

Extracting the Ω^- electric quadrupole moment from lattice QCD data

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The Ω^- has an extremely long lifetime, and is the most stable of the baryons with spin $3/2$. Therefore the Ω^- magnetic moment is very accurately known. Nevertheless, its electric quadrupole moment was never measured, although estimates exist in different formalisms. In principle, lattice QCD simulations provide at present the most appropriate way to estimate the Ω^- form factors, as function of the square of the transferred four-momentum, Q^2 , since it describes baryon systems at the physical mass for the strange quark. However, lattice QCD form factors, and in particular G_{E2} , are determined at finite Q^2 only, and the extraction of the electric quadrupole moment, $Q_{\Omega^-} = G_{E2}(0) \frac{e}{2M_{\Omega^-}}$, involves an extrapolation of the numerical lattice results. In this work, we reproduce the lattice QCD data with a covariant spectator quark model for Ω^- which includes a mixture of S and two D states for the relative quark-diquark motion. Once the model is calibrated, it is used to determine Q_{Ω^-} . Our prediction is $Q_{\Omega^-} = (0.96 \pm 0.02) \times 10^{-2} \text{ efm}^2$ [$G_{E2}(0) = 0.680 \pm 0.012$].

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I. INTRODUCTION

The prediction of the electromagnetic structure of baryons and mesons is an important challenge for quark models and, when compared to the available experimental results, it provides a test on the relevant hadronic degrees of freedom. For baryons with spin $1/2$ (as in the baryon octet) the charge and the magnetic moment are the only multipole moments to be defined, while for baryons with spin $3/2$ (as in the baryon decuplet) also the electric quadrupole moment exists. However, at present, there is no experimental measurement of the electric quadrupole moment for any of the baryons, although there are several model predictions for the Δ and the Ω^- , and other decuplet particles. For a summary of the Δ results see Refs. [1,2]. As for the Ω^- there are predictions based in quark models [3–9], chiral perturbation theory [10,11], large- N_c limit [12–14], and other formalisms [15–18].

Within all members of the baryon decuplet, the Ω^- is especially interesting. As it is composed solely by strange quarks in the valence sector, it can decay only by weak interaction and therefore its lifetime is extremely longer than the one of the other baryons. For this reason, experimentally, the Ω^- properties are easier to be determined than the ones of any other member of the decuplet. A good illustration of this is the accuracy of the Ω^- magnetic moment $\mu_{\Omega^-} = -(2.019 \pm 0.053)\mu_N$ [19–22], where μ_N is the nuclear magneton. It is also expected that the Q_{Ω^-} quadrupole moment will be measured in a near future [23–26].

Additionally, the valence quark content of the Ω^- is restricted to strange quarks, with a mass considerably larger than the light u and d quark masses. As now it is already possible to perform lattice QCD at the physical strange quark mass, the Ω^- magnetic moment [27–29],

and more recently also its electric charge G_{E0} , magnetic dipole G_{M1} , electric quadrupole G_{E2} and magnetic octupole G_{M3} [30,31] were calculated within lattice QCD. Another important issue is that in sea quark effects for the Ω^- only at most one single light quark participates, and therefore the pion has no role in this case. As in chiral perturbation theory loops involving mesons heavier than the pion are suppressed, the Ω^- becomes then a special case where meson cloud corrections to the valence quark core are expected to be small. A consequence of the smallness of the meson cloud effects is that lattice QCD simulations, quenched or unquenched, should be a good approximation to Ω^- form factors at the physical point. Therefore, in this work we take the lattice QCD simulations as good representations of the physical results, without any extrapolation of the lattice data to the physical pion mass.

The main limitation in obtaining the Ω^- electromagnetic form factors in lattice QCD simulations comes from these ones being restricted, for practical reasons, to finite nonzero values of Q^2 , while the determination of the quadrupole moment, for instance, is proportional to $G_{E2}(0)$. An extrapolation in the momentum transfer squared Q^2 , down to $Q^2 = 0$ is then required, and one has inevitably to resort to an analytical form to do it. It is at this point that it is reasonable to expect that a quark model is useful, in particular, for systems as the Ω^- , without light valence quarks and where meson loop corrections are expected to be small. Since the covariant spectator quark model was tested already for spin $1/2$ baryons [32–35], spin $3/2$ baryons [1,2,20,36,37], including strange quarks, and electromagnetic transitions between different baryon states [35,37–42], it is a good candidate to be used not only to interpolate between lattice

QCD data in Q^2 , but also to extrapolate the form factor data to $Q^2 = 0$.

The covariant spectator quark model was applied in the past to estimate successfully the leading order form factors of the Ω^- (G_{E0} and G_{M1}) neglecting D -state admixtures [20]. Here, we extend the formalism to the case where the D -state admixture coefficients are nonzero. Because these states are included, we have now contributions for the G_{E2} and G_{M3} form factors, and therefore we can use the model to extract the Ω^- electric quadrupole moment from lattice QCD data. By adjusting some parameters associated with the Ω^- wave function to the lattice data, our procedure has the advantage of incorporating into the phenomenological model the fundamental theory of the strong interaction, in its discrete version (lattice QCD). The information on the wave function parameters allows us then to calculate all the electromagnetic form factors G_{E0} , G_{M1} , G_{E2} and G_{M3} as functions of Q^2 . In particular, the model can be used to determine the electric quadrupole form factor at the $Q^2 = 0$ point. To constrain the model, we use the unquenched lattice QCD data from Ref. [30] for Ω^- at the physical Ω^- mass. Although the existing lattice data is unquenched, the valence quarks are expected to play the main role and meson dressing to be small, as mentioned before. This is why the adjustment of the quark model to the lattice QCD data is meaningful. To better constrain the parameters of the quark model, we use also the single data point for G_{M3} from Ref. [31], in addition to the lattice data from Ref. [30].

We start by using the spectator formalism to represent the Ω^- wave function, in a similar way to the one used before for the Δ [39]. Although both the Δ and the Ω^- have the same spin structure, they differ in their flavor content. Therefore, we begin with the SU(3) generalization of the spectator quark model for the overall study of the baryon decuplet [20]. Also, the quark momentum distributions for the two baryons are different, and our calculation at the end quantifies this difference. As for the electromagnetic current associated with the interaction of the photon with the strange quark, which is another aspect where the calculation differs from the calculation for the Δ baryon, we use the current based on vector meson dominance from Ref. [20].

Under the assumption that the D -state components are small, then we take only the electromagnetic current matrix elements which are in first order in the admixture coefficients as done in Ref. [2] for the Δ . Finally, in the process of adjusting the Ω^- electromagnetic form factor results to the lattice data, we determine the percentage of each D state present in the orbital quark-diquark part of the wave function of the Ω^- baryon. Our calculation enables us at the end to narrow the uncertainty in the extraction of the value of the quadrupole magnetic moment of the Ω^- from the lattice QCD data.

This work is organized as follows: In Sec. II, we give the formulas for the Ω^- electromagnetic form factors in first

order of the admixture coefficients. In Sec. III, we parametrize the Ω^- wave function and its momentum dependence. The results are presented in Sec. IV, and the conclusions and final remarks in Sec. V.

II. Ω^- FORM FACTORS

We use here the covariant spectator quark model, where relativity is implemented consistently. Within this framework a baryon is described as an off-shell quark and two *noninteracting* on-shell spectator quarks. Integrating over the on-mass-shell quarks degrees of freedom, one represents those quark states as a single on-shell particle (or diquark) with an average mass m_D [20,32]. With this reduction, the wave function associated with the baryon states including the spin, angular momentum, coordinate space and flavor structure, can then be written as the direct product of the diquark and quark states properly symmetrized.

The electromagnetic interaction with the Ω^- is, in relativistic impulse approximation, written as the sum over the terms in which the photon couples to each (off-shell) quark in turn with the other two (on-shell) quarks that compose the diquark. The electromagnetic structure of the quarks is parametrized in terms of form factors. One can rearrange the contributions to the electromagnetic current from the conveniently symmetrized wave functions in terms of the on-shell diquark states—quark pair (12), and the off-shell quark—quark 3 states. The final result for the current becomes then three times the current associated to the interaction with quark 3. See Ref. [20] for a detailed discussion. We write then the Ω^- wave function as $\Psi_\Omega(P, k)$ for total momentum P , as the combination of the diquark (on-shell) states, with momentum k , and the quark 3 (off-shell) states. In our notation, the indices for the diquark polarization $\epsilon = 0, \pm$ and the Ω^- spin projection are omitted for simplicity, and the matrix element of the electromagnetic current between the initial and final states of momentum P_+ and P_- , respectively, is written as

$$J^\mu = 3 \sum_\epsilon \int_k \bar{\Psi}_\Omega(P_+, k) j_q^\mu \Psi_\Omega(P_-, k), \quad (1)$$

where j_q^μ is the current operator for quark 3 and \int_k is the covariant integral is defined as $\int_k = \int \frac{d^3k}{(2\pi)^2 2E_D}$, where E_D is the diquark on-shell energy. In Eq. (1) the interactions with *all* quarks are counted, without including the coupling with the diquark [20,32].

In this work, the Ω^- wave function is represented as a combination of an S state and two D states for the quark-diquark relative motion [39]:

$$\Psi_\Omega(P, k) = N[\Psi_S(P, k) + a\Psi_{D3}(P, k) + b\Psi_{D1}(P, k)]. \quad (2)$$

In the previous equation, a and b are the mixture coefficients of the states: $D3$ (core spin $3/2$) and $D1$ (core spin $1/2$), respectively, and $N = 1/\sqrt{1 + a^2 + b^2}$ a normalization constant. Each of the three wave function terms

includes a scalar wave function, respectively ψ_S , ψ_{D3} , and ψ_{D1} , which is a function of the baryon momentum P and diquark momentum k , with a form and normalization given in the next section.

The quark current j_q^μ can be in general decomposed as

$$j_q^\mu(Q^2) = j_1(Q^2)\gamma^\mu + j_2(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2M_N}, \quad (3)$$

where M_N is the nucleon mass, and

$$j_i = \frac{1}{6}f_{i+}\lambda_0 + \frac{1}{2}f_{i-}\lambda_3 + \frac{1}{6}f_{i0}\lambda_8, \quad (4)$$

with $\lambda_0 = \text{diag}(1, 1, 0)$; $\lambda_3 = \text{diag}(1, -1, 0)$ and $\lambda_8 = \text{diag}(0, 0, -2)$ are SU(3) flavor operators acting in the particular quark flavor state $q = (uds)^T$ [20,34].

The quark form factors are normalized as $f_{1\pm}(0) = 1$, $f_{10}(0) = 1$, $f_{2\pm}(0) = \kappa_\pm$, and $f_{20}(0) = \kappa_s$. We model the electromagnetic structure of the quarks by means of a parametrization that is based on vector meson dominance. In particular, the strange quark form factors are represented [20] by

$$f_{10} = \lambda + (1 - \lambda)\frac{m_\phi^2}{m_\phi^2 + Q^2} + c_0\frac{M_h^2 Q^2}{(M_h^2 + Q^2)^2}, \quad (5)$$

$$f_{20} = \kappa_s \left\{ d_0 \frac{m_\phi^2}{m_\phi^2 + Q^2} + (1 - d_0) \frac{M_h^2}{M_h^2 + Q^2} \right\}, \quad (6)$$

where m_ϕ is a mass of the ϕ meson (system $\bar{s}s$), $M_h = 2M_N$ is an effective vector meson that simulates the short range structure and λ is fixed by deep inelastic scattering, and corresponds to the quark number density [32]. The coefficients defining the current were fixed as $c_0 = 4.427$ and $d_0 = -1.860$ in the study of the dominant form factors of the baryon decuplet [20]. As for the strange quark anomalous magnetic moment, one uses $\kappa_s = 1.462$ to reproduce the experimental value of μ_{Ω^-} [20]. The explicit expression for the remaining quark form factors are presented in Refs. [20,39,40].

Following Refs. [39,40], the current can be written in terms of *charge* \tilde{e}_Ω and *anomalous magnetic moment* $\tilde{\kappa}_\Omega$ functions

$$\tilde{e}_\Omega = -f_{10}(Q^2) \quad (7)$$

$$\tilde{\kappa}_\Omega = -f_{20}(Q^2)\frac{M_\Omega}{M_N}. \quad (8)$$

In the previous equation, for convenience we use a normalization that differs from the one presented in [20], by a factor $\frac{M_\Omega}{M_N}$. This redefinition does not change the results. We define also

$$\tilde{g}_\Omega = \tilde{e}_\Omega - \tau\tilde{\kappa}_\Omega \quad (9)$$

$$\tilde{f}_\Omega = \tilde{e}_\Omega + \tilde{\kappa}_\Omega, \quad (10)$$

where $\tau = Q^2/(4M_\Omega^2)$.

Working the algebra for the current as in Refs. [1,2], replacing \tilde{e}_Δ and $\tilde{\kappa}_\Delta$ by \tilde{e}_Ω and $\tilde{\kappa}_\Omega$, one obtains in first order in the admixture coefficients a and b :

$$G_{E0}(Q^2) = N^2\tilde{g}_\Omega I_S \quad (11)$$

$$G_{M1}(Q^2) = N^2\tilde{f}_\Omega \left[I_S + \frac{4}{5}aI_{D3} - \frac{2}{5}bI_{D1} \right] \quad (12)$$

$$G_{E2}(Q^2) = N^2\tilde{g}_\Omega(3a)\frac{I_{D3}}{\tau} \quad (13)$$

$$G_{M3}(Q^2) = \tilde{f}_\Omega N^2 \left[a\frac{I_{D3}}{\tau} + 2b\frac{I_{D1}}{\tau} \right], \quad (14)$$

where the overlap between the S states is

$$I_S = \int_k \psi_S(P_+, k) \psi_S(P_-, k), \quad (15)$$

and the overlap between the S and each one of the D states is

$$I_{D1} = \int_k b(\tilde{k}_+, \tilde{q}_+) \psi_{D1}(P_+, k) \psi_S(P_-, k) \quad (16)$$

$$I_{D3} = \int_k b(\tilde{k}_+, \tilde{q}_+) \psi_{D3}(P_+, k) \psi_S(P_-, k). \quad (17)$$

The function $b(\tilde{k}_+, \tilde{q}_+)$ is defined in Ref. [39] and includes the specific angular dependence of a D state, with \tilde{k}_+ defined as the three-momentum in the frame where $P_+ = (M_\Omega, 0, 0, 0)$: $\tilde{k}_+ = k - \frac{P_+ \cdot k}{M_\Omega^2} P_+$.

From Eq. (11), one obtains that the result for the charge form factor at $Q^2 = 0$ is $G_{E0}(0) = -N^2$, [note that $N^2 = 1/(1 + a^2 + b^2)$], which differs from the exact result (-1) , if $a, b \neq 0$. This deviation, although small if a and b are small, is a consequence of taking in the calculation of the current matrix elements only the terms in first order in these D state admixture coefficients a and b . Also, the magnetic dipole form factor G_{M1} , from Eq. (12), at $Q^2 = 0$ deviates slightly from the experimental value (which gives the Ω^- magnetic moment). Once the terms for the current matrix elements for the D - to D -state transitions are included, the exact results of both the charge and magnetic moment are recovered exactly. We will use this fact to estimate the error of our model, as explained in Sec. IV.

III. MODEL FOR THE SCALAR WAVE FUNCTIONS

To describe the momentum dependence of the scalar wave functions, one assumes a certain form (as done already for the Δ in Refs. [39,40])

$$\psi_S(P, k) = \frac{N_S}{m_D(\alpha_S + \chi_\Omega)^3} \quad (18)$$

$$\psi_{D3}(P, k) = \frac{N_{D3}}{m_D^3(\alpha_{D3} + \chi_\Omega)^4} \quad (19)$$

$$\psi_{D1}(P, k) = \frac{N_{D1}}{m_D^3(\alpha_{D1} + \chi_\Omega)^4}, \quad (20)$$

where

$$\chi_\Omega = \frac{(M_\Omega - m_D)^2 - (P - k)^2}{M_\Omega m_D}. \quad (21)$$

In this way, we introduce momentum scale parameters (α_S , α_{D3} , α_{D1}) for each angular momentum or orbital state. In contrast to the Δ case, one does not have to impose the orthogonality of the spin core $S = 1/2$ D state with an S state with the same spin, as a spin $1/2$ S state with three strange quarks is forbidden. Therefore, the expression for the state $D1$ differs from the one used for the Δ baryon [39,40], and one may consider a simpler parameterization for the Ω^- $D1$ state.

The normalization conditions are given by

$$\int_k |\psi_S(\bar{P}, k)|^2 = 1 \quad (22)$$

$$\int_k |\tilde{k}^2 \psi_{D3}(\bar{P}, k)|^2 = 1 \quad (23)$$

$$\int_k |\tilde{k}^2 \psi_{D1}(\bar{P}, k)|^2 = 1, \quad (24)$$

where $\bar{P} = (M_\Omega, 0, 0, 0)$ is the momentum of the Ω^- in the rest frame. The parameters α_S , α_{D3} , α_{D1} , and the admixture coefficients a , b are the five adjustable parameters of the model.

IV. RESULTS

In this section, we present the results for the Ω^- electromagnetic form factor data obtained from the quark model described in the previous sections, calibrated by a fit to the available lattice data. The available Ω^- form factor data as function of Q^2 , is restricted to the unquenched lattice simulations of the G_{E0} , G_{M1} and G_{E2} form factors [30]. Under these conditions, and given that the term from the S state dominates in G_{E0} and G_{M1} , the most important constraint for the D states comes from G_{E2} (to which only the interference term between S and $D3$

state contributes), and a decisive constraint on the $D1$ state (through G_{M3}) is not available yet, except for the quenched calculation of Ref. [31] for $Q^2 = 0.23$ GeV^2 . That point is also considered in our fit.

In principle, one could assume that the $D1$ state would not be important for the Ω^- structure—we evoke that it was checked earlier that the lattice data for the Δ baryon can be described with very small ($\approx 1\%$) D state admixtures [40]. Still, this needs to be investigated, and therefore we check here whether the $D1$ state can be important to G_{E0} and G_{M1} data and helps to improve the overall description of the form factor data. This is why in this work we leave the $D1$ state mixture free and use the lattice data (mainly G_{M1}) to fix that contribution. Future lattice QCD simulations for G_{M3} may then confirm or contradict our model.

The only limitation of our calculation is that the amount of D -states admixture is assumed to be small, since the calculation of the form factor proceeds by taking only into account the first order terms in a and b (neglecting transitions between D states). That limitation will be quantified at the end, comparing the result of $G_{E0}(0)$ with the Ω^- charge (-1), obtained when all states are include in the electromagnetic transition current.

A. Lattice data

We use the lattice QCD data from Alexandrou *et al.* [30] and the G_{M3} data point from Ref. [31]. All the lattice QCD simulations from Ref. [30] are unquenched but two different methods are used: hybrid action and domain wall fermions (DWF). The single data point from Ref. [31] is quenched. The data from Ref. [30] goes almost to 4 GeV^2 [43], but only the data for $Q^2 < 1.6$ GeV^2 is relatively precise. The simulations were performed for one pion mass using the hybrid method ($m_\pi = 0.353$ GeV) and three pion masses with the DWF method ($m_\pi = 0.297$, 0.330 and 0.355 GeV).

The results of G_{E0} and G_{M1} in the DWF case show a weak dependence in the pion mass, suggesting that meson cloud effects (K and η) may be negligible at least for those form factors as expected from a three strange valence quark system. The hybrid simulation for G_{E0} and G_{M1} shows a systematic deviation from the results from DWF (slower falloff) for large Q^2 with similar pion mass ($m_\pi = 0.355$ GeV).

For the hybrid simulation, one has results for G_{E0} , G_{M1} and G_{E2} for pion mass value of $m_\pi = 0.353$ GeV . As for the DWF simulations, one has data for G_{M1} , with $m_\pi = 0.355$ GeV ; for G_{E0} and G_{M1} with $m_\pi = 0.330$ GeV ; and

TABLE I. Results of the χ^2 in the fit to the lattice data. The results correspond to the values: $a = 0.0341$, $b = 0.2666$, $\alpha_S = 0.1793$, $\alpha_{D3} = 0.5394$ and $\alpha_{D1} = 0.4674$.

$\chi^2(G_{E0})$	$\chi^2(G_{M1})$	$\chi^2(G_{E2})$	$\chi^2(G_{M3})$	$\chi^2(tot)$	$G_{E2}(0)$
5.28	2.66	0.203	2.49	2.58	0.674

finally for data for G_{E0} , G_{M1} and also for G_{E2} with $m_\pi = 0.297$ GeV. The difference between the two methods, particularly to high Q^2 , can be a lattice artifact (cutoff effect) [30]. More lattice QCD simulations with smaller lattice spacing and larger volumes are necessary to clarify the differences between the two methods [30].

The Ω^- form factors can also be obtained from a simulation of the Δ^- in the SU(3) symmetry limit. This was done by Boinepalli *et al.* [31] for pion mass $m_\pi = 0.697$ GeV, leading to a slightly heavier mass for the Ω^- (1.732 GeV to be compared with 1.672 GeV of the physical case [19]). This simulation is quenched and the results are given only for one value of Q^2 [$Q^2 = 0.23$ GeV²]. Importantly, that work provides the only existing clue for the behavior of the octupole magnetic moment form factor: $G_{M3}(0.23 \text{ GeV}^2) = 1.25 \pm 7.50$.

B. Calibrating the model (fit to lattice data)

Ideally, to extrapolate the Ω^- form factors from lattice QCD, one should take the set of lattice QCD simulations performed as close as possible to the physical limit—the lowest pion mass ($m_\pi = 0.297$ GeV) considered in the DWF case. Unfortunately, the available DWF data for G_{E2} is restricted to 7 data points, for that mass, below 2 GeV², and there is no data for $Q^2 < 0.4$ GeV², which is a severe limitation to extrapolate the results down to $Q^2 = 0$.

To obtain a more accurate extrapolation, we took also the hybrid data (with 13 data points for G_{E2} below 2 GeV²). To consider simultaneously the hybrid and DWF data increases the statistics, which in principle leads to more accurate constraint of the model. However, the differences between the two methods, in particular, the high accuracy of the G_{E0} data, make a good fit difficult, particularly for the G_{E0} and G_{M1} form factors. Since the difference between the two methods is amplified as Q^2 increases (particularly for $Q^2 > 1$ GeV²), to improve the quality of our fit we took only the G_{E0} and G_{M1} data below $Q^2 \leq 1$ GeV². This is justified since the lattice simulations have larger numerical error for higher Q^2 and also because we are focused in the extrapolation for the $Q^2 = 0$ point, which leads us to reduce the weight of high Q^2 data. This procedure is frequently used in lattice QCD studies at low Q^2 [44]. The inclusion of the region $Q^2 > 1$ GeV² in the fit will be possible once a more homogeneous set of data with sufficient statistics is provided. As for G_{E2} we took the data for $Q^2 < 2$ GeV², since the error bars are more significant and we want to keep the statistics as large as possible.

The results for the χ^2 of the fit are presented in the Table I. The large χ^2 for the G_{E0} data is a consequence of the great precision of the data and also of the different behavior of the hybrid and DWF data. The parameters of the fit are indicated in the caption of Table I. The mixture of

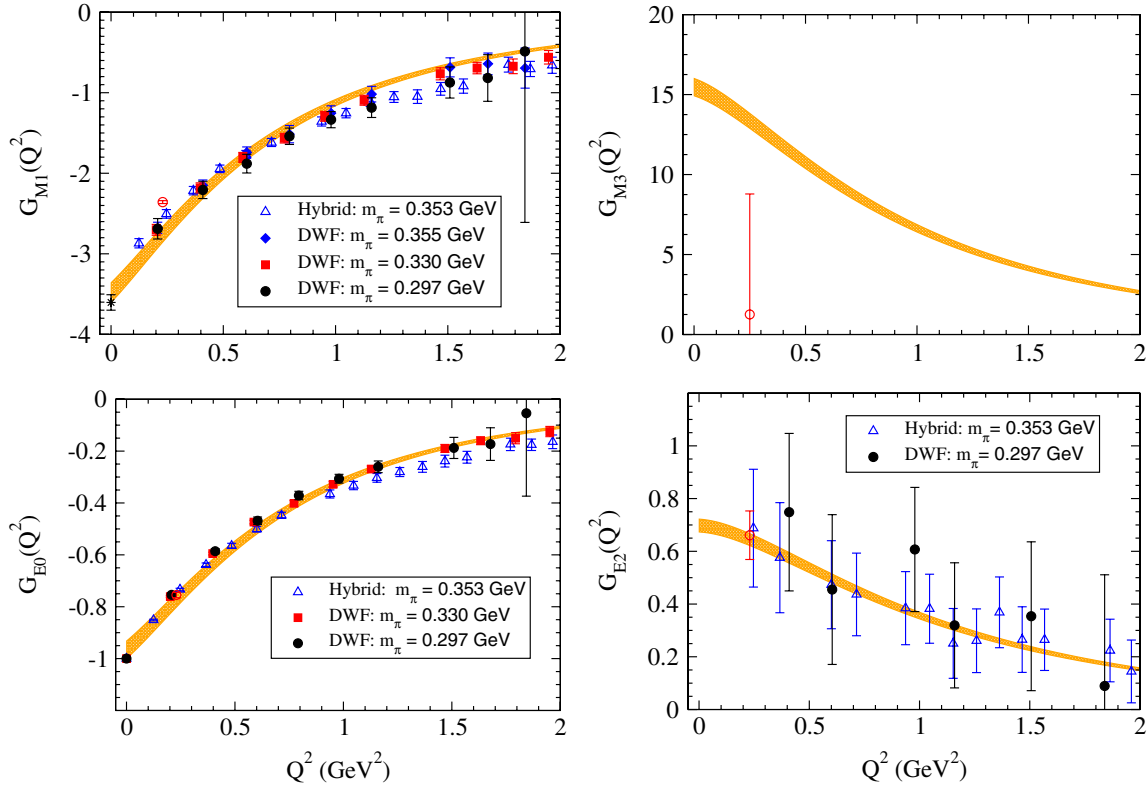


FIG. 1 (color online). Best fit with a D1 and D3 mixture. Lattice QCD data from Alexandrou *et al.* [30]. For G_{M1} , we include also the experimental result $G_{M1}(0) = -3.604 \pm 0.096$ from PDG [19] (*). The open circles represent the result for $Q^2 = 0.23$ GeV² from Boinepalli *et al.* [31].

D states that we obtain is 0.11% (for $D3$) and 6.6% (for $D1$). There is therefore a significant mixture of the $D1$ state.

In detail, the value for the momentum-range parameter α_S (0.1793) obtained in this fit (with S and D states) suggests that D states improve in fact the description of G_{E0} and G_{M1} , when compared with the predictions from Ref. [20], prior to the simulations of Ref. [30] and where α_S was smaller (0.1630).

The results of our fit to the lattice form factors are presented in Fig. 1. The upper (lower) results for G_{E0} and G_{M1} (G_{E2} and G_{M3}) are given by Eqs. (11)–(14), with the factor $N^2 = 1/(1 + a^2 + b^2)$ defined by the normalization of the Ω^- wave function,¹ given by Eq. (2). As mentioned in Sec. II, those equations are derived in first order in the D -state admixture parameters a and b , and differ from the final result because the transitions between D states are neglected. We can calculate the effect of this approximation by evaluating the correction to $G_{E0}(Q^2)$ needed to reproduce exactly the Ω^- charge at $Q^2 = 0$. From Eq. (11) the exact result for $G_{E0}(0)$ is obtained by setting $N^2 = 1$, which then defines our lowest estimation of G_{E0} . The band between the upper and the lower result measures the effect in the final result of the neglected D to D -state transitions. The width of the band is small because a and b turn out to be small from the fit to the lattice data. As for the uncertainty in the calculation of the form factors, G_{M1} , G_{E2} and G_{M3} we use the same procedure, for consistency.

Our results presented in Fig. 1, for the four form factors, are consistent with the overall lattice data within 1 standard deviation, to the exception of the form factor G_{M3} . In that case, although we overestimate the lattice data point [31], our result is still inside an uncertainty interval of 1.5 standard deviations. Note that in our formalism, the magnitude of G_{M3} is a consequence of the relatively large $D1$ state admixture (6.6%), a prediction to be tested by future and more precise lattice QCD simulations. The extracted value of the electric quadrupole form factor at $Q^2 = 0$ is $G_{E2}(0) = 0.680 \pm 0.012$.

V. CONCLUSIONS

The Ω^- is the most stable baryon with spin 3/2. Yet, only some of its properties are known. As a spin 3/2 particle with a long lifetime, it is the first candidate for the experimental determination of the electric quadrupole moment, since the nucleon (spin 1/2) has no quadrupole moment, and the lifetime of the Δ is much shorter. Several experimental

methods have been proposed to measure the still unknown quadrupole moment Q_{Ω^-} , which is a signature of distortion and is likely to be measured in a very near future. This makes its *a priori* prediction so challenging.

As the Ω^- is essentially a three strange valence quark system, it is possible nowadays to simulate the electromagnetic coupling with the Ω^- in a discrete lattice for physical strange quark masses with light sea quarks ($m_\pi \simeq 300$ MeV) and to determine the Ω^- electromagnetic form factors. In the absence of experimental information, lattice QCD provides then the more reliable method to unveil the Ω^- electromagnetic structure.

However, to determine the electric quadrupole moment, an extrapolation of the G_{E2} form factor down to $Q^2 = 0$ must be done, and some analytical form near the origin must be assumed. In this work, we provide then a method to extract information from the lattice QCD data, without assuming any special analytical form, as a dipole, tripole, or exponential function. We start by requiring an overall consistency between the prediction of our model for the four form factors and the lattice QCD data for the physical Ω^- mass. Then, the model extends naturally down to the $Q^2 = 0$ limit, inferring the behavior of the Ω^- system in the region not covered by the lattice data. As a bonus relatively to an *ad hoc* parameterization, we obtain even information about the microscopic structure such as the admixture percentage of each D state and the momentum distribution of the wave function.

The procedure assumes that meson cloud dressing is not significant in the Ω^- system. Although the present lattice QCD results cannot rule out that sea quark dressing (meson cloud) may be important, they suggest that dependence in the light quark (pion mass) is small at least for the charge and magnetic dipole form factors. This topic will be investigated in the future.

Our final results imply an unexpected large $D1$ state admixture (6.6%). The confirmation or disproof of this result will be possible once lattice data for G_{M3} becomes available. More precise data for G_{M1} (G_{E0} is already very precise), particularly at high Q^2 , where the weight of the D states is larger, will also be useful for an even better estimate of the both D -state admixtures.

Our final result for the electric quadrupole moment of the Ω^- is $Q_{\Omega^-} = (0.96 \pm 0.02) \times 10^{-2} \text{ efm}^2$. In the literature, the existing results for Q_{Ω^-} correspond to the interval $(0.4 - 4.0) \times 10^{-2} \text{ efm}^2$ [3–8,10–12,14,15,18], and our result has a magnitude consistent with this range. The value extracted directly from lattice QCD data, assuming an exponential dependence [30], corresponds to $(1.18 \pm 0.12) \times 10^{-2} \text{ efm}^2$. Note that our result satisfies simultaneously the constraints of the three form factors (G_{E0} , G_{M1} and G_{E2}), and therefore can be given with a smaller band of uncertainty.

The numerical value of $Q_{\Omega^-}/e_{\Omega^-}$ ($e_{\Omega^-} = -1$), because it is a positive number, can be interpreted in a

¹Note that in the present work, by taking $N^2 = 1/(1 + a^2 + b^2)$, we are not modifying the normalization of the Ω^- wave function relative to Ref. [20]. In previous calculations, we had $N^2 = 1$ because only the S state (normalized to 1) was considered. Once the D states are added, N^2 is redefined accordingly to the D -state admixture coefficients to $N^2 = 1/(1 + a^2 + b^2)$.

nonrelativistic formalism as the charge distribution being extended and flattened along the equatorial region, as it was also predicted for the Δ^+ baryon [1]. Note however that this interpretation has been questioned, and some authors suggest a different concept of deformation, based on the transverse electric quadrupole moment in the infinite momentum frame [45–47].

Comparing the Ω^- with the Δ^+ baryon, in the covariant spectator quark model it is interesting to notice that the difference in the $D3$ admixture, 0.72% for the Δ and 0.11% for the Ω^- does not correspond to a reduction of $G_{E2}(0)$ in the same proportion. This shows that the momentum distribution in the overlap integral between S and $D3$ states is very different in both systems and it has to be taken into consideration.

An intrinsic limitation of our calculation is the determination of the form factors only in first order of the admixture coefficients. The exact calculation of the form factors including the contributions from transitions between D states is in progress [48]. Nevertheless, the effect of these transitions can be estimated approximately by the correction implied by the deviation of the normalization constant N from 1. In the present case, this correction is about 3.4%.

Finally, in the future, a precise calculation of the magnetic octupole form factor of the Ω^- using lattice QCD will be very useful for a better understanding of the Ω^- electromagnetic structure. The magnetic octupole

moment $\mathcal{O}_{\Omega^-} = G_{M3}(0) \frac{e}{2M_{\Omega}^3}$, was estimated already in Refs. [11,17,49]. Our result corresponds to $(1.27 \pm 0.04) \times 10^{-2} \text{ efm}^3$. Although the experimental determination of \mathcal{O}_{Ω^-} may not be possible in practice, an evaluation based on lattice QCD in a regime where meson excitations are expected to be small, may help to select models. In particular for our model, it can allow a more accurate determination of the wave function structure, which carry information on the shape of the electromagnetic distributions inside that baryon, and also the relative contribution of the D -wave components in the wave function of the Ω^- .

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