} as low as C^{12}:C^{13} = 100:1 is sufficient for the thermalization. Note that the WIMP-C^{13} cross section scales as $\sim\sigma_{\mathrm{SD}}^{p}{\mu_C}^2/\mu_p^2$ times other nuclear spin factors, so the abundance of C^{13} can be even 10^4 smaller compared to C^{12}. We should also emphasize here that C^{13} has an excess of a neutron, and therefore we have assumed that the WIMP couples equally to protons and neutrons.

For our constraints to be valid, we have to make sure that the black hole formed inside the white dwarf can eat/destroy the star within at most 1×10^9 years (smaller than the age of the older white dwarfs observed in globular clusters). First note that once the WIMPs form a black hole inside a white dwarf, regular nuclear matter starts falling in it. This process has been considered in Ref. [51]. It has been argued that for black holes of a size exceeding the atomic size the accretion proceeds in the Bondi regime which is characterized by a quasistationary matter flow into the black hole. The total rate of matter accretion can be expressed in terms of the matter density and sound speed far from the black hole. The accretion rate is proportional to the square of the black hole mass, so the change of the black hole mass with time is described by the equation

$$M(t) = \frac{M_0}{1 - t/t_*},$$

where M_0 is the initial black hole mass and t_* is the characteristic time scale over which the star is destroyed,

$$t_* = \frac{c_s^3}{\pi G^2 \rho_c M_0}.$$

Here c_s and ρ_c are the sound speed and the density in the core of the white dwarf, respectively. Numerically, the time t_* is

$$t_* \sim 8\times 10^3\ \mathrm{yr}\left(\frac{M_0}{10^{-12}M_\odot}\right)^{-1},$$

where we have estimated the value $c_s = 0.03c$ which is consistent with [51] and the core density $\rho = 10^8\ \mathrm{g/cm^3}$. The heavier the WIMP, the smaller the amount of dark matter accumulated, and the smaller the mass of the black hole created when the WIMPs collapse. For black holes made of collapsing WIMPs up to masses 10^9 GeV or slightly lower, the time it takes for the full destruction of the star (i.e., the time it takes for the black hole to eat the whole star) is less than a billion years.

The gray "no thermalization" area in Fig. 1 (see Eq. (7) and the discussion following) corresponds to cases where the captured WIMPs in the progenitor did not have enough time to settle within the radius of a white dwarf, meaning that although most of them will be gravitationally bound to the white dwarf when it is formed, their orbits do not necessarily intersect with it, and therefore there is the danger of not thermalizing and collapsing gravitationally. Although in principle in such a case WIMPs might not collapse gravitationally, the situation is far from clear and we leave this for a future study. This is because WIMP-WIMP interactions can redistribute the angular momentum of the WIMPs and might eventually lead a large fraction of them to be captured fast by the white dwarf. In addition, if the white dwarf is part of a binary system, there is again the possibility of allowing WIMPs to intersect with the white dwarfs. We should emphasize that to the left of the grey area, there is no ambiguity about the outcome, since WIMPs have enough time to concentrate in a radius smaller than that of the would-be white dwarf, and therefore once the white dwarf is formed, they are already inside.

B. Constraints on spin-dependent cross section from neutron stars

Constraints on the spin-dependent cross section can be imposed also directly by accretion of WIMPs into neutron stars [36], or by accretion of WIMPs in a progenitor supermassive star that later collapses to a neutron star. We consider supermassive stars that accrete dark matter during the hydrogen-burning stage. At other stages of the star evolution the accretion is negligible since the star is mainly composed of spin-zero nuclei like He^4, C^{12}, O^{16}, etc., and also, these stages are much shorter. After the last (silicon) stage, the supermassive star collapses to a neutron star. The supernova explosion cannot blow out the accumulated WIMPs because the spin-dependent interaction with silicon is zero. After the formation of the neutron star, the accumulated WIMPs are located mostly outside the neutron star, simply because the thermal radius of a supermassive star where WIMPs concentrate during the hydrogen-burning stage is several orders of magnitude larger than the radius of a neutron star. Upon assuming that WIMP-WIMP interactions or a binary system can redistribute the angular momentum and randomize the velocity of the WIMPs trapped by the supermassive progenitor, after the neutron star's formation, the WIMPs are captured by it very quickly [41]. Inside the neutron star the WIMPs continue to scatter on neutrons until coming to a thermal equilibrium with the star. We should emphasize that we consider asymmetric WIMPs with spin-dependent cross section with both protons and neutrons here.

There is an issue to address in the first place, which is why to consider constraints which are due to the WIMP accumulation in the progenitor of a neutron star and not directly those which arise due to capture by the neutron star itself. The latter constraints have been first considered in [36]. For the spin-independent cross sections compatible with the experimental limits, a neutron star can accumulate a similar amount of WIMPs as a supermassive star. This is because the spin-independent cross sections consistent with the experiments are many orders of magnitude smaller than the critical cross section of a massive star, but not necessarily smaller compared to the critical cross section of a neutron star. Given the fact that massive stars live (and accrete dark matter) for a much smaller time compared to neutron stars, the latter are more efficient in accumulating WIMPs overall. However, in the case of spin-dependent interaction, experimental limits on the cross section are comparable to the critical cross section of a supermassive star. In such a case, a massive star can accumulate more WIMPs in the 10×10^6 years of the hydrogen-burning stage than a neutron star would capture in 10×10^9 years. Therefore, the constraints resulting from the former process are stronger.

Several pulsars have been detected in globular clusters close to (or inside) the core [52]. In addition, neutron stars older than 10×10^9 years have also been observed (as, for example, the B1620-26 in M4). In Fig. 2 we present the constraints on the spin-dependent WIMP-nucleon cross section that arise from the nongravitational collapse of WIMPs inside neutron stars in dark-matter-rich environments such as the globular cluster M4. The grey rectangular area corresponds to exclusion due to direct neutron star accretion of WIMPs for roughly 10×10^9 years (with $\rho_{\rm dm} = 10^3\ {\rm GeV/cm^3}$) similarly to what previously has been derived in [36,39,41]. The area appears as a rectangle due to the fact that it excludes cross sections down to the critical cross section of a neutron star ($\sim 10^{-45}\ {\rm cm^2}$ [41]). The triangle area in purple is excluded (if one assumes the redistribution of WIMP velocities we have already discussed) due to WIMPs that have been accreted by the progenitor during its lifetime, and collapse in the core of the neutron star once they are "sucked" inside the neutron star. As one can see from the figure, the existence of a neutron star in the core of a globular cluster can impose in principle constraints competitive to those from the Earth-based dark matter search experiments for WIMP masses

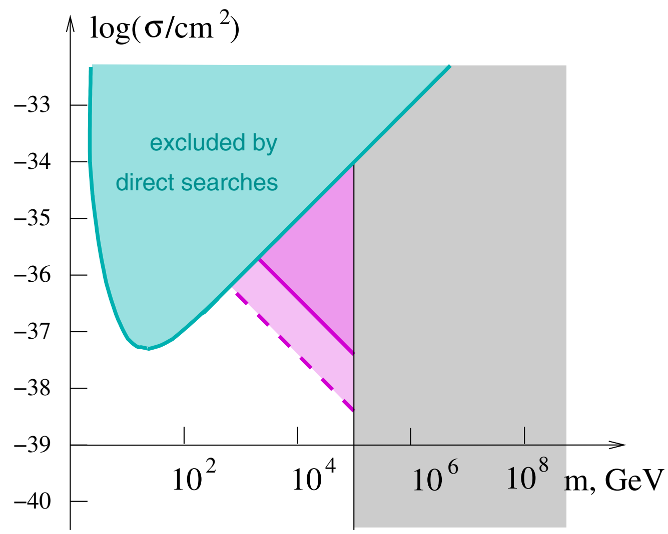

FIG. 2 (color online). Constraints on the spin-dependent WIMP-nucleon cross section that follow from the existence of old neutron stars in globular clusters [52]. The grey area is excluded by direct accretion of WIMPs (for 10 Gyr) in an old neutron star with local dark matter density $10^3\ {\rm GeV/cm^3}$. The purple area shows potential exclusion upon making an assumption regarding the distribution of WIMP velocities (see text), with the solid (dashed) line corresponding to local dark matter density $10^3(10^4)\ {\rm GeV/cm^3}$. We also show the exclusion area by direct dark matter search experiments.

above 3 to 10 TeV depending on the local dark matter density. Our constraints become better than the direct ones when the mass increases further, since at larger WIMP masses fewer particles are needed for the collapse, whereas for the Earth-based experiments it simply means fewer events in the detector.

In order to make sure that our constraints are valid, we have to estimate the time it takes for the WIMPs to collapse once the neutron star is formed. As we mentioned earlier, the time it takes for the WIMPs accumulated by the progenitor to be captured by the neutron star is negligible. Once inside the neutron star, WIMPs have to lose energy via collisions in order to concentrate in the center and collapse. This time has been estimated in [41] and for all relevant cases is at most of the order of a year. In addition, we have checked that for the whole relevant range of WIMP masses, self-gravitation of the WIMPs sets on even before they all concentrate within the thermal radius of a neutron star. This means that the collapse is even faster.

We should emphasize that if a mechanism for randomizing the velocities of the WIMPs is operating (and therefore our constraints are valid), we exclude parameter space (purple triangle) that is not excluded by the Earth-based experiments or by the gravitational collapse of WIMPs accumulated by the neutron star itself. Such a constraint can also be to some extent more robust than the one derived previously because no assumption is made regarding the age of the neutron star (as long as there is enough time for the black hole to destroy the star). On the contrary, in the constraints that come from direct accretion of WIMPs in the neutron star, one has to know with enough precision the age of the star in order to estimate the amount of WIMPs accreted.

V. CONCLUSIONS

We derived constraints on the spin-dependent cross section of asymmetric fermionic dark matter WIMPs based on the existence of white dwarfs and neutron stars in globular clusters. Our constraints are competitive to direct dark matter search experiments, excluding a large parameter space of cross sections and masses as low as TeV (or slightly lower than TeV).

In the case of white dwarfs, we were able to exclude a range of spin-dependent cross sections and WIMP masses because for these parameters, WIMPs that have been captured during the lifetime of the progenitor have enough time to concentrate within the core of the star that is inherited by the white dwarf, and eventually collapse gravitationally forming a black hole that destroys the star. This constraint is robust in the sense that it depends only on the local dark matter density of the globular cluster and no other hypothesis. We demonstrated that asymmetric WIMP candidates with only spin-dependent interactions with masses even lower than 1 TeV, trapped during the lifetime of the progenitor, can easily thermalize inside the white dwarf due to expected small abundances of isotopes of carbon or oxygen that carry spin.

If we now relax the strict condition of thermalization, i.e., to assume that WIMPs are gravitationally trapped by the progenitor but are not necessarily confined within the radius of a white dwarf, our constraints can be extended to higher masses and lower cross sections. However, in this case we have to make an extra assumption of a mechanism that redistributes the WIMP velocities in order for WIMPs which are on orbits around the white dwarf to intersect with it. Such a mechanism can be possibly provided by binaries or WIMP-WIMP interactions. Further investigation of this possibility is needed.

In the case of neutron stars, we exclude an area of WIMP masses and cross sections due to direct accretion of WIMPs in the neutron star. The constraints depend only on the local dark matter density of the globular cluster and the age of the neutron star. Upon assuming an extra mechanism of WIMP velocity redistribution, extra parameter space may be excluded down to the TeV scale.

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