Twist-4 contributions to the azimuthal asymmetry in semi-inclusive deeply inelastic scattering

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We calculate the differential cross section for the unpolarized semi-inclusive deeply inelastic scattering process $e^- + N \rightarrow e^- + q + X$ in leading order of perturbative QCD and up to twist-4 in power corrections and study, in particular, the azimuthal asymmetry $\langle\cos 2\phi\rangle$. The final results are expressed in terms of transverse momentum dependent parton matrix elements of the target nucleon up to twist-4. We also apply it to $e^- + A \rightarrow e^- + q + X$ and illustrate numerically the nuclear dependence of the azimuthal asymmetry $\langle\cos 2\phi\rangle$ by using a Gaussian ansatz for the transverse momentum dependent parton matrix elements.

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I. INTRODUCTION

Inclusive and semi-inclusive deep-inelastic scatterings (SIDIS) are important tools to understand the structures of nucleon and nucleus governed by the Quantum Chromodynamics (QCD) for the strong interaction. The azimuthal asymmetries and their spin and/or nuclear dependences of the SIDIS cross sections are directly related to the parton distribution and polarization inside nucleon or nuclei and therefore are the subjects of intense studies both theoretically [1–13] and experimentally [14–25]. They provide us with a glimpse into the dynamics of strong interaction within nucleons or nuclei and a baseline for the study of parton dynamics in other extreme conditions at high temperature and baryon density.

In the unpolarized SIDIS experiments, the azimuthal angle ϕ of the final hadrons is defined with respect to the leptonic plane and is directly related to the transverse momentum of the hadron from either parton fragmentation or the initial and final state interaction of the parton before hadronization. In this paper we will restrict our study to SIDIS $e^- + N(A) \rightarrow e^- + q + X$ of quark jet production so that we do not need to deal with the azimuthal asymmetry resulting from parton fragmentation and have no need to consider the Boer-Mulders effect [26]. We instead focus primarily on the effect of initial and final state interaction. In the large transverse momentum region, the azimuthal asymmetries arise predominately from hard gluon bremsstrahlung that can be calculated using perturbative QCD (pQCD) [1], and are clearly observed in experiments [14–18]. On the other hand, in the small transverse momentum region $p_{h\perp} \sim k_\perp \leq 1$ GeV$/c$, the asymmetry was shown [2] to arise mainly from the intrinsic transverse momentum of quarks in nucleon and is a higher twist effect proportional to $k_\perp/Q$ for $\langle\cos\phi\rangle$ and to $k_\perp^2/Q^2$ for $\langle\cos 2\phi\rangle$. (Here, $p_{h\perp}$ denotes the transverse momentum of the hadron produced, $k_\perp$ is the intrinsic transverse momentum of the quark in the nucleon, $Q^2 = -q^2$ and q is the four-momentum transfer from the lepton). The calculations in [2] are based on a generalization of the naive parton model to include intrinsic transverse momentum. To go beyond the naive parton model, one has to consider multiple soft gluon interaction between the struck quark and the remanent of the target nucleon or nucleus. Inclusion of such soft gluon interaction ensures the gauge invariance of the final results and relates the azimuthal asymmetry to the transverse momentum dependent (TMD) parton matrix elements of the nucleon or nucleus.

Within the framework of TMD parton distributions and correlations, the intrinsic transverse momentum of partons arises naturally from multiple soft gluon interaction inside the nucleon or nucleus. The TMD parton distributions and correlations can be in fact expressed in terms of the expectation values of matrix elements related to the accumulated total transverse momentum as a result of the color Lorentz force enforced upon the parton through soft gluon exchange [27]. These soft gluon interactions are responsible for the single-spin asymmetries observed in SIDIS, pp and $\bar{p}p$ collisions. They also lead to the transverse momentum broadening [27] of hadron production in deep-inelastic lepton-nucleus scattering [28–30] as well as the jet quenching observed at the Relativistic Heavy Ion Collider [31–36]. Such transverse momentum broadening inside the nucleus is directly related to the gluon saturation scale [27,37] and can be studied directly through the nuclear dependence of the azimuthal asymmetry in SIDIS.

Higher twist contributions in inclusive DIS have been studied systematically using the collinear expansion technique [38–40] which not only provides a useful tool to study the higher twist contributions but also is a necessary procedure to ensure gauge invariance of the parton distribution and/or correlation functions. In Ref. [11], such collinear expansion is extended to the SIDIS process $e^- + N \rightarrow e^- + q + X$ and calculation of the TMD differential cross section and the azimuthal asymmetries up to twist-3. Taking multiple gluon scattering into account, the study found the azimuthal asymmetry $\langle \cos\phi \rangle$ proportional to a twist-3 TMD parton correlation function $f_{q\perp}(x, k_\perp)$ defined as,

$$f^N_{q\perp}(x, k_\perp) = \int \frac{p^+ dy^- d^2 y_\perp}{(2\pi)^3} e^{ixp^+y^- - i\vec{k}_\perp \cdot \vec{y}_\perp} \langle N | \bar{\psi}(0) \times \frac{k_\perp}{2k^2_\perp} \mathcal{L}(0; y) \psi(y) | N \rangle, \tag{1}$$

where $\mathcal{L}(0; y)$ is the gauge link,

$$\mathcal{L}(0; y) = \mathcal{L}^\dagger_\parallel(\infty, \vec{0}_\perp; 0, \vec{0}) \mathcal{L}_\perp(\infty, \vec{0}_\perp; \infty, \vec{y}_\perp) \times \mathcal{L}_\parallel(\infty, \vec{y}_\perp; y^-, \vec{y}_\perp),$$
$$\mathcal{L}_\parallel(\infty, \vec{y}_\perp; y^-, \vec{y}_\perp) \equiv P e^{-ig\int_{y^-}^{\infty} d\xi^- A^+(\xi^-, \vec{y}_\perp)},$$
$$\mathcal{L}_\perp(\infty, \vec{0}_\perp; \infty, \vec{y}_\perp) \equiv P e^{-ig\int_{\vec{0}_\perp}^{\vec{y}_\perp} d\vec{\xi}_\perp \cdot \vec{A}_\perp(\infty, \vec{\xi}_\perp)}, \tag{2}$$

from the resummation of multiple soft gluon interaction that ensures the gauge invariance of the twist-3 parton correlation function in Eq. (1) under any gauge transformation. The asymmetry obtained within this generalized collinear expansion method reduces to that in the naive parton model [2] if and only if one neglects the soft gluon interaction as contained in the gauge link or equivalently by setting the strong coupling constant $g = 0$ in the final result. Measurements of $\langle \cos\phi \rangle$ in $e^- + N \to e^- + q + X$ and its $k_\perp$-dependence therefore provide a unique determination of this new parton correlation function in Eq. (1). Furthermore, the nuclear dependence of the asymmetry [27] from multiple soft gluon interaction within the target nucleus can probe the transverse momentum broadening or the jet quenching parameter in cold nuclear matter [13] which also determines the gluon saturation scale in cold nuclei.

In this paper, we present a complete calculation of the hadronic tensor and the differential cross section for $e^- + N \to e^- + q + X$ up to twist-4. We study, in particular, the azimuthal asymmetry $\langle \cos 2\phi \rangle$ in terms of the corresponding TMD quark correlation functions and its nuclear dependence. Our calculations in this paper are limited to the process $e^- + N \to e^- + q + X$ or $e^- + A \to e^- + q + X$, where no fragmentation is taken into account. This corresponds to the semi-inclusive (current) jet production process in experiments. Since the results depend only on the structure functions, they provide a nice place to study the correlation functions. However, such current jets with moderate transverse momenta are difficult to measure in experiments. Complete calculations up to twist-4 taking fragmentation into account are important to practical experimental studies.

The rest of the paper is arranged as follows. For completeness, in Sec. II, we present the formulae for calculating the hadronic tensor and differential cross sections within the framework of generalized collinear expansion. In Sec. III, we present the cross section and discuss azimuthal asymmetry $\langle \cos 2\phi \rangle$ including its nuclear dependence with a Gaussian ansatz for the TMD correlation functions. A summary is given in Sec. IV.

II. HADRONIC TENSOR $W_{\mu\nu}$ IN $e^- + N \to e^- + q + X$ UP TO TWIST-4

We consider the SIDIS process $e^- + N \to e^- + q + X$ with unpolarized beam and target. The differential cross section is given by,

$$d\sigma = \frac{\alpha^2_{\rm em} e^2_q}{sQ^4} L^{\mu\nu}(l, l') \frac{d^2 W_{\mu\nu}}{d^2 k'_\perp} \frac{d^3 l' d^2 k'_\perp}{2E_{l'}}, \tag{3}$$

where l and l' are, respectively, the four-momenta of the incoming and outgoing leptons, p is the four-momentum of the incoming nucleon N, k' is the four-momentum of the outgoing quark. We neglect the masses and use the light-cone coordinates. The unit vectors are taken as, $\bar{n}^\mu = (1, 0, 0, 0)$, $n^\mu = (0, 1, 0, 0)$, $n^\mu_{\perp 1} = (0, 0, 1, 0)$, $n^\mu_{\perp 2} = (0, 0, 0, 1)$. We chose the coordinate system in this way so that, $p = p^+ \bar{n}$, $q = -x_B p + nQ^2/(2x_B p^+)$, $l_\perp = |\vec{l}_\perp| n_{\perp 1}$, and $k_\perp = (0, 0, \vec{k}_\perp)$; where $x_B = Q^2/2p \cdot q$ is the Bjorken-x and $y = p \cdot q / p \cdot l$. The leptonic tensor $L^{\mu\nu}$ is defined as usual,

$$L^{\mu\nu}(l, l') = 4[l^\mu l'^\nu + l^\nu l'^\mu - (l \cdot l') g^{\mu\nu}], \tag{4}$$

and the differential hadronic tensor is,

$$\frac{d^2 W_{\mu\nu}}{d^2 k'_\perp} = \int \frac{dk'_z}{(2\pi)^3 2E_{k'}} W^{(\rm si)}_{\mu\nu}(q, p, k'), \tag{5}$$

$$W^{(\rm si)}_{\mu\nu}(q, p, k') = \frac{1}{2\pi} \sum_X \langle N | J_\mu(0) | k', X \rangle \langle k', X | J_\nu(0) | N \rangle \times (2\pi)^4 \delta^4(p + q - k' - p_X), \tag{6}$$

where the superscript (si) denotes SIDIS. It has been shown [11] that, after collinear expansion, the hadronic tensor can be expressed in an expansion series characterized by the number of covariant derivatives in the parton matrix elements in each term,

$$\frac{d^2 W_{\mu\nu}}{d^2 k_\perp} = \sum_{j=0}^{\infty} \frac{d^2 \tilde{W}^{(j)}_{\mu\nu}}{d^2 k_\perp}, \tag{7}$$

$$\frac{d\tilde{W}^{(0)}_{\mu\nu}}{d^2 k'_\perp} = \frac{1}{2\pi} \int dx d^2 k_\perp {\rm Tr}[\hat{H}^{(0)}_{\mu\nu}(x) \hat{\Phi}^{(0)N}(x, k_\perp)] \delta^{(2)}(\vec{k}_\perp - \vec{k}'_\perp); \tag{8}$$

$$\frac{d\tilde{W}^{(1)}_{\mu\nu}}{d^2k'_\perp} = \frac{1}{2\pi}\int dx_1 d^2k_{1\perp} dx_2 d^2k_{2\perp} \sum_{c=L,R} \mathrm{Tr}[\hat{H}^{(1,c)\rho}_{\mu\nu}(x_1,x_2)\omega_\rho{}^{\rho'}\hat{\Phi}^{(1)N}_{\rho'}(x_1,k_{1\perp},x_2,k_{2\perp})]\delta^{(2)}(\vec{k}_{c\perp}-\vec{k}'_\perp); \tag{9}$$

$$\frac{d\tilde{W}^{(2)}_{\mu\nu}}{d^2k'_\perp} = \frac{1}{2\pi}\int dx_1 d^2k_{1\perp} dx_2 d^2k_{2\perp} dx d^2k_\perp \sum_{c=L,R,M} \mathrm{Tr}[\hat{H}^{(2,c)\rho\sigma}_{\mu\nu}(x_1,x_2,x)\omega_\rho{}^{\rho'}\omega_\sigma{}^{\sigma'}\hat{\Phi}^{(2)N}_{\rho'\sigma'}(x_1,k_{1\perp},x_2,k_{2\perp},x,k_\perp)]\delta^{(2)}(\vec{k}_{c\perp}-\vec{k}'_\perp), \tag{10}$$

where, for different cuts $c = L$, R or M, $\vec{k}_{c\perp}$ denotes $\vec{k}_{L\perp} = \vec{k}_{1\perp}$, $\vec{k}_{R\perp} = \vec{k}_{2\perp}$, and $\vec{k}_{M\perp} = \vec{k}_\perp$; $\omega_\rho{}^{\rho'} = g_\rho{}^{\rho'} - \bar{n}_\rho n^{\rho'}$ is a projection operator. The matrix elements are defined as,

$$\hat{\Phi}^{(0)N}(x,k_\perp) = \int \frac{p^+dy^-d^2y_\perp}{(2\pi)^3} e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp}\langle N|\bar{\psi}(0)\mathcal{L}(0;y)\psi(y)|N\rangle, \tag{11}$$

$$\begin{aligned}&\hat{\Phi}^{(1)N}_\rho(x_1,k_{1\perp},x_2,k_{2\perp})\\ &= \int \frac{p^+dy^-d^2y_\perp}{(2\pi)^3}\frac{p^+dz^-d^2z_\perp}{(2\pi)^3} e^{ix_2p^+z^- - i\vec{k}_{2\perp}\cdot\vec{z}_\perp + ix_1p^+(y^- - z^-) - i\vec{k}_{1\perp}\cdot(\vec{y}_\perp - \vec{z}_\perp)}\langle N|\bar{\psi}(0)\mathcal{L}(0;z)D_\rho(z)\mathcal{L}(z;y)\psi(y)|N\rangle,\end{aligned} \tag{12}$$

$$\begin{aligned}&\hat{\Phi}^{(2)N}_{\rho\sigma}(x_1,k_{1\perp},x_2,k_{2\perp},x,k_\perp)\\ &= \int \frac{p^+dz^-d^2z_\perp}{(2\pi)^3}\frac{p^+dy^-d^2y_\perp}{(2\pi)^3}\frac{p^+dy'^-d^2y'_\perp}{(2\pi)^3} e^{ix_2p^+z^- - i\vec{k}_{2\perp}\cdot\vec{z}_\perp + ixp^+(z'^- - z^-) - i\vec{k}_\perp\cdot(\vec{z}'_\perp - \vec{z}_\perp) + ix_1p^+(y^- - z'^-) - i\vec{k}_{1\perp}\cdot(\vec{y}_\perp - \vec{z}'_\perp)}\\ &\quad\times\langle N|\bar{\psi}(0)\mathcal{L}(0;z)D_\rho(z)\mathcal{L}(z;z')D_\sigma(z')\mathcal{L}(z';y)\psi(y)|N\rangle,\end{aligned} \tag{13}$$

where $\mathcal{L}(0;y)$ is the gauge link as defined in Eq. (1), and also in the remainder of this paper, for brevity, unless explicitly specified, the coordinate y in the field operator denotes $(0, y^-, \vec{y}_\perp)$.

The hard parts after the collinear expansion are given as [11],

$$\hat{H}^{(0)}_{\mu\nu}(x) = \frac{2\pi}{2q\cdot p}\gamma_\mu(\not{q} + x\not{p})\gamma_\nu\delta(x - x_B), \tag{14}$$

$$\hat{H}^{(1,L)\rho}_{\mu\nu}(x_1,x_2) = \frac{2\pi}{(2q\cdot p)^2}\frac{\gamma_\mu(\not{q}+x_2\not{p})\gamma^\rho(\not{q}+x_1\not{p})\gamma_\nu}{x_2 - x_B - i\varepsilon}\times\delta(x_1 - x_B), \tag{15}$$

$$\begin{aligned}&\hat{H}^{(2,L)\rho\sigma}_{\mu\nu}(x_1,x_2,x)\\ &= \frac{2\pi}{(2q\cdot p)^3}\frac{\gamma_\mu(\not{q}+x_2\not{p})\gamma^\rho(\not{q}+x\not{p})\gamma^\sigma(\not{q}+x_1\not{p})\gamma_\nu}{(x - x_B - i\varepsilon)(x_2 - x_B - i\varepsilon)}\\ &\quad\times\delta(x_1 - x_B).\end{aligned} \tag{16}$$

These equations form the basis for calculating the hadronic tensor in $e^- + N \to e^- + q + X$. Because of the existence of the projection operators $\omega_\rho{}^{\rho'}$ and $\omega_\sigma{}^{\sigma'}$, the hard parts can be simplified to,

$$\hat{H}^{(0)}_{\mu\nu}(x) = \pi\hat{h}^{(0)}_{\mu\nu}\delta(x - x_B), \tag{17}$$

$$\hat{H}^{(1,L)\rho}_{\mu\nu}(x_1,x_2)\omega_\rho{}^{\rho'} = \frac{\pi}{2q\cdot p}\hat{h}^{(1)\rho}_{\mu\nu}\omega_\rho{}^{\rho'}\delta(x_1 - x_B), \tag{18}$$

$$\begin{aligned}&\hat{H}^{(2,L)\rho\sigma}_{\mu\nu}(x_1,x_2,x)\omega_\rho{}^{\rho'}\omega_\sigma{}^{\sigma'}\\ &= \frac{2\pi}{(2q\cdot p)^2}\Big[\bar{n}^\rho\hat{h}^{(1)\sigma}_{\mu\nu} + \frac{\hat{N}^{(2)\rho\sigma}_{\mu\nu}}{x_2 - x_B - i\varepsilon}\Big]\omega_\rho{}^{\rho'}\omega_\sigma{}^{\sigma'}\delta(x_1 - x_B),\end{aligned} \tag{19}$$

$$\begin{aligned}&\hat{H}^{(2,M)\rho\sigma}_{\mu\nu}(x_1,x_2,x)\omega_\rho{}^{\rho'}\omega_\sigma{}^{\sigma'}\\ &= \frac{2\pi}{(2q\cdot p)^2}\hat{h}^{(2)\rho\sigma}_{\mu\nu}\omega_\rho{}^{\rho'}\omega_\sigma{}^{\sigma'}\delta(x - x_B),\end{aligned} \tag{20}$$

where $\hat{h}^{(0)}_{\mu\nu} = \gamma_\mu\not{n}\gamma_\nu/p^+$, $\hat{h}^{(1)\rho}_{\mu\nu} = \gamma_\mu\not{\bar{n}}\gamma^\rho\not{n}\gamma_\nu$, $\hat{h}^{(2)\rho\sigma}_{\mu\nu} = p^+\gamma_\mu\not{\bar{n}}\gamma^\rho\not{n}\gamma^\sigma\not{\bar{n}}\gamma_\nu/2$ and $\hat{N}^{(2)\rho\sigma}_{\mu\nu} = q^-\gamma_\mu\gamma^\rho\not{n}\gamma^\sigma\gamma_\nu$. We insert them into Eqs. (8)–(10) and obtain,

$$\frac{d^2\tilde{W}^{(0)}_{\mu\nu}}{d^2k_\perp} = \frac{1}{2}\mathrm{Tr}[\hat{h}^{(0)}_{\mu\nu}\hat{\Phi}^{(0)N}(x_B,k_\perp)], \tag{21}$$

$$\frac{d^2\tilde{W}^{(1,L)}_{\mu\nu}}{d^2k_\perp} = \frac{1}{4q\cdot p}\mathrm{Tr}[\hat{h}^{(1)\rho}_{\mu\nu}\omega_\rho{}^{\rho'}\hat{\varphi}^{(1,L)N}_{\rho'}(x_B,k_\perp)], \tag{22}$$

$$\begin{aligned}\frac{d^2\tilde{W}^{(2,L)}_{\mu\nu}}{d^2k_\perp} &= \frac{1}{(2q\cdot p)^2}\{\mathrm{Tr}[\hat{h}^{(1)\rho}_{\mu\nu}\omega_\rho{}^{\rho'}\hat{\varphi}^{(2,L)N}_{\rho'}(x_B,k_\perp)]\\ &\quad + \mathrm{Tr}[\hat{N}^{(2)\rho\sigma}_{\mu\nu}\omega_\rho{}^{\rho'}\omega_\sigma{}^{\sigma'}\hat{\varphi}^{(2,L)N}_{\rho'\sigma'}(x_B,k_\perp)]\},\end{aligned} \tag{23}$$

$$\frac{d^2\tilde{W}^{(2,M)}_{\mu\nu}}{d^2k_\perp} = \frac{1}{(2q\cdot p)^2}\,\mathrm{Tr}[\hat{h}^{(2)\rho\sigma}_{\mu\nu}\omega_\rho{}^{\rho'}\omega_\sigma{}^{\sigma'}\hat{\varphi}^{(2,M)N}_{\rho'\sigma'}(x_B,k_\perp)]. \tag{24}$$

The correlation matrices are defined as,

$$\hat{\varphi}^{(1,L)N}_\rho(x_1,k_{1\perp}) \equiv \int dx_2 d^2k_{2\perp}\hat{\Phi}^{(1)N}_\rho(x_1,k_{1\perp},x_2,k_{2\perp}), \tag{25}$$

$$\hat{\varphi}^{(2,L)N}_{\rho\sigma}(x_1,k_{1\perp}) \equiv \int dx d^2k_\perp \frac{dx_2 d^2k_{2\perp}}{x_2-x_1-i\varepsilon} \times \hat{\Phi}^{(2)N}_{\rho\sigma}(x_1,k_{1\perp},x_2,k_{2\perp},x,k_\perp), \tag{26}$$

$$\hat{\varphi}^{(2,M)N}_{\rho\sigma}(x,k_\perp) \equiv \int dx_1 d^2k_{1\perp}dx_2 d^2k_{2\perp} \times \hat{\Phi}^{(2)N}_{\rho\sigma}(x_1,k_{1\perp},x_2,k_{2\perp},x,k_\perp), \tag{27}$$

$$\hat{\phi}^{(2,L)N}_{\sigma}(x_1,k_{1\perp}) \equiv \int dx d^2k_\perp dx_2 d^2k_{2\perp}\bar{n}^\rho \times \hat{\Phi}^{(2)N}_{\rho\sigma}(x_1,k_{1\perp},x_2,k_{2\perp},x,k_\perp). \tag{28}$$

They are given by,

$$\hat{\varphi}^{(1,L)N}_\rho(x,k_\perp) = \int \frac{p^+dy^-d^2y_\perp}{(2\pi)^3}e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp} \times \langle N|\bar{\psi}(0)D_\rho(0)\mathcal{L}(0;y)\psi(y)|N\rangle, \tag{29}$$

$$\hat{\varphi}^{(2,L)N}_{\rho\sigma}(x,k_\perp) = \int \frac{dx_2}{x_2-x-i\varepsilon}\frac{p^+dy^-d^2y_\perp}{(2\pi)^3} \times \frac{p^+dz^-}{2\pi}e^{ix_2p^+z^- + ixp^+(y^--z^-) - i\vec{k}_\perp\cdot\vec{y}_\perp} \times \langle N|\bar{\psi}(0)\mathcal{L}(0;z^-,y_\perp)D_\rho(z^-,y_\perp) \times D_\sigma(z^-,y_\perp)\mathcal{L}(z^-,\vec{y}_\perp;y)\psi(y)|N\rangle, \tag{30}$$

$$\hat{\varphi}^{(2,M)N}_{\rho\sigma}(x,k_\perp) = \int \frac{p^+dy^-d^2y_\perp}{(2\pi)^3}e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp} \times \langle N|\bar{\psi}(0)D_\rho(0)\mathcal{L}(0;y)D_\sigma(y)\psi(y)|N\rangle, \tag{31}$$

$$\hat{\phi}^{(2,L)N}_{\sigma}(x,k_\perp) = \int \frac{p^+dy^-d^2y_\perp}{(2\pi)^3}e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp} \times \langle N|\bar{\psi}(0)D^-(0)D_\sigma(0)\mathcal{L}(0;y)\psi(y)|N\rangle. \tag{32}$$

We note that, $\tilde{W}^{(0)*}_{\mu\nu} = \tilde{W}^{(0)}_{\nu\mu}$, $\tilde{W}^{(2,M)*}_{\mu\nu} = \tilde{W}^{(2,M)}_{\nu\mu}$, $\tilde{W}^{(1,R)}_{\mu\nu} = \tilde{W}^{(1,L)*}_{\nu\mu}$, and $\tilde{W}^{(2,R)}_{\mu\nu} = \tilde{W}^{(2,L)*}_{\nu\mu}$. Hence, if we divide $W_{\mu\nu}$ into a $\mu \leftrightarrow \nu$ symmetric part and an antisymmetric part, and denote $W_{\mu\nu} = W_{S,\mu\nu} + iW_{A,\mu\nu}$, we obtain,

$$\frac{d^2W_{S,\mu\nu}}{d^2k_\perp} = \frac{d^2\tilde{W}^{(0)}_{S,\mu\nu}}{d^2k_\perp} + 2\,\mathrm{Re}\frac{d^2\tilde{W}^{(1,L)}_{S,\mu\nu}}{d^2k_\perp} + 2\,\mathrm{Re}\frac{d^2\tilde{W}^{(2,L)}_{S,\mu\nu}}{d^2k_\perp} + \frac{d^2\tilde{W}^{(2,M)}_{S,\mu\nu}}{d^2k_\perp}, \tag{33}$$

$$\frac{d^2W_{A,\mu\nu}}{d^2k_\perp} = \frac{d^2\tilde{W}^{(0)}_{A,\mu\nu}}{d^2k_\perp} + 2\mathrm{Im}\frac{d^2\tilde{W}^{(1,L)}_{S,\mu\nu}}{d^2k_\perp} + 2\mathrm{Im}\frac{d^2\tilde{W}^{(2,L)}_{S,\mu\nu}}{d^2k_\perp} + \frac{d^2\tilde{W}^{(2,M)}_{A,\mu\nu}}{d^2k_\perp}. \tag{34}$$

The antisymmetric part contributes only in reactions with a polarized lepton. In this paper, we concentrate on the unpolarized reactions and calculate the symmetric part in the following.

Now, we continue with a complete calculation of the hadronic tensor $d^2W_{\mu\nu}/d^2k_\perp$ in the unpolarized $e^- + N \to e^- + q + X$ up to twist-4 level. For this purpose, we need to calculate $d^2W_{\mu\nu}/d^2k_\perp$ up to $d^2\tilde{W}^{(2)}_{\mu\nu}/d^2k_\perp$ and we now present the calculations of each term in the following.

The contribution from $d^2\tilde{W}^{(0)}_{\mu\nu}/d^2k_\perp$ is the easiest one to calculate. Because $\hat{H}^{(0)}_{\mu\nu}(x)$ contains 3 γ-matrices, only the γ^α term of $\hat{\Phi}^{(0)}(x,k_\perp)$ contributes in the unpolarized case so we need only to consider $\hat{\Phi}^{(0)N}(x,k_\perp) = \gamma^\alpha\Phi^{(0)N}_\alpha(x,k_\perp)/2$,

$$\Phi^{(0)N}_\alpha(x,k_\perp) = \int \frac{p^+dy^-d^2y_\perp}{(2\pi)^3}e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp}\langle N|\bar{\psi}(0) \times \frac{\gamma_\alpha}{2}\mathcal{L}(0;y)\psi(y)|N\rangle = p_\alpha f^N_q + k_{\perp\alpha}f^N_{q\perp} + \frac{M^2}{p^+}n_\alpha f^N_{q(-)}, \tag{35}$$

and obtain the result for $d^2\tilde{W}^{(0)}_{\mu\nu}/d^2k_\perp$ as,

$$\frac{d^2\tilde{W}^{(0)}_{\mu\nu}}{d^2k_\perp} = -d_{\mu\nu}f^N_q(x_B,k_\perp) + \frac{1}{q\cdot p}k_{\perp\{\mu}(q+x_Bp)_{\nu\}}f^N_{q\perp}(x_B,k_\perp) + 2\left(\frac{M}{q\cdot p}\right)^2(q+x_Bp)_\mu(q+x_Bp)_\nu f^N_{q(-)}(x_B,k_\perp), \tag{36}$$

where $d^{\mu\nu} = g^{\mu\nu} - \bar{n}^\mu n^\nu - \bar{n}^\nu n^\mu$ and $A_{\{\mu}B_{\nu\}} \equiv A_\mu B_\nu + A_\nu B_\mu$, $A_{[\mu}B_{\nu]} \equiv A_\mu B_\nu - A_\nu B_\mu$. The TMD quark distribution/correlation functions are given by,

$$f_q^N(x, k_\perp) = \frac{n^\alpha}{p^+}\Phi_\alpha^{(0)N}(x, k_\perp) = \int \frac{dy^- d^2y_\perp}{(2\pi)^3} e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp} \times \langle N|\bar{\psi}(0)\frac{\gamma^+}{2}\mathcal{L}(0;y)\psi(y)|N\rangle, \quad (37)$$

$$k_\perp^\alpha f_{q\perp}^N(x_B, k_\perp) = d^{\alpha\beta}\Phi_\beta^{(0)N}(x, k_\perp) = \int \frac{p^+ dy^- d^2y_\perp}{(2\pi)^3} e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp} \times \langle N|\bar{\psi}(0)\frac{\gamma_\perp^\alpha}{2}\mathcal{L}(0;y)\psi(y)|N\rangle, \quad (38)$$

$$f_{q(-)}^N(x_B, k_\perp) = \frac{p^+}{M^2}\bar{n}^\alpha\Phi_\alpha^{(0)N}(x, k_\perp) = \frac{p^+}{M^2}\int \frac{p^+ dy^- d^2y_\perp}{(2\pi)^3} e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp} \times \langle N|\bar{\psi}(0)\frac{\gamma^-}{2}\mathcal{L}(0;y)\psi(y)|N\rangle. \quad (39)$$

Our notation for the leading twist parton distribution $f_q^N(x, k_\perp)$ is the same as that usually used in the literature (see e.g. [1–13,41]); the twist-3 parton correlation function $f_{q\perp}^N(x_B, k_\perp)$ is denoted the same as those used in e.g. [11,13] but slightly different from that used in e.g. [12,41] where $\perp$ is used as a superscript; the twist-4 term is new here.

Because $\hat{h}_{\mu\nu}^{(1)\rho}$ contains 5γ-matrices, we have contributions from γ_α and $\gamma_5\gamma_\alpha$ in terms of $\varphi_\rho^{(1,L)N}$, i.e., we need to consider $\hat{\varphi}_\rho^{(1,L)N}(x, k_\perp) = [\gamma^\alpha\varphi_{\rho\alpha}^{(1)N}(x, k_\perp) - \gamma_5\gamma^\alpha\tilde{\varphi}_{\rho\alpha}^{(1)N}(x, k_\perp)]/2$ and obtain,

$$\frac{d^2\tilde{W}_{\mu\nu}^{(1,L)}}{d^2k_\perp} = \frac{1}{2p\cdot q}[h_{\mu\nu}^{(1)\rho\alpha}\omega_\rho{}^{\rho'}\varphi_{\rho'\alpha}^{(1)N}(x_B, k_\perp) - \tilde{h}_{\mu\nu}^{(1)\rho\alpha}\omega_\rho{}^{\rho'}\tilde{\varphi}_{\rho'\alpha}^{(1)N}(x_B, k_\perp)], \quad (40)$$

where $h_{\mu\nu}^{(1)\rho\alpha} \equiv \mathrm{Tr}[\gamma^\alpha\hat{h}_{\mu\nu}^{(1)\rho}]/4$, $\tilde{h}_{\mu\nu}^{(1)\rho\alpha} \equiv \mathrm{Tr}[\gamma_5\gamma^\alpha\hat{h}_{\mu\nu}^{(1)\rho}]/4$ and,

$$\varphi_{\rho\alpha}^{(1)N}(x, k_\perp) = \int \frac{p^+ dy^- d^2y_\perp}{(2\pi)^3} e^{ixp^+y^- - i\vec{y}_\perp\cdot\vec{k}_\perp} \times \langle N|\bar{\psi}(0)\frac{\gamma_\alpha}{2}\mathcal{L}(0;y)D_\rho(y)\psi(y)|N\rangle, \quad (41)$$

$$\tilde{\varphi}_{\rho\alpha}^{(1)N}(x, k_\perp) = \int \frac{p^+ dy^- d^2y_\perp}{(2\pi)^3} e^{ixp^+y^- - i\vec{y}_\perp\cdot\vec{k}_\perp} \times \langle N|\bar{\psi}(0)\frac{\gamma_5\gamma_\alpha}{2}\mathcal{L}(0;y)D_\rho(y)\psi(y)|N\rangle. \quad (42)$$

After evaluating the two traces in $h_{\mu\nu}^{(1)\rho\alpha}$ and $\tilde{h}_{\mu\nu}^{(1)\rho\alpha}$, we obtain the symmetric parts as,

$$h_{S,\mu\nu}^{(1)\rho\alpha} = -g^\alpha{}_\mu d^\rho{}_\nu - g^\alpha{}_\nu d^\rho{}_\mu + g_{\mu\nu}d^{\rho\alpha}, \quad (43)$$

$$\tilde{h}_{S,\mu\nu}^{(1)\rho\alpha} = ig^\alpha{}_\mu\varepsilon_{\perp\nu}^\rho + ig^\alpha{}_\nu\varepsilon_{\perp\mu}^\rho - ig_{\mu\nu}\varepsilon_\perp^{\rho\alpha}, \quad (44)$$

where $\epsilon_{\perp\rho\gamma} \equiv \epsilon_{\alpha\beta\rho\gamma}\bar{n}^\alpha n^\beta$. Up to twist-4, the contributing terms of $\varphi_{\rho\alpha}^{(1)N}(x, k_\perp)$ and $\tilde{\varphi}_{\rho\alpha}^{(1)N}(x, k_\perp)$ are, respectively,

$$\varphi_{\rho\alpha}^{(1)N}(x, k_\perp) = p_\alpha k_{\perp\rho}\varphi_\perp^{(1)N}(x, k_\perp) + \Big(k_{\perp\alpha}k_{\perp\rho} - \frac{k_\perp^2}{2}d_{\rho\alpha}\Big)\varphi_{\perp 2}^{(1)N}(x, k_\perp) + \frac{k_\perp^2}{2}(\bar{n}_{\{\alpha}n_{\rho\}} - d_{\rho\alpha})\varphi_{\perp 3}^{(1)N}(x, k_\perp), \quad (45)$$

$$\tilde{\varphi}_{\rho\alpha}^{(1)N}(x, k_\perp) = ip_\alpha\varepsilon_{\perp\rho\gamma}k_\perp^\gamma\tilde{\varphi}_\perp^{(1)N}(x, k_\perp) + \frac{i}{2}k_{\perp\{\alpha}\varepsilon_{\perp\rho\}\gamma}k_\perp^\gamma\tilde{\varphi}_{\perp 2}^{(1)N}(x, k_\perp) + \frac{i}{2}k_{\perp[\alpha}\varepsilon_{\perp\rho]\gamma}k_\perp^\gamma\tilde{\varphi}_{\perp 3}^{(1)N}(x, k_\perp). \quad (46)$$

The result for $d^2\tilde{W}_{S,\mu\nu}^{(1,L)}/d^2k_\perp$ is,

$$\frac{d^2\tilde{W}_{S,\mu\nu}^{(1,L)}}{d^2k_\perp} = -\frac{1}{2q\cdot p}\{(p_\mu k_{\perp\nu} + p_\nu k_{\perp\mu}) \times[\varphi_\perp^{(1)N}(x_B, k_\perp) - \tilde{\varphi}_\perp^{(1)N}(x_B, k_\perp)] + (2k_{\perp\mu}k_{\perp\nu} - k_\perp^2 d_{\mu\nu})[\varphi_{\perp 2}^{(1)N}(x_B, k_\perp) - \tilde{\varphi}_{\perp 2}^{(1)N}(x_B, k_\perp)] + k_\perp^2(g_{\mu\nu} - d_{\mu\nu})[\varphi_{\perp 3}^{(1)N}(x_B, k_\perp) - \tilde{\varphi}_{\perp 3}^{(1)N}(x_B, k_\perp)]\}. \quad (47)$$

Up to twist-4 level, we need only to consider $\not{p}$ and the $\gamma_5\not{p}$-term in the calculations of $d\tilde{W}_{\mu\nu}^{(2)}/d^2k_\perp$. For the first term in Eq. (23), because of $\omega_\rho{}^{\rho'}$ and $n_\rho\hat{h}_{\mu\nu}^{(1)\rho} = 0$, we need only to consider the $k_{\perp\rho}$ terms and we found out that they contribute only at twist-5 or higher level. For the second term, because $n_\rho\hat{N}_{\mu\nu}^{(2)\rho\sigma} = n_\sigma\hat{N}_{\mu\nu}^{(2)\rho\sigma} = 0$ and $\hat{\varphi}_{\rho\sigma}^{(2,L)N} = \hat{\varphi}_{\sigma\rho}^{(2,L)N}$, we need to consider only $k_{\perp\rho}k_{\perp\sigma}$ and $k_\perp^2 d_{\rho\alpha}$ for the tensor term and $k_{\perp\{\rho}\varepsilon_{\perp\sigma\}\gamma}k_\perp^\gamma$ for the pseudotensor term. Furthermore,

$$k_\perp^2\hat{N}_{\mu\nu}^{(2)\rho\sigma}d_{\rho\sigma} = 2\hat{N}_{\mu\nu}^{(2)\rho\sigma}k_{\perp\rho}k_{\perp\sigma} = -2k_\perp^2\gamma_\mu\not{n}\gamma_\nu, \quad (48)$$

$$\hat{N}_{\mu\nu}^{(2)\rho\sigma}k_{\perp\rho}\varepsilon_{\perp\sigma\gamma}k_\perp^\gamma = -\hat{N}_{\mu\nu}^{(2)\rho\sigma}k_{\perp\sigma}\varepsilon_{\perp\rho\gamma}k_\perp^\gamma = k_\perp^2\gamma_\mu\not{n}\not{n}_{\perp 1}\not{n}_{\perp 2}\gamma_\nu, \quad (49)$$

we need only to consider,

$$\hat{\varphi}_{\rho\sigma}^{(2,L)N}(x, k_\perp) = \frac{\not{p}}{2}\Big(-\frac{1}{2}k_\perp^2 d_{\rho\sigma}\Big)\varphi_\perp^{(2,L)N}(x, k_\perp) + \ldots \quad (50)$$

and obtain the results for $d^2\tilde{W}_{\mu\nu}^{(2,L)}/d^2k_\perp$ up to $1/Q^2$ as,

$$\frac{d^2\tilde{W}_{\mu\nu}^{(2,L)}}{d^2k_\perp} = -\frac{1}{2q\cdot p}k_\perp^2 d_{\mu\nu}\varphi_\perp^{(2,L)N}(x_B, k_\perp) + \ldots. \quad (51)$$

Similarly, to calculate $d^2\tilde{W}^{(2,M)}_{\mu\nu}/d^2k_\perp$ up to $1/Q^2$ level, we need to consider

$$\hat{\varphi}^{(2,M)N}_{\rho\sigma}(x,k_\perp)=\frac{\not{p}}{2}\Big(-\frac{1}{2}k_\perp^2 d_{\rho\sigma}\Big)\varphi^{(2,M)N}_\perp(x,k_\perp) -\frac{i}{4}\gamma_5\not{p}k_{\perp[\rho}\varepsilon_{\perp\sigma]\gamma}k_\perp^\gamma\tilde{\varphi}^{(2,M)N}_\perp(x,k_\perp), \quad (52)$$

and the results for $d^2\tilde{W}^{(2,M)}_{\mu\nu}/d^2k_\perp$ are given by,

$$\frac{d^2\tilde{W}^{(2,M)}_{\mu\nu}}{d^2k_\perp}=\frac{k_\perp^2}{(q\cdot p)^2}p_\mu p_\nu[\varphi^{(2,M)N}_\perp(x_B,k_\perp) - \tilde{\varphi}^{(2,M)N}_\perp(x_B,k_\perp)]. \quad (53)$$

The QCD equation of motion relates matrix elements with a different number of D_ρ and gives

$$xf^N_{q\perp}(x,k_\perp)=-[\varphi^{(1)N}_\perp(x,k_\perp)-\tilde{\varphi}^{(1)N}_\perp(x,k_\perp)], \quad (54)$$

$$2(xM)^2f^N_{q(-)}(x,k_\perp)=k_\perp^2[\varphi^{(2,M)N}_\perp(x,k_\perp)-\tilde{\varphi}^{(2,M)N}_\perp(x,k_\perp)], \quad (55)$$

$$x[\varphi^{(1)N}_{\perp 3}(x,k_\perp)-\tilde{\varphi}^{(1)N}_{\perp 3}(x,k_\perp)] =-[\varphi^{(2,M)N}_\perp(x,k_\perp)-\tilde{\varphi}^{(2,M)N}_\perp(x,k_\perp)], \quad (56)$$

where, as well as in the rest of this paper, all the correlation functions in the results of the hadronic tensors and/or cross section stand for their real parts. The final results for $d^2W_{\mu\nu}/d^2k_\perp$ up to twist-4 level are given by,

$$\frac{d^2W_{\mu\nu}}{d^2k_\perp}=-\frac{1}{q\cdot p}\Big\{(q\cdot p)d_{\mu\nu}f^N_q(x_B,k_\perp) +\frac{2M^2}{q\cdot p}(q+2x_Bp)_\mu(q+2x_Bp)_\nu f^N_{q(-)}(x_B,k_\perp) -(q+2x_Bp)_{\{\mu}k_{\perp\nu\}}f^N_{q\perp}(x_B,k_\perp) +(2k_{\perp\mu}k_{\perp\nu}-k_\perp^2d_{\mu\nu})[\varphi^{(1)N}_{\perp 2}(x_B,k_\perp) -\tilde{\varphi}^{(1)N}_{\perp 2}(x_B,k_\perp)]+k_\perp^2d_{\mu\nu}\varphi^{(2,L)N}_{\perp 2}(x_B,k_\perp)\Big\}. \quad (57)$$

We emphasize that this is the result of the hadronic tensor up to twist-4 for the semi-inclusive deeply inelastic scattering process $e^- + N \to e^- + q + X$ with an *unpolarized* beam and an *unpolarized* target. In the polarized cases, the calculations are much more involved and the results are also much more complicated. Such results are interesting and the corresponding calculations are underway.

III. DIFFERENTIAL CROSS SECTION AND $\langle\cos 2\phi\rangle$ UP TO THE $1/Q^2$

Making the Lorentz contraction of the result for $d^2W_{\mu\nu}/d^2k_\perp$ with the leptonic tensor $L_{\mu\nu}$ given in Eq. (3), we obtain the differential cross section as,

$$\frac{d\sigma}{dx_Bdyd^2k_\perp}=\frac{2\pi\alpha^2_{\rm em}e_q^2}{Q^2y}\Big\{[1+(1-y)^2]f^N_q(x_B,k_\perp)-4(2-y)\sqrt{1-y}\frac{|\vec{k}_\perp|}{Q}x_Bf^{(1)N}_{q\perp}(x_B,k_\perp)\cos\phi-4(1-y) \times\frac{|\vec{k}_\perp|^2}{Q^2}x_B[\varphi^{(1)N}_{\perp 2}(x_B,k_\perp)-\tilde{\varphi}^{(1)N}_{\perp 2}(x_B,k_\perp)]\cos 2\phi+8(1-y)\Big(\frac{|\vec{k}_\perp|^2}{Q^2}x_B[\varphi^{(1)N}_{\perp 2}(x_B,k_\perp)-\tilde{\varphi}^{(1)N}_{\perp 2}(x_B,k_\perp)] +\frac{2x_B^2M^2}{Q^2}f^N_{q(-)}(x_B,k_\perp)\Big)-2[1+(1-y)^2]\frac{|\vec{k}_\perp|^2}{Q^2}x_B\varphi^{(2,L)N}_{\perp 2}(x_B,k_\perp)\Big\}. \quad (58)$$

From Eq. (58), we can calculate the azimuthal asymmetries $\langle\cos\phi\rangle$ and $\langle\cos 2\phi\rangle$. The result for $\langle\cos\phi\rangle$ and its nuclear dependence are discussed in [13]. We now discuss the result for $\langle\cos 2\phi\rangle$. At fixed $k_\perp$, it is given by,

$$\langle\cos 2\phi\rangle_{eN}=-\frac{2(1-y)}{1+(1-y)^2}\frac{|\vec{k}_\perp|^2}{Q^2} \times\frac{x_B[\varphi^{(1)N}_{\perp 2}(x_B,k_\perp)-\tilde{\varphi}^{(1)N}_{\perp 2}(x_B,k_\perp)]}{f^N_q(x_B,k_\perp)}. \quad (59)$$

Integrating over the magnitude of $\vec{k}_\perp$, we obtain,

$$\langle\langle\cos 2\phi\rangle\rangle_{eN}=-\frac{2(1-y)}{1+(1-y)^2} \times\frac{\int|\vec{k}_\perp|^2d^2k_\perp x_B[\varphi^{(1)N}_{\perp 2}(x_B,k_\perp)-\tilde{\varphi}^{(1)N}_{\perp 2}(x_B,k_\perp)]}{Q^2f^N_q(x_B)},$$

where $f^N_q(x)=\int d^2k_\perp f^N_q(x,k_\perp)$ is the usual quark distribution in the nucleon. The new quark correlation functions involved are given by,

$$|\vec{k}_\perp|^2\varphi^{(1)N}_{\perp 2}(x,k_\perp)=(2\hat{k}^\alpha_\perp\hat{k}^\rho_\perp+d^{\alpha\rho})\varphi^{(1)N}_{\rho\alpha}(x,k_\perp), \quad (60)$$

$$|\vec{k}_\perp|^2\tilde{\varphi}^{(1)N}_{\perp 2}(x,k_\perp)=-i\hat{k}^{\{\alpha}_\perp\varepsilon^{\rho\}\sigma}_\perp\hat{k}_{\perp\sigma}\tilde{\varphi}^{(1)N}_{\rho\alpha}(x,k_\perp), \quad (61)$$

where $\hat{k}_\perp=k_\perp/|\vec{k}_\perp|$ denotes the unit vector. If we consider only "free parton with intrinsic transverse momentum", i.e., the same case as considered in [2], we need to just set $g=0$ in the results mentioned above. In this case, $\mathcal{L}=1$ and $x[\varphi^{(1)N}_{\perp 2}(x,k_\perp)-\tilde{\varphi}^{(1)N}_{\perp 2}(x,k_\perp)]=f^N_q(x,k_\perp)$, so that,

$$\langle \cos 2\phi \rangle_{eN}|_{g=0} = -\frac{2(1-y)}{1+(1-y)^2}\frac{|\vec{k}_\perp|^2}{Q^2}, \quad (62)$$

which is just the result obtained in [2].

In general, we need to take QCD multiple parton scattering into account, thus $\langle \cos 2\phi \rangle_{eN}$ is given by Eq. (59) where new quark correlation functions are involved. Measurements of $\langle \cos 2\phi \rangle_{eN}$, in particular, whether the results deviate from Eq. (62), can provide useful information on the new parton correlation functions and on multiple parton scattering as well.

If we consider $e^- + A \to e^- + q + X$, i.e. instead of a nucleon but a nucleus target, all the calculations given above apply and we obtain similar results with only a replacement of the state $|N\rangle$ by $|A\rangle$ in the definitions of the matrix elements and/or parton distribution/correlation functions. The multiple gluon scattering now can be connected to different nucleons in the nucleus A thus givng rise to nuclear dependence. It has been shown that, under the ''maximal two-gluon approximation'', a TMD quark distribution $\Phi_\alpha^A(x, k_\perp)$ in the nucleus that is defined in the form,

$$\Phi_\alpha^A(x, k_\perp) \equiv \int \frac{p^+ dy^- d^2y_\perp}{(2\pi)^3} e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp} \times \langle A \mid \bar{\psi}(0)\Gamma_\alpha \mathcal{L}(0;y)\Psi(y) \mid A\rangle, \quad (63)$$

is given by a convolution of the corresponding distribution $\Phi_\alpha^N(x, k_\perp)$ in the nucleon and a Gaussian broadening,

$$\Phi_\alpha^A(x, k_\perp) \approx \frac{A}{\pi\Delta_{2F}} \int d^2\ell_\perp e^{-(\vec{k}_\perp - \vec{\ell}_\perp)^2/\Delta_{2F}} \Phi_\alpha^N(x, \ell_\perp), \quad (64)$$

where Γ_α is any gamma matrix, $\Psi(y)$ is a field operator; Δ_{2F} is the broadening width given by,

$$\Delta_{2F} = \int d\xi_N^- \hat{q}_F(\xi_N) = \frac{2\pi^2\alpha_s}{N_c} \int d\xi_N^- \rho_N^A(\xi_N)[xf_g^N(x)]_{x=0}, \quad (65)$$

where $\rho_N^A(\xi_N)$ is the spatial nucleon number density inside the nucleus and $f_g^N(x)$ is the gluon distribution function in the nucleon.

We note that both $\varphi_{\rho\alpha}(x, k_\perp)$ and $\tilde{\varphi}_{\rho\alpha}(x, k_\perp)$ have the form of $\Phi_\alpha^A(x, k_\perp)$. Hence,

$$\varphi_{\rho\alpha}^{(1)A}(x, k_\perp) \approx \frac{A}{\pi\Delta_{2F}} \int d^2\ell_\perp e^{-(\vec{k}_\perp - \vec{\ell}_\perp)^2/\Delta_{2F}} \varphi_{\rho\alpha}^{(1)N}(x, \ell_\perp), \quad (66)$$

$$\tilde{\varphi}_{\rho\alpha}^{(1)A}(x, k_\perp) \approx \frac{A}{\pi\Delta_{2F}} \int d^2\ell_\perp e^{-(\vec{k}_\perp - \vec{\ell}_\perp)^2/\Delta_{2F}} \tilde{\varphi}_{\rho\alpha}^{(1)N}(x, \ell_\perp). \quad (67)$$

Making the Lorentz contraction of both sides of these two equations with $2\hat{k}_\perp^\rho \hat{k}_\perp^\alpha + d^{\rho\alpha}$ and $\hat{k}_\perp^{\{\alpha}\varepsilon_\perp^{\rho\}\sigma}$ respectively, we obtain,

$$|\vec{k}_\perp|^2 \varphi_{\perp 2}^{(1)A}(x, k_\perp) \approx \frac{A}{\pi\Delta_{2F}} \int d^2\ell_\perp e^{-(\vec{k}_\perp - \vec{\ell}_\perp)^2/\Delta_{2F}} \times [2(\ell_\perp \cdot \hat{k}_\perp)^2 + \ell_\perp^2]\varphi_{\perp 2}^{(1)N}(x, \ell_\perp), \quad (68)$$

$$|\vec{k}_\perp|^2 \tilde{\varphi}_{\perp 2}^{(1)A}(x, k_\perp) \approx \frac{A}{\pi\Delta_{2F}} \int d^2\ell_\perp e^{-(\vec{k}_\perp - \vec{\ell}_\perp)^2/\Delta_{2F}} \times [2(\ell_\perp \cdot \hat{k}_\perp)^2 + \ell_\perp^2]\tilde{\varphi}_{\perp 2}^{(1)N}(x, \ell_\perp). \quad (69)$$

Adopting a Gaussian ansatz, i.e.,

$$f_q^N(x, k_\perp) = \frac{1}{\pi\alpha} f_q^N(x) e^{(-\vec{k}_\perp^2/\alpha)}, \quad (70)$$

$$\varphi_{\perp 2}^{(1)N}(x, k_\perp) = \frac{1}{\pi\beta} \varphi_{\perp 2}^{(1)N}(x) e^{(-\vec{k}_\perp^2/\beta)}, \quad (71)$$

$$\tilde{\varphi}_{\perp 2}^{(1)N}(x, k_\perp) = \frac{1}{\pi\tilde{\beta}} \tilde{\varphi}_{\perp 2}^{(1)N}(x) e^{(-\vec{k}_\perp^2/\tilde{\beta})}, \quad (72)$$

we obtain, for those functions in the nucleus,

$$f_q^A(x, k_\perp) \approx \frac{A}{\pi\alpha_A} f_q^N(x) e^{(-\vec{k}_\perp^2/\alpha_A)}, \quad (73)$$

$$\varphi_{\perp 2}^{(1)A}(x, k_\perp) \approx \frac{A}{\pi\beta_A}\left(\frac{\beta}{\beta_A}\right)^2 \varphi_{\perp 2}^{(1)N}(x) e^{(-\vec{k}_\perp^2/\beta_A)}, \quad (74)$$

$$\tilde{\varphi}_{\perp 2}^{(1)A}(x, k_\perp) \approx \frac{A}{\pi\tilde{\beta}_A}\left(\frac{\tilde{\beta}}{\tilde{\beta}_A}\right)^2 \tilde{\varphi}_{\perp 2}^{(1)N}(x) e^{(-\vec{k}_\perp^2/\tilde{\beta}_A)}, \quad (75)$$

where $\alpha_A = \alpha + \Delta_{2F}$, $\beta_A = \beta + \Delta_{2F}$ and $\tilde{\beta}_A = \tilde{\beta} + \Delta_{2F}$. The azimuthal asymmetry is given by,

$$\frac{\langle \cos 2\phi \rangle_{eA}}{\langle \cos 2\phi \rangle_{eN}} \approx \frac{\alpha}{\alpha_A} e^{-\vec{k}_\perp^2/\alpha_A + \vec{k}_\perp^2/\alpha} \times \frac{\frac{\beta^2}{\beta_A^3}\varphi_{\perp 2}^{(1)N}(x_B) e^{(-\vec{k}_\perp^2/\beta_A)} - \frac{\tilde{\beta}^2}{\tilde{\beta}_A^3}\tilde{\varphi}_{\perp 2}^{(1)N}(x_B) e^{(-\vec{k}_\perp^2/\tilde{\beta}_A)}}{[\frac{1}{\beta}\varphi_{\perp 2}^{(1)N}(x_B) e^{(-\vec{k}_\perp^2/\beta)} - \frac{1}{\tilde{\beta}}\tilde{\varphi}_{\perp 2}^{(1)N}(x_B) e^{(-\vec{k}_\perp^2/\tilde{\beta})}]},$$

which reduces to

$$\frac{\langle \cos 2\phi \rangle_{eA}}{\langle \cos 2\phi \rangle_{eN}} = \left(\frac{\beta}{\beta + \Delta_{2F}}\right)^2, \quad (76)$$

in the case that $\alpha = \beta = \tilde{\beta}$. We see that, in this case, for given x_B, Q^2 and $|\vec{k}_\perp|$, $\langle \cos 2\phi \rangle_{eA}$ in deep-inelastic eA scattering is suppressed compared to that in eN scattering with a suppression factor $\beta^2/(\beta + \Delta_{2F})^2$. Comparing with the result of [13], we can see that $\langle \cos 2\phi \rangle_{eA}$ is more suppressed than $\langle \cos\phi \rangle_{eA}$. In general, $\beta, \tilde{\beta}$ can be different from α, and the ratio can also be different at different $k_\perp$ and Δ_{2F}. As an example, we show the results for a few cases in Figs. 1(a) and 1(b) with $\beta = \tilde{\beta}$.

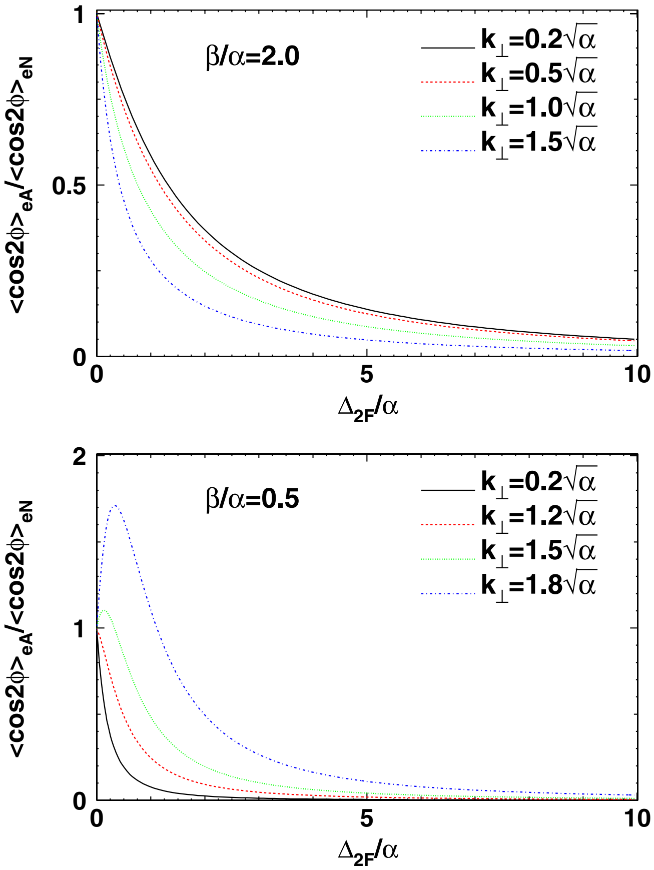

FIG. 1 (color online). Ratio $\langle\cos 2\phi\rangle_{eA}/\langle\cos 2\phi\rangle_{eN}$ as a function of Δ_{2F}/α for different $k_\perp$ and, in the upper panel, denoted as (a), for $\beta/\alpha = 2.0$, and in the lower panel, denoted as (b), for $\beta/\alpha = 0.5$.

We see that the asymmetry can be suppressed or enhanced depending on the values of $k_\perp$ and Δ_{2F}, and the magnitude is smaller than $\langle\cos\phi\rangle$ case.

If we integrate over the magnitude of $\vec{k}_\perp$, we obtain,

$$\frac{\langle\langle\cos\phi\rangle\rangle_{eA}}{\langle\langle\cos\phi\rangle\rangle_{eN}} \approx \frac{(\frac{\beta}{\beta_A})^2\beta_A\varphi^{(1)N}_{\perp 2}(x_B) - (\frac{\tilde\beta}{\tilde\beta_A})^2\tilde\beta_A\tilde\varphi^{(1)N}_{\perp 2}(x_B)}{\beta\varphi^{(1)N}_{\perp 2}(x_B) - \tilde\beta\tilde\varphi^{(1)N}_{\perp 2}(x_B)}, \quad (77)$$

which reduces to $\beta/(\beta + \Delta_{2F})$ for the special case $\beta = \tilde\beta$.

IV. SUMMARY AND DISCUSSIONS

We calculated the hadronic tensor and differential cross section for the unpolarized SIDIS process $e^- + N \to e^- + q + X$ in leading order pQCD and up to twist-4 contributions The results depend on a number of new TMD parton correlation functions. We showed that measurements of the azimuthal asymmetry $\langle\cos 2\phi\rangle$ and its $k_\perp$-dependence provide information on these TMD correlation functions which in turn can shed light on the properties of multiple gluon interaction in hadronic processes. Under two-gluon correlation approximation, we also show the relationship between these TMD correlation functions inside large nuclei and that of a nucleon. One can therefore study the nuclear dependence of the azimuthal asymmetry $\langle\cos 2\phi\rangle$ which is determined by the jet transport parameter $\hat{q}$ inside nuclei. With a Gaussian ansatz for the TMD parton correlation functions inside the nucleon, we also illustrate numerically that the asymmetry $\langle\cos 2\phi\rangle$ is suppressed in the corresponding SIDIS with a nuclear target.

There exist experimental measurements of the azimuthal asymmetries in both unpolarized and polarized DIS [14–25]. More results are expected from CLAS at JLab and COMPASS at CERN. The available data seem to be consistent with the Gaussian ansatz for the transverse momentum dependence of the TMD matrix elements [42]. However, these data are still not adequate enough to provide any precise constraints on the form of the higher twist matrix elements. Our calculations of the azimuthal asymmetries are most valid in the small transverse momentum region where next to the leading order pQCD corrections are not dominant. The high twist effects are also most accessible in the intermediate region of Q^2. One expects that future experiments such as those at the proposed Electron-Ion-Collider [43] will be better equipped to study these high twist effects in detail.

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