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Color suppressed contribution to $\bar{B}_d^0 \to \pi^0\pi^0$

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The decay modes of the type $B \to \pi\pi$ are dynamically different. For the case $\bar{B}_d^0 \to \pi^+\pi^-$ there is a substantial factorized contribution which dominates. In contrast, the decay mode $\bar{B}_d^0 \to \pi^0\pi^0$ has a small factorized contribution, being proportional to a small Wilson coefficient combination. However, for the decay mode $\bar{B}_d^0 \to \pi^0\pi^0$ there is a sizeable nonfactorizable (color suppressed) contribution due to soft (long distance) interactions, which dominate the amplitude. We estimate the branching ratio for the mode $\bar{B}_d^0 \to \pi^0\pi^0$ in the heavy quark limit for the b quark. In order to estimate color suppressed contributions we treat the energetic light (u, d, s) quark within a variant of Large Energy Effective Theory combined with a recent extension of chiral quark models in terms of model- dependent gluon condensates. We find that our calculated color suppressed amplitude is suppressed by a factor of order $\Lambda_{\rm QCD}/m_b$ with respect to the factorizable amplitude, as it should according to QCD-factorization. Further, for reasonable values of the constituent quark mass and the gluon condensate, the calculated nonfactorizable amplitude for $\bar{B}_d^0 \to \pi^0\pi^0$ can easily accommodate the experimental value. Unfortunately, the color suppressed amplitude is very sensitive to the values of these model-dependent parameters. Therefore fine-tuning is necessary in order to obtain an amplitude compatible with the experimental result for $\bar{B}_d^0 \to \pi^0\pi^0$. A possible link to the triangle anomaly is discussed.

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I. INTRODUCTION

Because of numerous experimental results coming from *BaBar* and Belle, there is presently great interest in decays of B mesons. LHC will also provide us with more data for such processes. B decays of the type $B \to \pi\pi$ and $B \to K\pi$, where the energy release is big compared to the light meson masses (heavy to light transitions), has been treated within *QCD-factorization* [1] and *Soft Collinear Effective Theory* (SCET) [2]. In the high energy limit, the amplitudes for such decay modes factorize into products of two matrix elements of weak currents, and some nonfactorizable corrections of order α_s can be calculated perturbatively. However, there are additional contributions of order $\Lambda_{\rm QCD}/m_b$ which cannot be reliably calculated within perturbative theory [1]. The so-called pQCD-model and QCD sum rules have also been used for B-meson decays [3,4].

For decay modes which are of the heavy to heavy type, involving b and c quarks, the decay amplitudes have been described within *Heavy Quark Effective Field Theory* (HQEFT) [5]. Some transitions of *heavy to heavy* type in the heavy quark limits $(1/m_b) \to 0$ like $B - \bar{B}$ mixing [6] has been studied within *Heavy-Light Chiral Perturbation Theory* (HLχPT) [7]. Furthermore, other transitions which are formally *heavy to heavy* in the heavy quark limits $(1/m_b) \to 0$ *and* $(1/m_c) \to 0$, like the Isgur-Wise function [8] for $B \to D$, have been studied within HLχPT [7]. The cases $\bar{B} \to D\bar{D}$ [9] and $B \to D^*\gamma$ [10,11] have also been studied within such a framework, even if the energy release in these processes is above the chiral symmetry breaking scale. Still this framework gives amplitudes of the right order of magnitude. The calculation of such transitions have in addition been supplemented with calculations within a *Heavy-Light Chiral Quark Model* (HLχQM) to determine quantities which are not determined within HLχPT itself [9,11,12].

As pointed out in a series of papers [9,11–13], there are processes which have factorized amplitudes multiplied by a very small Wilson coefficient combination, such that nonfactorized amplitudes are expected to dominate. Examples are $\bar{B}_{d,s}^0 \to D^0\bar{D}^0$ [9], $\bar{B}^0 \to D^0\eta'$ [12] and $\bar{B}_d^0 \to D^0\pi^0$. The latter process $\bar{B}_d^0 \to D^0\pi^0$ was considered recently [13,14]. In that case a heavy b quark decaying to a light, but energetic quark was involved. Then the light energetic quark might be described by an effective theory. The first version of such a framework was *Large Energy Effective Theory* (LEET) [15,16]. The HQEFT covers processes where the heavy quarks carry the main part of the momentum in each hadron. To describe processes where energetic light quarks emerge from decays of heavy b quarks, LEET was introduced [15] and used to study the current for $B \to \pi$ [16].

The idea was that LEET should do for energetic light quarks what HQEFT did for heavy quarks. In HQEFT one splits off the heavy motion from the full heavy quark field, thus obtaining a reduced field depending on the velocity of the heavy quark. Similarly, in LEET one splits off the large energy from the full field of the energetic light quark, thus obtaining an effective description for a reduced light quark which depends on a lightlike four vector. It was later shown that LEET in its initial formulation was incomplete and did not fully reproduce QCD physics [17]. Then LEET was further developed to be fully consistent with QCD and became the Soft Collinear Effective Theory (SCET) [2].

In the present paper we consider decay modes of the type $B \to \pi\pi$. The decay mode $\bar{B}^0_d \to \pi^-\pi^+$ has a substantial factorized amplitude, given by the current matrix element for $\bar{B}^0_d \to \pi^+$ transition times the matrix element of the weak current for the outgoing π^-, which is proportional to the pion decay constant f_π. The relevant Wilson coefficient is also the maximum possible, namely, of order 1 times the relevant Cabibbo-Kobayashi-Maskawa (CKM) quark mixing factors and the Fermi coupling constant. This is in contrast to the process $\bar{B}^0_d \to \pi^0\pi^0$ which is color suppressed. As said above, decays of the type $B \to 2\pi$ have been extensively studied within QCD-factorization, SCET, and QCD sum rule methods [18]. In spite of tremendous efforts it has not been possible to obtain an amplitude compatible with the experimental result $\bar{B}^0_d \to \pi^0\pi^0$. The purpose of this paper is study this decay mode within an alternative model-dependent framework.

First we point out that the factorized contribution to the decay mode $\bar{B}^0_d \to \pi^0\pi^0$, which is given by the $B \to \pi$ transition amplitude times the decay constant of the π^0 meson, is almost zero because it is proportional to a very small Wilson coefficient combination. For the dominant nonfactorizable (color suppressed) amplitude for $\bar{B}^0_d \to \pi^0\pi^0$ we will, as mentioned above, use a model named *Large Energy Chiral Quark Model* (LEχQM) recently constructed and used to handle the process $\bar{B}^0_d \to \pi^0 D^0$ [13,14]. Here a variant of LEET was combined with ideas from previous chiral quark model (χQM) calculations similarly to what has been done for other nonleptonic decays [6,11,12,19,20].

A priori it might look strange to use the framework of chiral quark models when the energy release is big compared to the chiral symmetry breaking scale Λ_χ. The point is that the motion of the heavy quark or energetic light quark can be split off, and the various versions of heavy-light or large energy chiral quark models and a corresponding chiral perturbation theory (χPT) can be used to describe the redundant strong interactions corresponding to momenta of order 1 GeV and below.

It might be argued that we should have used the full SCET theory as the basis our new model. However, the purpose of our paper is to estimate, in analogy with previous papers [6,11,12,19–23], the effects of soft-gluon emission in terms of gluon condensates, where transverse quark momenta and collinear gluons will not play an essential role. In any case this construction [13] will be a model. Therefore it suffices for our purpose to use the more simple formulation of LEET. We will combine LEET with chiral quark models (χQM) [21,24–27], containing only soft gluons making condensates. In LEχQM [13] an energetic quark is bound to a soft quark with an *a priori* unknown coupling, as proposed in [21]. The unknown coupling is determined by calculating the known $B \to \pi$ current matrix element within the model [13]. This fixes the unknown coupling because the matrix element of this current is known [16]. Then, in the next step, we use this coupling to calculate the nonfactorized (color suppressed) amplitude contribution to $\bar{B}^0_d \to \pi^0\pi^0$ in terms of the lowest dimension gluon condensate, as have been done for other nonleptonic decays [6,11,12,19,20]. After the quarks have been integrated out, we obtain an effective theory containing soft light mesons as in HLχPT, but also fields describing energetic light mesons. A similar idea with a combination of SCET with HLχPT is considered in [28]. The LEχQM was constructed in analogy with the previous *Heavy-Light Chiral Quark Model* (HLχQM) [20] and may be considered to be an extension of that model.

One might think that to be completely consistent, we should also have calculated the Wilson coefficients within a relevant large energy framework. For this purpose the use of LEET would be dubious because it is an incomplete theory as mentioned above. However, as we will see below, the main uncertainty in our final amplitude will be due to uncertainty in our model-dependent gluon condensate due to emission of soft gluons. Therefore the Wilson coefficients calculated within full QCD as in [29] will be appropriate for our purpose.

In the next Sec. II we present the weak four quark Lagrangian and its factorized and nonfactorizable matrix elements. In Sec. III we present our version of LEET, and in Sec. IV we present the new model LEχQM to include energetic light quarks and mesons. In Sec. V we calculate the nonfactorizable matrix elements due to soft gluons expressed through the (model-dependent) quark condensate. In Sec. VI we give the results and conclusion.

II. THE EFFECTIVE LAGRANGIAN AT QUARK LEVEL

We will study decays of $\bar{B}^0_d$ generated by the weak quark process $b \to u\bar{u}d$. We restrict ourselves to processes where the b quark decays. This means the quark level processes $b \to du\bar{u}$. Processes where the anti- b quark decays proceed analogously. The effective weak Lagrangian at quark level is [29] (neglecting penguin operators)

$$\mathcal{L}_{\rm eff} = -\frac{G_F}{\sqrt{2}} V_{\rm ub} V^*_{\rm ud}[c_A Q_A + c_B Q_B], \tag{1}$$

where the subscript L denotes the left-handed fields: $q_L \equiv Lq$, where $L \equiv (1-\gamma_5)/2$ is the left-handed projector in Dirac-space. The local operator products $Q_{A,B}$ are defined as

$$Q_A = 4\bar{u}_L\gamma_\mu b_L \bar{d}_L\gamma^\mu u_L; \qquad Q_B = 4\bar{u}_L\gamma_\mu u_L \bar{d}_L\gamma^\mu b_L. \tag{2}$$

In these operators summation over color is implied. In Eq. (1), c_A and c_B are Wilson coefficients. At tree level

$c_A = 1$ and $c_B = 0$. At one loop level, a contribution to c_B is also generated, and c_A is slightly increased. These effects are handled in terms of the *Renormalization Group Equations* (RGE) [29], and the coefficients can be calculated at for instance $\mu = m_b$ or $\mu = 1$ GeV. Using the color matrix identity

$$2t^a_{in}t^a_{lj} = \delta_{ij}\delta_{ln} - \frac{1}{N_c}\delta_{in}\delta_{lj},$$

and Fierz rearrangement, the amplitudes for the processes $\bar{B}^0 \to \pi^+\pi^-$ may be written as

$$\mathcal{M}_{\pi^+\pi^-} = 4\frac{G_F}{\sqrt{2}}V_{\rm ub}V^*_{\rm ud}\bigg[\bigg(c_A + \frac{1}{N_{\rm c}}c_B\bigg)\langle\pi^-|\bar{d}_L\gamma_\mu u_L|0\rangle \times\langle\pi^+|\bar{u}_L\gamma^\mu b_L|\bar{B}^0\rangle + 2c_B\langle\pi^+\pi^-|\bar{d}_L\gamma_\mu t^a u_L\bar{u}_L\gamma^\mu t^a b_L|B^0\rangle\bigg], \quad (3)$$

and for $\bar{B}^0 \to \pi^0\pi^0$

$$\mathcal{M}_{\pi^0\pi^0} = 4\frac{G_F}{\sqrt{2}}V_{\rm ub}V^*_{\rm ud}\bigg[\bigg(c_B + \frac{1}{N_{\rm c}}c_A\bigg)\langle\pi^0|\bar{u}_L\gamma_\mu u_L|0\rangle \times\langle\pi^0|\bar{d}_L\gamma^\mu b_L|\bar{B}^0\rangle + 2c_A\langle\pi^0\pi^0|\bar{d}_L\gamma_\mu t^a b_L\bar{u}_L\gamma^\mu t^a u_L|B^0\rangle\bigg]. \quad (4)$$

Here the terms proportional to $2c_A$ and $2c_B$ with color matrices inside the matrix elements are the genuinely nonfactorizable contributions.

Since c_A is of order one and c_B of order $-1/3$ [12,13], we refer to the coefficients

$$c_f \equiv \bigg(c_A + \frac{1}{N_{\rm c}}c_B\bigg) \simeq 1.1; \qquad c_{nf} \equiv \bigg(c_B + \frac{1}{N_{\rm c}}c_A\bigg) \simeq 0, \quad (5)$$

as favorable (c_f) and nonfavorable (c_{nf}) coefficients, respectively. Thus, the decay mode $\bar{B}^0_d \to \pi^+\pi^-$ has a sizeable factorized amplitude proportional to c_f. In contrast, the decay mode $\bar{B}^0_d \to \pi^0\pi^0$ has a factorized amplitude proportional to the nonfavorable coefficient c_{nf} which is close to zero. In this case we expect the nonfactorizable term (involving color matrices) proportional to $2c_A$ to be dominant, i.e. the last line of Eq. (4) dominates. A substantial part of this paper is dedicated to the calculation of this nonfactorizable contribution to the $\bar{B}^0_d \to \pi^0\pi^0$ decay amplitude.

Thus the main task of this paper will be to calculate the matrix element of the operator Q_C consisting of the product of two colored currents occurring in the last line of Eq. (4):

$$Q_C = (\bar{d}_L\gamma_\mu t^a b_L)(\bar{u}_L\gamma^\mu t^a u_L) \quad (6)$$

for the color suppressed process $\bar{B}^0_d \to \pi^0\pi^0$. This matrix element will be estimated in Sec. V where we use the LEχQM to estimate nonfactorizable amplitudes in terms of emission of soft gluons making gluon condensates.

III. AN ENERGETIC LIGHT QUARK EFFECTIVE DESCRIPTION (LEETδ)

An energetic light quark might, similarly to a heavy quark, carry practically all the energy E of the meson it is a part of. The difference is that now the mass of the energetic quark is close to zero compared to the heavy quark mass m_Q and E, which are assumed to be of the same order of magnitude. We assume that the energetic light quark is emerging from the decay of a heavy quark Q with momentum $p_Q = m_Q v + k$. The heavy quark is described by the HQEFT Lagrangian for the reduced quark field Q_v [5]:

$$\mathcal{L}_{\rm HQET} = \bar{Q}_v(iv\cdot D)Q_v + \mathcal{O}(1/m_Q), \quad (7)$$

where Q_v is the reduced heavy quark field (often named h_v in the literature), v its four velocity and m_Q the mass of the heavy quark.

The momentum of the light energetic light quark q can be written

$$p^\mu_q = En^\mu + k^\mu, \qquad |k^\mu| \ll |En^\mu|, \qquad m_q \ll E, \quad (8)$$

where E, which is of order m_Q, is the energy of the energetic light quark, m_q is the light quark mass. Further, n is the lightlike four vector which might be chosen to have the space part along the z-axis, $n^\mu = (1; 0, 0, 1)$, in the frame of the heavy quark where $v = (1, \underline{0})$. Then $(v\cdot n) = 1$ and $n^2 = 0$. Inserting this in the regular quark propagator, in the limit where the approximations in (8) are valid, we obtain the propagator

$$S(p_q) = \frac{\gamma\cdot p_q + m_q}{p_q^2 - m_q^2} \to \frac{\gamma\cdot n}{2n\cdot k}. \quad (9)$$

This propagator is the starting point for the Large Effective Theory (LEET) constructed in Ref. [16].

Unfortunately, the combination of LEET with χQM will lead to infrared divergent loop integrals for $n^2 = 0$ (see Sec. IV). Therefore, the formalism was modified [13,14] and instead of $n^2 = 0$, we use $n^2 = \delta^2$, with $\delta = \nu/E$ where $\nu \sim \Lambda_{\rm QCD}$, such that $\delta \ll 1$. An expansion in δ will then within our model be equivalent to an expansion in $\Lambda_{\rm QCD}/m_b$.

In the following we describe the modified LEET [16] where we keep $\delta \neq 0$ with $\delta \ll 1$. We call this construction LEETδ [13] and define the *almost* lightlike vectors

$$n = (1, 0, 0, +\eta), ; \qquad \tilde{n} = (1, 0, 0, -\eta), \quad (10)$$

where $\eta = \sqrt{1-\delta^2}$. This means that

$$n^\mu + \tilde{n}^\mu = 2v^\mu, \qquad n^2 = \tilde{n}^2 = \delta^2, \qquad v\cdot n = v\cdot\tilde{n} = 1, \qquad n\cdot\tilde{n} = 2 - \delta^2. \quad (11)$$

In the following we use the projection operators given by

$$\mathcal{P}_+ = \frac{1}{N^2}\gamma\cdot n(\gamma\cdot\tilde{n}+\delta), \quad \mathcal{P}_- = \frac{1}{N^2}(\gamma\cdot\tilde{n}-\delta)\gamma\cdot n, \tag{12}$$

where $N = \sqrt{2n\cdot\tilde{n}} = 2 + \mathcal{O}(\delta^2)$. One factors out the main energy dependence, just as was analogously done in HQEFT, and define the projected reduced quark fields [16]

$$q_\pm(x) = e^{iEn\cdot x}\mathcal{P}_\pm q(x), \quad q(x) = e^{-iEn\cdot x}[q_+(x)+q_-(x)]. \tag{13}$$

As in [16], the field q_- was eliminated and one obtained for $q_+ \equiv q_n$ the effective Lagrangian [13]:

$$\mathcal{L}_{\mathrm{LEET}\delta} = \bar{q}_n\left(\frac{\gamma\cdot\tilde{n}+\delta}{N}\right)(in\cdot D)q_n + \frac{1}{E}\bar{q}_n X q_n + \mathcal{O}(E^{-2}), \tag{14}$$

which (for $\delta = 0$) is the first part of the SCET Lagrangian. The operator X is given in [13]. Equation (14) yields the LEETδ quark propagator

$$S_n(k) = \mathcal{P}_+\left[\frac{\gamma\cdot\tilde{n}+\delta}{N}(n\cdot k)\right]^{-1} = \frac{\gamma\cdot n}{N(n\cdot k)}, \tag{15}$$

which reduces to (9) in the limit $\delta \to 0$. In addition, for a light energetic quark, the propagator within SCET [2] will for small transverse quark momenta $p_\perp \to 0$ coincide with Eq. (15).

Based on LEET, it was found [16] in the formal limits $M_H \to \infty$ and $E \to \infty$, that a heavy $H = (B, D)$ meson decaying by the weak hadronic vector current V^μ to a light pseudoscalar meson is described by a matrix element $\langle P|V^\mu|H\rangle$ of the form

$$\langle P|V^\mu|H\rangle = 2E[\zeta^{(v)}(M_H, E)n^\mu + \zeta_1^{(v)}(M_H, E)v^\mu], \tag{16}$$

where

$$\zeta^{(v)} = C\frac{\sqrt{M_H}}{E^2}, \qquad C \sim (\Lambda_{\mathrm{QCD}})^{3/2}, \qquad \frac{\zeta_1^{(v)}}{\zeta^{(v)}} \sim \frac{1}{E}. \tag{17}$$

This behavior is consistent with the energetic quark having x close to 1, where x is the quark momentum fraction of the outgoing pion [16].

IV. EXTENDED CHIRAL QUARK MODEL FOR HEAVY AND ENERGETIC LIGHT QUARKS (LEχQM)

The chiral quark model (χQM) [24,25] and the Heavy-Light Chiral Quark Model (HLχQM) [20], include meson-quark couplings and thereby allow us to calculate amplitudes and chiral Lagrangians for processes involving heavy quarks and low-energy light quarks. In this section we will extend these models to include also hard, energetic light quarks.

For the pure light and soft sector the χQM Lagrangian can be written as [19,24]

$$\mathcal{L}_{\chi\mathrm{QM}} = \bar{\chi}[\gamma\cdot(iD+\mathcal{V}) + \gamma\cdot\mathcal{A} - m]\chi, \tag{18}$$

where m is the constituent mass term being due to chiral symmetry breaking. The small current mass term is neglected here. Here we have introduced the flavor rotated fields $\chi_{L,R}$:

$$\chi_L = \xi^\dagger q_L, \qquad \chi_R = \xi q_R, \tag{19}$$

where q is the light quark flavor triplet and

$$\xi = \exp\{i\Pi/f\}, \quad \Pi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}. \tag{20}$$

Further, $\mathcal{V}_\mu$ and $\mathcal{A}_\mu$ are vector and axial vector fields, given by

$$\mathcal{V}_\mu \equiv \frac{i}{2}(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger), \quad \mathcal{A}_\mu \equiv -\frac{i}{2}(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger). \tag{21}$$

To couple the heavy quarks to mesons there are additional meson-quark couplings within HLχQM [20]:

$$\mathcal{L}_{\mathrm{int}} = -G_H[\bar{\chi}_a\bar{H}_v^a Q_v + \bar{Q}_v H_v^a\chi_a], \tag{22}$$

where Q_v is the (reduced) heavy quark field and H is the heavy $(0^-, 1^-)$ meson field(s)

$$H_v^{(+)} = P_+(v)(\gamma\cdot P^* - i\gamma_5 P_5), \tag{23}$$

P_μ^* being the 1^- and P_5 the 0^- fields, and $P_+(v) = (1+\gamma\cdot v)/2$. The quark-meson coupling G_H is determined within the HLχQM to be [20]

$$G_H^2 = \frac{2m}{f_\pi^2}\rho, \qquad \rho = \frac{(1+3g_A)}{4\left(1 + \frac{m^2 N_c}{8\pi f_\pi^2} - \frac{\eta_H}{2m^2 f_\pi^2}\langle\frac{\alpha_s}{\pi}G^2\rangle\right)}, \tag{24}$$

where $\eta_H = (8-\pi)/64$. The quantity ρ is of order one.

For hard light quarks and chiral quarks coupling to a hard light meson multiplet field M, one extends the ideas of χQM and HLχQM, and assume that the energetic light mesons couple to light quarks with a derivative coupling to an axial current [13]:

$$\mathcal{L}_{\mathrm{int}q} \sim \bar{q}\gamma_\mu\gamma_5(i\partial^\mu M)q. \tag{25}$$

One combines LEETδ with the χQM and assume that the ingoing light quark and the outgoing meson are energetic and have the behavior $\exp(\pm iEn\cdot x)$ as in (13). To describe (outgoing) light energetic mesons, we use an octet 3×3 matrix field $M = \exp(+iEn\cdot x)M_n$ of the same form as Π in (20):

$$M_n = \begin{pmatrix} \frac{\pi_n^0}{\sqrt{2}} + \frac{\eta_n}{\sqrt{6}} & \pi_n^+ & K_n^+ \\ \pi_n^- & -\frac{\pi_n^0}{\sqrt{2}} + \frac{\eta_n}{\sqrt{6}} & K_n^0 \\ K_n^- & \bar{K}_n^0 & -\frac{2\eta_n}{\sqrt{6}} \end{pmatrix}, \quad (26)$$

where π_n^0, π_n^+, K_n^+ etc. are the energetic light meson fields with momentum $\sim En^\mu$.

Combining (25) with the use of the rotated soft quark fields in (19) and using $\partial^\mu \rightarrow iEn^\mu$ one arrives at the ansatz for the LEχQM interaction Lagrangian:

$$\mathcal{L}_{\text{int}q\delta} = G_A E \bar{\chi}(\gamma \cdot n) Z q_n + \text{H.c.}, \quad (27)$$

where q_n represents an energetic light quark having momentum fraction close to 1 and χ represents a soft quark (see Eq. (19)). Further, the coupling G_A is determined by physical requirements [13,16], and

$$Z = \xi M_R R - \xi^\dagger M_L L. \quad (28)$$

Here M_L and M_R are both equal to M_n, but they have formally different transformation properties. This is analogous to the use of quark mass matrices $\mathcal{M}_q$ and $\mathcal{M}_q^\dagger$ in standard *Chiral Perturbation Theory* (χPT). They are in practice equal, but have formally different transformation properties.

The axial vector coupling introduces a factor $\gamma \cdot n$ to the vertex (see (27)), which simplifies the Dirac algebra within the loop integrals. In order to calculate the nonfactorizable contribution, one must first find a value for the large energy light quark bosonization coupling G_A. This was done [13] by requiring that our model should be consistent with the Eqs. (16) and (17). Applying the Feynman rules of LEχQM [13] we obtain the following bosonized current (before soft-gluon emission forming a condensate is taken into account):

$$J_0^\mu(H_{v_b} \rightarrow M_n) = -N_c \int đk \,\text{Tr}\{\gamma^\mu L i S_v(k)[-iG_H H_{v_b}^{(+)}] \times iS_\chi(k)[iEG_A \gamma \cdot n Z] i S_n(k)\}, \quad (29)$$

where $đk \equiv d^D k/(2\pi)^D$ (D being the dimension of space-time), and

$$S_v(k) = \frac{P_+(v)}{v \cdot k}, \quad S_\chi(k) = \frac{(\gamma \cdot k + m)}{k^2 - m^2}, \quad S_n(k) = \frac{\gamma \cdot n}{Nn \cdot k}, \quad (30)$$

are the propagators for heavy quarks described by (18), for light constituent quarks, and (14) for light energetic quarks. The presence of the left projection operator L in Z ensures that we only get contributions from the left-handed part of the interaction in (27), that is, $Z \longrightarrow -\xi^\dagger M_L L$. The contribution in (29) corresponding to the $B \rightarrow \pi$ current is illustrated by the lower part of the diagram in Fig. 1.

Loop diagrams within LEχQM depend on momentum integrals of the form

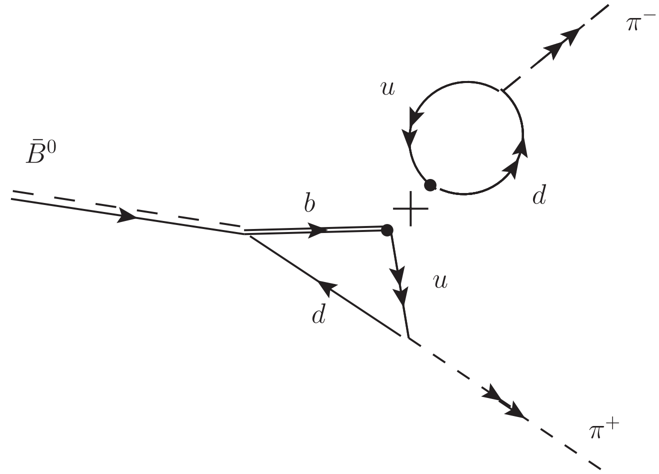

FIG. 1. The factorized contribution to the $B^0 \rightarrow \pi^+\pi^-$ decay, as described in combined HLχQM and LEχQM. Double lines, single lines and the single line with two arrows are representing heavy quarks, light soft quarks and light energetic quarks, respectively. Heavy mesons are represented by a single line combined with a parallel dashed line, and a light energetic pion is represented by a dashed line with a double arrow.

$$K_{rst} = \int \frac{đk}{(v \cdot k)^r (k \cdot n)^s (k^2 - m^2)^t}, \quad (31)$$

$$K_{rst}^\mu = \int \frac{đk k^\mu}{(v \cdot k)^r (k \cdot n)^s (k^2 - m^2)^t} = K_{rst}^{(v)} v^\mu + K_{rst}^{(n)} n^\mu. \quad (32)$$

These integrals have the important property that $K_{rst}^{(n)}$ dominates over $K_{rst}^{(v)}$ and K_{rst} with one power of $1/\delta$. In the present model, we choose $\nu = m$ which is of order Λ_{QCD}. Thus the constituent light quark mass m is the equivalent of Λ_{QCD} within our model. Some details of the calculation of the $B \rightarrow \pi$ is given in Ref. [13].

To calculate emission of soft gluons we have used the framework of Novikov *et al.* [30]. In that framework the ordinary vertex containing the gluon field A_μ^a will be replaced by the soft-gluon version containing the soft-gluon field tensor $G_{\mu\nu}^a$:

$$i g_s t^a \Gamma^\mu A_\mu^a \rightarrow -\frac{1}{2} g_s t^a \Gamma^\mu G_{\mu\nu}^a \frac{\partial}{\partial k_\nu} \ldots|_{k=0} \quad , \quad (33)$$

where k is the momentum of the soft-gluon. (Using this framework one has to be careful with the momentum routing because the gauge where $x^\mu A_\mu^a = 0$ has been used.) Here $\Gamma^\mu = \gamma^\mu$, v^μ, or $n^\mu(\gamma \cdot \tilde{n} + \delta)/N$ for a light soft quark, heavy quark, or light energetic quark, respectively. Our loop integrals are *a priori* depending on the gluon momenta $k_{1,2}$ which are sitting in some propagators. These gluon momenta disappear after having used the procedure in (33). (Note that the derivative has to be taken with respect to the whole loop integral).

Emision from the heavy quark or light energetic quark are expected to be suppressed. This will be realized in most cases because the gluon tensor is antisymmetric, and therefore such contributions are often proportional to

$$G^a_{\mu\nu}v^\mu v^\nu = 0, \quad \text{or} \quad G^a_{\mu\nu}n^\mu n^\nu = 0. \tag{34}$$

However, there are also contributions proportional to

$$G^a_{\mu\nu}v^\mu n^\nu \neq 0, \tag{35}$$

analogous to what happens in some diagrams for the Isgur-Wise diagram where there are two different velocities v_b and v_c [31]. Such contributions appear within our calculation when two soft gluons are emitted from the heavy quark line.

Using the prescription [19,20,25,30]

$$g_s^2 G^a_{\mu\nu}G^a_{\rho\lambda} \to 4\pi^2\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\frac{1}{12}(g_{\mu\rho}g_{\nu\lambda} - g_{\mu\lambda}g_{\nu\rho}), \tag{36}$$

for the gluon condensate one obtains the leading bosonized current [13]

$$J^\mu_{\text{tot}}(H\to M) = -i\frac{G_H G_A}{2}m^2 F\,\text{Tr}\{\gamma^\mu L H_v^{(+)}[\gamma\cdot n]\xi^\dagger M_L\}, \tag{37}$$

where the quantity F obtained from loop integration is *a priori* containing a linearly divergent integral, which is related to the axial coupling $g_{\mathcal{A}}$, and can be traded for $g_{\mathcal{A}}$. One obtains [13] for the quantity F:

$$F = \frac{3f_\pi^2}{8m^2\rho}(1 - g_A) + \frac{N_c}{16\pi} - \frac{(24-7\pi)}{768m^4}\langle\frac{\alpha_s}{\pi}G^2\rangle. \tag{38}$$

Note that F is dimensionless. The parameter ρ is given in (24). Numerically, it was found [13] that $F \simeq 0.08$.

In order to obtain the HLχPT Lagrangian terms $\text{Tr}(\bar{H}^a H^b v_\mu \mathcal{V}^\mu_{ba})$ and $\text{Tr}(\bar{H}^a H^b \gamma_\mu\gamma_5\mathcal{A}^\mu_{ba})$, having coefficients $+1$ and $-g_{\mathcal{A}}$ respectively, one calculates quark loops with attached heavy meson fields and vector and axial vector fields $\mathcal{V}^\mu$ or $\mathcal{A}^\mu$. Then logarithmic and linearly divergent integrals obtained within the loop diagrams are identified with physical quantities or quantities of the model [19,20,24,25].

In order to fix G_A in (27), we compare (16) with (37). In our case where no extra soft pions are going out, we put $\xi \to 1$, and for the momentum space $M_L \to k_M\sqrt{E}$, with the isospin factor $k_M = 1/\sqrt{2}$ for π^0 (while $k_M = 1$ for charged pions). Moreover for the B meson with spin-parity 0^- we have $H_v^{(+)} \to P_+(v)(-i\gamma_5)\sqrt{M_H}$. Using this, the involved traces are easily calculated, and we obtain $J^\mu_{\text{tot}}(H\to M)$ for the case $\bar{B}^0_d \to \pi^+$:

$$J^\mu_{\text{tot}x}(\bar{B}^0_d \to \pi^+) = \frac{G_H G_A}{2}(\sqrt{M_H E})m^2 F n^\mu. \tag{39}$$

Using the Eqs. (16), (38), and (39), one obtains [13]

$$G_A = \frac{4\zeta^{(v)}}{m^2 G_H F}\sqrt{\frac{E}{M_H}}, \tag{40}$$

where $\zeta^{(v)}$ is numerically known [32]. Within our model, the analogue of Λ_{QCD} is the constituent light quark mass m. To see the behavior of G_A in terms of the energy E, the quantity C in (17) is written as $C \equiv \hat{c}m^{3/2}$, which gives

$$G_A = \left(\frac{4\hat{c}f_\pi}{mF\sqrt{2\rho}}\right)\frac{1}{E^{3/2}}, \tag{41}$$

which explicitly displays the behavior $G_A \sim E^{-3/2}$. In terms of the number N_c of colors, $f_\pi \sim \sqrt{N_c}$ and $F \sim N_c$ which gives the behavior $G_A \sim 1/\sqrt{N_c}$, i.e. the same behavior as the coupling G_H in (22).

The bosonized current in (37) can now be written as

$$J^\mu_{\text{tot}}(H\to M) = -2i\zeta^{(v)}\sqrt{\frac{E}{M_H}}\text{Tr}\{\gamma^\mu L H_v^{(+)}[\gamma\cdot n]\xi^\dagger M_L\}. \tag{42}$$

V. NONFACTORIZABLE PROCESSES IN LEχQM

In this section we calculate the nonfactorizable contribution to $\bar{B}^0_d \to \pi^0\pi^0$ in Eq. (4). This will be formulated as a quasifactorized product of two colored currents, as illustrated in Fig. 2. Then the nonfactorized aspects enters through color correlation between the two parts, using Eq. (36). Such a calculation within HLχQM and HLχPT is done previously [9] for $\bar{B}^0_{d,s} \to D^0\bar{D}^0$. Here we will use the colored current for $B \to \pi$, within the LEχQM presented in the preceding section; see the diagram in Fig. 2. Using the G_A value from the preceding section, we may now calculate the nonfactorizable contribution to the process by adding one soft-gluon to each loop. Then we

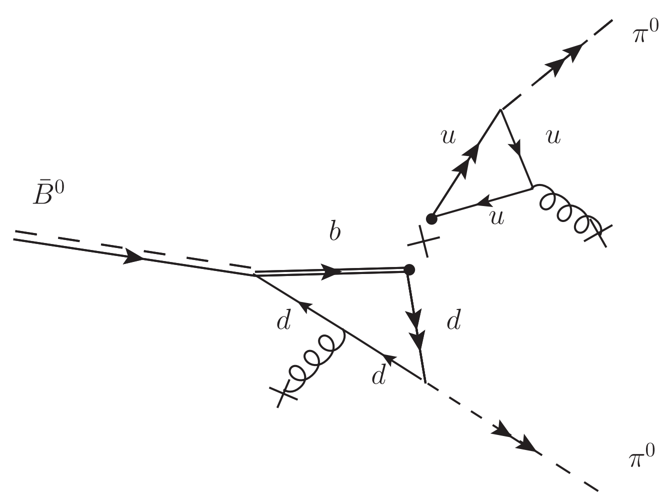

FIG. 2. Nonfactorizable contribution containing large energy light fermions and mesons. There is also a corresponding diagram where the outgoing antiquark $\bar{u}$ is hard.

calculate the decay width for $\bar{B}_d^0 \to \pi^0\pi^0$ from this nonfactorizable amplitude, and compare our results with a experiment.

For a low-energy quark interacting with one soft-gluon, one might in simple cases use the effective propagator [19,33]

$$S_1^G(k) = \frac{g_s}{4} t^a G^a_{\mu\nu} \frac{(2m\sigma^{\mu\nu} + \{\sigma^{\mu\nu}, \gamma \cdot k\})}{(k^2 - m^2)^2}, \tag{43}$$

where $\{a, b\} \equiv ab + ba$ denotes the anticommutator. This expression is consistent with the prescription in (33), and can be used for the diagram in Fig. 2.

Then one gets [13] the following contribution to the bosonized colored $B \to \pi$ current, shown in the lower part of the diagram in Fig. 2:

$$J^\mu_{1G}(H \to M)^a = -\int d\!\!{}^-k \,\mathrm{Tr}\{\gamma^\mu L t^a i S_v(k)[-iG_H H_v^{(+)}] \times iS_1^G(k)[iEG_A \gamma \cdot nZ] i S_n(k)\}, \tag{44}$$

where a is a color octet index. Once more, we deal with the momentum integrals of the types in (31) and (32). Taking the color trace, rewriting (44), we obtain a contribution of the form

$$J^\mu_{1G}(H_b \to M)^a = g_s G^a_{\alpha\beta} T^{\mu;\alpha\beta}(H_b \to M), \tag{45}$$

where the contribution from the lower part of the diagram in Fig. 2 alone is to leading order in δ:

$$T^{\mu;\alpha\beta}(H_b \to M) = \frac{G_H G_A}{128\pi} \epsilon^{\sigma\alpha\beta\lambda} n_\sigma \mathrm{Tr}(\gamma^\mu L H_v^{(+)} \gamma_\lambda \xi^\dagger M_L), \tag{46}$$

where $E \cdot \delta = m$ has been explicitly used.

There are also other diagrams not shown. In one case the gluon is emitted from the energetic quark. This diagram is zero due to (34). Furthermore, there is a diagram not shown where the gluon is emitted from the heavy quark which contains a nonzero part due to (35). This gives an additional contribution to the colored $B \to \pi$ current which is nonzero. However, this one will be projected out because it should be proportional to the Levi-Civita tensor to give a nonzero result for the $\bar{B}_d^0 \to \pi^0\pi^0$ amplitude as a whole, as will be seen from Eq. (49) below.

The colored current for an outgoing π^0 should now be calculated in the LEχQM (see the upper part of the diagram in Fig. 2), and we find

$$J^\mu_{1G}(M_{\tilde{n}})^a = -\int d\!\!{}^-k \,\mathrm{Tr}\{\gamma^\mu L t^a i S_1^G(k)[iEG_A \gamma \cdot \tilde{n} Z] i S_{\tilde{n}}(k)\}. \tag{47}$$

This colored π^0 current has the general form

$$J^\mu_{1G}(M_{\tilde{n}})^a = g_s G^a_{\alpha\beta} T^{\mu;\alpha\beta}(M_{\tilde{n}}), \tag{48}$$

where the tensor T is given by

$$T^{\mu;\alpha\beta}(M_{\tilde{n}}) = 2\left(-\frac{G_A E}{4}\right) Y \tilde{n}_\sigma \epsilon^{\sigma\alpha\beta\mu} \mathrm{Tr}[\lambda^X M_{\tilde{n}}], \tag{49}$$

where the λ^X within the trace is the appropriate Gell-Mann SU(3) flavor matrix. For an outgoing hard π^0 this trace has the value $\sqrt{E/2}$ when going to the momentum space. The explicit factor 2 in front of this expression comes from the corresponding diagram, where in the upper part of the diagram the antiquark could be hard and the quark could be soft and emit a soft-gluon. The factor Y contains the result of loop momentum integration. The relevant loop integral is now

$$K^\mu_{012} = \int \frac{d\!\!{}^-k k^\mu}{(k \cdot n)(k^2 - m^2)^2} = \frac{I_2}{\delta^2} n^\mu, \tag{50}$$

which gives

$$Y = -iI_2 = \frac{f_\pi^2}{4m^2 N_c}\lambda \equiv Y_\lambda \equiv \frac{1}{4m^2 N_c}\left(f_\pi^2 - \frac{1}{24m^2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\right). \tag{51}$$

Here the parameter λ is of order 10^{-2} to 10^{-1} and very sensitive to small variations in the model-dependent parameters m and $\langle\frac{\alpha_s}{\pi}G^2\rangle$.

It is easily seen that the experimental value of the $\bar{B}_d^0 \to \pi^0\pi^0$ amplitude can be accommodated for a constituent mass m around 220 Mev and a value for $\langle\frac{\alpha_s}{\pi}G^2\rangle^{1/4}$ around 315 MeV. These values are of the same order as used in previous articles [6,9,11–13,20–22]. But in contrast to these previous cases the present amplitude for $\bar{B}_d^0 \to \pi^0\pi^0$ is very sensitive to variations of the model-dependent parameters m and $\langle\frac{\alpha_s}{\pi}G^2\rangle$. Or more specific, the colored current $J^\mu_{1G}(M_{\tilde{n}})^a$ in (47)–(49) is very sensitive to these parameters. In other words, Y_λ has to be fine-tuned in order to produce the experimental result.

In a recent paper [31] an extra mass parameter was introduced in the propagator of heavy quarks. One might do the same for propagator of the light energetic quark, and use

$$S_n = \frac{\gamma \cdot n}{N(n \cdot k + \Delta_n)}. \tag{52}$$

This would also bring this propagator more in harmony with the SCET propagator if $\Delta_n \sim p_\perp^2/E$. This will to first order in Δ_n give an extra contribution in the loop integral obtained from the diagram in Fig. 2. However, also taking into account the corresponding diagram where the light antiquark is the energetic one, this first order term in Δ_n cancels. But there will be terms of second order in Δ_n, which are of order δ^2. Such contributions have to be considered together with higher order (in δ) terms obtained from the interaction given by the operator X in (14).

One should note that the colored current given by (48) and (49) is determined by a triangle diagram. Thus one might speculate if it can in some way be related to the triangle anomaly. Namely, the diagram in Fig. 2 would have, for standard full propagators, the mathematical properties of the diagram relevant for the triangle anomaly. Using dimensional regularization in this case, with dimension $D = 4 - 2\epsilon$, the loop integration gives an divergent result $\sim I_2 \sim 1/\epsilon$ while the corresponding Dirac trace is $\sim\epsilon$. Thereby one obtains a finite expression for the triangle diagram in that case. However, in the present case we have replaced one of the standard (full) quark propagators by the SCET-like propagator $S_{\tilde{n}}$. Then the trace will not be $\sim\epsilon$ while the corresponding loop integral is still divergent $\sim 1/\epsilon$. This means that the diagram is in total divergent. Within our various chiral quark models including heavy quarks and light energetic quarks, the naive dimensional regularization (NDR) has been used, and divergent integrals have been identified with physical parameters [6,9,11–13,20–22,31]. Using other schemes additional finite terms of type ϵ/ϵ might appear [19], and some parameters might have to be redefined.

We also note that the description of the anomaly is rather tricky when going from the low-energy process $\pi^0 \to 2\gamma$ to higher energies where some cancellations occur [34,35]. In [34] the high energy processes $Z \to \pi^0\gamma$ and $\gamma^* \to \pi^0\gamma$ was studied. (Here the high energy virtual photon γ^* is coming from an energetic e^+e^- pair). In this case a part of the amplitude corresponding to low-energy decay $\pi^0 \to 2\gamma$ is cancelled. But there is a remaining *anomaly tail* relevant for some high energy processes [34,35]. Trying to adapt such a description in our case, the tensor T in (49) for an outgoing π^0 and soft-gluon would be replaced by

$$T^{\mu;\alpha\beta}(An) = \frac{I_{An}}{4\pi^2 f_\pi\sqrt{2}} p^\pi_\sigma \epsilon^{\sigma\alpha\beta\mu}, \tag{53}$$

where we have taken into account that couplings and color traces are different from the calculations in [34,35]. The quantity I_{An} is an integral given by

$$I_{An} = \int_0^1 \frac{x dx}{\eta x(1-x) - 1}, \tag{54}$$

where $\eta \equiv p_\pi^2/m^2$. Using, as before, m as a constituent mass and $p^\pi = E\tilde{n}$ would give $\eta = 1$ leading to $I_{An} \simeq 0.6$. However, as the anomaly tail is of perturbative character [34,35] one might think that it is more relevant to use masses closer to the current masses of order 5 to 10 MeV. In this case one has an asymptotic behavior $I_{An} \simeq ln(\eta)/\eta$, and this would give values for I_{An} of order 10^{-2}.

Now we use (36) and also include the Fermi coupling the Cabibbo-Kobayashi-Maskawa matrix elements, and the coefficient $2c_A$ for the nonfactorizable contributions to the amplitude, where c_A is the Wilson coefficient for the $\mathcal{O}_A$ local operator. Using Eqs. (45) and (47) we find the effective Lagrangian at mesonic level for the nonfactorizable contribution to $\bar{B}^0_d \to \pi^0\pi^0$:

$$\mathcal{L}^{\mathrm{LE}\chi\mathrm{QM}}_{\mathrm{Non.fact.}} = \frac{4\pi^2 c_A}{3}\left(4\frac{G_F}{\sqrt{2}} V_{\mathrm{ub}} V^*_{\mathrm{ud}}\right)\langle\frac{\alpha_s}{\pi} G^2\rangle S(H_b \to M_n M_{\tilde{n}}), \tag{55}$$

where $S(H_b \to M_n M_{\tilde{n}})$ is the tensor product

$$S(H_b \to M_n M_{\tilde{n}}) \equiv T^{\mu;\alpha\beta}(H_b \to M_n) T_{\mu;\alpha\beta}(M_{\tilde{n}}). \tag{56}$$

Using Eqs. (46) and (49), and $n \cdot \tilde{n} \simeq 2$, we find the amplitude expressed entirely by known parameters, we find an explicit expression for $S(H_b \to M M_{\tilde{n}})$ in the case $\bar{B}^0_d \to \pi^0\pi^0$:

$$S(\bar{B}^0_d \to \pi^0\pi^0) = 6(\frac{1}{\sqrt{2}})^2 \frac{G_A^2 G_H}{128\pi} Y E^2 \sqrt{M_B}. \tag{57}$$

We will now compare this nonfactorizable amplitude for $\bar{B}^0_d \to \pi^0\pi^0$ with the factorized amplitude which dominates $\bar{B}^0_d \to \pi^+\pi^-$:

$$\mathcal{M}_{\pi^+\pi^-} = \left(4\frac{G_F}{\sqrt{2}} V_{\mathrm{ub}} V^*_{\mathrm{ud}}\right) \cdot c_f \cdot \left(\frac{1}{2} J_\mu(\pi^-)\right) \cdot \left(\frac{1}{2} J^\mu(\bar{B}^0_d \to \pi^+)\right), \tag{58}$$

where

$$J_\mu(\pi^-) = f_\pi E \tilde{n}_\mu,\; J^\mu(\bar{B}^0_d \to \pi^+) = 2En^\mu \zeta^{(v)}. \tag{59}$$

The form factor $\zeta^{(v)}$ is defined in (16) and (17).

Using the Eqs. (40) and (55)–(59), we find the following ratio between the nonfactorized for $\bar{B}^0_d \to \pi^0\pi^0$ and the factorized amplitudes $\bar{B}^0_d \to \pi^+\pi^-$ is

$$r \equiv \frac{\mathcal{M}(\bar{B}^0_d \to \pi^0\pi^0)_{\mathrm{Non-Fact}}}{\mathcal{M}(\bar{B}^0_d \to \pi^+\pi^-)_{\mathrm{Fact}}} = \frac{c_A}{c_f}\frac{\kappa}{N_c}\frac{E\zeta^{(v)}}{\sqrt{mM_B}}, \tag{60}$$

where κ is a model-dependent hadronic factor

$$\kappa = \frac{\pi N_c \langle\frac{\alpha_s}{\pi} G^2\rangle Y}{2F^2 m^4 \sqrt{2\rho}}. \tag{61}$$

It will be interesting how the ratio r scales with energy E. Using the scaling behavior for $\zeta^{(v)}$ with $C = \hat{c} m^{3/2}$ in (17) we find for the ratio r:

$$r \simeq \frac{c_A}{c_f}\frac{\kappa\hat{c}}{N_c}\frac{m}{E}. \tag{62}$$

Our calculations show that the ratio r of the amplitudes are suppressed by $1/N_c$, as it should. The ratio is also scaling like m/E. Because $E \simeq m_b/2$ and m is the equivalent of Λ_{QCD} in our model, we have found that the nonfactorized amplitude is suppressed by $\Lambda_{\mathrm{QCD}}/m_b$ as required by the analysis in Ref. [1].

Concerning numerical predictions from our model, we have to stick to Eq. (60). The measured branching ratios for $\bar{B}^0_d \to \pi^-\pi^+$ and $\bar{B}^0_d \to \pi^0\pi^0$ are $(5.13 \pm 0.24) \times 10^{-6}$ and $(1.62 \pm 0.31) \times 10^{-6}$, respectively [36]. In order to predict the experimental value solely with the mechanism considered in this section, we should have $r \simeq 0.56 \pm 0.11$.

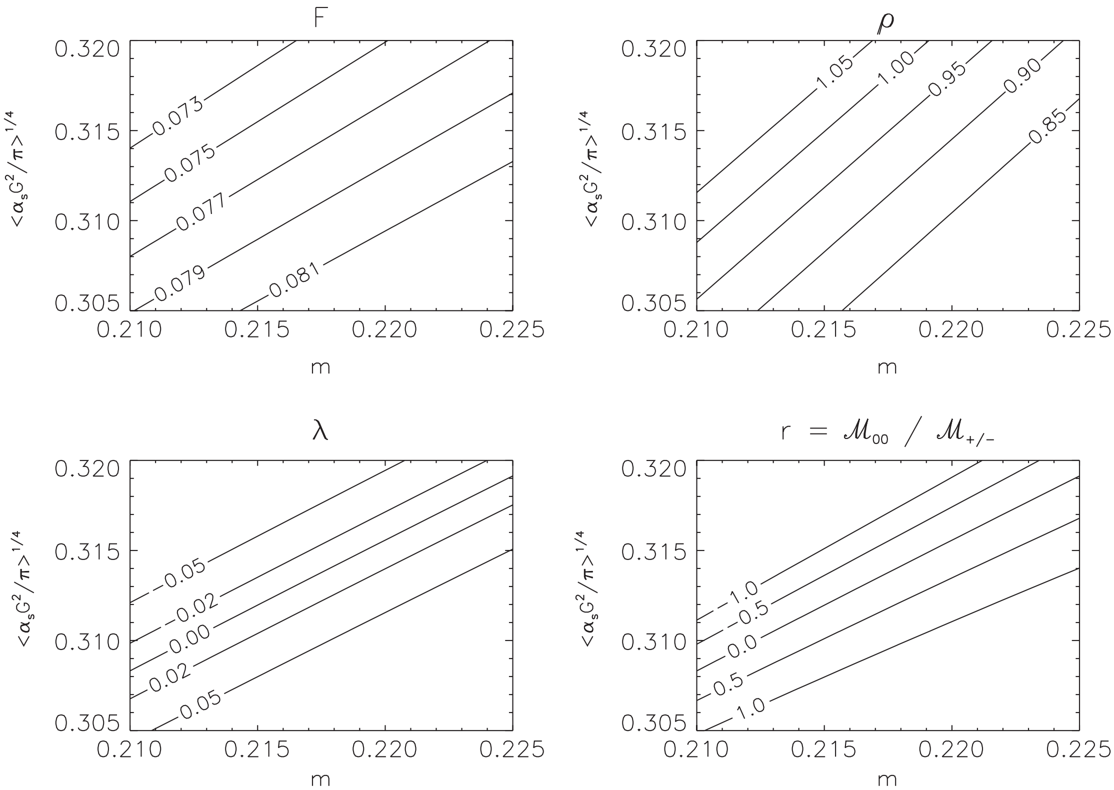

FIG. 3. Plots for the quantities F, ρ, λ and r in terms of m and $\langle\frac{\alpha_s}{\pi}G^2\rangle^{1/4}$. We observe that for reasonable values of these parameters the ratio r can take a wide range of values such that fine-tuning is required to reproduce the experimental value.

Numerically, we use $\zeta^{(v)} \simeq 1/3$ [32]. In previous papers on the heavy-light chiral quark model constituent masses $m \sim 220$ MeV and $\langle\frac{\alpha_s}{\pi}G^2\rangle^{1/4} \sim 315$ MeV has been used. From the plot of r in Fig. 3, we observe that the experimental value of r can easily be accommodated by values of such orders. The bad news is that in our case the value of Y_λ and thereby κ and r is very sensitive to the explicit choice of m and $\langle\frac{\alpha_s}{\pi}G^2\rangle^{1/4}$. Thus fine-tuning has to be used.

We also find that the perturbative anomaly tail will numerically reproduce the amplitude for $I_{An} \simeq 3.2\times10^{-2}$, corresponding to a quark mass $m_0 \simeq 11$ MeV, i.e. of same order of magnitude as typical current quark masses. Using a hybrid description with a quark model with constituent quark masses for the colored $\bar{B}^0 \to \pi^0$ current in (44)–(46), and the anomaly tail description [34,35] for the colored π^0 current in (47)–(49), is not preferable. Also, such a hybrid description also fails to show the behavior $\Lambda_{\rm QCD}/m_b$ required by QCD-factorization. Still it might be interesting that we can numerically match the colored current for outgoing π^0 with the anomaly tail description.

Note that there are also mesonic loop contributions similar to those contributing to processes of the type $B \to D\bar{D}$ and $B \to \gamma D$ [9,11]. For those processes intermediate $D^*(1^-)$ mesons contributed. In the present case the analogous contributions would involve energetic vector

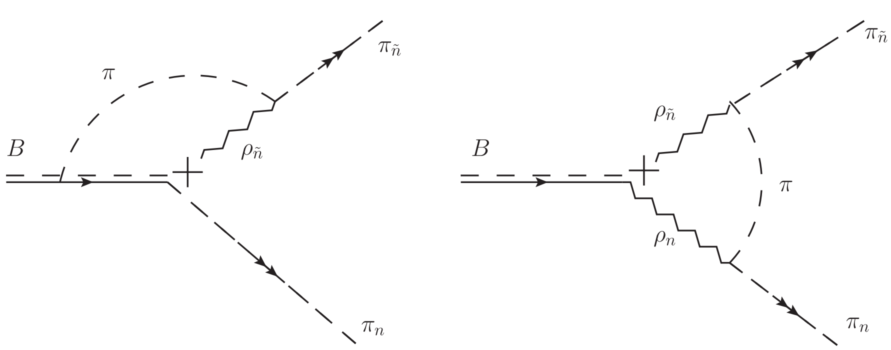

FIG. 4. Meson loops for $\bar{B}^0_d \to \pi\pi$. The zigzag lines represent energetic ρ mesons. The dashed lines with double arrows are energetic light mesons and the dashed line with no arrow is a soft pion.

mesons ρ_n, and we would need the amplitudes for $B \to \rho_n \rho_{\bar{n}}$. Such loops are shown in Fig. 4. The diagram to the right would be calculable within an extended theory involving energetic vector mesons. Unfortunately while the diagram to the left would be dubious because typical loop momenta would significantly exceed 1 GeV, and would require insertion of *ad hoc* form factors or should be handled within dispersion relation techniques. Both diagrams would of course require knowledge of the $\rho_n \pi_n \pi$ coupling in Fig. 4. In any case such calculations are beyond the scope of this paper.

VI. CONCLUSION

We have pointed out that the factorized amplitude for process $\bar{B}^0_d \to \pi^0\pi^0$ is proportional to a Wilson coefficient combination close to zero. Thus the nonfactorizable contributions dominate the amplitude for this decay mode. To handle the nonfactorizable contributions we have extended previous chiral quark models for the pure light quark case [24] used in [19,23,25], and the heavy-light case [20] used in [6,9,11,12,21,22], to include also energetic light quarks.

We have found that within our model we can account for the amplitude needed to explain the experimental branching ratio for $\bar{B}^0_d \to \pi^0\pi^0$ [36]. In addition, the ratio r between the nonfactorizable and factorized amplitude scales as $\Lambda_{\rm QCD}/m_b$ in agreement with QCD-factorization [1]. However, the bad news is that the calculated amplitude is very sensitive to our model-dependent parameters, i.e. the constituent quark mass m, and the gluon condensate $\langle \frac{\alpha_s}{\pi} G^2 \rangle$. Anyway, final state interactions should be present [37].

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