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Flavor-changing top-charm associated productions at the ILC in the littlest Higgs model with *T* parity

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The littlest Higgs model with T parity has new flavor-changing couplings with the standard model quarks, which do not suffer strong constraints from electroweak precision data. So these flavor-changing interactions may enhance the cross sections of some flavor-changing neutral-current processes. In this work, we study the flavor-changing top-charm associated productions via the $e^-\gamma$ collision at the ILC. We find that the cross sections are sensitive to the mirror quark masses. With reasonable values of the parameters, the cross sections may reach the detectable level and provide useful information about the relevant parameters in the littlest Higgs model with T parity, especially in setting an upper limit on the mirror quark masses.

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I. INTRODUCTION

An interesting solution to the hierarchy problem of the standard model (SM) is the little Higgs theory [1]. In this theory the Higgs boson is regarded as a pseudo-Goldstone boson (PGB) which can be naturally ''little'' in the current reincarnation of the PGB idea called collective symmetry breaking. Through such a collective symmetry breaking mechanism, the one-loop quadratic divergences in the Higgs boson mass can be avoided. The littlest Higgs model (LH) [2] is the most economical implementation of the little Higgs idea, which, however, suffers strong constraints from electroweak precision data [3] due to the tree-level mixing of heavy and light mass eigenstates. So the LH model would require raising the mass scale and thus reintroduce the fine-tuning in the Higgs potential [4]. To solve this problem, a Z_2 discrete symmetry called T parity is introduced [5]. Under this T parity the SM particles are even while most of the new particles at the TeV scale are odd. T parity explicitly forbids any tree-level contribution from the heavy gauge bosons to the observables involving only SM particles as external states. Since in the littlest Higgs model with T parity (LHT) the corrections to the precision electroweak observables are generated at loop level and suppressed, the fine-tuning can be avoided [6].

It is well known that the flavor-changing neutral-current (FCNC) interactions are absent at tree level and extremely small at loop levels in the SM due to the Glashow-Iliopoulos-Maiani mechanism. However, in the LHT model, the flavor-changing (FC) interactions between the SM fermions and the mirror fermions, which are parameterized by the newly Cabibbo-Kobayashi-Maskawa (CKM)-like unitary mixing matrices, may have significant contributions to some FC processes. A great deal of attention has been paid to the FC interactions in the LHT model in recent years. First, the LHT flavor structure was analyzed and some constraints on the mirror fermion mass spectrum were obtained from a one-loop analysis of neutral meson mixing in the K, B, and D systems [7]. Then an extensive study of FC transitions in the LHT model was performed in [8–10], which considered all prominent rare K and B decays and presented a collection of Feynman rules to the order of v^2/f^2. Motivated by the experimental evidence of meson oscillations in the D system, the impact of $D^0 - \bar{D}^0$ mixing on the LHT flavor structure was investigated in [11]. Furthermore, the LHT flavor study was extended to the lepton flavor violating decays in [12].

The International Linear Collider (ILC) with a center of mass (c.m.) energy from 200 GeV to 1.0 TeV and high luminosity has been proposed [13]. Because of its rather clean environment and high luminosity, the ILC will be an ideal machine for probing new physics. In such a collider, in addition to the e^+e^- collision, we may also realize a $\gamma\gamma$ or $e^-\gamma$ collision with the photon beams generated by the backward Compton scattering of incident electron- and laser-beams [14]. In particular, as the heaviest fermion with a mass of the order of the electroweak scale, the top quark is naturally regarded to be more sensitive to new physics than other fermions. Therefore the top quark FCNC processes at the ILC would provide an important test for new physics. This stimulates many attempts in probing new physics via rare top quark decays [15] or FC production processes at the ILC [16–26]. The FC couplings between the SM fermions and the mirror fermions can also induce the loop-level $tcV (V = \gamma, Z, g)$ couplings in the LHT model. Studies [27–29] have shown that some processes induced by such tcV couplings in the LHT model can be significantly enhanced. In this paper, we will study the process $e^-\gamma \to e^- t\bar{c}$ induced by the tcV couplings in the LHT model and compare it with the process $e^+e^-(\gamma\gamma) \to t\bar{c}$ studied previously [28]. Note that these processes have been studied thoroughly in other models, such as the minimal supersymmetric standard model (MSSM) [16,17], the two Higgs doublet model

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(2HDM) [18,19], and the TC2 model [20–22], and also in the model-independent way [23–25]. They showed that the production rates of such processes could be significantly enhanced by several orders compared to the SM predictions [16,30]. As found in other new physics models [16,20], the process $e^-\gamma \to e^- t\bar{c}$ has a much larger rate than $e^+e^- \to t\bar{c}$ for some part of the parameter space. In our study we will compare the LHT prediction with those predicted by other new physics models. Such an analysis will help to distinguish different models once the measurements are observed at the ILC.

This paper is organized as follows. In Sec. II, we briefly review the LHT model. In Sec. III, we present the detailed calculations for the production processes. The numerical results and the discussion are shown in Sec. IV. Finally, we give our conclusion in the last section.

II. THE LITTLEST HIGGS MODEL WITH T PARITY

The LHT model [5] is based on a nonlinear σ model describing an $SU(5)/SO(5)$ symmetry breaking with a locally gauged subgroup $[SU(2)\times U(1)]^2$. The $SU(5)$ symmetry spontaneously breaks down to $SO(5)$ at the scale $f \sim \mathcal{O}(TeV)$. From the $SU(5)/SO(5)$ breaking, there arise 14 Nambu-Goldstone bosons which are described by the matrix Π, given explicitly by

$$\Pi = \begin{pmatrix} -\frac{\omega^0}{2}-\frac{\eta}{\sqrt{20}} & -\frac{\omega^+}{\sqrt{2}} & -i\frac{\pi^+}{\sqrt{2}} & -i\phi^{++} & -i\frac{\phi^+}{\sqrt{2}} \\ -\frac{\omega^-}{\sqrt{2}} & \frac{\omega^0}{2}-\frac{\eta}{\sqrt{20}} & \frac{v+h+i\pi^0}{2} & -i\frac{\phi^+}{\sqrt{2}} & \frac{-i\phi^0+\phi^P}{\sqrt{2}} \\ i\frac{\pi^-}{\sqrt{2}} & \frac{v+h-i\pi^0}{2} & \sqrt{4/5}\eta & -i\frac{\pi^+}{\sqrt{2}} & \frac{v+h+i\pi^0}{2} \\ i\phi^{--} & i\frac{\phi^-}{\sqrt{2}} & i\frac{\pi^-}{\sqrt{2}} & -\frac{\omega^0}{2}-\frac{\eta}{\sqrt{20}} & -\frac{\omega^-}{\sqrt{2}} \\ i\frac{\phi^-}{\sqrt{2}} & \frac{i\phi^0+\phi^P}{\sqrt{2}} & \frac{v+h-i\pi^0}{2} & -\frac{\omega^+}{\sqrt{2}} & \frac{\omega^0}{2}-\frac{\eta}{\sqrt{20}} \end{pmatrix}. \tag{1}$$

Here, $H = (-i\pi^+/\sqrt{2}, (v+h+i\pi^0)/2)^T$ is the SM Higgs doublet and Φ is a physical scalar triplet with

$$\Phi = \begin{pmatrix} -i\phi^{++} & -i\frac{\phi^+}{\sqrt{2}} \\ -i\frac{\phi^+}{\sqrt{2}} & \frac{-i\phi^0+\phi^P}{\sqrt{2}} \end{pmatrix}. \tag{2}$$

In the LHT model, a T-parity discrete symmetry is introduced to make the model consistent with the electroweak precision data. Under the T parity, the fields Φ, ω, and η are odd, and the SM Higgs doublet H is even.

For the gauge subgroup $[SU(2)\times U(1)]^2$ of the global symmetry $SU(5)$, from the first step of symmetry breaking $[SU(2)\times U(1)]^2 \to SU(2)_L \times U(1)_Y$, which is identified as the SM electroweak gauge group, the Goldstone bosons ω^0, $\omega^\pm$, and η are, respectively, eaten by the new T-odd gauge bosons Z_H, W_H, and A_H, which obtain masses at the order of $\mathcal{O}(v^2/f^2)$

$$M_{Z_H} \equiv M_{W_H} = fg\left(1-\frac{v^2}{8f^2}\right), \quad M_{A_H} = \frac{fg'}{\sqrt{5}}\left(1-\frac{5v^2}{8f^2}\right), \tag{3}$$

with g, g' being the corresponding coupling constants of $SU(2)_L$ and $U(1)_Y$.

From the second step of symmetry breaking $SU(2)_L \times U(1)_Y \to U(1)_{\rm em}$, the masses of the SM T-even gauge bosons Z and W are generated through eating the Goldstone bosons π^0 and $\pi^\pm$. They are given at $\mathcal{O}(v^2/f^2)$ by

$$M_{W_L} = \frac{gv}{2}\left(1-\frac{v^2}{12f^2}\right), \quad M_{Z_L} = \frac{gv}{2cos\theta_W}\left(1-\frac{v^2}{12f^2}\right), \tag{4}$$
$$M_{A_L} = 0.$$

A consistent and phenomenologically viable implementation of T parity in the fermion sector requires the introduction of mirror fermions. The T-even fermion section consists of the SM quarks, leptons, and an additional heavy quark T_+. The T-odd fermion sector consists of three generations of mirror quarks and leptons and an additional heavy quark T_-. Only the mirror quarks (u_H^i, d_H^i) are involved in this paper. The mirror fermions get masses

$$m_{H_i}^u = \sqrt{2}\kappa_i f\left(1-\frac{v^2}{8f^2}\right) \equiv m_{H_i}\left(1-\frac{v^2}{8f^2}\right), \tag{5}$$

$$m_{H_i}^d = \sqrt{2}\kappa_i f \equiv m_{H_i},$$

where the Yukawa couplings κ_i can in general depend on the fermion species i.

The mirror fermions induce a new flavor structure and there are four CKM-like unitary mixing matrices in the mirror fermion sector:

$$V_{H_u}, \qquad V_{H_d}, \qquad V_{H_l}, \qquad V_{H_\nu}. \tag{6}$$

These mirror mixing matrices are involved in the FC interactions between the SM fermions and the T-odd mirror fermions which are mediated by the T-odd heavy gauge bosons or the Goldstone bosons. V_{H_u} and V_{H_d} satisfy the relation

$$V_{H_u}^\dagger V_{H_d} = V_{\rm CKM}. \tag{7}$$

We parameterize the V_{H_d} with three angles θ_{12}^d, θ_{23}^d, θ_{13}^d and three phases δ_{12}^d, δ_{23}^d, δ_{13}^d

$$V_{H_d} = \begin{pmatrix} c_{12}^d c_{13}^d & s_{12}^d c_{13}^d e^{-i\delta_{12}^d} & s_{13}^d e^{-i\delta_{13}^d} \\ -s_{12}^d c_{23}^d e^{i\delta_{12}^d} - c_{12}^d s_{23}^d s_{13}^d e^{i(\delta_{13}^d-\delta_{23}^d)} & c_{12}^d c_{23}^d - s_{12}^d s_{23}^d s_{13}^d e^{i(\delta_{13}^d-\delta_{12}^d-\delta_{23}^d)} & s_{23}^d c_{13}^d e^{-i\delta_{23}^d} \\ s_{12}^d s_{23}^d e^{i(\delta_{12}^d+\delta_{23}^d)} - c_{12}^d c_{23}^d s_{13}^d e^{i\delta_{13}^d} & -c_{12}^d s_{23}^d e^{i\delta_{23}^d} - s_{12}^d c_{23}^d s_{13}^d e^{i(\delta_{13}^d-\delta_{12}^d)} & c_{23}^d c_{13}^d \end{pmatrix}. \tag{8}$$

The matrix V_{H_u} is then determined through $V_{H_u} = V_{H_d} V_{\rm CKM}^\dagger$. As in the case of the CKM matrix the angles θ_{ij}^d can all be made to lie in the first quadrant with $0 \le \delta_{12}^d, \delta_{23}^d, \delta_{13}^d < 2\pi$.

III. TOP-CHARM QUARK ASSOCIATED PRODUCTIONS IN THE LHT MODEL

In the LHT model, there are FC interactions between SM quarks and T-odd mirror quarks which are mediated by the heavy T-odd gauge bosons or Goldstone bosons. With these FC couplings, the loop-level FC couplings $t\bar{c}\gamma(Z)$ can be induced and the relevant Feynman diagrams are shown in Fig. 1.

The effective one-loop-level couplings $t\bar{c}\gamma(Z)$ can be directly calculated by the method introduced in Ref. [31]. The relevant Feynman rules can be found in Ref. [9]. We list the explicit forms of $\Gamma^{\mu}_{t\bar{c}\gamma}(p_t, p_{\bar{c}})$ and $\Gamma^{\mu}_{t\bar{c}Z}(p_t, p_{\bar{c}})$ in the Appendix.

The FC couplings $t\bar{c}\gamma(Z)$ can contribute to the top-charm associated productions via the $e^-\gamma$ collision at the ILC. The process $e^-\gamma \rightarrow e^- t\bar{c}$ proceeds through the process $e^-\gamma \rightarrow e^-\gamma^*(Z^*)\gamma \rightarrow e^- t\bar{c}$, where the γ beam is generated by the backward Compton scattering of incident electron- and laser-beam and the $\gamma^*(Z^*)$ is radiated from e^- beam. The corresponding Feynman diagrams are shown in Fig. 2(a)–2(e) and the invariant production amplitudes of the process can be written as

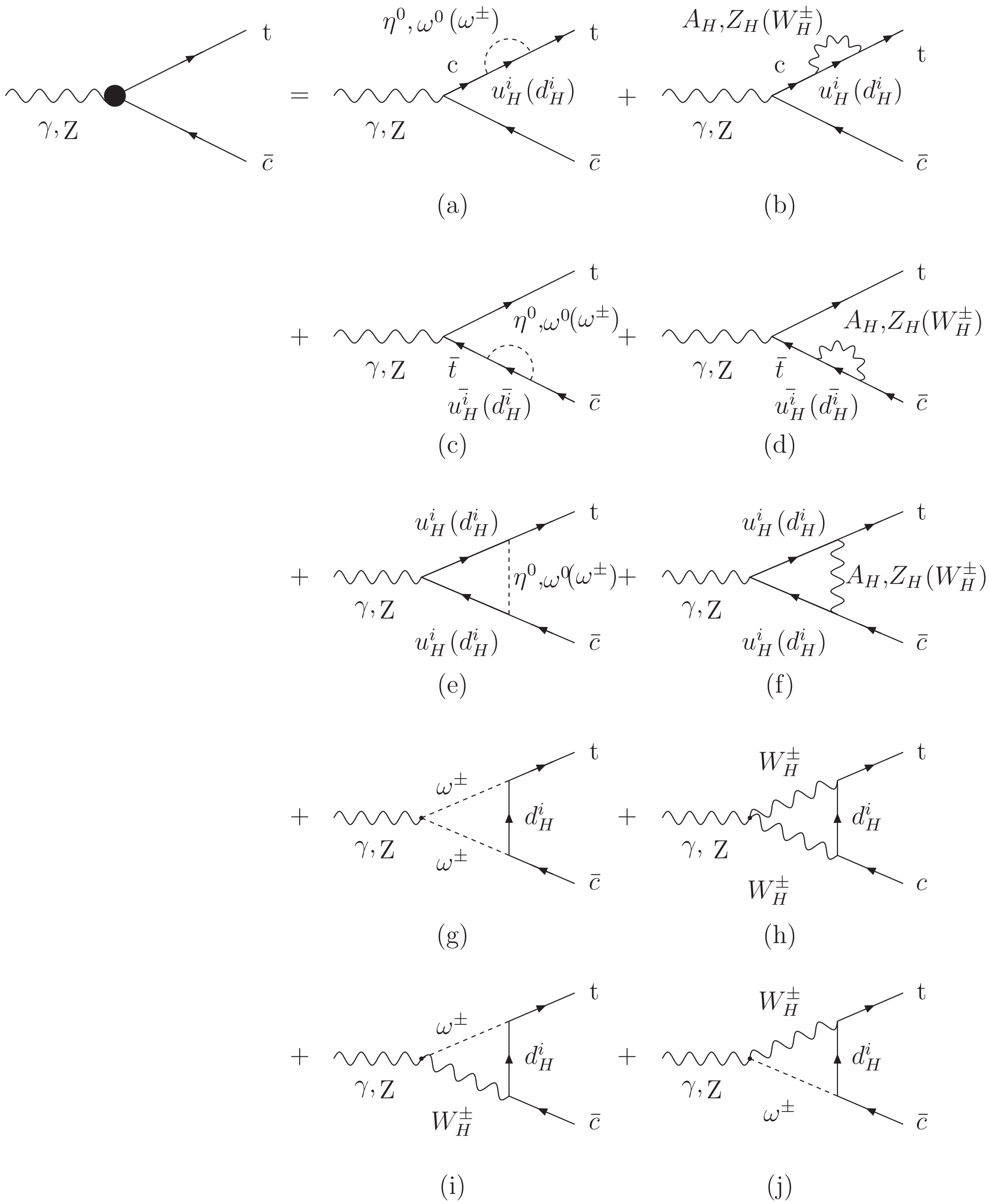

FIG. 1. One-loop contributions of the LHT model to the couplings $t\bar{c}\gamma(Z)$.

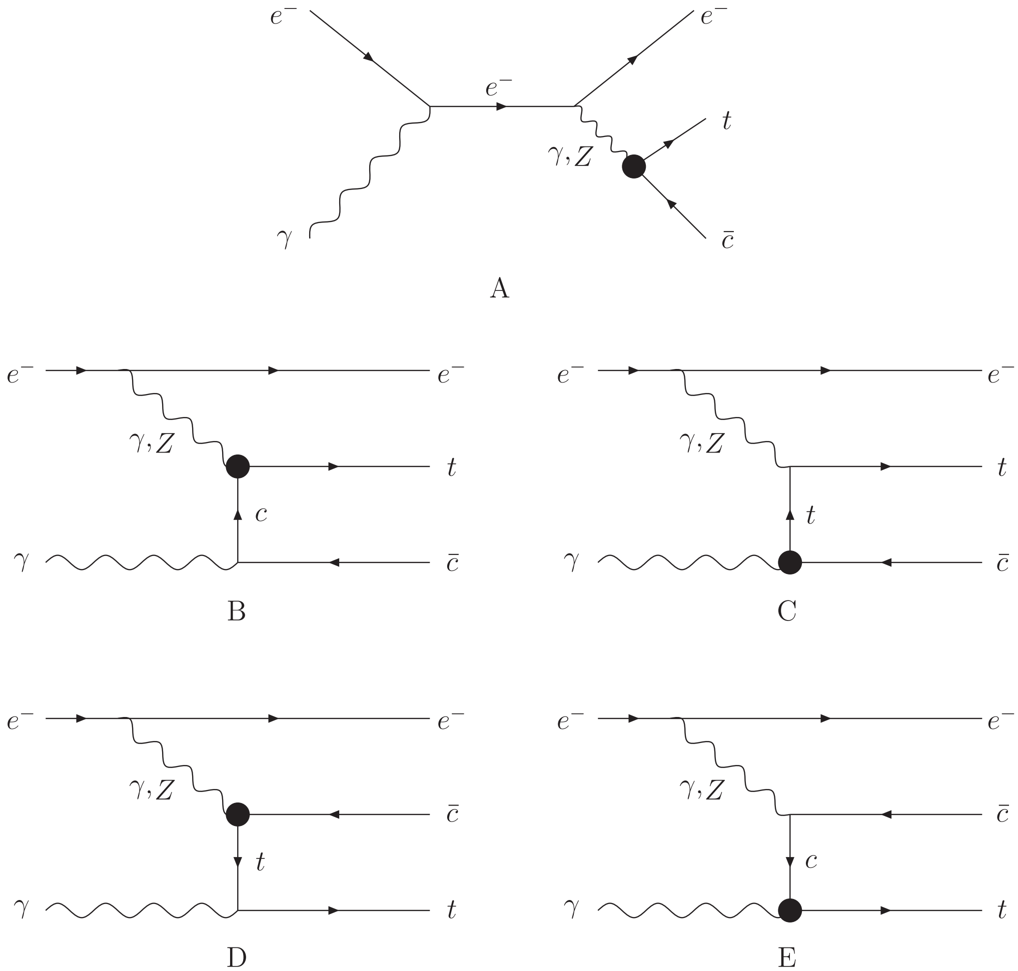

FIG. 2. The Feynman diagrams for $e^-\gamma \to e^- t\bar{c}$ in the LHT model.

$$M_A^\gamma = -e^2 G(p_1+p_2,0)G(p_3+p_4,0)\bar{u}_{e^-}(p_5)\gamma_\mu(\not{p}_1+\not{p}_2)\not{\epsilon}(p_2)u_{e^-}(p_1)\times\bar{u}_t(p_3)\Gamma^\mu_{t\bar{c}\gamma}(p_3,p_4)v_{\bar{c}}(p_4), \tag{9}$$

$$M_A^Z = \frac{eg}{\cos\theta_W}G(p_1+p_2)G(p_3+p_4,M_Z)\bar{u}_{e^-}(p_5)\gamma_\mu\bigg[\bigg(-\frac{1}{2}+\sin^2\theta_W\bigg)P_L+(\sin^2\theta_W)P_R\bigg](\not{p}_1+\not{p}_2)\not{\epsilon}(p_2)u_{e^-}(p_1)\bar{u}_t(p_3) \times\Gamma^\mu_{t\bar{c}Z}(p_3,p_4)v_{\bar{c}}(p_4), \tag{10}$$

$$M_B^\gamma = \frac{2e^2}{3}G(p_1-p_5,0)G(p_2-p_4,m_c)\bar{u}_{e^-}(p_5)\gamma_\mu u_{e^-}(p_1)\times\bar{u}_t(p_3)\Gamma^\mu_{t\bar{c}\gamma}(p_3,p_4-p_2)(\not{p}_2-\not{p}_4+m_c)\not{\epsilon}(p_2)v_{\bar{c}}(p_4), \tag{11}$$

$$M_B^Z = -\frac{2eg}{3\cos\theta_W}G(p_1-p_5,M_Z)G(\not{p}_2-\not{p}_4,m_c)\bar{u}_{e^-}(p_5)\gamma_\mu\bigg[\bigg(-\frac{1}{2}+\sin^2\theta_W\bigg)P_L+(\sin^2\theta_W)P_R\bigg]u_{e^-}(p_1)\bar{u}_t(p_3) \times\Gamma^\mu_{t\bar{c}Z}(p_3,p_4-p_2)(\not{p}_2-\not{p}_4+m_c)\not{\epsilon}(p_2)v_{\bar{c}}(p_4), \tag{12}$$

$$M_C^\gamma = \frac{2e^2}{3}G(p_1-p_5,0)G(p_2-p_4,m_t)\bar{u}_{e^-}(p_5)\gamma_\mu u_{e^-}(p_1)\times\bar{u}_t(p_3)\gamma^\mu(\not{p}_2-\not{p}_4+m_t)\Gamma^\nu_{t\bar{c}\gamma}(p_2-p_4,p_4)\epsilon_\nu(p_2)v_{\bar{c}}(p_4), \tag{13}$$

$$M_C^Z = -\frac{g^2}{\cos^2\theta_W}G(p_1-p_5,M_Z)G(p_2-p_4,m_t)\bar{u}_{e^-}(p_5)\gamma_\mu\bigg[\bigg(-\frac{1}{2}+\sin^2\theta_W\bigg)P_L+(\sin^2\theta_W)P_R\bigg]u_{e^-}(p_1)\bar{u}_t(p_3)\gamma^\mu \times\bigg[\bigg(\frac{1}{2}-\frac{2}{3}\sin^2\theta_W\bigg)P_L-\frac{2}{3}(\sin^2\theta_W)P_R\bigg](\not{p}_2-\not{p}_4+m_t)\Gamma^\nu_{t\bar{c}\gamma}(p_2-p_4,p_4)\epsilon_\nu(p_2)v_{\bar{c}}(p_4), \tag{14}$$

$$M_D^\gamma = \frac{2e^2}{3} G(p_1 - p_5, 0) G(p_3 - p_2, m_t) \bar{u}_{e^-}(p_5) \gamma_\mu u_{e^-}(p_1) \times \bar{u}_t(p_3) \epsilon(p_2)(\not{p}_3 - \not{p}_2 + m_t) \Gamma^\mu_{t\bar{c}\gamma}(p_3 - p_2, p_4) v_{\bar{c}}(p_4), \quad (15)$$

$$M_D^Z = -\frac{2eg}{3\cos\theta_W} G(p_1 - p_5, M_Z) G(p_3 - p_2, m_t) \bar{u}_{e^-}(p_5) \gamma_\mu \left[\left(-\frac{1}{2} + \sin^2\theta_W\right) P_L + (\sin^2\theta_W) P_R\right] \times u_{e^-}(p_1) \bar{u}_t(p_3) \epsilon(p_2)(\not{p}_3 - \not{p}_2 + m_t) \Gamma^\mu_{t\bar{c}Z}(p_3 - p_2, p_4) v_{\bar{c}}(p_4), \quad (16)$$

$$M_E^\gamma = \frac{2e^2}{3} G(p_1 - p_5, 0) G(p_3 - p_2, m_c) \bar{u}_{e^-}(p_5) \gamma_\mu u_{e^-}(p_1) \times \bar{u}_t(p_3) \Gamma^\nu_{t\bar{c}\gamma}(p_3, p_2 - p_3) \epsilon_\nu(p_2)(\not{p}_3 - \not{p}_2 + m_c) \gamma^\mu v_{\bar{c}}(p_4), \quad (17)$$

$$M_E^Z = -\frac{g^2}{\cos^2\theta_W} G(p_1 - p_5, M_Z) G(p_3 - p_2, m_c) \bar{u}_{e^-}(p_5) \gamma_\mu \left[\left(-\frac{1}{2} + \sin^2\theta_W\right) P_L + (\sin^2\theta_W) P_R\right] \times u_{e^-}(p_1) \bar{u}_t(p_3) \Gamma^\nu_{t\bar{c}\gamma}(p_3, p_2 - p_3) \epsilon_\nu(p_2)(\not{p}_3 - \not{p}_2 + m_c) \gamma^\mu \times \left[\left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right) P_L - \frac{2}{3}(\sin^2\theta_W) P_R\right] v_{\bar{c}}(p_4), \quad (18)$$

where $P_L = \frac{1}{2}(1 - \gamma_5)$ and $P_R = \frac{1}{2}(1 + \gamma_5)$ are the left and right chirality projectors. p_1, p_2 are the momenta of the incoming e^-, γ, and p_3, p_4, p_5 are the momenta of the outgoing final states top quark, anticharm quark and electron, respectively. We also define $G(p, m)$ as $\frac{1}{p^2 - m^2}$.

With the above amplitudes, we can directly obtain the production cross section $\hat{\sigma}(\hat{s})$ for the subprocess $e^-\gamma \to e^- t\bar{c}$ and the total cross sections at the e^+e^- linear collider can be obtained by folding $\hat{\sigma}(\hat{s})$ with the photon distribution function $F(x)$ [32]:

$$\sigma_{e^-\gamma \to e^- t\bar{c}}(s_{e^+e^-}) = \int_{(m_t+m_c)^2/s_{e^+e^-}}^{x_{\max}} dx F(x) \hat{\sigma}(\hat{s}), \quad (19)$$

where s is the c.m. energy squared for e^+e^-. The subprocess occurs effectively at $\hat{s} = xs$, and x is the fractions of the electron energies carried by the photons. The explicit form of the photon distribution function $F(x)$ is

$$F(x) = \frac{1}{D(\xi)}\left[1 - x + \frac{1}{1-x} - \frac{4x}{\xi(1-x)} + \frac{4x^2}{\xi^2(1-x)^2}\right], \quad (20)$$

with

$$D(\xi) = \left(1 - \frac{4}{\xi} - \frac{8}{\xi^2}\right)\ln(1 + \xi) + \frac{1}{2} + \frac{8}{\xi} - \frac{1}{2(1 + \xi)^2}, \quad (21)$$

and

$$\xi = \frac{4E_0\omega_0}{m_e^2}. \quad (22)$$

E_0 and ω_0 are the incident electron and laser light energies, and $x = \omega/E_0$. The energy ω of the scattered photon depends on its angle θ with respect to the incident electron beam and is given by

$$\omega = \frac{E_0\left(\frac{\xi}{1+\xi}\right)}{1 + \left(\frac{\theta}{\theta_0}\right)^2}. \quad (23)$$

Therefore, at $\theta = 0$, $\omega = E_0\xi/(1 + \xi) = \omega_{\max}$ is the maximum energy of the back-scattered photon, and $x_{\max} = \frac{\omega_{\max}}{E_0} = \frac{\xi}{1+\xi}$.

To avoid unwanted e^+e^- pair production from the collision between the incident and back-scattered photons, we should not choose too large ω_0. The threshold for e^+e^- pair creation is $\omega_{\max}\omega_0 > m_e^2$, so we require $\omega_{\max}\omega_0 \le m_e^2$. Solving $\omega_{\max}\omega_0 = m_e^2$, we find

$$\xi = 2(1 + \sqrt{2}) = 4.8. \quad (24)$$

For the choice $\xi = 4.8$, we obtain $x_{\max} = 0.83$ and $D(\xi_{\max}) = 1.8$.

In the above we have ignored the possible polarization for the photon and electron beams and we also assume that the number of the back-scattered photons produced per electron is one.

IV. NUMERICAL RESULTS AND DISCUSSION

In our numerical calculations, the charge conjugate $\bar{t}c$ production channel has also been included. To obtain the numerical results, we take the SM parameters as $m_t = 171.2$ GeV, $m_c = 1.25$ GeV, $\sin^2\theta_W = 0.231$, $M_Z = 91.2$ GeV, $\alpha_e = 1/128$. Moreover, the LHT model has several free parameters which are related to our study. They are the breaking scale f, 6 parameters $(\theta^d_{12}, \theta^d_{13}, \theta^d_{23}, \delta^d_{12}, \delta^d_{13}, \delta^d_{23})$ in the mixing matrix V_{H_u} and V_{H_d}, and the masses of the mirror quarks. For the mirror quark masses, we get $m^u_{H_i} = m^d_{H_i} = m_{H_i} (i = 1, 2, 3)$ at $\mathcal{O}(v/f)$ from Eq. (5). For the matrices V_{H_u} and V_{H_d}, considering the regions of parameter space that only loosely constrain the mass spectrum of the mirror fermions [7], we choose two scenarios as in Ref. [28].

Case I: $V_{H_d} = 1$, $V_{H_u} = V^\dagger_{\text{CKM}}$,

Case II: $s^d_{23} = 1/\sqrt{2}$, $s^d_{12} = s^d_{13} = 0$, $\delta^d_{12} = \delta^d_{23} = \delta^d_{13} = 0$.

In both cases, the constraints on the mass spectrum of the mirror fermions are very relaxed. On the other hand, Ref. [6] has shown that the experimental bounds on four-fermi interactions involving SM fields provide an upper bound on the mirror fermion masses and this yields $m_{H_i} \leq 4.8f^2$. We also consider such constraint in our calculation. For the breaking scale f, we take two typical values: 500 GeV and 1000 GeV.

For the c.m. energies of the ILC, we choose $\sqrt{s} = 500$, 1000 GeV as examples. Taking into account the detector acceptance, we have taken the basic cuts on the transverse momentum (p_T) and the pseudorapidity (η) for the final state particles

$$p_T \geq 20 \text{ GeV}, \qquad |\eta| \leq 2.5.$$

The numerical results of the cross sections are summarized in Figs. 3–5. Figures 3 and 4 show the cross sections of the processes $e^+e^-(\gamma\gamma) \to t\bar{c}$ and $e^-\gamma \to e^-t\bar{c}$ as a function of m_{H_3} for Case I and Case II, respectively.

In Case I, due to the absence of the mixing in the down type gauge and Goldstone boson interactions, there are no constraints on the masses of the mirror quarks at one-loop-level from the K and B systems and the constraints come only from the D system. The constraints on the mass of the third-generation mirror quark are very weak. Considering the constraint $m_{H_i} \leq 4.8f^2$, we take m_{H_3} to vary in the range of 500–1200 GeV for $f = 500$ GeV and 500–4800 GeV for $f = 1000$ GeV, and fix $m_{H_1} = m_{H_2} = 500$ GeV.

As shown in Fig. 3, the cross sections of the three different production processes rise with the increase of m_{H_3}. The reason is that the couplings between the mirror quarks and the SM quarks are proportional to the masses of the mirror quarks. The masses of the heavy gauge bosons and the mirror quarks, M_{V_H} and m_{H_i}, are proportional to f, but the scale f is insensitive to the cross sections of these processes because the production amplitudes are

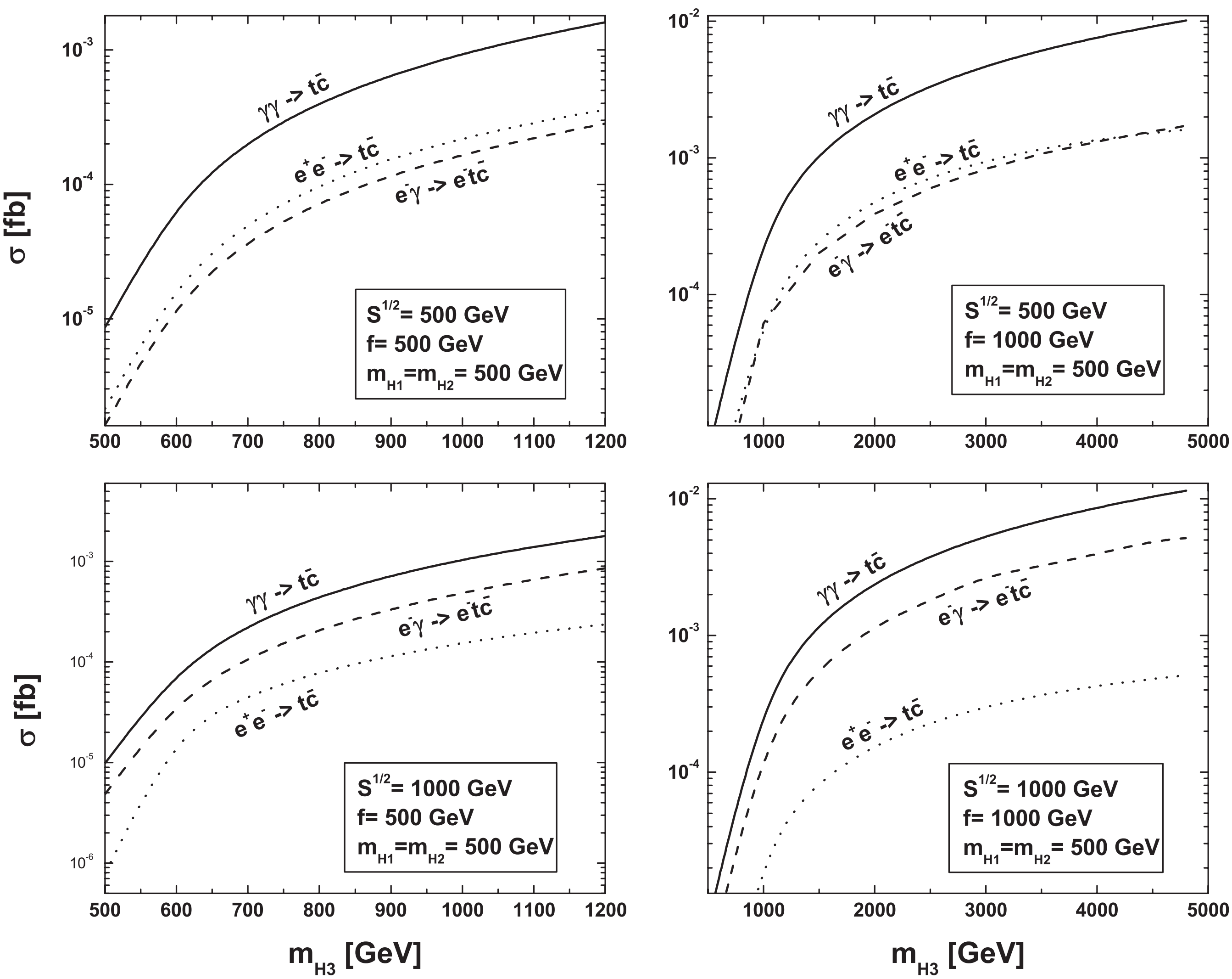

FIG. 3. The cross sections of top-charm associated production processes versus m_{H_3} in the LHT model for Case I.

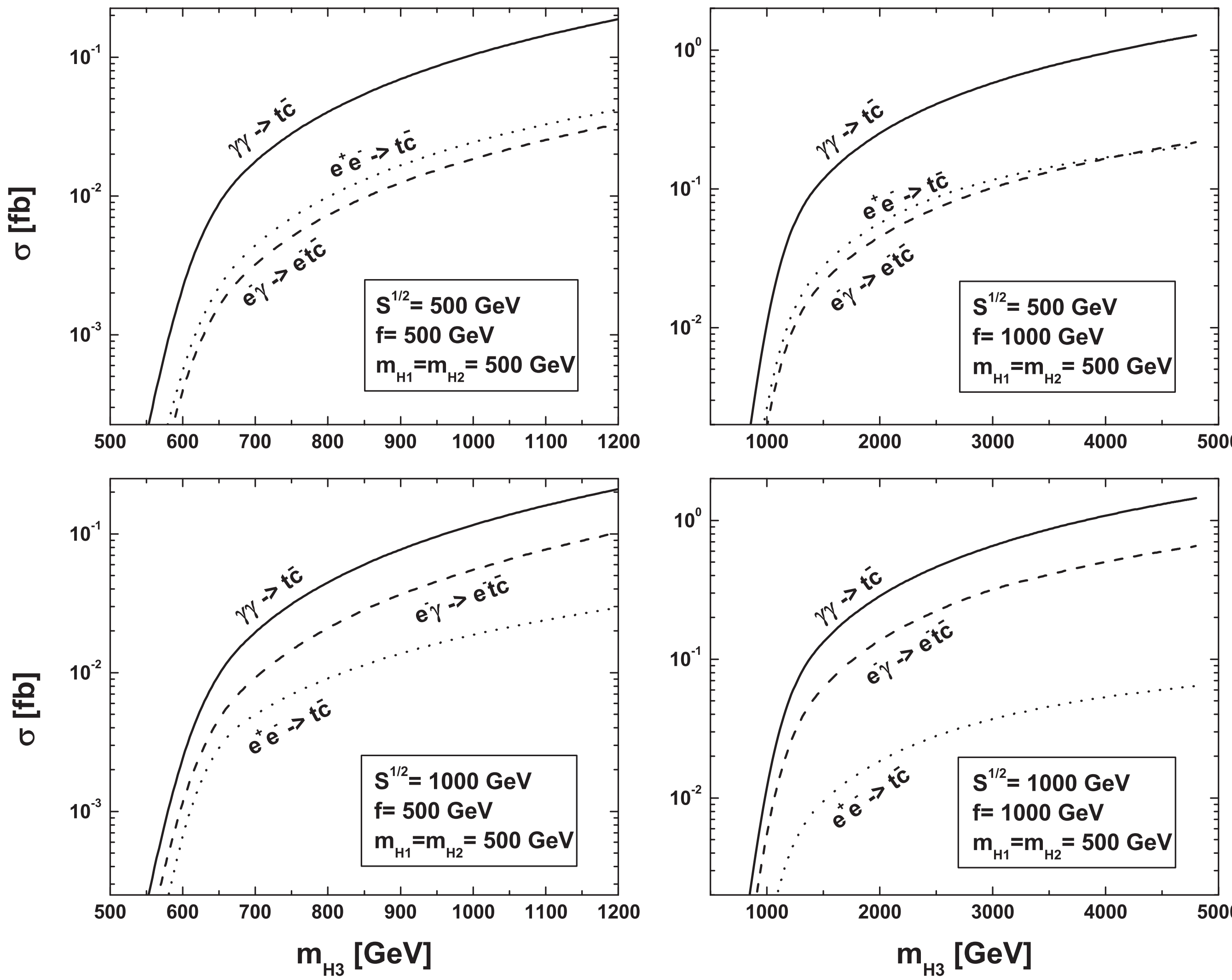

FIG. 4. The cross sections of top-charm associated production processes versus m_{H_3} in the LHT model for Case II.

represented in the form of m_{H_i}/M_{V_H}, which cancels the effect of f. In the case of $\sqrt{s} = 1000$ GeV, our calculations show that this case has slightly larger effects relative to the case of $\sqrt{s} = 500$ GeV.

For Case II, the dependence of the cross sections on m_{H_3} is presented in Fig. 4. In this case, the constraints from the K and B systems are also very weak. Compared to Case I, the mixing between the second and third generations is enhanced with the choice of a bigger mixing angle s_{23}^d. Here, we take the same values of $\sqrt{s}$, f and m_{H_i} as in Case I. Even with stricter constraints on the masses of the mirror quarks, the large masses of the mirror quarks can also enhance the cross sections significantly. The dependence of the cross sections on the c.m. energy is similar to that in Case I.

Compared to the three processes from Case I and Case II, we find that the cross section of process $\gamma\gamma \to t\bar{c}$ is the largest with the reasonable values of the parameter in the LHT model. The optimum value of $\sigma(\gamma\gamma \to t\bar{c})$ can reach $\mathcal{O}(10^0)$fb. On the other hand, the maximal value of cross section for process $e^-\gamma \to e^- t\bar{c}$ can reach 0.7 fb, which is higher than that in some models such as the MSSM model and the type III two Higgs doublet models, but lower than that in the TC2 model [20].

In Fig. 5 we show the behavior of the cross sections for $\gamma\gamma \to t\bar{c}$, $e^-\gamma \to e^- t\bar{c}$, and $e^+e^- \to t\bar{c}$ versus the collider energy for two cases. We see that the cross section of $e^+e^- \to t\bar{c}$ drops quickly with the increase of collider energy. This is because that the contribution of the LHT model comes from the s-channel, so the large collider energy depresses the cross section. However, for the process $\gamma\gamma \to t\bar{c}$, there is only t-channel contribution, so the large c.m. energy can enhance the cross section. So far as $e^-\gamma \to e^- t\bar{c}$ is concerned, the cross section increases with collider energy increasing, although its contribution arises from both the s-channel and t-channel.

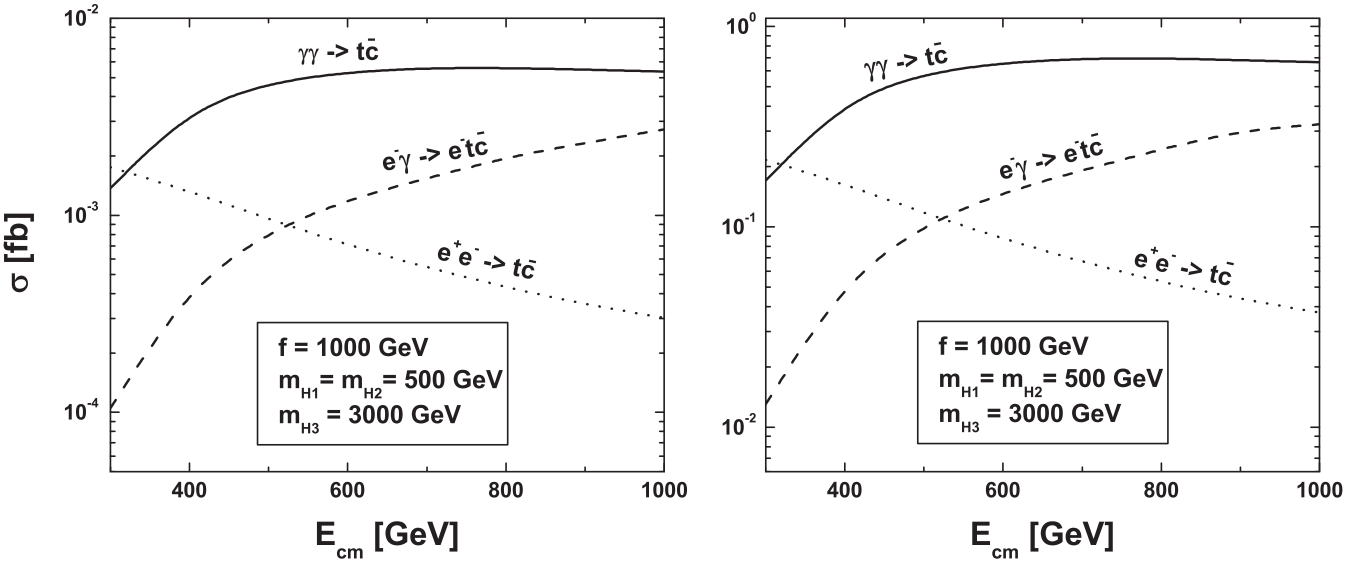

FIG. 5. Cross sections versus collider energy $E_{\rm cm} = \sqrt{s_{ee}}$ in the LHT model. The left diagram is for Case I and the right diagram is for Case II.

Theoretical predictions for the top-charm FCNC processes $\gamma\gamma \to t\bar{c}$, $e^-\gamma \to e^- t\bar{c}$, and $e^+e^- \to t\bar{c}$ in the different new physics models are summarized in Table I.

From Table I, we conclude that the new physics models can enhance the SM rates of the FCNC top-charm production processes by several orders because the tree-level FCNC is absent in the SM. As can be seen, the relation of the cross section is $\sigma(\gamma\gamma \to t\bar{c}) > \sigma(e^-\gamma \to e^- t\bar{c}) > \sigma(e^+e^- \to t\bar{c})$ in every new physics model. Because of the different values of the cross sections, the top-charm production also provides a good way to distinguish the LHT model from other new physics models.

Given the predictions listed in Table I, we now discuss the observability at the ILC. The possible signal for the FC top-charm production at the ILC is $b\bar{c}j_1j_2$ with j_1 and j_2 being light jets coming from the hadronic decay of W or $b\bar{c}l^+\nu_l$ with $l^+\nu_l$ coming from the leptonic decay of W boson. Since the observability of the processes $e^+e^-(\gamma\gamma) \to t\bar{c}$ has been analyzed in detail [28], in the following we only consider the process $e^-\gamma \to e^- t\bar{c}$. Note that $Br(W^+ \to j_1j_2)/Br(W^+ \to l^+\nu_l) \gtrsim 2$ and the maximal signal cross section is

$$\sigma(e^-\gamma \to e^- t\bar{c} \to e^- b\bar{c}j_1j_2) = \sigma(e^-\gamma \to e^- t\bar{c}) \times Br(t \to W^+ b) \times Br(W^+ \to j_1j_2) \simeq 0.474 \text{ fb}$$

in the favorable parameter regions, we consider the hadronic final states in order to retain an increased signal. The main irreducible background arises from $e^-\gamma \to e^- W^+ W^-$ with one W decaying to $b\bar{c}$ and the other decaying to two light jets. We calculate such background and find its rate is about 2.35 fb for $\sqrt{s} = 1000$ GeV without any cuts. Such background can be further reduced by considering kinematic distribution of the signal. For example, for the signal the b-jet is usually accompanied with two jets in one direction, while for the background, the b-jet is usually accompanied by one jet. This character may help us to further eliminate the contamination of the background on the signal. Since this process depends on detailed Monte Carlo simulation which is beyond the scope of our discussion, for a simple estimation, we assume that the signal is reduced to 10% with no significant background [33]. After considering the b-tagging efficiency (60%), we get about 47 events in the optimum case for 1000 fb^{-1} integrated luminosity [13].

TABLE I. The maximal predictions of the cross sections of the processes $\gamma\gamma \to t\bar{c}$, $e^-\gamma \to e^- t\bar{c}$ and $e^+e^- \to t\bar{c}$ in various models (in fb).

	SM	2HDM-III	MSSM	TC2	LHT
$\gamma\gamma \to t\bar{c}$	$\mathcal{O}(10^{-8})$ [16]	$\mathcal{O}(10^{-1})$ [19]	$\mathcal{O}(10^{-1})$ [16]	$\mathcal{O}(10)$ [20]	$\mathcal{O}(10^{0})$
$e^-\gamma \to e^- t\bar{c}$	$\mathcal{O}(10^{-9})$ [16]	$\mathcal{O}(10^{-2})$ [20]	$\mathcal{O}(10^{-2})$ [16]	$\mathcal{O}(1)$ [20]	$\mathcal{O}(10^{-1})$
$e^+e^- \to t\bar{c}$	$\mathcal{O}(10^{-10})$ [30]	$\mathcal{O}(10^{-3})$ [18]	$\mathcal{O}(10^{-2})$ [16]	$\mathcal{O}(10^{-1})$ [21]	$\mathcal{O}(10^{-2})$

V. CONCLUSION

We studied the top-charm associated productions via $e^-\gamma$ collisions in the framework of the LHT model at the ILC and compared with the process $e^+e^-(\gamma\gamma) \to t\bar{c}$. The numerical results showed that the cross sections of the processes increase sharply as the mirror quark masses increase, and in a large part of the allowed parameter space, the cross sections of $\gamma\gamma \to t\bar{c}$ and $e^-\gamma \to e^-t\bar{c}$ may reach the detectable level at the ILC. If these processes can be observed, some information about the FC couplings can be obtained in order to distinguish the LHT model from other new physics. If these FC processes are not to be observed, the upper limit on mirror quark masses can then be given.

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APPENDIX: THE EXPLICIT EXPRESSIONS OF THE EFFECTIVE $t\bar{c}\gamma(Z)$ COUPLINGS

The effective $t\bar{c}\gamma(Z)$ couplings $\Gamma^{\mu}_{t\bar{c}\gamma}$, $\Gamma^{\mu}_{t\bar{c}Z}$ can be directly calculated based on Fig. 1, and they can be represented in form of 2-point and 3-point standard functions B_0, B_1, C_{ij}. In our calculations, the higher order v^2/f^2 terms in the masses of new gauge bosons and in the Feynman rules are ignored. $\Gamma^{\mu}_{t\bar{c}\gamma}$, $\Gamma^{\mu}_{t\bar{c}Z}$ depend on the momenta of top quark and anticharm quark $(p_t, p_{\bar{c}})$. Here p_t and $p_{\bar{c}}$ are both outgoing momenta. The explicit expressions of them are

$$\Gamma^{\mu}_{t\bar{c}\gamma}(p_t, p_{\bar{c}}) = \Gamma^{\mu}_{t\bar{c}\gamma}(\eta^0) + \Gamma^{\mu}_{t\bar{c}\gamma}(\omega^0) + \Gamma^{\mu}_{t\bar{c}\gamma}(\omega^{\pm}) + \Gamma^{\mu}_{t\bar{c}\gamma}(A_H) + \Gamma^{\mu}_{t\bar{c}\gamma}(Z_H) + \Gamma^{\mu}_{t\bar{c}\gamma}(W_H^{\pm}) + \Gamma^{\mu}_{t\bar{c}\gamma}(W_H^{\pm}\omega^{\pm}),$$

$$\Gamma^{\mu}_{t\bar{c}\gamma}(\eta^0) = \frac{i}{16\pi^2}\frac{eg'^2}{150M^2_{A_H}}(V_{Hu})^*_{it}(V_{Hu})_{ic}(A+B+C)$$

$$\begin{aligned}
A &= \frac{1}{p_t^2 - m_c^2}[m^2_{Hi}(m_c^2B_0^a + p_t^2B_1^a)\gamma^{\mu}P_L + m_tm_c(m^2_{Hi}B_0^a + p_t^2B_1^a)\gamma^{\mu}P_R \\
&\quad + m_t(m^2_{Hi}B_0^a + m_c^2B_1^a)\not{p}_t\gamma^{\mu}P_L + m_cm^2_{Hi}(B_0^a + B_1^a)\not{p}_t\gamma^{\mu}P_R], \\
B &= \frac{1}{p_{\bar{c}}^2 - m_t^2}[m^2_{Hi}(m_t^2B_0^b + p_{\bar{c}}^2B_1^b)\gamma^{\mu}P_L + m_tm_c(m^2_{Hi}B_0^b + p_{\bar{c}}^2B_1^b)\gamma^{\mu}P_R \\
&\quad - m_tm^2_{Hi}(B_0^b + B_1^b)\gamma^{\mu}\not{p}_{\bar{c}}P_L - m_c(m^2_{Hi}B_0^b + m_t^2B_1^b)\gamma^{\mu}\not{p}_{\bar{c}}P_R], \\
C &= m^2_{Hi}[-\gamma^{\alpha}\gamma^{\mu}\gamma^{\beta}C^a_{\alpha\beta} + \gamma^{\alpha}\gamma^{\mu}(\not{p}_t + \not{p}_{\bar{c}})C^a_{\alpha} + 2m_tC^a_{\mu} - m_t\gamma^{\mu}(\not{p}_t + \not{p}_{\bar{c}})C^a_0 - m^2_{Hi}\gamma^{\mu}C^a_0]P_L \\
&\quad + m_c[-m_t\gamma^{\alpha}\gamma^{\mu}\gamma^{\beta}C^a_{\alpha\beta} + m_t\gamma^{\alpha}\gamma^{\mu}(\not{p}_t + \not{p}_{\bar{c}})C^a_{\alpha} + 2m^2_{Hi}C^a_{\mu} \\
&\quad + m_c[-m_t\gamma^{\alpha}\gamma^{\mu}\gamma^{\beta}C^a_{\alpha\beta} + m_t\gamma^{\alpha}\gamma^{\mu}(\not{p}_t + \not{p}_{\bar{c}})C^a_{\alpha} + 2m^2_{Hi}C^a_{\mu} - m^2_{Hi}\gamma^{\mu}(\not{p}_t + \not{p}_{\bar{c}})C^a_0 - m_tm^2_{Hi}\gamma^{\mu}C^a_0]P_{R,}
\end{aligned}$$

$$\Gamma^{\mu}_{t\bar{c}\gamma}(\omega^0) = \frac{i}{16\pi^2}\frac{eg^2}{6M^2_{Z_H}}(V_{Hu})^*_{it}(V_{Hu})_{ic}(D+E+F) \qquad D = A(B_0^a \to B_0^c, B_1^a \to B_1^c),$$

$$E = B(B_0^b \to B_0^d, B_1^b \to B_1^d), \qquad F = C(C^a_{\alpha\beta} \to C^b_{\alpha\beta}, C^a_{\alpha} \to C^b_{\alpha}, C^a_0 \to C^b_0),$$

$$\Gamma^{\mu}_{t\bar{c}\gamma}(\omega^{\pm}) = \frac{i}{16\pi^2}\frac{eg^2}{2M^2_{W_H}}(V_{Hu})^*_{it}(V_{Hu})_{ic}\left(\frac{2}{3}G + \frac{2}{3}H - \frac{1}{3}I + J\right) \qquad G = A(B_0^a \to B_0^e, B_1^a \to B_1^e),$$

$$\begin{aligned}
H &= B(B_0^b \to B_0^f, B_1^b \to B_1^f), \qquad I = C(C^a_{\alpha\beta} \to C^c_{\alpha\beta}, C^a_{\alpha} \to C^c_{\alpha}, C^a_0 \to C^c_0), \\
J &= m^2_{Hi}[2\gamma^{\alpha}C^d_{\mu\alpha} - 2(m_t - \not{p}_t)C^d_{\mu} + (P_t + P_{\bar{c}})^{\mu}\gamma^{\alpha}C^d_{\alpha} - (P_t + P_{\bar{c}})^{\mu}(m_t - \not{p}_t)C^d_0]P_L \\
&\quad + m_c[2m_t\gamma^{\alpha}C^d_{\mu\alpha} - 2(m^2_{Hi} - m_t\not{p}_t)C^d_{\mu} + m_t(P_t + P_{\bar{c}})^{\mu}\gamma^{\alpha}C^d_{\alpha} - (P_t + P_{\bar{c}})^{\mu}(m^2_{Hi} - m_t\not{p}_t)C^d_0]P_R,
\end{aligned}$$

$$\Gamma^{\mu}_{t\bar{c}\gamma}(A_H) = \frac{i}{16\pi^2}\frac{eg'^2}{75}(V_{Hu})^*_{it}(V_{Hu})_{ic}(K+L+M) \qquad K = \frac{1}{p_t^2 - m_c^2}[p_t^2B_1^a\gamma^{\mu}P_L + m_cB_1^a\not{p}_t\gamma^{\mu}P_R],$$

$$L = \frac{1}{p_{\bar{c}}^2 - m_t^2}[p_{\bar{c}}^2B_1^b\gamma^{\mu}P_L - m_tB_1^b\gamma^{\mu}\not{p}_{\bar{c}}P_L], \qquad M = [-\gamma^{\alpha}\gamma^{\mu}\gamma^{\beta}C^a_{\alpha\beta} + (\not{p}_t + \not{p}_{\bar{c}})\gamma^{\mu}\gamma^{\alpha}C^a_{\alpha} - m^2_{Hi}\gamma^{\mu}C^a_0]P_L,$$

$$\Gamma^{\mu}_{t\bar{c}\gamma}(Z_H) = \frac{i}{16\pi^2}\frac{eg^2}{3}(V_{Hu})^*_{it}(V_{Hu})_{ic}(N + O + P) \qquad N = K(B^a_1 \to B^c_1),$$
$$O = L(B^b_1 \to B^d_1), \qquad P = M(C^a_{\alpha\beta} \to C^b_{\alpha\beta}, C^a_\alpha \to C^b_\alpha, C^a_0 \to C^b_0),$$

$$\Gamma^{\mu}_{t\bar{c}\gamma}(W^{\pm}_H) = \frac{i}{16\pi^2}\frac{eg^2}{2}(V_{Hu})^*_{it}(V_{Hu})_{ic}\left(\frac{4}{3}Q + \frac{4}{3}R - \frac{2}{3}S - T\right) \qquad Q = K(B^a_1 \to B^e_1),$$
$$R = L(B^b_1 \to B^f_1), \qquad S = M(C^a_{\alpha\beta} \to C^c_{\alpha\beta}, C^a_\alpha \to C^c_\alpha, C^a_0 \to C^c_0),$$
$$T = \{4\gamma^\alpha C^d_{\mu\alpha} + [\gamma^\mu \not{p}_t \gamma^\alpha - \gamma^\mu\gamma^\alpha \not{p}_t + 2\not{p}_t\gamma^\alpha\gamma^\mu + \gamma^\alpha\not{p}_t\gamma^\mu - \gamma^\mu\gamma^\alpha\not{p}_{\bar{c}}$$
$$+ 2\not{p}_{\bar{c}}\gamma^\alpha\gamma^\mu + 2(p_t + p_{\bar{c}})^\mu\gamma^\alpha]C^d_\alpha + 4\not{p}_t C^d_\mu + 2(B^g_0 + m^2_{W_H}C^d_0)\gamma^\mu$$
$$+ [2\not{p}_{\bar{c}}\not{p}_t\gamma^\mu - \gamma^\mu\not{p}_t\not{p}_{\bar{c}} + 2\not{p}_t(p_t + p_{\bar{c}})^\mu + p_t^2\gamma^\mu]C^d_0\}P_L,$$

$$\Gamma^{\mu}_{t\bar{c}\gamma}(W^{\pm}_H\omega^{\pm}) = \frac{i}{16\pi^2}\frac{eg^2}{2}(V_{Hu})^*_{it}(V_{Hu})_{ic} \times \{[m_t\not{p}_t C^d_0 + m_t\gamma^\alpha C^d_\alpha - 2m^2_{Hi}C^d_0]\gamma^\mu P_L + m_c[\gamma^\mu\not{p}_t C^d_0 + \gamma^\mu\gamma^\alpha C^d_\alpha]P_R\},$$

$$\Gamma^{\mu}_{t\bar{c}Z}(p_t, p_{\bar{c}}) = \Gamma^{\mu}_{t\bar{c}Z}(\eta^0) + \Gamma^{\mu}_{t\bar{c}Z}(\omega^0) + \Gamma^{\mu}_{t\bar{c}Z}(\omega^\pm) + \Gamma^{\mu}_{t\bar{c}Z}(A_H) + \Gamma^{\mu}_{t\bar{c}Z}(Z_H) + \Gamma^{\mu}_{t\bar{c}Z}(W^\pm_H) + \Gamma^{\mu}_{t\bar{c}Z}(W^\pm_H\omega^\pm),$$

$$\Gamma^{\mu}_{t\bar{c}Z}(\eta^0) = \frac{i}{16\pi^2}\frac{g}{\cos\theta_W}\frac{g'^2}{100M^2_{A_H}}(V_{Hu})^*_{it}(V_{Hu})_{ic}(A' + B' + C')$$
$$A' = \frac{1}{p_t^2 - m_c^2}\left[\left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right)m^2_{Hi}(m^2_c B^a_0 + p^2_t B^a_1)\gamma^\mu P_L - \frac{2}{3}\sin^2\theta_W m_t m_c(m^2_{Hi}B^a_0 + p^2_t B^a_1)\gamma^\mu P_R\right.$$
$$\left. + \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right)m_t(m^2_{Hi}B^a_0 + m^2_c B^a_1)\not{p}_t\gamma^\mu P_L - \frac{2}{3}\sin^2\theta_W m_c m^2_{Hi}(B^a_0 + B^a_1)\not{p}_t\gamma^\mu P_R\right],$$
$$B' = \frac{1}{p_{\bar{c}}^2 - m_t^2}\left[\left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right)m^2_{Hi}(m^2_t B^b_0 + p^2_{\bar{c}} B^b_1)\gamma^\mu P_L - \frac{2}{3}\sin^2\theta_W m_t m_c(m^2_{Hi}B^b_0 + p^2_{\bar{c}} B^b_1)\gamma^\mu P_R\right.$$
$$\left. + \frac{2}{3}\sin^2\theta_W m_t m^2_{Hi}(B^b_0 + B^b_1)\gamma^\mu\not{p}_{\bar{c}}P_L - \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right)m_c(m^2_{Hi}B^b_0 + m^2_t B^b_1)\gamma^\mu\not{p}_{\bar{c}}P_R\right],$$
$$C' = \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right)C,$$

$$\Gamma^{\mu}_{t\bar{c}Z}(\omega^0) = \frac{i}{16\pi^2}\frac{g}{\cos\theta_W}\frac{g^2}{4M^2_{Z_H}}(V_{Hu})^*_{it}(V_{Hu})_{ic}(D' + E' + F') \qquad D' = A'(B^a_0 \to B^c_0, B^a_1 \to B^c_1),$$
$$E' = B'(B^b_0 \to B^d_0, B^b_1 \to B^d_1), \qquad F' = C'(C^a_{\alpha\beta} \to C^b_{\alpha\beta}, C^a_\alpha \to C^b_\alpha, C^a_0 \to C^b_0),$$

$$\Gamma^{\mu}_{t\bar{c}Z}(\omega^\pm) = \frac{i}{16\pi^2}\frac{g}{\cos\theta_W}\frac{g^2}{2M^2_{W_H}}(V_{Hu})^*_{it}(V_{Hu})_{ic}(G' + H' + I' + J') \qquad G' = A'(B^a_0 \to B^e_0, B^a_1 \to B^e_1),$$
$$H' = B'(B^b_0 \to B^f_0, B^b_1 \to B^f_1), \qquad I' = \left(-\frac{1}{2} + \frac{1}{3}\sin^2\theta_W\right)I, \qquad J' = \cos^2\theta_W J,$$

$$\Gamma^{\mu}_{t\bar{c}Z}(A_H) = \frac{i}{16\pi^2}\frac{g}{\cos\theta_W}\frac{g'^2}{50}(V_{Hu})^*_{it}(V_{Hu})_{ic}(K' + L' + M')$$
$$K' = \frac{1}{p_t^2 - m_c^2}\left[\left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right)p^2_t B^a_1\gamma^\mu P_L - \frac{2}{3}\sin^2\theta_W m_c B^a_1\not{p}_t\gamma^\mu P_R\right],$$
$$L' = \frac{1}{p_{\bar{c}}^2 - m_t^2}\left[\left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right)p^2_{\bar{c}} B^b_1\gamma^\mu P_L + \frac{2}{3}\sin^2\theta_W m_t B^b_1\gamma^\mu\not{p}_{\bar{c}}P_L\right],$$
$$M' = \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right)[-\gamma^\alpha\gamma^\mu\gamma^\beta C^a_{\alpha\beta} + (\not{p}_t + \not{p}_{\bar{c}})\gamma^\mu\gamma^\alpha C^a_\alpha - m^2_{Hi}C^a_0\gamma^\mu]P_L,$$

$$\Gamma^{\mu}_{t\bar{c}Z}(Z_H) = \frac{i}{16\pi^2}\frac{g}{\cos\theta_W}\frac{g^2}{2}(V_{Hu})^*_{it}(V_{Hu})_{ic}(N' + O' + P') \qquad N' = K'(B^a_1 \to B^c_1),$$
$$O' = L'(B^b_1 \to B^d_1), \qquad P' = M'(C^a_{\alpha\beta} \to C^b_{\alpha\beta}, C^a_\alpha \to C^b_\alpha, C^a_0 \to C^b_0),$$

$$\Gamma^{\mu}_{t\bar{c}Z}(W^\pm_H) = \frac{i}{16\pi^2}\frac{g}{\cos\theta_W}g^2(V_{Hu})^*_{it}(V_{Hu})_{ic}(Q' + R' + S' + T') \qquad Q' = K'(B^a_1 \to B^e_1),$$
$$R' = L'(B^b_1 \to B^f_1), \qquad S' = \left(-\frac{1}{2} + \frac{1}{3}\sin^2\theta_W\right)S, \qquad T' = -\frac{1}{2}\cos^2\theta_W T,$$

$$\Gamma^{\mu}_{t\bar{c}Z}(W^\pm_H\omega^\pm) = \frac{i}{16\pi^2}g\cos\theta_W\frac{g^2}{2}(V_{Hu})^*_{it}(V_{Hu})_{ic}\times\{[m_t\not{p}_tC^d_0 + m_t\gamma^\alpha C^d_\alpha - 2m^2_{Hi}C^d_0]\gamma^\mu P_L + m_c[\gamma^\mu\not{p}_tC^d_0 + \gamma^\mu\gamma^\alpha C^d_\alpha]P_R\}.$$

For the two-point and three-point standard loop functions B_0, B_1, C_0, C_{ij} in the above expressions are defined as

$$B^a = B^a(-p_t, m_{Hi}, M_{A_H}), \qquad B^b = B^b(-p_{\bar{c}}, M_{Hi}, M_{A_H}), \qquad B^c = B^c(-p_t, m_{Hi}, M_{Z_H}),$$
$$B^d = B^d(-p_{\bar{c}}, M_{Hi}, M_{Z_H}), \qquad B^e = B^e(-p_t, m_{Hi}, M_{W_H}), \qquad B^f = B^f(-p_{\bar{c}}, M_{Hi}, M_{W_H}),$$
$$B^g = B^g(p_{\bar{c}}, M_{Hi}, M_{W_H}), \qquad C^a_{ij} = C^a_{ij}(-p_t, -p_{\bar{c}}, m_{Hi}, M_{A_H}, m_{Hi}), \qquad C^b_{ij} = C^b_{ij}(-p_t, -p_{\bar{c}}, m_{Hi}, M_{Z_H}, m_{Hi}),$$
$$C^c_{ij} = C^c_{ij}(-p_t, -p_{\bar{c}}, m_{Hi}, M_{W_H}, m_{Hi}), \qquad C^d_{ij} = C^d_{ij}(p_t, p_{\bar{c}}, M_{W_H}, m_{Hi}, M_{W_H}).$$

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