

**Glue condensation and scaling exponents for the propagators in Yang-Mills theory**Astrid Eichhorn,<sup>1</sup> Holger Gies,<sup>1</sup> and Jan M. Pawłowski<sup>2,3</sup><sup>1</sup>*Theoretisch-Physikalisches Institut, Friedrich-Schiller-Universität Jena, D-07743 Jena, Germany*<sup>2</sup>*Institut für Theoretische Physik, University of Heidelberg, Philosophenweg 16, 62910 Heidelberg, Germany*<sup>3</sup>*ExtreMe Matter Institute EMMI, GSI Helmholtzzentrum für Schwerionenforschung, Planckstr. 1, 64291 Darmstadt, Germany*  
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We investigate the infrared (strong-coupling) regime of  $SU(N)$ -Yang-Mills theory on a self-dual background. We present an evaluation of the full effective potential for the field-strength invariant  $F_{\mu\nu}F^{\mu\nu}$  from nonperturbative gauge correlation functions and find a nontrivial minimum corresponding to the existence of a dimension four gluon condensate in the vacuum. We also relate the infrared asymptotic form of the  $\beta$  function of the running background-gauge coupling to the asymptotic behavior of Landau-gauge gluon and ghost propagators. Consistency between both gauges in the infrared imposes a new upper bound on the infrared exponents of the propagators. For the *scaling solution*, this bound reads  $\kappa_c < 23/38$  which, together with Zwanziger's horizon condition  $\kappa_c > 1/2$ , defines a rather narrow window for this critical exponent. Current estimates from functional methods indeed satisfy these bounds.

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**I. INTRODUCTION**

The understanding of low-energy QCD has continuously been advancing over the recent decades. Since a variety of methods, gauges and pictures have been used to investigate QCD in detail, progress has never been linear but many scenarios have been developed abreast. Particularly for the challenging problem of confinement, a number of successful scenarios exist in parallel. Contemporary research therefore has to clarify whether these scenarios mutually support, exclude or simply coexist with one another. In this work, we pursue this question for two scenarios of pure Yang-Mills gauge theory based on two different gauges: the Landau gauge and the background gauge.

In the Landau gauge, information about the strongly coupled sector and confinement is drawn from low-order correlation functions which have been studied with functional methods such as Dyson-Schwinger equations (DSE) [1–5], the functional renormalization group (FRG) [4–8], and stochastic quantization [9,10], as well as with lattice gauge theory [10,11]. Many results support the Kugo-Ojima, or Gribov-Zwanziger confinement scenarios [12–14] where the impossibility of colored asymptotic states imposes conditions on the infrared (IR) behavior of ghost and gluon propagators (see Sec. III). Reviews on the different approaches and numerical results can be found in [5,15,16]; for further work see [17,18].

The background gauge is particularly useful for computing the full quantum effective action in an expansion in terms of gauge-invariant operators. As the quantum effective action governs the dynamics of field expectation values, information about confinement is contained in the resulting nonlinear and nonlocal quantum equations of motion. Simple scenarios for the full effective action, such as the leading-log model [19–21] or more sophisticated dielectric confinement models [22–24], indeed find

confining potentials for the solution of the field equations for static color sources.

Even though the Landau gauge and the background gauge have formally much in common, results from one gauge generally cannot easily be transferred to the other. The background gauge reduces to covariant Lorenz-type gauges including the Landau gauge in the limit of zero background. However, most results for the background gauge are obtained for zero fluctuation field instead, so that direct Landau-gauge information is lost. In turn, the full background-field dependence in the background gauge cannot be reconstructed solely from Landau-gauge correlation functions. For special cases, however, such a reconstruction in certain parametric limits is indeed possible. This has recently been shown for the case of the Polyakov loop [25], where Landau-gauge propagators have been used to determine a variant of the background Polyakov-loop potential. These results compare well with a related study in Polyakov-gauge, [26].

The resulting connection between the two gauges works in both ways [25]: given the Landau-gauge propagators, an estimate of the deconfinement phase transition temperature has been computed in excellent agreement with lattice simulations. The other way around, a confining Polyakov-loop potential in the background gauge imposes a new confinement criterion on gluon and ghost propagators in the Landau gauge. This criterion is indeed found to be satisfied by these propagators.

In the present work, we further explore the connection between the two gauges, concentrating on the running coupling and the effective potential for the square of the Yang-Mills field-strength. In particular, we focus on the interrelation between the asymptotic behavior of the gluon and ghost propagator for low momenta and the  $\beta$  function of the background running coupling. Consistency of the latter with a strongly interacting infrared regime already

imposes severe restrictions on the scaling exponents of the propagators. Furthermore, we report on a nonperturbative calculation of the full effective potential without any polynomial truncation. This allows to search for a nontrivial minimum, indicating the condensation of gluons in the vacuum.

This work is structured as follows: In Sec. II, we introduce a nonperturbative Renormalization Group equation, the Wetterich equation for the case of nonzero background and elaborate on the extension of Landau-gauge propagators to the background gauge. We then specialize to the case of a self-dual background in Sec. III, which allows to evaluate the asymptotic form of the  $\beta$  function of the running coupling. This can be related to the asymptotic behavior of the propagators, see Sec. IV). Finally, we report on our results for a numerical evaluation of the full effective potential in Sec. V. Conclusions and discussion are summarized in Sec. VI.

## II. BACKGROUND-FIELD FLOW

Information about correlation functions and effective potentials is encoded in the effective action  $\Gamma$ , being the generating functional of 1PI Green's functions. Nonperturbative access to the effective action is given by the FRG in which the full  $\Gamma$  is constructed successively by integrating out quantum fluctuations momentum shell by momentum shell. For a generic quantum field theory, this procedure leads to a flow equation for a scale-dependent effective action  $\Gamma_k$ . Here,  $k$  denotes a momentum scale above which all quantum fluctuations have been integrated out. The dependence of  $\Gamma_k$  on this momentum scale is determined by the Wetterich equation [27], being an exact functional differential equation. Reviews on this topic can be found in [28]; for gauge theories see [29,30]. The application of the background-field method to flow equations in gravity is reviewed in e.g. [31].

In gauge theories, momentum-shell integrations in continuum quantum field theory can most conveniently be performed in gauge-fixed formulations. An effective action constructed from only gauge-invariant building blocks can nevertheless be obtained by use of the background-field formalism [32,33]. Here, the full gauge field  $\mathcal{A}_\mu$  is split into a background  $A_\mu$  and a fluctuation field  $a_\mu$  according to  $\mathcal{A}_\mu = A_\mu + a_\mu$ . The gauge of the fluctuations is then fixed with respect to the background field, whereas the whole action remains invariant under an auxiliary gauge transformation of the full gauge field  $\mathcal{A}_\mu$  and the background field  $A_\mu$ . Let us stress that this transformation is not the physical, but just an auxiliary gauge transformation. It allows, however, to retain invariance of the standard effective action  $\Gamma[A] = \Gamma[a = 0, A]$  under physical gauge transformations. Within the setting,  $a = 0$  is allowed only once all fluctuations have been integrated out, hence the price to be paid for the construction of a gauge-invariant effective action is its dependence on two gauge fields, [34,35].

The Wetterich equation in background-field formalism reads [33,36]

$$\partial_t \Gamma_k[a, A] = \frac{1}{2} \text{STr}(\Gamma_k^{(2,0)}[a, A] + R_k)^{-1} \partial_t R_k, \quad (1)$$

where  $\Gamma_k^{(n,m)}[a, A] = \frac{\delta^n}{(\delta a)^n} \frac{\delta^m}{(\delta A)^m} \Gamma_k[a, A]$  and  $\partial_t \equiv k \frac{d}{dk}$ . The action  $\Gamma_k$  is a functional which interpolates between the microscopic action  $S_\Lambda$  at the UV cutoff  $\Lambda$  and the full quantum effective action  $\Gamma$ , i.e.,  $\Gamma_{k \rightarrow \Lambda} \rightarrow S_\Lambda$  and  $\Gamma_{k \rightarrow 0} \rightarrow \Gamma$ . The solution to the flow Eq. (1) provides an RG trajectory of action functionals  $\Gamma_k$  that interconnects the microscopic action with the full effective action. The quantity  $R_k$  is a regulator function depending on an infrared (IR) cutoff  $k$  that suppresses propagation of momenta smaller than  $k$ . The trace (STr) runs over all internal indices, momenta and field components, i.e., gluon and ghost degrees of freedom, including a negative sign for the ghosts.

As we will eventually be interested in the background-field action

$$\Gamma_k[A] = \Gamma_k[a = 0, A], \quad (2)$$

simplifications arise from the fact that only the fluctuation field propagators in a background, being the inverse of  $\Gamma_k^{(2,0)}[0, A]$ , are required on the right-hand side in order to extract the flow of  $\Gamma_k[a = 0, A]$  on the left-hand side. In the following, we will concentrate on determining these inverse propagators in the background gauge with gauge parameter  $\alpha = 0$ , i.e., in the so-called Landau-DeWitt gauge. These are directly related to a covariantization of the well-studied (inverse) Landau-gauge propagators  $\Gamma_{\text{Landau}}^{(2)}(p^2)$ , which is a well-suited gauge for FRG calculations as it is a fixed point of the RG flow [37]. Moreover, FRG flows for correlation functions in the Landau-DeWitt gauge directly relate to those obtained within fully gauge-invariant flows for the geometrical effective action [38,39]. It is interesting to note that in such a setting the difference between background and fluctuation correlation functions is controlled by Nielsen identities [39], and indeed supports the above relation between Landau-gauge propagators and background gauge propagators.

The relation between the inverse propagators in these two gauges can be parameterized by

$$\Gamma_k^{(2,0)}[0, A] = \Gamma_{k, \text{Landau}}^{(2)}(\mathcal{D}[A]) + F_{\mu\nu} f_{\mu\nu}(D), \quad (3)$$

where  $\mathcal{D}[A]$  is a suitable background covariant differential operator, reducing to the Laplacian for vanishing background field. The covariant derivative with respect to the background field is given by  $D_\mu^{ab} = \partial_\mu \delta^{ab} + g f^{abc} A_\mu^c$ . The function  $f_{\mu\nu}$  occurring in combination with the Yang-Mills field strength  $F_{\mu\nu}$  in the second term cannot be determined from the knowledge of the Landau-gauge

propagators alone. It is nonsingular for vanishing argument in order to guarantee the proper Landau-gauge limit. For backgrounds with vanishing field strength (such as a pure Polyakov loop as studied in [25]), the background propagators are exactly determined.

In this work, we will particularly be interested in backgrounds of nonzero field strengths. For this, we approximate the  $f$  terms by a minimal reconstruction in terms of replacing the dependence on the momentum  $p^2$  of the Landau-gauge propagators by the corresponding background-covariant Laplacians for the transversal, longitudinal and ghost modes,  $p^2 \rightarrow \mathcal{D}_T, \mathcal{D}_L, \mathcal{D}_{\text{gh}}$ , respectively,

$$\mathcal{D}_{T\mu\nu} = -D^2\delta_{\mu\nu} + 2igF_{\mu\nu}, \quad \mathcal{D}_{L\mu\nu} = -D_\mu D_\nu, \quad (4)$$

and  $\mathcal{D}_{\text{gh}} = -D^2$ . Here, the transversal Laplacian also contains the spin-1 coupling to the background field. This leads to the following construction of the background field inverse propagators

$$\Gamma_{k\text{Landau}}^{(2)}(p^2) \rightarrow \Gamma_k^{(2,0)}[0, A] = \Gamma_{k\text{Landau}}^{(2)}(\mathcal{D}), \quad (5)$$

where  $\mathcal{D} = \mathcal{D}_{T,L,\text{gh}}$ , respectively.

For the flow Eq. (1), also the regulator needs to be specified. Background-gauge invariance requires a dependence of the regulator on the background-covariant momentum. RG invariance for nonvanishing IR cutoff, [29,34], and spectral considerations [40], then lead to

$$R_k = \Gamma_k^{(2,0)}(k^2)r(y), \quad y = \frac{\mathcal{D}}{k^2}. \quad (6)$$

The regulator shape function  $r(y)$  encodes the detailed prescription of the momentum-shell integration and will be chosen as  $r(y) = e^{-y}$  here.

It has been shown that the form of the regulator can be used to maximize the physics content within a given approximation [29,41], very similarly to the construction of improved and perfect actions on the lattice. For the numerical computations in the present paper, we use results for Landau gauge propagators [5,42] that have been obtained within functional optimization [29]. For the general considerations, we leave the shape function  $r(y)$  in (5) unspecified.

Finally, it should be emphasized that the use of background field flows is not limited to the pure Yang-Mills sector, but has also successfully been applied in QCD calculations including quark fluctuations [43–46].

### III. EFFECTIVE-ACTION FLOW IN A SELF-DUAL BACKGROUND

We are interested in computing the flow of  $\Gamma_k$  in a derivative expansion, keeping all operators of the type  $(F_{\mu\nu}^a F_{\mu\nu}^a)^n$ , with  $n \in \mathbb{N}$ . This can be summarized in terms of an effective potential  $W(F^2)$  which is an analytic func-

tion of  $F^2 = F_{\mu\nu}^a F_{\mu\nu}^a$ . For this computation, it suffices to evaluate the flow equation in a background of covariantly-constant field strength that allows for a unique identification of  $(F_{\mu\nu}^a F_{\mu\nu}^a)^n$  operators.

To understand why the flow is independent of the specific choice of background, consider theory space, which is the space of all couplings of operators of a given field content, compatible with the chosen symmetries. The flow equation defines a vector field in this space by determining a  $\beta$  function for each running coupling. After a basis is chosen in this space, i.e. the operators are specified uniquely as functions of the field content, a specific field configuration will only serve to distinguish different operators. The  $\beta$  function of a specific running coupling can be evaluated by projecting the flow equation onto a suitable field configuration, but it will not depend on this configuration as long as it uniquely determines the desired operator.

In principle, a covariantly-constant colormagnetic field would be sufficient to project onto the effective potential of the field strength invariant. However, the background has a second meaning in the present formalism: the flow is regularized with respect to the Laplace-type spectra in the given background, see Eq. (6). Purely magnetic backgrounds now suffer from the problem of the tachyonic Nielsen-Olesen mode in the spectrum of fluctuations [47] typically spoiling perturbative computations [48]. The tachyonic mode indicates the instability of the colormagnetic background, which may be quantified by an imaginary part of the effective action as a decay rate. The FRG can indeed deal with this tachyonic mode problem owing to its well-controlled IR regularization [40,44]. Still, the technical complications are substantial and the physics of the tachyonic mode in terms of, e.g., decay rates of constant magnetic backgrounds is not of primary interest.

Instead, we propose to use the only known stable constant background-field, a covariantly-constant self-dual background with  $F_{\mu\nu}^a \equiv \tilde{F}_{\mu\nu}^a$ , as first analyzed in [49–51]. The covariant Laplacian  $\mathcal{D}_T$  has nontrivial zero modes  $a_0$  (“chromons”), as discussed in detail in [50,51], which carry important perturbative contributions, such as a significant contribution to the one-loop  $\beta$  function. As a disadvantage, the self-dual background will not uniquely project onto the desired operators, since  $F_{\mu\nu}^a F_{\mu\nu}^a$  and  $F_{\mu\nu}^a \tilde{F}_{\mu\nu}^a$  become indistinguishable. Whereas this can potentially lead to systematic errors in the determination of the effective potential  $W(F^2)$ , our important conclusions drawn from the  $\beta$  function of the running coupling will be unaffected, see below.

The covariantly-constant self-dual background is given by the gauge potential

$$A_\mu^a = -\frac{1}{2}F_{\mu\nu}x_\nu n^a, \quad n^a = \text{const.}, \quad n^2 = 1, \quad (7)$$

where

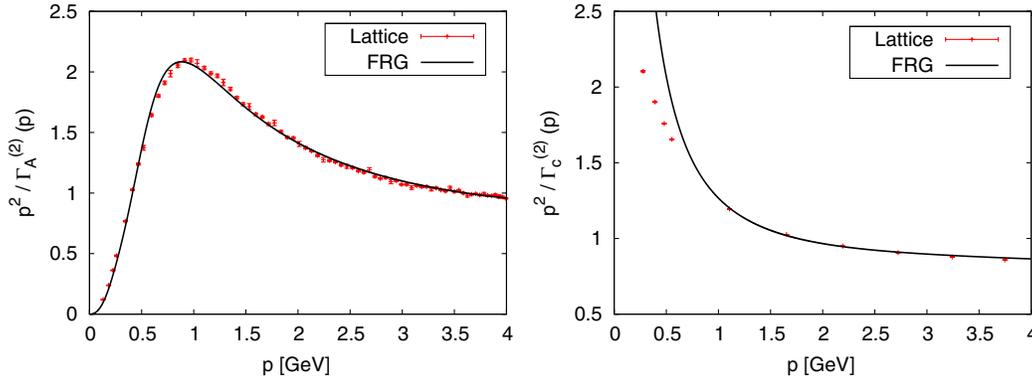


FIG. 1 (color online). Momentum dependence of the gluon (left panel) and ghost (right panel) 2-point functions. We show the FRG results from Ref. [5] (black solid line) and from lattice simulations from Ref. [11] (red points).

$$F_{12} = F_{34} \equiv f = \text{const.} \quad (8)$$

Apart from antisymmetric partner components, all other components are zero and the field strength is of Abelian type as the commutator of two gauge potentials vanishes. Here,  $n^a$  is a constant vector in color space that can be rotated into the Cartan subalgebra. For later use, we define  $\nu_l$  (with  $l = 1, \dots, N_c^2 - 1$ ) as the eigenvalues of  $n^a (T^a)^{bc}$  and  $f_l = |\nu_l| f$ . For the traces, we need the spectral properties of the relevant operators  $\mathcal{D}_T$ ,  $-D^2$  in 4 dimensions, which can be found in Appendix .

In the following, we treat the zero-mode  $a_{0\mu}$ , satisfying  $(\mathcal{D}_T)_{\mu\nu}^a a_{0\nu}^b = 0$ , separately, as it requires special attention for the construction of a suitable regulator. The gauge field is decomposed according to

$$\mathcal{A}_\mu = A_\mu + a_{T\mu} + a_{L\mu} = A_\mu + a_{T\mu}^l + a_{L\mu} + a_{0\mu}, \quad (9)$$

where  $A_\mu$  is the self-dual background field,  $a_{T,L}$  are transversal and longitudinal gluon modes, and  $a_T^l$  denotes the transversal modes except for the zero mode. Our truncation can then be written as

$$\Gamma_k[a, A] = \int d^4x \left[ \frac{1}{2} a_{T\mu}^a (\Gamma_{k,T}^{(2,0)}(\mathcal{D}_T))_{\mu\nu}^{ab} a_{T\nu}^b + \bar{c}^a (\Gamma_{k,\text{gh}}^{(2,0)})^{ab} c^b + \frac{1}{2} a_{L\mu}^a (\Gamma_{k,L}^{(2,0)}(-D^2))_{\mu\nu}^{ab} a_{L\nu}^b \right], \quad (10)$$

where  $c, \bar{c}$  denote the ghost fields.

The inverse propagators for gluons and ghosts are taken from Landau-gauge calculations. In the deep infrared, a family of solutions exists that is parameterized by the infrared boundary condition for the ghost propagator [5,52]. They can be written as

$$\begin{aligned} \Gamma_{k \rightarrow 0,A}^{(2,0)}[0,0](p^2) &= p^2 Z_A(p^2) \Pi_T(p) \mathbb{1}, \\ \Gamma_{k \rightarrow 0,c}^{(2,0)}[0,0](p^2) &= p^2 Z_c(p^2) \mathbb{1}. \end{aligned} \quad (11)$$

The transversal projector satisfies  $\Pi_{T\mu\nu}(p) = \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}$ , and the identity is understood to apply to color indices. Herein the wave function renormalizations

$$Z_A(p^2 \rightarrow 0) \simeq (p^2)^{\kappa_A}, \quad Z_c(p^2 \rightarrow 0) \simeq (p^2)^{\kappa_c} \quad (12)$$

are parameterized by critical exponents  $\kappa_A$  and  $\kappa_c$  in the deep infrared. In the Landau gauge, the longitudinal propagator remains trivial as the longitudinal modes decouple completely from the flow.

The so-called *decoupling solution*, found in lattice studies of Landau-gauge Yang-Mills theory [11], and also in continuum studies, [5,17], is characterized by the critical exponents

$$\kappa_A = -1, \quad \text{and} \quad \kappa_c = 0, \quad (13)$$

which corresponds to a positivity-violating gluon propagator [5,53], indicating the confinement of gluons.

The scaling solution, first identified in [1], has only one independent critical exponent  $\kappa_c$ , as the nonrenormalization theorem for the ghost-gluon vertex [54] (proven to all-orders in perturbation theory) implies the sum rule

$$\kappa_A = -2\kappa_c \quad (14)$$

in  $d = 4$  dimensions [2,4]. In most functional computations, we are led to ( $d = 4$ )

$$\kappa_c = 0.59535 \dots \quad \text{and} \quad \kappa_A = -2\kappa_c = -1.1907 \dots, \quad (15)$$

being the value for the optimized regulator [7]. The regulator dependence in FRG computations leads to a  $\kappa_c$  range of  $\kappa_c \in [0.539, 0.595]$ , see [7]; for a specific flow, see [8]. In Fig. 1, we show the momentum dependence of the ghost- and gluon propagator as obtained from an FRG study [5] in comparison to lattice results [11].

The scaling solution satisfies both the Kugo-Ojima confinement criterion  $\kappa_c > 0$  featuring an infrared enhanced ghost [12], as well as the Gribov-Zwanziger scenario [13,14]. The latter relates color confinement to horizon

conditions for gluons and ghosts, which for the scaling solution is satisfied if

$$\kappa_c > \frac{1}{2}. \quad (16)$$

The gauge correlation functions can also be used to relate color confinement to quark confinement: as shown in [25], the infrared behavior of the propagators determines the effective potential of a Polyakov-loop order parameter for the deconfinement phase transition. From the behavior of the effective potential, a sufficient criterion for quark confinement can be deduced, implying

$$\kappa_c > \frac{1}{4} \quad (17)$$

for the scaling solution. The decoupling solution also satisfies a corresponding confinement criterion. Hence, both the decoupling and the scaling solution for the propagators correspond to scenarios where all color charges are confined. The scaling solution, however, is the only solution which is compatible with global BRST invariance, see [5,55]. For further work on the infrared behavior of Landau-gauge Yang-Mills theory see, e.g., [18].

The asymptotic forms of the propagators can be parameterized as

$$\Gamma_{k,A}^{(2,0)}(p^2) = \Gamma_{k,A}^{(2)}(p^2)\Pi_T(p)\mathbb{1}, \quad \Gamma_{k,c}^{(2,0)}(p^2) = \Gamma_{k,c}^{(2)}(p^2)\mathbb{1}, \quad (18)$$

where the scalar functions  $\Gamma_{k,A/c}^{(2)}(p^2)$  are the IR-regularized generalizations of Eq. (11) [42],

$$\begin{aligned} \Gamma_{k,A}^{(2)}(p^2) &= \gamma_A \frac{(p^2 + c_A k^2)^{1+\kappa_A}}{(\Lambda_{\text{QCD}}^2)^{\kappa_A}}, \\ \Gamma_{k,c}^{(2)}(p^2) &= \gamma_c \frac{p^2(p^2 + c_c k^2)^{\kappa_c}}{(\Lambda_{\text{QCD}}^2)^{\kappa_c}}. \end{aligned} \quad (19)$$

Here, the  $\gamma_{A,c}$ 's are simple proportionality constants which account for the difference between  $\Lambda_{\text{QCD}}$  and the scale where this asymptotic form takes over. The constants  $c_{A,c}$  are manifestations of the regulator dependence in the asymptotic regime; generically, they are of order  $\mathcal{O}(1)$ . In the absence of any IR regularization, i.e.,  $k \rightarrow 0$  or  $c_{A,c} \rightarrow 0$ , Eq. (19) reduces to the standard form, cf. Eqs (11) and (12).

For the regularization of the zero modes, some care is required, since the seeming standard choice  $\mathcal{D} = \mathcal{D}_T$  would imply that the regulator function  $r(y)$  is always evaluated at the singular point  $y = \mathcal{D}/k^2 \rightarrow 0$  (because  $\mathcal{D}_T = 0$  on the zero-mode subspace). In particular, this would not properly account for the decoupling of the zero mode, once it acquires a field-dependent mass, which already happens perturbatively [50]. Instead, we choose  $\mathcal{D} = -D^2$  as the argument of the regulator on the zero-mode subspace which makes the regulator satisfy all

standard requirements. On the zero-mode subspace, we have  $\mathcal{D} = -D^2 \rightarrow 2f_l$ , cf. first line of Eq. (A1) for  $n = m = 0$ .

The flow equation for the background potential in the self-dual background can finally be written as

$$\begin{aligned} \partial_t \Gamma_k[A] &= \frac{3}{2} \text{tr} \partial_t R_{k,A}(-D^2)(\Gamma_{k,A}^{(2)} + R_{k,A}(-D^2))^{-1} \\ &\quad - \text{tr} \partial_t R_{k,c}(-D^2)(\Gamma_{k,c}^{(2)} + R_{k,c}(-D^2))^{-1} \\ &\quad + \frac{1}{2} \text{tr} \partial_t R_{k,L}(-D^2)(\Gamma_{k,L}^{(2)} + R_{k,L}(-D^2))^{-1} \\ &\quad + \frac{1}{2} \text{tr} P_0 \partial_t R_{k,A}(-D^2)(\Gamma_{k,A}^{(2)} + R_{k,A}(-D^2))^{-1}, \end{aligned} \quad (20)$$

where  $P_0$  denotes the projector onto the zero-mode subspace. The zero-mode trace simply corresponds to the terms  $m = n = 0$  in the spectrum (A1). The subscripts  $A$ ,  $c$  and  $L$  in the regulator function imply the use of the appropriate transversal, ghost or longitudinal two-point function in its argument.

The trace measure factor in Eq. (20) corresponds to the density of states in a self-dual background field, given by  $[f_l/(2\pi)]^2$  [50].

#### IV. RELATION BETWEEN THE $\beta$ FUNCTION AND THE CRITICAL EXPONENTS

In the following, we will investigate the asymptotic form of the  $\beta$  function of the background running coupling, defined as the prefactor of the  $F_{\mu\nu}^a F_{\mu\nu}^a$  term:

$$\Gamma_k[A] = \int d^4x \frac{1}{4g^2} F_{\mu\nu}^a F_{\mu\nu}^a + \dots \rightarrow \Gamma[f] = \Omega \frac{f^2}{g^2}, \quad (21)$$

where  $\Omega$  denotes the spacetime volume. For asymptotic freedom, the Gaussian fixed point of the  $\beta$  function has to be UV attractive (i.e. IR repulsive). On the other hand, a minimum requirement for the confinement of color charges is an interacting theory in the long-range limit. This requirement is incompatible with an IR-attractive Gaussian fixed point. As we show below, this seemingly elementary condition imposes a new confinement criterion on the exponents  $\kappa_A$  and  $\kappa_c$ .

According to Eq. (21), the  $\beta$  function can be evaluated from the first term in a Taylor expansion of the effective action in powers of the background field

$$\beta_{g^2} := \partial_t g^2 = -g^4 \partial_t \frac{1}{g^2} = -\frac{g^4}{\Omega} \partial_t \Gamma_k[A]|_{f^2}. \quad (22)$$

Here,  $|_{f^2}$  denotes the projection onto the Taylor coefficient at order  $f^2$ . Although there can be ambiguities in the projection onto  $(F_{\mu\nu} F_{\mu\nu})^n$  and  $(F_{\mu\nu} \tilde{F}_{\mu\nu})^n$  for a self-dual background, this does not affect the  $\beta$  function, as the second operator is only nonzero for even powers  $n$  due to parity conservation.

For the evaluation of Eq. (22), some care is needed as the trace over the Laplace-type spectra and the projection onto the  $f^2$  order do not commute. As the degeneracy factor in the trace already carries a factor of  $\frac{f^2}{(2\pi)^2}$ , all  $f$  dependence outside the operator trace can already be ignored due to the projection, yielding

$$\begin{aligned} \partial_t \Gamma_k[A] \Big|_{f^2} = & \left[ \left( \frac{3}{2} \frac{\partial_t \Gamma_{kA}^{(2)}(k^2)}{\Gamma_{kA}^{(2)}(k^2)} \left( \frac{\Gamma_{kA}^{(2)}(0)}{\Gamma_{kA}^{(2)}(k^2)} + 1 \right) \right)^{-1} \right. \\ & - \frac{\partial_t \Gamma_{k,c}^{(2)}(k^2)}{\Gamma_{k,c}^{(2)}(k^2)} \left( \frac{\Gamma_{k,c}^{(2)}(0)}{\Gamma_{k,c}^{(2)}(k^2)} + 1 \right)^{-1} + 1 \Big] \text{tr} e^{-(-D^2/k^2)} \\ & + \frac{\partial_t \Gamma_{kA}^{(2)}(k^2)}{\Gamma_{kA}^{(2)}(k^2)} \left( \frac{\Gamma_{kA}^{(2)}(0)}{\Gamma_{kA}^{(2)}(k^2)} + 1 \right)^{-1} \text{tr} e^{-2f_l/k^2} \Big] \Big|_{f^2}. \end{aligned} \quad (23)$$

To obtain the  $\beta$  function, we extract the coefficient of the expansion of the heat-kernel trace over coordinate and color space at second order in  $f$ ,

$$\begin{aligned} \text{Tr}_{xc} e^{-(-D^2/k^2)} &= \frac{\Omega}{(4\pi)^2} \sum_{l=1}^{N_c^2-1} \frac{f_l^2}{\sinh^2(\frac{f_l}{k})} \\ &= \frac{\Omega}{(4\pi)^2} \sum_l \left( 1 - \frac{f_l^2}{3} + \mathcal{O}(f^4) \right). \end{aligned} \quad (24)$$

Evaluating  $\partial_t \Gamma_{k,A/c}^{(2)}$  with the infrared asymptotic form of the inverse propagator Eq. (19) yields the infrared form of the  $\beta$  function, whereas using the perturbative form of the inverse propagators  $\Gamma^{(2)}(p^2) \sim p^2$  results in the standard one-loop form of the  $\beta$  function. Recovering this universal term within our setting can be viewed as a simple check of our formalism.

Focusing on the infrared asymptotic form of the propagators Eq. (19), we obtain to lowest order in the background-field coupling

$$\begin{aligned} \beta_{g^2} = & -\frac{N_c g^4}{(4\pi)^2} \left( -\frac{(1+\kappa_A)}{c_\kappa+1} + \frac{2}{3}(1+\kappa_c) \right. \\ & \left. - \frac{1}{3} + 8 \frac{1+\kappa_A}{c_\kappa+1} \right), \end{aligned} \quad (25)$$

where we have used that  $\sum_l^{N_c^2-1} \nu_l^2 = N_c$ . Here, we have defined

$$c_\kappa = \frac{\Gamma_{kA}^{(2)}(0)}{\Gamma_{kA}^{(2)}(k^2)} = \left( \frac{c_A}{1+c_A} \right)^{1+\kappa_A}. \quad (26)$$

Incidentally, the perturbative one-loop  $\beta$  function  $\beta_{g^2} = -\frac{22}{3} \frac{N_c g^4}{(4\pi)^2}$  is obtained from Eq. (25) in the limit of perturbative critical exponents  $\kappa_{A,c} \rightarrow 0$  and in the absence of an additional IR regularization  $c_{A,\kappa} \rightarrow 0$ .

For retaining an interacting theory in the long-range limit, the  $\beta$  function near the Gaussian fixed point must not change sign. This gives us a new bound on the IR values of the  $\kappa$ 's,

$$-\frac{(1+\kappa_A)}{c_\kappa+1} + \frac{2}{3}(1+\kappa_c) - \frac{1}{3} + 8 \frac{1+\kappa_A}{c_\kappa+1} > 0. \quad (27)$$

For the accidental case of an equality, the sign of the  $\beta$  function would have to be determined from higher-order terms in  $g^2$  and a corresponding additional bound would follow from the prefactor of these higher-order terms.

Let us first concentrate on the scaling solution: Using the sum rule  $\kappa_A = -2\kappa_c$ , this bound yields a critical value of  $\kappa_{c,\text{crit}}(c_A)$  as a function of the regulator-dependent constant  $c_A$ . For an interacting IR theory, the true critical exponent has to satisfy

$$\kappa_c < \kappa_{c,\text{crit}}(c_A), \quad (28)$$

such that our criterion imposes for the first time a relevant upper bound on the Landau-gauge exponents. The strongest bound (minimum upper bound) is achieved in the limit of  $c_A \rightarrow \infty$ , where

$$\kappa_{c,\text{crit}}(c_A \rightarrow \infty) = \frac{23}{38} \simeq 0.6053. \quad (29)$$

The maximum upper limit for  $\kappa_{c,\text{crit}}$  is reached for  $c_A \approx 0.1073$ , where  $\kappa_{c,\text{crit}} \approx 0.72767$ . For completeness, let us mention that, for values of  $c_A \lesssim 0.2$ , the inequality Eq. (27) can also be satisfied if  $\kappa_c > \kappa_{c,\text{crit}2}$ , as the non-linear inequality Eq. (27) bifurcates. We find that allowed values for  $\kappa_c$  lie to the left of the red/upper curve (see right panel of Fig. 2). For  $0 \leq c_A \lesssim 0.1073$ , only the bound  $\kappa_c < 1$  from unitarity [2] remains. However, let us stress that the limit  $c_A \rightarrow 0$  corresponds to a highly asymmetric regularization as the contribution from transverse gluons to the  $\beta$  function is removed in this limit (note that  $c_\kappa \rightarrow \infty$  for  $c_A \rightarrow 0$  and  $\kappa_A < -1$ ). For this case, the Gaussian fixed point is naturally IR repulsive for all values of  $\kappa_c > \frac{1}{2}$ .

To summarize, our argument based on the mere existence of an interacting IR regime imposes a bound on the IR critical exponent  $\kappa_c < \kappa_{c,\text{crit}}$ . Both sides of this inequality are regulator-dependent, such that the bound has to be satisfied in every regularization scheme as a necessary criterion. A sufficient criterion is therefore given by imposing that  $\kappa_c < \kappa_{c,\text{crit},\text{min}} = 23/38 \simeq 0.6053$  holds for all regularization schemes.

This bound is furthermore required to be satisfied due to the following argument: an IR fixed point for the background-field running coupling is observed in the background-field gauge in a derivative expansion [40] as well as for the running coupling fixed at the ghost-gluon vertex in the Landau gauge in a vertex expansion [1,2,4]. Both running couplings are defined with the aid of non-renormalization theorems arising from gauge invariance. From the conjecture that both running couplings are

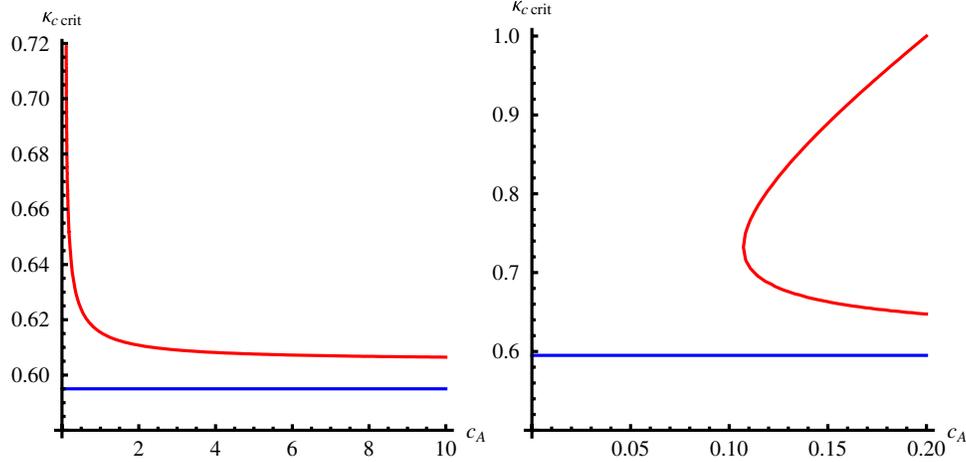


FIG. 2 (color online).  $\kappa_{c,\text{crit}}$  as a function of  $c_A$ . Left panel: continuous branch of  $\kappa_{c,\text{crit}}$  that exists for all  $c_A$  (all regularization schemes); for the  $c_A$  values displayed here,  $c_A \gtrsim \mathcal{O}(1)$ ,  $\kappa_{c,\text{crit}}$  represents an upper bound for the ghost critical exponent  $\kappa_c$ . Right panel: for  $0.1 \leq c_A \leq 0.2$ , the bound bifurcates, such that all  $\kappa_c$  values to the left of the red/upper curve are allowed. The horizontal line marks the value  $\kappa_c \simeq 0.595$  as found in most functional calculations for the scaling solution which obviously satisfies our confinement criterion.

actually linked on all scales, consistency requires the existence of the fixed point in both gauges. Even though such a fixed point is not observed in the simplified truncation leading to Eq. (22), it is expected to arise in a larger truncation as used in [40]. But already without knowing the full IR flow of the background field coupling, consistency of the leading-order result of Eq. (25) with the Landau-gauge coupling gives rise to the bound on the scaling exponent. This consistency would be spoiled if the above first coefficient of the  $\beta$  function changed sign. Therefore, we infer that  $\kappa_c < \frac{23}{38}$  is also required for the consistency between the background-gauge and the Landau-gauge results, independently of higher-order terms that generate an IR fixed point of the coupling.

Physically, our upper bound for  $\kappa_c$  may be interpreted as a criterion for an IR interacting theory and thus ultimately for confinement. We conclude that for  $\kappa_c$  larger than some critical value Yang-Mills theory would not be confining. Therefore, it is very reassuring that all results for  $\kappa_c$  obtained by functional methods satisfy the sufficient criterion bound  $\kappa_c < \frac{23}{38}$ , c.f. Eq. (15).

It is important to note that also the decoupling solution for the propagators  $\kappa_A = -1$ ,  $\kappa_c = 0$  that has been found in many lattice simulations (and can possibly be understood as a different way to deal with the gauge ambiguity in the Landau-gauge, see [52]) fulfills our confinement criterion which in this case is given by the more general bound Eq. (27).

## V. GLUON CONDENSATION FROM THE EFFECTIVE POTENTIAL

Dimensional transmutation and the scale anomaly going along with the quantization of Yang-Mills theory require a

nonzero expectation value for the energy-momentum tensor. This implies a nontrivial vacuum structure:

$$\langle F^2 \rangle \neq 0. \quad (30)$$

Indeed, phenomenological estimates indicate a nonzero value [56], being interpreted as a condensation of gluons in the vacuum. These findings have been corroborated by other methods [57,58], as well as lattice gauge theory [59].

The effective action for a colormagnetic background field has been evaluated at one-loop order in [60], where a nontrivial minimum has been found (Savvidy vacuum). Because of the tachyonic mode of the gluon propagator on such a background, this configuration is unstable [47], which can be amended by introducing a spatial inhomogeneity into the field configuration. This vacuum configuration is referred to as the Copenhagen vacuum [47,61–63]. A more severe problem is posed by the fact that the minimum lies in the nonperturbative domain, i.e. beyond the perturbative Landau pole of the Yang-Mills coupling, questioning the relevance of a perturbative estimate.

First nonperturbative studies from functional methods indeed provided indications for gluon condensation [36].

Here we will further pursue this question, based on the knowledge of full correlation functions. As a simple parametrization of these correlation functions, we still use the asymptotic form displayed in Eq. (19), which we amend with the  $k$ -dependent critical exponents  $\kappa_A(k)$  and  $\kappa_c(k)$  in accordance with the propagators in [42]. A suitable interpolation between  $\kappa_{A,c}(k \rightarrow \infty) \rightarrow 0$  and the corresponding IR values  $\kappa_{A,c}(k \rightarrow 0) \rightarrow \kappa_{A,c}$  of Eq. (15) can parameterize the full momentum dependence of the correlation functions. This allows us to evaluate the effective potential from an integrated form of the flow equation

$$W_k(F^2) = -\frac{1}{\Omega} \int_0^{k_{\text{UV}}} \frac{dk}{k} \frac{1}{2} \text{STr} \partial_t R_k (\Gamma_k^{(2)} + R_k)^{-1} + W_{k_{\text{UV}}}(F^2), \quad (31)$$

where  $k_{\text{UV}}$  is an initial ultraviolet scale, and  $F^2 = F_{\mu\nu}^a F_{\mu\nu}^a = 4f^2$ . The initial condition  $W_{k_{\text{UV}}}(F^2)$  is fixed deep inside the perturbative regime at  $k_{\text{UV}} = 10 \text{ GeV}$ ,

$$W_{\text{UV}} = W_{k=10 \text{ GeV}} = \frac{F^2}{16\pi \cdot 0.2294}, \quad (32)$$

where we have used the peak of the maximum of the gluon dressing function  $1/Z_A(p^2)$ , see left panel of Fig. 1, for relating our Yang-Mills (YM) scales to that used in lattice computations. The normalization is such that the related string-tension  $\sigma$  (computed on the lattice) is given by  $\sqrt{\sigma} = 440 \text{ MeV}$ . Our initial condition is thus self-consistently determined from our main input, the Landau-gauge ghost and gluon propagators, which goes along with a coupling  $\alpha_s(k = 10 \text{ GeV}) \simeq 0.2294$  at the initial scale. At this initial cutoff scale,  $k = 10 \text{ GeV}$ , higher-order operators do not contribute significantly to the effective potential, which is determined exclusively by the functional form of the classical action. Alternatively, an RG-improved initial condition in the form of the one-loop effective potential could be used.

Equation (31) can then directly be integrated numerically. For SU(3), we find a nontrivial minimum at

$$F^2 \approx 0.93 \text{ GeV}^4 = (3.46 \Lambda_{\text{QCD}})^4, \quad (33)$$

where we have used  $\Lambda_{\text{QCD}} = 284 \text{ MeV}$ , cf. solid line in Fig. 3.

We emphasize that the formation of the nontrivial minimum is mainly driven by the dynamics in the midmomentum regime, the form of the propagators in the deep infrared is not crucial here. The existence of a gluon condensate in the vacuum thereby occurs for both the scaling and the decoupling solution, and its value does not significantly depend on the infrared asymptotics. This agrees with the observation in [25], where quark confinement can also be inferred from the propagators, independent of their infrared asymptotics.

Note that on the right-hand side of the flow equation, the color eigenvalues  $|\nu_l|$  enter. We may choose these along one of the two directions of the Cartan subalgebra, which yields an uncertainty of about 10% in the value of the minimum.

As a simple parametrization, we fit the effective potential to a function of the form

$$W(F^2) = aF^2 \ln(bF^2), \quad (34)$$

which is inspired by the corresponding one-loop results [60] with two fit parameters  $a$  and  $b$ , cf. dashed line in Fig. 3). The fit yields  $a = 0.00528$  and  $b = 0.433 \text{ GeV}^{-4}$ .

With some reservations, the one-loop result might thus be interpreted as providing indeed a reasonable qualitative

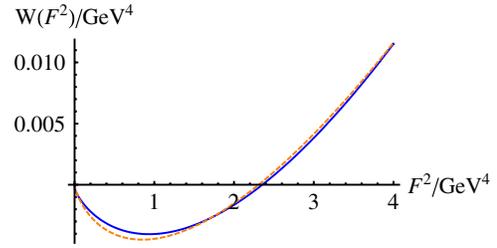


FIG. 3 (color online). The effective potential for SU(3) as a function of  $F^2$  (thick blue line), and the one-loop inspired fit to the numerical data of the form  $aF^2 \ln bF^2$  (orange dashed line).

estimate of the true nonperturbative functional form of the effective potential. Let us stress once more that the one-loop calculation is, of course, not reliable, as the predicted minimum lies in a regime inaccessible to perturbation theory. It is only its qualitative prediction of the functional form of the potential which seems to be altered little by nonperturbative corrections.

A few comments on the numerical values for the gluon condensate are in order: first of all, as the self-dual background does not distinguish between the invariants  $F^2$  and  $F\tilde{F}$ , our result for the condensate receives contributions from both operators; under the assumption that condensates of both types exist, the value given in Eq. (33) should be considered as an upper bound for the phenomenologically more relevant  $F^2$  condensate. In this sense, our result  $\langle F^2 \rangle / 4\pi \simeq 0.074 \text{ GeV}^4$  is well compatible with recent phenomenological estimates,  $\langle F^2 \rangle / 4\pi \simeq 0.068(13) \text{ GeV}^4$  from spectral sum rules [64] (note that our field definition differs from that of [64] by a rescaling with the coupling, cf. Eq. (21)). The good agreement might even indicate that the contribution from  $F\tilde{F}$  operators is rather small. Also, we expect the inclusion of dynamical quark degrees of freedom to decrease our condensate value slightly, owing to their screening property.

We have used a background that can only be considered as an approximation of the true ground state locally. For studies involving a more realistic gauge-field configuration than the one considered here, see [65] and references therein.

Within the *leading-log* model which is based on the assumption that the infrared effective action is essentially given by its one-loop functional form (34), the condensate value is related to the string tension in a simple manner [19]: the static potential between two opposite color charges arising from the nonlinear field equations following from Eq. (34) grows linearly with distance,  $V = \sigma r$ , where the string-tension  $\sigma$  and the minimum of the action obey,  $\sqrt{\sigma} = (Q\kappa)^{1/4}$ . Here,  $Q = 4/3$  is a simple color factor for SU(3), and  $\kappa = (1/4)F^2|_{\text{min}}$  denotes the minimum of the action, i.e. the condensate value. From our result (33), we obtain the estimate

$$\sigma^{1/2} \simeq 747 \text{ MeV} \quad (35)$$

which, in view of the rather restrictive truncation and the simplicity of the leading-log model, compares rather favorably with the value  $\sigma^{1/2} \simeq 440$  MeV which we used for fixing the scale of the propagators in the first place. We should stress that the string-tension  $\sigma$  appears in two very different meanings in our calculation: It first occurs as an input scale for fixing the initial condition for the flow equation, i.e., it fixes the absolute scale of the propagators. We then derive the *physical* string tension in a nontrivial way from the minimum of the effective potential via the leading-log model. This output is linked to a mechanism of confinement, and has therefore acquired a physical meaning beyond pure scale fixing. With regard to the approximations involved in our calculation and, in particular, in the confinement model used to map our results onto the physical string tension, the order-of-magnitude equivalence of “input scale” and “output physics” is satisfactory.

## VI. CONCLUSIONS

Based on the knowledge of low-order gauge correlation functions from Landau-gauge calculations, we have used an approximate mapping onto propagators in the Landau-DeWitt gauge in a background field. This allowed us to extract nonperturbative information about the effective action of Yang-Mills theories. We have specialized to a self-dual constant background which—apart from technical simplicity—is known to be stable against fluctuations, and thus a candidate for at least local approximations of the Yang-Mills vacuum.

We have concentrated on two properties of the effective action: the running of the background field coupling and the form of the effective potential for the self-dual field strength. From the simple requirement that the background field coupling should remain finite, and the theory thus interacting, in the long-range limit, we have derived a nontrivial criterion for the infrared behavior of Landau-gauge correlation functions. As interactions are mandatory for confinement, our criterion may be viewed as a confinement criterion. For the scaling solution of these correlation functions which is characterized by a single critical exponent  $\kappa_c$ , this criterion translates into a new upper bound on  $\kappa_c$ . In its sufficient version, the confinement criterion is satisfied if

$$\kappa_c < \kappa_{c,\text{crit},\text{min}} = \frac{23}{38} \simeq 0.6053, \quad (36)$$

where  $\kappa_{c,\text{crit},\text{min}}$  is a minimum value obtained in a specific regularization scheme. If  $\kappa_c$  satisfies this bound in any scheme, the system remains interacting in the infrared. In general, both sides of the inequality are regularization-scheme dependent, such that the condition that  $\kappa_c[R_k] < \kappa_{c,\text{crit}}[R_k]$  is a necessary criterion for confinement which then has to be satisfied for each regulator  $R_k$ .

Read together with the confinement criterion for quark confinement as derived from the Polyakov loop potential in [25],  $\kappa_c > \frac{1}{4}$ , and the horizon condition for color confinement [14],  $\kappa_c > \frac{1}{2}$ , our new criterion defines a window for the infrared critical exponent,  $0.5 < \kappa_c \leq 0.6053$ , thereby tightly fixing the possible infrared behavior of the gauge correlation functions for the scaling solution in the infrared. It is reassuring that the result  $\kappa_c \simeq 0.595$  obtained from many functional calculations lies precisely in this window.

Furthermore, we numerically investigate the full effective potential for the field strength invariant  $F^2 = F_{\mu\nu}^a F_{\mu\nu}^a$  and find a nontrivial minimum at  $F^2 \simeq 0.93$  GeV<sup>4</sup>. Our calculation is fully nonperturbative, and therefore supports the conclusions drawn from the one-loop effective action [60] in a nontrivial way.

Most importantly, effective models of confinement such as the leading-log model or dielectric confinement models receive strong support from our results. These models map the nontrivial vacuum structure onto classical field equations. For instance, it can be shown rather straightforwardly in the leading-log model that a nontrivial minimum of the effective action at nonzero  $F^2$  is already sufficient to produce a linearly rising potential for static color charges at long distances [19]. Our calculation therefore provides for an explicit example how different pictures of confinement can not only exist in parallel but actually support each other, potentially being two sides of the same coin. A similar observation has been made for the Coulomb gauge in the Hamiltonian approach and the Gribov-Zwanziger confinement scenario [66].

Quantitatively, our estimate of the self-dual gluon condensate  $F^2 \simeq 0.93$  GeV<sup>4</sup> compares favorably with phenomenological estimates [64] ( $F^2 \simeq 0.85$  GeV<sup>4</sup>). It is also in agreement with the large-distance limit of the static potential between a quark and antiquark, yielding a string tension of  $\sigma^{1/2} \simeq 747$  MeV within the leading-log model of confinement [19]. Our nonperturbative calculations, starting from the microscopic theory, thus can be interpreted as providing a fundamental justification of the two basic assumptions of the leading-log model: the functional form of the effective action (34) and the free parameter given in terms of the minimum of the effective action, i.e., the gluon condensate. Our studies therefore provide an explicit example of how seemingly different confinement scenarios, the Kugo-Ojima and Gribov-Zwanziger scenario on the one hand, and the leading-log model on the other hand, can not only exist in parallel but actually support each other.

Our results are affected by several sources of uncertainties: first of all, the mapping of Landau-gauge propagators onto background-field propagators is not unique, but has been realized in a minimal-coupling fashion. Whereas we have concentrated onto lowest-order correlation functions in the Landau gauge, also higher orders contribute to the

full effective potential in the background gauge. In principle, it is straightforward to include such higher-order correlation functions, as, e.g., computed in [67], into our formalism. Also, differences between gluonic field invariants, such as  $(F^2)^n$  and  $(F\tilde{F})^n$ , which have been ignored for the self-dual background can be treated with suitable heat-kernel expansion techniques in future studies.

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### APPENDIX A: SPECTRAL PROPERTIES OF LAPLACE-TYPE OPERATORS

Let us summarize the spectral properties of the Laplace-type operators involved. We concentrate here on the particularities of the self-dual background. The general trace technology in covariantly-constant backgrounds as it is relevant for the computations in this paper can be found in [36,40]. The self-dual-background spectra needed in this work are

$$\begin{aligned} \text{Spec}\{-D^2\} &= 2f_l(n+m+1), & n, m &= 0, 1, 2, \dots \\ \text{Spec}\{\mathcal{D}_T\} &= \begin{cases} 2f_l(n+m+2), & \text{multiplicity } 2 \\ 2f_l(n+m), & \text{multiplicity } 2' \end{cases} \quad (\text{A1}) \end{aligned}$$

where  $f_l = |\nu_l|f$ , and  $\nu_l$  denotes the eigenvalues of the adjoint color matrix  $n^a T^a$ . The covariant spin-1 Laplacian  $\mathcal{D}_T$  has a double zero mode for  $n = m = 0$ , which is due to the symmetry between colorelectric and colormagnetic field. Using the trace technology of [36,40], the spectral problem of the longitudinal Laplacian  $\mathcal{D}_L$  can be mapped onto that of  $-D^2$ , such that Eq. (A1) is sufficient for the calculation in the main part of the paper.

Defining  $\text{Tr}'$  as the trace without the zero mode, we make the following useful observation for the trace over some function  $\mathcal{F}$ :

$$\begin{aligned} \text{Tr}'_{xcL} \mathcal{F}(\mathcal{D}_T) &= 2 \sum_{l=1}^{N_c^2-1} \left(\frac{f_l}{2\pi}\right)^2 \left\{ \sum_{n,m=0}^{\infty} \mathcal{F}(2f_l(n+m+2)) \right. \\ &\quad \left. + \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \mathcal{F}(2f_l(n+m)) + \sum_{n=1}^{\infty} \mathcal{F}(2f_l n) \right\} \\ &= 4 \sum_{l=1}^{N_c^2-1} \left(\frac{f_l}{2\pi}\right)^2 \sum_{n,m=0}^{\infty} \mathcal{F}(2f_l(n+m+1)) \\ &= 4 \text{Tr}_{xc} \mathcal{F}(-D^2), \quad (\text{A2}) \end{aligned}$$

where the trace subscripts denote traces over coordinate space ‘‘x,’’ color space c and Lorentz indices ‘‘L.’’ In other words, there exists an isospectrality relation between  $-D^2$  and the nonzero eigenvalues of  $\mathcal{D}_T$ . Similar isospectrality properties are also known for the Dirac operator in a self-dual homogeneous background [68]. As a consequence, all gluon and ghost modes except for the two zero modes couple in the same fashion to the self-dual background. This basic observation has been used for the decomposition of terms in Eq. (20).

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