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Modified dispersion relations and the response of the rotating Unruh-DeWitt detector

Sashideep Gutti,* Shailesh Kulkarni,† and L. Sriramkumar‡

Harish-Chandra Research Institute, Chhatnag Road, Jhunsi, Allahabad 211 019, India

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We study the response of a rotating monopole detector that is coupled to a massless scalar field which is described by a nonlinear dispersion relation in flat spacetime. Since it does not seem to be possible to evaluate the response of the rotating detector analytically, we resort to numerical computations. Interestingly, unlike the case of the uniformly accelerated detector that has been considered recently, we find that defining the transition probability rate of the rotating detector poses no difficulties. Further, we show that the response of the rotating detector can be computed *exactly* (albeit, numerically) even when it is coupled to a field that is governed by a nonlinear dispersion relation. We also discuss the response of the rotating detector in the presence of a cylindrical boundary on which the scalar field is constrained to vanish. While superluminal dispersion relations hardly affect the standard results, we find that subluminal dispersion relations can lead to relatively large modifications.

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I. PLANCK SCALE MODIFICATIONS, THE UNRUH EFFECT, AND A VARIANT

Over the last decade or two, there has been intermittent efforts towards understanding the possible Planck scale corrections to a variety of nonperturbative, quantum field theoretic effects in flat as well as curved spacetimes. While the trans-Planckian nature of the modes that are encountered (when the initial conditions are imposed on a quantum field) in the context of black hole evaporation and inflationary cosmology have cornered most of the attention (see, for instance, Refs. [1,2]), a few other problems have been investigated as well (see, for example, Refs. [3–5]). Needless to say, in the absence of a workable theory of quantum gravity, it seems imperative to extend such phenomenological analyses to as large a set of physical situations as possible (in this context, see Ref. [6], and references therein).

The Unruh effect—viz. the thermal nature of the Minkowski vacuum when viewed by an observer in motion along a uniformly accelerated trajectory (see any of the following texts [7], or the recent review [8])—has certain similarities with Hawking radiation from black holes. Because of this reason, the Unruh effect and its variants provide another interesting arena to investigate the quantum gravitational effects [3,9–11].

However, due to the lack of a viable theory of quantum gravity, to study the Planck scale effects, one is forced to consider phenomenological models constructed by hand. These models often attempt to capture one or more features expected of the actual effective theory obtained by integrating out the gravitational degrees of freedom. An approach that has been extensively considered both in the context of black holes and inflationary cosmology are the models that are based on modified dispersion relations. These models effectively introduce a fundamental scale into the theory by breaking local Lorentz invariance [4,12,13]. Though it is true that there does not seem to exist any experimental or observational reason to believe that Lorentz invariance could be violated at high energies, theoretically, the hallmark of these models is that they often allow quantum field theories to be constructed and calculations to be carried out in a consistent and systematic fashion.

In this work, we shall adopt the approach due to the modified dispersion relations to analyze the Planck scale corrections to the response of the so-called Unruh-DeWitt detector [14,15], when it is set in motion on a rotating trajectory in flat spacetime [16–21]. In fact, the case of the uniformly accelerated detector has been studied recently [10]. The analysis indicates two possible difficulties. First, evaluating the response of the accelerated detector requires resorting to certain approximations to compute the Wightman function associated with the massless scalar field that is governed by a nonlinear dispersion relation. However, the validity of these approximations under which the Wightman function can be evaluated in a closed form does not seem to be completely clear. Second, defining the corresponding transition probability rate of the accelerating detector poses peculiar problems (for a detailed discussion on these two points, see Ref. [11]). As we shall see, the rotating trajectory turns out to be a special case wherein the second difficulty does not arise; the transition probability rate of the rotating detector can be defined in precisely the same fashion as in the standard case of the linear dispersion relation. Further, we find that the first of the two difficulties mentioned above can be easily overcome in the case of the rotating trajectory by adopting a slightly different method (which avoids having to initially evaluate the Wightman function) to calculate the response of the

*sashideep@hri.res.in
†skulkarni@hri.res.in
‡sriram@hri.res.in

detector. But, it is known that, even in the conventional case of the linear dispersion relation, it is not possible to calculate the response of the rotating detector analytically, and one has to resort numerical computations to arrive at the transition probability rate [16]. We shall illustrate that, the response of the rotating detector can be computed *exactly*, although, numerically, even when the field it is coupled to is described by a nonlinear dispersion relation. In addition, we shall also discuss the response of the rotating detector in the presence of a cylindrical boundary on which the scalar field is constrained to vanish. We find that, whereas superluminal dispersion relations scarcely affect the standard results, subluminal dispersion relations can lead to large modifications.

The plan of the paper is as follows. In the following section, after briefly sketching the concept of a detector, we shall outline as to how the response of the Unruh-DeWitt detector can be expressed in terms of the Fourier transform of the Wightman function with respect to the differential proper time in the frame of the detector. We shall then illustrate the transition probability rate of the rotating detector in the Minkowski vacuum by numerically computing the integral involved. In Sec. III, we shall consider the response of the rotating detector when it is coupled to a massless scalar field that is governed by a modified dispersion relation. However, since it seems difficult to express the Wightman function corresponding to a scalar field that is described by a nonlinear dispersion relation in a closed form, we shall adopt another method to evaluate the response of the detector. We shall first rederive the standard result for the rotating detector using the method, and then consider the case wherein the detector is coupled to a field that is described by a nonlinear dispersion relation. Importantly, we find that, though we have to resort to a numerical computation of a particular sum, the response of the detector can be evaluated exactly even when it is coupled to a scalar field governed by a modified dispersion relation. In Sec. IV, we shall discuss the response of a rotating detector when the scalar field is assumed to vanish on a cylindrical boundary that is located at a radius within the static limit in the rotating frame. Finally, in Sec. V, we shall conclude with a few comments on the results of our analysis.

In what follows, we shall set $\hbar = c = 1$, and shall work in the $(3+1)$-dimensional Minkowski spacetime with the metric signature of $(+,-,-,-)$. For convenience, we shall denote the set of four spacetime coordinates $x^\mu \equiv (t, \mathbf{x})$ as $\tilde{x}$. Also, an overbar shall refer to a suitable dimensionless quantity.

II. THE STANDARD RESPONSE OF THE ROTATING UNRUH-DEWITT DETECTOR

In this section, we shall rapidly summarize the essential aspects of the Unruh-DeWitt detector [14,15], and numerically compute the standard response of the rotating detector, i.e., when it is coupled to a massless scalar field governed by the conventional, linear dispersion relation [16].

By a detector, one has in mind a pointlike object that can be described by a classical worldline, but which nevertheless possesses internal energy levels. The detectors are essentially described by the interaction Lagrangian for the coupling between the internal degrees of freedom of the detector and the quantum field. In our discussion below, we shall consider the quantum field to be a massless scalar field, say, ϕ. The Unruh-DeWitt detector is coupled to the scalar field by a monopole interaction of the form: $(\mathcal{C}\mu(\tau)\phi[\tilde{x}(\tau)])$, where $\mathcal{C}$ is the coupling constant, $\mu(\tau)$ is the quantity that describes the monopole moment of the detector and $\tilde{x}(\tau)$ is the trajectory of the detector, with τ being the proper time in the detector's frame. Up to the first order in the perturbation theory, the amplitude of transition of the detector from its ground state $|E_0\rangle$ (with energy E_0) to an excited state $|E\rangle$ (with energy E) can easily be shown to be [7]

$$\mathcal{A}(\mathcal{E}) = \mathcal{M}\int_{-\infty}^{\infty} d\tau e^{i\mathcal{E}\tau}\langle\Psi|\hat{\phi}[\tilde{x}(\tau)]|0\rangle, \tag{1}$$

where $\mathcal{M} = [i\mathcal{C}\langle E|\hat{\mu}(0)|E_0\rangle]$, $\mathcal{E} = (E - E_0)$, $|\Psi\rangle$ is the final state of the quantum field, and we have assumed that the field was initially in the vacuum state $|0\rangle$. Note that the quantity $\mathcal{M}$ depends only on the internal structure of the detector, and not on its motion. Therefore, as is the usual practice, we shall drop the quantity hereafter. The transition probability to all possible final states $|\Psi\rangle$ of the quantum field is then given by [7]

$$\begin{aligned}\mathcal{P}(\mathcal{E}) &= \sum_{|\Psi\rangle}|\mathcal{A}(\mathcal{E})|^2 \\ &= \int_{-\infty}^{\infty} d\tau\int_{-\infty}^{\infty} d\tau' e^{-i\mathcal{E}(\tau-\tau')}G^+[\tilde{x}(\tau),\tilde{x}(\tau')],\end{aligned} \tag{2}$$

where $G^+[\tilde{x}(\tau),\tilde{x}(\tau')]$ denotes the Wightman function that is defined as

$$G^+[\tilde{x}(\tau),\tilde{x}(\tau')] = \langle 0|\hat{\phi}[\tilde{x}(\tau)]\hat{\phi}[\tilde{x}(\tau')]|0\rangle. \tag{3}$$

When the Wightman function is invariant under time translations in the frame of the detector—as it occurs, for instance, in the cases wherein the scalar field is described by the conventional dispersion relation and the detector is moving along the integral curves of timelike Killing vector fields [16–18,20]—we have

$$G^+[\tilde{x}(\tau),\tilde{x}(\tau')] = G^+(\tau-\tau'). \tag{4}$$

In such situations, the transition probability of the detector simplifies to

$$\mathcal{P}(\mathcal{E}) = \lim_{T\to\infty}\int_{-T}^{T}\frac{dv}{2}\int_{-\infty}^{\infty} du e^{-i\mathcal{E}u}G^+(u), \tag{5}$$

where

$$u = (\tau - \tau') \quad \text{and} \quad v = (\tau + \tau'). \tag{6}$$

The above expression then allows us to define the transition probability *rate* of the detector to be [7]

$$\mathcal{R}(\mathcal{E}) = \lim_{T\to\infty}[\mathcal{P}(\mathcal{E})/T] = \int_{-\infty}^{\infty} du e^{-i\mathcal{E}u} G^{+}(u). \tag{7}$$

When the massless scalar field is described by the standard, linear dispersion relation, the Wightman function $G^{+}(\tilde{x}, \tilde{x}')$ in the Minkowski vacuum is given by [7]

$$G^{+}(\tilde{x}, \tilde{x}') = -\left(\frac{1}{4\pi^2}\right)\left(\frac{1}{(\Delta t - i\epsilon)^2 - \Delta \mathbf{x}^2}\right), \tag{8}$$

where $\Delta t = (t - t')$, $\Delta \mathbf{x} = (\mathbf{x} - \mathbf{x}')$, and $\epsilon \to 0^{+}$. Given a trajectory $\tilde{x}(\tau)$, the response of the detector is obtained by substituting the trajectory in this Wightman function and evaluating the transition probability rate (7).

The timelike Killing vector field ξ^{μ} that generates the rotational motion is given by [16–18,20]

$$\xi^{\mu} = (\gamma, -\lambda y, \lambda x, 0). \tag{9}$$

The integral curve corresponding to this Killing vector can be expressed in terms of the proper time, say, τ, as follows:

$$\tilde{x}(\tau) = [(\gamma\tau), \sigma\cos(\gamma\Omega\tau), \sigma\sin(\gamma\Omega\tau), 0], \tag{10}$$

where we have set $\lambda = (\gamma\Omega)$. The constants σ and Ω denote the radius of the circular path along which the detector is moving and the angular velocity of the detector, respectively. The quantity $\gamma = [1 - (\sigma\Omega)^2]^{-1/2}$ is the Lorentz factor that relates the Minkowski time to the proper time in the frame of the detector. For such a rotational motion, we find that

$$\Delta t^2 = \gamma^2(\tau - \tau')^2 = \gamma^2 u^2, \tag{11}$$

$$\Delta \mathbf{x}^2 = 4\sigma^2 \sin^2[\gamma\Omega(\tau - \tau')/2] = 4\sigma^2\sin^2(\gamma\Omega u/2). \tag{12}$$

Upon substituting these quantities in the expression (8), the Wightman function along the rotating trajectory can be obtained to be

$$G^{+}[\tilde{x}(\tau), \tilde{x}(\tau')] = G^{+}(u) = -\left(\frac{1}{4\pi^2\sigma^2}\right) \times \left(\frac{1}{(\gamma/\sigma)^2(u - i\epsilon)^2 - 4\sin^2(\gamma\Omega u/2)}\right). \tag{13}$$

However, unfortunately, it does not seem to be possible to evaluate the corresponding transition probability rate $\mathcal{R}(\mathcal{E})$ analytically. We have arrived at the response of the rotating detector by substituting the Wightman function (13) in the expression (7), and numerically computing the integral involved. If we define the dimensionless energy to be $\bar{\mathcal{E}} = (\mathcal{E}/\gamma\Omega)$, we find that the dimensionless transition probability rate $\bar{\mathcal{R}}(\bar{\mathcal{E}}) \equiv [\sigma\mathcal{R}(\bar{\mathcal{E}})]$ of the detector depends only on the dimensionless quantity $(\sigma\Omega)$ that describes the linear velocity of the detector. In Fig. 1, we have plotted the transition probability rate of the detector for three different values of the quantity $(\sigma\Omega)$ [16]. We should mention here that, in order to check the accuracy of the numerical procedure that we have used to evaluate the integral (7) for the rotating trajectory, we have compared the results from the numerical code with the analytical ones that are available for the case of the uniformly accelerated detector [14–16,18,20]. This comparison clearly indicates that the numerical procedure we have adopted to evaluate the integral (7) is quite accurate (for details, see the Appendix).

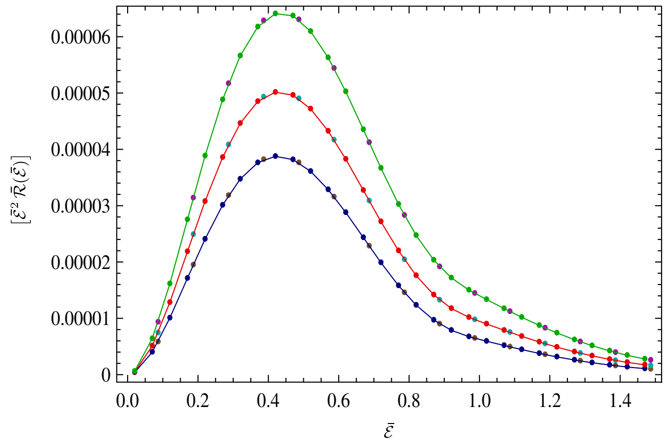

FIG. 1 (color online). The transition probability rate of the rotating Unruh-DeWitt detector that is coupled to a massless scalar field governed by the standard dispersion relation. The blue, the red and the green dots denote the numerical results obtained through the computation of the integral (7) along the rotating trajectory, and they correspond to the following three choices of the quantity $(\sigma\Omega) = 0.325$, 0.350 and 0.375, respectively. The solid curves simply join the dots to guide the eye. The intervening dots of an alternate color that appear on these curves denote the corresponding numerical results that have been arrived at by another method as described in the following section [they actually correspond to the sum (28)]. It is evident that the results from the two methods match very well.

III. RESPONSE OF THE ROTATING DETECTOR COUPLED TO A SCALAR FIELD GOVERNED BY A MODIFIED DISPERSION RELATION

In this section, we shall calculate the response of the rotating detector when it is coupled to a massless, scalar field that is governed by a modified dispersion relation of the following form:

$$\omega = k[1 + \alpha(k/k_{\mathrm{P}})^2 + \beta(k/k_{\mathrm{P}})^4]^{1/2}, \tag{14}$$

where ω is the frequency corresponding to the mode $\mathbf{k}$, $k = |\mathbf{k}|$, k_{P} denotes the fundamental scale beyond which the modifications to the linear dispersion relation become important, while α and β are dimensionless constants whose magnitudes are of order unity. Note that the above dispersion relation is superluminal or subluminal depending upon whether the constants α and β are positive or negative. Evidently, if we can evaluate Wightman function associated with the scalar field described by the nonlinear dispersion relation (14), we may then be able to evaluate the corresponding transition probability rate of the detector as we did the previous section. However, unlike the standard case, it turns out to be difficult to compute the Wightman function of such a scalar field exactly. As a result, one has to either take recourse to some approximations to arrive at an analytic expression for the Wightman function or adopt another method to evaluate the response of the detector. Before going on to calculate the response of the rotating detector through another method, we shall briefly discuss the approximations that are typically made in the evaluation of the Wightman function for the quadratic dispersion relation (i.e. the case wherein $\beta = 0$) [10], and the difficulties that can be encountered in defining the corresponding transition probability rate [11].

The equation of motion of the scalar field ϕ that is described by the dispersion relation (14) is given by

$$\Box\phi + \left(\frac{\alpha}{k_{\rm P}^2}\right)\nabla^2(\nabla^2\phi) - \left(\frac{\beta}{k_{\rm P}^4}\right)\nabla^2[\nabla^2(\nabla^2\phi)] = 0, \quad (15)$$

where $\Box$ is the D'Alembertian corresponding to the four dimensional Minkowski spacetime, while ∇^2 is the three dimensional, spatial Laplacian. Whereas the first term in the above equation is the standard one, the remaining terms arise due to the nonlinearity in the dispersion relation. Such terms can be generated by adding suitable terms to the original action describing the scalar field [4,12]. These additional terms preserve rotational invariance, but break Lorentz invariance and, in fact, this property is common to all the theories that are described by a nonlinear dispersion relation. The normal modes of the scalar field in flat spacetime remain plane waves as in the standard case (with the frequency and the wavenumber related by the modified dispersion relation), and the quantization of the field can be carried out in the same fashion. In the Minkowski vacuum, the Wightman function for any such field can be expressed as (see, for example, Refs. [4])

$$G_{\rm M}^{+}(\tilde{x}, \tilde{x}') = \int \frac{d^3\mathbf{k}}{(2\pi)^3(2\omega)} e^{-i\omega(t-t')} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')} \quad (16)$$

with ω being related to $k = |\mathbf{k}|$ by the given nonlinear dispersion relation.

As we have said before, it does not seem to be possible to calculate the above integral exactly for a modified dispersion relation of the form (14). (In fact, we are not aware of any nonlinear dispersion relation for which the Wightman function can be evaluated exactly in a closed form.) To arrive at an analytic expression for the Wightman function, two approximations are resorted to. One first expands the quantity ω in the denominator and in the exponential, say, up to the first order in the inverse power of $k_{\rm P}^2$. Then, the exponential term containing the correction is expanded up to the same order in $k_{\rm P}^{-2}$ as well. Evidently, such an expansion will be valid only for $k < k_{\rm P}$. So, a cutoff at large momentum (at $k \simeq k_{\rm P}$) is further assumed in order to carry out the integral over k. Under these conditions, for the case wherein $\beta = 0$, it can be shown that the resulting Wightman function consists of two terms, with the leading term being the standard, Lorentz invariant, Wightman function (8), whereas the correction is given by [10,11]

$$G_{\rm C}^{+}(\tilde{x}, \tilde{x}') = -\left(\frac{\alpha}{4\pi^2 k_{\rm P}^2}\right)\left(\frac{15\Delta t^4 + 10\Delta t^2\Delta\mathbf{x}^2 - \Delta\mathbf{x}^4}{[(\Delta t - i\epsilon)^2 - \Delta\mathbf{x}^2]^4}\right). \quad (17)$$

It is important to note that, as opposed to the leading term, the above correction $G_{\rm C}^{+}(\tilde{x}, \tilde{x}')$ is *not* Lorentz invariant. Because of the lack of Lorentz invariance, unlike the original Wightman function, the quantity $G_{\rm C}^{+}[\tilde{x}(\tau), \tilde{x}(\tau')]$ may not be invariant under time translations in the frame of the detector, even if it is moving along the integral curves of timelike Killing vector fields, such as, for instance, the popular, uniformly accelerated trajectory [10,11]. In other words, generically, $G_{\rm C}^{+}[\tilde{x}(\tau), \tilde{x}(\tau')]$ will be a function of u *as well as* v along the noninertial trajectory under consideration. Since the transition probability of the detector is linear in the Wightman function, clearly, the response of the detector will be the sum of the standard response, and a term that is proportional to $k_{\rm P}^{-2}$, given by

$$\mathcal{P}_{\rm C}(\mathcal{E}) = \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau' e^{-i\mathcal{E}(\tau-\tau')} G_{\rm C}^{+}[\tilde{x}(\tau), \tilde{x}(\tau')]. \quad (18)$$

However, because of the dependence of the term $G_{\rm C}^{+}[\tilde{x}(\tau), \tilde{x}(\tau')]$ on v as well, in general, it turns out to be more involved to define the corresponding transition probability rate (for a detailed discussion in this context, see Ref. [11]).

Recall that, along the rotating trajectory, the quantities Δt and $\Delta\mathbf{x}$ depend only on u [cf. Eqs. (11) and (12)]. Therefore, the rotating trajectory turns out to be a special case where, as in the original Wightman function, the correction term $G_{\rm C}^{+}[\tilde{x}(\tau), \tilde{x}(\tau')]$ proves to be a function only of u, and not of v. This feature essentially arises due to the fact that the term $G_{\rm C}^{+}(\tilde{x}, \tilde{x}')$, though it is not Lorentz invariant, preserves rotational invariance. Hence, we can define the corresponding transition probability rate just as in the standard case. However, some concerns have been raised about the validity of the approximations that have been made to arrive at the above form for $G_{\rm C}^{+}(\tilde{x}, \tilde{x}')$ [11]. Therefore, we shall not use the term (17) to evaluate the corrections to the response of the rotating detector, as has been done recently in the uniformly accelerated case [10]. Instead, in what follows, we shall compute the Planck scale modifications to the response of the rotating detector by adopting a slightly different method to compute the transition probability rate. As we shall illustrate, the method allows us to evaluate the response of the detector *exactly*, even for the case of a scalar field that is described by a modified dispersion relation.

A. Another method to evaluate the standard response of the rotating detector

In the previous section, we had obtained the standard response of the rotating detector by evaluating the Fourier transform of the Wightman function with respect to the differential proper time in the frame of the detector. Here, we shall firstly rederive the result in a slightly different fashion, a method which, in retrospect, would seem evident in the rotating case. It essentially involves expressing the Wightman function as a sum over the normal modes, and first evaluating the integral over the differential proper time u before computing the sum. As we shall see, the method can be extended in a straightforward manner to the case wherein the scalar field is described by a modified dispersion relation.

To start with, we shall work in the cylindrical polar coordinates, say, (t, ρ, θ, z), instead of the Cartesian coordinates, as they turn out to be more convenient. In terms of the cylindrical coordinates, the trajectory (10) of the rotating detector can be written in terms of the proper time τ as follows:

$$\tilde{x}(\tau) = [(\gamma\tau), \sigma, (\gamma\Omega\tau), 0]. \tag{19}$$

Using certain well established properties of the Bessel functions, it can be easily shown that, along the trajectory of the rotating detector, the standard Minkowski Wightman function (8) can be expressed as

$$G^+[\tilde{x}(\tau), \tilde{x}(\tau')] = G^+(u) = \sum_{m=-\infty}^{\infty} \int_0^\infty \frac{dqq}{(2\pi)^2} \int_{-\infty}^{\infty} \frac{dk_z}{(2\omega)} J_m^2(q\sigma) \times e^{-i\gamma(\omega - m\Omega)u}, \tag{20}$$

where $J_m(q\sigma)$ denote the Bessel functions of order m, with ω given by

$$\omega = (q^2 + k_z^2)^{1/2}. \tag{21}$$

The corresponding transition probability rate of the rotating detector is then given by

$$\mathcal{R}(\mathcal{E}) = \sum_{m=-\infty}^{\infty} \int_0^\infty \frac{dqq}{(2\pi)} \int_{-\infty}^{\infty} \frac{dk_z}{(2\omega)} J_m^2(q\sigma) \times \delta^{(1)}[\mathcal{E} + \gamma(\omega - m\Omega)]. \tag{22}$$

Since, $\mathcal{E} > 0$, $\omega \geq 0$ (as is appropriate for positive frequency modes), and Ω too is a positive definite quantity by assumption, the delta function in the above expression will be nonzero only when $m \geq \bar{\mathcal{E}}$, where $\bar{\mathcal{E}} = [\mathcal{E}/(\gamma\Omega)]$ is the dimensionless energy. Hence, the response of the detector can be expressed as

$$\mathcal{R}(\bar{\mathcal{E}}) = \sum_{m\geq\bar{\mathcal{E}}}^{\infty} \int_0^\infty \frac{dqq}{(2\pi)} \int_{-\infty}^{\infty} \frac{dk_z}{(2\omega)} J_m^2(q\sigma) \times \left[\frac{\delta^{(1)}(k_z - \kappa_z)}{\gamma|(d\omega/dk_z)|_{\kappa_z}}\right], \tag{23}$$

where κ_z are the two roots of k_z from the following equation:

$$\omega = (m - \bar{\mathcal{E}})\Omega. \tag{24}$$

The roots are given by

$$\kappa_z = \pm(\delta^2 - q^2)^{1/2}, \tag{25}$$

where, for convenience, we have set

$$\delta = (\bar{\delta}\Omega) = (m - \bar{\mathcal{E}})\Omega. \tag{26}$$

As both the positive and negative roots of κ_z contribute equally, we obtain the dimensionless transition probability rate of the rotating detector to be

$$\bar{\mathcal{R}}(\bar{\mathcal{E}}) \equiv \sigma\mathcal{R}(\bar{\mathcal{E}}) = \left(\frac{\sigma}{2\pi\gamma}\right) \sum_{m\geq\bar{\mathcal{E}}}^{\infty} \int_0^\delta dqq \left(\frac{J_m^2(q\sigma)}{(\delta^2 - q^2)^{1/2}}\right), \tag{27}$$

where we have set the upper limit on q to be δ since κ_z is a real quantity [cf. Eq. (25)]. We find that the integral over q can be expressed in terms of the hypergeometric functions (see, for instance, Ref. [22]). Therefore, the transition probability rate of the rotating detector can be written as

$$\bar{\mathcal{R}}(\bar{\mathcal{E}}) = \left(\frac{1}{2\pi\gamma}\right) \sum_{m\geq\bar{\mathcal{E}}}^{\infty} \left(\frac{(\sigma\Omega\bar{\delta})^{(2m+1)}}{\Gamma(2m+2)}\right) {}_1F_2\Big[[m + (1/2)]; [m + (3/2)], (2m + 1); -(\sigma\Omega\bar{\delta})^2\Big], \tag{28}$$

where ${}_1F_2(a; b, c; x)$ denotes the hypergeometric function, while $\Gamma(x)$ is the usual Gamma function. It does not seem to be possible to arrive at a closed form expression for this sum, but the sum converges very quickly, and hence proves to be easy to evaluate numerically. In Fig. 1, we have plotted the numerical results for the above sum for the same values of the linear velocity $(\sigma\Omega)$ that we had plotted the results obtained from Fourier transforming the Wightman function (13) along the rotating trajectory. It is clear from the figure that the results from the two different methods match each other rather well.

B. The case of a scalar field described by a quadratic dispersion relation

When the scalar field is governed by a modified dispersion relation, using the expression (16) for the corresponding Wightman function, it is straightforward to show that, along the rotating trajectory, the function can be expressed exactly as in Eq. (20), with the frequency ω being related to the wavenumbers q and k_z by the nonlinear dispersion relation. Clearly, in such a case, the corresponding transition probability rate of the detector will again be given by Eq. (23) with ω suitably defined. In fact, we should stress here that the result is actually valid for *any* nonlinear dispersion relation.

Let us now consider the response of the rotating detector for the dispersion relation (14), but *with* β *set to zero*. In such a case, ω is related to the wave numbers q and k_z as follows:

$$\omega = (q^2 + k_z^2)^{1/2}\left[1 + \left(\frac{\alpha}{k_{\rm P}^2}\right)(q^2 + k_z^2)\right]^{1/2}. \qquad (29)$$

Also, we find that the roots κ_z [from Eq. (24)] are given by

$$\kappa_z^2 = \pm\left(\frac{k_{\rm P}^2}{2\alpha}\right)\left[1 + \left(\frac{4\alpha\delta^2}{k_{\rm P}^2}\right)\right]^{1/2} - \left(\frac{k_{\rm P}^2}{2\alpha}\right) - q^2, \qquad (30)$$

with δ defined as in Eq. (26). Note that κ_z^2 has to be positive definite, since κ_z is a real quantity.

Let us first consider the case when α is positive. When, say, $\alpha = 1$, the two roots that contribute to the delta function in Eq. (23) can be written as

$$\kappa_z = \pm(\delta_+^2 - q^2)^{1/2}, \qquad (31)$$

where δ_+^2 is given by the expression

$$\delta_+^2 = \left(\frac{k_{\rm P}^2}{2}\right)\left(\left[1 + \left(\frac{4\delta^2}{k_{\rm P}^2}\right)\right]^{1/2} - 1\right) = \left(\frac{\bar{\delta}_+^2}{\sigma^2}\right) = \left(\frac{\bar{k}_{\rm P}^2}{2\sigma^2}\right)\left(\left[1 + \left(\frac{4(\sigma\Omega\bar{\delta})^2}{\bar{k}_{\rm P}^2}\right)\right]^{1/2} - 1\right), \qquad (32)$$

$\bar{k}_{\rm P} = (\sigma k_{\rm P})$ denotes the dimensionless fundamental scale, and the subscript in δ_+ refers to the fact that we are considering a superluminal dispersion relation. Further, as κ_z is real, we require that $q \leq \delta_+$. As in the standard case, the positive and negative roots of κ_z above contribute equally. Therefore, the response of the rotating detector is given by

$$\bar{\mathcal{R}}_{\rm M}(\bar{\mathcal{E}}) = \sigma\mathcal{R}_{\rm M}(\bar{\mathcal{E}}) = \left(\frac{\sigma}{2\pi\gamma}\right)\sum_{m\geq\bar{\mathcal{E}}}^{\infty}\left[1 + \left(\frac{2\delta_+^2}{k_{\rm P}^2}\right)\right]^{-1} \times \int_0^{\delta_+} dq\, q\left(\frac{J_m^2(q\sigma)}{(\delta_+^2 - q^2)^{1/2}}\right), \qquad (33)$$

and the integral over q can be carried out as earlier to arrive at the result

$$\bar{\mathcal{R}}_{\rm M}(\bar{\mathcal{E}}) = \left(\frac{1}{2\pi\gamma}\right)\sum_{m\geq\bar{\mathcal{E}}}^{\infty}\left(\frac{\bar{\delta}_+^{(2m+1)}}{\Gamma(2m+2)}\right) \times\left[1 + \left(\frac{2\bar{\delta}_+^2}{\bar{k}_{\rm P}^2}\right)\right]^{-1} {}_1F_2[[m + (1/2)]; [m + (3/2)], (2m + 1); -\bar{\delta}_+^2]. \qquad (34)$$

It should be stressed that this expression for the transition probability rate is exact.

Let us now turn to understanding the behavior of the above transition probability rate for large $\bar{k}_{\rm P}$. It is clear that, as $\bar{k}_{\rm P} \to \infty$, $\bar{\delta}_+ \to (\sigma\Omega\bar{\delta})$ and, hence, the transition probability rate (34) reduces to the expression that we had arrived at earlier for the standard dispersion relation [viz. Eq. (28)], as required. Let us now expand the transition probability rate (34) retaining terms up to $\mathcal{O}[(\delta/k_{\rm P})^2]$. Note that, in such a case, δ_+ reduces to

$$\delta_+ \simeq \delta\left[1 - \left(\frac{\delta^2}{2k_{\rm P}^2}\right)\right], \qquad (35)$$

so that we have

$$\delta_+^{(2m+1)} \simeq \delta^{(2m+1)} - (2m + 1)\left(\frac{\delta^{(2m+3)}}{2k_{\rm P}^2}\right) \quad \text{and} \quad \left[1 + \left(\frac{2\delta_+^2}{k_{\rm P}^2}\right)\right]^{-1} \simeq 1 - \left(\frac{2\delta^2}{k_{\rm P}^2}\right). \qquad (36)$$

Moreover, in the limit of our interest, the hypergeometric function in Eq. (34) can be written as

$${}_1F_2[[m + (1/2)]; [m + (3/2)], (2m + 1); -\bar{\delta}_+^2] \simeq {}_1F_2\left[[m + (1/2)]; [m + (3/2)], (2m + 1); -(\sigma\Omega\bar{\delta})^2\right] + \left(\frac{(\sigma\Omega\bar{\delta})^2}{\bar{k}_{\rm P}^2}\right)\left(\frac{[m + (1/2)](\sigma\Omega\bar{\delta})^2}{[m + (3/2)](2m + 1)}\right) \times {}_1F_2\left[[m + (3/2)]; [m + (5/2)], (2m + 2); -(\sigma\Omega\bar{\delta})^2\right]. \qquad (37)$$

Upon using the above expansions, we obtain the response of the detector at $\mathcal{O}[(\delta/k_{\rm P})^2]$ to be

$$\begin{aligned}\bar{\mathcal{R}}_{\rm M}(\bar{\mathcal{E}}) \simeq &\left(\frac{1}{2\pi\gamma}\right)\sum_{m\geq\bar{\mathcal{E}}}^{\infty}\left(\frac{(\sigma\Omega\bar{\delta})^{(2m+1)}}{\Gamma(2m+2)}\right){}_1F_2[[m+(1/2)];[m+(3/2)],(2m+1);-(\sigma\Omega\bar{\delta})^2]\\ &-\left(\frac{1}{2\pi\gamma}\right)\left(\frac{(\sigma\Omega\bar{\delta})^2}{\bar{k}_{\rm P}^2}\right)\sum_{m\geq\bar{\mathcal{E}}}^{\infty}\left(\frac{[m+(5/2)](\sigma\Omega\bar{\delta})^{(2m+1)}}{\Gamma(2m+2)}\right){}_1F_2[[m+(1/2)];[m+(3/2)],(2m+1);-(\sigma\Omega\bar{\delta})^2]\\ &+\left(\frac{1}{2\pi\gamma}\right)\left(\frac{(\sigma\Omega\bar{\delta})^2}{\bar{k}_{\rm P}^2}\right)\sum_{m\geq\bar{\mathcal{E}}}^{\infty}\left(\frac{[m+(1/2)](\sigma\Omega\bar{\delta})^{(2m+3)}}{[m+(3/2)](2m+1)\Gamma(2m+2)}\right)\\ &\times{}_1F_2[[m+(3/2)];[m+(5/2)],(2m+2);-(\sigma\Omega\bar{\delta})^2]. \end{aligned} \tag{38}$$

Evidently, the first term in this expression corresponds to the conventional transition probability rate [cf. Eq. (28)], while the other two terms represent the leading corrections to the standard result.

Let us now turn to considering the subluminal dispersion relation. When α is negative, say, $\alpha = -1$ (and β is again zero), the roots κ_z are given by

$$\kappa_z = \pm(\delta_-^2 - q^2)^{1/2} \tag{39}$$

with δ_-^2 defined as

$$(\delta_-^{\pm})^2 = \left(\frac{k_{\rm P}^2}{2}\right)\left(1\pm\left[1-\left(\frac{4\delta^2}{k_{\rm P}^2}\right)\right]^{1/2}\right) = \left(\frac{(\bar{\delta}_-^{\pm})^2}{\sigma^2}\right) = \left(\frac{\bar{k}_{\rm P}^2}{2\sigma^2}\right)\left(1\pm\left[1-\left(\frac{4(\sigma\Omega\bar{\delta})^2}{\bar{k}_{\rm P}^2}\right)\right]^{1/2}\right), \tag{40}$$

where the minus sign in the subscript represents that it corresponds to the subluminal case (i.e. when α is negative), while the superscripts denote the two different possibilities of δ_-. As in the superluminal case (i.e. when $\alpha = 1$), we require $q \leq \delta_-^{\pm}$, if κ_z is to remain real. Moreover, note that, unlike the superluminal case, there also arises an upper limit on the sum over m. We require that $\delta \leq (k_{\rm P}/2)$, in order to ensure that $\delta_-^{\pm}$ is real. This corresponds to $m \leq [\bar{\mathcal{E}} + (\bar{k}_{\rm P}/2\sigma\Omega)]$. Therefore, for the subluminal dispersion relation, we find that we can write the response of the rotating detector as follows:

$$\begin{aligned}\bar{\mathcal{R}}_{\rm M}(\bar{\mathcal{E}}) = &\left(\frac{1}{2\pi\gamma}\right)\sum_{m\geq\bar{\mathcal{E}}}^{\bar{\mathcal{E}}+(\bar{k}_{\rm P}/2\sigma\Omega)}\left(\frac{(\bar{\delta}_-^-)^{(2m+1)}}{\Gamma(2m+2)}\right)\left[\left|1-\left(\frac{2(\bar{\delta}_-^-)^2}{\bar{k}_{\rm P}^2}\right)\right|\right]^{-1}\times{}_1F_2[[m+(1/2)];[m+(3/2)],(2m+1);-(\bar{\delta}_-^-)^2]\\ &+\left(\frac{1}{2\pi\gamma}\right)\sum_{m\geq\bar{\mathcal{E}}}^{\bar{\mathcal{E}}+(k_{\rm P}/2\sigma\Omega)}\left(\frac{(\bar{\delta}_-^+)^{(2m+1)}}{\Gamma(2m+2)}\right)\left[\left|1-\left(\frac{2(\bar{\delta}_-^+)^2}{\bar{k}_{\rm P}^2}\right)\right|\right]^{-1}\times{}_1F_2[[m+(1/2)];[m+(3/2)],(2m+1);-(\bar{\delta}_-^+)^2]. \end{aligned} \tag{41}$$

The origin of the upper limit on m as well as the second term in the above expression for the response of the rotating detector can be easily understood. In the case of the superluminal dispersion relation, ω is a monotonically increasing function of q and k_z. So, there exist only two real roots of k_z corresponding to a given ω. Also, ω^2 remains positive definite for all the modes. But, in the subluminal case, after a rise, ω begins to decrease for sufficiently large values of q and k_z. In fact, ω^2 even turns negative at a suitably large value [4]. It is this feature of the subluminal dispersion relation which leads to the upper limit on m. (The upper limit ensures that we avoid complex frequencies. Such a cut-off can be achieved if we assume that, say, the detector is not coupled to modes with m beyond a certain value, when the frequency turns complex.) The additional two roots of k_z that contribute to the detector response in the subluminal case arise as a result of the decreasing ω at large q and k_z. The second term in the above transition probability rate of the rotating detector corresponds to the contributions from these two extra roots.

When we plot the result (34) for the response of the rotating detector when it is coupled to a field that is governed by a superluminal dispersion relation, we find that it does not differ from the standard result (as plotted in Fig. 1) even for an unnaturally small value of $\bar{k}_{\rm P}$ such that $(\bar{k}_{\rm P}/\bar{\mathcal{E}}) \simeq 10$. In other words, superluminal dispersion relations do not alter the conventional result to any extent. It is worthwhile pointing out that similar conclusions have been arrived at earlier in the context of black holes as well as inflationary cosmology. In these contexts, it has been shown that Hawking radiation and the inflationary perturbation spectra remain unaffected due to superluminal modifications to the conventional, linear, dispersion relation [1,2]. In Fig. 2, we have plotted the transition probability rate (41) of the rotating Unruh-DeWitt detector corresponding to the subluminal dispersion relation that we have considered. Again, we have plotted the result for a rather small value of $\bar{k}_{\rm P} = 50$. It is clear from the figure that the subluminal dispersion relation can lead to substantial modifications to the standard result. We believe that the modifications from the standard result will be considerably smaller (than exhibited in the figure) for much larger and more realistic values of $\bar{k}_{\mathrm{P}}$ such that, say, $(\bar{k}_{\mathrm{P}}/\bar{\mathcal{E}}) > 10^{10}$.

C. The case of the superluminal, quartic dispersion relation

In the previous subsection, we had considered the response of the rotating detector when the scalar field is governed by a quadratic dispersion relation (i.e. as described by Eq. (14), *but with* β *set to zero*). As we have emphasized earlier, the result (23) holds for *any* dispersion relation. In this subsection, we shall briefly discuss the situation involving a higher order dispersion relation, viz. the case when $\beta \neq 0$. We have already seen that, even in the quadratic case, a subluminal dispersion relation leads to substantial deviations from the standard result. Therefore, we shall restrict ourselves to the superluminal, quartic, dispersion relation, and examine if it leads to any significant modifications, in contrast to the quadratic case.

When $\alpha = \beta = 1$, in the cylindrical coordinates, the quartic dispersion relation (14) is given by

$$\omega = (q^2 + k_z^2)^{1/2}\left[1 + \left(\frac{1}{k_{\mathrm{P}}^2}\right)(q^2 + k_z^2) + \left(\frac{1}{k_{\mathrm{P}}^4}\right)(q^2 + k_z^2)^2\right]^{1/2}. \tag{42}$$

In this case, Eq. (24) proves to be cubic in k_z^2. We find that it admits one positive, and two imaginary roots for k_z^2. The positive root leads to

$$\kappa_z = \pm(\delta_{\mathrm{c}}^2 - q^2)^{1/2}, \tag{43}$$

where $\delta_{\mathrm{c}} = (\bar{\delta}_{\mathrm{c}}/\sigma)$, with $\bar{\delta}_{\mathrm{c}}^2$ given by

$$\bar{\delta}_{\mathrm{c}}^2 = -\left(\frac{\bar{k}_{\mathrm{P}}^2}{3}\right) + \left(\frac{(2^{4/3}\bar{k}_{\mathrm{P}}^2/3)}{(-7 - 27(\sigma\Omega\bar{\delta}/\bar{k}_{\mathrm{P}})^2 + 3^{3/2}[3 + 14(\sigma\Omega\bar{\delta}/\bar{k}_{\mathrm{P}})^2 + 27(\sigma\Omega\bar{\delta}/\bar{k}_{\mathrm{P}})^4]^{1/2})^{1/3}}\right) - \left(\frac{\bar{k}_{\mathrm{P}}^2}{3\times 2^{1/3}}\right)(-7 - 27(\sigma\Omega\bar{\delta}/\bar{k}_{\mathrm{P}})^2 + 3^{3/2}[3 + 14(\sigma\Omega\bar{\delta}/\bar{k}_{\mathrm{P}})^2 + 27(\sigma\Omega\bar{\delta}/\bar{k}_{\mathrm{P}})^4]^{1/2})^{1/3}. \tag{44}$$

Also, for κ_z to be real, we require that $q \leq \delta_{\mathrm{c}}$. Therefore, the dimensionless transition probability rate of the detector is given by

$$\bar{\mathcal{R}}_{\mathrm{M}}(\bar{\mathcal{E}}) = \left(\frac{\sigma}{2\pi\gamma}\right)\sum_{m\geq\bar{\mathcal{E}}}^{\infty}\left[1 + \left(\frac{2\delta_{\mathrm{c}}^2}{k_{\mathrm{P}}^2}\right) + \left(\frac{3\delta_{\mathrm{c}}^4}{k_{\mathrm{P}}^4}\right)\right]^{-1} \times \int_0^{\delta_{\mathrm{c}}} dq q\left(\frac{J_m^2(q\sigma)}{(\delta_{\mathrm{c}}^2 - q^2)^{1/2}}\right) \tag{45}$$

and, upon performing the integral over q, we arrive at the result

$$\bar{\mathcal{R}}_{\mathrm{M}}(\bar{\mathcal{E}}) = \left(\frac{1}{2\pi\gamma}\right)\sum_{m\geq\bar{\mathcal{E}}}^{\infty}\left(\frac{(\bar{\delta}_{\mathrm{c}})^{(2m+1)}}{\Gamma(2m+2)}\right) \times\left[1 + \left(\frac{2\bar{\delta}_{\mathrm{c}}^2}{\bar{k}_{\mathrm{P}}^2}\right) + \left(\frac{3\bar{\delta}_{\mathrm{c}}^4}{\bar{k}_{\mathrm{P}}^4}\right)\right]^{-1} {}_1F_2[[m + (1/2)]; [m + (3/2)], (2m + 1); -(\bar{\delta}_{\mathrm{c}})^2]. \tag{46}$$

Note that, as $\bar{k}_{\mathrm{P}} \to \infty$, $\bar{\delta}_{\mathrm{c}} \to (\sigma\Omega\bar{\delta})$, and the above expression reduces to the standard result (28), as expected. Further, as in the case of the quadratic, superluminal dispersion relation that we had considered in the last subsection, we find that the above result hardly differs from the standard result even for a rather small value of $\bar{k}_{\mathrm{P}}$.

IV. THE CASE OF THE ROTATING DETECTOR IN THE PRESENCE OF A BOUNDARY

We shall now consider the response of the rotating detector in the presence of an additional boundary condition that is imposed on the scalar field on a cylindrical surface in flat spacetime. Because of symmetry of the problem, in this case too, it proves to be more convenient to work in the cylindrical coordinates, as we did in the last section.

Consider the timelike Killing vector associated with an observer who is rotating with an angular velocity Ω in flat spacetime. Notice that the Killing vector becomes spacelike for radii greater than $\rho_{\mathrm{SL}} = (1/\Omega)$. As a result, it was argued that one has to impose a boundary condition on the quantum field at a radius $\rho < \rho_{\mathrm{SL}}$ when evaluating the response of the rotating detector [23]. Curiously, in the presence of such a boundary, it was found that the rotating detector coupled to the standard scalar field ceases to respond. It is then interesting to examine whether this result holds true even when we assume that the scalar field is governed by a modified dispersion relation.

In the cylindrical coordinates, along the rotating trajectory (19), the Wightman function corresponding to a scalar field that is assumed to vanish at, say, $\rho = a(<\rho_{\mathrm{SL}})$, can be expressed as a sum over the normal modes of the field as follows [23]:

$$G_{\mathrm{M}}^{+}[\tilde{x}(\tau), \tilde{x}(\tau')] = G_{\mathrm{M}}^{+}(u) = \sum_{m=-\infty}^{\infty}\sum_{n=1}^{\infty}\int_{-\infty}^{\infty}\frac{dk_z}{(2\pi)^2(2\omega)} \times[\mathcal{N}J_m(\xi_{mn}\sigma/a)]^2 e^{-i\gamma(\omega - m\Omega)u}, \tag{47}$$

where ξ_{mn} denotes the nth zero of the Bessel function $J_m(\xi_{mn}\sigma/a)$, while $\mathcal{N}$ is a normalization constant that is given by

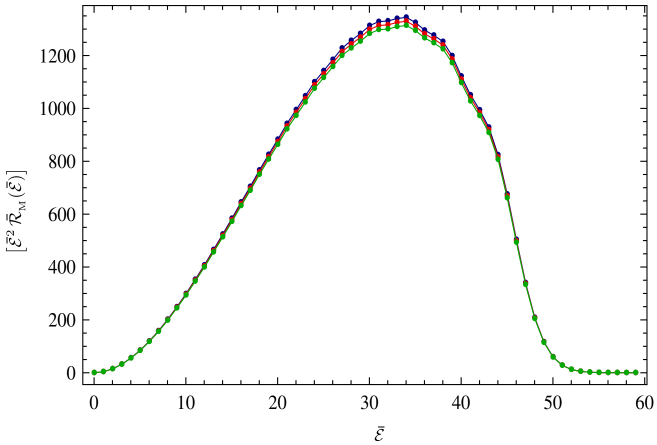

FIG. 2 (color online). The transition probability rate of the rotating Unruh-DeWitt detector that is coupled to a massless scalar field governed by the modified dispersion relation (14), with $\alpha = -1$ and $\beta = 0$. The blue, the red and the green dots denote the numerical results corresponding to the following three values of the quantity $(\sigma\Omega) = 0.325$, 0.350 and 0.375, that we had worked with in the previous figure. As in the last figure, the curves simply link the dots. We have set $\bar{k}_{\rm P}$ to be 50, and it should be stressed that this is an extremely small value for $\bar{k}_{\rm P}$. For such a value, as is evident, the modifications to the standard result (cf. Fig. 1) due to the subluminal dispersion relation prove to be substantial. Actually, one has to work with reasonably large and more realistic values of $\bar{k}_{\rm P}$ such that, say, $(\bar{k}_{\rm P}/\bar{\mathcal{E}}) > 10^{10}$. However, numerically, it proves to be difficult to sum the contributions in the expression (41) up to such large values of $\bar{k}_{\rm P}$. We believe that it would be reasonable to conclude that the modifications to standard result due to the subluminal dispersion relation can be expected to be much smaller if we assume $\bar{k}_{\rm P}$ to be sufficiently large. Nevertheless, our analysis unambiguously points to the fact that, as is known to occur in other contexts, a subluminal dispersion relation modifies the standard result considerably more than a similar superluminal dispersion relation.

$$\mathcal{N} = \left(\frac{\sqrt{2}}{a|J_{m+1}(\xi_{mn})|}\right). \tag{48}$$

Note that, as in the case without a boundary, m is a real integer, whereas k_z is a continuous real number. But, due to the imposition of the boundary condition at $\rho = a$, the spectrum of the radial modes is now discrete, and is described by the positive integer n. It should be pointed out that the expression (47) is in fact valid for any dispersion relation, with ω suitably related to the quantities ξ_{mn} and k_z. For instance, in the case of the modified dispersion relation (14) with β set to zero, the quantity ω is given by

$$\omega = [(\xi_{mn}/a)^2 + k_z^2]^{1/2}\left(1 + \left(\frac{\alpha}{k_{\rm P}^2}\right)[(\xi_{mn}/a)^2 + k_z^2]\right)^{1/2}, \tag{49}$$

where, it is evident that, while the overall factor corresponds to the standard, linear, dispersion relation, the term involving α within the brackets arises due to the modifications to it. Since the Wightman function depends only u, the transition probability rate of the detector simplifies to

$$\mathcal{R}_{\rm M}(\mathcal{E}) = \sum_{m=-\infty}^{\infty}\sum_{n=1}^{\infty}\int_{-\infty}^{\infty}\frac{dk_z}{(2\pi)(2\omega)} \times [\mathcal{N} J_m(\xi_{mn}\sigma/a)]^2 \delta^{(1)}[\mathcal{E} + \gamma(\omega - m\Omega)]. \tag{50}$$

For exactly the same reasons that we had presented in the last section, the delta function in the above expression can be nonzero only when $m > 0$. In fact, the detector will respond only under the condition

$$(m\Omega) > (\xi_{m1}/a)\left[1 + \alpha(\xi_{m1}/k_{\rm P}a)^2\right]^{1/2}, \tag{51}$$

where the right-hand side is the lowest possible value of ω corresponding to $n = 1$ and $k_z = 0$. However, from the properties of the Bessel function, it is known that $\xi_{mn} > m$, for all m and n (see, for instance, Ref. [24]). Therefore, *when α is positive*, (Ωa) has to be greater than unity, if the rotating detector has to respond. But, this is not possible since we have assumed that the boundary at a is located *inside* the static limit $\rho_{\rm SL} = (1/\Omega)$. This is exactly the same conclusion that one arrives at in the standard case [8,23].

In fact, it is straightforward to see that the above conclusion would apply for all superluminal dispersion relations. However, it seems that, under the same conditions, the rotating detector would be excited by a certain range of modes if we consider the scalar field to be described by a subluminal (such as, when $\alpha < 0$) dispersion relation! Actually, this feature is rather easy to understand. Consider a frequency, say, ω, associated with a mode through the linear dispersion relation. Evidently, a superluminal dispersion relation raises the energy of all the modes, while the subluminal dispersion relation lowers it. Therefore, if the interaction of the detector with a standard field does not excite a particular mode of the quantum field, clearly, the mode is unlikely to be excited if its energy has been raised further, as in a superluminal dispersion relation. However, the motion of the detector mode may be able to excite a mode of the field, if the energy of certain modes are lowered when compared to the standard case, as the subluminal dispersion relation does.

V. DISCUSSION

In this work, we have studied the response of a rotating Unruh-DeWitt detector that is coupled to a massless scalar field which is described by a nonlinear dispersion relation in flat spacetime. Unlike, say, the case of the uniformly accelerating detector [10,11], defining the transition probability rate of the rotating detector does not lead to any difficulties and, we find that, it can be defined in exactly the manner as in the standard case. Since it seems to be impossible to evaluate the modified Wightman function in a closed form, we had adopted a new method to evaluate the response of the rotating detector. However, as the transition probability rate for the rotating detector proves to be difficult to evaluate analytically, we had to calculate the response of the detector numerically. We have shown that the response of the rotating detector can be computed *exactly* (albeit, numerically) even when it is coupled to a field that is governed by a nonlinear dispersion relation. We have illustrated that the Planck scale modifications due to the nonlinear dispersion relation turn out to be extremely negligible when the dispersion relation is superluminal. However, we find that there can be a reasonable extent of changes to the standard results when one considers a subluminal dispersion relation. In addition, we have also considered the response of the rotating detector when the field is subjected to a boundary condition on a cylindrical surface located inside the static limit in the rotating frame. It is known that, in the standard case, the rotating detector fails to respond in such a situation [23]. We have shown that the null result remains true even for the case with a modified dispersion relation, provided the dispersion relation is a superluminal one.

As we have discussed earlier, scalar fields that are governed by nonlinear dispersion relations in flat spacetime are described by actions that break Lorentz invariance. The lack of Lorentz invariance implies that not all inertial frames are equivalent, and there exists a special inertial frame with respect to which the dispersion relation describing the field has been specified [4,12]. Therefore, the calculation that we have carried out is applicable in this special frame. Also, as we had pointed out, when the scalar field breaks Lorentz invariance, the modified Wightman function, in general, ceases to be invariant under time translations in noninertial frames that are integral curves of timelike Killing vector fields. However, if the modified theory possess rotational invariance, then, possibly, the corresponding Wightman function can be expected to be time translational invariant along trajectories that respect this symmetry, as it occurs in the case of the rotating coordinates. It is interesting to examine whether there also exist other noninertial trajectories that possess such a property. We are currently investigating such issues.

APPENDIX: ACCURACY OF THE NUMERICAL COMPUTATIONS

Since the response of the rotating detector can not be evaluated analytically, we had initially computed the response by numerically evaluating the integral (7) along the rotating trajectory, with the Wightman function being given by Eq. (13). In order to illustrate the accuracy of our numerical procedure to evaluate the integral, in this Appendix, we shall compare the results for the transition probability rate from our numerical code with the analytical result that is available for the uniformly accelerated motion [14,15]. We shall carry out the comparison for the case wherein the scalar field is described by the standard, linear dispersion relation. In such a case, the dimensionless transition probability rate of a detector that is moving along the trajectory

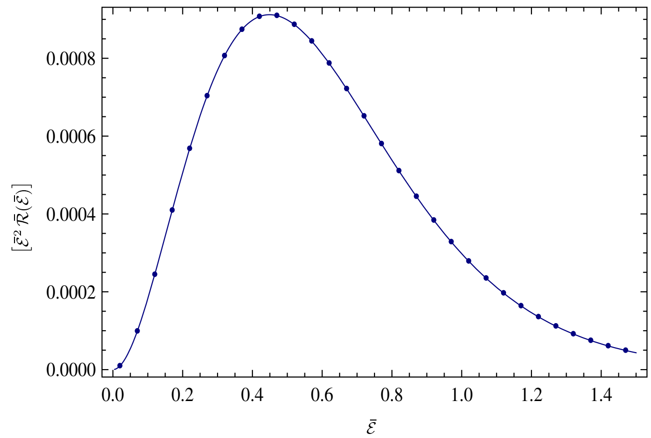

FIG. 3 (color online). The numerical and the analytical results for the transition probability rate of the uniformly accelerated detector that is coupled to a scalar field which is described by the standard, linear dispersion relation have been plotted. While the solid blue curve denotes the analytical result (A2), the dots lying on the curves denote the corresponding results from our numerical computation. Note that the plot does not explicitly depend on the acceleration parameter g. Evidently, the numerical and the analytical results are in good agreement.

$$\tilde{x}(\tau) = g^{-1}[\sinh(g\tau), \cosh(g\tau), 0, 0], \qquad \text{(A1)}$$

where g is the proper acceleration in the comoving frame, is well known to be (see, for instance, Ref. [20])

$$\bar{\mathcal{R}}(\bar{\mathcal{E}}) = [\mathcal{R}(\bar{\mathcal{E}})/g] = \left(\frac{1}{2\pi}\right)\left(\frac{\bar{\mathcal{E}}}{e^{(2\pi\bar{\mathcal{E}})} - 1}\right), \qquad \text{(A2)}$$

with $\bar{\mathcal{E}} = (\mathcal{E}/g)$. In Fig. 3, we have plotted the numerical as well as the above analytical result. It is obvious from the plot that the numerical result matches the analytical one quite well.

[1] T. Jacobson, Phys. Rev. D **48**, 728 (1993); **53**, 7082 (1996); W. G. Unruh, *ibid.* **51**, 2827 (1995); R. Brout, S. Massar, R. Parentani, and Ph. Spindel, *ibid.* **52**, 4559 (1995); N. Hambli and C. P. Burgess, *ibid.* **53**, 5717 (1996); S. Corley and T. Jacobson, *ibid.* **54**, 1568 (1996); T. Jacobson, Prog. Theor. Phys. Suppl. **136**, 1 (1999); R. Brout, Cl. Gabriel, M. Lubo, and Ph. Spindel, Phys. Rev. D **59**, 044005 (1999); C. Barrabes, V. Frolov,

and R. Parentani, *ibid.* **59**, 124010 (1999); **62**, 044020 (2000); R. Parentani, *ibid.* **63**, 041503 (2001); R. Casadio, P.H. Cox, B. Harms, and O. Micu, *ibid.* **73**, 044019 (2006); I. Agullo, J. Navarro-Salas, and G.J. Olmo, Phys. Rev. Lett. **97**, 041302 (2006); I. Agullo, J. Navarro-Salas, G.J. Olmo, and L. Parker, Phys. Rev. D **76**, 044018 (2007); R. Schutzhold and W.G. Unruh, *ibid.* **78**, 041504 (2008); D.A. Kothawala, S. Shankaranarayanan, and L. Sriramkumar, J. High Energy Phys. 09 (2008) 095.

[2] R. Brandenberger and J. Martin, Mod. Phys. Lett. A **16**, 999 (2001); C.S. Chu, B.R. Greene, and G. Shiu, *ibid.* **16**, 2231 (2001); J. Martin and R. Brandenberger, Phys. Rev. D **63**, 123501 (2001); J.C. Niemeyer, *ibid.* **63**, 123502 (2001); A. Kempf and J.C. Niemeyer, *ibid.* **64**, 103501 (2001); J.C. Niemeyer and R. Parentani, *ibid.* **64**, 101301 (2001); R. Easther, B.R. Greene, W.H. Kinney, and G. Shiu, *ibid.* **64**, 103502 (2001); A.A. Starobinsky, Pis'ma Zh. Eksp. Teor. Fiz. **73**, 415 (2001); M. Lemoine, M. Lubo, J. Martin, and J.P. Uzan, Phys. Rev. D **65**, 023510 (2001); J. Martin and R. Brandenberger, *ibid.* **65**, 103514 (2002); U.H. Danielsson, *ibid.* **66**, 023511 (2002); R. Brandenberger and P.M. Ho, *ibid.* **66**, 023517 (2002); R. Easther, B.R. Greene, W.H. Kinney, and G. Shiu, *ibid.* **66**, 023518 (2002); N. Kaloper, M. Kleban, A.E. Lawrence, and S. Shenker, *ibid.* **66**, 123510 (2002); F. Lizzi, G. Mangano, G. Miele, and M. Peloso, J. High Energy Phys. 06 (2002) 049; U.H. Danielsson, *ibid.* 12 (2002) 025; L. Bergstrom and U.H. Danielsson, *ibid.* 12 (2002) 038; R. Brandenberger and J. Martin, Int. J. Mod. Phys. A **17**, 3663 (2002); J. Martin and R. Brandenberger, Phys. Rev. D **68**, 063513 (2003); S. Shankaranaryanan, Classical Quantum Gravity **20**, 75 (2003); C.P. Burgess, J.M. Cline, F. Lemieux, and R. Holman, J. High Energy Phys. 02 (2003) 048; S.F. Hassan and M.S. Sloth, Nucl. Phys. **B674**, 434 (2003); J. Martin and C. Ringeval, Phys. Rev. D **69**, 083515 (2004); S. Shankaranarayanan and L. Sriramkumar, *ibid.* **70**, 123520 (2004); L. Sriramkumar and T. Padmanabhan, *ibid.* **71**, 103512 (2005); R. Easther, W.H. Kinney, and H. Peiris, J. Cosmol. Astropart. Phys. 05 (2005) 009; 08 (2005) 001; L. Sriramkumar and S. Shankaranarayanan, J. High Energy Phys. 12 (2006) 050.

[3] K. Srinivasan, L. Sriramkumar, and T. Padmanabhan, Phys. Rev. D **58**, 044009 (1998).

[4] T. Jacobson and D. Mattingly, Phys. Rev. D **63**, 041502 (2001); **64**, 024028 (2001).

[5] U. Harbach and S. Hossenfelder, Phys. Lett. B **632**, 379 (2006); S. Hossenfelder, Phys. Rev. D **73**, 105013 (2006); Classical Quantum Gravity **25**, 038003 (2008); D.A. Kothawala, L. Sriramkumar, S. Shankaranarayanan, and T. Padmanabhan, Phys. Rev. D **80**, 044005 (2009).

[6] S. Hossenfelder and L. Smolin, Phys. Canada **66**, 99 (2010).

[7] N.D. Birrell and P.C.W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, England, 1982); V.F. Mukhanov and S. Winitzki, *Introduction to Quantum Effects in Gravity* (Cambridge University Press, Cambridge, England, 2007); L. Parker and D.J. Toms, *Quantum Field Theory in Curved Spacetime: Quantized Fields and Gravity* (Cambridge University Press, Cambridge, England, 2009).

[8] L.C.B. Crispino, A. Higuchi, and G.E.A. Matsas, Rev. Mod. Phys. **80**, 787 (2008).

[9] I. Agullo, J. Navarro-Salas, G.J. Olmo, and L. Parker, Phys. Rev. D **77**, 104034 (2008); **77**, 124032 (2008).

[10] M. Rinaldi, Phys. Rev. D **77**, 124029 (2008).

[11] D. Campo and N. Obadia, arXiv:1003.0112v1.

[12] D. Mattingly, Living Rev. Relativity **8**, 5 (2005); T. Jacobson, S. Liberati, and D. Mattingly, Ann. Phys. (N.Y.) **321**, 150 (2006).

[13] D. Raetzel, S. Rivera, and F.P. Schuller, arXiv:1010.1369v2.

[14] W.G. Unruh, Phys. Rev. D **14**, 870 (1976).

[15] B.S. DeWitt, in *General Relativity: An Einstein Centenary Survey*, edited by S.W. Hawking and W. Israel (Cambridge University Press, Cambridge, England, 1979).

[16] J.R. Letaw, Phys. Rev. D **23**, 1709 (1981).

[17] J.R. Letaw and J.D. Pfautsch, Phys. Rev. D **24**, 1491 (1981).

[18] T. Padmanabhan, Astrophys. Space Sci. **83**, 247 (1982).

[19] J.S. Bell and J.M. Leinaas, Nucl. Phys. **B212**, 131 (1983); **B284**, 488 (1987).

[20] L. Sriramkumar and T. Padmanabhan, Int. J. Mod. Phys. D **11**, 1 (2002).

[21] J.I. Korsbakken and J.M. Leinaas, Phys. Rev. D **70**, 084016 (2004).

[22] A.P. Prudnikov, Yu.A. Brychkov, and O.I. Marichev, *Integrals and Series* (Gordon and Breach Science Publishers, New York, 1986), Vol. 2, p. 212.

[23] P.C.W. Davies, T. Dray, and C.A. Manogue, Phys. Rev. D **53**, 4382 (1996).

[24] M. Abramowitz and I.A. Stegun, *Handbook of Mathematical Functions* (Dover, New York, 1965).