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Four curious supergravities

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We consider four supergravities with 16 + 16, 32 + 32, 64 + 64, 128 + 128 degrees of freedom displaying some curious properties: (1) They exhibit minimal supersymmetry ($\mathcal{N} = 1, 2, 2, 1$) but maximal rank ($r = 7, 6, 4, 0$) of the scalar coset in $D = 4, 5, 7, 11$. (2) They couple naturally to supermembranes and admit these membranes as solutions. (3) Although the $D = 4, 5, 7$ supergravities follow from truncating the maximally supersymmetric ones, there nevertheless exist M-theory compactifications with G_2, $SU(3)$, $SU(2)$ holonomy having these supergravities as their massless sectors. (4) They reduce to $\mathcal{N} = 1, 2, 4, 8$ theories all with maximum rank 7 in $D = 4$ which (5) correspond to 0, 1, 3, 7 lines of the Fano plane and hence admit a division algebra ($\mathbb{R}, \mathbb{C}, \mathbb{H}, \mathbb{O}$) interpretation consistent with the black-hole/qubit correspondence, (6) are generalized self-mirror, and hence (7) have a vanishing on-shell trace anomaly.

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I. INTRODUCTION

In the early 1980s Green and Schwarz [1] showed that spacetime supersymmetry allows classical superstrings moving in spacetime dimensions 3, 4, 6, and 10, with the $D = 10$ case being anomaly-free at the quantum level. However, following the brane scan [2] of Table I, which pinpointed those 12 (p, D) slots consistent with kappa-symmetric Green-Schwarz type actions,[1] it was realized that these 1-branes in $D = 3, 4, 6$, and 10 should now be viewed as the endpoints of four sequences of p-branes. Moving diagonally down the brane scan corresponds to a simultaneous dimensional reduction of spacetime and world volume [5]. In particular, supermembranes exist in $D = 4, 5, 7, 11$ with minimal supersymmetry $\mathcal{N} = 1, 2, 2, 1$, respectively.

In $D = 11$, it is known that the membrane [6] couples to the $D = 11$ supergravity background $(g_{MN}; \psi_M; A_{MNP})$, and that this supergravity in turn admits the membrane as a solution [7]. Yet, little attention has been paid to the corresponding supergravities that couple to membranes in $D = 4, 5$, and 7 (or strings in $D = 3, 4$, and 6). This hitherto lack of interest[2] in these minimal supergravity theories is no doubt due to the perception that they describe the low-energy limit of noncritical quantum inconsistent string or M theories.

In this paper, however, we consider those supergravities that emerge as the massless sectors of compactifications of M theory on manifolds/orbifolds X^4, X^6, and X^7 with reduced holonomy $SU(2)$, $SU(3)$, and G_2 and with special Betti numbers:

$$X^4\colon (1, 0, 6, 0, 1),$$
$$X^6\colon (1, 0, 3, 8, 3, 0, 1),$$
$$X^7\colon (1, 0, 0, 7, 7, 0, 0, 1), \tag{1}$$

respectively. This means that the resulting theories in $D = 7, 5, 4$ with $\mathcal{N} = 2, 2, 1$ are just as interesting as their counterpart in $D = 11$ with $\mathcal{N} = 1$. Denoting the fields that result from the $D = 11$ (metric; gravitino; 3-form) by $(g_{\mu\nu}, \mathcal{A}_\mu, \mathcal{A}; \psi_\mu, \chi; A_{\mu\nu\rho}, A_{\mu\nu}, A_\mu, A)$, and their numbers of degrees of freedom by f, we show in Sec. II that the four supergravities are

(i) $D = 11$: $\mathcal{N} = 1$ graviton, $f = 128 + 128$

$$(g_{\mu\nu}; \psi_\mu; A_{\mu\nu\rho})$$

(ii) $D = 7$: $\mathcal{N} = 2$ graviton + 3 vector, $f = 40 + 40 + 3(8 + 8) = 64 + 64$

$$(g_{\mu\nu}, \mathcal{A}; 2\psi_\mu, 2\chi; A_{\mu\nu\rho}, 3A_\mu) + 3(3\mathcal{A}; 2\chi; A_\mu),$$

with rank 4 scalar coset

$$\frac{G}{H} = SO(1, 1) \times \frac{SL(4, R)}{SO(4)}$$

(iii) $D = 5$: $\mathcal{N} = 2$ graviton + 2 vector + 3 hyper + 1 3-form, $f = 8 + 8 + 2(4 + 4) + 3(4 + 4) + (4 + 4) = 32 + 32$

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[1]It will be sufficient for our purposes to focus on this old brane-scan with just scalars and spinors on the worldvolume, as opposed to the new one [3,4] with vectors (D-branes) and tensors (M5-branes) also.

[2]Exceptions may be found in [8–11] and in John Baez's blog [12].

TABLE I. p-branes described by Green-Schwarz actions.

$D\uparrow$								
11	.			o				
10	.		o				o	
9	.					o		
8	.				o			
7	.			o				
6	.		o		o			
5	.			o				
4	.		o	o				
3	.		o					
2	.							
1	.							
0	.	.	.	.	.	.	.	
	0	1	2	3	4	5	6	$p+1\rightarrow$

$$(g_{\mu\nu};2\psi_\mu;A_\mu)+2(\mathcal{A};2\chi;A_\mu)$$
$$+3(2\mathcal{A};2\chi;2A)+(\mathcal{A};2\chi;A_{\mu\nu\rho},2A),$$

with rank 6 scalar coset

$$\frac{G}{H}=SO(1,1)^3\times\frac{SO(3,4)}{SO(3)\times SO(4)}\ltimes R^2$$

(iv) $D=4$: $\mathcal{N}=1$ graviton + 7WZ (where WZ refers to Wess-Zumino), $f=2+2+7(2+2)=16+16$

$$(g_{\mu\nu};\psi_\mu;A_{\mu\nu\rho})+7(\mathcal{A};\chi;A),$$

with rank 7 scalar coset

$$\frac{G}{H}=\frac{SL(2)^7}{SO(2)^7}$$

The U-duality is only $SL(2)^6\times SO(1,1)\ltimes R$, however, because of the coupling of the scalars to $A_{\mu\nu\rho}$.

These theories may also be derived by compactifying M theory on T^4, T^6, and T^7 and truncating the massless sectors so as to obtain minimal supersymmetry ($\mathcal{N}=2$, 2, and 1, respectively) while preserving the maximum rank of the scalar coset ($r=4$, 6, and 7, respectively). They exhibit several other remarkable properties. For example, they admit membranes as elementary (electric) solutions by virtue of the universal presence of a 3-form $A_{\mu\nu\rho}$ and by virtue of the correct dilaton exponent which follows from the maximum rank condition. We therefore expect that they will be compatible with the superspace constraints enforced by kappa symmetry on the world volume of the Green-Schwarz membranes [6], but we do not address this problem here.

It will be important for our purposes to distinguish between the Lagrangians obtained directly in these two ways and the conventional supergravity Lagrangians obtained after dualization of p-forms. For example, the latter has no $A_{\mu\nu\rho}$ field in $D=4$ and so not only has a different symmetry, namely, $SL(2)^7$ as opposed to the $SL(2)^6\times SO(1,1)\ltimes R$ of the former, but also admits no electric membrane (domain wall) solution.

As described in Sec. III, these four theories may be further reduced to $\mathcal{N}=1$, 2, 4, and 8 theories all with maximum rank $r=7$ in $D=4$, corresponding to compactification on X^7 with independent Betti numbers

$$(b_0,b_1,b_2,b_3)=(1,\mathcal{N}-1,3\mathcal{N}-3,4\mathcal{N}+3). \quad (2)$$

Compactifications with such Betti numbers may indeed be found in [13] for $\mathcal{N}=8$, in [14,15] for $\mathcal{N}=4$ and in [16–18] for $\mathcal{N}=2$ and $\mathcal{N}=1$. We show that the corresponding supergravities before dualization are:

(i) T^7: (1, 7, 21, 35)
$\mathcal{N}=8$ graviton, $f=128+128$,

$$(g_{\mu\nu},7\mathcal{A}_\mu,28\mathcal{A};8\psi_\mu,56\chi;A_{\mu\nu\rho},7A_{\mu\nu},21A_\mu,35A),$$

with rank 7 scalar coset

$$\frac{G}{H}=SO(1,1)\times\frac{SL(7,R)}{SO(7)}\ltimes R^{35}$$

(ii) $X^4\times T^3$: (1, 3, 9, 19)
$\mathcal{N}=4$ graviton + 3 vector + 3 2-form, $f=16+16+3(8+8)+3(8+8)=64+64$,

$$(g_{\mu\nu},3\mathcal{A}_\mu,\mathcal{A};4\psi_\mu,4\chi;A_{\mu\nu\rho},3A_\mu,A)$$
$$+3(3\mathcal{A};4\chi;A_\mu,3A)$$
$$+3(2\mathcal{A};4\chi;A_{\mu\nu},A_\mu,3A),$$

with rank 7 scalar coset

$$\frac{G}{H}=\frac{SL(2,R)}{SO(2)}\times\frac{SO(3,6)}{SO(3)\times SO(6)}\times SO(1,1)\times\frac{SL(3)}{SO(3)}\ltimes R^9$$

(iii) $X^6\times S^1$: (1, 1, 3, 11)
$\mathcal{N}=2$ graviton + 3 vector + 3 hyper + 1 linear, $f=4+4+3(4+4)+3(4+4)+(4+4)=32+32$,

$$(g_{\mu\nu},\mathcal{A}_\mu;2\psi_\mu;A_{\mu\nu\rho})+3(\mathcal{A};2\chi;A_\mu,A)$$
$$+3(2\mathcal{A};2\chi;2A)+(\mathcal{A};2\chi;A_{\mu\nu},2A),$$

with rank 7 scalar coset

$$\frac{G}{H}=SO(1,1)\times\frac{SL(2,R)^3}{SO(2)^3}\times\frac{SO(3,4)}{SO(3)\times SO(4)}\ltimes R^2$$

(iv) X^7: (1, 0, 0, 7)
$\mathcal{N}=1$ graviton + 7WZ, $f=2+2+7(2+2)=16+16$,

$$(g_{\mu\nu};\psi_\mu;A_{\mu\nu\rho}) + 7(\mathcal{A};\chi;A),$$

with rank 7 scalar coset

$$\frac{G}{H} = \frac{SL(2)^7}{SO(2)^7}$$

Interestingly enough, the cases $\mathcal{N}=8$, $\mathcal{N}=4$, and $\mathcal{N}=2$ (albeit without the three hyper and one linear multiplet [19]) have already made an appearance in the context of the *black-hole/qubit correspondence* [20–22], where their 56, 24, 8 black-hole charges correspond to 7, 3, 1 lines of the Fano plane [23–25]. In particular, the $\mathcal{N}=2$ supergravity is just the STU model whose black holes have a Bekenstein-Hawking entropy given by Cayley's hyperdeterminant, the same quantity that describes the entanglement of three qubits. The 7, 3, 1 lines of the Fano plane in turn provide the multiplication table of the imaginary octonion, quaternion, and complex numbers, respectively. The fourth $\mathcal{N}=1$ supergravity completes the set with 0 lines, corresponding to the real numbers.

Earlier work on the brane scan gave an $\mathbb{O}$, $\mathbb{H}$, $\mathbb{C}$, $\mathbb{R}$ division algebra interpretation to the four sequences appearing in Table I, which have 8 + 8, 4 + 4, 2 + 2, 1 + 1 world volume degrees of freedom. See [26–28] and references therein. Since our supergravities are obtained by compactification, however, the corresponding membranes all have 8 + 8.

Furthermore, we recall that in [29] we defined a generalized mirror symmetry

$$(b_0, b_1, b_2, b_3) \to (b_0, b_1, b_2 - \rho/2, b_3 + \rho/2), \tag{3}$$

under which

$$\rho \equiv 7b_0 - 5b_1 + 3b_2 + b_3 \tag{4}$$

changes sign

$$\rho \to -\rho. \tag{5}$$

Generalized self-mirror theories are defined to be those for which ρ vanishes. In the case of G_2 manifolds with $b_1 = 0$, Joyce [16,17] refers to $\rho = 0$ as an "axis of symmetry." For related work on mirror symmetry and Joyce manifolds, see [18,30,31].

Moreover, the quantity ρ also shows up in the on-shell Weyl anomaly [32,33], before dualization [34], which is given by

$$g_{\mu\nu}\langle T^{\mu\nu}\rangle = A\frac{1}{32\pi^2}R^{*\mu\nu\rho\sigma}R^*_{\mu\nu\rho\sigma}, \tag{6}$$

where

$$A = -\frac{1}{24}\rho. \tag{7}$$

Since our four curious supergravities all have $\rho = 0$, they are self-mirror in the above sense and hence have a vanishing Weyl anomaly.

Finally, we note that a spacelike reduction gives the four Type IIA supergravities that couple to superstrings in $D = 3, 4, 6, 10$. They yield $\mathcal{N} = 16, 8, 4, 2$ supergravities in $D = 3$. While a timelike reduction from $D = 4$ to $D = 3$ yields after dualization the four cosets that play a role in the four-way entanglement of eight qubits [25,35–38], namely, $E_{8(8)}/SO^*(16)$, $SO(8,8)/SO(4,4)^2$, $SO(4,4)^2/SO(2,2)^4$, $SO(2,2)^4/SO(1,1)^8$.

II. MINIMAL SUPERGRAVITIES IN $D = 4, 5, 7, 11$

A. Compactifications

To derive the $D = 4, 5, 7, 11$ theories we begin with compactification on generic manifolds, tori, and manifolds of special holonomy as shown in Tables II, III, IV, and V.

B. Supermultiplets

Here we group the individual fields into supermutiplets as shown in Tables VI, VII, VIII, and IX:

C. Lagrangians

The bosonic sector of the toroidally compactified $D = 11$ supergravity prior to dualization may be found in [39,40]. It will be useful to split the metric scalars $\mathcal{A}$ into $\vec{\phi}$, the $(11 - D)$ vector of dilatonic scalar fields coming from the diagonal components of the internal metric, and the rest, which we continue to describe by the letter $\mathcal{A}$.

TABLE II. $D = 11$ fields.

Field	f	
g_{MN}	44	1
ψ_M	128	1
A_{MNP}	84	1
Total f		256

TABLE III. Compactify to $D = 7$ on X^4 with Betti numbers: generic ($d_0 = d_4 = 1$; $d_1 = d_3$; d_2); torus (1; 4; 6); and $SU(2)$ holonomy (1; 0; 6).

	Field	f	Generic	Torus	Special
g_{MN}	$g_{\mu\nu}$	14	d_0	1	1
	$\mathcal{A}_\mu$	5	d_1	4	0
	$\mathcal{A}$	1	$-8d_0 + 3d_2$	10	10
ψ_M	ψ_μ	16	$2d_0 + d_1/2$	4	2
	χ	4	$-4d_0 + 2d_1 + 2d_2$	16	8
A_{MNP}	$A_{\mu\nu\rho}$	10	d_0	1	1
	$A_{\mu\nu}$	10	d_1	4	0
	A_μ	5	d_2	6	6
	A	1	d_1	4	0
Total f			$16(2d_0 + 2d_1 + d_2)$	256	128
$\chi(X^4)$			$2d_0 - 2d_1 + d_2$	0	8

TABLE IV. Compactify to $D=5$ on X^6 with Betti numbers: generic ($c_0=c_6=1$; $c_1=c_5$; $c_2=c_4$; c_3), torus (1; 6; 15; 20), and $SU(3)$ holonomy (1; 0; 3; 8).

	Field	f	Generic	Torus	Special
g_{MN}	$g_{\mu\nu}$	5	c_0	1	1
	$\mathcal{A}_\mu$	3	c_1	6	0
	$\mathcal{A}$	1	$-2c_0-2c_1+c_2+c_3$	21	9
ψ_M	ψ_μ	4	$2c_0+c_1$	8	2
	χ	2	$-2c_0+2c_2+c_3$	48	12
A_{MNP}	$A_{\mu\nu\rho}$	1	c_0	1	1
	$A_{\mu\nu}$	3	c_1	6	0
	A_μ	3	c_2	15	3
	A	1	c_3	20	8
Total f			$4(2c_0+2c_1+2c_2+c_3)$	256	64
$\chi(X^6)$			$2c_0-2c_1+2c_2-c_3$	0	0

TABLE V. Compactify to $D=4$ on X^7 with Betti numbers: generic ($b_0=b_7=1$; $b_1=b_6$; $b_2=b_5$, $b_3=b_4$), torus (1; 7; 21; 35), and G_2 holonomy (1; 0; 0; 7).

	Field	f	Generic	Torus	Special
g_{MN}	$g_{\mu\nu}$	2	b_0	1	1
	$\mathcal{A}_\mu$	2	b_1	7	0
	$\mathcal{A}$	1	$-b_1+b_3$	28	7
ψ_M	ψ_μ	2	b_0+b_1	8	1
	χ	2	b_2+b_3	56	7
A_{MNP}	$A_{\mu\nu\rho}$	0	b_0	1	1
	$A_{\mu\nu}$	1	b_1	7	0
	A_μ	2	b_2	21	0
	A	1	b_3	35	7
Total f			$4(b_0+b_1+b_2+b_3)$	256	32
$\rho(X^7)$			$7b_0-5b_1+3b_2-b_3$	0	0

TABLE VI. The $D=11$ multiplet in the minimal $\mathcal{N}=1$ basis.

$N=1$	Multiplet	f
Graviton	$(g_{MN};\psi_M;A_{MNP})$	128 + 128

The original 11-dimensional fields g_{MN} and A_{MNP} will give then rise to the following fields in D dimensions:

$$\begin{aligned} g_{MN} &\to g_{\mu\nu}, \quad \vec{\phi}, \quad \mathcal{A}^i_\mu, \quad \mathcal{A}^i{}_j, \\ A_{MNP} &\to A_{\mu\nu\rho}, \quad A_{\mu\nu k}, \quad A_{\mu jk}, \quad A_{ijk}, \end{aligned} \tag{8}$$

where the indices i, j, k run over the $(11-D)$ internal toroidally compactified dimensions. If we denote the rank $(p+1)$-field strength of the rank p potentials by a subscript $(p+1)$, the Lagrangian is

$$\begin{aligned} \frac{\mathcal{L}}{\sqrt{-g}} &= R - \frac{1}{2}(\partial\vec{\phi})^2 - \frac{1}{48}e^{\vec{a}\cdot\vec{\phi}}F_{(4)}^2 - \frac{1}{12}\sum_i e^{\vec{a}_i\cdot\vec{\phi}}(F_{(3)i})^2 \\ &\quad - \frac{1}{4}\sum_{i<j} e^{\vec{a}_{ij}\cdot\vec{\phi}}(F_{(2)ij})^2 - \frac{1}{4}\sum_i e^{\vec{b}_i\cdot\vec{\phi}}(\mathcal{F}^i_{(2)})^2 \\ &\quad - \frac{1}{2}\sum_{i<j<k} e^{\vec{a}_{ijk}\cdot\vec{\phi}}(F_{(1)ijk})^2 - \frac{1}{2}\sum_{i<j} e^{\vec{b}_{ij}\cdot\vec{\phi}}(\mathcal{F}^i_{(1)j})^2 \\ &\quad + \mathcal{L}_{FFA}, \end{aligned} \tag{9}$$

where the "dilaton vectors" $\vec{a}$, $\vec{a}_i$, $\vec{a}_{ij}$, $\vec{a}_{ijk}$, $\vec{b}_i$, $\vec{b}_{ij}$ are constants that characterize the couplings of the dilatonic scalars $\vec{\phi}$ to the various gauge fields [41]

$$F_4\colon \vec{a} = -\vec{g}, \tag{10}$$

$$F_3\colon \vec{a}_i = \vec{f}_i - \vec{g}, \tag{11}$$

$$F_2\colon \vec{a}_{ij} = \vec{f}_i + \vec{f}_j - \vec{g}, \tag{12}$$

$$F_1\colon \vec{a}_{ijk} = \vec{f}_i + \vec{f}_j + \vec{f}_k - \vec{g}, \tag{13}$$

$$\mathcal{F}_2\colon \vec{b}_i = -\vec{f}_i, \tag{14}$$

$$\mathcal{F}_1\colon b_{ij} = -\vec{f}_i + \vec{f}_j, \tag{15}$$

where the vectors $\vec{g}$ and $\vec{f}_i$ have $(11-D)$) components in D dimensions, and are given by

$$\begin{aligned} \vec{g} &= 3(s_1, s_2, \ldots, s_{11-D}), \\ \vec{f}_i &= (\underbrace{0, 0, \ldots, 0}, (10-i)s_i, s_{i+1}, s_{i+2}, \ldots, s_{11-D}), \end{aligned} \tag{16}$$

where $s_i = \sqrt{2/((10-i)(9-i))}$. Note that the 4-dimensional metric is related to the 11-dimensional one by

$$ds_{11}^2 = e^{1/3\vec{g}\cdot\vec{\phi}}ds_4^2 + \sum_i e^{2\vec{\gamma}_i\cdot\vec{\phi}}(h^i)^2, \tag{17}$$

where

$$\vec{\gamma}_i = \tfrac{1}{6}\vec{g} - \tfrac{1}{2}\vec{f}_i \tag{18}$$

and

TABLE VII. The $D=7$ multiplets in the minimal $\mathcal{N}=2$ basis.

$N=2$	Multiplet	f	$N=2d_0+d_1/2$	$N=4$	$N=2$
Graviton	$(g_{\mu\nu},\mathcal{A};2\psi_\mu,2\chi;A_{\mu\nu\rho},3A_\mu)$	40 + 40	d_0	1	1
Gravitino	$(4\mathcal{A}_\mu;2\psi_\mu,8\chi;4A_{\mu\nu},4A)$	64 + 64	$d_1/4$	1	0
Vector	$(3\mathcal{A};2\chi;A_\mu)$	8 + 8	$-3d_0+d_2$	3	3

TABLE VIII. The $D = 5$ multiplets in the minimal $\mathcal{N} = 2$ basis.

$N = 2$	Multiplet	f	$N = 2c_0 + c_1$	$N = 8$	$N = 2$
Graviton	$(g_{\mu\nu}; 2\psi_\mu; A_\mu)$	$8 + 8$	c_0	1	1
Gravitino	$(2\mathcal{A}_\mu; 2\psi_\mu, 2\chi; 2A_\mu)$	$12 + 12$	$c_1/2$	3	0
Vector	$(\mathcal{A}; 2\chi; A_\mu)$	$4 + 4$	$-c_0 - c_1 + c_2$	8	2
Hyper	$(2\mathcal{A}; 2\chi; 2A)$	$4 + 4$	$-c_0 - c_1/2 + c_3/2$	6	3
2-form	$(2\chi; A_{\mu\nu}; A)$	$4 + 4$	c_1	6	0
3-form	$(\mathcal{A}; 2\chi; A_{\mu\nu\rho}; 2A)$	$4 + 4$	c_0	1	1

TABLE IX. The $D = 4$ multiplets in the minimal $\mathcal{N} = 1$ basis.

$N = 1$	Multiplet	f	$N = b_0 + b_1$	$N = 8$	$N = 1$
Graviton	$(g_{\mu\nu}; \psi_\mu; A_{\mu\nu\rho})$	$2 + 2$	b_0	1	1
Gravitino	$(\mathcal{A}_\mu; \psi_\mu)$	$2 + 2$	b_1	7	0
Vector	$(\chi; A_\mu)$	$2 + 2$	b_2	21	7
WZ	$(\mathcal{A}; \chi; A)$	$2 + 2$	$-b_1 + b_3$	28	7
Linear	$(\chi; A_{\mu\nu}; A)$	$2 + 2$	b_1	1	1

$$h^i = dz^i + \mathcal{A}^i + \mathcal{A}^i{}_j dz^j. \tag{19}$$

In general, the field strengths appearing in the kinetic terms are not simply the exterior derivatives of their associated potentials, but have nonlinear Kaluza-Klein modifications as well. On the other hand the terms included in $\mathcal{L}_{FFA}$, which denotes the dimensional reduction of the $F_{(4)} \wedge F_{(4)} \wedge A_{(3)}$ term in $D = 11$, are best expressed purely in terms of the potentials and their exterior derivatives. The complete details may be found in [41], where it is shown that the symmetry of the Lagrangian is

$$GL(11 - D, R) \ltimes R^q, \tag{20}$$

with

$$q = \tfrac{1}{6}(11 - D)(10 - D)(9 - D). \tag{21}$$

Our minimal Lagrangians in $D = 7, 5, 4$ are followed by appropriate truncations that nevertheless keep all the $\vec{\phi}$. We shall not show these explicitly.

D. Membrane solutions

According to [4,41,42], the existence of an elementary membrane solution in D dimensions requires a metric, 3-form potential and dilaton described by the action

$$\frac{\mathcal{L}}{\sqrt{-g}} = R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{48}e^{a\phi}F^2_{(4)}. \tag{22}$$

Moreover, the dilaton coupling must be such that

$$a^2 = \frac{2(11 - D)}{(D - 2)}, \tag{23}$$

or $a = 0, 8/5, 4, 7$ in $D = 11, 7, 5, 4$. But if we start with

$$\frac{\mathcal{L}}{\sqrt{-g}} = R - \frac{1}{2}(\partial\vec{\phi})^2 - \frac{1}{48}e^{\vec{a}\cdot\vec{\phi}}F^2_{(4)}, \tag{24}$$

and make the ansatz

$$a\vec{\phi} = \vec{a}\phi, \tag{25}$$

noting that

$$\vec{a}.\vec{a} = \frac{2(11 - D)}{(D - 2)} = a^2, \tag{26}$$

then the two Lagrangians coincide. Note that this ansatz would not have worked had we failed to implement the maximum rank condition by omitting some of the components of $\vec{\phi}$.

E. Cosets

The scalar cosets, before and after dualization, are shown in Tables X and XI.

TABLE X. $D = 4, 5, 7, 11$ cosets before dualization.

Theory	Charges	G/H	Dim	Rank	Max G/H	Dim	Rank
$D = 11$	32	0	0	0	$\subset 0$	0	0
$D = 7$	16	$SO(1,1) \times SL(4,R)/SO(4)$	10	4	$\subset SL(5,R)/SO(5)$	14	4
$D = 5$	8	$SO(1,1)^3 \times SO(4,3)/[SO(4) \times SO(3)] \ltimes R^2$	17	6	$\subset SO(1,1) \times SL(6,R)/SO(6) \ltimes R^{20}$	41	6
$D = 4$	4	$SL(2,R)^7/SO(2)^7$	14	7	$\subset SO(1,1) \times SL(7,R)/SO(7) \ltimes R^{35}$	63	7

TABLE XI. $D = 4, 5, 7, 11$ cosets after dualization.

Theory	Charges	G/H	Dim	Rank	Max G/H	Dim	Rank
$D = 11$	32	0	0	0	$\subset 0$	0	0
$D = 7$	16	$SO(1,1) \times SL(4,R)/SO(4)$	10	4	$\subset SL(5,R)/SO(5)$	14	4
$D = 5$	8	$SO(1,1)^2 \times SO(4,4)/SO(4)^2$	18	6	$\subset E_{6(6)}/Usp(8)$	42	6
$D = 4$	4	$SL(2,R)^7/SO(2)^7$	14	7	$\subset E_{7(7)}/SU(8)$	70	7

TABLE XII. X^7, $X^6 \times S^1$, $X^4 \times T^3$, T^7 compactification of $D = 11$ supergravity.

	Field	f	$360A$	X^7	$X^6 \times S^1$	$X^4 \times T^3$	T^7
g_{MN}	$g_{\mu\nu}$	2	848	b_0	c_0	d_0	1
	$\mathcal{A}_\mu$	2	-52	b_1	$c_0 + c_1$	$3d_0 + d_1$	7
	$\mathcal{A}$	1	4	$-b_1 + b_3$	$-c_0 - c_1 + c_2 + c_3$	$-2d_0 + 3d_1 + 3d_2$	28
ψ_M	ψ_μ	2	-233	$b_0 + b_1$	$2c_0 + c_1$	$4d_0 + d_1$	8
	χ	2	7	$b_2 + b_3$	$c_1 + 2c_2 + c_3$	$4d_0 + 7d_1 + 4d_2$	56
A_{MNP}	$A_{\mu\nu\rho}$	0	-720	b_0	d_0	c_0	1
	$A_{\mu\nu}$	2	364	b_1	$c_0 + c_1$	$3d_0 + d_1$	7
	A_μ	2	-52	b_2	$c_1 + c_2$	$3d_0 + 3d_1 + d_2$	21
	A	1	4	b_3	$c_2 + c_3$	$d_0 + 4d_1 + 3d_2$	35
				$A = -\rho/24$	$A = -\chi/24$	$A = 0$	$A = 0$

III. $\mathcal{N} = 1, 2, 4, 8$ IN $D = 4$

A. Betti numbers

These four theories may be further reduced to $\mathcal{N} = 1$, 2, 4 and 8 theories all with maximum rank $r = 7$ in $D = 4$, corresponding to compactification on $X^{(8-\mathcal{N})} \times T^{(\mathcal{N}-1)}$. Denote the Betti numbers of X^7, X^6, X^4 by b, c, d, respectively. The Betti numbers of S^1 are (1, 1), of T^3 are (1, 3, 3, 1), of T^4 are (1, 4, 6, 4, 1), of T^7 are (1, 7, 21, 35, 21, 7, 1), so we have

$$X^7\colon (b_0, b_1, b_2, b_3),$$
$$X^6 \times S^1\colon (c_0, c_0 + c_1, c_1 + c_2, c_2 + c_3),$$
$$X^4 \times T^3\colon (d_0, 3d_0 + d_1, 3d_0 + 3d_1 + d_2, d_0 + 4d_1 + 3d_2). \tag{27}$$

The number of fields in $D = 4$ is given by Table XII.

B. Self-mirror with vanishing trace anomaly

Finally, we note that in [29] we defined a generalized mirror symmetry

$$(b_0, b_1, b_2, b_3) \rightarrow (b_0, b_1, b_2 - \rho/2, b_3 + \rho/2), \tag{28}$$

under which

$$\rho \equiv 7b_0 - 5b_1 + 3b_2 + b_3 \tag{29}$$

changes sign

$$\rho \rightarrow -\rho. \tag{30}$$

Moreover, the quantity ρ also shows up in the on-shell trace anomaly (before dualization), which is given by

$$g_{\mu\nu}\langle T^{\mu\nu}\rangle = A\frac{1}{32\pi^2}R^{*\mu\nu\rho\sigma}R^{*}_{\mu\nu\rho\sigma}. \tag{31}$$

The value of the A coefficients for each field is given in Table XII, which shows compactification on X^7, $X^6 \times S^1$, $X^4 \times T^3$, and T^7. We adopt the interpretation of [34] that assigns different anomalies to $A_{\mu\nu}$ and $\mathcal{A}$ even though they are naively dual to one another and the nonzero anomaly to $A_{\mu\nu\rho}$. Remarkably, we find that the total anomaly depends on ρ

TABLE XIII. The $D = 4$ multiplets in an $\mathcal{N} = 1$ basis.

$\mathcal{N} = 1$	Multiplet	f	$360A$	$\mathcal{N} = b_0 + b_1$	$\mathcal{N} = 8$	$\mathcal{N} = 1$
Graviton	$(g_{\mu\nu}; \psi_\mu; A_{\mu\nu\rho})$	$2+2$	-105	b_0	1	1
Gravitino	$(\mathcal{A}_\mu; \psi_\mu)$	$2+2$	-285	b_1	7	0
Vector	$(\chi; A_\mu)$	$2+2$	-45	b_2	21	0
WZ	$(\mathcal{A}; \chi; A)$	$2+2$	15	$-b_1 + b_3$	28	7
Linear	$(\chi; A_{\mu\nu}; A)$	$2+2$	375	b_1	7	0
total f				$4(b_0 + b_1 + b_2 + b_3)$	256	32
total A				$-(7b_0 - 5b_1 + 3b_2 - b_3)/24$	0	0

TABLE XIV. The $D = 4$ multiplets in an $\mathcal{N} = 2$ basis.

$\mathcal{N} = 2$	Multiplet	f	$360A$	$\mathcal{N} = 2c_0 + c_1$	$\mathcal{N} = 8$	$\mathcal{N} = 2$
Graviton	$(g_{\mu\nu}, \mathcal{A}_\mu; 2\psi_\mu; A_{\mu\nu\rho})$	$4 + 4$	-390	c_0	1	1
Gravitino	$(\mathcal{A}_\mu; \psi_\mu, \chi; A_\mu)$	$4 + 4$	-330	c_1	6	0
Vector	$(\mathcal{A}, 2\chi; A_\mu, A)$	$4 + 4$	-30	c_2	15	3
Hyper	$(2\mathcal{A}; 2\chi; 2A)$	$4 + 4$	30	$-c_0 - c_1 + c_3/2$	3	3
Linear	$(\mathcal{A}; 2\chi; A_{\mu\nu}, 2A)$	$4 + 4$	390	$c_0 + c_1$	7	1
Total f				$4(2c_0 + 2c_1 + 2c_2 + c_3)$	256	64
Total A				$-(2c_0 - 2c_1 + 2c_2 - c_3)/24$	0	0

TABLE XV. The $D = 4$ multiplets in an $\mathcal{N} = 4$ basis.

$\mathcal{N} = 4$	multiplet	f	$360A$	$\mathcal{N} = 4d_0 + d_1$	$\mathcal{N} = 8$	$\mathcal{N} = 4$
Graviton	$(g_{\mu\nu}, 3\mathcal{A}_\mu, \mathcal{A}, 4\psi_\mu, 4\chi, A_{\mu\nu\rho}, 3A_\mu, A)$	$16 + 16$	-1080	d_0	1	1
Gravitino	$(\mathcal{A}_\mu, 3\mathcal{A}, \psi_\mu, 7\chi, A_{\mu\nu}, 3A_\mu, 4A)$	$16 + 16$	0	d_1	4	0
Vector	$(3\mathcal{A}; 4\chi; A_\mu, 3A)$	$8 + 8$	0	$-3d_0 + d_2$	3	3
2-form	$(2\mathcal{A}; 4\chi; A_{\mu\nu}, A_\mu, 3A)$	$8 + 8$	360	$3d_0$	3	3
Total f				$16(2d_0 + 2d_1 + d_2)$	256	128
Total A				0	0	0

TABLE XVI. The $D = 4$ multiplets in an $\mathcal{N} = 8$ basis.

$\mathcal{N} = 8$	Multiplet	f	$360A$	$\mathcal{N} = 8$
Graviton	$(g_{\mu\nu}, 7\mathcal{A}_\mu, 28\mathcal{A}; 8\psi_\mu, 56\chi; A_{\mu\nu\rho}, 7A_{\mu\nu}, 21A_\mu, 35A)$	256	0	1
Total f				256
Total A				0

$$A = -\tfrac{1}{24}\rho. \tag{32}$$

So the anomaly flips sign under generalized mirror symmetry and vanishes for generalized self-mirror theories.

In the case of ($\mathcal{N} = 1$, $D = 11$) on $X^6 \times S^1$, or equivalently (Type IIA, $D = 10$) on X^6,

$$A = -\tfrac{1}{24}\chi, \tag{33}$$

where χ is the Euler number of X^6.

Here we group the individual fields into supermultiplets as shown in Tables XIII, XIV, XV, and XVI.

C. Cosets

The $D = 4$ scalar cosets, before and after dualization are given in Tables XVII and XVIII

D. Fano plane and $\mathbb{O}$, $\mathbb{C}$, $\mathbb{H}$, $\mathbb{R}$

Next we turn to the black-hole/qubit correspondence [20–25]. The number of electric and magnetic black-hole charges of these $\mathcal{N} = 8, 4, 2, 1$ theories are 56, 24, 8, 0, respectively. These correspond to 7, 3, 1, 0 lines of the Fano plane of Fig. 1, which in turn admit an interpretation in terms of entangled qubits, as may be seen by writing them all in an $SL(2)^7$ basis:

TABLE XVII. $D = 4$ cosets before dualization.

Theory	Charges	G/H	Dim	Rank
$N = 8$	32	$SO(1,1) \times SL(7,R)/SO(7) \ltimes R^{35}$	63	7
$N = 4$	16	$SL(2)/SO(2) \times SO(6,3)/[SO(6) \times SO(3)] \times SL(3,R)/SO(3) \ltimes R^9$	35	7
$N = 2$	8	$SO(1,1) \times SL(2)^3/SO(2)^3 \times SO(4,3)/[SO(4) \times SO(3)] \ltimes R^2$	21	7
$N = 1$	4	$SL(2,R)^7/SO(2)^7$	14	7

TABLE XVIII. $D = 4$ cosets after dualization.

Theory	Charges	G/H	Dim	Rank
$N = 8$	32	$E_7/SU(8)$	70	7
$N = 4$	16	$SL(2)/SO(2) \times SO(6,6)/SO(6)^2$	38	7
$N = 2$	8	$SL(2)^3/SO(2)^3 \times SO(4,4)/SO(4)^2$	22	7
$N = 1$	4	$SL(2,R)^7/SO(2)^7$	14	7

(i) $\mathcal{N} = 8$

$$E_{7(7)} \supset SL(2)_A \times SL(2)_B \times SL(2)_C \times SL(2)_D \times SL(2)_E \times SL(2)_F \times SL(2)_G, \quad (34)$$

and the **56** decomposes as

$$\mathbf{56} \to (\mathbf{2,2,1,2,1,1,1}) + (\mathbf{1,2,2,1,2,1,1}) + (\mathbf{1,1,2,2,1,2,1}) + (\mathbf{1,1,1,2,2,1,2}) + (\mathbf{2,1,1,1,2,2,1}) + (\mathbf{1,2,1,1,1,2,2}) + (\mathbf{2,1,2,1,1,1,2}), \quad (35)$$

corresponding to the seven lines of the Fano plane describing a tripartite entanglement of seven qubits (Alice, Bob, Charlie, Daisy, Emma, Fred, and George):

$$|\psi\rangle_{56} = a_{ABD}|ABD\rangle + b_{BCE}|BCE\rangle + c_{CDF}|CDF\rangle + d_{DEG}|DEG\rangle + e_{EFA}|EFA\rangle + f_{FGB}|FGB\rangle + g_{GAC}|GAC\rangle \quad (36)$$

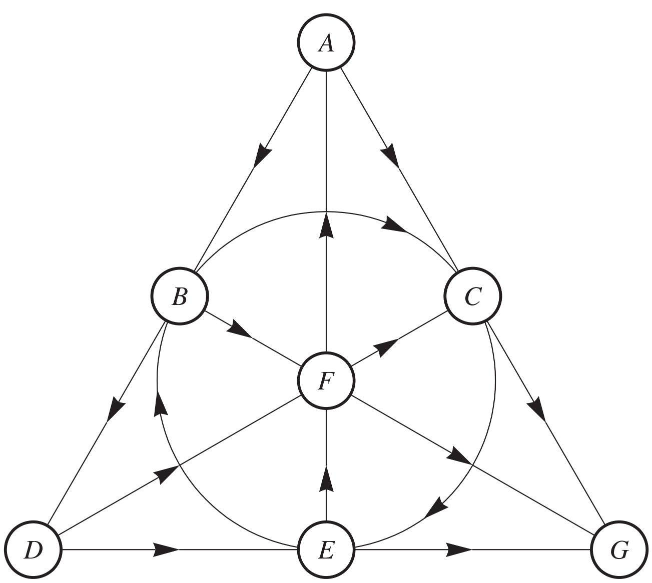

FIG. 1. The Fano plane has seven points and seven lines (the circle counts as a line) with three points on every line and three lines through every point. The points A, B, C, D, E, F, G represent the seven qubits and the seven lines; ABD, BCE, CDF, DEG, EFA, FGB, GAC represent the tripartite entanglement.

(ii) $\mathcal{N} = 4$

$$SL(2)_A \times SO(6,6) \supset SL(2)_A \times SL(2)_B \times SL(2)_C \times SL(2)_D \times SL(2)_E \times SL(2)_F \times SL(2)_G, \quad (37)$$

and the $(\mathbf{2}, \mathbf{12})$ decomposes as

$$(\mathbf{2,12}) \to (\mathbf{2,2,1,2,1,1,1}) + (\mathbf{2,1,1,1,2,2,1}) + (\mathbf{2,1,2,1,1,1,2}), \quad (38)$$

corresponding to the three lines of the Fano plane describing a tripartite entanglement of Alice with Bob and Daisy, Alice with Emma and Fred, Alice with Charlie and George:

$$|\psi\rangle_{24} = a_{ABD}|ABD\rangle + e_{EFA}|EFA\rangle + g_{GAC}|GAC\rangle \quad (39)$$

(iii) $\mathcal{N} = 2$

$$SL(2)_A \times SL(2)_B \times SL(2)_D \times SO(4,4) \supset SL(2)_A \times SL(2)_B \times SL(2)_C \times SL(2)_D \times SL(2)_E \times SL(2)_F \times SL(2)_G, \quad (40)$$

and the $(\mathbf{2,2,2,1})$ decomposes as

$$(\mathbf{2,2,2,1}) \to (\mathbf{2,2,1,2,1,1,1}), \quad (41)$$

corresponding to the one line of the Fano plane describing a tripartite entanglement of three qubits, Alice, Bob and Daisy:

$$|\psi\rangle_8 = a_{ABD}|ABD\rangle \quad (42)$$

TABLE XIX. Vanishing anomaly in $\mathbb{O}$, $\mathbb{H}$, $\mathbb{C}$ $\mathbb{R}$ theories.

Field	f	$360A$	$\mathbb{A}$	$\mathbb{O}$	$\mathbb{H}$	$\mathbb{C}$	$\mathbb{R}$
$g_{\mu\nu}$	2	848	1	1	1	1	1
$\mathcal{A}_\mu$	2	−52	$\mathcal{N} - 1$	7	3	1	0
$\vec{\phi}$	1	4	7	7	7	7	7
$\mathcal{A}$	1	4	$3(\mathcal{N} - 1)$	21	9	3	0
ψ_μ	2	−233	$\mathcal{N}$	8	4	2	1
χ	2	7	$7\mathcal{N}$	56	28	14	7
$A_{\mu\nu\rho}$	0	−720	1	1	1	1	1
$A_{\mu\nu}$	1	364	$\mathcal{N} - 1$	7	3	1	0
A_μ	2	−52	$3(\mathcal{N} - 1)$	21	9	3	0
A	1	4	$4\mathcal{N} + 3$	35	19	11	7
total f			$32\mathcal{N}$	256	128	64	32
total A			0	0	0	0	0

TABLE XX. The $D = 4$ multiplets in an $\mathcal{N} = 1$ basis from X^7 with $(b_0, b_1, b_2, b_3) = (1, \mathcal{N} - 1, 3\mathcal{N} - 3, 4\mathcal{N} + 3)$.

$\mathcal{N} = 1$	Multiplet	f	360A	$\mathbb{A}$	$\mathbb{O}$	$\mathbb{H}$	$\mathbb{C}$	$\mathbb{R}$
Graviton	$(g_{\mu\nu}; \psi_\mu; A_{\mu\nu\rho})$	2 + 2	−105	1	1	1	1	1
Gravitino	$(\mathcal{A}_\mu; \psi_\mu)$	2 + 2	−285	$\mathcal{N} - 1$	7	3	1	0
Vector	$(\chi; A_\mu)$	2 + 2	−45	$3(\mathcal{N} - 1)$	21	9	3	0
WZ_ϕ	$(\vec{\phi}; \chi; A)$	2 + 2	15	7	7	7	7	7
$WZ_{\mathcal{A}}$	$(\mathcal{A}; \chi; A)$	2 + 2	15	$3(\mathcal{N} - 1)$	21	9	3	0
Linear	$(\chi; A_{\mu\nu}, A)$	2 + 2	375	$\mathcal{N} - 1$	7	3	1	0
Total f				$32\mathcal{N}$	256	128	64	32
Total A				0	0	0	0	0

(iv) $\mathcal{N} = 1$

$$SL(2)_A \times SL(2)_B \times SL(2)_C \times SL(2)_D \times SL(2)_E \times SL(2)_F \times SL(2)_G \supset SL(2)_A \times SL(2)_B \times SL(2)_C \times SL(2)_D \times SL(2)_E \times SL(2)_F \times SL(2)_G, \quad (43)$$

$$(\mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{0}) \rightarrow (\mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{0}), \quad (44)$$

corresponding to no lines.

The black-hole entropies are given by the qubit entanglement measures which are quartic polynomials in the a, b, c, d, e, f, g coefficients, namely, Cartan's E_7 invariant, the analogous $SL(2) \times SO(6, 6)$ invariant, and Cayley's $SL(2)^3$ hyperdeterminant.

Since the 7, 3, 1, 0 lines of the Fano plane also describe the multiplication table of the octonions, quaternions, complex and real, it was conjectured in [25] that there is an $\mathbb{O}$, $\mathbb{H}$, $\mathbb{C}$, $\mathbb{R}$ interpretation not just for the charges but also for the entire theories. Their field content and trace anomalies are given in Tables XIX and XX.

IV. $\mathcal{N} = 2, 4, 8, 16$ IN $D = 3$

A. Compactifications

Consider Type IIA in $D = 10$. In the Neveu-Schwarz sector we have the fields $(g_{MN}, \Phi; \psi_M, \chi; A_{MN})$ with $f = 64 + 64$; in the Ramond-Ramond sector we have the fields $(\mathcal{A}_M; \psi_M, \chi; A_{MNP})$ also with $f = 64 + 64$. We compactify on generic X^7 with independent Betti numbers (b_0, b_1, b_2, b_3), $X^6 \times S^1$ with independent X^6 Betti numbers (c_0, c_1, c_2, c_3), $X^4 \times S^3$ with independent X^4 Betti numbers (d_0, d_1, d_2) and on T^7 with (1, 7, 21, 35). The results for Neveu-Schwarz and Ramond-Ramond combined are shown in Table XXI. In Table XXII, we group into $\mathcal{N} = 2$ multiplets.

TABLE XXI. X^7, $X^6 \times S^1$, $X^4 \times T^3$, T^7 compactification of Type IIA.

	Field	f	X^7	$X^6 \times S^1$	$X^4 \times T^3$	T^7
g_{MN}	$g_{\mu\nu}$	0	b_0	c_0	d_0	1
	$\mathcal{A}_\mu$	1	$b_0 + b_1$	$2c_0 + c_1$	$4d_0 + d_1$	8
	$\mathcal{A}$	1	b_3	$c_2 + c_3$	$d_0 + 4d_1 + 3d_2$	35
Φ	Φ	1	b_0	c_0	d_0	1
ψ_M	ψ_μ	0	$2b_0 + 2b_1$	$4c_0 + 2c_1$	$8d_0 + 2d_1$	16
	χ	1	$2b_0 + 2b_1 + 2b_2 + 2b_3$	$4c_0 + 4c_1 + 4c_2 + 2c_3$	$16d_0 + 16d_1 + 8d_2$	128
A_{MNP}	$A_{\mu\nu\rho}$	0	b_0	c_0	d_0	1
	$A_{\mu\nu}$	0	$b_0 + b_1$	$2c_0 + c_1$	$3d_0 + d_1$	8
	A_μ	1	$b_1 + b_2$	$c_0 + 2c_1 + c_2$	$6d_0 + 4d_1 + d_2$	28
	A	1	$b_2 + b_3$	$c_1 + 2c_2 + c_3$	$4d_0 + 7d_1 + 4d_2$	56
Total f			$4(b_0 + b_1 + b_2 + b_3)$	$4(2c_0 + 2c_1 + 2c_2 + c_3)$	$16(2d_0 + 2d_1 + d_2)$	256

TABLE XXII. The $D = 3$ multiplets in an $\mathcal{N} = 2$ basis.

$\mathcal{N} = 2$ multiplet	Content	f	$\mathcal{N} = 2b_0 + 2b_1$	$\mathcal{N} = 16$	$\mathcal{N} = 8$	$\mathcal{N} = 4$	$\mathcal{N} = 2$
Graviton	$(g_{\mu\nu}, \mathcal{A}_\mu, \mathcal{A}; 2\psi_\mu, 2\chi; A_{\mu\nu\rho}, A_{\mu\nu})$	2 + 2	b_0	1	1	1	1
Gravitino	$(\mathcal{A}_\mu, \mathcal{A}; 2\psi_\mu, 2\chi)$	2 + 2	b_1	7	3	1	0
Vector	$(2\chi; A_\mu; A)$	2 + 2	b_2	21	9	3	0
Hyper	$(\mathcal{A}; 2\chi; A)$	2 + 2	$-b_1 + b_3$	28	16	10	7
Linear	$(2\chi; A_{\mu\nu}, A_\mu, A)$	2 + 2	b_1	7	3	1	0
Total f			$4(b_0 + b_1 + b_2 + b_3)$	256	128	64	32

TABLE XXIII. $D = 3$ cosets after dualization.

Theory	G/H	Dim	Rank
$\mathcal{N} = 16$	$E_8/SO(16)$	128	8
$\mathcal{N} = 8$	$SO(8,8)/SO(8)^2$	64	8
$\mathcal{N} = 4$	$SO(4,4)^2/SO(4)^4$	32	8
$\mathcal{N} = 2$	$SL(2,R)^8/SO(2)^8$	16	8

TABLE XXIV. $D = 3$ cosets from timelike reduction.

Theory	G/H	Dim	Rank
$\mathcal{N} = 16$	$E_8/SO^*(16)$	128	8
$\mathcal{N} = 8$	$SO(8,8)/SO(4,4)^2$	64	8
$\mathcal{N} = 4$	$SO(4,4)^2/SO(2,2)^4$	32	8
$\mathcal{N} = 2$	$SL(2,R)^8/SO(1,1)^8$	16	8

B. Cosets

The $D = 3$ scalar cosets after dualization for spacelike and timelike reductions are given in Tables XXIII and XXIV.

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