Mass varying neutrinos, quintessence, and the accelerating expansion of the Universe

Gennady Y. Chitov,[1] Tyler August,[1] Aravind Natarajan,[2] and Tina Kahniashvili[2,1,3]

[1]*Department of Physics, Laurentian University, Ramsey Lake Road, Sudbury, ON, P3E 2C6, Canada*

[2]*McWilliams Center for Cosmology and Department of Physics, Carnegie Mellon University, 5000 Forbes Ave, Pittsburgh, Pennsylvania 15213, USA*

[3]*National Abastumani Astrophysical Observatory, Ilia Chavchavadze State University, 2A Kazbegi Ave, Tbilisi, GE-0160, Georgia*

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We analyze the mass varying neutrino scenario. We consider a minimal model of massless Dirac fermions coupled to a scalar field, mainly in the framework of finite-temperature quantum field theory. We demonstrate that the mass equation we find has nontrivial solutions only for special classes of potentials, and only within certain temperature intervals. We give most of our results for the Ratra-Peebles dark energy (DE) potential. The thermal (temporal) evolution of the model is analyzed. Following the time arrow, the stable, metastable, and unstable phases are predicted. The model predicts that the present Universe is below its critical temperature and accelerates. At the critical point, the Universe undergoes a first-order phase transition from the (meta)stable oscillatory regime to the unstable rolling regime of the DE field. This conclusion agrees with the original idea of quintessence as a force making the Universe roll towards its true vacuum with a zero Λ term. The present mass varying neutrino scenario is free from the coincidence problem, since both the DE density and the neutrino mass are determined by the scale M of the potential. Choosing $M \sim 10^{-3}$ eV to match the present DE density, we can obtain the present neutrino mass in the range $m \sim 10^{-2}$–1 eV and consistent estimates for other parameters of the Universe.

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I. INTRODUCTION

Neutrino mass related questions are of great interest for particle physics as well as for cosmology (for reviews, see Ref. [1], and references therein). Current upper limits on the sum of neutrino masses from cosmological observations are of the order of 1 eV [2–4], while neutrino oscillations give a lower bound of roughly 0.01 eV [5,6], making neutrino mass an established element of particle physics. Furthermore, understanding the origin of neutrino mass opens a window into understanding physical processes beyond the standard model of particle physics [7–10].

It is now well established that about 74% of the Universe is comprised of dark energy (DE) (for reviews, see Ref. [11], and citations therein). The present stage of evolution of the Universe is governed by this dominant DE contribution, and the Universe experiences an accelerating expansion [12,13]. The nature of DE is still unknown, and it is one of the major questions of modern cosmology. There are, broadly speaking, three major possibilities proposed to explain the DE [11]. Most straightforwardly, and in good agreement with the current observational data, it can be present just as the cosmological constant [11]. Second, the DE can be accommodated in some framework of the modified non-Einsteinian gravity theories (see, e.g., Refs. [14,15]). And last, following the original proposals [16,17] on the DE originating from a scalar field action similar to the inflaton field, there has been a lot of activity in constructing and analyzing various trial scalar field Lagrangians to model the DE [13]. Note, that it is even unclear what kind of scalar field potential governs the inflationary expansion of the Universe [18], and as the result, the effective quantum field that adequately describes inflation is still under debate [19]. A similar observation can be drawn from analyzing many potentials proposed for the DE action [13].

On the other hand, several cosmological and astrophysical observations imply that about 22% of the Universe consists of dark matter (DM) [11], if we admit the general relativity theory of gravity. Most likely, DM is formed through massive weakly interacting particles (WIMPs), and the nature of these particles is also still unknown. There are several recent observations performed by PAMELA [20] and GLAST missions which indicate DM particle annihilations [21]. Recently, it was proposed that both of these observations could be used to test baryogenesis [22] which is one of the important problems of the standard particle physics model.

Another puzzling question in modern cosmology is the coincidence problem—the density of DE is comparable to the present energy density of DM. In turn, the latter is comparable (within the order of magnitude), to the energy density of cosmological neutrinos [1,2]. Is there a mechanism explaining this coincidence? A very convincing answer to this question is given by the mechanism of DM mass generation via various types of DM-DE couplings, ranging from Yukawa to more exotic ones [23–28]. The mass of the DM particle in this approach is naturally time-dependent, and they were coined varying mass particles (VAMPs). Various DE-DM interaction models have been constrained by observations of supernovae type Ia [29], the age of the Universe [30–32], cosmic microwave background anisotropies [33,34], and large scale structure formation [35].

Fardon, Nelson, and Weiner elaborated on the VAMP mechanism in the context of neutrinos [36].[1] In their model the relic neutrinos, i.e., fermionic field(s), interact with a scalar field via the Yukawa coupling. If the decoupled neutrino field is initially massless, then the coupling generates a (varying) mass of neutrinos in this DE-neutrinos model. This mass varying neutrino (MaVaN) scenario is quite compelling, since it connects the origin of neutrino mass to the DE, and solves the additional coincidence problem of why the neutrino mass and DE are of comparable scales [38]. (For more on the coincidence, see, e.g. [39]). To consider neutrinos as particles that get their mass through the coupling is attractive for particle physics, as well as for its cosmological consequences. However, there are significant issues that have to be resolved for the sake of viability of the MaVaN scenario. Most notably, it has been shown [40] that the model of Ref. [36] suffers from a strong instability due to the negative sound speed squared of the DE-neutrino fluid (see also [41]).

Any DM-DE coupling induces observable changes in large scale structure formation [42]. The main reason for this is due to the presence of additional DM contributions (perturbations) in the equation of motion which determines the dynamics of the scalar field. The changes in the dynamics are drastic when massive neutrinos are coupled to DE [40]. In this case the squared sound speed of the DE-neutrino fluid defined as $c_s^2 = \delta P/\delta\rho$ (where δ represents the variation, and P and ρ are pressure and energy density of the DE-neutrino fluid) is negative. The negative squared sound speed results in an exponential growth of scalar perturbations [43–46].

After the critique in Ref. [40], the issue of stability of the DE-neutrinos fluid has been addressed by many authors [41,46–53]. Various physical assumptions were made in those references in order to avoid the exponential clustering of neutrinos. In particular, to achieve stability, proposals were put forward to make the DE-DM model more complicated, e.g., by extending it to a multicomponent scalar field, or by promoting its supersymmetry [49,51]. We however are not inclined to pursue this line of thought and will explore the simplest possible "minimal" model. As we will demonstrate, the occurrence of the instability in the coupled DE-neutrinos model is meaningful, and we will explore the physical implications of this phenomenon. Note that Wetterich and coworkers [46] have already analyzed various implications of the instability in the MaVaN model on the dynamics of neutrino clustering.

In this paper we again address the analysis of the DE-neutrinos coupled model. What is really new in our results, to the best of our knowledge, apart from a consistent equation for the equilibrium condition, is the analysis of the thermal (i.e. temporal) evolution of the MaVaN model and prediction of its stable, metastable, and unstable phases. The analysis of the dynamics in the unstable phase results in, for the first time in the framework of the MaVaN scenario, a picture of the present-time Universe totally consistent with observations. Our findings are in line with the original proposal [16,17] of the DE potential (quintessence) to model the Universe slowly rolling towards its true vacuum ($\Lambda = 0$). As it turns out, the present Universe, seen as a system of the coupled DE (quintessence) field and fermions (neutrinos) is below its critical temperature. It is similar to a supercooled liquid which has not crystallized yet: its high-temperature (meta)stable phase became unstable, but the new low-temperature stable phase ($\Lambda = 0$) is still to be reached. The Afshordi-Zaldarriaga-Kohri instability corresponding to $c_S^2 < 0$ is just telling us this.

The rest of the paper is organized as follows: In Sec. II we give the outlook of the model and formalism applied and derive the basic equations for the coupled model. In Sec. III we present the qualitative analysis of the equation which yields the fermionic (neutrino) mass. Section IV contains analysis of the coupled model with the Ratra-Peebles DE potential at equilibrium. The dynamics of the model applied to the whole universe is studied in Sec. V. The results are summarized in the concluding Sec. VI.

II. MODEL AND FORMALISM. BASIC EQUATIONS

A. Outlook

In this paper we focus on the case when the scalar field potential $U(\varphi)$ does not have a nontrivial minimum, and the generation of the fermion mass is due to the breaking of chiral symmetry in the Dirac sector of the Lagrangian. A nontrivial solution of the fermionic mass equation is a result of the interplay between the scalar and fermionic contributions. We consider the most natural and intuitively plausible Yukawa coupling between the Dirac and the scalar fields.

The key assumption is that the fermionic mass generation can be obtained from minimization of the thermodynamic potential. That is, the coupled system of the scalar bosonic and fermionic fields is at equilibrium, at least at some temperatures. This will be analyzed below more specifically. We assume the cosmological evolution, governed by the scale factor $a(t)$ to be slow enough that the coupled system is at equilibrium at a given temperature $T(a)$. Then the methods of thermal quantum field theory [54,55] can be applied.

This problem is rather well studied with quantum field theory and statistical physics in different contexts [54–56]. The major conceptual difficulty in applying quantum field-theoretical methods for the dark-energy scalar field is the lack of "well-behaved" potentials interesting for cosmological applications. For instance, a class of the

[1] The DE-neutrino coupling and the baryogenesis constraints have been studied also in Ref. [37].

very popular inverse power law slow-rolling quintessence potentials [13] are singular at the origin. Consequently, the field theory should be understood as a sort of effective theory, and we plan to address this issue more deeply in our future work.

As far as the fermionic sector of the theory is concerned, one needs to distinguish two different cases pertinent for neutrino applications:

(i) an equal number of fermions and antifermions, i.e., zero chemical potential $\mu = 0$;

(ii) a surplus of particles over antiparticles, and small nonzero chemical potential.

For the bounds on the neutrino chemical potential, see Refs. [1,57]. If experiments confirm neutrinoless double beta decay, i.e., that neutrinos are Majorana fermions, then the lepton number is not conserved [8], and one cannot introduce a (nonzero) chemical potential. Then case (i) above is applicable, proviso that the Majorana fields are utilized instead of the Dirac ones. For the case (i) with Dirac fermions, the ground state corresponds to a complete annihilation of fermion-antifermion pairs, i.e. the fermions completely vanish in the zero-temperature limit.

Assumption of the fermion-antifermion asymmetry and (conserving) particle surplus, i.e., of a nonzero chemical potential, results in the fermionic contributions which survive the zero-temperature limit. However, the smallness of the zero-temperature contribution renders this issue rather academic. Indeed, for the neutrinos we are interested in this study, by assuming the *maximal* particle surplus $n_\circ \sim 115\ \mathrm{cm}^{-3}$, one gets the Fermi momentum $k_F \sim 3 \cdot 10^{-4}$ eV. For $m \sim 10^{-2}$ eV, one obtains $\mu(T=0) = \varepsilon_F = \sqrt{k_F^2 + m^2} = m + \mathcal{O}(10^{-4}\ \mathrm{eV})$. This results in a nontrivial vacuum with the particle surplus frozen within an extremely narrow Fermi shell $m \le \varepsilon \le \varepsilon_F$. Thus, trying to grasp the essential physics in this study from possibly the simplest "minimal model," we assume the fermions to be described by a Dirac spinor field with zero chemical potential.

In this work we will use the standard methods of general relativity and finite-temperature quantum field theory extended for fields living in a spatially flat universe with the Friedmann-Lemaître-Robertson-Walker (FLRW) metric where the line element is $ds^2 = dt^2 - a^2(t)d\mathbf{x}^2$. Here t is the physical time and $a(t)$ is the scale factor, which can be obtained from the Friedmann equations [9,10]

$$H^2(t) = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_{\mathrm{tot}}, \tag{1}$$

$$\dot{H}(t) + H^2(t) = \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_{\mathrm{tot}} + 3P_{\mathrm{tot}}). \tag{2}$$

Equations (1) and (2) also lead to the continuity equation

$$\dot{\rho}_{\mathrm{tot}} + \frac{3\dot{a}}{a}(\rho_{\mathrm{tot}} + P_{\mathrm{tot}}) = 0. \tag{3}$$

Here the dot represents the physical time derivative, and ρ_{tot} and P_{tot} are the total energy density and pressure of the Universe. In accordance with the (standard) ΛCDM model, the Universe is assumed to consist of (1) DE, (2) cold DM (CDM) made of weakly interacting massive particles, presumably $M_{\mathrm{DM}} > 1 \sim 10$ GeV, (3) photons, and (4) baryons. The DM and baryon density parameters today are $\Omega_{\mathrm{DM}} = \rho_{\mathrm{DM}}(t_{\mathrm{now}})/\rho_{\mathrm{cr}} \approx 0.22$ and $\Omega_b = \rho_b(t_{\mathrm{now}})/\rho_{\mathrm{cr}} \approx 0.04$. Here $\rho_{\mathrm{cr}} = 3H_0^2/(8\pi G) = 8.1h^2 \times 10^{-47}\ \mathrm{GeV}^4$ is the critical density today, t_{now} defines the current time, $H_0 = 2.1h \times 10^{-42}$ GeV is the present Hubble parameter, G is the Newton constant, and $h \approx 0.72$ is the Hubble parameter in units of 100 km/ sec /Mpc. The photon contribution to the energy density today can be neglected. The flatness of the Universe leads to the relative energy density of the DE-neutrino coupled fluid $\Omega_{\varphi\nu} \approx 0.74$. To ensure the accelerated expansion of the Universe today, the right-hand side (r.h.s.) of Eq. (2) must be positive at $t = t_{\mathrm{now}}$.

In this paper we will not assume the existence of the cosmological constant Λ, as the ΛCDM model suggests. Instead, we accept the hypothesis of the dynamical dark energy described by a scalar field. This is a bold assumption and a highly debatable issue. We vindicate our approach *a posteriori* by the consistent picture we arrive at the end. For a review and/or alternative approaches, see, e.g., Refs. [13,58,59]. The massless neutrinos are described by the conventional Dirac Lagrangian. The resulting model is given by the coupled Dirac and scalar fields. The grand thermodynamic potential of the coupled model can be derived from the Euclidian functional integral representation of the grand partition function. The dynamics of the coupled model is governed by the Friedmann equations.

Throughout the paper, we use natural units where $\hbar = c = k_B = 1$.

B. Bosonic scalar field

The bosonic scalar field Hamiltonian in the FLRW metric reads as [9,60]

$$H_B = \int a^3 d^3x \left[\frac{1}{2}\dot{\varphi}^2 + \frac{1}{2a^2}(\nabla\varphi)^2 + U(\varphi)\right], \tag{4}$$

where the comoving volume $V = \int d^3x$, while the physical volume $V_{\mathrm{phys}} = a^3(t)V$. Since this field does not carry a conserved charge (number), the chemical potential $\mu = 0$. The grand partition function in the functional integral representation:

$$\mathcal{Z}_B \equiv \mathrm{Tr} e^{-\beta\hat{H}} = \int \mathcal{D}\varphi e^{-S_B^E} \tag{5}$$

with the bosonic Euclidian action

$$S_B^E = \int_0^\beta d\tau \int a(t)^3 d^3x \left[\frac{1}{2}(\partial_\tau\varphi)^2 + \frac{1}{2a^2}(\nabla\varphi)^2 + U(\varphi)\right], \tag{6}$$

where $\varphi = \varphi(\mathbf{x}, \tau)$.

It is instructive to find the partition function of the free scalar field $U(\varphi) = \frac{1}{2} M_b^2 \varphi^2$ following the methods explained by Kapusta and Gale [54] for the case of the Minkowski metric. Rescaling of the field

$$\tilde{\varphi} = a^{3/2} \varphi \tag{7}$$

changes the partition function (5) by a thermodynamically irrelevant prefactor. The functional integration over $\tilde{\varphi}$ of the Gaussian action gives

$$\log \mathcal{Z}_B = -V \int \frac{d^3k}{(2\pi)^3} \Big[\beta \sqrt{M_b^2 + k^2/a^2} + \log(1 - e^{-\beta \sqrt{M_b^2 + k^2/a^2}}) \Big]. \tag{8}$$

Then the density (with respect to the physical volume) of the thermodynamic potential is given by

$$\Omega_B \equiv -\frac{1}{\beta a^3 V} \log \mathcal{Z}_B = -P_B = \int \frac{d^3k}{(2\pi)^3} \Big[\varepsilon + \frac{1}{\beta} \log(1 - e^{-\beta \varepsilon}) \Big], \tag{9}$$

where $\varepsilon = \sqrt{M_b^2 + k^2}$ and P_B is the pressure due to the bosonic field.

C. Free dirac spinor field

The Dirac Hamiltonian in the FLRW metric is [60]

$$H_D = \int a^3 d^3x \bar{\psi} \Big(-\frac{\imath}{a} \boldsymbol{\gamma} \cdot \boldsymbol{\nabla} + m \Big) \psi. \tag{10}$$

The grand partition function is given by the following Grassmann functional integral:

$$\mathcal{Z}_D \equiv \mathrm{Tr} e^{-\beta(\hat{H} - \mu \hat{Q})} = \int \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{-S_D^E}, \tag{11}$$

where the conserved charge (lepton number) operator $\hat{Q} = \int a^3 d^3x \psi^\dagger \psi$ and the Euclidian action

$$S_D^E = \int_0^\beta d\tau \int a(t)^3 d^3x \bar{\psi}(\mathbf{x}, \tau) \times \Big(\gamma^o \frac{\partial}{\partial \tau} - \frac{\imath}{a} \boldsymbol{\gamma} \cdot \boldsymbol{\nabla} + m - \mu \gamma^o \Big) \psi(\mathbf{x}, \tau). \tag{12}$$

By rescaling the Grassmann fields (7) and using the standard techniques [54], we get the thermodynamic potential density (pressure) as a function of the chemical potential and temperature:

$$\Omega_D \equiv -\frac{1}{\beta a^3 V} \log \mathcal{Z}_D = -P_D = -2 \int \frac{d^3k}{(2\pi)^3} \Big[\varepsilon + \frac{1}{\beta} \log(1 + e^{-\beta \varepsilon_-}) + \frac{1}{\beta} \log(1 + e^{-\beta \varepsilon_+}) \Big], \tag{13}$$

where

$$\varepsilon(k) = \sqrt{m^2 + k^2}, \tag{14}$$

and $\varepsilon_\pm = \varepsilon(k) \pm \mu$. The first term on the r.h.s. of Eq. (13) corresponds to the vacuum contribution to the thermodynamic potential (pressure):

$$-\Omega_0 = P_0 = 2 \int \frac{d^3k}{(2\pi)^3} \varepsilon(k). \tag{15}$$

Introducing the notation for the Fermi distribution function

$$n_F(x) \equiv \frac{1}{e^{\beta x} + 1}, \tag{16}$$

Equation (13) can be brought to the following form:

$$-\Omega_D = P_D = P_0 + \frac{1}{3\pi^2} \int_0^\infty \frac{k^4 dk}{\varepsilon(k)} [n_F(\varepsilon_-) + n_F(\varepsilon_+)]. \tag{17}$$

D. Coupled model: Scalar field and dirac massless fermions

Let us consider a scalar bosonic field interacting via a Yukawa coupling with massless Dirac fermions. The Euclidian action of the model in the FLRW metric reads

$$\mathcal{S} = S_B^E + S_D^E|_{m=0} + g \int_0^\beta d\tau \int a^3 d^3x \varphi \bar{\psi} \psi. \tag{18}$$

The path integral for the partition function of the coupled model is

$$\mathcal{Z} = \int \mathcal{D}\varphi \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{-\mathcal{S}}. \tag{19}$$

The Grassmann fields can be formally integrated out resulting in

$$\mathcal{Z} = \int \mathcal{D}\varphi e^{-\mathcal{S}(\varphi)} = \int \mathcal{D}\varphi \exp[-S_B^E + \log\mathrm{Det}\hat{D}(\varphi)], \tag{20}$$

where the Dirac operator is

$$\hat{D}(\varphi) = \gamma^o \frac{\partial}{\partial \tau} - \frac{\imath}{a} \boldsymbol{\gamma} \cdot \boldsymbol{\nabla} + g\varphi(\mathbf{x}, \tau) - \mu \gamma^o. \tag{21}$$

The thermodynamic potential Ω of the model (18) at tree level can be found by evaluating the path integral (20) in the saddle-point approximation. Assuming the existence of a constant $(\mathbf{x}, \tau)$-independent field ϕ_c which minimizes the action $\mathcal{S}(\varphi)$, the term $\log\mathrm{Det}\hat{D}$ can be evaluated exactly, and fermionic contribution to the thermodynamic potential is given by Eq. (13) or Eq. (17) with the fermionic mass

$$m = g\phi_c. \tag{22}$$

The bosonic contribution to the partition function in this approximation is simply $\mathcal{Z} \propto \exp[-\beta a^3 V U(\phi_c)]$. The thermodynamic potential density is given then by

$$\Omega(\phi_c) = U(\phi_c) + \Omega_D(\phi_c). \tag{23}$$

Self-consistency of the employed saddle-point approximation naturally coincides with the condition of minimum of the thermodynamic potential at equilibrium (at fixed temperature and chemical potential):

$$\left.\frac{\partial\Omega(\varphi)}{\partial\varphi}\right|_{\varphi=\phi_c}=0, \tag{24}$$

and

$$\left.\frac{\partial^2\Omega(\varphi)}{\partial\varphi^2}\right|_{\varphi=\phi_c}>0. \tag{25}$$

Note that a nontrivial solution ϕ_c of Eq. (24) (if it exists) is called the classical field: it is the average of the bosonic field, i.e., $\phi_c=\langle\varphi\rangle$. Equations (22)–(24) can be brought to the equivalent form:

$$U'(\phi_c)+g\rho_s=0, \tag{26}$$

where the scalar fermionic density (a.k.a. the chiral density) ρ_s is given by the following expression:

$$\begin{aligned}\rho_s &\equiv \frac{\langle\hat N\rangle}{V}=\frac{\partial\Omega_D}{\partial m}\\ &=\rho_0+\frac{m}{\pi^2}\int_0^\infty\frac{k^2dk}{\varepsilon(k)}[n_F(\varepsilon_-)+n_F(\varepsilon_+)],\end{aligned} \tag{27}$$

and $\hat N=\int d^3x\bar\psi\psi$. Here ρ_0 stands for the vacuum contribution to the chiral condensate:

$$\rho_0\equiv\frac{\partial\Omega_0}{\partial m}=-\frac{m}{\pi^2}\int_0^\infty\frac{k^2dk}{\varepsilon(k)}. \tag{28}$$

Note that even if the time, i.e., $a(t)$, does not enter *explicitly* in the equations for the thermodynamic quantities of the coupled, fermionic or bosonic models (9), (13), (23), (26), and (27), and they look like their counterparts in a flat static universe, such parameters as, e.g., the temperature and chemical potential in those equations are time-dependent, i.e., $T=T(a)$ and $\mu=\mu(a)$. The particular form of the dependencies $T(a)$ and $\mu(a)$ must be determined from the Friedmann continuity Eq. (3) which relates the energy density $\rho(T)$ and pressure $P(T)$ to the evolution of $a(t)$ [9,10]. In addition, the fermionic mass $m\propto\phi_c$ in the coupled model is also time varying, since the time enters into ϕ_c (26) via T, μ, and all three functions $m(a)$, $T(a)$, and $\mu(a)$ are governed by the Friedmann Eqs. (1)–(3).

The present theory works consistently for the physical quantities (bosonic or fermionic) measured with respect to their vacuum contributions. So, in the rest of the paper we will employ the thermodynamic quantities with subtracted vacuum contributions, keeping however, the same notations, e.g.:

$$\begin{gathered}\Omega_D\mapsto\Omega_D-\Omega_0,\qquad P_D\mapsto P_D-P_0,\\ \rho_s\mapsto\rho_s-\rho_0.\end{gathered} \tag{29}$$

Then, according to Volovik [61], the pressure and energy of the pure and equilibrium vacuum is exactly zero. (The renormalization of the vacuum terms is, of course, a very subtle issue. There are alternative approaches to this problem known from the literature. See, e.g., [62,63].)

III. ANALYSIS OF THE MASS (GAP) EQUATION: GENERAL PROPERTIES

In cases interesting for cosmological applications, the scalar field potential $U(\varphi)$ does not have a nontrivial minimum, and the generation of the fermion mass [i.e. a solution of (24) $0<\phi_c<\infty$] is due to the interplay between the scalar and fermionic contributions to the total thermodynamic potential (23).

From now on, we adapt our equations for the case of equal number of fermions and antifermions and $\mu=0$, as discussed in Sec. II A. Keeping in mind the neutrinos, we assume an extra flavor index of fermions with the number of flavors $\mathfrak{s}$. (For neutrinos, $\mathfrak{s}=3$.) We also assume the flavor degeneracy of the fermionic sector.

Before proceeding further, we need to make some important observations regarding the behavior of the coupled model in two limiting cases. Assuming that a nontrivial solution of (24) with finite m exists, the fermionic contribution to the thermodynamic potential (pressure) (17) can be written as

$$-\Omega_D=P_D=\frac{2\mathfrak{s}}{3\pi^2\beta^4}\mathcal{I}_p(\beta m),\qquad \mu=0, \tag{30}$$

where the integral defined as

$$\mathcal{I}_p(\kappa)\equiv\int_\kappa^\infty\frac{(z^2-\kappa^2)^{3/2}}{e^z+1}dz \tag{31}$$

can be evaluated analytically in two cases:

$$\mathcal{I}_p(\kappa)=\begin{cases}\frac{7\pi^4}{120}-\frac{\pi^2}{8}\kappa^2+\mathcal{O}(\kappa^4), & \kappa<1\\ 3\kappa^2K_2(\kappa)+\mathcal{O}(e^{-2\kappa}), & \kappa\gtrsim 1\end{cases}, \tag{32}$$

where $K_\nu(x)$ is the modified Bessel function of the second kind.

In the (classical) low-temperature regime

$$\beta m\equiv\frac{m}{T}\gg 1, \tag{33}$$

the above equation results in

$$-\Omega_D=P_D=\frac{2\mathfrak{s}m^2}{\pi^2\beta^2}K_2(\beta m)+\mathcal{O}(e^{-2\beta m}). \tag{34}$$

To leading order,

$$-\Omega_D=P_D\approx\frac{\sqrt{2}\mathfrak{s}}{\pi^{3/2}}T(Tm)^{3/2}e^{-m/T}. \tag{35}$$

The chiral condensate density (27)

$$\rho_s=\frac{2\mathfrak{s}m}{\pi^2\beta^2}\int_{\beta m}^\infty\frac{(z^2-(\beta m)^2)^{1/2}}{e^z+1}dz,\qquad \mu=0 \tag{36}$$

can be also evaluated in the low-temperature limit as

$$\rho_s = \frac{2\mathfrak{s}m^2}{\pi^2\beta}K_1(\beta m) + \mathcal{O}(e^{-2\beta m}), \tag{37}$$

which gives to leading order

$$\rho_s \approx \frac{\sqrt{2}\mathfrak{s}}{\pi^{3/2}}(Tm)^{3/2}e^{-m/T}. \tag{38}$$

In this limit the fermions enter the regime of *a classical ideal gas.* Indeed, the fermionic particle (antiparticle) density

$$n_+ = n_- = \frac{\mathfrak{s}}{\pi^2\beta^3}\int_{\beta m}^{\infty}\frac{z(z^2-(\beta m)^2)^{1/2}}{e^z+1}dz \tag{39}$$

in the low-temperature limit yields

$$n_\pm = \frac{\mathfrak{s}m^2}{\pi^2\beta}K_2(\beta m) + \mathcal{O}(e^{-2\beta m}), \tag{40}$$

and to leading order:

$$n_\pm \approx \frac{\mathfrak{s}}{\sqrt{2}\pi^{3/2}}(Tm)^{3/2}e^{-m/T}. \tag{41}$$

We see from Eqs. (34) and (40) that up to terms $\mathcal{O}(e^{-2\beta m})$, the fermions satisfy the ideal gas equation of state

$$P_D \approx (n_+ + n_-)T, \tag{42}$$

and the chiral density is equal to the total particle density n:

$$\rho_s \approx n \equiv n_+ + n_-. \tag{43}$$

In the (ultrarelativistic) high-temperature regime

$$\frac{m}{T} \ll 1, \tag{44}$$

one obtains

$$-\Omega_D = P_D \approx \frac{7\pi^2\mathfrak{s}}{180}T^4 - \frac{\mathfrak{s}}{12}(mT)^2. \tag{45}$$

To leading order, the chiral condensate is

$$\rho_s \approx \frac{\mathfrak{s}}{6}mT^2, \tag{46}$$

while the particle density is

$$n_\pm \approx \frac{3\mathfrak{s}\zeta(3)}{2\pi^2}T^3. \tag{47}$$

Now we can make some general observations of the fermionic mass generation in the coupled model:

(i) It is obvious from the sign of ρ_s [cf. (27) and (36)] that *nontrivial solutions of* (26) *are impossible for a monotonically increasing potential* $U(\varphi)$. That rules out some popular potentials, e.g., $U \propto \log(1 + \varphi/M)$ [13,36] for this Yukawa-coupling driven scenario of the mass generation.

(ii) The monotonously decreasing slow-rolling DE potentials ([16,17] and for reviews, see [11,13]), e.g., $U \propto \varphi^{-\alpha}$ or $U \propto \exp[-A\varphi^\gamma]$, do have *a window of parameters* wherein nontrivial solutions of (26) exist. As we can see from (38), for those decreasing potentials the mass equation (26) always has a trivial solution $m = g\phi_c = \infty$ for the minimum of the thermodynamic potential (23).[2] This solution corresponds to a "doomsday" vacuum state [61], when the Universe reached its true ground state with zero dark-energy density and completely frozen out fermions. A nontrivial solution of (26), corresponding to another minimum of the potential (23), is totally due to the fermionic contribution. Since the latter freezes out in the limit $T \to 0$, it is clear qualitatively that such a solution $0 < m < \infty$ can exist only above a certain temperature. For a more quantitative account of these phenomena, we need to assume some specific form of the DE potential. This will be done in the following section.

(iii) To explain the differences between the present study and earlier related work on mass varying fermions (see [23,24,36,40], and more references therein), some clarifications are warranted. It is usually assumed in the literature that the low-temperature regime formulas are applicable, and according to (43) $\rho_s = n$. The approximation for (26) then can be written as $\partial U/\partial m + n = 0$. The latter is interpreted as a result of minimization of some effective potential $U_{\text{eff}} = U + nm$ with fixed n, which always has a nontrivial minimum $0 < m < \infty$ for the class of decreasing potentials U; see, e.g., [23,24]. It turns out that such an approximation changes the picture *qualitatively.*

In what follows, we explore in detail the predictions of the consistent mass Eq. (26) on the mass varying scenario for the coupled model with a specific DE potential ansatz.

IV. COUPLED MODEL WITH THE RATRA-PEEBLES QUINTESSENCE POTENTIAL

A. Mass equation and critical temperature

Now we analyze in detail our coupled model for a particular choice of $U(\varphi)$, the so-called Ratra-Peebles quintessence potential [16]:

$$U(\varphi) = \frac{M^{\alpha+4}}{\varphi^\alpha}, \tag{48}$$

where $\alpha > 0$. It is convenient to introduce the dimensionless parameters

$$\Delta \equiv \frac{M}{T}, \qquad \kappa \equiv \frac{g\varphi}{T}, \qquad \Omega_R \equiv \frac{\Omega}{M^4}. \tag{49}$$

[2] Recall that the grand thermodynamical potential is equal to the free energy for the case $\mu = 0$.

Then the mass Eq. (26) can be written as

$$\frac{\alpha\pi^2}{2\mathfrak{F}}g^\alpha\Delta^{\alpha+4} = \mathcal{I}_\alpha(\kappa), \tag{50}$$

where we introduced

$$\mathcal{I}_\alpha(\kappa) \equiv \kappa^{\alpha+2}\int_\kappa^\infty \frac{\sqrt{z^2-\kappa^2}}{e^z+1}dz. \tag{51}$$

According to the relation (22) between the fermionic mass m and the classical field, we get $m = T\kappa_c$, where κ_c is the solution of Eq. (50) corresponding to the minimum of the thermodynamic potential which reads now as ([cf. Equation (31)]:

$$\Omega_R = g^\alpha\left(\frac{\Delta}{\kappa}\right)^\alpha - \frac{2}{3\pi^2}\frac{1}{\Delta^4}\mathcal{I}_p(\kappa). \tag{52}$$

The dimensionless Yukawa coupling constant $g \sim 1$. To reduce the number of model parameters, we can set $g = 1$. This is equivalent to the simultaneous rescaling $g\varphi \mapsto \tilde{\varphi}$ and $Mg^{((\alpha)/(\alpha+4))} \mapsto \tilde{M}$.[3] For simplicity, we also restrict the number of flavors $\mathfrak{F} = 1$.

We define the mass of the scalar field as

$$m_\phi^2 = \frac{\partial^2 U(\varphi)}{\partial\varphi^2}\bigg|_{\varphi=\phi_c}. \tag{53}$$

In terms of the dimensionless parameters, it reads

$$\frac{m_\phi}{M} = \sqrt{\alpha(\alpha+1)}\left(\frac{\Delta}{\kappa_c}\right)^{(\alpha+2)/2}. \tag{54}$$

It is important to realize that the integral $\mathcal{I}_\alpha(\kappa)$ on the r.h.s. of the mass equation is bounded. The quantitative parameters of the function $\mathcal{I}_\alpha(\kappa)$ depend on α, but its shape is always similar to the curve shown in Fig. 1 for $\alpha = 1$. So, there exists a maximal Δ_{crit} (critical temperature T_{crit}) such that for $\Delta > \Delta_{\text{crit}}$ ($T < T_{\text{crit}}$) only a trivial solution $m = \infty$ exists, and the stable vacuum has zero energy and pressure.

The mass equation Eq. (50) is solved numerically for various values of its parameters, and the characteristic results are shown in Fig. 1. The numerical results can be complemented by an approximate analytical treatment of the problem. The latter turns out to be quite accurate and greatly helps in gaining intuitive understanding of the results.

It is easy to evaluate $\mathcal{I}_\alpha(\kappa)$ to leading order:

$$\mathcal{I}_\alpha(\kappa) \approx \begin{cases} \frac{\pi^2}{12}\kappa^{\alpha+2}, & \kappa < 1 \\ \kappa^{\alpha+3}K_1(\kappa), & \kappa \gtrsim 1 \end{cases} \tag{55}$$

For the critical point where $\mathcal{I}'_\alpha(\kappa_{\text{crit}}) = 0$, we obtain

[3] One can check this scaling also holds for the dynamics of the model, considered in Sec. V. In particular, the neutrino masses do not depend on the value of g. To avoid cluttering of notations, we will drop tildes in the rescaled parameters.

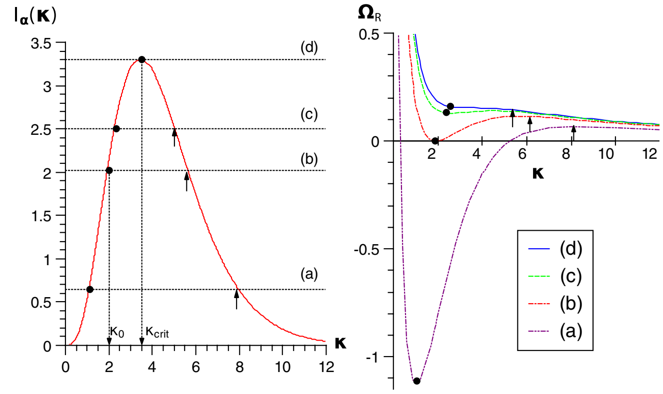

FIG. 1 (color online). Left: Graphical solutions of the mass Eq. (50) for different values of $\Delta \equiv M/T$ ($\alpha = 1$). Right: dimensionless density of the thermodynamic potential (52). The thermodynamically stable solutions of Eq. (50) indicated by the large dots correspond to the minima of the potential. The arrows indicate the unstable solutions of the mass equation, corresponding to the maxima of the potential.

$$\kappa_{\text{crit}} \approx \nu, \qquad \nu \equiv \alpha + \tfrac{5}{2}; \tag{56}$$

$$\mathcal{I}_\alpha(\kappa_{\text{crit}}) \approx \sqrt{\frac{\pi}{2}}\nu^\nu e^{-\nu}. \tag{57}$$

The most important conclusion we draw from Fig. 1 is that *there are three phases in the model's phase diagram.* We analyze each of them in the following subsections.

1. Stable (massive) phase: $\Delta < \Delta_\circ (T_\circ < T < \infty)$

In this range of parameters the Eq. (50) has two nontrivial solutions. The root $\kappa_c < \kappa_\circ$ indicated with a large dot in Fig. 1 (case a) gives the fermionic mass and corresponds to a global minimum of the potential. So it is a thermodynamically *stable state*. In this phase $\Omega(\kappa_c) < 0$, so the pressure is positive $P > 0$. Another nontrivial root of (50) corresponds to a thermodynamically *unstable state* (maximum of Ω indicated with an arrow in Fig. 1). There is a trivial third root of the mass equation $\kappa = \infty$. At these temperatures it corresponds to the *metastable vacuum state* $\Omega = 0$.

In the high-temperature region of this phase where $\Delta \ll 1$ the fermionic mass is small (see Fig. 2),

$$\frac{m}{M} \approx \left(\sqrt{6\alpha}\frac{M}{T}\right)^{2/(\alpha+2)} \propto T^{-2/(\alpha+2)}. \tag{58}$$

The fermionic contribution to the thermodynamic potential is dominant, and it behaves to leading order as the potential of the ultrarelativistic fermion gas [cf. Equation (45)]:

$$\Omega = -P = -\frac{7\pi^2}{180}T^4 + \mathcal{O}(T^{2\alpha/(\alpha+2)}). \tag{59}$$

One can check that the subleading term in the above expression combines the DE potential contribution and

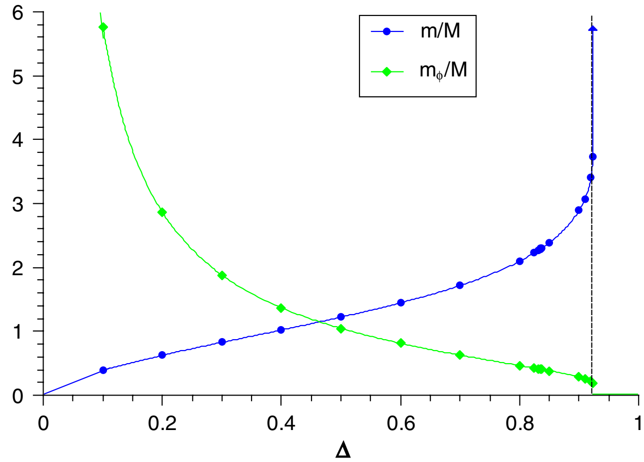

FIG. 2 (color online). Masses of the fermionic and scalar fields (m and m_ϕ, respectively) as functions of $\Delta \equiv M/T$, $\alpha = 1$. At $\Delta > \Delta_{\rm crit}(T < T_{\rm crit})$, the stable phase corresponds to $m = \infty$ and $m_\phi = 0$.

the first fermionic mass correction, which are both of the same order.

It is important to stress that in this coupled model with the slow-rolling potential Eq. (48), the mass generation does not follow a conventional Landau thermal phase transition scenario. There is no critical temperature below which the chiral symmetry is spontaneously broken and the mass is generated. Instead, the mass grows smoothly as $\kappa_c \propto \Delta^{(\alpha+4)/(\alpha+2)}$, albeit starting from the "point" $T = \infty$. From physical grounds, we expect the applicability of the model to have the upper temperature bound:

$$T \lesssim T_{\rm RD}, \tag{60}$$

where $T_{\rm RD}$ is roughly the temperature of the boundary between inflation and the radiation-dominated era. The high-temperature result (59) shows that the stable massive phase of the present model can indeed be extended up to those temperatures.

The scalar field and fermionic masses demonstrate opposite temperature dependencies. The scalar field is "heavy" at high temperatures:

$$m_\phi \approx \sqrt{\frac{\alpha+1}{6}}T, \qquad \Delta \ll 1; \tag{61}$$

however, its mass decreases together with the temperature. In contrary, the fermionic mass m monotonously increases with decreasing temperature. The exact numerical results for the two masses are shown in Fig. 2

2. *Metastable (massive) phase:* $\Delta_\circ < \Delta < \Delta_{\rm crit}(T_{\rm crit} < T < T_\circ)$

Upon increasing Δ, we reach a certain value $\Delta_\circ$ corresponding to a critical temperature $T_\circ$ when the thermodynamic potential has two degenerate minima $\Omega(\kappa_\circ) = P(\kappa_\circ) = \Omega(\infty) = 0$. This is shown in Fig. 1 (case b). After this point, when the temperature decreases further in the range $\Delta_\circ < \Delta < \Delta_{\rm crit}$ [here $\Delta_{\rm crit}$ stands for the maximal value of Δ when a nontrivial solution of the gap Eq. (50) exists, see Fig. 1], the two minima of the thermodynamic potential exchange their roles. The root κ_c now becomes a metastable state with $\Omega(\kappa_c) > 0$, i.e., with the negative pressure $P(\kappa_c) < 0$, while the stable state of the system corresponds to the true stable vacuum of the Universe [61] $\Omega(\infty) = P(\infty) = 0$. See Fig. 1 (case c). The system's state in the local minimum $\Omega(\kappa_c)$ is analogous to a metastable supercooled liquid. We disregard the exponentially small probability of tunneling of the fermions from the metastable state $\Omega(\kappa_c)$ into the vacuum state $\Omega(\infty) = 0$ [18]. Accordingly, the fermionic mass in this phase is determined by the root κ_c of (50).

In the metastable phase $\kappa_c \gtrsim 1$, so by using Eqs. (52), (32), and (50), we obtain the potential:

$$\Omega_R \approx \left(\frac{\Delta}{\kappa_c}\right)^\alpha \left\{1 - \frac{\alpha}{\kappa_c} - \frac{3\alpha}{2\kappa_c^2}\right\}. \tag{62}$$

From the above result, we can find the metastability point $\Omega(\kappa_\circ) = 0$ as

$$\kappa_\circ \approx \frac{\alpha}{2}\left(1 + \sqrt{1 + \frac{6}{\alpha}}\right). \tag{63}$$

Expanding $\mathcal{I}_\alpha(\kappa)$ near its maximum and using Eqs. (55)–(57) along with the gap equation, (50), we obtain the following equation:

$$\frac{(\kappa_c - \kappa_{\rm crit})^2}{2\nu} \approx 1 - \left(\frac{\Delta}{\Delta_{\rm crit}}\right)^{\alpha+4}. \tag{64}$$

On finds from the above equation, e.g., how the mass approaches its critical value:

$$m_{\rm crit} - m \propto \left(\frac{T}{T_{\rm crit}} - 1\right)^{1/2}, \tag{65}$$

or the ratios of temperatures and masses at the metastable and critical points. These latter parameters are given in Table I.

3. *Critical point:* $\Delta = \Delta_{\rm crit}(T = T_{\rm crit})$ *and phase transition*

The critical point of the model corresponds to the case when the two roots of the mass equation Eq. (50) merge, and the minimum of the potential disappears. One can check that instead of the minimum this is an inflection point of the potential, i.e., $\Omega_R''(\kappa_{\rm crit}) = 0$. This situation is shown in Fig. 1 (case d). At this point the system is in the *unstable* state with the fermionic mass

$$\frac{m_{\rm crit}}{T_{\rm crit}} = \kappa_{\rm crit} \approx \nu. \tag{66}$$

In particular, this implies that the fermions are nonrelativistic at the critical temperature. From Eqs. (57) and (50),

TABLE I. Masses, critical temperatures, and potentials for various values of α. All the parameters used in this table are defined in the text.

α	$\frac{T_{\rm crit}}{T_\circ}$	$\Delta_{\rm crit}$	$\frac{m_\circ}{m_{\rm crit}}$	$\frac{m_{\rm crit}}{M}$	$\frac{m_\phi^{\rm crit}}{M}$	$\frac{T_{\rm crit}}{M}$	$\frac{\Omega_{\rm crit}}{M^4}$	$\frac{\rho_{\rm crit}}{M^4}$	$w(T_{\rm crit})$
1	0.90	0.91	0.558	3.86	0.187	1.10	0.15	0.84	−0.18
2	0.95	1.04	0.70	4.35	0.130	0.97	0.02	0.25	−0.09
4	0.98	1.44	0.81	4.52	0.048	0.70	6×10^{-4}	0.02	−0.03
10	0.99	3.00	0.91	4.16	2×10^{-3}	0.33	7×10^{-8}	9×10^{-6}	−0.008

we find the critical parameter (see Table I for its numerical values)

$$\Delta_{\rm crit} \approx \left(\frac{\sqrt{2}}{\alpha\pi^{3/2}}\nu^\nu e^{-\nu}\right)^{1/(\alpha+4)}, \tag{67}$$

which allows us to evaluate the critical temperature

$$T_{\rm crit} = \frac{M}{\Delta_{\rm crit}}. \tag{68}$$

We can also find the potential at $T_{\rm crit}$:

$$\Omega_{\rm crit} \approx \frac{5}{2\nu}\left(\frac{\Delta_{\rm crit}}{\nu}\right)^\alpha M^4. \tag{69}$$

Thus, from the viewpoint of *equilibrium thermodynamics* at $T = T_{\rm crit}$, the model must undergo a *first-order (discontinuous) phase transition* and reach its third *thermodynamically stable* (*at* $T < T_{\rm crit}$) *phase* corresponding to the vacuum $\Omega(\kappa = \infty) = P(\kappa = \infty) = 0$. During this transition, the fermionic mass given at the critical point by Eq. (66) and the scalar field mass

$$m_\phi^{\rm crit} \approx \sqrt{\alpha(\alpha+1)}\left(\frac{\Delta_{\rm crit}}{\nu}\right)^{(\alpha+2)/2} M \tag{70}$$

both jump to their values in the vacuum state $m = \infty$ and $m_\phi = 0$. See Fig. 2.

However, the above arguments are based on the minimization of the thermodynamic potential (i.e. maximization of entropy) at equilibrium. To address the question of how such a system as the Universe evolves towards the new equilibrium vacuum state, we need to analyze the dynamics of this phase transition. More qualitatively, we need to study how the particle at the point $\kappa_{\rm crit}$ at the critical temperature (see Fig. 1) rolls down towards its equilibrium at infinity. This issue will be addressed in Sec. V.

B. Equation of state

We define the equation of state in the standard form:

$$P = w\rho, \tag{71}$$

where the total pressure in this model is obtained from Eq. (52), while the total energy density (ρ) and its dimensionless counterpart (ρ_R) are determined by the following equation:

$$\rho_R \equiv \frac{\rho}{M^4} = \left(\frac{\Delta}{\kappa}\right)^\alpha + \frac{2}{\pi^2}\frac{1}{\Delta^4}\mathcal{I}_\varepsilon(\kappa). \tag{72}$$

Here we define the integral

$$\mathcal{I}_\varepsilon(\kappa) \equiv \int_\kappa^\infty \frac{z^2\sqrt{z^2-\kappa^2}}{e^z+1}dz, \tag{73}$$

which can be evaluated in two limits of our interest:

$$\mathcal{I}_\varepsilon(\kappa) = \begin{cases} \frac{7\pi^4}{120} - \frac{\pi^2}{24}\kappa^2 + \mathcal{O}(\kappa^4), & \kappa < 1 \\ 3\kappa^2 K_2(\kappa) + \kappa^3 K_1(\kappa) + \mathcal{O}(e^{-2\kappa}), & \kappa \gtrsim 1 \end{cases}. \tag{74}$$

In the high-temperature region of the stable massive phase where $\Delta \ll 1$, the fermionic contribution is dominant, and the energy density to leading order is that of the ultrarelativistic fermion gas [cf. Equation (59)]

$$\rho = \frac{7\pi^2}{60}T^4 + \mathcal{O}(T^{2\alpha/(\alpha+2)}). \tag{75}$$

Thus, in this regime the model follows approximately the equation of state of a relativistic gas with $w \approx \frac{1}{3}$.

In the region $\kappa_c \gtrsim 1$ which includes the metastable phase and the critical point, we obtain by using Eqs. (72), (74), (50), and (62):

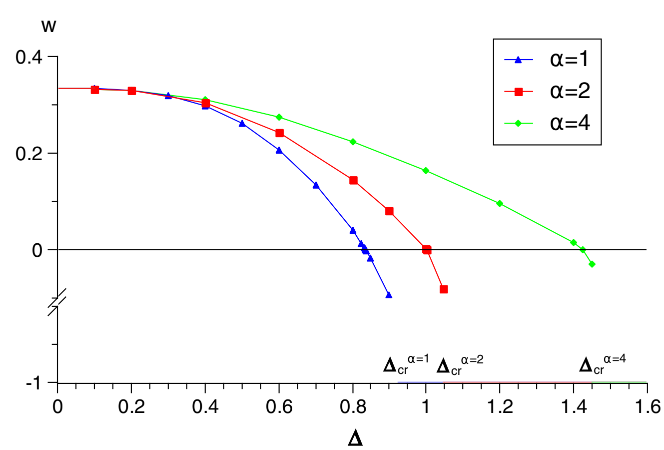

FIG. 3 (color online). $w \equiv P/\rho$ for several values of α. At $\Delta > \Delta_{\rm crit}(\alpha)$, i.e., $T < T_{\rm crit}(\alpha)$, the equilibrium value $w = -1$ exactly.

$$\rho \approx \left(\frac{\Delta}{\kappa_c}\right)^{\alpha}\left\{1+\alpha+\frac{3\alpha}{\kappa_c}+\frac{9\alpha}{2\kappa_c^2}\right\}, \tag{76}$$

and

$$w \approx -\frac{1-\frac{\alpha}{\kappa_c}-\frac{3\alpha}{2\kappa_c^2}}{1+\alpha+\frac{3\alpha}{\kappa_c}+\frac{9\alpha}{2\kappa_c^2}}. \tag{77}$$

The last equation follows very closely the results of the exact numerical calculations shown in Fig. 3. At the critical point, we evaluate

$$\rho_{\rm crit} \approx \left(\frac{\Delta_{\rm crit}}{\nu}\right)^{\alpha}\left\{1+\alpha+\frac{3\alpha}{\nu}\right\}M^4, \tag{78}$$

and making a rough estimate, we get a lower bound:

$$w \approx -\frac{5}{2}\frac{1}{\nu(1+\alpha+\frac{3\alpha}{\nu})} \geq -\frac{1}{4}, \quad \forall\ \alpha \geq 1. \tag{79}$$

Thus for any power law $\alpha \geq 1$, the parameter w of this model at equilibrium cannot cross the bound $w < -\frac{1}{3}$, necessary for accelerating expansion of the Universe $\ddot{a} > 0$.[4]

At $T < T_{\rm crit}$, we obtain the equilibrium value of w in the stable vacuum state from Eqs. (52) and (72):

$$w = \lim_{\kappa\to\infty}\frac{P(\kappa)}{\rho(\kappa)} = -1. \tag{80}$$

So the true vacuum in this model corresponds to the Universe with a cosmological constant in the limit $\Lambda \to 0$.

C. Speed of sound

We define the sound velocity as

$$c_s^2 = \frac{\frac{dP}{dt}}{\frac{d\rho}{dt}} = \frac{\frac{dP}{d\Delta}}{\frac{d\rho}{d\Delta}}, \tag{81}$$

where to obtain the second expression, we used the fact that the time enters our formulas only through the temperature $T(a(t))$, so

$$\frac{d}{dt} = \frac{d\Delta}{dt}\frac{d}{d\Delta}. \tag{82}$$

Let us first consider the temperatures $T \geq T_{\rm crit}$, i.e., $\Delta \leq \Delta_{\rm crit}$. Then

$$\frac{d\rho}{d\Delta} = \frac{\partial\rho}{\partial\Delta} + \frac{\partial\rho}{\partial\kappa}\cdot\frac{d\kappa}{d\Delta}\bigg|_{\kappa=\kappa_c}, \tag{83}$$

where κ is related to Δ through the gap Eq. (50):

$$\frac{d\kappa}{d\Delta}\bigg|_{\kappa=\kappa_c} \equiv \dot{\kappa}_c = \frac{\alpha+4}{\Delta}\frac{\mathcal{I}_\alpha(\kappa_c)}{\mathcal{I}'_\alpha(\kappa_c)} = \frac{\alpha+4}{\Delta}\left(\frac{d\log\mathcal{I}_\alpha(\kappa_c)}{d\kappa}\right)^{-1}. \tag{84}$$

Note that for the pressure, the following relation

$$\frac{dP}{d\Delta} = \frac{\partial P}{\partial\Delta} \tag{85}$$

holds, since

$$\frac{\partial P}{\partial\kappa}\bigg|_{\kappa=\kappa_c} = 0 \tag{86}$$

is just another form of the gap Eq. (24). Thus

$$c_s^2 = \frac{\frac{\partial P}{\partial\Delta}}{\frac{\partial\rho}{\partial\Delta}+\frac{\partial\rho}{\partial\kappa}\dot{\kappa}_c}\bigg|_{\kappa=\kappa_c}. \tag{87}$$

In the high-temperature regime $\Delta \ll 1$ ($\kappa_c \ll 1$), it is even easier to use the explicit asymptotic expansions for $P(\Delta)$ and $\rho(\Delta)$ in the definition (81) instead of the above formula (87). A straightforward calculation gives the result

$$c_s^2 \approx \tfrac{1}{3} - b\Delta^{2(\alpha+4)/(\alpha+2)}, \qquad b > 0, \tag{88}$$

consistent with the earlier observation that for $\Delta \ll 1$, the model behaves as an ultrarelativistic Fermi gas.

In the case $\kappa_c \gtrsim 1$ we find

$$\dot{\kappa}_c \approx \frac{\alpha+4}{\Delta}\frac{\kappa_c}{\nu-\kappa_c}, \tag{89}$$

and

$$c_s^2 \approx \frac{\nu-\kappa_c}{\alpha(\alpha+4)(1+\frac{4}{\alpha\nu})}. \tag{90}$$

Everywhere at $T > T_{\rm crit}$, including the stable and metastable massive phases $c_s^2 > 0$, so the model is stable with respect to the density fluctuations. The sound velocity vanishes in the limit $T \to T_{\rm crit}^+$ as

$$c_s \propto \sqrt{\nu-\kappa_c} \to 0. \tag{91}$$

Qualitatively, the vanishing speed of sound is due to divergent $\dot{\kappa}_c$ (84) and (89) at the critical point.

The above analytical results are in excellent agreement with the numerical calculations of c_s^2 from the formula (87) shown in Fig. 4. At the temperatures $T < T_{\rm crit}$, there is no gap equation relating κ and Δ, so the sound velocity is easily calculated to yield the value in the equilibrium vacuum state:

$$c_s^2 = \lim_{\kappa\to\infty}\frac{\frac{\partial P}{\partial\Delta}}{\frac{\partial\rho}{\partial\Delta}} = -1. \tag{92}$$

That what is expected for a barotropic perfect liquid with a constant w, where $c_s^2 = w$.

[4]The relation (79) $w(T_{\rm crit}) \gtrsim -\frac{1}{4}$ holds for the model which contains only the DE-neutrino coupled fluid. In a more realistic model for the Universe, baryons and DM also contribute to the total energy density, and as a consequence $w(T_{\rm crit})$ increases; see Sec. V.

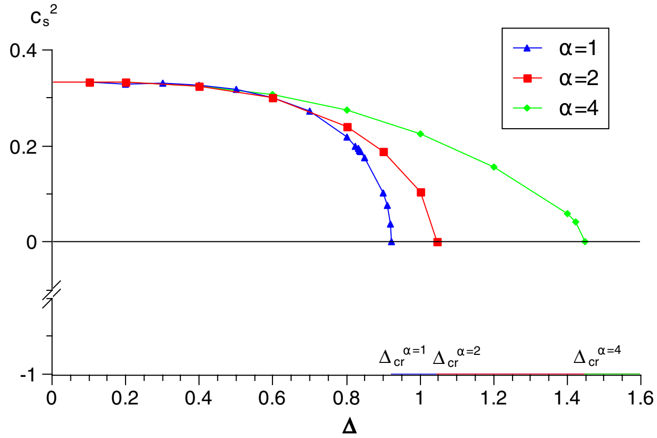

FIG. 4 (color online). The square of the sound velocity for several values of α. At $\Delta > \Delta_{\rm crit}(\alpha)$, i.e., $T < T_{\rm crit}(\alpha)$, the equilibrium value $c_s^2 = -1$ exactly.

V. DYNAMICS OF THE COUPLED MODEL AND OBSERVABLE UNIVERSE

A. Scales and observable universe

In order to make a connection between the above model results and the observable universe, we need to first conclude where we are now with respect to the critical temperatures $T_\circ$ and $T_{\rm crit}$. As one can see from Table I for $\alpha \sim 1$, the model has $T_\circ \sim T_{\rm crit} \sim M$. We identify the current equilibrium temperature of the Universe with the cosmic background radiation temperature $T = 2.275\ {\rm K} = 2.4 \times 10^{-4}$ eV. Then we see right away that we cannot be above the critical temperature of the coupled model, since:

(i) assuming $T > T_{\rm crit}$ leads to $M \lesssim 10^{-4}$ eV, which in turn implies too small densities $\rho \sim M^4 \sim 10^{-16}\ {\rm eV}^4$, i.e, 4 orders of magnitude less than the observable density;

(ii) At $T > T_{\rm crit}$, the equation of state has $w > -\frac{1}{4}$ (see Fig. 3), which is not even enough to get a positive acceleration $\ddot{a} > 0$, while the observable value $w \approx -1$ [13].

So, the first qualitative conclusion is that we are currently below the critical temperature. The Universe has already passed the stable and metastable phases and is now unstable, i.e. it is in the transition toward the stable doomsday vacuum $m = \infty$ and $\Omega = 0$.

Since at the temperature of metastability $P_\circ = 0$, the transition occurs somewhere between the beginning of the matter-dominated era ($T_{\rm MD} \approx 16\,500\ {\rm K} \approx 1.42$ eV) and now, i.e., $1.4\ {\rm eV} \gtrsim T_{\rm crit} > T_{\rm now} \sim 2.4 \times 10^{-4}$ eV. Because of Eq. (68), this inequality gives us the possible range of the model's parameter M:

$$2.4 \times 10^{-4}\ {\rm eV} < M \lesssim 1.4\ {\rm eV}. \tag{93}$$

As we will show in the following, other consistency checks of the model bring the upper bound of M much lower.

B. Universe before the phase transition

In order to apply the results of the coupled model for the calculation of the parameters of the observable universe, we need to incorporate the matter (we will just add up the dark and conventional baryonic matter together) and the radiation. Assuming a spatially flat universe, the total energy density is critical, so

$$\rho_{\rm tot} = \rho_{\gamma,{\rm now}}/a^4 + \rho_{M,{\rm now}}/a^3 + \rho_{\varphi\nu}(\Delta) = \rho_{\rm cr} = \frac{3H^2}{8\pi G}, \tag{94}$$

where from now on we denote $\rho_{\varphi\nu}$ the energy density of the coupled model given by Eq. (72). To relate our model's parameters to the standard cosmological notations, we assume that the temperature is evolving as that of the blackbody radiation, i.e., $T = T_{\rm now}/a$. Then

$$\Delta \equiv \frac{M}{T} = \frac{Ma}{T_{\rm now}} = \frac{M}{T_{\rm now}(1+z)}. \tag{95}$$

We know that

$$\rho_\gamma = \frac{\pi^2}{15} T^4, \tag{96}$$

and we set the current density of the coupled scalar field to the observable value of the dark energy, i.e., 3/4 of the critical density:

$$\rho_{\varphi\nu,{\rm now}} = \frac{3}{4} \cdot \frac{3H_0^2}{8\pi G} \approx 31 \cdot (10^{-3}\ {\rm eV})^4, \tag{97}$$

and

$$\rho_{M,{\rm now}} \approx \frac{1}{4} \cdot \frac{3H_0^2}{8\pi G}. \tag{98}$$

The equations above allow us to plot the relative energy densities

$$\Omega_\# \equiv \rho_\#/\rho_{\rm tot} \tag{99}$$

as functions of redshift (or temperature) up to the critical point; see Fig. 5.[5] In the high-temperature limit, the matter term is subleading and

$$\rho_{\rm tot} \approx \rho_\gamma + \rho_{\varphi\nu} \approx \frac{\pi^2}{15}\left(1 + \frac{7}{4}\right) T^4. \tag{100}$$

In this limit, then

$$\Omega_{\varphi\nu} = \frac{7}{11} \approx 0.636, \qquad \Omega_\gamma = \frac{4}{11} \approx 0.363, \tag{101}$$

which agrees well with the numerical results displayed in Fig. 5. At the critical point the matter strongly dominates and $\rho_M/\rho_{\gamma,\varphi\nu} \gtrsim 10^2$.

[5] We apologize for some abuse of notations, but using the same Greek letter for the grand thermodynamic potential and relative densities seems to be standard now. Since these quantities are mainly discussed in different sections of the paper, we hope the reader will not be confused.

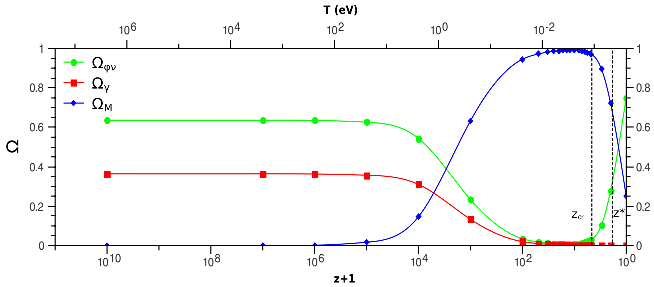

FIG. 5 (color online). Relative energy densities plotted up to the current redshift (temperature, upper axis): $\Omega_{\varphi\nu}$—coupled DE and neutrino contribution; Ω_γ—radiation; Ω_M—combined baryonic and dark matters. Parameter $M = 2.39 \times 10^{-3}$ eV ($\alpha = 0.01$), chosen to fit the current densities, determines the critical point of the phase transition $z_{cr} \approx 3.67$. The crossover redshift $z^* \approx 0.83$ corresponds to the point where the Universe starts its accelerating expansion.

The equation of state parameter of the entire universe, w_{tot}, is given by $P_{tot} = w_{tot}\rho_{tot}$. Since the matter contribution $P_M = 0$, then $P_{tot} = P_\gamma + P_{\varphi\nu}$, where $P_\gamma = \frac{1}{3}\rho_\gamma$ and the pressure of the coupled model $P_{\varphi\nu}$ is obtained from Eq. (52). The numerical results of w_{tot} are given in Fig. 6.

To analyze the dynamics of the coupled model, we need, in principle, to go beyond the saddle-point approximation applied in the previous sections and solve the equation of motion:

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{\partial\Omega}{\partial\varphi} = 0. \tag{102}$$

Above the transition point ($T > T_{crit}$) the dynamics is quite simple. Let us analyze perturbations to the saddle-point solution of (24):

$$\varphi(t) \equiv \phi_c + \psi(t). \tag{103}$$

Taylor-expanding the thermodynamic potential of the coupled model

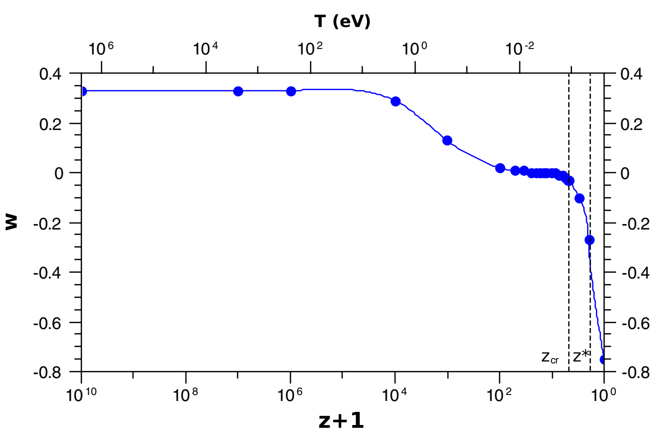

FIG. 6 (color online). Equation of state parameter $w_{tot} = P_{tot}/\rho_{tot}$ for $M = 2.39 \times 10^{-3}$ eV ($\alpha = 0.01$) plotted up to the current redshift (temperature, upper axis).

$$\frac{\partial\Omega}{\partial\varphi} = \omega^2\psi + \frac{1}{2}\Omega'''(\phi_c)\psi^2 + \cdots \tag{104}$$

with $\omega^2 \equiv \Omega''(\phi_c)$, we obtain from (102) the equation of a damped harmonic oscillator to the leading order:

$$\ddot{\psi} + 3H\dot{\psi} + \omega^2\psi = 0. \tag{105}$$

So, the quintessence field $\varphi(t)$ oscillates around its saddle-point value ϕ_c with $\psi(t) \propto e^{\imath\omega t-(3/2)Ht}$. The damping is very small, since as one can check

$$\omega \gg \tfrac{3}{2}H. \tag{106}$$

The violation of the above condition and breaking down of the oscillating regime occurs in the vicinity of the critical point, which is the inflection point of the potential ($\omega = 0$). This is the well-known phenomenon of the critical slowing down near phase transition. Retaining the first nonvanishing term in (104), the equation of motion in the vicinity of the critical point reads

$$\ddot{\psi} + 3H\dot{\psi} + \tfrac{1}{2}\Omega'''(\phi_c)\psi^2 = 0. \tag{107}$$

Neglecting the small damping term in this equation, its solution can be found analytically via a hypergeometric function. Since the explicit form of this solution is not very interesting at this point, we just emphasize the qualitative conclusion of the analysis: the fluctuation $\psi(t)$ oscillates near the classical field ϕ_c in the stable (metastable) phase at $T > T_{crit}$, and it enters the run-away (power-law) regime when $T \to T_{crit}^+$ [64].

C. Late-time acceleration of the Universe. Toward the end of times

The equilibrium methods are not applicable below the phase transition, and we study the dynamics of the model from the equation of motion (102) together with the Friedmann Eqs. (1)–(3). Solution of the Dirac equations yields $\rho_s \propto a^{-3}$ for the chiral density [65], so the equation of motion (102) at $a \leq a_{crit}$ reads

$$\ddot{\varphi} + 3H\dot{\varphi} = -\frac{\partial U}{\partial\varphi} - \rho_{s,crit}\left(\frac{a_{crit}}{a}\right)^3. \tag{108}$$

From the results of the previous section, we evaluate the chiral density at the critical point:

$$\rho_{s,crit} \approx \alpha\left(\frac{\Delta_{crit}}{\nu}\right)^{\alpha+1} M^3. \tag{109}$$

The system of the integro-differential equations (1)–(3) and (108) was solved numerically. All the quantities entering those equations are defined in the previous subsection, except that one needs to include the extra term $\frac{1}{2}\dot{\varphi}^2$ in the computation of both ρ_{tot} and P_{tot}. However, the numerical results show that in the regimes of the parameters we are interested, the kinetic term can be safely neglected. Since the critical point of the model lies in the matter-dominated regime (cf. Fig. 5), we start with the Hubble parameter

$H = 2/3t$ ($a \propto t^{2/3}$). At the latest times ($z \lesssim 1$), the Hubble parameter was determined self-consistently from the numerical solution of the Friedmann equations.

We find numerically that the quintessence field $\varphi(t)$ from the critical point to the present time oscillates quickly (with the period $\tau \sim 10^{-27}$ Gyr) around the smooth ("mean value") solution $\bar{\varphi}(t)$, where the "mean" $\bar{\varphi}$ nullifies the r.h.s. of the equation of motion (108). Relating the mean values with the physically relevant observable quantities, we can easily obtain the key results analytically. (They are checked against direct numerical calculations and found to be accurate within 5% at most). Thus, we get

$$\bar{\varphi} = \varphi_{\rm crit} \cdot \left(\frac{1 + z_{\rm crit}}{1 + z}\right)^{3/(\alpha+1)}, \tag{110}$$

$$\rho_{\bar{\varphi}} = \rho_{\varphi,{\rm crit}} \cdot \left(\frac{1 + z}{1 + z_{\rm crit}}\right)^{3\alpha/(\alpha+1)}, \tag{111}$$

where $\varphi_{\rm crit} \approx \frac{\nu}{\Delta_{\rm crit}} M$ and $\rho_{\varphi,{\rm crit}} \approx (\frac{\Delta_{\rm crit}}{\nu})^{\alpha} M^4$. Having a free model parameter M, we will set it by matching the current density of the scalar field $\rho_{\varphi,{\rm now}}$ to the observable value of the DE density (97), so

$$M = (\nu^{\alpha} \rho_{\varphi,{\rm now}})^{(\alpha+1)/(\alpha+4)} \Delta_{\rm crit}^{-\alpha} T_{\rm now}^{-3\alpha/(\alpha+4)}. \tag{112}$$

The exponent of the quintessence potential α is now the only parameter which can be varied. We define the time-dependent mass via the solution of the motion equation as $m(t) = \bar{\varphi}(t)$, thus obtaining an estimate for the present-time neutrino mass. Results for various α are given in Table II. There, we also calculate the critical points parametrized by the redshifts $z_{\rm crit}$ and the crossover points z^*. The latter is defined as the redshift at which the Universe starts its late-time acceleration, i.e., where $w_{\rm tot} = -\frac{1}{3}$. For the present time, we find

$$w_{\rm tot}^{\rm now} \approx -\tfrac{3}{4}. \tag{113}$$

As we infer from the data of Table II, the range of exponents $\alpha \ll 1$ corresponds to more realistic predictions for the neutrino mass [1,7,8] and for the crossover redshift z^* [66]. For $\alpha = 0.01$, we plot the evolution of the relative energy densities, the equation of state parameter, and the neutrino mass in Figs. 5–7.

TABLE II. Model's parameters and observables for various α. All the entries in this table are defined in the text.

α	M (eV)	$m_{\rm now}$ (eV)	$z_{\rm crit}$	z^*
2	9.75×10^{-2}	167	392	4.9
1	1.69×10^{-2}	44.6	76.6	2.3
1/2	6.33×10^{-3}	17.0	27.7	1.5
10^{-1}	2.81×10^{-3}	2.82	8.73	0.93
10^{-2}	2.39×10^{-3}	0.27	3.67	0.83
10^{-3}	2.36×10^{-3}	0.027	1.60	0.82

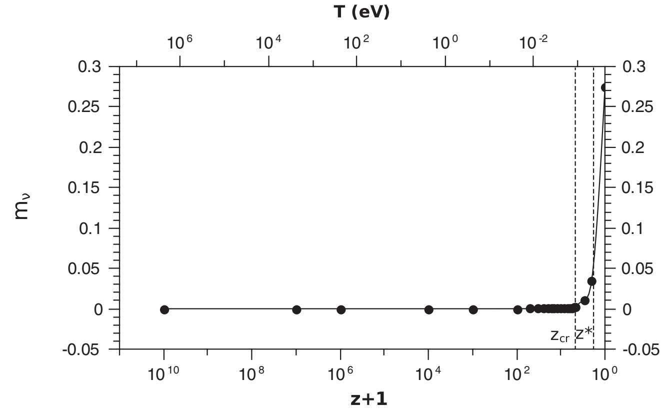

FIG. 7. Neutrino mass m for $M = 2.39 \times 10^{-3}$ eV ($\alpha = 0.01$) plotted up to the current redshift (temperature, upper axis).

We consider the quite artificial case of small quintessence exponent α as an ansatz crossing over smoothly from physically plausible potentials with, say, $\alpha = 1$ or 2 to the logarithmic potential

$$U(\varphi) = M^4\left(1 + \alpha \log\frac{M}{\varphi}\right). \tag{114}$$

The latter often appears in various contexts [13,36].[6]

VI. CONCLUSIONS

In this paper we analyzed the MaVaN scenario in a framework of a simple minimal model with only one species of the (initially) massless Dirac fermions coupled to the scalar quintessence field. By using the methods of thermal quantum field theory, we derived for the first time (in the context of the MaVaN or, even more broadly, the VAMP models) a consistent equation for fermionic mass generation in the coupled model.

We demonstrated that the mass equation has nontrivial solutions only for special classes of potentials and only within certain temperature intervals. It appears that these results have not been reported in the literature on VAMPs before now.

We gave most of the results for the particular choice of a trial DE potential—the Ratra-Peebles quintessence potential. This potential has all the necessary properties we needed for our task: it is simple, it satisfies the criteria we found for nontrivial solutions of the mass equation to exist, and it has only one dimensionful parameter—the energy scale M to tune. Also, at small values of the exponent α, it effectively crosses over to the case of a logarithmic potential. We have checked that other

[6] The numerical results for small parameter α, as e.g. $\alpha = 0.01$ taken for the plots, are virtually indistinguishable for the cases of the Ratra-Peebles (48) or logarithmic (114) potentials. However, the Ratra-Peebles potential at more "natural" $\alpha = 1, 2$ allows to probe the coupled fermionic-quintessence models in the search of heavy DM particle candidates.

potentials, e.g., exponential, lead to a qualitatively similar picture, but they have at least one more energy scale to handle, which we consider as an unnecessary complication at this point.

We analyzed the thermal (i.e. temporal) evolution of the model, following the time arrow. Contrary to what one might expect from analogies with other contexts, like, e.g., condensed matter, the model does not generate the mass via a conventional spontaneous symmetry breaking below a certain temperature. Instead it has a nontrivial solution for the fermionic mass evolving "smoothly" from zero at the point $T = \infty$. The scalar field is infinitely heavy at the same point. More realistically, we assumed the model is applicable starting at the temperatures somewhere in the beginning of the radiation-dominated era. We found that the DE contribution in this regime is subleading, and the model behaves as an ultrarelativistic Fermi gas at those temperatures.

This regime corresponds to a stable phase of the model given by a global minimum of the thermodynamic potential $\Omega(\varphi)$. The temperature/time-dependent minimum $\langle\varphi\rangle$ generates the varying fermionic mass $m \propto \langle\varphi\rangle$.

With increase in time, as the temperature decreases, the model reaches the point of metastability where its pressure (P) vanishes. From our estimates of the model's scales, we showed that this happens during the matter-dominated era of the Universe. At this point the system's ground state becomes doubly degenerate, and the potential $\Omega = 0$ at the nontrivial (finite) minimum $\langle\varphi\rangle$ as well as at the trivial vacuum $\varphi = \infty$.

Further on, at lower temperatures the system stays in the metastable (supercooled) state until it reaches the critical point where the local minimum of the thermodynamic potential disappears and it becomes an inflexion point. At this critical temperature, the model undergoes a first-order (discontinuous) phase transition. At the critical point, *the equilibrium values* of the fermionic and the scalar field masses discontinuously jump to the doomsday vacuum state values $m = \infty$ and $m_\phi = 0$, respectively. The square of the sound velocity and equation of state parameter w have the equilibrium values corresponding to the de Sitter universe with a cosmological constant, i.e. $c_s^2 = w = -1$. It is worth pointing out that $c_s^2 > 0$ in both the stable and metastable phases, and the sound velocity vanishes reaching the critical temperature from above.

Since the equilibrium approach is not applicable below the critical temperature, we find parameters of the model from direct numerical solution of the equation of motion and the Friedmann equations. The single scale M of the quintessence potential is chosen to match the present DE density, then other parameters of the Universe are determined. We obtain a consistent picture: the phase transition has occurred rather recently at $z_{\rm crit} \lesssim 5$ during the matter-dominated era, and the Universe is now being driven towards the stable vacuum with zero Λ term. The expansion of the Universe accelerates starting from $z^* \approx 0.83$. Setting $\alpha = 0.01$ for $M \approx 2.4 \times 10^{-3}$ eV, we end up with the neutrino mass $m \approx 0.27$ eV.

The present results allow us to propose a completely new viewpoint not only on the MaVaN, but on the quintessence scenario for the Universe as well. The common concerns about the slow-rolling mechanism for the DE relaxation toward the $\Lambda = 0$ vacuum are related to the question of what is the mechanism to set the initial value of the scalar field φ where it evolves (rolls down) from. Our results demonstrate that up to recent times (i.e. above the critical temperature) the quintessence field was locked around its average (classical) value $\langle\varphi\rangle$. Its value is determined by the scale M and the temperature. The average $\langle\varphi\rangle$ gives the fermionic mass at the same time. The scalar field is rigid (i.e. massive), although it softens (i.e., its mass decreases) as the system approaches the critical temperature. Above the critical temperature, the scalar field can only oscillate around its equilibrium value $\langle\varphi\rangle$. At the critical point, the minimum of the thermodynamic potential becomes the inflexion point, the scalar field looses its rigidity (mass). Then the field can only roll down towards the new stable ground state $\Omega = 0$ at $\varphi = \infty$. So physically, the critical point corresponds to the transition of the Universe from the stable oscillatory to the unstable rolling regime.

A more sophisticated numerical study of the kinetics after the critical point is warranted in order to address such issues as the detailed description of the crossover between different regimes, and the clustering of neutrinos. These and some other questions are relegated to our future work.

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