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Twisted supersymmetry: Twisted symmetry versus renormalizability

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We discuss a deformation of superspace based on a Hermitian twist. The twist implies a $\star$-product that is noncommutative, Hermitian and finite when expanded in a power series of the deformation parameter. The Leibniz rule for the twisted supersymmetry transformations is deformed. A minimal deformation of the Wess-Zumino action is proposed and its renormalizability properties are discussed. There is no tadpole contribution, but the two-point function diverges. We speculate that the deformed Leibniz rule, or more generally the twisted symmetry, interferes with renormalizability properties of the model. We discuss different possibilities to render a renormalizable model.

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I. INTRODUCTION

It is well known that quantum field theory encounters problems at high energies and short distances. This suggests that the structure of space-time has to be modified at these scales. One possibility to modify the structure of space-time is to deform the usual commutation relations between coordinates; this gives a noncommutative (NC) space [1]. Different models of noncommutativity have been discussed in the literature; see [2–4] for references. A version of the standard model on the canonically deformed space-time was constructed in [5], and its renormalizability properties were discussed in [6]. Renormalizability of different noncommutative field theory models was discussed in [7].

A natural further step is modification of the superspace and introduction of non(anti)commutativity. A strong motivation for this comes from string theory. Namely, it was discovered that a noncommutative superspace can arise when a superstring moves in a constant gravitino or graviphoton background [8,9]. Since that discovery there has been a lot of work on this subject and different ways of deforming superspace have been discussed. Here we mention some of them.

The authors of [10] combine supersymmetry (SUSY) with the κ-deformation of space-time, while in [11] SUSY is combined with the canonical deformation of space-time. In [8] a version of non(anti)commutative superspace is defined and analyzed. The anticommutation relations between fermionic coordinates are modified in the following way:

$$\{\theta^\alpha \stackrel{\star}{,} \theta^\beta\} = C^{\alpha\beta}, \qquad \{\bar\theta_{\dot\alpha} \stackrel{\star}{,} \bar\theta_{\dot\beta}\} = \{\theta^\alpha \stackrel{\star}{,} \bar\theta_{\dot\alpha}\} = 0, \tag{1.1}$$

where $C^{\alpha\beta} = C^{\beta\alpha}$ is a complex, constant symmetric matrix. This deformation is well defined only when undotted and dotted spinors are not related by the usual complex conjugation. The notion of chirality is preserved in this model, i.e., the deformed product of two chiral superfields is again a chiral superfield. On the other hand, one half of $\mathcal{N} = 1$ supersymmetry is broken, and this is the so-called $\mathcal{N} = 1/2$ supersymmetry. Another type of deformation is introduced in [12,13]. There the product of two chiral superfields is not a chiral superfield but the model is invariant under the full supersymmetry. Renormalizability of different models (both scalar and gauge theories) has been discussed in [13–16]. The twist approach to non-anticommutativity was discussed in [17].

In our previous paper [18] we introduced a Hermitian deformation of the usual superspace. The non(anti)commutative deformation was introduced via the twist

$$\mathcal{F} = e^{(1/2)C^{\alpha\beta}\partial_\alpha \otimes \partial_\beta + (1/2)\bar C_{\dot\alpha\dot\beta}\bar\partial^{\dot\alpha}\otimes\bar\partial^{\dot\beta}}. \tag{1.2}$$

Here $C^{\alpha\beta} = C^{\beta\alpha}$ is a complex constant matrix, $\bar C^{\dot\alpha\dot\beta}$ its complex conjugate and $\partial_\alpha = \frac{\partial}{\partial\theta^\alpha}$ are fermionic partial derivatives. The twist (1.2) is Hermitian under the usual complex conjugation. Because of this choice of the twist, the coproduct of the SUSY transformations becomes deformed, leading to the deformed Leibniz rule. The inverse of (1.2) defines the $\star$-product. It is obvious that the $\star$-product of two chiral fields will not be a chiral field. Therefore we have to use the method of projectors to decompose the $\star$-products of fields into their irreducible components. Collecting the terms invariant under the twisted SUSY transformations, we construct the deformed Wess-Zumino action.

Being interested in implications of the twisted symmetry on renormalizability properties, in this paper we calculate the divergent part of the one-loop effective action. More precisely, we calculate divergent parts of the one-point and the two-point functions. The plan of the paper is as follows: In Sec. II we summarize the most important properties of our model; more details of the construction are given in [18]. In Sec. III we describe the method we use to calculate divergent parts of the n-point Green functions: the background field method and the supergraph technique.

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In Sec. IV the tadpole diagram and the divergent part of the two-point function are calculated. Finally, we discuss renormalizability of the model in Sec. V. We give some comments and compare our results with the results already present in the literature. Some details of our calculations are presented in the Appendix.

II. CONSTRUCTION OF THE MODEL

There are different ways to realize a noncommutative and/or a nonanticommutative space and to formulate a physical model on it; see [2,4]. We shall follow the approach of [3,18].

Let us first fix the notation and the conventions which we use. The superspace is generated by supercoordinates x^m, θ^α, and $\bar\theta_{\dot\alpha}$ which fulfill

$$[x^m, x^n] = [x^m, \theta^\alpha] = [x^m, \bar\theta_{\dot\alpha}] = 0,\\ \{\theta^\alpha, \theta^\beta\} = \{\bar\theta_{\dot\alpha}, \bar\theta_{\dot\beta}\} = \{\theta^\alpha, \bar\theta_{\dot\alpha}\} = 0, \tag{2.1}$$

with $m = 0, \ldots, 3$ and $\alpha, \beta = 1, 2$. We refer to x^m as bosonic and to θ^α and $\bar\theta_{\dot\alpha}$ as fermionic coordinates. We work in Minkowski space-time with the metric $(-,+,+,+)$ and $x^m x_m = -(x^0)^2 + (x^1)^2 + (x^2)^2 + (x^3)^2$.

A general superfield $\mathrm{F}(x, \theta, \bar\theta)$ can be expanded in powers of θ and $\bar\theta$:

$$\mathrm{F}(x,\theta,\bar\theta) = f(x) + \theta\phi(x) + \bar\theta\bar\chi(x) + \theta\theta m(x) + \bar\theta\bar\theta n(x) + \theta\sigma^m\bar\theta v_m(x) + \theta\theta\bar\theta\bar\lambda(x) + \bar\theta\bar\theta\theta\varphi(x) + \theta\theta\bar\theta\bar\theta d(x). \tag{2.2}$$

Under the infinitesimal $\mathcal{N} = 1$ SUSY transformations, it transforms as

$$\delta_\xi \mathrm{F} = (\xi Q + \bar\xi\bar Q)\mathrm{F}, \tag{2.3}$$

where ξ^α and $\bar\xi_{\dot\alpha}$ are constant anticommuting parameters, and the SUSY generators Q^α and $\bar Q_{\dot\alpha}$ are given by

$$Q_\alpha = \partial_\alpha - i\sigma^m{}_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_m, \qquad \bar Q_{\dot\alpha} = -\bar\partial_{\dot\alpha} + i\theta^\alpha\sigma^m{}_{\alpha\dot\alpha}\partial_m. \tag{2.4}$$

Transformations (2.3) close in the algebra:

$$[\delta_\xi, \delta_\eta] = -2i(\eta\sigma^m\bar\xi - \xi\sigma^m\bar\eta)\partial_m. \tag{2.5}$$

The product of two superfields is a superfield again; its transformation law is given by

$$\delta_\xi(\mathrm{F}\cdot\mathrm{G}) = (\xi Q + \bar\xi\bar Q)(\mathrm{F}\cdot\mathrm{G}) = (\delta_\xi\mathrm{F})\cdot\mathrm{G} + \mathrm{F}\cdot(\delta_\xi\mathrm{G}). \tag{2.6}$$

The last line is the undeformed Leibniz rule for the infinitesimal SUSY transformation δ_ξ.

Nonanticommutativity is introduced following the twist approach [3]. For the twist $\mathcal{F}$ we choose

$$\mathcal{F} = e^{(1/2)C^{\alpha\beta}\partial_\alpha\otimes\partial_\beta + (1/2)\bar C_{\dot\alpha\dot\beta}\bar\partial^{\dot\alpha}\otimes\bar\partial^{\dot\beta}}, \tag{2.7}$$

with the complex constant matrix $C^{\alpha\beta} = C^{\beta\alpha}$. Note that $C^{\alpha\beta}$ and $\bar C^{\dot\alpha\dot\beta}$ are related by the usual complex conjugation. It can be shown [19]that the twist (2.7) satisfies all necessary requirements [20].

The inverse of the twist (2.7)

$$\mathcal{F}^{-1} = e^{-(1/2)C^{\alpha\beta}\partial_\alpha\otimes\partial_\beta - (1/2)\bar C_{\dot\alpha\dot\beta}\bar\partial^{\dot\alpha}\otimes\bar\partial^{\dot\beta}} \tag{2.8}$$

defines a new product in the algebra of superfields called the $\star$-product. For two arbitrary superfields F and G, the $\star$-product is defined as follows:

$$\begin{aligned}\mathrm{F}\star\mathrm{G} &= \mu_\star\{\mathrm{F}\otimes\mathrm{G}\} = \mu\{\mathcal{F}^{-1}\mathrm{F}\otimes\mathrm{G}\} = \mu\{e^{-(1/2)C^{\alpha\beta}\partial_\alpha\otimes\partial_\beta - (1/2)\bar C_{\dot\alpha\dot\beta}\bar\partial^{\dot\alpha}\otimes\bar\partial^{\dot\beta}}\mathrm{F}\otimes\mathrm{G}\}\\ &= \mathrm{F}\cdot\mathrm{G} - \frac{1}{2}(-1)^{|\mathrm{F}|}C^{\alpha\beta}(\partial_\alpha\mathrm{F})\cdot(\partial_\beta\mathrm{G}) - \frac{1}{2}(-1)^{|\mathrm{F}|}\bar C_{\dot\alpha\dot\beta}(\bar\partial^{\dot\alpha}\mathrm{F})(\bar\partial^{\dot\beta}\mathrm{G}) - \frac{1}{8}C^{\alpha\beta}C^{\gamma\delta}(\partial_\alpha\partial_\gamma\mathrm{F})\cdot(\partial_\beta\partial_\delta\mathrm{G})\\ &\quad - \frac{1}{8}\bar C_{\dot\alpha\dot\beta}\bar C_{\dot\gamma\dot\delta}(\bar\partial^{\dot\alpha}\bar\partial^{\dot\gamma}\mathrm{F})(\bar\partial^{\dot\beta}\bar\partial^{\dot\delta}\mathrm{G}) - \frac{1}{4}C^{\alpha\beta}\bar C_{\dot\alpha\dot\beta}(\partial_\alpha\bar\partial^{\dot\alpha}\mathrm{F})(\partial_\beta\bar\partial^{\dot\beta}\mathrm{G}) + \frac{1}{16}(-1)^{|\mathrm{F}|}C^{\alpha\beta}C^{\gamma\delta}\bar C_{\dot\alpha\dot\beta}(\partial_\alpha\partial_\gamma\bar\partial^{\dot\alpha}\mathrm{F})(\partial_\beta\partial_\delta\bar\partial^{\dot\beta}\mathrm{G})\\ &\quad + \frac{1}{16}(-1)^{|\mathrm{F}|}C^{\alpha\beta}\bar C_{\dot\alpha\dot\beta}\bar C_{\dot\gamma\dot\delta}(\partial_\alpha\bar\partial^{\dot\alpha}\bar\partial^{\dot\gamma}\mathrm{F})(\partial_\beta\bar\partial^{\dot\beta}\bar\partial^{\dot\delta}\mathrm{G}) + \frac{1}{64}C^{\alpha\beta}C^{\gamma\delta}\bar C_{\dot\alpha\dot\beta}\bar C_{\dot\gamma\dot\delta}(\partial_\alpha\partial_\gamma\bar\partial^{\dot\alpha}\bar\partial^{\dot\gamma}\mathrm{F})(\partial_\beta\partial_\delta\bar\partial^{\dot\beta}\bar\partial^{\dot\delta}\mathrm{G}),\end{aligned} \tag{2.9}$$

where $|\mathrm{F}| = 1$ if F is odd (fermionic) and $|\mathrm{F}| = 0$ if F is even (bosonic), and the pointwise multiplication μ is the bilinear map from the tensor product to the space of superfields (functions). The definition of the multiplication $\mu_\star$ is given in the first line. No higher powers of $C^{\alpha\beta}$ and $\bar C_{\dot\alpha\dot\beta}$ appear since the derivatives ∂_α and $\bar\partial^{\dot\alpha}$ are Grassmannian. Expansion of the $\star$-product (2.9) ends after the fourth order in the deformation parameter. This $\star$-product is different from the Moyal-Weyl $\star$-product [21] where the expansion in powers of the deformation parameter leads to an infinite power series. One should also note that the $\star$-product (2.9) is Hermitian,

$$(\mathrm{F}\star\mathrm{G})^* = \mathrm{G}^*\star\mathrm{F}^*, \tag{2.10}$$

where $*$ denotes the usual complex conjugation.

The $\star$-product (2.9) implies

$$\begin{aligned}\{\theta^\alpha \overset{\star}{,} \theta^\beta\} &= C^{\alpha\beta}, & \{\bar\theta_{\dot\alpha} \overset{\star}{,} \bar\theta_{\dot\beta}\} &= \bar C_{\dot\alpha\dot\beta},\\ \{\theta^\alpha \overset{\star}{,} \bar\theta_{\dot\alpha}\} &= 0, & [x^m \overset{\star}{,} x^n] &= 0,\\ [x^m \overset{\star}{,} \theta^\alpha] &= 0, & [x^m \overset{\star}{,} \bar\theta_{\dot\alpha}] &= 0.\end{aligned} \tag{2.11}$$

Relations (2.11) enable us to define the deformed superspace or "nonanticommutative superspace." It is generated by the usual bosonic and fermionic coordinates (2.1) while the deformation is contained in the new product (2.9).

The next step is to apply the twist (2.7) to the Hopf algebra of SUSY transformations. We will not give details here; they can be found in [18]. We just state the most important results.

The deformed infinitesimal SUSY transformation is defined in the following way:

$$\delta^\star_\xi \mathrm{F} = (\xi Q + \bar{\xi}\bar{Q})\mathrm{F}. \tag{2.12}$$

The twist (2.7) leads to a deformed Leibniz rule for the deformed SUSY transformations (2.12). This ensures that the $\star$-product of two superfields is again a superfield. Its transformation law is given by

$$\delta^\star_\xi(\mathrm{F}\star\mathrm{G}) = (\xi Q + \bar{\xi}\bar{Q})(\mathrm{F}\star\mathrm{G}), \tag{2.13}$$

$$\begin{aligned} &= (\delta^\star_\xi \mathrm{F})\star\mathrm{G} + \mathrm{F}\star(\delta^\star_\xi\mathrm{G}) \\ &\quad + \frac{i}{2}C^{\alpha\beta}(\bar{\xi}^{\dot\gamma}\sigma^m{}_{\alpha\dot\gamma}(\partial_m\mathrm{F})\star(\partial_\beta\mathrm{G}) + (\partial_\alpha\mathrm{F})\star\bar{\xi}^{\dot\gamma}\sigma^m{}_{\beta\dot\gamma}(\partial_m\mathrm{G})) \\ &\quad - \frac{i}{2}\bar{C}_{\dot\alpha\dot\beta}(\xi^\alpha\sigma^m{}_{\alpha\dot\gamma}\varepsilon^{\dot\gamma\dot\alpha}(\partial_m\mathrm{F})\star(\bar\partial^{\dot\beta}\mathrm{G}) + (\bar\partial^{\dot\alpha}\mathrm{F})\star\xi^\alpha\sigma^m{}_{\alpha\dot\gamma}\varepsilon^{\dot\gamma\dot\beta}(\partial_m\mathrm{G})). \end{aligned} \tag{2.14}$$

Note that we have to enlarge the algebra (2.5) by introducing the fermionic derivatives ∂_α and $\bar\partial_{\dot\alpha}$. Since these derivatives commute with the generators of Poincaré algebra ∂_m and M_{mn}, the super Poincaré algebra does not change. Especially the Leibniz rule for ∂_m and M_{mn} does not change.

Being interested in a deformation of the Wess-Zumino model, we need to analyze properties of the $\star$-products of chiral fields. A chiral field Φ fulfills $\bar{D}_{\dot\alpha}\Phi = 0$, with the supercovariant derivative $\bar{D}_{\dot\alpha} = -\bar\partial_{\dot\alpha} - i\theta^\alpha\sigma^m{}_{\alpha\dot\alpha}\partial_m$.

In terms of component fields the chiral superfield Φ is given by

$$\begin{aligned}\Phi(x,\theta,\bar\theta) &= A(x) + \sqrt{2}\theta^\alpha\psi_\alpha(x) + \theta\theta F(x) + i\theta\sigma^l\bar\theta\partial_l A(x) \\ &\quad - \frac{i}{\sqrt{2}}\theta\theta\partial_m\psi^\alpha(x)\sigma^m{}_{\alpha\dot\alpha}\bar\theta^{\dot\alpha} + \frac{1}{4}\theta\theta\bar\theta\bar\theta\Box A(x).\end{aligned} \tag{2.15}$$

It is easy to calculate the $\star$-product of two chiral fields from (2.9). It is given by

$$\begin{aligned}\Phi\star\Phi &= A^2 - \frac{C^2}{2}F^2 + \frac{1}{4}C^{\alpha\beta}\bar{C}^{\dot\alpha\dot\beta}\sigma^m{}_{\alpha\dot\alpha}\sigma^l{}_{\beta\dot\beta}(\partial_m A)(\partial_l A) + \frac{1}{64}C^2\bar{C}^2(\Box A)^2 \\ &\quad + \theta^\alpha\Big(2\sqrt{2}\psi_\alpha A - \frac{1}{\sqrt{2}}C^{\gamma\beta}\bar{C}^{\dot\alpha\dot\beta}\varepsilon_{\gamma\alpha}(\partial_m\psi^\rho)\sigma^m{}_{\rho\dot\beta}\sigma^l{}_{\beta\dot\alpha}(\partial_l A)\Big) \\ &\quad - \frac{i}{\sqrt{2}}C^2\bar\theta_{\dot\alpha}\bar\sigma^{m\dot\alpha\alpha}(\partial_m\psi_\alpha)F + \theta\theta(2AF - \psi\psi) + \bar\theta\bar\theta\Big(-\frac{C^2}{4}\Big(F\Box A - \frac{1}{2}(\partial_m\psi)\sigma^m\bar\sigma^l(\partial_l\psi)\Big)\Big) \\ &\quad + i\theta\sigma^m\bar\theta\Big((\partial_m A^2) + \frac{1}{4}C^{\alpha\beta}\bar{C}^{\dot\alpha\dot\beta}\sigma_{m\alpha\dot\alpha}\sigma^l{}_{\beta\dot\beta}(\Box A)(\partial_l A)\Big) + i\sqrt{2}\theta\theta\bar\theta_{\dot\alpha}\bar\sigma^{m\dot\alpha\alpha}(\partial_m(\psi_\alpha A)) + \frac{1}{4}\theta\theta\bar\theta\bar\theta(\Box A^2),\end{aligned} \tag{2.16}$$

where $C^2 = C^{\alpha\beta}C^{\gamma\delta}\varepsilon_{\alpha\gamma}\varepsilon_{\beta\delta}$ and $\bar{C}^2 = \bar{C}_{\dot\alpha\dot\beta}\bar{C}_{\dot\gamma\dot\delta}\varepsilon^{\dot\alpha\dot\gamma}\varepsilon^{\dot\beta\dot\delta}$. One sees that due to the $\bar\theta$-term and the $\bar\theta\bar\theta$-term (2.16) is not a chiral field. But, in order to write an action invariant under the deformed SUSY transformations (2.12), we need to preserve the notion of chirality. This can be done in different ways. One possibility is to use a different $\star$-product, the one which preserves chirality [8]. However, a chirality-preserving $\star$-product implies working in a space where $\bar\theta \neq (\theta)^*$. Since we want to work in Minkowski space-time and keep the usual complex conjugation, we use the $\star$-product (2.9) and decompose the $\star$-products of superfields into their irreducible components using the projectors defined in [22]. In that way, (2.16) becomes

$$\Phi\star\Phi = P_1(\Phi\star\Phi) + P_2(\Phi\star\Phi) + P_T(\Phi\star\Phi), \tag{2.17}$$

with antichiral, chiral, and transversal projectors given by

$$P_1 = \frac{1}{16}\frac{D^2\bar{D}^2}{\Box}, \tag{2.18}$$

$$P_2 = \frac{1}{16}\frac{\bar{D}^2D^2}{\Box}, \tag{2.19}$$

$$P_T = -\frac{1}{8}\frac{D\bar{D}^2D}{\Box}. \tag{2.20}$$

Finally, the deformed Wess-Zumino action is constructed, requiring that the action is invariant under the deformed SUSY transformations (2.12) and that in the commutative limit it reduces to the undeformed Wess-Zumino action. In addition, we require that deformation is minimal: We deform only those terms that are present in the commutative Wess-Zumino model. We do not, for the time being, add the terms whose commutative limit is zero.

Taking these requirements into account, we propose the following action:

$$S=\int d^4x\Big\{\Phi^+\star\Phi|_{\theta\theta\bar\theta\bar\theta} +\Big[\frac{m}{2}P_2(\Phi\star\Phi)|_{\theta\theta}+\frac{\lambda}{3}P_2(\Phi\star P_2(\Phi\star\Phi))|_{\theta\theta}+c.c.\Big]\Big\}, \tag{2.21}$$

where m and λ are real constants. To rewrite (2.21) in terms of component fields and as compactly as possible, we introduce the following notation:

$$C_{\alpha\beta}=K_{ab}(\sigma^{ab}\varepsilon)_{\alpha\beta}, \tag{2.22}$$

$$\bar C_{\dot\alpha\dot\beta}=K^*_{ab}(\varepsilon\bar\sigma^{ab})_{\dot\alpha\dot\beta}, \tag{2.23}$$

where $K_{ab}=-K_{ba}$ is an antisymmetric self-dual complex constant matrix. Then we have

$$C^2=2K_{ab}K^{ab},\qquad \bar C^2=2K^*_{ab}K^{*ab},\qquad K^{ab}K^*_{ab}=0, \tag{2.24}$$

$$K^*_{cd}K_{ab}(\sigma^n\bar\sigma^{cd}\bar\sigma^m\sigma^{ab})_\alpha{}^\beta=-4\delta^\beta_\alpha K^{ma}K^{*n}{}_a + 8K^{ma}K^{*nb}(\sigma_{ba})_\alpha{}^\beta, \tag{2.25}$$

$$C^{\alpha\beta}\bar C^{\dot\alpha\dot\beta}\sigma^m{}_{\alpha\dot\alpha}\sigma^l{}_{\beta\dot\beta}=8K^{am}K^{*l}_a. \tag{2.26}$$

Using these formulas and expanding (2.21) in component fields, we obtain

$$\begin{aligned}S&=\int d^4x\Big\{\Phi^+\star\Phi|_{\theta\theta\bar\theta\bar\theta}+\Big[\frac{m}{2}P_2(\Phi\star\Phi)|_{\theta\theta}+\frac{\lambda}{3}P_2(\Phi\star P_2(\Phi\star\Phi))|_{\theta\theta}+c.c\Big]\Big\}\\
&=\int d^4x\Big\{A^*\Box A+i(\partial_m\bar\psi)\bar\sigma^m\psi+F^*F+\Big[\frac{m}{2}(2AF-\psi\psi)+\lambda(FA^2-A\psi\psi)-\frac{\lambda}{3}(K^m{}_aK^{*na}\psi(\partial_n\psi)\\
&\quad-2K^m{}_aK^{*n}{}_b(\partial_n\psi)\sigma^{ba}\psi)(\partial_mA)-\frac{\lambda}{12}K^{mn}K_{mn}F^3+\frac{\lambda}{6}K^m{}_lK^{*nl}F(\partial_mA)(\partial_nA)+\frac{\lambda}{3}K^m{}_lK^{*nl}F\frac{1}{\Box}\partial_m((\partial_nA)\Box A)\\
&\quad+\frac{\lambda}{192}K^{ab}K_{ab}K^{*cd}K^*_{cd}F(\Box A)^2+c.c.\Big]\Big\}.\end{aligned} \tag{2.27}$$

Partial integration was used to rewrite some of the terms in (2.27) in a more compact way. Note that this is the complete action; there are no higher order terms in the deformation parameter K^{ab}. However, for simplicity in the following sections we shall keep only terms up to second order in the deformation parameter.

III. ONE-LOOP EFFECTIVE ACTION

In this section we look at the quantum properties of our model. We calculate the one-loop divergent part of the one-point and the two-point functions up to second order in the deformation parameter. We use the background field method, dimensional regularization, and the supergraph technique. The supergraph technique significantly simplifies calculations. However, we cannot directly apply this technique since our action (2.27) is not written as an integral over the whole superspace and in terms of the chiral field Φ and its derivatives. This is a consequence of the particular deformation (2.2) and differs from [13].

In order to be able to use the supergraph technique, we notice the following: From (2.15) (see also [22]), it follows that the fields A, ψ, and F can be written as

$$A=\Phi|_{\theta,\bar\theta=0},\qquad \psi_\alpha=\frac{1}{\sqrt2}D_\alpha\Phi|_{\theta,\bar\theta=0},\qquad F=-\frac14D^2\Phi|_{\theta,\bar\theta=0}. \tag{3.1}$$

Inserting this in (2.27) we obtain

$$\begin{aligned}S&=\int d^8z\Big\{\Phi^+\Phi+\Big[-\frac{m}{8}\Phi\frac{D^2}{\Box}\Phi-\lambda\Phi^2\frac{D^2}{12\Box}\Phi+\lambda\theta\theta\bar\theta\bar\theta\Big(\frac{1}{768}K^{mn}K_{mn}(D^2\Phi)^3-\frac16(K^m{}_aK^{*na}(D^\alpha\Phi)(\partial_nD_\alpha\Phi)\\
&\quad-2K^m{}_aK^{*n}{}_b(\partial_nD^\alpha\Phi)(\sigma^{ba})_\alpha{}^\beta D_\beta\Phi)(\partial_m\Phi)-\frac{1}{24}K^m{}_aK^{*na}(D^2\Phi)(\partial_m\Phi)(\partial_n\Phi)\\
&\quad-\frac{1}{12}K^m{}_aK^{*na}(D^2\Phi)\frac{1}{\Box}\partial_m((\partial_n\Phi)(\Box\Phi))\Big)+c.c.\Big]\Big\},\end{aligned} \tag{3.2}$$

with $f(x)\frac{1}{\Box}g(x) = f(x)\int d^4yG(x-y)g(y)$. Notice that two spurion fields

$$U^{mn}_{(1)\,ab} = K^m{}_a K^{*n}{}_b \theta\theta\bar\theta\bar\theta, \qquad U_{(2)} = K^{mn}K_{mn}\theta\theta\bar\theta\bar\theta \quad (3.3)$$

appear in (3.2). This is a consequence of rewriting the action (2.27) as an integral over the whole superspace.

Now we can start the machinery of the background field method. First we split the chiral and antichiral superfields into their classical and quantum parts,

$$\Phi \to \Phi + \Phi_q, \qquad \Phi^+ \to \Phi^+ + \Phi_q^+, \quad (3.4)$$

and integrate over the quantum superfields in the path integral. Since Φ_q and Φ_q^+ are chiral and antichiral fields, they are constrained by

$$\bar D_{\dot\alpha}\Phi_q = D_\alpha \Phi_q^+ = 0.$$

To simplify the supergraph technique, we introduce the unconstrained superfields Σ and Σ^+,

$$\Phi_q = -\frac{1}{4}\bar D^2\Sigma, \qquad \Phi_q^+ = -\frac{1}{4}D^2\Sigma^+. \quad (3.5)$$

Note that we do not express the background superfields Φ and Φ^+ in terms of Σ and Σ^+, only the quantum parts Φ_q and Φ_q^+. After the integration of quantum superfields, the result is expressed in terms of the (anti)chiral superfields. This is a big advantage of the background field method and of the supergraph technique. The unconstrained superfields are determined up to a gauge transformation

$$\Sigma \to \Sigma + \bar D_{\dot\alpha}\bar\Lambda^{\dot\alpha}, \qquad \Sigma^+ \to \Sigma^+ + D^\alpha\Lambda_\alpha, \quad (3.6)$$

with the gauge parameter Λ. This additional symmetry has to be fixed, so we add a gauge-fixing term to the action. For the gauge functions, we choose

$$\chi_\alpha = D_\alpha\Sigma, \qquad \bar\chi_{\dot\alpha} = \bar D_{\dot\alpha}\Sigma^+. \quad (3.7)$$

The product $\delta(\chi)\delta(\bar\chi)$ in the path integral is averaged by the weight $e^{-i\xi\int d^8z\bar f M f}$:

$$\int df d\bar f \delta(\chi_\alpha - f_\alpha)\delta(\bar\chi^{\dot\alpha} - \bar f^{\dot\alpha}) e^{-i\xi\int d^8z \bar f^{\dot\alpha}M_{\dot\alpha\alpha}f^\alpha}, \quad (3.8)$$

where

$$\bar f^{\dot\alpha}M_{\dot\alpha\alpha}f^\alpha = \frac{1}{4}\bar f^{\dot\alpha}\Big(D_\alpha\bar D_{\dot\alpha} + \frac{3}{4}\bar D_{\dot\alpha}D_\alpha\Big)f^\alpha, \quad (3.9)$$

and the gauge-fixing parameter is denoted by ξ. The gauge-fixing term becomes

$$S_{gf} = -\xi\int d^8z(\bar D_{\dot\alpha}\bar\Sigma)\Big(\frac{3}{16}\bar D^{\dot\alpha}D^\alpha + \frac{1}{4}D^\alpha\bar D^{\dot\alpha}\Big)(D_\alpha\Sigma). \quad (3.10)$$

One can easily show that the ghost fields are decoupled.

After the gauge-fixing, the part of the classical action quadratic in quantum superfields is given by

$$S^{(2)} = S_0^{(2)} + S_{\rm int}^{(2)}, \quad (3.11)$$

with

$$S_0^{(2)} = \frac{1}{2}\int d^8z(\Sigma \quad \Sigma^+)\mathcal{M}\begin{pmatrix}\Sigma\\ \Sigma^+\end{pmatrix} \quad (3.12)$$

and

$$S_{\rm int}^{(2)} = \frac{1}{2}\int d^8z d^8z'(\Sigma \ \ \Sigma^+)(z)\mathcal{V}(z,z')\begin{pmatrix}\Sigma\\ \Sigma^+\end{pmatrix}(z'). \quad (3.13)$$

Kinetic and interaction terms are collected in the matrices $\mathcal{M}$ and $\mathcal{V}$, respectively. The matrix $\mathcal{M}$ is given by

$$\mathcal{M} = \begin{pmatrix} -m\Box^{1/2}P_- & \Box(P_2 + \xi(P_1 + P_T)) \\ \Box(P_1 + \xi(P_2 + P_T)) & -m\Box^{1/2}P_+ \end{pmatrix}, \quad (3.14)$$

with

$$P_+ = \frac{D^2}{4\Box^{1/2}}, \qquad P_- = \frac{\bar D^2}{4\Box^{1/2}}. \quad (3.15)$$

The interaction matrix $\mathcal{V}$ is

$$\mathcal{V} = \begin{pmatrix} F & 0 \\ 0 & \bar F \end{pmatrix}. \quad (3.16)$$

There are two types of elements in $\mathcal{V}$, local and nonlocal. We split them into F_1 and F_2:

$$F(z,z') = F_1(z)\delta(z-z') + F_2(z,z'). \quad (3.17)$$

Elements of F_1 are given by

$$\begin{aligned}F_1(z)&=\sum_{i=0}^{10}F^{(i)}\\&=-\frac{\lambda}{2}\Phi\bar{D}^2-\frac{\lambda}{48}K^m{}_aK^{*na}\bar{D}^2\overleftarrow{D}^\alpha(\partial_m\Phi)\theta\theta\bar{\theta}\bar{\theta}\partial_nD_\alpha\bar{D}^2-\frac{\lambda}{48}K^m{}_aK^{*na}\bar{D}^2\overleftarrow{D}^\alpha(\partial_mD_\alpha\Phi)\theta\theta\bar{\theta}\bar{\theta}\partial_n\bar{D}^2\\&\quad-\frac{\lambda}{48}K^m{}_aK^{*na}\partial_m\overleftarrow{\bar{D}}^2(D^\alpha\Phi)\theta\theta\bar{\theta}\bar{\theta}\partial_nD_\alpha\bar{D}^2+\frac{\lambda}{24}K^m{}_aK^{*n}{}_b\bar{D}^2\overleftarrow{D}^\alpha(\sigma^{ab})_\alpha{}^\beta(\partial_m\Phi)\theta\theta\bar{\theta}\bar{\theta}\partial_nD_\beta\bar{D}^2\\&\quad+\frac{\lambda}{24}K^m{}_aK^{*n}{}_b\partial_m\overleftarrow{\bar{D}}^2(D^\alpha\Phi)(\sigma^{ab})_\alpha{}^\beta\theta\theta\bar{\theta}\bar{\theta}\partial_nD_\beta\bar{D}^2+\frac{\lambda}{24}K^m{}_aK^{*n}{}_b\partial_m\overleftarrow{\bar{D}}^2(\partial_nD^\alpha\Phi)(\sigma^{ba})_\alpha{}^\beta\theta\theta\bar{\theta}\bar{\theta}D_\beta\bar{D}^2\\&\quad-\frac{\lambda}{512}K^{mn}K_{mn}\overleftarrow{\bar{D}}^2D^2\Phi\bar{\theta}\bar{\theta}D^2\bar{D}^2-\frac{\lambda}{96}K^m{}_aK^{*na}\partial_m\overleftarrow{\bar{D}}^2(\partial_n\Phi)\theta\theta\bar{\theta}\bar{\theta}D^2\bar{D}^2-\frac{\lambda}{192}K^m{}_aK^{*na}\partial_m\overleftarrow{\bar{D}}^2(D^2\Phi)\theta\theta\bar{\theta}\bar{\theta}\partial_n\bar{D}^2\\&\quad+\frac{\lambda}{96}K^m{}_aK^{*na}\Box\overleftarrow{\bar{D}}^2\Big(\int d^8z'(\partial_mD^2\Phi)(z')\frac{1}{\Box_{z'}}\delta(z'-z)\Big)\theta\theta\bar{\theta}\bar{\theta}\partial_n\bar{D}^2,\end{aligned}\tag{3.18}$$

while the elements of F_2 read

$$\begin{aligned}F_2(z,z')&=\sum_{i=11}^{12}F^{(i)}\\&=\frac{\lambda}{96}K^m{}_aK^{*na}\partial_m\overleftarrow{\bar{D}}^2D^2\\&\quad\times\frac{1}{\Box_{z'}}\delta(z'-z)\theta\theta\bar{\theta}\bar{\theta}((\partial_n\Phi)\Box\bar{D}^2)(z')\\&\quad+\frac{\lambda}{96}K^m{}_aK^{*na}\partial_m\overleftarrow{\bar{D}}^2D^2\\&\quad\times\frac{1}{\Box_{z'}}\delta(z'-z)\theta\theta\bar{\theta}\bar{\theta}(\Box\Phi\partial_n\bar{D}^2)(z').\end{aligned}\tag{3.19}$$

The one-loop effective action is then

$$\Gamma=S_0+S_{\rm int}+\frac{i}{2}\,{\rm Tr}\log(1+\mathcal{M}^{-1}\mathcal{V}).\tag{3.20}$$

The last term in (3.20) is the one-loop correction to the effective action, and $\mathcal{M}^{-1}$ is the inverse of (3.14) given by

$$\begin{aligned}\mathcal{M}^{-1}&=\begin{pmatrix}A&B\\\bar{B}&\bar{A}\end{pmatrix}\\&=\begin{pmatrix}\frac{mD^2}{4\Box(\Box-m^2)}&\frac{D^2\bar{D}^2}{16\Box(\Box-m^2)}+\frac{\bar{D}^2D^2-2\bar{D}D^2\bar{D}}{16\xi\Box^2}\\\frac{\bar{D}^2D^2}{16\Box(\Box-m^2)}+\frac{D^2\bar{D}^2-2D\bar{D}^2D}{16\xi\Box^2}&\frac{m\bar{D}^2}{4\Box(\Box-m^2)}\end{pmatrix}.\end{aligned}\tag{3.21}$$

Expansion of the logarithm in (3.20) leads to the one-loop corrections

$$\Gamma_1=\frac{i}{2}\,{\rm Tr}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}(\mathcal{M}^{-1}\mathcal{V})^n=\sum_{n=1}^{\infty}\Gamma_1^{(n)}.\tag{3.22}$$

IV. ONE-POINT AND TWO-POINT FUNCTIONS

The first term in the expansion (3.22) gives the divergent part of the one-point functions, the tadpole contribution. We obtain

$$\Gamma_1^{(1)}=\frac{i}{2}\,{\rm Tr}(\mathcal{M}^{-1}\mathcal{V})=\frac{i}{2}\,{\rm Tr}(AF+\bar{A}\bar{F})=0.\tag{4.1}$$

Therefore just like in the commutative Wess-Zumino model there is no tadpole contribution.

Next we calculate the divergent part of the two-point functions. It is given by

$$\begin{aligned}\Gamma_1^{(2)}&=-\frac{i}{4}\,{\rm Tr}(\mathcal{M}^{-1}\mathcal{V})^2\\&=-\frac{i}{4}\,{\rm Tr}(AFAF+2\bar{B}FB\bar{F}+\bar{A}\bar{F}\bar{A}\bar{F}).\end{aligned}\tag{4.2}$$

First we calculate the $AFAF$ contributions. They are given by [remember that $F^{(i)}$ is the i-th element of the expansions (3.18) and (3.19)]

$$\begin{aligned}{\rm Tr}(AF^{(0)}AF^{(0)})&=0,\\{\rm Tr}(AF^{(1)}AF^{(0)})|_{d.p.}&=-\frac{im^2\lambda^2K^m{}_aK^{*na}}{6\pi^2\varepsilon}\int d^4x\partial_mA\partial_nA,\\{\rm Tr}(AF^{(2)}AF^{(0)})&=0,\\{\rm Tr}(AF^{(3)}AF^{(0)})&=0,\\{\rm Tr}(AF^{(4)}AF^{(0)})&=0,\\{\rm Tr}(AF^{(5)}AF^{(0)})&=0,\\{\rm Tr}(AF^{(6)}AF^{(0)})&=0,\\{\rm Tr}(AF^{(7)}AF^{(0)})|_{d.p.}&=-\frac{im^2\lambda^2K^{mn}K_{mn}}{8\pi^2\varepsilon}\int d^4xF^2,\\{\rm Tr}(AF^{(8)}AF^{(0)})|_{d.p.}&=\frac{im^2\lambda^2K^m{}_aK^{*na}}{12\pi^2\varepsilon}\int d^4x\partial_mA\partial_nA,\\{\rm Tr}(AF^{(9)}AF^{(0)})&=0,\\{\rm Tr}(AF^{(10)}AF^{(0)})&=0,\\{\rm Tr}(AF^{(11)}AF^{(0)})|_{d.p.}&=0,\\{\rm Tr}(AF^{(12)}AF^{(0)})|_{d.p.}&=0.\end{aligned}$$

Adding these terms, we obtain

$$\mathrm{Tr}\,(AFAF)|_{d.p.} = \mathrm{Tr}(AF^{(0)}AF^{(0)})|_{d.p.} + 2\sum_{i=1}^{12}\mathrm{Tr}(AF^{(i)}AF^{(0)})|_{d.p.} = -\frac{im^2\lambda^2 K^m{}_a K^{*na}}{6\pi^2\varepsilon}\int d^4x\partial_m A\partial_n A - \frac{im^2\lambda^2 K^{mn}K_{mn}}{4\pi^2\varepsilon}\int d^4xF^2. \tag{4.3}$$

The $\bar{B}FB\bar{F}$ term is more difficult to calculate. Some of the identities we use are given in the Appendix. We obtain the following contributions:

$$\mathrm{Tr}(\bar{B}F^{(0)}B\bar{F}^{(0)})|_{d.p.} = \frac{i\lambda^2}{2\pi^2\varepsilon}\int d^8z\Phi^\dagger\Phi,$$

$$\mathrm{Tr}(\bar{B}F^{(1)}B\bar{F}^{(0)})|_{d.p.} = -\frac{i\lambda^2 K^m{}_a K^{*na}}{12\pi^2\varepsilon}\int d^4xA^*(\Box - 4m^2)\partial_m\partial_n A,$$

$$\mathrm{Tr}(\bar{B}F^{(2)}B\bar{F}^{(0)})|_{d.p.} = -\frac{\lambda^2 K^m{}_a K^{*na}}{36\pi^2\varepsilon}\int d^4x\bar{\psi}\bar{\sigma}^l\partial_l\partial_m\partial_n\psi + \frac{\lambda^2 K^m{}_a K^{*na}}{12\pi^2\varepsilon}\int d^4x\bar{\psi}\bar{\sigma}_n\Big(m^2 - \frac{\Box}{6}\Big)\partial_m\psi,$$

$$\mathrm{Tr}(\bar{B}F^{(3)}B\bar{F}^{(0)})|_{d.p.} = \frac{\lambda^2 K^m{}_a K^{*na}}{72\pi^2\varepsilon}\int d^4x\bar{\psi}\bar{\sigma}^l\partial_l\partial_m\partial_n\psi,$$

$$\mathrm{Tr}(\bar{B}F^{(4)}B\bar{F}^{(0)})|_{d.p.} = -\frac{i\lambda^2 K^m{}_a K^{*na}}{2\pi^2\varepsilon}\int d^4xA^*\Big(m^2 - \frac{\Box}{6}\Big)\partial_m\partial_n A,$$

$$\mathrm{Tr}(\bar{B}F^{(5)}B\bar{F}^{(0)})|_{d.p.} = -\frac{\lambda^2 K^m{}_a K^{*n}_b}{72\pi^2\varepsilon}\int d^4x\bar{\psi}(\bar{\sigma}^b\partial^a - \bar{\sigma}^a\partial^b + i\varepsilon^{abcd}\bar{\sigma}_d\partial_c)\partial_m\partial_n\psi + \frac{\lambda^2 K^m{}_a K^{*na}}{12\pi^2\varepsilon}\int d^4x\bar{\psi}\bar{\sigma}_n\Big(m^2 - \frac{\Box}{6}\Big)\partial_m\psi,$$

$$\mathrm{Tr}(\bar{B}F^{(6)}B\bar{F}^{(0)})|_{d.p.} = -\frac{\lambda^2 K^m{}_a K^{*n}_b}{36\pi^2\varepsilon}\int d^4x\bar{\psi}(\bar{\sigma}^b\partial^a - \bar{\sigma}^a\partial^b + i\varepsilon^{abcd}\bar{\sigma}_d\partial_c)\partial_m\partial_n\psi - \frac{\lambda^2 K^m{}_a K^{*na}}{12\pi^2\varepsilon}\int d^4x\bar{\psi}\bar{\sigma}_n\Big(m^2 - \frac{\Box}{6}\Big)\partial_m\psi,$$

$$\mathrm{Tr}(\bar{B}F^{(7)}B\bar{F}^{(0)}) = 0,$$

$$\mathrm{Tr}(\bar{B}F^{(8)}B\bar{F}^{(0)})|_{d.p.} = \frac{im^2\lambda^2 K^m{}_a K^{*na}}{12\pi^2\varepsilon}\int d^4x\partial_m A^*\partial_n A,$$

$$\mathrm{Tr}(\bar{B}F^{(9)}B\bar{F}^{(0)})|_{d.p.} = \frac{i\lambda^2 K^m{}_a K^{*na}}{72\pi^2\varepsilon}\int d^4xF^*\partial_m\partial_n F,$$

$$\mathrm{Tr}(\bar{B}F^{(10)}B\bar{F}^{(0)})|_{d.p.} = \frac{im^2\lambda^2 K^m{}_a K^{*na}}{12\pi^2\varepsilon}\int d^4xd^4y\partial_m\partial_n F(x)\Box_x^{-1}\delta(x-y)F^*(y),$$

$$\mathrm{Tr}(\bar{B}F^{(11)}B\bar{F}^{(0)})|_{d.p.} = -\frac{im^2\lambda^2 K^m{}_a K^{*na}}{12\pi^2\varepsilon}\int d^4x\partial_m A^*\partial_n A,$$

$$\mathrm{Tr}(\bar{B}F^{(12)}B\bar{F}^{(0)})|_{d.p.} = -\frac{i\lambda^2 K^m{}_a K^{*na}}{36\pi^2\varepsilon}\int d^4xA^*\partial_m\partial_n\Box A.$$

Collecting all contributions, we have

$$\mathrm{Tr}(\bar{B}FB\bar{F})|_{d.p.} = \mathrm{Tr}(\bar{B}F^{(0)}B\bar{F}^{(0)})|_{d.p.} + 2\sum_{i=1}^{12}\mathrm{Tr}(\bar{B}F^{(i)}B\bar{F}^{(0)})|_{d.p.}$$
$$= \frac{i\lambda^2}{2\pi^2\varepsilon}\int d^8z\Phi^+\Phi - \frac{i\lambda^2 K^m{}_a K^{*na}}{3\pi^2\varepsilon}\int d^4xA^*\Big(m^2 + \frac{\Box}{6}\Big)\partial_m\partial_n A - \frac{\lambda^2 K^m{}_a K^{*n}_b}{12\pi^2\varepsilon}\int d^4x\bar{\psi}(\bar{\sigma}^b\partial^a - \bar{\sigma}^a\partial^b + i\varepsilon^{abcd}\bar{\sigma}_d\partial_c)\partial_m\partial_n\psi + \frac{\lambda^2 K^m{}_a K^{*na}}{6\pi^2\varepsilon}\int d^4x\bar{\psi}\bar{\sigma}_n\partial_m\Big(m^2 - \frac{\Box}{6}\Big)\psi - \frac{\lambda^2 K^m{}_a K^{*na}}{36\pi^2\varepsilon}\int d^4x\bar{\psi}\bar{\sigma}^l\partial_l\partial_m\partial_n\psi + \frac{i\lambda^2 K^m{}_a K^{*na}}{36\pi^2\varepsilon}\int d^4xF^*\partial_m\partial_n F + \frac{im^2\lambda^2 K^m{}_a K^{*na}}{6\pi^2\varepsilon}\int d^4xd^4y\partial_m\partial_n F(x)\Box_x^{-1}\delta(x-y)F^*(y). \tag{4.4}$$

Finally, adding (4.3) and (4.4) we obtain the divergent part of the two-point function:

$$\Gamma_1^{(2)}|_{d.p.} = -\frac{m^2\lambda^2 K^m{}_a K^{*na}}{24\pi^2\varepsilon}\int d^4x(\partial_m A\partial_n A + \partial_m A^*\partial_n A^*) - \frac{m^2\lambda^2 K^{mn}K_{mn}}{16\pi^2\varepsilon}\int d^4x F^2 - \frac{m^2\lambda^2 K^{*mn}K^*_{mn}}{16\pi^2\varepsilon}\int d^4x F^{*2}$$
$$+ \frac{\lambda^2}{4\pi^2\varepsilon}\int d^8z\Phi^+\Phi - \frac{\lambda^2 K^m{}_a K^{*na}}{6\pi^2\varepsilon}\int d^4x A^*\Big(m^2+\frac{\Box}{6}\Big)\partial_m\partial_n A + \frac{i\lambda^2 K^m{}_a K^{*n}_b}{24\pi^2\varepsilon}\int d^4x\bar\psi(\bar\sigma^b\partial^a - \bar\sigma^a\partial^b$$
$$+ i\varepsilon^{abcd}\bar\sigma_d\partial_c)\partial_m\partial_n\psi - \frac{i\lambda^2 K^m{}_a K^{*na}}{12\pi^2\varepsilon}\int d^4x\bar\psi\bar\sigma_n\partial_m\Big(m^2-\frac{\Box}{6}\Big)\psi + \frac{i\lambda^2 K^m{}_a K^{*na}}{72\pi^2\varepsilon}\int d^4x\bar\psi\bar\sigma^l\partial_l\partial_m\partial_n\psi$$
$$+ \frac{\lambda^2 K^m{}_a K^{*na}}{72\pi^2\varepsilon}\int d^4x F^*\partial_m\partial_n F + \frac{m^2\lambda^2 K^m{}_a K^{*na}}{12\pi^2\varepsilon}\int d^4x d^4y\partial_m\partial_n F(x)\Box_x^{-1}\delta(x-y)F^*(y). \tag{4.5}$$

We immediately see that the divergences appearing in (4.5) cannot be absorbed by counterterms since the terms appearing in (4.5) do not exist in the classical action. All terms in (4.5) quadratic in the deformation parameter are also quadratic in fields. However, the deformation of the classical action (3.2) is only present in the interaction term, and terms in the action quadratic in the deformation parameter will always be of the third order in fields. We have to conclude that our model, as it stands, is not renormalizable.

V. DISCUSSION AND CONCLUSIONS

Let us now summarize what we have done so far and discuss the obtained results in more detail.

In order to see how different deformations (different twists) affect renormalizability of the Wess-Zumino model, we considered one special example of twist, (2.7). The main advantage of this twist is that it is Hermitian and therefore implies the Hermitian $\star$-product. Compared with the undeformed SUSY Hopf algebra, the twisted SUSY Hopf algebra changes. In particular, the Leibniz rule (2.13) becomes deformed. The notion of chirality is lost, and we had to apply the method of projectors introduced in [18] to obtain the action. A nonlocal deformation of the commutative Wess-Zumino action invariant under the deformed SUSY transformations (2.12) and with a good commutative limit was introduced, and its renormalizability properties were investigated. Notice that the nonlocality comes from the application of the chiral projector P_2.[1]

To calculate the divergent part of the effective action, we used the background field method and the supergraph technique. Like in the commutative Wess-Zumino model, there is no tadpole contribution. There is no mass counterterm, which is again the same as in the undeformed Wess-Zumino model. However, the divergent part of the two-point function cannot be canceled, and we have to conclude that our model is not renormalizable. Calculating divergent parts of the three-point and higher functions does not make sense, and it is technically very demanding.

Having in mind results of [23], we also investigated on-shell renormalizability of our model. In general, on-shell renormalizability leads to a one-loop renormalizable S-matrix. On the other hand, one-loop on-shell renormalizable Green functions may spoil renormalizability at higher loops. After using the equations of motion which follow from the action (3.2) to obtain the on-shell divergent terms, we see that the divergences in the two-point function remain, and therefore the model is also not on-shell renormalizable.

In our previous work [13], we had a similar problem, a deformed model which was not renormalizable. To obtain a renormalizable model, we had to relax the condition of minimality of deformation and to include nonminimal terms. Also, in [15], new terms of the form $\int d^8z\theta\theta\bar\theta\bar\theta D^2\Phi$ and $\int d^8z\theta\theta\bar\theta\bar\theta(D^2\Phi)^2$ were added in order to absorb divergences produced by the $\int d^4xF^3 = \int d^8z\theta\theta\bar\theta\bar\theta(D^2\Phi)^3$ term. Since the model we work with is more complicated than the models of [13,15], it is not obvious which terms should be added. Let us list possible terms. Note that the new terms have to be invariant under the deformed SUSY transformations (2.12). This requirement gives three possibilities:

$$T_1 = \int d^4x P_1(\Phi\star\Phi)|_{\bar\theta\bar\theta} = \frac{1}{2}K^{ab}K_{ab}\int d^4x\Big(\frac{1}{2}(\partial_m\psi)\sigma^m\bar\sigma^n(\partial_n\psi) - F\Box A\Big). \tag{5.1}$$

$$T_2 = \int d^4x P_1(\Phi\star P_2(\Phi\star\Phi))|_{\bar\theta\bar\theta} = \frac{1}{4}K^{ab}K_{ab}\int d^4x\Big(-AF\Box A - \frac{1}{2}F\Box A^2 + \frac{1}{2}\psi\psi\Box A + \partial_m(A\psi)\sigma^m\bar\sigma^n(\partial_n\psi)\Big). \tag{5.2}$$

[1] Unlike the Moyal-Weyl $\star$-product, the $\star$-product (2.9) is finite and it does not introduce nonlocality.

$$T_3 = \int d^4x \Phi \star P_1(\Phi \star \Phi))|_{\theta\theta\bar\theta\bar\theta}$$
$$= \frac{3}{4} K^{ab} K_{ab} \int d^4x (F(\partial_m \psi)\sigma^m \bar\sigma^l (\partial_l \psi) - F^2 \Box A)$$
$$+ K^m{}_a K^{*na} \int d^4x (A(\Box A)(\partial_m \partial_n A)$$
$$+ A(\partial_m \partial_l A)(\partial_n \partial^l A)). \quad (5.3)$$

The term T_1 produces divergences of the type $\int d^4x \Phi^+ \Phi$, so it would not spoil the renormalizability of the model. However, it cannot improve renormalizability, since divergences appearing in (4.5) are not of the type T_1. The term T_2 produces additional divergences that cannot be absorbed, so we have to ignore it. The T_3 term does not cancel any of the terms present in the action (2.27). Additionally, it produces new divergent terms. However these terms might look, they can never cancel all the divergences in (4.5), as divergences proportional to $K^m{}_a K^{*n}{}_b$ will remain. This analysis forces us to conclude that even with a nonminimal deformation our model remains nonrenormalizable.

Let us make a remark about the nonrenormalization theorem and its modifications in the case of deformed SUSY. One easily sees that the divergent terms of the effective action (4.5) can be rewritten as

$$\Gamma_1^{(2)}|_{d.p.} = \int d^4x_1 d^4x_2 d^2\theta d^2\bar\theta G_2(x_1, x_2, U_{(1)}, U_{(2)})$$
$$\times f_1(x_1, \theta, \bar\theta) f_2(x_2, \theta, \bar\theta), \quad (5.4)$$

with $f_i = f_i(\Phi, \Phi^+, D\Phi, \bar D\Phi, D\Phi^+, \bar D\Phi^+, \ldots)$. The nonlocal term in (5.4) appears as a consequence of nonlocality in the classical action (3.2). The result (5.4) confirms the modified nonrenormalization theorem [15]. The appearance of the spurion fields in (5.4) signals breaking of the undeformed SUSY. In our case, symmetry which remains after the breaking is the twisted SUSY (2.12). However, it seems that the twisted SUSY is not enough to guarantee renormalizability.

It is obvious that different deformations obtained from different twists lead to models with different quantum properties. In our previous work [13], we studied a deformation which preserves the full undeformed SUSY. There, after relaxing the condition of minimality of deformation, we obtained a renormalizable Wess-Zumino model. In this paper we work with a deformation given in terms of the non-SUSY-covariant derivatives. The Leibniz rule for the SUSY transformation (2.12) changes, and the deformed action (2.27), though invariant under twisted SUSY transformations, is not invariant under the undeformed SUSY transformations. For example, the term $K^{mn} K_{mn} F^3$ breaks the undeformed SUSY. On the other hand, the twisted SUSY allows this term as a part of the invariant term $P_2(\Phi \star P_2(\Phi \star \Phi))|_{\theta\theta}$; see Eqs. (5.13) and (5.14) in [18].

The classical properties of theories with twisted symmetries are not fully understood [3,24]. For example, one cannot apply standard methods to find conserved charges, and the modification of the Noether theorem in the case of twisted symmetry has not yet been formulated. In this paper we analyze quantum properties of the theory with the twisted SUSY. This is the first time that renormalizability of a theory with a twisted symmetry has been analyzed. Even after relaxing the condition of minimality of deformation, our model remains nonrenormalizable. This indicates that theories with twisted symmetries do not have the same quantum properties as theories with undeformed symmetries. In our example, we see that the twisted SUSY is not enough to guarantee renormalizability of the Wess-Zumino model. It is obvious that a better understanding of the twisted symmetry and its consequences, both classical and quantum, is needed.

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APPENDIX: CALCULATION

In this appendix we collect details of some calculations and some important side results.

(1) Transformation laws of the component fields of the superfield F (2.2):

$$\delta_\xi f = \xi^\alpha \phi_\alpha + \bar\xi_{\dot\alpha} \bar\chi^{\dot\alpha}, \quad (A1)$$

$$\delta_\xi \phi_\alpha = 2\xi_\alpha m + \sigma^m{}_{\alpha\dot\alpha} \bar\xi^{\dot\alpha} (v_m + i(\partial_m f)), \quad (A2)$$

$$\delta_\xi \bar\chi^{\dot\alpha} = 2\bar\xi^{\dot\alpha} n + \bar\sigma^{m\dot\alpha\alpha} \xi_\alpha (-v_m + i(\partial_m f)), \quad (A3)$$

$$\delta_\xi m = \bar\xi_{\dot\alpha} \bar\lambda^{\dot\alpha} + \frac{i}{2} \bar\xi_{\dot\alpha} \bar\sigma^{m\dot\alpha\alpha} (\partial_m \phi_\alpha), \quad (A4)$$

$$\delta_\xi n = \xi^\alpha \varphi_\alpha + \frac{i}{2} \xi^\alpha \sigma^m{}_{\alpha\dot\alpha} (\partial_m \bar\chi^{\dot\alpha}), \quad (A5)$$

$$\sigma^m{}_{\alpha\dot\alpha} \delta_\xi v_m = -i(\partial_m \phi_\alpha)\xi^\beta \sigma^m{}_{\beta\dot\alpha} + 2\xi_\alpha \bar\lambda_{\dot\alpha}$$
$$+ i\sigma^m{}_{\alpha\dot\beta} \bar\xi^{\dot\beta} (\partial_m \bar\chi_{\dot\alpha}) + 2\varphi_\alpha \bar\xi_{\dot\alpha}, \quad (A6)$$

$$\delta_\xi \bar\lambda^{\dot\alpha} = 2\bar\xi^{\dot\alpha} d + i\bar\sigma^{l\dot\alpha\alpha} \xi_\alpha (\partial_l m)$$
$$+ \frac{i}{2} \bar\sigma^{l\dot\alpha\alpha} \sigma^m{}_{\alpha\dot\beta} \bar\xi^{\dot\beta} (\partial_m v_l), \quad (A7)$$

$$\delta_\xi \varphi_\alpha = 2\xi_\alpha d + i\sigma^l{}_{\alpha\dot\alpha} \bar\xi^{\dot\alpha} (\partial_l n)$$
$$- \frac{i}{2} \sigma^l{}_{\alpha\dot\alpha} \bar\sigma^{m\dot\alpha\beta} \xi_\beta (\partial_m v_l), \quad (A8)$$

$$\delta_\xi d = \frac{i}{2} \xi^\alpha \sigma^m{}_{\alpha\dot\alpha} (\partial_m \bar\lambda^{\dot\alpha}) - \frac{i}{2} (\partial_m \varphi^\alpha) \sigma^m{}_{\alpha\dot\alpha} \bar\xi^{\dot\alpha}. \quad (A9)$$

(2) Irreducible components of the superfield F:

$$P_2\mathrm{F} = \frac{1}{16}\frac{\bar{D}^2 D^2}{\Box}\mathrm{F}$$
$$= \frac{1}{\Box}\Big(d - \frac{i}{2}(\partial_m v^m) + \frac{1}{4}\Box f\Big) + \sqrt{2}\theta^\alpha\Big(\frac{i}{\sqrt{2}\Box}\sigma^m{}_{\alpha\dot\alpha}(\partial_m\bar\lambda^{\dot\alpha}) + \frac{1}{2\sqrt{2}}\phi_\alpha\Big) + \theta\theta m + i\theta\sigma^l\bar\theta\partial_l\Big(\frac{d}{\Box} - \frac{i}{2\Box}(\partial_m v^m) + \frac{1}{4}f\Big)$$
$$+ \frac{1}{\sqrt{2}}\theta\theta\bar\theta_{\dot\alpha}\Big(\frac{1}{\sqrt{2}}\bar\lambda^{\dot\alpha} + \frac{i}{2\sqrt{2}}\bar\sigma^{m\dot\alpha\alpha}(\partial_m\phi_\alpha)\Big) + \frac{1}{4}\theta\theta\bar\theta\bar\theta\Big(d - \frac{i}{2}(\partial_m v^m) + \frac{1}{4}\Box f\Big). \tag{A10}$$

$$P_1\mathrm{F} = \frac{1}{16}\frac{D^2\bar{D}^2}{\Box}\mathrm{F}$$
$$= \frac{1}{\Box}\Big(d + \frac{i}{2}(\partial_m v^m) + \frac{1}{4}\Box f\Big) + \sqrt{2}\bar\theta_{\dot\alpha}\Big(\frac{i}{\sqrt{2}\Box}\bar\sigma^{m\dot\alpha\alpha}(\partial_m\varphi_\alpha) + \frac{1}{2\sqrt{2}}\bar\chi^{\dot\alpha}\Big) + \bar\theta\bar\theta n - i\theta\sigma^l\bar\theta\partial_l\Big(\frac{d}{\Box} + \frac{i}{2\Box}(\partial_m v^m) + \frac{1}{4}f\Big)$$
$$+ \frac{1}{\sqrt{2}}\bar\theta\bar\theta\theta^\alpha\Big(\frac{1}{\sqrt{2}}\varphi_\alpha + \frac{i}{2\sqrt{2}}\sigma^m{}_{\alpha\dot\alpha}(\partial_m\bar\chi^{\dot\alpha})\Big) + \frac{1}{4}\theta\theta\bar\theta\bar\theta\Big(d + \frac{i}{2}(\partial_m v^m) + \frac{1}{4}\Box f\Big), \tag{A11}$$

$$P_T\mathrm{F} = \frac{1}{2}f - \frac{2}{\Box}d + \theta^\alpha\Big(\frac{1}{2}\phi_\alpha - i\frac{1}{\Box}\sigma^m{}_{\alpha\dot\alpha}\partial_m\bar\lambda^{\dot\alpha}\Big) + \bar\theta_{\dot\alpha}\Big(\frac{1}{2}\bar\chi^{\dot\alpha} - i\frac{1}{\Box}\bar\sigma^{m\dot\alpha\alpha}\partial_m\varphi_\alpha\Big) + \theta\sigma^m\bar\theta\Big(v_m - \frac{1}{\Box}\partial_m\partial_l v^l\Big)$$
$$+ \theta\theta\bar\theta_{\dot\alpha}\Big(\frac{1}{2}\bar\lambda^{\dot\alpha} - \frac{i}{4}\bar\sigma^{m\dot\alpha\alpha}(\partial_m\phi_\alpha)\Big) + \bar\theta\bar\theta\theta^\alpha\Big(\frac{1}{2}\varphi_\alpha - \frac{i}{4}\sigma^m{}_{\alpha\dot\alpha}(\partial_m\bar\chi^{\dot\alpha})\Big) + \frac{1}{4}\theta\theta\bar\theta\bar\theta\Big(2d - \frac{1}{2}\Box f\Big). \tag{A12}$$

The following identity holds:

$$P_T = I - P_1 - P_2. \tag{A13}$$

(3) Some general formulas for the divergent parts of traces, where $K = \Box - m^2$:

$$\mathrm{Tr}\,(K^{-1}f) = \frac{i}{8\pi^2\epsilon}m^2\int d^4x f, \tag{A14}$$

$$\mathrm{Tr}\,(\partial_a K^{-1}f) = 0, \tag{A15}$$

$$\mathrm{Tr}\,(\Box K^{-1}f) = \frac{im^4}{8\pi^2\epsilon}\int d^4x f, \tag{A16}$$

$$\mathrm{Tr}\,(\Box^2 K^{-1}f) = \frac{im^6}{16\pi^2\epsilon}\int d^4x f, \tag{A17}$$

$$\mathrm{Tr}\,(K^{-1}fK^{-1}g) = \frac{i}{8\pi^2\epsilon}\int d^4x fg, \tag{A18}$$

$$\mathrm{Tr}\,(\partial_n K^{-1}fK^{-1}g) = \frac{i}{16\pi^2\epsilon}\int d^4x\partial_n fg, \tag{A19}$$

$$\mathrm{Tr}\,(\partial_n K^{-1}f\partial_m K^{-1}g) = -\frac{i}{16\pi^2\epsilon}\int d^4x f\Big(\frac{1}{3}\partial_n\partial_m + \frac{1}{6}\eta_{mn}\Box - \eta_{mn}m^2\Big)g, \tag{A20}$$

$$\mathrm{Tr}\,(\partial_n K^{-1}f\partial_m\partial_p K^{-1}g) = -\frac{i}{32\pi^2\epsilon}\int d^4x f\Big(\frac{1}{3}\partial_n\partial_m\partial_p + (\eta_{mp}\partial_n - \eta_{np}\partial_m - \eta_{nm}\partial_p)\Big(m^2 - \frac{1}{6}\Box\Big)\Big)g. \tag{A21}$$

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