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Bethe-Salpeter equation for doubly heavy baryons in the covariant instantaneous approximation

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In the heavy quark limit, a doubly heavy baryon is regarded as composed of a heavy diquark and a light quark. We establish the Bethe-Salpeter equations for the heavy diquarks and the doubly heavy baryons, respectively, to leading order in a $1/m_Q$ expansion. The Bethe-Salpeter equations are solved numerically under the covariant instantaneous approximation with the kernels containing scalar confinement and one-gluon-exchange terms. The masses for the heavy diquarks and the doubly heavy baryons are obtained, and the nonleptonic decay widths for the doubly heavy baryons emitting a pseudoscalar meson are calculated within the model.

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I. INTRODUCTION

The past few years have seen many important developments concerning hadron colliders, especially the advent of the LHC. Recently, the discovery of and searches for double charm baryons have been reported by various experimental collaborations [1–4]. One is convinced that more and more doubly heavy baryons will be observed in the near future. Consequently, it is an urgent task for theorists to investigate the properties of these states.

On the other hand, the existence of three valence quarks in a baryon makes the theoretical study much more complicated than the case of mesons. People have suggested the presence of diquark structure in a baryon and have studied the properties of heavy baryons in such a picture [5–9]. No matter whether the diquark is a real physical object or simply a theoretical approximation, this picture reduces the three body system to a two body problem which is much simpler for investigation.

In recent years, heavy quark effective theory has been widely used in the study of doubly heavy baryons [10–17]. Two heavy quarks are reasonably bound into a color-antitriplet heavy diquark whose radius is much smaller than the typical scale ($1/\Lambda_{\rm QCD}$) of the nonperturbative QCD interactions in the heavy quark limit ($m_Q \gg \Lambda_{\rm QCD}$, m_Q denotes the heavy quark mass). The leftover light quark involved in the baryon moves in the color field induced by the heavy diquark. Unlike the heavy quark and light quark system, the internal motion in the heavy diquark cannot be ignored even at leading order in the $1/m_Q$ expansion. This is because the relative momentum in the heavy diquark is not simply $\mathcal{O}(\Lambda_{\rm QCD})$, but $\sim\alpha_s^2 m_Q$ as calculated in the Coulomb potential model [11,18].

As a formally exact equation to describe the relativistic bound system, the Bethe-Salpeter (BS) equation was initially formulated in Minkowski space based on the relativistic quantum theory [19,20]. However, it is difficult to solve the BS equation in Minkowski space due to its singular behavior. In order to overcome this difficulty, with the so-called "Wick rotation", the formalism for the BS equation in Euclidean space was developed and investigated in detail [21–24]. With the perturbation theory integral representation, the BS equation was solved in Minkowski space for the scalar and fermion systems [25–30]. Recently, based on the Nakanishi integral representation of the BS amplitude and the projection of the BS equation on the light-front plane, a new method for solving the BS equation in Minkowski space was proposed and was applied to study the electromagnetic form factor [31–36]. In another formalism to solve the BS equation, the covariant instantaneous approximation is adopted in the kernel. In recent decades, this formalism has been successfully used to investigate heavy mesons, heavy baryons, and exotic states [7–9,12,37–44].

Based on the diquark picture of the composition of the doubly heavy baryon, we will establish the BS equations for both the heavy diquarks and the doubly heavy baryons in the leading order of a $1/m_Q$ expansion. Motivated by the potential model, the kernel for the BS equation is assumed to be composed of the scalar confinement and one-gluon-exchange terms [45]. We will solve the BS equations numerically under the covariant instantaneous approximation [41–44]. Since the heavy diquark is not really a point object, a few form factors for the effective vertex of the heavy diquark coupling to the gluon are introduced to reflect the inner structure of the heavy diquark. These form factors will be expressed in terms of the BS wave functions obtained for the heavy diquarks. Finally, we will calculate the nonleptonic decay widths for the doubly heavy baryons emitting a pseudoscalar meson in the BS formalism.

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The remainder of this paper is organized as follows. In Sec. II, we establish the BS equation for the heavy diquarks in the leading order of $1/m_Q$ expansion. We also give the normalization conditions for the BS wave functions for the heavy diquarks in this section. In Sec. III, the form factors for the effective vertex of the heavy diquark coupling to the gluon are derived from the BS wave functions obtained for the heavy diquarks. In Sec. IV, we establish the BS equation for the doubly heavy baryons at leading order in the $1/m_Q$ expansion. The normalization conditions for the BS wave functions for the doubly heavy baryons are also given in this section. In Sec. V, the nonleptonic decay widths for the doubly heavy baryons emitting a pseudoscalar meson are calculated in the BS formalism. Section VI is reserved for our summary and some discussions.

II. BS EQUATION FOR HEAVY DIQUARKS

In general, the parity of a ground state baryon is positive. Since the parity of quark is supposed to be positive, the parity of the diquark involved in a ground state baryon should be positive. Because of the Pauli principle, two quarks with the same flavor can only constitute an axial-vector diquark. On the other hand, two quarks with different flavors can constitute either a scalar diquark or an axial-vector diquark. It can be easily shown that a heavy diquark which is in the ground state can not be a tensor diquark.

Suppose two heavy quarks Q_1 and Q_2 (with masses m_{Q_1} and m_{Q_2}, respectively) compose a ground state heavy diquark. Define two ratios $\lambda_1 = m_{Q_1}/(m_{Q_1} + m_{Q_2})$ and $\lambda_2 = m_{Q_2}/(m_{Q_1} + m_{Q_2})$. The BS wave function for the heavy diquark is defined as follows:

$$\chi_{P_D}(x_1, x_2)_{\alpha\beta} = \varepsilon^{ijk}\langle 0|T\psi_1(x_1)^i_\alpha \psi_2(x_2)^j_\beta|P_D, k\rangle = e^{-iP_D X}\int \frac{d^4p}{(2\pi)^4}\chi_{P_D}(p)_{\alpha\beta}e^{-ipx}, \quad (1)$$

$$\bar{\chi}_{P_D}(x_2, x_1)_{\beta\alpha} = \varepsilon^{ijk}\langle P_D, k|T\psi^*_2(x_2)^j_\beta \psi^*_1(x_1)^i_\alpha|0\rangle = e^{iP_D X}\int \frac{d^4p}{(2\pi)^4}\bar{\chi}_{P_D}(p)_{\beta\alpha}e^{ipx}, \quad (2)$$

where ψ_1 and ψ_2 stand for the field operators of the heavy quarks Q_1 and Q_2, respectively, i, j, k represent the color indices, α and β represent the spin indices, $X \equiv \lambda_1 x_1 + \lambda_2 x_2$ is the coordinate of the heavy diquark mass center, $x \equiv x_1 - x_2$ is the relative coordinate of the two heavy quarks, P_D is the momentum of the heavy diquark, and p is the relative momentum between the two heavy quarks.

The BS equation for the heavy diquark can be written in the following form (details can be found in Ref. [9]):

$$\chi_{P_D}(p) = S(p_1) \otimes S(p_2)\int \frac{d^4p'}{(2\pi)^4}[\gamma^\mu \otimes \gamma_\mu K^{(1g)}(p - p') + I \otimes I K^{(cf)}(p - p')]\chi_{P_D}(p'), \quad (3)$$

where $p_1 = \lambda_1 P_D + p$ and $p_2 = \lambda_2 P_D - p$ are the momenta of heavy quarks Q_1 and Q_2, respectively, and $S(p_1)$ and $S(p_2)$ are the propagators of heavy quarks Q_1 and Q_2, respectively. $K^{(1g)}$ and $K^{(cf)}$ are the one-gluon-exchange and scalar confinement terms of the kernel for the BS equation given by (after imposing the covariant instantaneous approximation [41–44]):

$$K^{(1g)}(p_t - p'_t) = -\frac{8i\pi}{3}\frac{\alpha_s}{(p_t - p'_t)^2 - \mu^2}, \quad (4)$$

and

$$K^{(cf)}(p_t - p'_t) = \frac{4i\pi\kappa}{[-(p_t - p'_t)^2 + \mu^2]^2} - (2\pi)^3\delta^3(p_t - p'_t) \times \int \frac{d^3k_t}{(2\pi)^3}\frac{4i\pi\kappa}{[-(p_t - k_t)^2 + \mu^2]^2}, \quad (5)$$

where α_s and κ are the coupling parameters related to one-gluon-exchange and scalar confinement terms, respectively, p_t is the transverse projection of the relative momentum (p) along the heavy diquark momentum (P_D) (see the definition below Eq. (8)), the second term of $K^{(cf)}$ is introduced to remove the infrared singularity near the point $p'_t = p_t$, and the small parameter μ is introduced to avoid the divergence in the numerical calculations. This kernel is motivated by the potential model which has been successfully applied in mesons [45]. Furthermore, we assume that the kernel of the heavy diquark is related to the meson by the one-half rule [46,47].

Equation (3) can be written in a more usual matrix form as [9]

$$\tilde{\chi}^T_{P_D}(p) = S(p_2)\int \frac{d^4p'}{(2\pi)^4}[-\gamma^\mu \tilde{\chi}^T_{P_D}(p')\gamma_\mu K^{(1g)}(p_t - p'_t) + \tilde{\chi}^T_{P_D}(p')K^{(cf)}(p_t - p'_t)]S(-p_1), \quad (6)$$

where $\tilde{\chi}_{P_D}(p) = \mathcal{C}\chi_{P_D}(p)$ ($\mathcal{C}$ is the charge conjugation matrix) and the superscript T represents the transpose of the spinor indices.

In the leading order of a $1/m_Q$ expansion, the heavy quark propagators [$S(p_1)$ and $S(p_2)$] can be written as

$$S(p_1) = i\frac{m_{Q_1}(\not{v}_D + 1)}{2\omega_{Q_1}(\lambda_1 m_D + p_l - \omega_{Q_1} + i\varepsilon)}, \quad (7)$$

and

$$S(p_2) = -i\frac{m_{Q_2}(\not{v}_D + 1)}{2\omega_{Q_2}(-\lambda_2 m_D + p_l + \omega_{Q_2} - i\varepsilon)}, \quad (8)$$

where m_D and v_D are the mass and velocity of the heavy diquark, respectively, $p_l = p \cdot v_D$ and $p^\mu_t = p^\mu - p_l v^\mu_D$

are the longitudinal and transverse projections of the relative momentum (p) along the heavy diquark momentum (P_D), respectively, the energy $\omega_{Q_{1(2)}} = \sqrt{m^2_{Q_{1(2)}} - p_t^2}$, and ε is the infinitesimal. As studied in Refs. [10,11], $|p_t|$ is $\mathcal{O}(\alpha_s^2 m_Q)$. Consequently, $|p_t|/m_Q$ terms should not be neglected in calculations carried out to leading order in the $1/m_Q$ expansion.

Substituting Eqs. (7) and (8) into Eq. (6), one finds the following two constraints for the BS wave function for the heavy diquarks:

$$\not{v}_D \tilde{\chi}^T_{P_D}(p) = \tilde{\chi}^T_{P_D}(p), \tag{9}$$

$$\tilde{\chi}^T_{P_D}(p)\not{v}_D = -\tilde{\chi}^T_{P_D}(p). \tag{10}$$

Then, taking these constraints into account in the BS equation for the positive parity and zero angular momentum ground state of the heavy diquark system, the BS wave functions for the scalar and the axial-vector heavy diquarks can be parametrized in the following forms, respectively:

$$\tilde{\chi}^T_{P_D}(p) = (\not{v}_D + 1)\gamma^5 f_1, \tag{11}$$

and

$$\tilde{\chi}^{(r)T}_{P_D}(p) = (\not{v}_D + 1)\not{\xi}^{(r)} f_2, \tag{12}$$

where $\xi^{(r)}_\mu$ is the rth polarization vector of the axial-vector heavy diquark, f_1 and f_2 are the Lorentz-scalar functions of p_t^2, p_l, and $P_D^2 = m_D^2$.

After some algebra, we find that the BS scalar wave functions for both the scalar heavy diquark (f_1) and the axial-vector heavy diquark (f_2) satisfy the same integral equation as follows:

$$\tilde{f}(p_t) = \frac{m_{Q_1} m_{Q_2}}{\omega_{Q_1}\omega_{Q_2}(-m_D + \omega_{Q_1} + \omega_{Q_2})} \times \int \frac{d^3 p_t'}{(2\pi)^3}[V^{(1g)}(p_t - p_t') + V^{(cf)}(p_t - p_t')]\tilde{f}(p_t'), \tag{13}$$

where we define $\tilde{f}(p_t) \equiv \int \frac{dp_l}{2\pi} f_{1(2)}(p)$, $V^{(1g)} \equiv -iK^{(1g)}$, and $V^{(cf)} \equiv -iK^{(cf)}$.

In general, the normalization condition for the heavy diquark can be written as (after imposing the covariant instantaneous approximation on the kernel) [9]

$$\frac{i}{36}\delta^{i_1 i_2}_{j_1 j_2}\int \frac{d^4 p d^4 p'}{(2\pi)^8}\bar{\chi}_{P_D}(p)\frac{\partial}{\partial P_D^0}[I_{P_D}(p,p')]^{i_1 i_2 j_2 j_1}\chi_{P_D}(p') = 1, \tag{14}$$

where $i_{1(2)}$ and $j_{1(2)}$ represent the color indices of the heavy quarks, $\delta^{i_1 i_2}_{j_1 j_2} = \delta^{i_1}_{j_1}\delta^{i_2}_{j_2} - \delta^{i_1}_{j_2}\delta^{i_2}_{j_1}$, and $I^{i_1 i_2 j_2 j_1}_{P_D}(p,p')$ stands for the inverse of the four point function,

$$I^{i_1 i_2 j_2 j_1}_{P_D}(p,p') = \delta^{i_1 j_1}\delta^{i_2 j_2}(2\pi)^4\delta^4(p-p') \times [S(p_1)\gamma_0]^{-1}[S(p_2)\gamma_0]^{-1}. \tag{15}$$

Now, it is straightforward to obtain the normalization condition for the BS wave function for the heavy diquark as the following:

$$-\frac{1}{6}\int \frac{d^4 p}{(2\pi)^4}\{\mathrm{Tr}[-\lambda_1 S(-p_1)\tilde{\bar{\chi}}^{(c)}_{P_D}(p_t)S(p_2)\tilde{\bar{\chi}}_{P_D}(p_t) \times S(-p_1)\not{\varepsilon}] + \mathrm{Tr}[\lambda_2 S(-p_1)\tilde{\bar{\chi}}^{(c)}_{P_D}(p_t)S(p_2)\not{\varepsilon}S(p_2) \times \tilde{\bar{\chi}}_{P_D}(p_t)]\} = 1, \tag{16}$$

where $\varepsilon = (1, \vec{0})$, $\tilde{\bar{\chi}}_{P_D}(p_t)$ and $\tilde{\bar{\chi}}^{(c)}_{P_D}(p_t)$ are the transverse projections of the BS wave function given by

$$\tilde{\bar{\chi}}_{P_D}(p_t) = -iS(p_2)^{-1}\tilde{\chi}^T_{P_D}(p)S(-p_1)^{-1}, \tag{17}$$

and

$$\tilde{\bar{\chi}}^{(c)}_{P_D}(p_t) = \mathcal{C}\tilde{\bar{\chi}}^T_{-P_D}(-p_t)\mathcal{C}^{-1}, \tag{18}$$

respectively.

For the scalar heavy diquark, the transverse projections of the BS wave function [$\tilde{\bar{\chi}}_{P_D}(p_t)$ and $\tilde{\bar{\chi}}^{(c)}_{P_D}(p_t)$] can be obtained from Eqs. (17) and (18) as follows, respectively:

$$\tilde{\bar{\chi}}_{P_D}(p_t) = \tilde{f}_{s_1}(p_t)\gamma^5 + \tilde{f}_{s_2}(p_t)\not{v}_D\gamma^5, \tag{19}$$

and

$$\tilde{\bar{\chi}}^{(c)}_{P_D}(p_t) = \tilde{f}_{s_1}(p_t)\gamma^5 - \tilde{f}_{s_2}(p_t)\not{v}_D\gamma^5, \tag{20}$$

where

$$\tilde{f}_{s_1}(p_t) = \int \frac{d^3 p_t'}{(2\pi)^3}[V^{(1g)}(p_t - p_t') + 4V^{(cf)}(p_t - p_t')]\tilde{f}(p_t'), \tag{21}$$

and

$$\tilde{f}_{s_2}(p_t) = \int \frac{d^3 p_t'}{(2\pi)^3}[V^{(1g)}(p_t - p_t') - 2V^{(cf)}(p_t - p_t')]\tilde{f}(p_t'). \tag{22}$$

After substituting Eqs. (19) and (20) into Eq. (16), carrying out the trace calculation, and integrating out the longitudinal momentum p_l, we obtain the normalization condition for the BS wave function for the scalar heavy diquark as the following:

$$\int \frac{d^3 p_t}{(2\pi)^3}\frac{E_D m_{Q_1} m_{Q_2}(\lambda_1 m_{Q_1}\omega_{Q_2} + \lambda_2 m_{Q_2}\omega_{Q_1})}{3m_D\omega^2_{Q_1}\omega^2_{Q_2}(-m_D + \omega_{Q_1} + \omega_{Q_2})^2} \times [\tilde{f}_{s_1}(p_t) + \tilde{f}_{s_2}(p_t)]^2 = 1, \tag{23}$$

where $E_D = P_D \cdot \varepsilon$.

For the axial-vector heavy diquark, the transverse projections of the BS wave function [$\tilde{\bar{\chi}}^{(r)}_{P_D}(p_t)$ and $\tilde{\bar{\chi}}^{(r)(c)}_{P_D}(p_t)$]

can be obtained from Eqs. (17) and (18) as follows, respectively:

$$\tilde{\tilde{\chi}}^{(r)}_{P_D}(p_t) = \tilde{f}_{v_1}(p_t)\xi^{(r)} + \tilde{f}_{v_2}(p_t)\not{v}_D\xi^{(r)}, \tag{24}$$

and

$$\tilde{\tilde{\chi}}^{(r)(c)}_{P_D}(p_t) = -\tilde{f}_{v_1}(p_t)\xi^{(r)} + \tilde{f}_{v_2}(p_t)\not{v}_D\xi^{(r)}, \tag{25}$$

where

$$\tilde{f}_{v_1}(p_t) = \int \frac{d^3p'_t}{(2\pi)^3}[V^{(1g)}(p_t - p'_t) + 2V^{(cf)}(p_t - p'_t)]\tilde{f}(p'_t), \tag{26}$$

and

$$\tilde{f}_{v_2}(p_t) = \int \frac{d^3p'_t}{(2\pi)^3} V^{(1g)}(p_t - p'_t)\tilde{f}(p'_t). \tag{27}$$

Analogously, the normalization condition for the BS wave function for the axial-vector heavy diquark is given by

$$\int \frac{d^3p_t}{(2\pi)^3} \frac{E_D m_{Q_1} m_{Q_2}(\lambda_1 m_{Q_1}\omega_{Q_2} + \lambda_2 m_{Q_2}\omega_{Q_1})}{3m_D\omega^2_{Q_1}\omega^2_{Q_2}(-m_D + \omega_{Q_1} + \omega_{Q_2})^2} \times [\tilde{f}_{v_1}(p_t) + \tilde{f}_{v_2}(p_t)]^2 = 1. \tag{28}$$

In the numerical calculations, we take the constituent masses of the heavy quarks to be $m_b = 4.88$ GeV and $m_c = 1.486$ GeV which were obtained by fitting the real spectra of charmonium and bottomonium in Ref. [48]. The parameters in the kernel $\alpha_s = 0.4$ and $\kappa = 0.18$ are determined by fitting the experimental data for heavy meson spectra [45]. In order to solve the integral Eq. (13), we discretize the integration region into n pieces (with n sufficiently large). Then Eq. (13) becomes an eigenvalue equation for the n dimensional vector $\tilde{f}$. After solving the eigenvalue equation, the heavy diquark masses are obtained and are displayed in Table I. We find that the heavy diquark masses are independent of the heavy diquark spin and only determined by the flavors of the constituent heavy quarks. In Fig. 1, the normalized BS scalar wave functions for the heavy diquarks are shown. It can be seen that the amplitudes of the BS scalar wave functions do not distinguish the different spins of the heavy diquarks. As discussed in Refs. [10,11], unlike the heavy quark and light quark system, only the spin symmetry survives (when the diquark mass is below ~10 GeV) in the leading order $1/m_Q$ expansion for the heavy diquark system. Our results are consistent with this statement.

TABLE I. Values of the heavy diquark masses used here.

m_{Q_1} (GeV)	4.88	4.88	1.486
m_{Q_2} (GeV)	4.88	1.486	1.486
m_D (GeV)	9.80	6.55	3.23

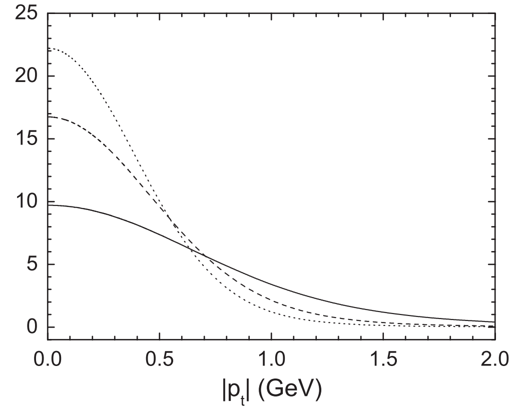

FIG. 1. The normalized BS scalar wave functions [$\tilde{f}(p_t)$] for the heavy diquarks. The solid, dashed, and dotted lines are for the heavy diquarks composed of double b quarks, b and c quarks, and double c quarks, respectively.

III. FORM FACTOR OF HEAVY DIQUARK COUPLING TO GLUON

Since the heavy diquark is not really a point object and its radius is enhanced by $\ln^2 m_Q$ with respect to $1/m_Q$, we introduce a few form factors for the effective vertex of the heavy diquark coupling to gluon to reflect the inner structure of the heavy diquark.

The effective current for the scalar heavy diquark coupling to a gluon is given as follows [5]:

$$J^\mu = ig_s\frac{\lambda^a}{2}(P^\mu_{D_f} + P^\mu_{D_i})F_s(Q^2), \tag{29}$$

where g_s is the coupling constant of the strong interaction, λ^a $(a = 1, 2, \ldots, 8)$ denote the Gell-Mann color matrices, $F_s(Q^2)$ is the form factor for the effective vertex, $P^\mu_{D_i}$ and $P^\mu_{D_f}$ are the momenta of the initial and final heavy diquarks, respectively, and Q^2 is the square of the momentum transfer.

On the other hand, the effective current for the scalar heavy diquark coupling to a gluon can be written as the following in the BS equation formalism (The Feynmann diagram for the vertex is shown in Fig. 2):

$$J^\mu = ig_s\frac{\lambda^a}{2}(M^\mu_1 + M^\mu_2), \tag{30}$$

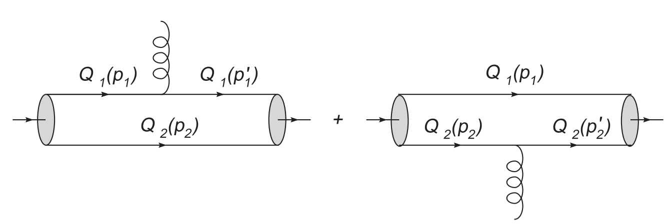

FIG. 2. The schematic diagram for the heavy diquark coupling to gluon. $Q_1(p_1^{(\prime)})$ and $Q_2(p_2^{(\prime)})$ stand for the heavy quarks Q_1 and Q_2 with momenta $p_1^{(\prime)}$ and $p_2^{(\prime)}$, respectively.

where

$$M_1^{\mu} = -\frac{1}{6}(2\pi)^4\delta^4(P_{D_i} - P_{D_f} - k)\int\frac{d^4p}{(2\pi)^4}\int\frac{d^4p'}{(2\pi)^4}(2\pi)^4\delta^4(p_2 - p_2') \times \mathrm{Tr}[S(-p_1)\tilde{\bar{\chi}}^{(c)}_{P_{D_f}}(p_t)S(p_2)\tilde{\bar{\chi}}_{P_{D_i}}(p_t')S(-p_1')v^{\mu}], \tag{31}$$

and

$$M_2^{\mu} = -\frac{1}{6}(2\pi)^4\delta^4(P_{D_i} - P_{D_f} - k)\int\frac{d^4p}{(2\pi)^4}\int\frac{d^4p''}{(2\pi)^4}(2\pi)^4\delta^4(p_1 - p_1'') \times \mathrm{Tr}[S(-p_1)\tilde{\bar{\chi}}^{(c)}_{P_{D_f}}(p_t)S(p_2)v^{\mu}S(p_2'')\tilde{\bar{\chi}}_{P_{D_i}}(p_t'')], \tag{32}$$

where k denotes the momentum carried by the gluon, p and $p'^{('')}$ in this section denote the relative momenta of final and initial heavy diquarks, respectively.

Then, comparing Eq. (29) with Eq. (30), one can get the form factor $[F_s(Q^2)]$ in the leading order of a $1/m_Q$ expansion as follows:

$$F_s(Q^2) = -\frac{(2\pi)^4}{3m_D}\int\frac{d^3p_t}{(2\pi)^3}\left\{\frac{m_{Q_1}^2 m_{Q_2}[\tilde{f}_{s1}(p_t) + \tilde{f}_{s2}(p_t)][\tilde{f}_{s1}(p_t') + \tilde{f}_{s2}(p_t')]}{\omega_{Q_1}\omega_{Q_2}\omega'_{Q_1}(m_D - \omega_{Q_1} - \omega_{Q_2})(m_D - \omega_{Q_2}\tilde{\omega} - |p_t|\sqrt{\tilde{\omega}^2 - 1}\cos\theta - \omega'_{Q_1})} + \frac{m_{Q_1}m_{Q_2}^2[\tilde{f}_{s1}(p_t) + \tilde{f}_{s2}(p_t)][\tilde{f}_{s1}(p_t'') + \tilde{f}_{s2}(p_t'')]}{\omega_{Q_1}\omega_{Q_2}\omega''_{Q_2}(m_D - \omega_{Q_1} - \omega_{Q_2})(m_D - \omega_{Q_1}\tilde{\omega} + |p_t|\sqrt{\tilde{\omega}^2 - 1}\cos\theta - \omega''_{Q_2})}\right\}, \tag{33}$$

where the velocity transfer $\tilde{\omega} = v_{D_i} \cdot v_{D_f}$ (v_{D_i} and v_{D_f} are the velocities of the initial and final heavy diquarks, respectively), $p_l'^{('')} = p'^{('')} \cdot v_{D_i}$ and $p_t'^{('')} = p'^{('')} - p_l'^{('')} v_{D_i}$ are the longitudinal and transverse projections of the initial heavy diquark relative momenta $[p'^{('')}]$ along their momenta (P_{D_i}), respectively, $\omega'_{Q_1} = \sqrt{m_{Q_1}^2 - p_t'^2}$, $\omega''_{Q_2} = \sqrt{m_{Q_2}^2 - p_t''^2}$, and θ is the angle between p_t and $P_{D_{it}}^{\mu} (= P_{D_i}^{\mu} - v_{D_f}^{\mu} P_{D_i} \cdot v_{D_f})$.

Now, let us turn to the effective current of the axial-vector heavy diquark coupling to gluon [5]:

$$\begin{aligned} J^{\alpha\mu\beta} = ig_s\frac{\lambda^a}{2}[&g^{\alpha\beta}(P_{D_f}^{\mu} + P_{D_i}^{\mu})F_{v_1}(Q^2) \\ &- (P_{D_f}^{\beta}g^{\mu\alpha} + P_{D_i}^{\alpha}g^{\mu\beta})F_{v_2}(Q^2) \\ &+ P_{D_i}^{\alpha}P_{D_f}^{\beta}(P_{D_f}^{\mu} + P_{D_i}^{\mu})F_{v_3}(Q^2)], \end{aligned} \tag{34}$$

where $F_{v_1}(Q^2)$, $F_{v_2}(Q^2)$, and $F_{v_3}(Q^2)$ are the form factors for the effective vertex.

Analogously, the effective current of the axial-vector heavy diquark coupling to the gluon can be written as follows in the BS equation formalism:

$$J^{\alpha\mu\beta} = ig_s\frac{\lambda^a}{2}(M_1^{\alpha\mu\beta} + M_2^{\alpha\mu\beta}), \tag{35}$$

where

$$\begin{aligned} M_1^{\alpha\mu\beta} = &-\frac{1}{6}(2\pi)^4\delta^4(P_{D_i} - P_{D_f} - k)\int\frac{d^4p}{(2\pi)^4}\int\frac{d^4p'}{(2\pi)^4} \\ &\times (2\pi)^4\delta^4(p_2 - p_2') \times \mathrm{Tr}[S(-p_1)\tilde{\bar{\chi}}^{(c)\alpha}_{P_{D_f}}(p_t) \\ &\times S(p_2)\tilde{\bar{\chi}}^{\beta}_{P_{D_i}}(p_t')S(-p_1')v^{\mu}], \end{aligned} \tag{36}$$

and

$$\begin{aligned} M_2^{\alpha\mu\beta} = &-\frac{1}{6}(2\pi)^4\delta^4(P_{D_i} - P_{D_f} - k)\int\frac{d^4p}{(2\pi)^4}\int\frac{d^4p''}{(2\pi)^4} \\ &\times (2\pi)^4\delta^4(p_1 - p_1'') \times \mathrm{Tr}[S(-p_1)\tilde{\bar{\chi}}^{(c)\alpha}_{P_{D_f}}(p_t) \\ &\times S(p_2)v^{\mu}S(p_2'')\tilde{\bar{\chi}}^{\beta}_{P_{D_i}}(p_t'')]. \end{aligned} \tag{37}$$

As discussed in Ref. [5], the contribution of the $F_{v_3}(Q^2)$ term is suppressed at small and intermediate momentum transfer, Q^2, since such a term is multiplied by high powers of momenta. Consequently, $F_{v_3}(Q^2)$ is ignored in our calculation. Comparing Eq. (34) with Eq. (35), we derive the other two form factors $[F_{v_1}(Q^2)$ and $F_{v_2}(Q^2)]$ in the leading order of a $1/m_Q$ expansion as follows:

$$F_{v_1}(Q^2) = -\frac{(2\pi)^4}{12m_D}\int\frac{d^3p_t}{(2\pi)^3}\left\{\frac{m_{Q_1}^2 m_{Q_2}[\tilde{f}_{v_1}(p_t)+\tilde{f}_{v_2}(p_t)][\tilde{f}_{v_1}(p_t')+\tilde{f}_{v_2}(p_t')]}{\omega_{Q_1}\omega_{Q_2}\omega_{Q_1}'(m_D-\omega_{Q_1}-\omega_{Q_2})(m_D-\omega_{Q_2}\tilde{\omega}-|p_t|\sqrt{\tilde{\omega}^2-1}\cos\theta-\omega_{Q_1}')}\right.$$
$$\left.+\frac{m_{Q_1}m_{Q_2}^2[\tilde{f}_{v_1}(p_t)+\tilde{f}_{v_2}(p_t)][\tilde{f}_{v_1}(p_t'')+\tilde{f}_{v_2}(p_t'')]}{\omega_{Q_1}\omega_{Q_2}\omega_{Q_2}''(m_D-\omega_{Q_1}-\omega_{Q_2})(m_D-\omega_{Q_1}\tilde{\omega}+|p_t|\sqrt{\tilde{\omega}^2-1}\cos\theta-\omega_{Q_2}'')}\right\}, \tag{38}$$
$$F_{v_2}(Q^2) = 0.$$

It can be seen that the form factors for both the effective vertex of the scalar heavy diquark coupling to the gluon and the effective vertex of the axial-vector heavy diquark coupling to the gluon are equal to each other in the leading order of a $1/m_Q$ expansion. So, we redefine the form factors as $F(Q^2) \equiv F_s(Q^2) = F_{v_1}(Q^2)$ for convenience.

It is well known that when $Q^2 \to 0$, the heavy diquark is seen by the gluon as a point particle without the inner structure, and hence the form factor for the effective vertex should be normalized to unity. When $Q^2 \to \infty$, the gluon can see the individual quarks inside the diquark, and hence the form factor for the effective vertex should approach to zero. We calculate the form factors with the BS wave functions obtained numerically for the heavy diquarks. The dependence of the form factors on the square of the momentum transfer is shown in Fig. 3. We can see that the behavior of our results coincides with the tendency of the above physical picture.

IV. BS EQUATION FOR DOUBLY HEAVY BARYONS

As discussed in Sec. I, the doubly heavy baryon can be regarded as a bound state of a heavy diquark and a light quark in the heavy quark limit.

Let us define the ratios $\eta_1 = m_l/(m_l + m_D)$ and $\eta_2 = m_D/(m_l + m_D)$ (m_l is the light quark mass) for convenience. The BS wave function for the doubly heavy baryon composed of a scalar heavy diquark and a light quark is defined as the following:

$$\chi_P(y_1, y_2) = \langle 0|T\psi_l(y_1)\phi_D(y_2)|P\rangle = e^{-iPY}\int\frac{d^4q}{(2\pi)^4}\chi_P(q)e^{-iqy}, \tag{39}$$

where ψ_l and ϕ_D stand for the light quark field and the scalar heavy diquark field, respectively, $Y \equiv \eta_1 y_1 + \eta_2 y_2$ is the coordinate of the doubly heavy baryon mass center, $y \equiv y_1 - y_2$ is the relative coordinate, P is the momentum of the doubly heavy baryon, and q is the relative momentum between the heavy diquark and the light quark.

It is straightforward to derive the following BS equation for the doubly heavy baryon containing a scalar heavy diquark and a light quark:

$$\chi_P(q) = S_l(q_1)\int\frac{d^4q'}{(2\pi)^4}G(P,q,q')\chi_P(q')S_D(q_2), \tag{40}$$

where $q_1 = \eta_1 P + q$ and $q_2 = \eta_2 P - q$ are the momenta of the light quark and the heavy diquark, respectively, $S_l(q_1)$ and $S_D(q_2)$ are the propagators of the light quark and the heavy diquark, respectively, G is the kernel which is, motivated by the potential model, given by [45]

$$-iG(P,q,q') = I\otimes IV_1(q,q') + \gamma_\mu\otimes\Gamma^\mu V_2(q,q'), \tag{41}$$

where $\Gamma^\mu = (q_2^\mu + q_2'^\mu)F(Q^2)$ is the effective vertex of a gluon with two scalar heavy diquarks, which has been derived in Sec. III. V_1 and V_2 are the scalar confinement and one-gluon-exchange terms given in the following, respectively, (after imposing covariant instantaneous approximation [41–44]):

$$V_1(q_t - q_t') = \frac{8\pi\kappa'}{[-(q_t-q_t')^2+\mu^2]^2} - (2\pi)^3\delta^3(q_t-q_t') \times\int\frac{d^3k_t}{(2\pi)^3}\frac{8\pi\kappa'}{[-(q_t-k_t)^2+\mu^2]^2}, \tag{42}$$

and

$$V_2(q_t - q_t') = -\frac{16\pi}{3}\frac{\alpha_s}{(q_t-q_t')^2-\mu^2}, \tag{43}$$

where $q_t^{(\prime)}$ is the transverse projection of the relative momentum (q) along the baryon momentum (P), the second term of V_1 is introduced to remove the infrared singularity

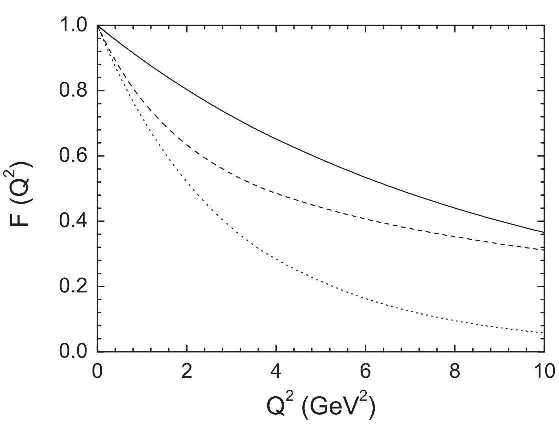

FIG. 3. The normalized form factors for the vertices of the heavy diquark coupling to gluon as a function of the square of the momentum transfer. The solid, dashed, and dotted lines represent the heavy diquark composed of double b quarks, b and c quarks, and double c quarks, respectively.

near the point $q_t = q'_t$, and the small parameter μ is introduced to avoid the divergence in the numerical calculations. As discussed in Ref. [7], the dimension of κ' is three and that of κ in the meson case is two. This extra dimension in κ' should be caused by nonperturbative diagrams which include the form factor effects at low momentum region. We expect that $\kappa' \sim \Lambda_{\rm QCD}\kappa$, since $\Lambda_{\rm QCD}$ is the only parameter related to confinement. In the numerical calculations, we let κ' vary between 0.01 GeV^3 and 0.06 GeV^3 [8,39,49].

The light quark propagator can be written as the following form [9,44]:

$$S_l(q_1) = i\not{v}\left[\frac{\Lambda_l^+}{\eta_1 M + q_l - \omega_l + i\varepsilon} + \frac{\Lambda_l^-}{\eta_1 M + q_l + \omega_l - i\varepsilon}\right], \tag{44}$$

where M and v are, respectively, the mass and velocity of the doubly heavy baryon, $q_l = q \cdot v$ and $q_t^\mu = q^\mu - q_l v^\mu$ are the longitudinal and transverse projections of the relative momentum (q) along the doubly heavy baryon momentum (P), respectively, $\omega_l = \sqrt{m_l^2 - q_t^2}$, and $\Lambda_l^\pm$ are the projection operators given by

$$\Lambda_l^\pm = \frac{\omega_l \pm \not{v}(\not{q}_t + m_l)}{2\omega_l}. \tag{45}$$

In the leading order of a $1/m_Q$ expansion, the propagator of the scalar heavy diquark can be written as

$$S_D(q_2) = \frac{i}{2m_D(\eta_2 M - q_l - m_D + i\varepsilon)}. \tag{46}$$

After writing down the most general form for the BS wave function and taking into account its property under parity transformation, we can parametrize the BS wave function for the doubly heavy baryon with a scalar heavy diquark and a light quark in the following form:

$$\chi_P(q) = (g_{s_1} + \not{q}_t g_{s_2})u(v), \tag{47}$$

where $u(v)$ is the spinor of the doubly heavy baryon, g_{s_1} and g_{s_2} are the Lorentz-scalar functions of q_t^2, q_l and $P^2 = M^2$.

Defining $\tilde{g}_{s_{1(2)}}(q_t) \equiv \int \frac{dq_l}{2\pi} g_{s_{1(2)}}$, one finds that the BS scalar wave functions satisfy the coupled integral equations as follows:

$$\begin{aligned}\tilde{g}_{s_1}(q_t) = &-\int \frac{d^3q'_t}{(2\pi)^3}\frac{(m_l+\omega_l)[V_1(q_t-q'_t)+2m_DF(Q^2)V_2(q_t-q'_t)]}{4\omega_l m_D(M-m_D-\omega_l)}\tilde{g}_{s_1}(q'_t)\\ &-\int \frac{d^3q'_t}{(2\pi)^3}\frac{V_1(q_t-q'_t)-2m_DF(Q^2)V_2(q_t-q'_t)}{4\omega_l m_D(M-m_D-\omega_l)}q_t\cdot q'_t\tilde{g}_{s_2}(q'_t),\end{aligned} \tag{48}$$

$$\begin{aligned}\tilde{g}_{s_2}(q_t) = &-\int \frac{d^3q'_t}{(2\pi)^3}\frac{V_1(q_t-q'_t)+2m_DF(Q^2)V_2(q_t-q'_t)}{4\omega_l m_D(M-m_D-\omega_l)}\tilde{g}_{s_1}(q'_t)\\ &-\int \frac{d^3q'_t}{(2\pi)^3}\frac{(m_l-\omega_l)[V_1(q_t-q'_t)-2m_DF(Q^2)V_2(q_t-q'_t)]}{4\omega_l m_D(M-m_D-\omega_l)}\frac{q_t\cdot q'_t}{q_t^2}\tilde{g}_{s_2}(q'_t).\end{aligned} \tag{49}$$

The normalization condition for the doubly heavy baryon with a scalar heavy diquark and a light quark is given by (after imposing the covariant instantaneous approximation on the kernel)

$$i\delta^{i_1i_2}_{j_1j_2}\int\frac{d^4qd^4q'}{(2\pi)^8}\bar{\chi}_P(q,s)\left[\frac{\partial}{\partial P_0}I_P(q,q')\right]^{i_1i_2j_2j_1} \times \chi_P(q',s') = \delta_{ss'}, \tag{50}$$

where $i_{1(2)}$ and $j_{1(2)}$ represent the color indices of the heavy diquark and the light quark, respectively, $s^{(\prime)}$ is the spin index for the doubly heavy baryon and $I_P^{i_1i_2j_2j_1}$ is the inverse of the four point propagator defined as follows:

$$I_P^{i_1i_2j_2j_1}(q,q') = \delta^{i_1j_1}\delta^{i_2j_2}(2\pi)^4\delta^4(q-q')S_l^{-1}(q_1)S_D^{-1}(q_2). \tag{51}$$

After some algebra, Eq. (50) can be written in the following form:

$$\begin{aligned}-\frac{i}{6}\int\frac{d^4q}{(2\pi)^4}\{&\mathrm{Tr}[\tilde{\chi}_P(q_t)\bar{\tilde{\chi}}_P(q_t)S_l(q_1)(-i\eta_1\not{\varepsilon})S_l(q_1)S_D(q_2)]\\ &+\mathrm{Tr}[\tilde{\chi}_P(q_t)\bar{\tilde{\chi}}_P(q_t)(-2i\eta_2)q_2\cdot\varepsilon S_l(q_1)S_D^2(q_2)]\} = 1,\end{aligned} \tag{52}$$

where $\varepsilon = (1, \vec{0})$, $\tilde{\chi}_P(q_t)$ and $\bar{\tilde{\chi}}_P(q_t)$ are the transverse projections of the BS wave functions given as follows:

$$\tilde{\chi}_P(q_t) = -iS_l(q_1)^{-1}\chi_P(q)S_D(q_2)^{-1}, \tag{53}$$

and

$$\bar{\tilde{\chi}}_P(q_t) = -iS_D(q_2)^{-1}\bar{\chi}_P(q)S_l(q_1)^{-1}, \tag{54}$$

respectively.

Then, one can derive the transverse projections of the BS wave functions from Eqs. (53) and (54), respectively:

$$\tilde{\chi}_P(q_t) = [\tilde{h}_{s_1}(q_t) + \not{q}_t\tilde{h}_{s_2}(q_t)]u(v), \tag{55}$$

and

$$\tilde{\bar{\chi}}_P(q_t) = \bar{u}(v)[\tilde{h}_{s_1}(q_t) + \not{q}_t \tilde{h}_{s_2}(q_t)], \quad (56)$$

where

$$\tilde{h}_{s_1}(q_t) = \int \frac{d^3 q'_t}{(2\pi)^3}[V_1(q_t - q'_t) + 2m_D F(Q^2) \times V_2(q_t - q'_t)]\tilde{g}_{s_1}(q'_t), \quad (57)$$

and

$$\tilde{h}_{s_2}(q_t) = \int \frac{d^3 q'_t}{(2\pi)^3}[V_1(q_t - q'_t) - 2m_D F(Q^2) V_2(q_t - q'_t)] \times \frac{q_t \cdot q'_t}{q_t^2} \tilde{g}_{s_2}(q'_t). \quad (58)$$

After substituting Eqs. (55) and (56) into Eq. (52) and integrating out the longitudinal momentum q_l, the normalization condition can be written in the following form:

$$\int \frac{d^3 q_t}{(2\pi)^3} \frac{1}{24 M m_D \omega_l^3(-M + m_D + \omega_l)^2}\{(m_l + \omega_l)(2\eta_2 M \omega_l^2 + E\eta_1(m_l m_D - \omega_l m_D + 2m_l\omega_l + M\omega_l - Mm_l) + E\eta_1(-M + m_D + 2\omega_l)q_t^2)\tilde{h}_{s_1}^2(q_t) + 4\omega_l^2 q_t^2(\eta_2 M + \eta_1 E)\tilde{h}_{s_1}(q_t)\tilde{h}_{s_2}(q_t) + (2\eta_2 M(m_l - \omega_l)q_t^2\omega_l^2 + E\eta_1(-M + m_D + 2\omega_l)q_t^4 + E\eta_1(m_l - \omega_l)(-m_l m_D - \omega_l m_D - 2m_l\omega_l + M(m_l + \omega_l))q_t^2\tilde{h}_{s_2}^2(q_t)\} = 1, \quad (59)$$

where $E = P \cdot \varepsilon$.

Now let us define the BS wave function for the doubly heavy baryon composed of an axial-vector heavy diquark and a light quark as follows:

$$\chi_P^\mu(y_1, y_2) = \langle 0|T\psi_l(y_1)A_D^\mu(y_2)|P\rangle = e^{-iPY}\int \frac{d^4 q}{(2\pi)^4}\chi_P^\mu(q)e^{-iqy}, \quad (60)$$

where $A_D^\mu(y_2)$ stands for the axial-vector heavy diquark field.

The BS equation for the doubly heavy baryon with an axial-vector heavy diquark and a light quark is given by

$$\chi_P^\mu(q) = S_l(q_1)\int \frac{d^4 q'}{(2\pi)^4} G_{\rho\nu}(P, q, q')\chi_P^\nu(q')S_D^{\mu\rho}(q_2), \quad (61)$$

where $S_D^{\mu\rho}(q_2)$ is the propagator of the axial-vector heavy diquark and $G_{\rho\nu}$ is the kernel for the BS equation given by

$$-iG_{\rho\nu}(P, q, q') = -g_{\rho\nu} I \otimes I V_1(q, q') - \gamma^\mu \otimes \Gamma_{\mu\rho\nu} V_2(q, q'), \quad (62)$$

where $\Gamma_{\mu\rho\nu} = (q_{2\mu} + q'_{2\mu})g_{\rho\nu}F(Q^2)$ is the effective vertex for the axial-vector heavy diquark coupling to the gluon, which was derived in Sec. III.

In the leading order of a $1/m_Q$ expansion, the propagator of the axial-vector heavy diquark can be written as

$$S_D^{\mu\nu}(q_2) = -i\frac{g^{\mu\nu} - v^\mu v^\nu}{2m_D(\eta_2 M - q_l - m_D + i\varepsilon)}. \quad (63)$$

Similar to the case of the doubly heavy baryon containing a scalar heavy diquark, we can parametrize the BS wave function for the doubly heavy baryon containing an axial-vector heavy quark and a light quark in the following form:

$$\chi_P^\mu(q) = (g_{v_1} + \not{q}_t g_{v_2})u^\mu(v), \quad (64)$$

where $u^\mu(v)$ is the spinor of the heavy baryon, g_{v_1} and g_{v_2} are the Lorentz-scalar functions of q_t^2, q_l and $P^2 = M^2$. When the spin of the doubly heavy baryon is $1/2$, $u^\mu(v) = \frac{1}{\sqrt{3}}(\gamma^\mu + v^\mu)\gamma^5 u(v)$, while the spin of the doubly heavy baryon is $3/2$, $u^\mu(v)$ is the Rarita-Schwinger vector spinor.

After defining $\tilde{g}_{v_{1(2)}}(q_t) = \int \frac{dp_l}{2\pi} g_{v_{1(2)}}$, one can find that the BS scalar wave functions satisfy the coupled integral equations in the following:

$$\tilde{g}_{v_1}(q_t) = -\int \frac{d^3 q'_t}{(2\pi)^3}\frac{(m_l + \omega_l)[V_1(q_t - q'_t) + 2m_D F(Q^2)V_2(q_t - q'_t)]}{4\omega_l m_D(M - m_D - \omega_l)}\tilde{g}_{v_1}(q'_t) - \int \frac{d^3 q'_t}{(2\pi)^3}\frac{V_1(q_t - q'_t) - 2m_D F(Q^2)V_2(q_t - q'_t)}{4\omega_l m_D(M - m_D - \omega_l)} q_t \cdot q'_t \tilde{g}_{v_2}(q'_t), \quad (65)$$

$$\tilde{g}_{v_2}(q_t) = -\int \frac{d^3 q'_t}{(2\pi)^3}\frac{V_1(q_t - q'_t) + 2m_D F(Q^2)V_2(q_t - q'_t)}{4\omega_l m_D(M - m_D - \omega_l)}\tilde{g}_{v_1}(q'_t) - \int \frac{d^3 q'_t}{(2\pi)^3}\frac{(m_l - \omega_l)[V_1(q_t - q'_t) - 2m_D F(Q^2)V_2(q_t - q'_t)]}{4\omega_l m_D(M - m_D - \omega_l)}\frac{q_t \cdot q'_t}{q_t^2}\tilde{g}_{v_2}(q'_t). \quad (66)$$

The normalization condition for the BS wave function for the doubly heavy baryon with an axial-vector heavy diquark and a light quark is given by

$$i\delta^{i_1 i_2}_{j_1 j_2}\int \frac{d^4 q d^4 q'}{(2\pi)^8}\bar{\chi}^{\mu}_P(q,s)\left[\frac{\partial}{\partial P_0} I_{P\mu\nu}(q,q')\right]^{i_1 i_2 j_2 j_1}\chi^{\nu}_P(q',s') = \delta_{ss'}, \tag{67}$$

where $I^{i_1 i_2 j_2 j_1}_{P\mu\nu}$ is the inverse of the four point propagator defined as follows:

$$I^{i_1 i_2 j_2 j_1}_{P\mu\nu}(q,q') = \delta^{i_1 j_1}\delta^{i_2 j_2}(2\pi)^4\delta^4(q-q')S_l^{-1}(q_1)S^{-1}_{D\mu\nu}(q_2). \tag{68}$$

After some algebra, Eq. (67) can be written in the following form:

$$\begin{aligned}-\frac{i}{6}\int \frac{d^4 q}{(2\pi)^4} &\times \{\mathrm{Tr}[\tilde{\chi}_{P\sigma}(q_t)\tilde{\bar{\chi}}_{P\rho}(q_t)S_l(q_1)(-i\eta_1\not{\varepsilon})S_l(q_1)S_D^{\sigma\rho}(q_2)]\\ &-\mathrm{Tr}[\tilde{\chi}_{P\sigma}(q_t)\tilde{\bar{\chi}}_{P\rho}(q_t)(i\eta_2)(2q_2\cdot\varepsilon g_{\mu\nu}-\varepsilon_\mu q_{2\nu}-q_{2\mu}\varepsilon_\nu)\\ &\times S_l(q_1)S_D^{\sigma\nu}(q_2)S_D^{\mu\rho}(q_2)]\} = 1,\end{aligned} \tag{69}$$

where $\tilde{\chi}_{P\sigma}(q_t)$ and $\tilde{\bar{\chi}}_{P\rho}(q_t)$ are the transverse projections of the BS wave functions given by

$$\tilde{\chi}_{P\sigma}(q_t) = -iS_l(q_1)^{-1}\chi^{\alpha}_P(q)S_{D\alpha\sigma}(q_2)^{-1}, \tag{70}$$

and

$$\tilde{\bar{\chi}}_{P\rho}(q_t) = -iS_{D\rho\beta}(q_2)^{-1}\bar{\chi}^{\beta}_P(q)S_l(q_1)^{-1}, \tag{71}$$

respectively.

From Eqs. (70) and (71), we get the expressions for the transverse projections of the BS wave functions as follows:

$$\tilde{\chi}_{P\sigma}(q_t) = [\tilde{h}_{v_1}(q_t) + \not{q}_t\tilde{h}_{v_2}(q_t)]u_\sigma(v), \tag{72}$$

and

$$\tilde{\bar{\chi}}_{P\rho}(q_t) = \bar{u}_\rho(v)[\tilde{h}_{v_1}(q_t) + \not{q}_t\tilde{h}_{v_2}(q_t)], \tag{73}$$

where

$$\tilde{h}_{v_1}(q_t) = \int\frac{d^3 q'_t}{(2\pi)^3}[V_1(q_t-q'_t) + 2m_D F(Q^2)\times V_2(q_t-q'_t)]\tilde{g}_{v_1}(q'_t), \tag{74}$$

and

$$\tilde{h}_{v_2}(q_t) = \int\frac{d^3 q'_t}{(2\pi)^3}[V_1(q_t-q'_t) - 2m_D F(Q^2)\times V_2(q_t-q'_t)]\frac{q_t\cdot q'_t}{q_t^2}\tilde{g}_{v_2}(q'_t). \tag{75}$$

After substituting Eqs. (72) and (73) into Eq. (69) and integrating out the longitudinal momentum q_l, the normalization condition can be written in the following form:

$$\begin{aligned}\int\frac{d^3 q_t}{(2\pi)^3}\frac{1}{24Mm_D\omega_l^3(-M+m_D+\omega_l)^2}&\{(m_l+\omega_l)[2\eta_2 M\omega_l^2 + E\eta_1(m_l m_D - \omega_l m_D + 2m_l\omega_l + M\omega_l - Mm_l)\\ &+E\eta_1(-M+m_D+2\omega_l)q_t^2]\tilde{h}^2_{v_1}(q_t) + 4\omega_l^2 q_t^2(\eta_2 M+\eta_1 E)\tilde{h}_{v_1}(q_t)\tilde{h}_{v_2}(q_t) + (2\eta_2 M(m_l-\omega_l)q_t^2\omega_l^2\\ &+E\eta_1(-M+m_D+2\omega_l)q_t^4 + E\eta_1(m_l-\omega_l)(-m_l m_D - \omega_l m_D - 2m_l\omega_l + M(m_l+\omega_l))q_t^2\tilde{h}^2_{v_2}(q_t)\} = 1.\end{aligned} \tag{76}$$

In our calculation, we take the constituent masses of the light quarks as $m_u = m_d = 0.33$ GeV, $m_s = 0.45$ GeV, and the scale of nonperturbative interaction $\Lambda_{\mathrm{QCD}} \simeq 0.2$ GeV. In order to solve the coupled integral Eqs. (48) and (49), we discretize the integration region into n pieces (with n sufficiently large). In this way the integral equations are transformed into coupled matrix equations for the n dimensional vectors $\tilde{g}_{s_{1(2)}}$. Then, it is easy to obtain the eigenvalue equations for $\tilde{g}_{s_{1(2)}}$. The same method is applied in dealing with the coupled integral Eqs. (65) and (66). After solving the eigenvalue equations, we obtain masses of the doubly heavy baryons shown in Table II. The mass of the Ξ_{cc} obtained in our model is consistent with the experimental value, 3518.9 ± 0.9 MeV [50]. The obtained BS scalar wave functions for the doubly heavy baryons composed of a heavy diquark and a light quark are displayed in Figs. 4 and 5. One finds that the masses and the amplitudes of BS scalar wave functions for the doubly heavy baryons are independent of the spins of both the heavy diquarks and the doubly heavy baryons. In fact, this is just the consequence of the heavy diquark spin symmetry. So, we redefine $\tilde{g}_1 \equiv \tilde{g}_{s(v)_1}$ and $\tilde{g}_2 \equiv \tilde{g}_{s(v)_2}$ in Figs. 4 and 5 for convenience.

TABLE II. Values of the masses of baryons containing two heavy quarks. The lower (upper) masses correspond to $\kappa' = 0.01\,(0.06)$ GeV.

	Ξ_{bb}	Ξ_{bc}	Ξ_{cc}	Ω_{bb}	Ω_{bc}	Ω_{cc}
M (GeV)	$10.08 \sim 10.10$	$6.83 \sim 6.85$	$3.52 \sim 3.56$	$10.18 \sim 10.19$	$6.94 \sim 6.95$	$3.62 \sim 3.65$

V. NONLEPTONIC DECAY OF DOUBLY HEAVY BARYONS

In this section, we will apply the obtained BS wave functions to calculate the nonleptonic decay widths for the doubly heavy baryons emitting a pseudoscalar meson in the BS formalism. The Hamiltonian describing such decays reads [51]

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{UD}^{*} (a_1 O_1 + a_2 O_2), \tag{77}$$

where V_{cb} and V_{UD}^* are the elements of Cabibbo-Kobayashi-Maskawa matrix, U and D stand for the fields of the light quarks involved in the decay, $O_1 = [\bar{D}\gamma^{\sigma}(1-\gamma^5)U][\bar{c}\gamma_{\sigma}(1-\gamma^5)b]$ and $O_2 = [\bar{c}\gamma^{\sigma}(1-\gamma^5)U][\bar{D}\gamma_{\sigma}(1-\gamma^5)b]$. a_1 and a_2 in Eq. (77) are defined as the linear combination of Wilson coefficients (c_1 and c_2), $a_1 = c_1 + c_2/N_c$ and $a_2 = c_2 + c_1/N_c$, where N_c is an effective number of colors which includes nonfactorizable color-octet effects in the hadronization process. Because of the lack of knowledge about

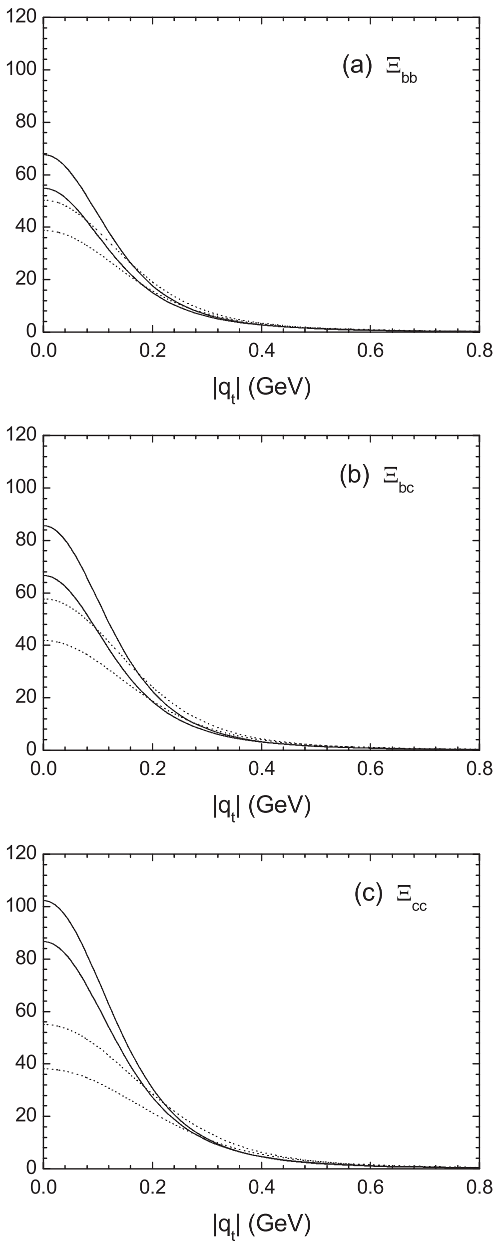

FIG. 4. The normalized BS scalar wave functions for the doubly heavy baryons ($\Xi_{QQ'}$) composed of a heavy diquark and a light quark (u, d). The solid (dotted) lines are for $\kappa' = 0.01$ (0.06) GeV. The upper solid and dotted lines are for $\tilde{g}_2(q_t)$ in unit of GeV^{-1}. The lower solid and dotted lines are for $\tilde{g}_1(q_t)$.

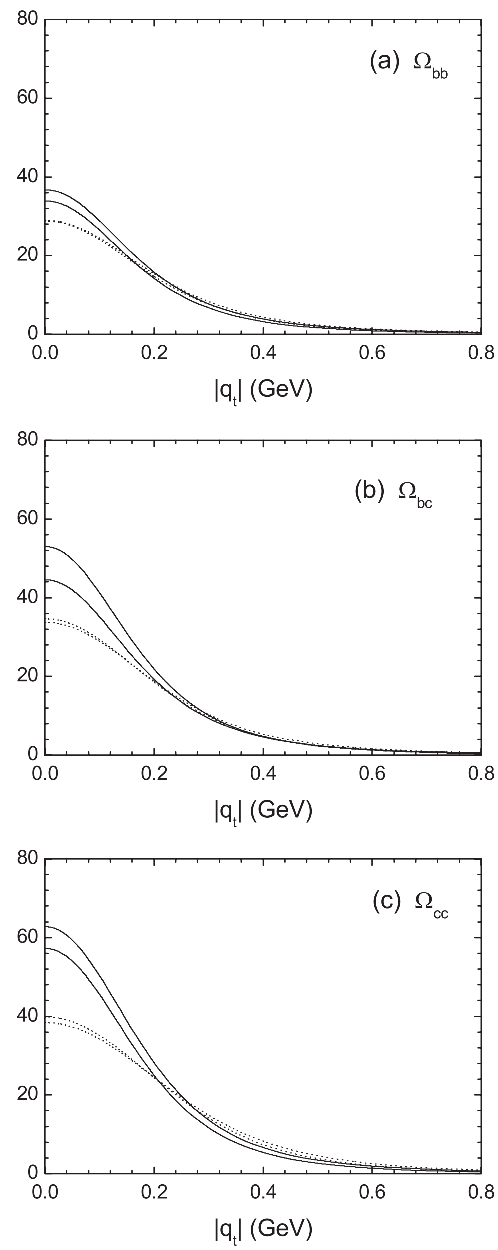

FIG. 5. The normalized BS scalar wave functions for the doubly heavy baryons ($\Omega_{QQ'}$) composed of a heavy diquark and a strange light quark (s). The solid (dotted) lines are for $\kappa' = 0.01$ (0.06) GeV. The upper solid and dotted lines are for $\tilde{g}_2(q_t)$ in unit of GeV^{-1}. The lower solid and dotted lines are for $\tilde{g}_1(q_t)$.

hadronization, a_1 and a_2 are treated as free parameters and determined by fitting experimental data [52,53]. Since $b \to c$ decays are energetic, the factorization assumption can be applied in our calculation. Hence the decay amplitude of the two body nonleptonic decay becomes the product of two matrix elements: one is related to the decay constant of the factorized pseudoscalar meson and the other is the weak transition matrix between the initial and final doubly heavy baryon states,

$$\begin{aligned}\langle \Xi_{Qc}^{(*)}(P_f)P(k)|\Xi_{Qb}^{(*)}(P_i)\rangle &= -\frac{i}{\sqrt{2}}G_F V_{cb}V_{UD}^* a_1 \int d^4z \langle P(k)|\bar{D}(z) \\ &\quad\times \gamma_\sigma(1-\gamma^5)U(z)|0\rangle e^{-ikz} \\ &\quad\times \langle \Xi_{Qc}^{(*)}(P_f)|\bar{b}(z)\gamma^\sigma(1-\gamma^5)c(z)|\Xi_{Qb}^{(*)}(P_i)\rangle, \end{aligned} \tag{78}$$

where $P(k)$ stands for the pseudoscalar meson with momentum k, $\Xi_{QQ'}^{(*)}$ stands for the doubly heavy baryon composed of a (an) scalar (axial-vector) heavy diquark and a light u (or d) quark.

The first matrix element on the right hand side of Eq. (78) is related to the decay constant of the pseudoscalar meson, f_P, which is defined as

$$\langle P(k)|\bar{D}(z)\gamma_\sigma(1-\gamma^5)U(z)|0\rangle = -if_P k_\sigma. \tag{79}$$

The decay amplitude [Eq. (78)] is classified into three cases according to different initial and final doubly heavy baryon states as follows:

$$\mathcal{M}_1 = \langle \Xi_{Qc}(P_f)P(k)|\Xi_{Qb}^*(P_i)\rangle, \tag{80}$$

$$\mathcal{M}_2 = \langle \Xi_{Qc}^*(P_f)P(k)|\Xi_{Qb}^*(P_i)\rangle, \tag{81}$$

$$\mathcal{M}_3 = \langle \Xi_{Qc}^*(P_f)P(k)|\Xi_{Qb}(P_i)\rangle. \tag{82}$$

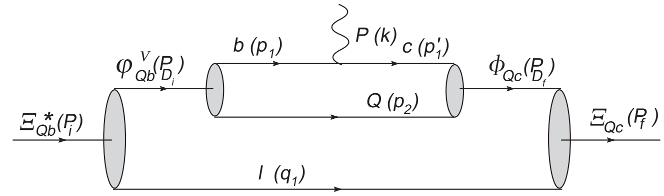

FIG. 6. The Feynman diagram for the nonleptonic decays of doubly heavy baryons emitting a pseudoscalar meson (P), taking the decay amplitude $\mathcal{M}_1$ for instance. $\Xi_{Qc}(P_f)$ and $\Xi_{Qb}^*(P_i)$ stand for the states of the final and initial doubly heavy baryons with momenta P_f and P_i, respectively. $\phi_{Qc}(P_{D_f})$ and $\varphi_{Qb}^\nu(P_{D_i})$ stand for the diquark states involved in the final and initial doubly heavy baryons with momenta P_{D_f} and P_{D_i}, respectively. $b(p_1)$, $c(p_1')$, $Q(p_2)$, and $l(q_1)$ stand for different quark fields with corresponding momenta. $P(k)$ stands for the emitted meson with momentum k.

Let us first calculate the decay amplitude $\mathcal{M}_1$. The Feynman diagram for $\mathcal{M}_1$ is shown in Fig. 6. $\mathcal{M}_1$ is related to the BS wave functions through the following equation:

$$\begin{aligned}\mathcal{M}_1 &= -\frac{1}{\sqrt{2}}G_F V_{cb}V_{UD}^* a_1 f_P k_\sigma \int d^4(x_1x_2y_1y_2z_1z_2z) \\ &\quad\times e^{-ikz}\bar{\chi}_{P_f}(x_2,x_1)S_l^{-1}(x_1-y_1)\chi_{P_i}^\mu(y_1,y_2) \\ &\quad\times S_D^{-1}(x_2-z_1)S_{D\mu\nu}^{-1}(z_2-y_2)\langle 0|\phi_{Qc}(z_1)\bar{b}(z) \\ &\quad\times \gamma^\sigma(1-\gamma^5)c(z)\varphi_{Qb}^\nu(z_2)|0\rangle, \end{aligned}\tag{83}$$

where $\bar{\chi}_{P_f}$ and $\chi_{P_i}^\mu$ stand for the BS wave functions for the final and initial doubly heavy baryons, respectively, ϕ_{Qc} and φ_{Qb}^ν are the diquark field operators involved in the final and initial doubly heavy baryons, respectively.

The matrix element in Eq. (83) is related to the BS wave functions for the heavy diquarks through the following equation:

$$\begin{aligned}\langle 0|\phi_{Qc}(z_1)\bar{b}(z)\gamma^\sigma(1-\gamma^5)c(z)\varphi_{Qb}^\nu(z_2)|0\rangle &= \sum_\lambda \int \frac{d^4P_{D_i}d^4P_{D_f}}{(2\pi)^8}(2\pi)^2\delta\Big(P_{D_fl}-\sqrt{m_{D_f}^2-P_{D_ft}^2}\Big)\delta\Big(P_{D_il}-\sqrt{m_{D_i}^2-P_{D_it}^2}\Big) \\ &\quad\times\frac{1}{6}\int\frac{d^4pd^4p'}{(2\pi)^8}(2\pi)^4\delta^4(p_2-p_2')\,\mathrm{Tr}[S(-p_1')\tilde{\bar{\chi}}_{P_{D_f}}^{(c)}(p_t')S(p_2)\tilde{\bar{\chi}}_{P_{D_i}}^{(\lambda)}(p_t) \\ &\quad\times S(-p_1)(1-\gamma^5)\gamma^\sigma]\xi^{\nu(\lambda)}\times e^{-iz(P_{D_f}-P_{D_i}+k)}e^{-iz_1P_{D_f}}e^{-iz_2P_{D_i}}, \end{aligned}\tag{84}$$

where $\tilde{\bar{\chi}}_{P_{D_f}}^{(c)}$ and $\tilde{\bar{\chi}}_{P_{D_i}}^{(\lambda)}$ are the BS wave functions for the heavy diquarks involved in the final and initial doubly heavy baryons, respectively, m_{D_f} and m_{D_i} are the masses of the heavy diquarks involved in the final and initial doubly heavy baryons, respectively, $P_{D_{f(i)}l} = P_{D_{f(i)}}\cdot v_i$ (v_i denotes the velocity of the initial doubly heavy baryon) and $P_{D_{f(i)}t}^\mu = P_{D_{f(i)}}^\mu - P_{D_{f(i)}l}v_i^\mu$ are the longitudinal and transverse projections of the final (initial) heavy diquark momentum ($P_{D_{f(i)}}$) along the initial doubly heavy baryon momentum (P_i), respectively, and $\xi^{\nu(\lambda)}$ is the λth polarization vector of the axial-vector heavy diquark involved in the initial doubly heavy baryon.

Substituting Eq. (84) into Eq. (83), the decay amplitude $\mathcal{M}_1$ can be written in momentum space as follows:

$$\mathcal{M}_1 = -\frac{1}{\sqrt{2}} G_F V_{cb} V_{UD}^* a_1 f_P k_\sigma (2\pi)^4 \delta^4(P_i - P_f - k) \int \frac{d^4q d^4q'}{(2\pi)^8} (2\pi)^4 \delta^4(q_1 - q_1') \times (2\pi)^2 \delta\left(P_{D_f l} - \sqrt{m_{D_f}^2 - P_{D_f t}^2}\right) \delta\left(P_{D_i l} - \sqrt{m_{D_i}^2 - P_{D_i t}^2}\right) \tilde{\bar{\chi}}_{P_f}(q_t') S_l(q_1) \tilde{\chi}_{P_{i\nu}}(q_t) \times \frac{1}{6} \sum_\lambda \int \frac{d^4pd^4p'}{(2\pi)^8} (2\pi)^4 \delta^4(p_2 - p_2') \mathrm{Tr}[S(-p_1') \tilde{\bar{\chi}}_{P_{D_f}}^{(c)}(p_t') S(p_2) \tilde{\bar{\chi}}_{P_{D_i}}^{(\lambda)}(p_t) S(-p_1)(1-\gamma^5)\gamma^\sigma] \xi^{\nu(\lambda)}. \tag{85}$$

Analogously, we can derive the other two decay amplitudes ($\mathcal{M}_2$ and $\mathcal{M}_3$) in the BS formalism as follows:

$$\mathcal{M}_2 = -\frac{1}{\sqrt{2}} G_F V_{cb} V_{UD}^* a_1 f_P k_\sigma (2\pi)^4 \delta^4(P_i - P_f - k) \int \frac{d^4q d^4q'}{(2\pi)^8} (2\pi)^4 \delta^4(q_1 - q_1') \times (2\pi)^2 \delta\left(P_{D_f l} - \sqrt{m_{D_f}^2 - P_{D_f t}^2}\right) \delta\left(P_{D_i l} - \sqrt{m_{D_i}^2 - P_{D_i t}^2}\right) \tilde{\bar{\chi}}_{P_{f\tau}}(q_t') S_l(q_1) \tilde{\chi}_{P_{i\nu}}(q_t) \times \frac{1}{6} \sum_{\lambda\lambda'} \int \frac{d^4pd^4p'}{(2\pi)^8} (2\pi)^4 \delta^4(p_2 - p_2') \mathrm{Tr}[S(-p_1') \tilde{\bar{\chi}}_{P_{D_f}}^{(\lambda')(c)}(p_t') S(p_2) \tilde{\bar{\chi}}_{P_{D_i}}^{(\lambda)}(p_t) S(-p_1)(1-\gamma^5)\gamma^\sigma] \xi^{\tau(\lambda')} \xi^{\nu(\lambda)}, \tag{86}$$

and

$$\mathcal{M}_3 = -\frac{1}{\sqrt{2}} G_F V_{cb} V_{UD}^* a_1 f_P k_\sigma (2\pi)^4 \delta^4(P_i - P_f - k) \int \frac{d^4q d^4q'}{(2\pi)^8} (2\pi)^4 \delta^4(q_1 - q_1') \times (2\pi)^2 \delta\left(P_{D_f l} - \sqrt{m_{D_f}^2 - P_{D_f t}^2}\right) \delta\left(P_{D_i} - \sqrt{m_{D_i}^2 - P_{D_i t}^2}\right) \tilde{\bar{\chi}}_{P_{f\tau}}(q_t') S_l(q_1) \tilde{\chi}_{P_i}(q_t) \times \frac{1}{6} \sum_{\lambda'} \int \frac{d^4pd^4p'}{(2\pi)^8} (2\pi)^4 \delta^4(p_2 - p_2') \mathrm{Tr}[S(-p_1') \tilde{\bar{\chi}}_{P_{D_f}}^{(\lambda')(c)}(p_t') S(p_2) \tilde{\bar{\chi}}_{P_{D_i}}(p_t) S(-p_1)(1-\gamma^5)\gamma^\sigma] \xi^{\tau(\lambda')}, \tag{87}$$

where $\xi^{\tau(\lambda')}$ is the λ'th polarization vector of the axial-vector heavy diquark involved in the final doubly heavy baryon.

The differential decay width for the two body decay reads [50]

$$d\Gamma = \frac{1}{32\pi^2} |\mathcal{M}|^2 \frac{|\mathbf{k}|}{M_i^2} d\tilde{\Omega}, \tag{88}$$

where $\tilde{\Omega}$ denotes the solid angle, $\mathcal{M}$ stands for the decay amplitude, M_i is the mass of the initial baryon, and $|\mathbf{k}|$ is the absolute value of the three momentum of the particles in the final state in the rest frame of the initial state.

Numerically, the parameters in the Hamiltonian (G_F, V_{cb}, and V_{UD}), and the masses and the decay constants of the pseudoscalar mesons are taken to have the following values [50]: $G_F = 1.16637 \times 10^{-5}$ GeV^{-2}, $V_{cb} = 0.0412$, $V_{ud} = 0.97418$, $V_{us} = V_{cd} = 0.2255$, $V_{cs} = 0.9742$, $m_\pi = 0.1396$ GeV, $m_K = 0.4937$ GeV, $m_D = 1.8696$ GeV, $m_{D_s} = 1.9685$ GeV, $f_\pi = 0.1304$ GeV, $f_K = 0.1555$ GeV, $f_D = 0.2058$ GeV, and $f_{D_s} = 0.273$ GeV. Our predictions for the nonleptonic decay widths for the doubly heavy baryons emitting a pseudoscalar meson are shown in Tables III, IV, V, and VI, where the superscripts $\frac{1}{2}$ and $\frac{3}{2}$ denote the spin of the doubly heavy baryons.

VI. SUMMARY AND DISCUSSION

In the heavy quark limit, a doubly heavy baryon can be regarded as a bound state composed of a heavy diquark and a light quark. We first establish the BS equations for both the heavy diquarks and the doubly heavy baryons,

TABLE III. Predictions for the nonleptonic decay widths for the doubly heavy baryons emitting π meson.

Process	Decay width ($10^{-14}a_1^2$ GeV)	Process	Decay width ($10^{-14}a_1^2$ GeV)
$\Gamma(\Xi_{bb}^{*(1/2)} \to \Xi_{bc}^{(1/2)}\pi)$	$0.343 \sim 0.362$	$\Gamma(\Omega_{bb}^{*(1/2)} \to \Omega_{bc}^{(1/2)}\pi)$	$0.591 \sim 0.607$
$\Gamma(\Xi_{bb}^{*(1/2)} \to \Xi_{bc}^{*(1/2)}\pi)$	$0.205 \sim 0.211$	$\Gamma(\Omega_{bb}^{*(1/2)} \to \Omega_{bc}^{*(1/2)}\pi)$	$0.380 \sim 0.381$
$\Gamma(\Xi_{bb}^{*(3/2)} \to \Xi_{bc}^{*(3/2)}\pi)$	$4.110 \sim 4.234$	$\Gamma(\Omega_{bb}^{*(3/2)} \to \Omega_{bc}^{*(3/2)}\pi)$	$7.606 \sim 7.643$
$\Gamma(\Xi_{bc}^{(1/2)} \to \Xi_{cc}^{*(1/2)}\pi)$	$0.848 \sim 1.101$	$\Gamma(\Omega_{bc}^{(1/2)} \to \Omega_{cc}^{*(1/2)}\pi)$	$1.708 \sim 1.876$
$\Gamma(\Xi_{bc}^{*(1/2)} \to \Xi_{cc}^{*(1/2)}\pi)$	$0.415 \sim 0.587$	$\Gamma(\Omega_{bc}^{*(1/2)} \to \Omega_{cc}^{*(1/2)}\pi)$	$0.965 \sim 1.019$
$\Gamma(\Xi_{bc}^{*(3/2)} \to \Xi_{cc}^{*(3/2)}\pi)$	$8.626 \sim 12.110$	$\Gamma(\Omega_{bc}^{*(3/2)} \to \Omega_{cc}^{*(3/2)}\pi)$	$19.435 \sim 20.529$

TABLE IV. Predictions for the nonleptonic decay widths for the doubly heavy baryons emitting K meson.

Process	Decay width ($10^{-15}a_1^2$ GeV)	Process	Decay width ($10^{-15}a_1^2$ GeV)
$\Gamma(\Xi_{bb}^{*(1/2)} \to \Xi_{bc}^{(1/2)} K)$	$0.265 \sim 0.267$	$\Gamma(\Omega_{bb}^{*(1/2)} \to \Omega_{bc}^{(1/2)} K)$	$0.469 \sim 0.482$
$\Gamma(\Xi_{bb}^{*(1/2)} \to \Xi_{bc}^{*(1/2)} K)$	$0.165 \sim 0.171$	$\Gamma(\Omega_{bb}^{*(1/2)} \to \Omega_{bc}^{*(1/2)} K)$	$0.307 \sim 0.308$
$\Gamma(\Xi_{bb}^{*(3/2)} \to \Xi_{bc}^{*(3/2)} K)$	$3.317 \sim 3.425$	$\Gamma(\Omega_{bb}^{*(3/2)} \to \Omega_{bc}^{*(3/2)} K)$	$6.150 \sim 6.170$
$\Gamma(\Xi_{bc}^{(1/2)} \to \Xi_{cc}^{*(1/2)} K)$	$0.649 \sim 0.845$	$\Gamma(\Omega_{bc}^{(1/2)} \to \Omega_{cc}^{*(1/2)} K)$	$1.313 \sim 1.557$
$\Gamma(\Xi_{bc}^{*(1/2)} \to \Xi_{cc}^{*(1/2)} K)$	$0.328 \sim 0.466$	$\Gamma(\Omega_{bc}^{*(1/2)} \to \Omega_{cc}^{*(1/2)} K)$	$0.767 \sim 0.806$
$\Gamma(\Xi_{bc}^{*(3/2)} \to \Xi_{cc}^{*(3/2)} K)$	$6.810 \sim 9.612$	$\Gamma(\Omega_{bc}^{*(3/2)} \to \Omega_{cc}^{*(3/2)} K)$	$15.436 \sim 16.237$

TABLE V. Predictions for the nonleptonic decay widths for the doubly heavy baryons emitting D meson.

Process	Decay width ($10^{-15}a_1^2$ GeV)	Process	Decay width ($10^{-15}a_1^2$ GeV)
$\Gamma(\Xi_{bb}^{*(1/2)} \to \Xi_{bc}^{(1/2)} D)$	$0.818 \sim 0.832$	$\Gamma(\Omega_{bb}^{*(1/2)} \to \Omega_{bc}^{(1/2)} D)$	$1.404 \sim 1.466$
$\Gamma(\Xi_{bb}^{*(1/2)} \to \Xi_{bc}^{*(1/2)} D)$	$0.693 \sim 0.741$	$\Gamma(\Omega_{bb}^{*(1/2)} \to \Omega_{bc}^{*(1/2)} D)$	$1.292 \sim 1.319$
$\Gamma(\Xi_{bb}^{*(3/2)} \to \Xi_{bc}^{*(3/2)} D)$	$13.885 \sim 14.834$	$\Gamma(\Omega_{bb}^{*(3/2)} \to \Omega_{bc}^{*(3/2)} D)$	$25.873 \sim 26.419$
$\Gamma(\Xi_{bc}^{(1/2)} \to \Xi_{cc}^{*(1/2)} D)$	$1.136 \sim 1.528$	$\Gamma(\Omega_{bc}^{(1/2)} \to \Omega_{cc}^{*(1/2)} D)$	$2.434 \sim 2.552$
$\Gamma(\Xi_{bc}^{*(1/2)} \to \Xi_{cc}^{*(1/2)} D)$	$0.945 \sim 1.464$	$\Gamma(\Omega_{bc}^{*(1/2)} \to \Omega_{cc}^{*(1/2)} D)$	$2.383 \sim 2.426$
$\Gamma(\Xi_{bc}^{*(3/2)} \to \Xi_{cc}^{*(3/2)} D)$	$19.525 \sim 30.028$	$\Gamma(\Omega_{bc}^{*(3/2)} \to \Omega_{cc}^{*(3/2)} D)$	$47.895 \sim 48.741$

TABLE VI. Predictions for the nonleptonic decay widths for the doubly heavy baryons emitting D_s meson.

Process	Decay width ($10^{-13}a_1^2$ GeV)	Process	Decay width ($10^{-13}a_1^2$ GeV)
$\Gamma(\Xi_{bb}^{*(1/2)} \to \Xi_{bc}^{(1/2)} D_s)$	$0.285 \sim 0.290$	$\Gamma(\Omega_{bb}^{*(1/2)} \to \Omega_{bc}^{(1/2)} D_s)$	$0.511 \sim 0.515$
$\Gamma(\Xi_{bb}^{*(1/2)} \to \Xi_{bc}^{*(1/2)} D_s)$	$0.253 \sim 0.271$	$\Gamma(\Omega_{bb}^{*(1/2)} \to \Omega_{bc}^{*(1/2)} D_s)$	$0.471 \sim 0.483$
$\Gamma(\Xi_{bb}^{*(3/2)} \to \Xi_{bc}^{*(3/2)} D_s)$	$5.065 \sim 5.435$	$\Gamma(\Omega_{bb}^{*(3/2)} \to \Omega_{bc}^{*(3/2)} D_s)$	$9.437 \sim 9.668$
$\Gamma(\Xi_{bc}^{(1/2)} \to \Xi_{cc}^{*(1/2)} D_s)$	$0.368 \sim 0.497$	$\Gamma(\Omega_{bc}^{(1/2)} \to \Omega_{cc}^{*(1/2)} D_s)$	$0.794 \sim 0.828$
$\Gamma(\Xi_{bc}^{*(1/2)} \to \Xi_{cc}^{*(1/2)} D_s)$	$0.328 \sim 0.513$	$\Gamma(\Omega_{bc}^{*(1/2)} \to \Omega_{cc}^{*(1/2)} D_s)$	$0.828 \sim 0.851$
$\Gamma(\Xi_{bc}^{*(3/2)} \to \Xi_{cc}^{*(3/2)} D_s)$	$6.763 \sim 10.520$	$\Gamma(\Omega_{bc}^{*(3/2)} \to \Omega_{cc}^{*(3/2)} D_s)$	$16.650 \sim 17.099$

respectively, in the leading order of a $1/m_Q$ expansion. The kernel for the BS equation contains the scalar confinement and one-gluon-exchange terms, which are motivated by the potential model and successfully used in the cases of mesons and heavy baryons containing a single heavy quark. Since the size of the heavy diquark is enhanced by $\ln^2 m_Q$ with respect to $1/m_Q$, we also introduce a few form factors to the effective vertex for the heavy diquark coupling to the gluon in order to reflect the inner structure of the heavy diquark.

The BS equations are solved numerically under the covariant instantaneous approximation, which is suitable for the weakly bound states of both the heavy diquark and the doubly heavy baryon. The obtained masses of the doubly heavy baryons are consistent with those from the lattice simulations [54,55]. It is found that the properties of both the heavy diquarks and the doubly heavy baryons are independent of their spin in the leading order of a $1/m_Q$ expansion.

As we know, the superflavor symmetry relates doubly heavy baryons to heavy mesons, and hence the form factors of the transitions of doubly heavy baryons are reduced to the Isgur-Wise function, which is well known for heavy mesons [10,11,56]. The calculation of the doubly heavy baryon transitions is greatly simplified under the superflavor symmetry at the cost of ignoring the derivation from nonpointlike spatial dispersion of the heavy diquark. In this work, we directly calculate the decay amplitudes for the doubly heavy baryons using the BS wave functions obtained for both the heavy diquark and the doubly heavy baryons, instead of employing the superflavor symmetry. We give the predictions for the nonleptonic decay widths of doubly heavy baryons emitting a pseudoscalar meson. Our results will be tested in the future experiments.

In our calculation, for the propagators of the heavy diquarks and the light quarks involved in the bound states, we simply assume the forms of free propagators with the masses of the heavy diquarks and the light quarks taken to be the constituent ones. Actually, the real propagators should be solved using the Dyson-Schwinger equation. In such an approach, one has to guarantee the consistency between the kernel of the BS equation and that of the Dyson-Schwinger equation as required by the axial-vector Ward-Takahashi identity [24]. This is a very complicated procedure and needs more careful investigations in the future.

At the HERA-B and Tevatron facilities more than 10^5 events involving double charm baryons are expected, while at the LHC one can expect about 10^9 events [57]. Since the available energy at the LHC is much higher than the masses of Ξ_{cc} and Ξ_{bb}, it is believed that their production rates should be comparable. The decay widths of the doubly heavy baryons we present in this work will be tested in the forthcoming experiments.

Since the BS equations are established at the leading order in a $1/m_Q$ expansion, we do not distinguish the different spins of both the heavy diquarks and the doubly heavy baryons. Such differences should happen at $\mathcal{O}(1/m_Q)$. $1/m_Q$ corrections will be studied in the future.

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