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Embedding $R + R^2$ inflation in supergravity

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We find the natural embedding of the $(R + R^2)$ inflationary model into the recently constructed $F(\mathcal{R})$ supergravity. It gives a simple and viable realization of chaotic inflation in supergravity. The only requirement for a slow-roll inflation is the existence of the $\mathcal{R}^3$ term with an anomalously large coefficient in Taylor expansion of the $F(\mathcal{R})$ function.

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I. INTRODUCTION

A natural realization of inflation in supergravity is known to be problematic [1,2] because of the factor $\exp(K/M_{\rm Pl}^2)$ in the (F-term generated) scalar potential [3], where K is the Kähler potential of the chiral scalar matter superfields Φ and $\bar{\Phi}$. The naive (tree-level) *Ansatz* $K = \bar{\Phi}\Phi$ gives rise to the scalar potential proportional to $\exp(\bar{\Phi}\Phi)$ that is too steep for a slow-roll inflation (the so-called η problem) with the unacceptable inflaton mass $|m^2| \sim V_0/M_{\rm Pl}^2 \approx H^2$.

To cure the above problem, the D-term mechanism was proposed [4], where the inflation is generated in the *gauge* sector and is highly sensitive to the gauge charges. Another proposal is to assume that the Kähler potential does not depend upon some scalars(= flat directions) and then add a desired scalar superpotential for the flat directions [5]. Both proposals are nongeometrical and nonuniversal because they refer to the matter sector (not gravity) and require the existence of extra fields too.

As has also been known for a long time [6,7], viable inflationary models can be easily constructed in (nonsupersymmetric) $f(R)$ gravity theories (see, e.g., Ref. [8] for a recent review) with the action

$$S = -\frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}f(R), \qquad (1)$$

whose function $f(R)$ begins with the scalar curvature R, and the difference $(f(R) - R)$ takes the form $R^2A(R)$ for $R \to \infty$, with a slowly varying function $A(R)$ (we assume that $\hbar = c = 1$). The simplest one of those models is given by (see Ref. [9] for our sign conventions)

$$f(R) = R - \frac{R^2}{6M^2}. \qquad (2)$$

The theory (2) is known as the excellent model of chaotic inflation [10]. The coefficient in front of the second term on the right-hand side of Eq. (2) is chosen so that M actually coincides with the rest mass of the scalar particle appearing in $f(R)$ gravity (dubbed *scalaron* in Ref. [6]) at low curvatures $|R| \ll M^2$ or in flat spacetime, in particular. The model fits the observed amplitude of scalar perturbations if $M/M_{\rm Pl} \approx 1.5 \times 10^{-5}(50/N_e)$, and gives rise to the spectral index $n_s - 1 \approx -2/N_e \approx -0.04(50/N_e)$ and the scalar-to-tensor ratio $r \approx 12/N_e^2 \approx 0.005(50/N_e)^2$, in terms of the e-foldings number $N_e \approx (50 \div 55)$ depending upon details of reheating after inflation [9,11]. Despite of the fact that it has been known for 30 years, the model (2) remains viable and is in agreement with the most recent WMAP7 observations of $n_s = 0.963 \pm 0.012$ and $r < 0.24$ (with 95% CL) [12].

The purpose of this paper is to show that there exists a natural embedding of the inflationary model (2) into supergravity.[1] For that purpose we use the supersymmetric extension of $f(R)$ gravity theories, called $F(\mathcal{R})$ supergravity, that was recently constructed in Ref. [16]. In Sec. II we briefly outline the $F(\mathcal{R})$ supergravity by focusing on its reduction to the more familiar $f(R)$ gravity. In Sec. III we propose a simple realization of chaotic inflation in supergravity via embedding of the bosonic model (2) into the particular $F(\mathcal{R})$ supergravity model. Sec. IV is our conclusion.

II. $F(\mathcal{R})$ SUPERGRAVITY AND $f(R)$ GRAVITY

The most succinct formulation of $F(\mathcal{R})$ supergravity exists in a chiral 4D, $N = 1$ superspace where it is defined by an action[2]

$$S = \int d^4xd^2\theta\mathcal{E}F(\mathcal{R}) + {\rm H.c.} \qquad (3)$$

[1]For completeness, it is worthwhile to mention some other microscopic approaches that are unrelated to supergravity but also lead to the $(R + R^2)$ model as the macroscopic (and approximate) theory with a high precision: (i) the Higgs inflation with a large nonminimal coupling of the Higgs field to gravity [13,14], and (ii) the so-called emergent gravity [15].

[2]For simplicity, we take $M_{\rm Pl} = 1$ in this section.

in terms of a holomorphic function $F(\mathcal{R})$ of the covariantly-chiral scalar curvature superfield $\mathcal{R}$, and the chiral superspace density $\mathcal{E}$. The chiral $N = 1$ superfield $\mathcal{R}$ has the scalar curvature R as the field coefficient at its θ^2 term (see, e.g., Ref. [17] for details about supergravity in superspace). The chiral superspace density $\mathcal{E}$ (in a WZ gauge) reads

$$\mathcal{E} = e(1 - 2i\theta\sigma_a\bar{\psi}^a + \theta^2 B), \tag{4}$$

where $e = \sqrt{-g}$, ψ^a is gravitino, and $B = S - iP$ is the complex scalar auxiliary field (it does not propagate in the theory (3) despite of the apparent presence of the higher derivatives). The full component structure of the action (3) is very complicated. Nevertheless, it is classically equivalent to the standard $N = 1$ Poincaré supergravity minimally coupled to the chiral scalar superfield, via the supersymmetric Legendre-Weyl-Kähler transform [16]. The chiral scalar superfield is given by the superconformal mode of the supervielbein (in Minkowski or AdS vacuum) which becomes dynamical in $F(\mathcal{R})$ supergravity.

A relation to the $f(R)$ gravity theories can be established by dropping the gravitino ($\psi^a = 0$) and restricting the auxiliary field B to its real (scalar) component, $B = 3X$ with $\bar{X} = X$. Then, as was shown in Ref. [18], the bosonic Lagrangian takes the form

$$L = 2F'\left[\frac{1}{3}R + 4X^2\right] + 6XF. \tag{5}$$

It follows that the auxiliary field X obeys an algebraic equation of motion,

$$3F + 11F'X + F''\left[\frac{1}{3}R + 4X^2\right] = 0. \tag{6}$$

In those equations $F = F(X)$ and the primes denote the derivatives with respect to X. Solving Eq. (6) for X and substituting the solution back into Eq. (5) results in the bosonic function $L = -\frac{1}{2}f(R)$.

It is natural to expand the input function $F(\mathcal{R})$ into power series of $\mathcal{R}$. For instance, when $F(\mathcal{R}) = f_0 - \frac{1}{2}f_1\mathcal{R}$ with some (nonvanishing and complex) coefficients f_0 and f_1, one recovers the standard *pure* $N = 1$ supergravity with a negative cosmological term [16].

A more interesting Ansatz is given by

$$F(\mathcal{R}) = -\frac{1}{2}f_1\mathcal{R} + \frac{1}{2}f_2\mathcal{R}^2 \tag{7}$$

with some real coefficients f_1 and f_2. It gives rise to the bosonic function (with $f_1 = 3/2$) [18]

$$\begin{aligned} f(R) &= \frac{5\times 17}{3^2\times 11}R - \frac{2^2\times 7}{3^2\times 11}(R - R_{\max})\left[1 - \sqrt{1 - R/R_{\max}}\right] \\ &= R - \frac{R^2}{6M^2} - \frac{11R^3}{252M^4} + \mathcal{O}(R^4), \end{aligned} \tag{8}$$

where $R_{\max} = \frac{3^2\times 7^2}{2^3\times 11}f_2^{-2}$ is the AdS bound automatically generated in the model, and $M^2 = \frac{11}{7}R_{\max}$. Unfortunately, the model (8) is not viable as the inflationary model because it suffers from the η problem arising due to the presence of the higher-order terms with respect to the scalar curvature in Eq. (8) [18].

III. OUR NEW MODEL

The Ansatz we propose in this paper is given by

$$F(\mathcal{R}) = -\frac{1}{2}f_1\mathcal{R} + \frac{1}{2}f_2\mathcal{R}^2 - \frac{1}{6}f_3\mathcal{R}^3, \tag{9}$$

whose real (positive) coupling constants $f_{1,2,3}$ are of (mass) dimension 2, 1, and 0, respectively. Our conditions on the coefficients are

$$f_3 \gg 1, \qquad f_2^2 \gg f_1. \tag{10}$$

The first condition is needed to have inflation at the curvatures much less than $M_{\rm Pl}^2$ (and to meet observations), while the second condition is needed to have the scalaron (inflaton) mass be much less than $M_{\rm Pl}$, in order to avoid large (gravitational) quantum loop corrections after the end of inflation up to the present time.

Stability of our bosonic embedding (5) in supergravity implies $F'(X) < 0$. In the case (9) it gives rise to the condition $f_2^2 < f_1 f_3$. For simplicity we will assume

$$f_2^2 \ll f_1 f_3. \tag{11}$$

Then the second term on the right-hand side of Eq. (9) will not affect inflation, as is shown below.

Equation (5) with the Ansatz (9) reads

$$\begin{aligned} L = &-5f_3X^4 + 11f_2X^3 - \left(7f_1 + \frac{1}{3}f_3R\right)X^2 \\ &+ \frac{2}{3}f_2RX - \frac{1}{3}f_1R \end{aligned} \tag{12}$$

and gives rise to a cubic equation on X,

$$X^3 - \left(\frac{33f_2}{20f_3}\right)X^2 + \left(\frac{7f_1}{10f_3} + \frac{1}{30}R\right)X - \frac{f_2}{30f_3}R = 0. \tag{13}$$

We find three consecutive (overlapping) regimes.

(i) The high curvature regime including inflation is given by

$$\delta R < 0 \quad \text{and} \quad \frac{|\delta R|}{R_0} \gg \left(\frac{f_2^2}{f_1 f_3}\right)^{1/3}, \tag{14}$$

where we have introduced the notation $R_0 = 21f_1/f_3 > 0$ and $\delta R = R + R_0$. With our sign conventions (Sec. I) we have $R < 0$ during the de Sitter and matter dominated stages. In the regime (14) the f_2-dependent terms in Eqs. (12) and (13) can be neglected, and we get

$$X^2 = -\frac{1}{30}\delta R \tag{15}$$

and

$$L = -\frac{f_1}{3} R + \frac{f_3}{180} (R + R_0)^2. \qquad (16)$$

It closely reproduces the inflationary model (2) since inflation occurs at $|R| \gg R_0$. So it is natural to denote $f_3 = 15M_{\rm Pl}^2/M^2$ (see Sec. I). It is worth mentioning that we cannot simply set $f_2 = 0$ in Eq. (9) because it would imply $X = 0$ and $L = -\frac{f_1}{3} R$ for $\delta R > 0$. As a result of that the scalar degree of freedom would disappear that would lead to the breaking of a regular Cauchy evolution. Therefore, the second term in Eq. (9) is needed to remove that degeneracy.

(ii) The intermediate (post-inflationary) regime is given by

$$\frac{|\delta R|}{R_0} \ll 1. \qquad (17)$$

In this case X is given by a root of the cubic equation

$$30X^3 + (\delta R)X + \frac{f_2 R_0}{f_3} = 0. \qquad (18)$$

It also implies that the 2nd term in Eq. (13) is always small. Equation (18) reduces to Eq. (15) under the conditions (14).

(iii) The low-curvature regime (up to $R = 0$) is given by

$$\delta R > 0 \quad \text{and} \quad \frac{\delta R}{R_0} \gg \left(\frac{f_2^2}{f_1 f_3}\right)^{1/3}. \qquad (19)$$

It yields

$$X = \frac{f_2 R}{f_3 (R + R_0)} \qquad (20)$$

and

$$L = -\frac{f_1}{3} R + \frac{f_2^2 R^2}{3 f_3 (R + R_0)}. \qquad (21)$$

It is now clear that f_1 should be equal to $3M_{\rm Pl}^2/2$ in order to obtain the correctly normalized Einstein gravity at $|R| \ll R_0$. In this regime the scalaron mass squared is given by

$$\frac{1}{3|f''(R)|} = \frac{f_3 R_0 M_{\rm Pl}^2}{4 f_2^2} = \frac{21 f_1}{4 f_2^2} M_{\rm Pl}^2 = \frac{63 M_{\rm Pl}^4}{8 f_2^2} \qquad (22)$$

in agreement with the case of the absence of the $\mathcal{R}^3$ term, studied in Sec. II. The scalaron mass squared (22) is much less than $M_{\rm Pl}^2$ indeed, due to the second inequality in Eq. (10), but it is much more than the one at the end of inflation ($\sim M^2$).

It is worth noticing that the corrections to the Einstein action in Eqs. (16) and (21) are of the same order (and small) at the borders of the intermediate region (17).

The roots of the cubic Eq. (13) are given by the textbook (Cardano) formula [19], though that formula is not very illuminating in a generic case. The Cardano formula greatly simplifies in the most interesting (high-curvature) regime where inflation takes place, and the Cardano discriminant is

$$D \approx \left(\frac{R}{90}\right)^3 < 0. \qquad (23)$$

It implies that all three roots are real and unequal. The Cardano formula yields the roots

$$X_{1,2,3} \approx \frac{2}{3}\sqrt{\frac{-R}{10}} \cos\left(\frac{27}{4 f_3 \sqrt{-10R/f_2^2}} + C_{1,2,3}\right) + \frac{11 f_2}{20 f_3}, \qquad (24)$$

where the constant $C_{1,2,3}$ takes the values $(\pi/6, 5\pi/6, 3\pi/2)$.

As regards the leading terms, Eqs. (12) and (24) result in the $(-R)^{3/2}$ correction to the $(R + R^2)$ terms in the effective Lagrangian in the high-curvature regime $|R| \gg f_2^2/f_3^2$. In order to verify that this correction does not change our results under the conditions, (14), let us consider the $f(R)$-gravity model with

$$f(R) = R - b(-R)^{3/2} - aR^2, \qquad (25)$$

whose parameters $a > 0$ and $b > 0$ are subject to the conditions $a \gg 1$ and $b/a^2 \ll 1$. It is easy to check that $f'(R) > 0$ for $R \in (-\infty, 0]$, as is needed for (classical) stability.

Any $f(R)$ gravity model is known to be classically equivalent to the scalar-tensor gravity with proper scalar potential [20]. The scalar potential can be calculated from a given function $f(R)$ along the standard lines (see, e.g., Refs. [8,9]). We find (in the high-curvature regime)

$$V(y) = \frac{1}{8a}(1 - e^{-y})^2 + \frac{b}{8\sqrt{2a}} e^{-2y}(e^y - 1)^{3/2} \qquad (26)$$

in terms of the inflaton field y. The first term of this equation is the scalar potential associated with the pure $(R + R^2)$ model, and the second term is the correction due to the $R^{3/2}$ term in Eq. (25). It is now clear that for large positive y the vacuum energy in the first term dominates and drives inflation until the vacuum energy is compensated by the y-dependent terms near $e^y = 1$.

It can be verified along the lines of Ref. [11] that the formula for scalar perturbations remains the same as for the model (2), i.e., $\Delta_{\mathcal{R}}^2 \approx N^2 M^2/(24\pi^2 M_{\rm Pl}^2)$, where N is the number of e-folds from the end of inflation. So, to fit the observational data, one has to choose $f_3 \approx 5N_e^2/(8\pi^2\Delta_{\mathcal{R}}^2) \approx 6.5 \times 10^{10}(N_e/50)^2$. Here the value of $\Delta_{\mathcal{R}}$ is taken from Ref. [12] and the subscript $\mathcal{R}$ has a different meaning from the rest of this paper.

IV. CONCLUSION

We conclude that the model (9) with a sufficiently small f_2 obeying the conditions (10) and (11) gives a viable realization of the chaotic $(R + R^2)$-type inflation in supergravity. The only significant difference with respect to the original $(R + R^2)$ inflationary model is the scalaron mass that becomes much larger than M in supergravity, soon after the end of inflation when δR becomes positive. However, it only makes the scalaron decay faster and creation of the usual matter (reheating) more effective.

The whole series in powers of $\mathcal{R}$ may also be considered, instead of the limited Ansatz (9). The only necessary condition for embedding inflation is that f_3 should be anomalously large. When the curvature grows, the $\mathcal{R}^3$ term should become important much earlier than the convergence radius of the whole series without that term. Of course, it means that viable inflation may not occur for any function $F(\mathcal{R})$ but only inside a small region of nonzero measure in the space of all those functions. However, the same is true for all known inflationary models, so the very existence of inflation has to be taken from the observational data, not from a pure thought.

We consider our results as the viable alternative to the earlier fundamental proposals [4,5] for realization of chaotic inflation in supergravity. However, inflation is not the only target of our construction. As is well known [6,7]—see also the recent paper Ref. [21]—the scalaron decays into pairs of particles and antiparticles of quantum matter fields, while its decay into gravitons is strongly suppressed [22]. It represents the universal mechanism of viable reheating after inflation and provides a transition to the subsequent hot radiation-dominated stage of the Universe's evolution and the characteristic temperature $T_{\text{reheating}} \approx 10^9$ GeV. In its turn, it leads to the standard primordial nucleosynthesis after. In $F(R)$ supergravity the scalaron has a pseudoscalar superpartner (or axion) that may be the source of a strong CP violation and then, subsequently, a leptogenesis and a baryogenesis that naturally lead to baryon (matter-antimatter) asymmetry [23].

Supersymmetry in $F(R)$ supergravity is already broken by inflation. It may give rise to a massive gravitino with $m_{3/2} \geq 10^7$ GeV. The gravitino is a natural candidate for the cold dark matter in our construction, *cf.* Ref. [24]. The gravitationally mediated supersymmetry breaking may serve as the important element for the new particle phenomenology (beyond the standard model) based on a matter-coupled $F(R)$ supergravity.

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