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# Gribov ambiguity in asymptotically AdS three-dimensional gravity

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In this paper the zero modes of the de Donder gauge Faddeev-Popov operator for three-dimensional gravity with negative cosmological constant are analyzed. It is found that the $AdS_3$ vacuum produces (infinitely many) normalizable smooth zero modes of the Faddeev-Popov operator. On the other hand, it is found that the Bañados-Teitelboim-Zanelli black hole (including the zero mass black hole) does not generate zero modes. This differs from the usual Gribov problem in QCD where, close to the maximally symmetric vacuum, the Faddeev-Popov determinant is positive definite while ''far enough'' from the vacuum it can vanish. This suggests that the zero mass Bañados-Teitelboim-Zanelli black hole could be a suitable ground state of three-dimensional gravity with negative cosmological constant. Because of the kinematic origin of this result, it also applies for other covariant gravity theories in three dimensions with $AdS_3$ as maximally symmetric solution, such as new massive gravity and topologically massive gravity. The relevance of these results for supersymmetry breaking is pointed out.

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## I. INTRODUCTION

The Yang-Mills interaction is paramount in the current understanding of the fundamental interactions. As is well known, the Yang-Mills Lagrangian takes the form

$$L = \mathrm{tr} F_{\mu\nu} F^{\mu\nu}, \qquad (F_{\mu\nu})^a = (\partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu])^a, \tag{1}$$

where the degrees of freedom of the theory are redundantly described by a Lie algebra valued one form, namely, the gauge potential $(A_\mu)^a$. The redundancies arise due to the invariance of the Lagrangian under finite gauge transformations, which acts on the gauge potential as

$$A_\mu \to U^\dagger A_\mu U + U^\dagger \partial_\mu U. \tag{2}$$

It immediately follows that any physical observable must be invariant under gauge transformations. As a dynamical system, the redundancy implies that the symplectic form has a fixed, nonmaximal rank, it being invertible only on the surface where the constraints hold, and the restriction of the symplectic form to that surface is achieved by fixing the gauge. Indeed, the gauge fixing is quite relevant in the classical theory since, when using the Dirac bracket formalism, the Faddeev-Popov determinant appears in the denominators of the Dirac-Poisson brackets (see, for instance, the detailed analysis in [1]).

It could be possible, in principle, to formulate the theory from the very beginning in terms of gauge-invariant variables such as Wilson loops. However, up to now, such an ambitious program has been carried out completely only in the cases of topological field theories in $2 + 1$ dimensions [2], while it is still far from clear how to perform practical computations, such as scattering amplitudes and correlation functions using the Wilson loop variables for Yang-Mills theories in $2 + 1$ and $3 + 1$ dimensions.

Therefore, gauge fixing seems to be unavoidable to properly describe the evolution of a gauge theory. In Yang-Mills theories, the most convenient choices are the Coulomb gauge and the Lorenz gauge:

$$\partial^i A_i = 0, \qquad \partial^\mu A_\mu = 0, \tag{3}$$

where $i = 1, \ldots, D$ are the spacelike indices while $\mu = 0, 1, \ldots, D$ are space-time indices. Indeed, other choices are possible, such as the axial gauge, the temporal gauge, et cetera. Nevertheless these gauge fixings have their own problems (see, for instance, [3]).

If there exists a proper gauge transformation[1] (2) that preserves any of the gauge conditions (3), then it would not fix the gauge freedom completely, making impossible the avoidance of some kind of overcounting in the gauge fixed path integral [5]: this phenomenon, called *Gribov ambiguity,* prevents one from obtaining a *proper gauge fixing*. Furthermore, it has been shown by Singer [6], that if Gribov ambiguities occur in the Coulomb gauge, they occur in any gauge fixing involving derivatives of the gauge field. Abelian gauge theories on flat space-time are

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[1]A proper gauge transformation has to be smooth everywhere, and it has to decrease fast enough at infinity such that a suitable norm, to be specified later, converges. The problem of defining a proper gauge transformation was first explored in [4].

 

devoid of this problem, since the Gribov copy equation for the smooth gauge parameter $\phi$ is

$$\partial_i \partial^i \phi = 0 \quad \text{or} \quad \partial_\mu \partial^\mu \phi = 0, \qquad (4)$$

which on flat space-times (once the time coordinate has been Wick-rotated: $t \rightarrow i\tau$) has no smooth nontrivial solutions fulfilling the physical boundary conditions.

The situation changes dramatically when we consider an Abelian gauge field propagating on a curved background due to the replacement of the partial derivatives by covariant derivatives: it was shown in [7] that, quite generically, a proper gauge fixing in the Abelian case cannot be achieved.

In the case of non-Abelian gauge theories, even on flat space-time the Lorenz or Coulomb gauge fixing are ambiguous. In the path integral formalism, an ambiguity in the gauge fixing corresponds to a smooth normalizable zero mode of the Faddeev-Popov (FP) determinant. In order to define the path integral in the presence of Gribov copies, it has been suggested to exclude classical $A_\mu$ backgrounds which generate zero modes of the FP operator (see, in particular, [5,8–12]). This possibility is consistent with the usual perturbative point of view since, in the case of $SU(N)$ Yang-Mills theories, for a "small enough" potential $A_\mu$ (with respect to a suitable functional norm [12]), there are no zero modes of the Faddeev-Popov operator in the Landau gauge.

It is therefore very important to analyze the problem of gauge fixing ambiguities in the context of a gravitational field. It is quite well known that Gribov ambiguities are also present in the gravitational case (see the review in [13,14]), but very few explicit cases have been considered. As will be shown below, the gravitational gauge fixing problem is quite peculiar and very interesting from many points of view.

A very interesting case which allows an explicit analysis showing, at the same time, many peculiar features is gravity in three dimensions [15]. Since the work by Witten [16], the standard lore has been that the Einstein-Hilbert action defines a quantum theory of gravity in three dimensions. In three dimensions, the traceless part of the Riemann tensor identically vanishes and all the geometrical information is thus encoded in the Ricci tensor. This implies that an Einstein space is locally of constant curvature. As was pointed out in [16], the theory under consideration is thus trivial modulo the existence of global obstructions. The seemingly uninteresting situation got closer to what one would expect from a realistic gravitational theory with the discovery that, when the cosmological constant is negative, one of the possible global obstructions is actually a black hole [17].

Moreover, it is possible to give a microscopic description of the entropy of the Bañados-Teitelboim-Zanelli (BTZ) black hole which has its roots in the well-known work of Brown and Henneaux [18], who showed that the asymptotic symmetry group of a three-dimensional space-time which matches $AdS_3$ at infinity (with a precise falloff) is the product of two copies of the de Witt algebra and that its canonical realization is projective. The existence of such central charge was connected, much later, with the entropy of this black hole through the Cardy formula [19,20]. The fact that this result holds even for hairy black holes [21] suggests that such a result is independent of the finiteness of three-dimensional quantum gravity. Indeed, since pure gravity is classically trivial, any possible counterterm is on-shell equivalent to the volume of the space-time and can be reabsorbed in the cosmological constant and a local redefinition of the metric tensor. Furthermore, the fact that three-dimensional gravity can be formulated as a Chern-Simons gauge theory implies that the cosmological constant is proportional to the structure functions of the gauge group, and the nonexistence of gauge anomalies in three dimensions can be used to argue that the gauge algebra holds at the quantum level. Therefore, no renormalization can affect the cosmological constant and the theory must be finite [22]; this argument supports the Strominger proposal that three-dimensional Einstein gravity *is* a conformal field theory. However, when there are matter fields, the first ingredient of the previous discussion no longer holds, namely, the fact that on-shell every space-time has constant curvature. Although it cannot be discarded that there is an ultraviolet completion of the theory analyzed in [21], the Cardy formula gives the right result without the arguments that Strominger proposed to use it.

Because of the triviality of the theory one would expect to obtain the thermodynamical properties of the BTZ black hole computing the partition function [23]; however, the sum of all the contributions coming from known classical geometries gives rise to an inconsistent result.

Our aim in writing this paper is to point out that the quantization process should be reanalyzed in the light of the existence of an infinite number of zero modes of the FP operator for the de Donder gauge in three dimensions when gravity is quantized around $AdS_3$ while it has no nontrivial zero mode for the BTZ black holes.

The paper is organized as follows: Sec. II presents the generalities of the Gribov problem associated with diffeomorphism invariance in a gravitational theory. In Sec. III we focus on the case of $AdS_3$, where we found a set of vector fields which generate Gribov copies for the de Donder Faddeev-Popov operator, which preserve the asymptotically $AdS_3$ behavior of the metric in the Brown-Henneaux sense. Section IV is devoted to analyzing the same problem on BTZ black holes, and we prove that, within the family of diffeomorphisms considered here, there are no normalizable copies, (suggesting then a massless BTZ black hole as a better ground state to perform a perturbative analysis). Finally, in Sec. V we give some further comments concerning other theories of gravity and a possible new approach for supersymmetry breaking.

## II. GRIBOV AMBIGUITY IN GRAVITATIONAL THEORIES

The degrees of freedom of the gravitational field are described by the metric tensor $g_{\mu\nu}$. Because of the diffeomorphism invariance, metric tensors related by a coordinate transformation describe the same space-time. In the framework of the path integral approach to the semiclassical quantization of gravity, the most commonly used gauge-fixing condition of the diffeomorphism invariance is the de Donder gauge, defined as follows.[2] Let us denote the classical background metric as $g^{(0)}_{\mu\nu}$, while a small fluctuation around $g^{(0)}_{\mu\nu}$ will be denoted as $h_{\mu\nu}$. Under a generic coordinate transformation, the metric fluctuation $h_{\mu\nu}$ transforms as

$$h_{\mu\nu} \to h_{\mu\nu} + \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu, \tag{5}$$

where $\xi_\mu$ is the vector field generating the diffeomorphism, and the covariant derivative is taken with the background metric $g^{(0)}_{\mu\nu}$. Then the de Donder gauge on the metric fluctuation $h_{\mu\nu}$ reads

$$\nabla^\mu h_{\mu\nu} = 0. \tag{6}$$

The field equations of general relativity in vacuum further imply that

$$h^\mu{}_\mu = 0; \tag{7}$$

on the other hand, if one is analyzing a diffeomorphism invariant theory different from general relativity, only the condition (6) should be taken into account. In the present paper, we will mainly focus on general relativity, and some comments on alternative theories of gravity will be presented in the last section.

In the context of three-dimensional quantum general relativity, the analysis of the gauge-fixing problem is even more relevant than in four-dimensional space-times because the theory is finite [16]. Furthermore, in the case of gravity in $2+1$ dimensions, it is also possible to consider explicitly topology change amplitudes [25] using the Lorentzian path integral. The interest in considering the issue of topology change amplitudes is based on the spin-statistics connection, which strongly suggests the necessity of topology changes in quantum gravity [26,27]. (For detailed reviews, see [28,29].) As discussed in the Introduction, three-dimensional AdS gravity has the special feature of having a infinite dimensional asymptotic symmetry algebra, which, as we will see, is closely related with the Gribov copies around the AdS background. Remarkably enough, the same structure exists in three- and four-dimensional asymptotically flat space-times, namely, the $BMS_3$ and $BMS_4$ algebra, both of them extended beyond the usual supertranslations in [30,31], respectively. It can be noted that the supertranslations (and the extension of this work to higher dimensions) disappear when a Wick rotation is performed. For these reasons, we will consider the Lorentzian signature in the present paper.

The main goal of this paper is to show that the gauge-fixing ambiguity for three-dimensional general relativity with negative cosmological constant and, more generically, for any diffeomorphism-invariant metric theory is actually quite different from the usual Gribov problem for $SU(N)$ Yang-Mills in a four-dimensional flat space-time. The maximally supersymmetric vacuum of the theory (which is $AdS_3$ space-time) generates a denumerable set of normalizable, smooth, zero modes of the FP operator in the de Donder gauge. Thus, even if following the QCD case in four dimensions, one would naturally expect [25] that the Gribov problem should not manifest itself at a perturbative level; even perturbative calculations in the path integral formalism around the maximally supersymmetric vacuum seem to be problematic. On the other hand, it is found that the BTZ black hole solutions of this theory [17] do not generate zero modes. In particular, the massless BTZ black hole appears to be a sensible ground state for perturbative analysis but *it preserves only half of the supersymmetries*.

It is also worth emphasizing that, even if one were to find another gauge fixing free of Gribov ambiguities, still the presence of Gribov ambiguities in the de Donder gauge would have very deep physical consequences. In particular, it is known that the presence of Gribov ambiguity can led to a breaking of the Becchi-Rouet-Stora-Tyutin (BRST) symmetry at a nonperturbative level (see, for instance, [32–35]). Therefore, even if one were to adopt an ambiguity-free gauge fixing, it would still be necessary to analyze the above issues carefully.

## III. ZERO MODES OF THE FP OPERATOR ON AN $AdS_3$ BACKGROUND

The metric of $AdS_3$ is

$$ds^2 = -\left(1 + \frac{r^2}{l^2}\right)dt^2 + \frac{dr^2}{1 + \frac{r^2}{l^2}} + r^2 d\phi^2, \tag{8}$$

where

$$-\infty < t < \infty, \qquad 0 \le r < \infty, \qquad 0 \le \phi < 2\pi. \tag{9}$$

The two copies of the de Witt algebra that arise as a nontrivial endomorphism of the space of solutions of general relativity with negative cosmological constant $\Lambda = -1/l^2$ in three dimensions, have a description in terms of the asymptotic Killing vectors which preserve the asymptotic behavior of the metric at spatial infinity [18]. The asymptotic Killing vectors read

$$\xi^t = l\left(T^+ + \frac{l^2}{2r^2}\partial_-^2 T^- + T^- + \frac{l^2}{2r^2}\partial_+^2 T^+\right) + O(r^{-4}), \tag{10}$$

$$\xi^\phi = T^+ - T^- + \frac{l^2}{2r^2}\partial_-^2 T^- - \frac{l^2}{2r^2}\partial_+^2 T^+ + O(r^{-4}), \tag{11}$$

[2] It is worth noting that this gauge is also the most common choice in the analysis of gravitational waves [24].

$$\xi^r = -r(\partial_+ T^+ + \partial_- T^-) + O(r^{-1}), \tag{12}$$

where $x^\pm = \frac{t}{l} \pm \phi$ ($\partial_\pm = \frac{l}{2}\partial_t \pm \frac{1}{2}\partial_\phi$), and

$$T^\pm := T^\pm\left(\frac{t}{l} \pm \phi\right), \tag{13}$$

which preserve the following asymptotic behavior of the metric,

$$h_{rr} \sim O(r^{-4}), \qquad h_{rm} \sim O(r^{-3}), \qquad h_{mn} \sim O(1), \tag{14}$$

where the indices $m$, $n$ stand for $\{t, \phi\}$, and $h_{\mu\nu}$ is the departure from the $AdS_3$ space-time.

Obviously, because of the periodicity of the coordinate $\phi$, both the coordinate $x^+ = \frac{t}{l} + \phi$ and the coordinate $x^- = \frac{t}{l} - \phi$ are periodic:

$$x^\pm \sim x^\pm + 2\pi. \tag{15}$$

Therefore, it is possible to Fourier analyze the functions $T^\pm(\frac{t}{l} \pm \phi)$. These modes furnish a realization of two copies of the de Witt algebra with the Lie bracket. On the other hand, if the periodicity in the coordinate $\phi$ is disregarded, the arbitrary functions in the Killing vectors can be expanded in Laurent series, and the same asymptotic algebra can be obtained [31]. In the following calculations, we will adopt the expansion in Fourier modes.

In order to construct zero modes of the FP operator in the de Donder gauge on the $AdS_3$ background metric (8), one has to solve the following system of equations,

$$\nabla^\mu(\nabla_\mu \eta_\nu + \nabla_\nu \eta_\mu) = 0, \tag{16}$$

together with the scalar equation

$$\nabla^\mu \eta_\mu = 0, \tag{17}$$

where $\nabla_\mu$ is the covariant derivative with respect to the $AdS_3$ metric in Eq. (8). A proper, linearized diffeomorphism, $\eta_\mu$, has to be smooth everywhere, and furthermore it has to possess a finite norm $\mathcal{N}(\eta)$:

$$\mathcal{N}(\eta) := \int \sqrt{-g} d^3x \nabla_{(\mu}\eta_{\nu)} \nabla^{(\mu}\eta^{\nu)} < \infty. \tag{18}$$

It is worth noting here that the above norm in Eq. (18) is the closest analogue of the functional norm used in the Yang-Mills path integral (see in particular [12]). In the case of the Yang-Mills path integral, the gauge potential[3] $A^a_\mu$ has to satisfy the following finite norm condition:

$$\mathcal{N}_{\rm YM}(A) = \int \sqrt{-g} d^{D+1}x\, {\rm Tr}(A_\mu A^\mu) < \infty. \tag{19}$$

As is well known, this induces a condition on the gauge transformation parameter $U$ in Eq. (2) by requiring that, whenever $A^a_\mu$ satisfies the condition in Eq. (19), the gauge transformed by $A^a_\mu$ must also satisfy it. In the gravitational case in $(2+1)$ dimensions, the path integral is taken on the metric fluctuation $h_{\mu\nu}$. The natural norm for $h_{\mu\nu}$ is then

$$\mathcal{N}(h) := \int \sqrt{-g} d^3x h_{\mu\nu} h^{\mu\nu}, \tag{20}$$

and the finite norm condition is $\mathcal{N}(h) < \infty$. This norm then induces the condition on the vector field $\eta_\mu$ given in Eq. (18).

Let us consider the following ansatz for a vector field $\eta^\mu$ which is designed in order to accommodate explicitly the Brown-Henneaux asymptotics in Eqs. (10)–(12):

$$\eta^+ = f_1(r)T^+ + f_2(r)\frac{l^2}{2r^2}\partial_-^2 T^-, \tag{21}$$

$$\eta^- = f_3(r)T^- + f_4(r)\frac{l^2}{2r^2}\partial_+^2 T^+, \tag{22}$$

$$\eta^r = -\frac{r}{2}(f_5(r)\partial_+ T^+ + f_6(r)\partial_- T^-). \tag{23}$$

We will search for zero modes of the Faddeev-Popov operator in the de Donder gauge within the family determined by (21)–(23), such that the functions $f_{(i)}(r)$ $(i = 1, \ldots, 6)$ are everywhere smooth and guarantee that the norm in Eq. (18) is finite. The diffeomorphisms generated by the vector field $\eta^\mu$ with components (21)–(23) will belong to the Brown-Henneaux class (10)–(12), provided the functions $f_{(i)}(r)$ satisfy the following asymptotic behavior,

$$f_{(i)}(r) \underset{r\to+\infty}{\longrightarrow} \begin{cases} \alpha + O(r^{-4}) & \text{for } i = 1, 3 \\ & \text{and} \\ \beta + O(r^{-2}) & \text{for } i = 2, 4, 5, 6 \end{cases}, \tag{24}$$

where $\alpha$ and $\beta$ are constant.

It is possible to Fourier expand $T^+(x^+)$ and $T^-(x^-)$ so that one can replace $T^+$ and $T^-$ as follows:

$$T^+ \to e^{inx^+}, \tag{25}$$

$$T^- \to e^{imx^-}, \tag{26}$$

where the expansion in Fourier modes is in the interval when the function is nonzero. Here we consider $n \geq 2$ and $m \geq 2$. The computation with negative $n$ and $m$ follows the same lines. The modes with $m, m \in \{0, \pm 1\}$, which generate the $sl(2, R) \times sl(2, R)$ subalgebra of the asymptotic Brown-Henneaux symmetries, will be discussed later.

With the above requirements, and with an ansatz for $\eta^\mu$ of the form (21)–(23), the relevant solutions for the system (16) and (17) are

[3]In the case of the Yang-Mills path integral, the gauge potential $A^a_\mu$ is "a small fluctuation." Namely, $A^a_\mu$ represents a small deviation from the maximally symmetric vacuum $A^a_\mu = 0$ (or any other classical background one is interested in). The norm conditions are supposed to describe mathematically the "smallness" of the fluctuations.

$$f_1(r) = C_3 \frac{r^{n-2}}{2(r^2+l^2)^{(n+2)/2}}(4r^4 + 2l^2(n+2)r^2 + nl^4(n+1)),$$

$$f_2(r) = -\frac{(4l^2+5r^2)r^3}{l^4m^2} f_6' - \frac{(l^2+r^2)r^4}{l^4m^2} f_6'' + \frac{r^2(l^2m^2 - 2l^2 - 2r^2)}{l^2m^2(l^2+r^2)} f_6,$$

$$f_3(r) = C_4 \frac{r^{m-2}}{2(r^2+l^2)^{(m+2)/2}}(4r^4 + 2l^2(m+2)r^2 + ml^4(m+1)),$$

$$f_4(r) = -\frac{(4l^2+5r^2)r^3}{l^4n^2} f_5' - \frac{(l^2+r^2)r^4}{l^4n^2} f_5'' + \frac{r^2(l^2n^2 - 2l^2 - 2r^2)}{l^2n^2(l^2+r^2)} f_5,$$

$$f_5(r) = \frac{C_3 r^{n-2}}{(r^2+l^2)^{n/2}}(nl^2 + l^2 + 2r^2),$$

$$f_6(r) = \frac{C_4 r^{m-2}}{(r^2+l^2)^{m/2}}(l^2m + l^2 + 2r^2),$$

where we used the notation $X' := \partial_r X$.

The asymptotic behavior at infinity of these functions is given by

$$f_1(r) = 2C_3 + C_3 \frac{l^4n^2}{r^4} + O(r^{-6}),$$

$$f_2(r) = 2C_4 + C_4 \frac{l^2(1-m-m^2)}{mr^2} + O(r^{-4}),$$

$$f_3(r) = 2C_4 + C_4 \frac{l^4m^2}{r^4} + O(r^{-6}),$$

$$f_4(r) = 2C_3 + C_3 \frac{l^2(1-n-n^2)}{nr^2} + O(r^{-4}),$$

$$f_5(r) = 2C_3 + \frac{C_3 l^2}{r^2} + O(r^{-4}),$$

$$f_6(r) = 2C_4 + \frac{C_4 l^2}{r^2} + O(r^{-4}),$$

fulfilling then the required behavior given by (24).

The radial part of the integral contributing to the norm in Eq. (18) is given by

$$\mathcal{N} \propto \frac{1}{l^2(m+n-2)} \frac{r^{m+n-2}}{(r^2+l^2)^{(m+n-2)/2}}\Bigg|_0^\infty, \tag{37}$$

which converges for $n, m \geq 2$, since, when one considers the variables $x^+$ and $x^-$ periodic as in Eq. (15), the integral in $x^+$ and $x^-$ in the norm is obviously finite. On the other hand, one could disregard the periodicity in $x^+$ and $x^-$ and consider the dependence of the copies on these variables to be given by a function of compact support. A careful analysis for the case $m, n \in \{0, \pm 1\}$, along the same lines as the one presented here, shows that, within the family we considered, there are no normalizable vector field $\eta$ which asymptotically match the Brown-Henneaux vector generating the $sl(2,R) \times sl(2,R)$ subalgebra[4] which would generate zero modes for the FP determinant.

## IV. BTZ BLACK HOLE BACKGROUND

In the previous section, it has been shown that, for three-dimensional gravity with a negative cosmological constant, there is a gauge-fixing ambiguity for the maximally (super)symmetric vacuum. As mentioned above, this situation is quite different from the usual Yang-Mills $SU(N)$ Gribov problem, where "near to the vacuum," the gauge fixing is well defined. Thus, in order to get a perturbatively well-defined theory, it is sufficient to restrict the path integral to this region. The boundary in the space of connections which delimits the region where the gauge fixing is not ambiguous is called the Gribov horizon. Since for the gravitational field in $(2+1)$ dimensions the most natural ground state manifests gauge-fixing ambiguities, one would naïvely expect that going "far away" from the ground state the situation could become worse. Surprisingly enough, this is not the case. Indeed, let us consider the BTZ black hole metric with Lorentzian signature

$$ds^2 = \frac{dr^2}{\frac{r^2}{l^2} - \mu} - \frac{l^2}{4}(dx^{-2} + dx^{+2}) - \left(r^2 - \frac{\mu l^2}{2}\right) dx^+ dx^-, \tag{28}$$

where $\mu$ is a mass parameter. (The $AdS_3$ vacuum turns out to have $\mu = -1$.) Solutions with $-1 < \mu < 0$ represent naked singularities and must therefore be discarded. The other physically sensible solutions are therefore given by $\mu > 0$, which are black holes, and the "zero mass black hole" is obtained when $\mu = 0$. None of these black hole solutions preserves all the supersymmetries; nonetheless, the case $\mu = 0$ is half Bogomol'nyi-Prasad-Sommerfeld (BPS). These states are separated from the AdS vacuum by a mass gap so that they are not connected to it, and therefore they are "far away" in the space of solutions.

Let us therefore analyze the existence of zero modes of the Faddeev-Popov operator for these other asymptotically AdS solutions. For the BTZ black holes with mass parameter larger than zero, the equation for the zero modes can again be integrated, and the functions $f_{(j)}$ of our ansatz (21)–(23) now acquire the form

$$f_{(j)}(r) = (r - r_+)^{i\alpha}, \tag{29}$$

$\alpha$ being a real constant and $r_+ := l\sqrt{\mu}$. Therefore, the function $f_{(j)}$ in this case poses an essential singularity.

[4]The vector fields that belong to the $sl(2,R) \times sl(2,R)$ subalgebra of the asymptotic symmetries are special in the sense that they generate all the nontrivial charges associated to the known solutions of three-dimensional general relativity.

Consequently, they do not describe a smooth proper gauge transformation. For the massless case, the function $f_{(j)}$ takes the form

$$f_{(j)}(r) = c_1 + \frac{c_2}{r^a} + c_3 r^b, \qquad (30)$$

with $c_i$ and $a$, $b$ real constants, which blows up at the origin or in the asymptotic region, and so again does not give rise to a well-defined gauge transformation $\eta$. Therefore, within the family of vector fields considered here, it is not possible to generate a zero mode for the FP operator of the diffeomorphism invariance on BTZ black hole, not even in the massless case. This means that in three dimensions the Gribov problem for the gravitational field seems to be reversed, in the sense that there exists a horizon around the natural ground state such that *inside* this horizon the gauge fixing is ambiguous whereas *outside* it is not. This suggests that the massless black hole could be a suitable vacuum for the theory, even if it preserves fewer (super)symmetries than the $AdS_3$ space-time.

## V. FURTHER COMMENTS

*Gauge fixing in alternative theories.*—The analysis of the Gribov ambiguity in other diffeomorphism invariant theories in (2 + 1) dimensions is very similar to the one presented in the previous sections. However, there is an important difference: Unlike the condition in Eq. (16) which is common to all the diffeomorphism invariant theories, the condition in Eq. (17) is particular to general relativity. Therefore, when one searches for zero modes of the Faddeev-Popov operator in the de Donder gauge, the condition in Eq. (17) has to be dropped. This could be relevant, for instance, in the analysis of chiral gravity [20], where the de Donder gauge has been used. In general, this implies that, in the cases of different covariant gravity theories, the Gribov copies would be less restricted than in general relativity.

*An effective mechanism of (partial) supersymmetry breaking?*—The present results are quite peculiar when compared with the usual Gribov problem in $SU(N)$ Yang-Mills theory in four dimensions. In the present case, near the maximally supersymmetric vacuum ($AdS_3$), there are gauge-fixing ambiguities, while ''far enough'' from it, and within the family of diffeomorphisms considered here, there are no gauge-fixing problems. Therefore, the Gribov problem could be used as an effective mechanism of partial supersymmetry breaking (at least in $2 + 1$ dimensions) since, in order to properly define the Faddeev-Popov operator, one should consider small fluctuations around a ground state which preserves only one half of the supersymmetry. In other words, the appearance of Gribov copies would select a different ground state than the one which would be selected according to the criterion of the maximum number of supersymmetries. This issue is even more apparent if one considers the common point of view (see, for instance, [5,8–12]) to cut from the path integral the classical backgrounds affected by the presence of copies; then the maximally supersymmetric background ($AdS_3$) should be excluded. In this case, one could have at most a classical background (the zero mass BTZ black hole) preserving one half of the supersymmetries. This is very interesting since the problem to find a satisfactory mechanism of supersymmetry breaking has not yet been solved (see, for instance, [36–38]).

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