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Path integral quantization of generalized quantum electrodynamics

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In this paper, a complete covariant quantization of generalized electrodynamics is shown through the path integral approach. To this goal, we first studied the Hamiltonian structure of the system following Dirac's methodology and, then, we followed the Faddeev-Senjanovic procedure to obtain the transition amplitude. The complete propagators (Schwinger-Dyson-Fradkin equations) of the correct gauge fixation and the generalized Ward-Fradkin-Takahashi identities are also obtained. Afterwards, an explicit calculation of one-loop approximations of all Green's functions and a discussion about the obtained results are presented.

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I. INTRODUCTION

The results that have been obtained for known quantum field theories using available theoretical tools are very impressive: the agreement of the QED_4 with experiments, the predictions of the standard model and the QCD_4, and so many others. A point that warrants comment is the effectiveness of such theories up to a determined energy scale. Usually, a physics problem involves widely separated energy scales; this allows us to study the low-energy dynamics independently of the details of the high-energy interactions. The main idea is to identify those parameters that are very large (small) compared to the relevant energy scale of the physical system and let them go to infinity (zero). This provides a sensible approximation to the problem, which can always be improved by taking into account the corrections induced by the neglected energy scales as small perturbations. Effective theories constitute the appropriate theoretical tools to describe low-energy physics, where low is defined with respect to some energy scale. This idea of effective theories was proposed by Weinberg [1].

The set of higher-order theories belongs to such effective theories. As it is known, the majority of physical systems described by Lagrangians depends, at most, on first-order derivatives. However, with the first development in formal aspects of higher-order derivative Lagrangians in classical mechanics by Ostrogradski [2], a new field of research was opened.

This branch of higher-order derivative theories becomes very interesting, due to the fact that these additional terms are constructed in such a way so as to preserve the original symmetries of the problem. As a remark, it is important to say that this kind of theory has been shown to be a powerful method for consistent regularization of the ultraviolet divergences of gauge-invariant and supersymmetric theories [3]. Also, the use of higher derivative terms becomes an interesting regulator, by the fact that it improves the convergence of the Feynman diagrams [4].

More examples of systems treated with higher-order Lagrangians that we can mention are: the study of the problem of color confinement on the infrared sector of QCD_4 [5], the attempts to solve the problem of the renormalization of the gravitational field [6], and a generalization of Utiyma's theory to second-order theories [7]. Although all these works improve the use of higher-order terms, the ones that most contributed to show the effectiveness of such terms in field theory were the contributions of Bopp [8] and Podolsky and Schwed [9], where they proposed a generalization of the Maxwell electromagnetic field. They wanted to get rid of the infinities of the theory, such as the electron self-energy (r^{-1} singularity) and the vacuum polarization current present in the Maxwell theory. The modification suggested by Podolsky and Schwed handles these unsolved problems and, also, gives a positive definite energy in the electrostatic case; also, as shown by Frenkel [10], it gives the correct expression for the self-force of charged particles. In [7], it was shown that the Podolsky Lagrangian is the only possible generalization of Maxwell electrodynamics that preserves invariance under $U(1)$.

On the theoretical and the experimental framework, efforts have been made to determine an upper-bound value for the mass of the photon [11], the existence of a massive sector being a prediction of generalized electrodynamics. Along this line of thought, we believe that a way to set limits over the Podolsky parameter will be to study the Podolsky photons interacting with standard model particles and compare the obtained results with high-energy experiments. This idea and other purposes led Podolsky

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and some of his students to study the interaction of electrons with the Podolsky photons, which they called generalized quantum electrodynamics (GQED_4) [12]. Among the points dealt with in their thesis, the most interesting was the calculation of the electron self-energy at a one-loop approximation. They expected that the contribution of massive photons would lead to a finite result; nevertheless, in the end, they found, as in the usual QED_4, a divergent expression. Analyzing, now, their thesis results, we found a mistake in their treatment of the theory, i.e., the choice of the usual Lorenz gauge condition. However, this analysis was only possible due to the contribution of Galvão and Pimentel [13], which gives the first consistent quantization to the Podolsky theory, where Dirac Hamiltonian formalism [14] was used with the correct choice of gauge condition, which they called the generalized Lorenz gauge condition. Also, they showed that, different from the usual Lorenz condition, the generalized one fulfills all the requirements for a good choice of gauge condition on the context of the Podolsky theory. Indeed, one of the aims of this paper is to quantize the GQED_4, now, in the generalized Lorenz gauge condition. The Podolsky electrodynamics, by itself, takes account of several classical problems of Maxwell's theory, and it should be expected that the addition of the Podolsky term into the QED_4 Lagrangian with an appropriate gauge choice should give rise to interesting results.

Based on all these facts pointed out above, we can conclude that higher-order theories deserve a deeper investigation. Therefore, this paper is intended to give a correct and transparent quantization of the GQED_4 and the interaction of electrons with the Podolsky photons in four-dimensional space-time. To improve our understanding of the features of the GQED_4, we proceeded to calculate the radiative corrections of Green's functions. The main results of the paper will be closed formulas to the complete propagators and the vertex function by functional methods (for an excellent review, see [15]), and it turns out that, with the correct gauge choice, the electron and vertex self-energy functions are finite at the e^2-order approximation.

The work is organized as follows. In Sec. II, we present a brief study of canonical structure of the theory and, then, construct the transition amplitude by the Faddeev-Senjanovic procedure [16], which we believe is the most appropriate for our interests. In Sec. III, we introduce the generating functional, which will generate all the Green's functions, the photon and electron propagators, and the vertex function; also, through it, we will derive the generalized Ward-Fradkin-Takahashi identities in Sec. IV. In Sec. V, we evaluate and discuss the self-energy functions of the theory at the e^2-order approximation. In order to avoid an awful reading, we place most of the calculation in the Appendices, and some useful identities, as well. Our remarks are given in Sec. VI.

II. TRANSITION AMPLITUDE

To construct the transition amplitude, we must first do a constraint analysis. Hence, before using the Faddeev-Senjanovic procedure, we will present a short study, showing the main points of the Hamiltonian structure of the GQED_4: the evaluation of canonical momenta, followed by the determination of first- and second-class constraints, and, at last, the choice of an appropriate set of gauge conditions. However, it will be necessary to use the Faddeev-Popov-DeWitt method to get a covariant expression for the transition amplitude. Thus, we start with the Lagrangian density of the GQED_4, defined by[1]

$$\mathcal{L} = \frac{i}{2}(\bar{\psi}\,\hat{\partial}\,\psi - \bar{\psi}\,\overleftarrow{\hat{\partial}}\,\psi) - m\bar{\psi}\psi + e\bar{\psi}\hat{A}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{a^2}{2}\partial_\mu F^{\alpha\mu}\partial^\beta F_{\alpha\beta}, \tag{1}$$

which, at the classical level, is invariant under the local gauge transformations

$$\psi'(x) = e^{i\lambda(x)}\psi(x), \quad A'_\mu(x) = A_\mu(x) + \frac{1}{e}\partial_\mu\lambda(x). \tag{2}$$

In the Lagrangian (1), we used the following definitions: the field-strength tensor $F_{\nu\mu} \equiv \partial_\nu A_\mu - \partial_\mu A_\nu$, and $\hat{O} \equiv \gamma_\mu O^\mu$. The Lagrangian $\mathcal{L}$ preserves all symmetries of the usual QED. The Euler-Lagrange equations following from the Hamiltonian principle with the corresponding boundary conditions are

$$(i\hat{\partial} + e\hat{A} - m)\psi = 0, \quad (1 + a^2\Box)\partial_\mu F^{\alpha\mu} = e\bar{\psi}\gamma^\alpha\psi. \tag{3}$$

The canonical momenta, π^β and ϕ^β, conjugate to A_α and Γ_α, respectively, where $\Gamma_\alpha \equiv \partial_0 A_\alpha$ are considered as independent variables, defined [13], and given by

$$\begin{aligned}\pi^\mu &\equiv \frac{\partial\mathcal{L}}{\partial\Gamma_\mu} - \partial_0\frac{\partial\mathcal{L}}{\partial(\partial_0\Gamma_\mu)} - 2\partial_k\frac{\partial\mathcal{L}}{\partial(\partial_k\Gamma_\mu)}\\ &= F^{\mu0} - a^2[\eta^{\mu k}\partial_k\partial_\lambda F^{0\lambda} - \partial_0\partial_\lambda F^{\mu\lambda}],\end{aligned} \tag{4}$$

$$\phi^\mu \equiv \frac{\partial\mathcal{L}}{\partial(\partial_0\Gamma_\mu)} = a^2[\eta^{\mu0}\partial_\lambda F^{0\lambda} - \partial_\lambda F^{\mu\lambda}]. \tag{5}$$

The canonical momenta associated with the fermion fields ψ and $\bar{\psi}$ are

$$p_A \equiv \frac{\partial\mathcal{L}}{\partial(\partial_0\bar{\psi}_A)} = \frac{i}{2}(\gamma^0\psi)_A, \tag{6}$$

$$\bar{p}_A \equiv \frac{\partial\mathcal{L}}{\partial(\partial_0\psi_A)} = \frac{i}{2}(\bar{\psi}\gamma^0)_A. \tag{7}$$

From the above momentum expressions, we shall study the constraint structure of the theory following Dirac's

[1] We shall adopt, here, the metric convention $\eta_{\mu\nu} = \mathrm{diag}(+,-,-,-)$; the Greek and Latin indices runs from 0 to 3 and 1 to 3, respectively, and the spinorial indices are represented by capital Latin letters.

approach to singular systems [14]. From Eqs. (4)–(7) and the linear independence of the constraints [14], it is possible to obtain the following set of first-class constraints:

$$\Omega_1 \equiv \phi_0 \approx 0, \qquad \Omega_2 \equiv \pi_0 - \partial_k \phi^k \approx 0, \\ \Omega_3 \equiv \partial_k \pi^k + e\bar{\psi}\gamma^0\psi \approx 0, \tag{8}$$

and a set of second-class ones,

$$\chi_A \equiv p_A - \frac{i}{2}(\gamma^0\psi)_A \approx 0, \\ \bar{\chi}_A \equiv \bar{p}_A - \frac{i}{2}(\bar{\psi}\gamma^0)_A \approx 0, \tag{9}$$

where $\approx$ represents the fact that Eqs. (8) and (9) are weak equations, according to Dirac's procedure. The constraint analysis presented here is justified by the Faddeev-Senjanovic procedure to get the transition amplitude [16]. This point will become clear below.

The transition amplitude in the Hamiltonian form is written in the following way:

$$Z = \int D\mu \exp\biggl(i\int d^4x[\pi^\mu(\partial_0 A_\mu) + \phi_\alpha(\partial_0\Gamma^\alpha) \\ - (\partial_0\psi)\bar{p} - (\partial_0\bar{\psi})p - \mathcal{H}_C]\biggr), \tag{10}$$

where the canonical Hamiltonian $\mathcal{H}_C$ is given by

$$\mathcal{H}_C = \pi_0\Gamma^0 + \pi_j\Gamma^j + \frac{\phi_l\phi^l}{2a^2} + \phi_l\partial^l\Gamma_0 + \phi_l\partial_k F^{lk} \\ - \frac{i}{2}\bar{\psi}\gamma^j\overset{\Leftrightarrow}{\partial}_j\psi + m\bar{\psi}\psi - e\bar{\psi}\hat{A}\psi + \frac{1}{4}F_{kj}F^{kj} \\ + \frac{1}{2}(\Gamma_j - \partial_j A_0)^2 - \frac{a^2}{2}(\partial^j\Gamma_j - \partial_j\partial^j A_0)^2. \tag{11}$$

The integration measure is defined by

$$D\mu = D\phi_\nu D\Gamma^\nu D\pi^\mu DA_\mu D\bar{\psi}D\psi D\bar{p}Dp\,\delta(\Theta_l) \\ \times \det\|\{\Omega_a, \Sigma_b\}_B\| \det\|\{\chi_A, \bar{\chi}_B\}_B\|^{-1/2}, \tag{12}$$

where $\Theta = \{\Omega, \Sigma, \chi, \bar{\chi}\}$, and the functionals Σ are the gauge conditions that fix the first-class constraints. Here, we will use the generalized radiation gauge condition

$$\Sigma_1 \equiv \Gamma_0 \approx 0, \qquad \Sigma_2 \equiv A_0 \approx 0, \\ \Sigma_3 \equiv (1 + a^2\Box)\partial^k A_k \approx 0, \tag{13}$$

which, as it is shown in [13], is an appropriate set of noncovariant gauge conditions for the first-class constraints (8). We notice that the determinant associated with the second-class constraints, $\det\|\{\chi_A, \bar{\chi}_B\}_B\|$, does not contain field variables, and so it can be absorbed in a normalization constant; we also show that the determinant between the first-class constraints (8) and the gauge-fixing conditions (13) has the form

$$\det\|\{\Omega_\alpha, \Sigma_\beta\}_B\| = -(1 + a^2\nabla^2)\nabla^2. \tag{14}$$

Therefore, through the following manipulations—combining Eqs. (12) and (14), substituting them into (10), and also carrying out momenta integrals and field variables—we find the following expression for the transition amplitude:

$$Z = \int DA_\mu D\bar{\psi}D\psi \det\| - (1 + a^2\nabla^2)\nabla^2\| \\ \times \delta[(1 + a^2\Box)\partial^k A_k]\exp\biggl(i\int d^4x\mathcal{L}\biggr). \tag{15}$$

Although Eq. (15) is correct, the noncovariant form is not good for calculation purposes. However, we can use the ansatz of Faddeev-Popov-DeWitt [17] to achieve the desired covariant form for the transition amplitude. Then, choosing the generalized Lorenz gauge condition [13]

$$\Omega[A] = (1 + a^2\Box)\partial^\mu A_\mu = 0, \tag{16}$$

we finally obtain a expression for the covariant vacuum-vacuum transition amplitude:

$$Z = \int DA_\mu D\bar{\psi}D\psi \det\| - (1 + a^2\Box)\Box\| \\ \times \exp\biggl(i\int d^4x\biggl[\bar{\psi}(i\hat{\partial} - m + e\hat{A})\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ + \frac{a^2}{2}\partial^\mu F_{\mu\beta}\partial_\alpha F^{\alpha\beta} - \frac{1}{2\xi}\{(1 + a^2\Box)\partial^\mu A_\mu\}^2\biggr]\biggr). \tag{17}$$

In this covariant gauge choice, we see that the Faddeev-Popov-DeWitt determinant does not contain field variables (the ghosts decouple from the gauge fields), and so, it can be absorbed into a normalization constant.

III. SCHWINGER-DYSON-FRADKIN EQUATIONS

There are a lot of ways to extract the physical content of quantum field models, but the most elegant one is from the Green's functions using functional derivatives, which is a natural way to obtain such functions. The method of functional derivatives, which has been largely used by Schwinger, among others [18,19], uses a generating functional from which all of Green's functions can be obtained by functional differentiation. These equations are also known as Schwinger-Dyson-Fradkin equations (SDFEs), and the motivation to construct the SDFEs is the non-perturbative information that is provided for the theory. However, if we regard these equations only as a source of obtaining formal expansions in powers of the coupling constant, we shall obtain nothing new in comparison with perturbation theory. The problem of finding an effective method of solving those equations not based on perturbation theory is, at present, still far from any sort of satisfactory solution.

In the present section, we will derive these relations for the photon and electron fields, and also for the vertex function. The first step is to define the generating functional

$$Z[\eta,\bar{\eta},J_\mu]=\int D\mu(\psi,\bar{\psi},A_\mu)\exp[i\mathcal{S}_{\rm eff}], \tag{18}$$

with the effective action given by

$$\mathcal{S}_{\rm eff}=\int d^4x\bigg[\bar{\psi}(i\hat{\partial}-m+e\hat{A})\psi-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{a^2}{2}\partial^\mu F_{\mu\beta}\partial_\alpha F^{\alpha\beta}-\frac{1}{2\xi}\{(1+a^2\Box)\partial^\mu A_\mu\}^2+\bar{\psi}\eta+\bar{\eta}\psi+A^\mu J_\mu\bigg], \tag{19}$$

where $\bar{\eta}$, η, and J_μ are the sources (auxiliary mathematical devices) for the fermion ψ, the antifermion $\bar{\psi}$, and the gauge A_μ fields, respectively. Let us stress that the components of fermionic fields $(\bar{\psi},\psi)$ and their sources $(\eta,\bar{\eta})$ are elements of the Grassmann algebra, and that A_μ and its source J_μ are c-numbers. From the generating functional (18), all the physical quantities of the theory can be obtained. Whenever possible, we will discuss the meaning of expressions of GQED$_4$ and also its points of equivalence or inequivalence with the known results of QED$_4$.

A. Schwinger-Dyson-Fradkin equation for the photon propagator

We will derive and discuss, here, the properties of the complete expression of the gauge-field propagator in interaction with electrons. First, to obtain the corresponding photon SDFE, we need to solve the following equation:

$$0=\bigg[\frac{\delta\mathcal{S}_{\rm eff}}{\delta A_\mu(x)}\bigg|_{(\delta/\delta i\eta),-(\delta/\delta i\bar{\eta}),(\delta/\delta iJ_\mu)}+J^\mu(x)\bigg]Z[\eta,\bar{\eta},J_\mu], \tag{20}$$

which, after evaluating the first term, can be written as

$$-J^\mu(x)=\bigg[\Box\eta^{\mu\nu}-\bigg\{1-\frac{1}{\xi}(1+a^2\Box)\bigg\}\partial^\mu\partial^\nu\bigg]\times(1+a^2\Box)\frac{\delta W}{\delta J^\nu(x)}+ie\frac{\delta W}{\delta\eta(x)}\gamma^\mu\frac{\delta W}{\delta\bar{\eta}(x)}+ie\frac{\delta}{\delta\eta(x)}\bigg[\gamma^\mu\frac{\delta W}{\delta\bar{\eta}(x)}\bigg]. \tag{21}$$

The last equation represents the compact form of the nonperturbative equivalent to the Podolsky field equation, subject to an external source J_μ. The functional W present in (21) is the generating functional for the connected Green's functions $W[\eta,\bar{\eta},J_\mu]$, which is defined by $W[\eta,\bar{\eta},J_\mu]=-i\ln Z[\eta,\bar{\eta},J_\mu]$. We also introduce the generating functional for one-particle irreducible ($1PI$) Green's functions $\Gamma[\bar{\psi},\psi,A_\mu]$ through the Legendre transformation

$$\Gamma[\bar{\psi},\psi,A_\mu]=W[\eta,\bar{\eta},J_\mu]-\int d^4x(\bar{\psi}\eta+\bar{\eta}\psi+A^\mu J_\mu). \tag{22}$$

From the above definitions, we obtain expressions for $(\bar{\psi},\psi,A_\mu)$ in terms of $(\eta,\bar{\eta},J_\mu)$, and vice versa, being given by

$$A_\mu=\frac{1}{i}\frac{\delta W}{\delta J^\mu},\qquad \psi=\frac{1}{i}\frac{\delta W}{\delta\bar{\eta}},\qquad \bar{\psi}=-\frac{1}{i}\frac{\delta W}{\delta\eta}, \tag{23}$$

$$J_\mu=-\frac{\delta\Gamma}{\delta A^\mu},\qquad \eta=-\frac{\delta\Gamma}{\delta\bar{\psi}},\qquad \bar{\eta}=\frac{\delta\Gamma}{\delta\psi}. \tag{24}$$

Assuming the case that the fermionic sources are null, Eq. (21) is written as

$$\frac{\delta\Gamma}{\delta A_\mu(x)}=\bigg[T^{\mu\beta}+\frac{1}{\xi}(1+a^2\Box)L^{\mu\beta}\bigg](1+a^2\Box)\Box A_\beta(x)+ie\frac{\delta}{\delta\eta_A(x)}\bigg[\gamma^\mu\frac{\delta W}{\delta\bar{\eta}(x)}\bigg], \tag{25}$$

where we have used the following set of projectors:

$$T^{\alpha\beta}+L^{\alpha\beta}=\eta^{\alpha\beta},\qquad L^{\alpha\beta}=\frac{\partial^\alpha\partial^\beta}{\Box}. \tag{26}$$

From identifying

$$\mathcal{S}(x,y)\equiv i\frac{\delta^2W[\eta,\bar{\eta},J_\mu]}{\delta\eta(y)\delta\bar{\eta}(x)}\bigg|_{\psi=\bar{\psi}=0} \tag{27}$$

as the complete electron propagator in an external field A_μ, which satisfies the following functional relation,

$$i\int d^4z\mathcal{S}_{BC}(x,z)\frac{\delta^2\Gamma}{\delta\psi_C(y)\delta\bar{\psi}_D(z)}=\delta_{BD}\delta(x-y), \tag{28}$$

we can express (25) as

$$\frac{\delta\Gamma}{\delta A_\mu(x)}=\bigg[T^{\mu\beta}+\frac{1}{\xi}(1+a^2\Box)L^{\mu\beta}\bigg](1+a^2\Box)\Box A_\beta(x)+e\,{\rm Tr}[\gamma^\mu\mathcal{S}(x,x)]. \tag{29}$$

Now, differentiating (29) with respect to $A_\nu(y)$ and setting $J_\mu(x)=0$, yields

$$\frac{\delta^2\Gamma}{\delta A_\nu(y)\delta A_\mu(x)}=\bigg[T^{\mu\beta}+\frac{1}{\xi}(1+a^2\Box)L^{\mu\beta}\bigg]\times\Box(1+a^2\Box)\delta(x-y)-ie\,{\rm Tr}\bigg\{\gamma^\mu\frac{\delta}{\delta A_\nu(y)}\bigg[\frac{\delta^2\Gamma}{\delta\psi(x)\delta\bar{\psi}(x)}\bigg]^{-1}\bigg\}. \tag{30}$$

The second term on the right-hand side of (30) can be evaluated immediately, giving a simple expression

$$\frac{\delta}{\delta A_\nu(y)}\bigg[\frac{\delta^2\Gamma}{\delta\psi(x)\delta\bar{\psi}(x)}\bigg]^{-1}=e\int d^4ud^4w\mathcal{S}(u,x)\times\Gamma^\nu(w,u;y)\mathcal{S}(x,w), \tag{31}$$

where we take into account the definition (27) and have introduced the complete electron-photon vertex function

$$e\Gamma_\mu(x, y; z) \equiv \frac{\delta^3\Gamma}{\delta A^\mu(z)\delta\psi(y)\delta\bar{\psi}(x)}\bigg|_{A=\psi=\bar{\psi}=0}. \tag{32}$$

Similar to the fermionic case (28), the second derivative of $\Gamma[\bar{\psi}, \psi, A_\mu]$, with respect to $A_\mu(x)$, generates the inverse of the photon propagator $\mathcal{D}_{\mu\nu}(x-y)$. From this fact, and substituting (30) into (31), then follows the SDFE for the inverse of the complete photon propagator:

$$\mathcal{D}^{-1}_{\mu\nu}(x-y) = \Pi_{\mu\nu}(x, y) + \left[T_{\mu\nu} + \frac{1}{\xi}(1 + a^2\Box)L_{\mu\nu}\right] \times (1 + a^2\Box)\Box\delta(x-y), \tag{33}$$

where the functional $\Pi_{\mu\nu}$ is known as the photon self-energy function, and is defined as

$$\Pi_{\mu\nu}(x, y) = -ie^2 \int d^4u d^4w \,\mathrm{Tr}[\mathcal{S}(u, x)\gamma_\mu \mathcal{S}(x, w) \times \Gamma_\nu(w, u; x)]. \tag{34}$$

The (-1) factor comes from the fermionic loop in the usual way. The $\Pi_{\mu\nu}$ tensor describes the interaction of a photon with the electron-positron field, and this interaction consists of the creation and annihilation of virtual pairs. Equation (34) in momentum representation assumes the form

$$\Pi_{\mu\nu}(k) = -\frac{ie^2}{(2\pi)^4}\int d^4p \,\mathrm{Tr}[\mathcal{S}(p)\gamma_\mu\mathcal{S}(p-k) \times \Gamma_\nu(p, k; p-k)]. \tag{35}$$

From expression (33), we can compute the gauge-field propagator in a perturbative way, order-by-order, in the coupling constant e. An explicit calculation, analysis, and discussion at the lowest order of radiative correction of the Green's functions will be presented in Sec. V. Indeed, since the bosonic $1PI$ function and the complete photon propagator satisfy the identity $\Gamma_{\mu\sigma}(k)\mathcal{D}^{\sigma\nu}(k) = -i\eta^\nu_\mu$, it is possible to find the following solution to the complete photon propagator:

$$i\mathcal{D}_{\mu\nu}(k) = \frac{\eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}}{k^2[\Pi(k) + (1 - a^2k^2)]} + \frac{\xi}{k^2(1-a^2k^2)^2}\frac{k_\mu k_\nu}{k^2}, \tag{36}$$

where Π is called scalar polarization, which is introduced due to the Lorentz invariance of $\Pi_{\mu\nu}$, and has the structure

$$\Pi_{\mu\nu}(k) \equiv (-\eta_{\mu\nu}k^2 + k_\mu k_\nu)\Pi(k). \tag{37}$$

It should be noted that Eq. (36) shows that the $\Pi(k)$ function is related to the transverse pole of the photon propagator in momentum representation. The diagrammatic representation of the SDFE for the photon propagator (36) is shown in Fig. 1.

FIG. 1. The SDFE for the photon propagator.

The photon propagator at the lowest order in perturbation theory, i.e., taking $\Pi(k) = 0$ in (36), can be conveniently written as

$$\begin{aligned} iD_{\mu\nu}(k) = &\left[\eta_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2}\right]\frac{1}{k^2} \\ &- \left[\eta_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2 - \frac{1}{a^2}}\right]\frac{1}{k^2 - \frac{1}{a^2}} \\ &+ (1-2\xi)\frac{k_\mu k_\nu}{k^2(k^2 - \frac{1}{a^2})} - \frac{k_\mu k_\nu}{(k^2 - \frac{1}{a^2})^2}. \end{aligned} \tag{38}$$

As it can be seen in (38), the beauty of this expression is the appearance of the second term on the right-hand side, which originated from the Podolsky term and the generalized Lorenz condition. Note that this term has a massive pole, $m^2 = a^{-2}$, which leads to a cancellation of the IR divergences that are present in the first term, the Maxwell's term. Furthermore, the separation of massless (usually QED_4) and massive modes in the propagator expression (38) in general gauge ξ is owing to the linearity of fields in the gauge terms of the Lagrangian (1). By the relation between the Podolsky parameter and the mass of photons, it is possible to set a bound value for the photon mass, once we evaluate the parameter a [11,20].

B. Schwinger-Dyson-Fradkin equation for the fermionic propagator

In what follows in this subsection, we present the derivation of an integral expression to the complete the electron propagator $\mathcal{S}$. We also introduce the mass operator $\mathcal{M}$, which contains all the radiative corrections to the motion of the electron (in the same sense as the polarization operator Π for photons). We guide the derivation of the SDFE for $\mathcal{S}$ in the same way as presented in the last subsection for the photon propagator. We recall that the functional equation

$$\begin{aligned} 0 &= \left[\frac{\delta\mathcal{S}_{\mathrm{eff}}}{\delta\bar{\psi}(x)}\bigg|_{(\delta/\delta i\eta), -(\delta/\delta i\bar{\eta}), (\delta/\delta iJ_\mu)} - \eta(x)\right]Z[\eta, \bar{\eta}, J_\mu] \\ &= \eta(x)Z + i\left[i\hat{\partial} - m - ie\gamma^\mu\frac{\delta}{\delta J^\mu(x)}\right]\frac{\delta Z}{\delta\bar{\eta}(x)}, \end{aligned} \tag{39}$$

which is equivalent to the Dirac equation in the presence of external sources, will define a relation between $\mathcal{S}$ and $\mathcal{M}$. Now, differentiating (39) with respect to $\eta(y)$ and taking the fermionic sources going to zero, one gets

$$i\delta(x-y) = \left[i\hat{\partial} - m + e\hat{A}(x) - ie\gamma^{\mu}\frac{\delta}{\delta J^{\mu}(x)}\right]\mathcal{S}(x,y), \tag{40}$$

where we have used the definition (27) for $\mathcal{S}$. Equation (40) defines the nonperturbative connected two-point fermionic Green's functions.

By means of a functional derivative identity, together with (24) and (31), and also taking the source J_{μ} going to zero, the last term of (40) reads

$$\frac{\delta}{\delta J^{\mu}(x)}\mathcal{S}(x,y;A) = e\int d^4u d^4z d^4w \mathcal{D}_{\mu\alpha}(x-u)\mathcal{S}(x,w) \times \Gamma^{\alpha}(w,z;u)\mathcal{S}(z,y). \tag{41}$$

Also, note that the electromagnetic potential A presented in the fourth term of (40) vanishes in the absence of an external source; that is, $A_{\mu}(x; J_{\mu} = 0) = 0$. Combining this fact with (41), Eq. (40) is rewritten as

$$i\delta(x-y) = (i\gamma^{\mu}\partial_{\mu} - m)\mathcal{S}(x,y) - \int d^4z \Sigma(x,z)\mathcal{S}(z,y), \tag{42}$$

where the electron self-energy operator Σ introduced above is defined by the following relation:

$$\Sigma(x-y) = ie^2\gamma^{\mu}\int d^4u d^4w \mathcal{S}(x,w)\mathcal{D}_{\mu\alpha}(u-x) \times \Gamma^{\alpha}(w,y;u), \tag{43}$$

which, in momentum representation, is written as

$$\Sigma(p) = \frac{ie^2}{(2\pi)^4}\gamma^{\mu}\int d^4k \mathcal{S}(p-k)\mathcal{D}_{\mu\alpha}(k)\Gamma^{\alpha}(p,k;p-k). \tag{44}$$

If we denote conveniently $\Sigma\mathcal{S}(x,y;A) = \int d^4z \Sigma(x,z)\mathcal{S}(z,y;A)$, then we can rewrite Eq. (42) in the following suitable form:

$$(i\gamma^{\mu}\partial_{\mu} - m - \Sigma)\mathcal{S}(x,y) = i\delta(x-y). \tag{45}$$

Moreover, introducing the so-called mass operator $\mathcal{M}$,

$$\mathcal{M}(x,y) = m\delta(x-y) + \Sigma(x,y), \tag{46}$$

into (45), we find that the complete electron propagator in momentum representation assumes the form

$$\mathcal{S}(p) = \frac{i}{\gamma^{\mu}p_{\mu} - \mathcal{M}(p)} = \frac{i}{\gamma^{\mu}p_{\mu} - m - \Sigma(p)}, \tag{47}$$

which states the relation between the electron propagator and the mass operator. The SDFE corresponding to the electron propagator is presented in Fig. 2. Equations (45) and (47) show that the electron propagator is the Green's function for an equation similar to the Dirac equation $(\hat{p} - m - \Sigma)\psi = 0$, but differing from the latter by the addition to the bare mass m of the quantity Σ. For this reason, $\mathcal{M}$ is called the *mass operator*.

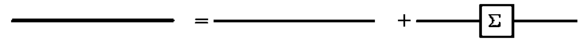

FIG. 2. The SDFE for the electron propagator.

In a similar way to the operator Π, we can say that the operator Σ describes the interaction of the electron with its own electromagnetic field. This interaction consists of the emission and absorption of virtual photons.

C. Schwinger-Dyson-Fradkin equation for the vertex

As it is already known [21], it is impossible to construct for QED_4 a closed integral equation that expresses the vertex function Γ in terms of $\mathcal{S}$ and $\mathcal{D}$ and that, together with Eqs. (36) and (47), would give us a complete system of equations determining the Green's functions. Nevertheless, it is possible to find a relation connecting the vertex function Γ with $\mathcal{S}$ and $\mathcal{D}$ [22]; however, different from other Green's functions, this relation contains only skeleton graphs [19], i.e., connected graphs. But, for our purposes here, it is enough to consider this kind of approximation, due to the fact that, here, we have only interest in the e^2-order calculation. Thus, recalling that the vertex function is formally obtained from

$$e\Gamma^{\mu}(x,y;z) = \frac{\delta[\mathcal{S}(x,y;A)]^{-1}}{\delta A(z)}, \tag{48}$$

with $\mathcal{S}^{-1}$ being the inverse of the fermionic propagator (47), the vertex function can be also decomposed as

$$\Gamma_{\mu}(x,y;z) = -i\gamma_{\mu}\delta(x-y)\delta(y-z) + \Lambda_{\mu}(x,y;z), \tag{49}$$

where Λ_{μ} is denoted as the vertex part of the graphs. The vertex function can be expressed in momentum space in terms of a new unknown quantity, the electron-positron kernel K, by means of an integral equation [22]

$$\Gamma_{\mu}(p,p';k) = -i\gamma_{\mu}\delta(p+p'-k) + \int \frac{d^4q}{(2\pi)^4}[i\mathcal{S}(p'+q)\Gamma_{\mu}(q+p',p+q) \times i\mathcal{S}(p+q)]K(p+q,p'+q,q), \tag{50}$$

where p' and p are, respectively, the momenta of the emerging and incident electrons, while $k = p - p'$ is the transferred momentum. K consists of graphs with two external electron and two external positron lines. Well, we have obtained, here, a closed integral equation for the vertex function; however, for practical calculations we did not accomplish much, because Γ_{μ} is expressed in terms of an unknown quantity—the kernel K. We shall write down the complete kernel K as a sum over skeleton graphs, which in first order yields [22]

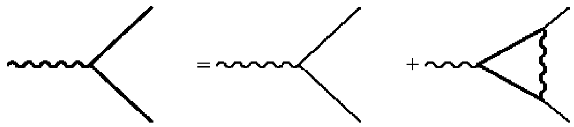

FIG. 3. The SDFE for the vertex function.

$$iK(p, p', k) = (ie)^2\Gamma^\mu(p, p-k)\mathcal{D}_{\mu\nu}(k)\Gamma^\nu(p'-k, p'). \tag{51}$$

Therefore, from (51), we find that the skeleton equation for the vertex function (49) written in the Fourier representation is given by

$$\begin{aligned}\Gamma_\mu(p, p';k) = &-i\gamma_\mu\delta(p-p'-k) + \frac{e^2}{(2\pi)^4}\int d^4q i\mathcal{S}(p'-q)\\ &\times\Gamma_\mu(p-q, p'-q;k)i\mathcal{S}(p-q)\\ &\times\Gamma^\alpha(p, p-q;q)i\mathcal{D}_{\nu\alpha}(q)\Gamma^\nu(p', p'-k, k).\end{aligned} \tag{52}$$

Figure 3 shows the vertex function. It is important to emphasize, here, that the operators Σ, Π, and Λ introduced above are functional of the Green's functions $\mathcal{S}$, $\mathcal{D}$, and Γ, which means that the self-energy functions are coupled, and one of the Green's functions depends on the other ones of lower order. Hence, we clearly see that this tower of equations is related.

IV. WARD-FRADKIN-TAKAHASHI IDENTITIES

As it is well known, the generalized Ward-Fradkin-Takahashi (WFT) identities are, in general, identities among Green's functions following from the existence of a symmetry. The goal of this section is to derive these gauge identities for GQED_4. First, we will show the WFT identity satisfied by the $1PI$ gauge function, which leads to the transverse character of the operator $\Pi_{\mu\nu}$. Next, we will derive the relation between the vertex function and the inverse of the complete electron propagator, which is known as the main WFT identity. At last, we will reproduce the main WFT identity in the $k \longrightarrow 0$ limit (null transferred momentum). The derivation of these identities is formally given as follows: starting from the generating functional (18) and performing the infinitesimal transformations

$$\begin{aligned}\psi'(x) &= \psi(x) + i\lambda(x)\psi(x),\\ A'_\mu(x) &= A_\mu(x) + \frac{1}{e}\partial_\mu\lambda(x),\end{aligned} \tag{53}$$

and noticing that neither gauge-fixing term nor the source terms are invariant under these transformations, we find that the generating functional $\mathcal{Z}[\eta, \bar{\eta}, J^\mu]$ satisfies the following equation of motion:

$$\begin{aligned}\Big[i\frac{\Box}{e\xi}(1+a^2\Box)^2\partial_\mu\frac{\delta}{\delta J_\mu(x)} - \bar{\eta}\frac{\delta}{\delta\bar{\eta}(x)}\\ + \eta\frac{\delta}{\delta\eta(x)} - \frac{1}{e}\partial_\mu J^\mu\Big]\mathcal{Z} = 0.\end{aligned} \tag{54}$$

The next step in deriving the WFT identities is to express (54) in terms of the connected Green's functions $W[\eta, \bar{\eta}, J^\mu]$ as

$$\begin{aligned}-\frac{\Box}{e\xi}(1+a^2\Box)^2\partial_\mu\frac{\delta W}{\delta J_\mu(x)} - i\bar{\eta}\frac{\delta W}{\delta\bar{\eta}(x)}\\ + i\eta\frac{\delta W}{\delta\eta(x)} - \frac{1}{e}\partial_\mu J^\mu = 0.\end{aligned} \tag{55}$$

Finally, one can obtain the main quantum equation of motion for the theory by writing (55) into an expression for the $1PI$-generating functional $\Gamma(\bar{\psi}, \psi, A_\mu)$ through (22). Thus, one has the general equation

$$\begin{aligned}-\frac{\Box}{e\xi}(1+a^2\Box)^2\partial_\mu A^\mu(x) - i\frac{\delta\Gamma}{\delta\psi(x)}\psi(x)\\ + i\frac{\delta\Gamma}{\delta\bar{\psi}(x)}\bar{\psi}(x) + \frac{1}{e}\partial_\mu\frac{\delta\Gamma}{\delta A_\mu(x)} = 0.\end{aligned} \tag{56}$$

From Eq. (56), it is possible to derive all WFT identities. Thus, the first identity comes by applying the functional derivative of $A_\nu(y)$ in Eq. (56) at $A_\mu = \psi = \bar{\psi} = 0$,

$$\partial_\mu\Gamma^{\mu\nu}(x, y) - \frac{\Box}{\xi}(1+a^2\Box)^2\partial^\nu\delta(x-y) = 0, \tag{57}$$

which, together with Eq. (33), implies that

$$k_\mu\Pi^{\mu\nu}(k) = 0, \tag{58}$$

which shows the transverse character of the operator $\Pi^{\mu\nu}$. Now, the main gauge WFT identity follows by taking the derivatives of Eq. (56) with respect to $\psi(y)$ and $\bar{\psi}(z)$ at $A_\mu = \psi = \bar{\psi} = 0$, which in momentum space takes the form

$$\begin{aligned}&k_\mu\tilde{\Gamma}^\mu(p, p'; k = p-p')\\ &\qquad = i(2\pi)^4[\mathcal{S}^{-1}(p) - \mathcal{S}^{-1}(p-p')],\end{aligned} \tag{59}$$

with $\mathcal{S}^{-1}$ being the inverse of the complete electron propagator (47).

Although the local gauge invariance at the classical level has been broken in the quantum theory through the gauge-fixing procedure and the source terms, the main WFT identity (59) holds, inheriting its essence, without which the renormalizability cannot be guaranteed.

In the limit of null transferred momentum, i.e., $k \longrightarrow 0$, Eq. (59) leads to a relation

$$i\tilde{\Gamma}^\mu(p, p; 0) = (2\pi)^4\frac{\partial}{\partial p_\mu}\mathcal{S}^{-1}(p); \tag{60}$$

from this limit, it also follows that

$$\tilde{\Lambda}^{\mu}(p,p;0) = -\frac{\partial}{\partial p_{\mu}}\Sigma(p). \tag{61}$$

Both relations, Eqs. (60) and (61), hold in the same way that they do for QED$_4$.

V. RADIATIVE CORRECTIONS OF THE SECOND ORDER

In the preceding sections, we have derived integral equations for the Green's functions, the electron and photon propagators, and the vertex function for the GQED$_4$. Now, we will investigate the corrections to these functions in the first nonvanishing order of perturbation theory. The expression for the operator $\Pi_{\mu\nu}$ at e^2 order does not differ from that of the QED$_4$. This divergent result implies, in the same way as for the QED$_4$ [21], the renormalization of the electronic charge e and the introduction of the renormalization constant Z_3 in the GQED$_4$. Although the electron self-energy function Σ and the vertex part Λ in e^2 order are different from the usual corrections for the QED$_4$, due to the presence of the Podolsky terms in the free photon propagator $D_{\mu\nu}$ (38), the structure of divergences at this order, by power counting, remains the same as in the QED$_4$, linearly and logarithmically divergent. At first glance, this fact seems to lead to infinity results for the other two self-energy functions of the GQED$_4$ at e^2 order; thus, an explicit calculation of Σ and Λ expressions becomes necessary. These calculations are also necessary to verify whether the main WFT identity (59) is still satisfied at this order.

In order to use the dimensional regularization procedure, the Lagrangian must have the right dimension (of internal loops); then, it is necessary to introduce the *t' Hooft mass* μ. Thus, also considering the case of $\xi = 1$, we have

$$\mathcal{L}_{\text{eff}} = \frac{i}{2}\bar{\psi}\overset{\Leftrightarrow}{\hat{\partial}}\psi - m\bar{\psi}\psi - e\mu^{4-d}\bar{\psi}\hat{A}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{a^2}{2}\partial_{\mu}F^{\mu\alpha}\partial_{\nu}F^{\nu}_{\alpha} - \frac{1}{2}[(1+a^2\square)\partial_{\mu}A^{\mu}]^2. \tag{62}$$

In the next two subsections, we shall compute the Σ and the Λ functions. We will show that both functions, Σ and Λ, can be separated into two distinct contributions, the well-known contribution from the QED$_4$ and a new one that we will call the Podolsky contribution. However, we can observe by power counting that the Podolsky sector presents a divergent share with the QED sector; thus, we expect that they may cancel out the divergence of the GQED$_4$. Now, we proceed to an explicit evaluation of the electron self-energy and the vertex part.

A. Electron self-energy

We begin by investigating the second-order electron self-energy function. This quantity corresponds to the diagram shown in Fig. 4.

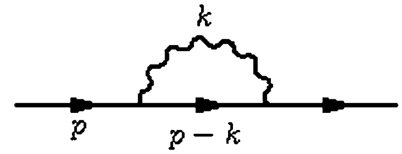

FIG. 4. Electron self-energy diagram.

In accordance with Eq. (44), the self-energy function Σ can be written as

$$\Sigma^{(2)}(p) \equiv \Sigma_{\text{QED}}(p) + \Sigma_{\text{Pod}}(p), \tag{63}$$

where

$$\Sigma_{\text{QED}}(p) = i\mu^{4-d}\int\frac{d^dk}{(2\pi)^d}\frac{1}{k^2}\gamma^{\nu}\frac{(\hat{p}-\hat{k}+m)}{[(p-k)^2-m^2]}\gamma_{\nu}, \tag{64}$$

and

$$\Sigma_{\text{Pod}}(p) = -i\mu^{4-d}\int\frac{d^dk}{(2\pi)^d}\gamma^{\lambda}\frac{(\hat{p}-\hat{k}+m)}{[(p-k)^2-m^2]}\gamma^{\nu} \times\frac{1}{(k^2-\frac{1}{a^2})}\left[\eta_{\lambda\nu}+\left(\frac{1}{k^2}-\frac{1}{k^2-\frac{1}{a^2}}\right)k_{\lambda}k_{\nu}\right]. \tag{65}$$

The separation of $\Sigma^{(2)}$ in two contributions is only made possible by the linear structure of the free photon propagator (38). First, the regularized QED$_4$ contribution for the electron self-energy is given by [21]

$$\Sigma_{\text{QED}}(p) = \frac{1}{8\pi^2}\frac{1}{\epsilon}(\hat{p}-4m) + \Sigma_{\text{QED Finite}}(p), \tag{66}$$

with

$$\Sigma_{\text{QED Finite}}(p) = -\frac{1}{16\pi^2}[\hat{p}(\gamma+1) - 2m(2\gamma+1)] - \frac{1}{8\pi^2}\int_0^1 dz[\hat{p}(1-z)-2m] \times\ln\left|\frac{(1-z)zp^2-m^2z}{4\pi\mu^2}\right|, \tag{67}$$

where $\epsilon = 4 - d$ is the dimensional-regularization parameter.

Now, to evaluate the Podolsky contribution Σ_{Pod} (65), it is suitable to write it as

$$\Sigma_{\text{Pod}}(p) = \sum_{\alpha=1}^{3}\Sigma^{(\alpha)}_{\text{Pod}}(p), \tag{68}$$

so that the quantities $\Sigma^{(i)}_{\text{Pod}}$ are defined by

$$\Sigma^{(1)}_{\text{Pod}}(p) \equiv -i\mu^{4-d}\int\frac{d^dk}{(2\pi)^d}\frac{\gamma^{\lambda}(\hat{p}-\hat{k}+m)\gamma_{\lambda}}{[(p-k)^2-m^2]}\frac{1}{(k^2-\frac{1}{a^2})}, \tag{69}$$

$$\Sigma^{(2)}_{\rm Pod}(p) \equiv -i\mu^{4-d}\int \frac{d^dk}{(2\pi)^d}\frac{\hat{k}(\hat{p}-\hat{k}+m)\hat{k}}{[(p-k)^2-m^2]} \times \frac{1}{k^2(k^2-\frac{1}{a^2})}, \tag{70}$$

$$\Sigma^{(3)}_{\rm Pod}(p) \equiv i\mu^{4-d}\int \frac{d^dk}{(2\pi)^d}\frac{\hat{k}(\hat{p}-\hat{k}+m)\hat{k}}{[(p-k)^2-m^2]}\frac{1}{(k^2-\frac{1}{a^2})^2}. \tag{71}$$

We are going now to calculate the expressions of $\Sigma^{(i)}_{\rm Pod}$, Eqs. (69)–(71). To solve conveniently the momentum integration, we will use the Feynman parametrization and the dimensional regularization. Using both procedures in Eq. (69), one can put it in the form

$$\Sigma^{(1)}_{\rm Pod}(p) = i\mu^{4-d}\int_0^1 dz\int\frac{d^dk}{(2\pi)^d}\frac{\gamma^\lambda(\hat{k}-\hat{p}-m)\gamma_\lambda}{[(k-pz)^2+b^2]^2}, \tag{72}$$

where $b^2=(1-z)(zp^2-\frac{1}{a^2})-m^2z$. Introducing the change of variables $k\longrightarrow k-pz$, we obtain

$$\Sigma^{(1)}_{\rm Pod}(p) = -i\mu^{4-d}\int_0^1 dz\gamma^\lambda[(1-z)\hat{p}+m]\gamma_\lambda \times\int\frac{d^dk}{(2\pi)^d}\frac{1}{[k^2+b^2]^2}. \tag{73}$$

The k integration is carried out by using the identity (A1), so that (73) reads

$$\Sigma^{(1)}_{\rm Pod}(p) = (-1)^{d/2}\frac{\mu^{4-d}}{(4\pi)^{d/2}}\Gamma\Big(2-\frac{d}{2}\Big) \times\int_0^1 dz\gamma^\lambda[(1-z)\hat{p}+m]\gamma_\lambda[b^2]^{(d/2)-2}. \tag{74}$$

Now, expanding (74) around $d=4$, we find that

$$\Sigma^{(1)}_{\rm Pod}(p) \underset{\epsilon\to 0}{=} -\frac{1}{\epsilon}\frac{1}{8\pi^2}(\hat{p}-4m)+\Sigma^{(1)}_{\rm Pod\,Finite}(p), \tag{75}$$

where

$$\Sigma^{(1)}_{\rm Pod\,Finite}(p) = \frac{1}{16\pi^2}[\hat{p}(1+\gamma)-2m(1+2\gamma)] + \frac{1}{8\pi^2}\int_0^1 dz[\hat{p}(1-z)-2m] \times\ln\left|\frac{(1-z)(zp^2-\frac{1}{a^2})-m^2z}{4\pi\mu^2}\right|. \tag{76}$$

We can evaluate the other terms in a similar way; however, to avoid an extensive calculus, we present here only the results, leaving the explicit calculation of these quantities and other extensive expressions in Appendix B. The evaluated expressions of them are

$$\Sigma^{(2)}_{\rm Pod}(p) = \frac{1}{\epsilon}\frac{1}{8\pi^2}(m-\hat{p})+\Sigma^{(2)}_{\rm Pod\,Finite}(p), \tag{77}$$

$$\Sigma^{(3)}_{\rm Pod}(p) = -\frac{1}{\epsilon}\frac{1}{8\pi^2}(m-\hat{p})+\Sigma^{(3)}_{\rm Pod\,Finite}(p), \tag{78}$$

with the finite parts given by (B4) and (B8), respectively.

Indeed, by combining the results of Eqs. (75), (77), and (78) into Eq. (68), it follows that the regularized contribution of the Podolsky sector for the electron self-energy function is given by

$$\Sigma_{\rm Pod}(p) = -\frac{1}{8\pi^2}\frac{1}{\epsilon}(\hat{p}-4m)+\Sigma_{\rm Pod\,Finite}(p), \tag{79}$$

where $\Sigma_{\rm Pod\,Finite}$ is given by (B9).

Therefore, it finally follows from a rearrangement of Eqs. (66) and (79) that the electron self-energy function (63), at e^2 order, has the following expression:

$$\Sigma^{(2)}(p) = \frac{1}{8\pi^2}\int_0^1 dz[(1-z)\hat{p}-2m]\ln\left|\frac{(1-z)zp^2-m^2z-\frac{1}{a^2}(1-z)}{(1-z)zp^2-m^2z}\right| + \frac{1}{16\pi^2}\int_0^1 dx\int_0^{1-x}dy[2m-(1+3y)\hat{p}]\boldsymbol{A}_1(p,x,y)+\frac{1}{16\pi^2}\int_0^1 dx\int_0^{1-x}dy[(1-y)\hat{p}+m]p^2y^2\boldsymbol{A}_2(p,x,y), \tag{80}$$

with the quantities $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$ defined as

$$\boldsymbol{A}_1(p,x,y) \equiv \ln\left|\frac{(1-y)yp^2-m^2y-\frac{1}{a^2}(1-y)}{(1-y)yp^2-m^2y-\frac{x}{a^2}}\right|,$$

$$\boldsymbol{A}_2(p,x,y) \equiv \frac{1}{(1-y)yp^2-m^2y-\frac{x}{a^2}} - \frac{1}{(1-y)(yp^2-\frac{1}{a^2})-m^2y}.$$

Equation (80) shows that the electron self-energy function $\Sigma^{(2)}$, at e^2 order, does not depend on μ, and that it is also free of divergences, which do not occur in such ordinary QED_4 as Eq. (66). This last feature is an interesting property of the theory. It seems that the Podolsky term in the Lagrangian (62) acts like a natural regulator of the theory, due to its massive character. Nevertheless, a better analysis shows that the Podolsky term is not the only one responsible for the finiteness of the electron self-energy in e^2 order; the choice of the generalized Lorenz gauge

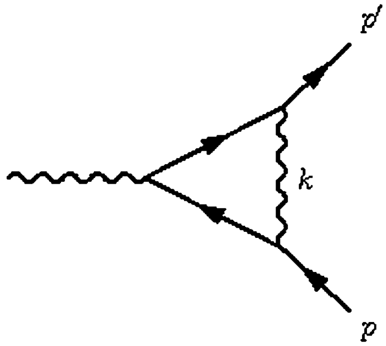

FIG. 5. Vertex part diagram.

condition (16) is also closely related to the finite result (80). Hence, we can conclude that the choice of the usual Lorenz condition for the $GQED_4$ leads to the divergent result for the self-energy of the electron evaluated in the thesis advised by Podolsky [12].

B. Vertex correction

We now turn to the calculation of the vertex part $\Lambda^\mu(p', p; q = p - p') \equiv \Lambda^\mu(p', p)$ (52), where, as usual, p' and p are, respectively, the momenta of the emerging and incident electrons, while $q = p - p'$ is the momentum of the incident photon. The diagram that corresponds to this quantity is shown in Fig. 5.

In the same way that it occurs in Eq. (63) for the electron self-energy function Σ, the vertex part (51) also shows the splitting of its expression into two distinct contributions:

$$\Lambda^{\mu(2)}(p', p) = \Lambda^\mu_{\rm QED}(p', p) + \Lambda^\mu_{\rm Pod}(p', p). \tag{81}$$

One contribution comes from the QED_4,

$$\Lambda^\mu_{\rm QED}(p', p) = -i\mu^{4-d}\int \frac{d^dk}{(2\pi)^d}\gamma^\alpha \frac{\hat{p}' - \hat{k} + m}{(p' - k)^2 - m^2} \gamma^\mu \times \frac{\hat{p} - \hat{k} + m}{(p - k)^2 - m^2}\gamma_\alpha \frac{1}{k^2}, \tag{82}$$

and another one from the Podolsky sector,

$$\Lambda^\mu_{\rm Pod}(p', p) = i\mu^{4-d}\int \frac{d^dk}{(2\pi)^d}\gamma^\alpha \frac{\hat{p}' - \hat{k} + m}{(p' - k)^2 - m^2} \times \gamma^\mu \frac{\hat{p} - \hat{k} + m}{(p - k)^2 - m^2}\gamma^\beta \frac{1}{k^2 - \frac{1}{a^2}} \times \left[\eta_{\alpha\beta} + \left(\frac{1}{k^2} - \frac{1}{k^2 - \frac{1}{a^2}}\right)k_\alpha k_\beta\right]. \tag{83}$$

The regularized QED_4 contribution (82) for the vertex part is known as [21]

$$\Lambda^\mu_{\rm QED}(p', p) = \frac{1}{\epsilon}\frac{1}{8\pi^2}\gamma^\mu + \Lambda^\mu_{\rm QED\,Finite}(p', p), \tag{84}$$

with $\Lambda^\mu_{\rm QED\,Finite}(p', p)$ given by

$$\Lambda^\mu_{\rm QED\,Finite}(p', p) = -\frac{1}{16\pi^2}\int_0^1 dx\int_0^{1-x} dy \frac{\Xi^\mu(x, y, \hat{p}', \hat{p})}{\Delta^2} - \frac{1}{8\pi^2}\gamma^\mu\left[1 + \frac{\gamma}{2} + \int_0^1 dx\int_0^{1-x} dy \ln\left|\frac{\Delta^2}{4\pi\mu^2}\right|\right], \tag{85}$$

where we have introduced the functions

$$\Xi^\mu(p', p, x, y) = 6(1 - x - y)\hat{p}'\gamma^\mu\hat{p} + 2mq_\nu[\gamma^\nu, \gamma^\mu] - 4(1 - x - y + 3xy)p.p'\gamma^\mu + 2m^2\gamma^\mu + 2x(1 - x)\gamma^\mu\hat{p}^2 + 2y(1 - y)(\hat{p}')^2\gamma^\mu - 4y(1 - y)\hat{p}'(p')^\mu - 4x(1 - x)\hat{p}p^\mu - 4(1 - x - y - xy)[\hat{p}'p^\mu + \hat{p}(p')^\mu] \tag{86}$$

and

$$\Delta^2 = xp^2(1 - x) + yp'^2(1 - y) - 2xypp' - m^2(x + y) \tag{87}$$

to simplify the notation of integrals.

Again, as it has happened with the Podolsky contribution for the electron self-energy function $\Sigma_{\rm Pod}$, the vertex part $\Lambda^\mu_{\rm Pod}$ (83) can also be written as three terms,

$$\Lambda^\mu_{\rm Pod}(p', p) = \sum_{\alpha=1}^{3}\Lambda^{\mu(\alpha)}_{\rm Pod}(p', p), \tag{88}$$

where we have defined each term in the following way:

$$\Lambda^{\mu(1)}_{\rm Pod}(p', p) \equiv i\mu^{4-d}\int \frac{d^dk}{(2\pi)^d}\gamma^\alpha \frac{\hat{p}' - \hat{k} + m}{(p' - k)^2 - m^2}\gamma^\mu \frac{\hat{p} - \hat{k} + m}{(p - k)^2 - m^2}\gamma_\alpha \frac{1}{k^2 - \frac{1}{a^2}}, \tag{89}$$

$$\Lambda^{\mu(2)}_{\rm Pod}(p', p) \equiv i\mu^{4-d}\int \frac{d^dk}{(2\pi)^d}\hat{k} \frac{\hat{p}' - \hat{k} + m}{(p' - k)^2 - m^2}\gamma^\mu \frac{\hat{p} - \hat{k} + m}{(p - k)^2 - m^2}\hat{k} \frac{1}{k^2(k^2 - \frac{1}{a^2})}, \tag{90}$$

$$\Lambda^{\mu(3)}_{\rm Pod}(p', p) \equiv -i\mu^{4-d}\int \frac{d^dk}{(2\pi)^d}\hat{k}\frac{\hat{p}' - \hat{k} + m}{(p' - k)^2 - m^2}\gamma^\mu \frac{\hat{p} - \hat{k} + m}{(p - k)^2 - m^2}\hat{k}\frac{1}{(k^2 - \frac{1}{a^2})^2}. \tag{91}$$

To evaluate such integrals, we will proceed as we presented in the last subsection for the electron self-energy function $\Sigma_{\rm Pod}$. In this subsection, we will only calculate one term, Eq. (89), and present the results for the others, Eqs. (90) and (91), leaving the calculation of the last two terms in Appendix C. Also, we exhibit there some extensive expressions that appear throughout this subsection. Hence, recalling the Feynman parametrization, Eq. (89) can be expressed as

$$\Lambda^{\mu(1)}_{\rm Pod}(p', p) = 2i\mu^{4-d}\int_0^1 dx \int_0^{1-x} dy \int \frac{d^dk}{(2\pi)^d} \times \frac{N^\mu(k, p', p, x, y)}{[k^2 + \Delta^2 - \frac{1}{a^2}(1 - x - y)]^3}, \tag{92}$$

where we have replaced k by $k - xp - yp'$ and defined the function

$$N^\mu(k, p', p, x, y) \equiv \gamma^\sigma[(1 - y)\hat{p}' - \hat{k} - x\hat{p} + m] \times \gamma^\mu[(1 - x)\hat{p} - \hat{k} - y\hat{p}' + m]\gamma_\sigma$$

for convenience.

We now attend to the k integration of (92). Using the properties of the Dirac matrices (A8) to separate the different k terms in the numerator and performing the momentum integration with the aid of Eqs. (A1) and (A2), we find that

$$\begin{aligned}\Lambda^{\mu(1)}_{\rm Pod}(p', p) = &-\frac{(-1)^{d/2}}{2}\frac{\mu^{4-d}}{(4\pi)^{d/2}}(2 - d)^2\gamma^\mu\Gamma\Big(2 - \frac{d}{2}\Big)\int d\varsigma \frac{1}{[\Delta^2 - \frac{1}{a^2}(1 - x - y)]^{2-(d/2)}}\\ &- (-1)^{d/2}\frac{\mu^{4-d}}{(4\pi)^{d/2}}(2 - d)\Gamma\Big(3 - \frac{d}{2}\Big)\int d\varsigma \frac{M^\mu(p', p, x, y)}{[\Delta^2 - \frac{1}{a^2}(1 - x - y)]^{3-(d/2)}}\\ &- (-1)^{d/2}\frac{\mu^{4-d}}{(4\pi)^{d/2}}\Gamma\Big(3 - \frac{d}{2}\Big)\int d\varsigma \frac{\Pi^\mu(x, y, \hat{p}', \hat{p})}{[\Delta^2 - \frac{1}{a^2}(1 - x - y)]^{3-(d/2)}},\end{aligned} \tag{93}$$

with

$$M^\mu(p', p, x, y) = [(1 - y)\hat{p}' - x\hat{p} - m] \times \gamma^\mu[(1 - x)\hat{p} - y\hat{p}' - m], \tag{94}$$

$$\begin{aligned}\Pi^\mu(x, y, \hat{p}', \hat{p}) = &4[(1 - y)p'^\sigma - xp^\sigma][(1 - x)p_\sigma - yp'_\sigma]\gamma^\mu\\ &- 2[(1 - y)\hat{p}' - x\hat{p} - m][(1 - x)\hat{p} - y\hat{p}']\gamma^\mu\\ &- 2\gamma^\mu[(1 - y)\hat{p}' - x\hat{p}][(1 - x)\hat{p} - y\hat{p}' - m],\end{aligned} \tag{95}$$

and the measure

$$\int d\varsigma \equiv \int_0^1 dx \int_0^{1-x} dy. \tag{96}$$

Equation (93), expanded around $d = 4$, gives the following expression:

$$\Lambda^{\mu(1)}_{\rm Pod}(p', p) = -\frac{1}{\epsilon}\frac{1}{8\pi^2}\gamma^\mu + \Lambda^{\mu(1)}_{\rm Pod\,Finite}(p', p), \tag{97}$$

with

$$\begin{aligned}\Lambda^{\mu(1)}_{\rm Pod\,Finite}(p', p) &= \frac{1}{16\pi^2}\int d\varsigma \frac{\Xi^\mu(p', p, x, y)}{[\Delta^2 - \frac{1}{a^2}(1 - x - y)]}\\ &+ \frac{1}{8\pi^2}\gamma^\mu\Big[1 + \frac{\gamma}{2} + \int d\varsigma \ln\Big|\frac{\Delta^2 - \frac{1}{a^2}(1 - x - y)}{4\pi\mu^2}\Big|\Big].\end{aligned} \tag{98}$$

The evaluated expressions for $\Lambda^{\mu(2)}_{\rm Pod}(p', p)$ and $\Lambda^{\mu(3)}_{\rm Pod}(p', p)$ are (see Appendix C)

$$\Lambda^{\mu(2)}_{\rm Pod}(p', p) = -\frac{1}{\epsilon}\frac{1}{8\pi^2}\gamma^\mu + \Lambda^{\mu(2)}_{\rm Pod\,Finite}(p', p), \tag{99}$$

$$\Lambda^{\mu(3)}_{\rm Pod}(p', p) = \frac{1}{\epsilon}\frac{1}{8\pi^2}\gamma^\mu + \Lambda^{\mu(3)}_{\rm Pod\,Finite}(p', p), \tag{100}$$

where the finite parts are given by (C4) and (C8), respectively.

Therefore, when Eqs. (97), (99), and (100) are combined, we determine the regularized expression for the Podolsky contribution to the vertex part $\Lambda^\mu_{\rm Pod}$ (88):

$$\Lambda^\mu_{\rm Pod}(p', p) = -\frac{1}{\epsilon}\frac{1}{8\pi^2}\gamma^\mu + \Lambda^\mu_{\rm Pod\,Finite}(p', p), \tag{101}$$

where $\Lambda^\mu_{\rm Pod\,Finite}$ is given by Eq. (C9).

Substituting the results of Eqs. (84) and (101) into the definition (81), we obtain that the vertex part Λ^μ at e^2 order has the following expression:

$$\begin{aligned}\Lambda^{\mu(2)}(p', p) &= \frac{1}{8\pi^2}\gamma^\mu \int d\varsigma \ln\left|\frac{\Delta^2 - \frac{1}{a^2}(1-x-y)}{\Delta^2}\right| + \frac{3}{8\pi^2}\gamma^\mu \int d\xi \ln\left|\frac{\Delta^2 - \frac{1}{a^2}z}{\Delta^2 - \frac{1}{a^2}(1-x-y)}\right| \\ &\quad + \frac{1}{16\pi^2}\int d\varsigma \boldsymbol{\Xi}^\mu(p', p, x, y)\left[\frac{1}{\Delta^2 - \frac{1}{a^2}(1-x-y)} - \frac{1}{\Delta^2}\right] + \frac{1}{16\pi^2}\int d\xi \boldsymbol{\Sigma}_\mathbf{1}(x, y, \hat{p}', \hat{p})\gamma^\mu \boldsymbol{\Sigma}_\mathbf{2}(x, y, \hat{p}', \hat{p}) \\ &\quad \times \left\{\frac{\mathbf{1}}{[\Delta^2 - \frac{1}{a^2}(1-x-y)]^2} - \frac{\mathbf{1}}{[\Delta^2 - \frac{1}{a^2}z]^2}\right\} + \frac{1}{8\pi^2}\int d\xi \Big[\boldsymbol{\Sigma}_\mathbf{1}(x, y, \hat{p}', \hat{p})\gamma^\mu + \gamma^\mu \boldsymbol{\Sigma}_\mathbf{2}(x, y, \hat{p}', \hat{p}) \\ &\quad - \frac{1}{4}\boldsymbol{\Sigma}^\mu_\mathbf{3}(x, y, \hat{p}', \hat{p}) + \frac{1}{2}(\hat{p}' - m)\gamma^\mu(\hat{p} - m)\Big]\left[\frac{\mathbf{1}}{\Delta^2 - \frac{1}{a^2}z} - \frac{\mathbf{1}}{\Delta^2 - \frac{1}{a^2}(1-x-y)}\right],\end{aligned} \quad (102)$$

where we have defined the functions $\boldsymbol{\Sigma}_\mathbf{1}$, $\boldsymbol{\Sigma}_\mathbf{2}$, and $\boldsymbol{\Sigma}^\mu_\mathbf{3}$ as

$$\boldsymbol{\Sigma}_\mathbf{1}(x, y, \hat{p}', \hat{p}) = (x\hat{p} + y\hat{p}')[(1-y)\hat{p}' - x\hat{p} + m], \quad (103)$$

$$\boldsymbol{\Sigma}_\mathbf{2}(x, y, \hat{p}', \hat{p}) = [(1-x)\hat{p} - y\hat{p}' + m](x\hat{p} + y\hat{p}'), \quad (104)$$

$$\begin{aligned}\boldsymbol{\Sigma}^\mu_\mathbf{3}(x, y, \hat{p}', \hat{p}) = &-2\gamma^\mu[(1-y)\hat{p}' - x\hat{p}](\hat{p} - m) \\ &+ 4[(1-y)p'^\sigma - xp^\sigma][(1-x)p_\sigma - yp'_\sigma]\gamma^\mu \\ &- 2(\hat{p}' - m)[(1-x)\hat{p} - y\hat{p}']\gamma^\mu,\end{aligned} \quad (105)$$

and the measure

$$\int d\xi \equiv \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz, \quad (106)$$

to simplify the notation of integrals, as we have done with the functions Ξ and Δ.

As stated in the beginning of this section, we have shown that both radiative corrections, the electron self-energy and the vertex part, are finite at e^2 order. Equation (102) indicates the independence of the vertex part with the t' Hooft mass μ, and, again, as it has happened with the electron self-energy function, the finiteness of Λ^μ is due to the Podolsky term plus the choice of the generalized Lorenz gauge condition. Another important point is that the finiteness of the vertex part Λ^μ and the electron self-energy function Σ implies that the main WFT identity (59) is still satisfied at e^2 order.

VI. REMARKS AND CONCLUSIONS

In this paper, the effects of the Podolsky term in the quantum theory of electron and photon interactions were analyzed. After a constraint analysis, the covariant transition amplitude was derived with the aid of the Faddeev-Popov-DeWitt ansatz in the generalized Lorenz gauge condition. The choice of this gauge is of great importance to the obtained results. Then, we proceeded by deriving the SDFE of the theory by functional methods, and three Green's functions have been determined: the photon $\mathcal{D}_{\mu\nu}$ and electron $\mathcal{S}$ propagators and the vertex function Γ_μ, Eqs. (36), (47), and (52), respectively. Through these functions, we introduced the self-energy functions that contain the radiative corrections in all orders in perturbation theory: the polarization tensor $\Pi_{\mu\nu}$, the mass operator $\mathcal{M}$, and the vertex part Λ_μ, respectively. However, modifications of these expressions compared to the ones for QED_4 were observed only in the mass operator and the vertex part, resulting from the contributions of the Podolsky electrodynamics. Although such modifications are presented in all Green's functions, only the photon propagator (38) presents changes at tree level. Moreover, the most interesting feature of this expression is the fact that we could separate the usual contribution for the QED_4 in a general gauge ξ from the one that arises from the Podolsky theory, and that the IR divergences presented in the QED_4 terms are suppressed by the massive terms of the Podolsky contribution.

The derivation of WFT identities was also presented. The first identity (57) showed that the transverse character of the polarization tensor $\Pi_{\mu\nu}$ is also preserved in the $GQED_4$ as in the QED_4. Immediately, we found the main WFT identity that relates the $1PI$ vertex function and the complete electron propagator. The main WFT identity (59) is responsible for holding the essence of the gauge symmetry in quantum level, without which the renormalizability of the theory cannot be guaranteed.

The last part of the article was devoted to the analysis of what the Podolsky contribution brings to the quantum theory at e^2 order in perturbation theory. At this order of approximation, we verified that the photon self-energy function is divergent, showing that, if we claim the renormalization theory, the electronic charge needs to be renormalized. Now, for the other two corrections, interesting features appeared, and the free photon propagator performs an important role in this analysis, giving origin to a splitting of the correction functions in two distinct contributions: one from the usual QED_4 and another from the Podolsky theory. This splitting makes it possible to study each contribution independently. Thus, since the QED_4 contribution is well-known in the literature, our task here was to calculate the Podolsky contribution to the electron self-energy function and to the vertex part. And, the obtained expressions for the Podolsky contribution $\Sigma_{\rm Pod}$ and $\Lambda^\mu_{\rm Pod}$, Eqs. (79) and (101), respectively, present the same divergent terms of the QED_4, Eqs. (66) and (84), but with oppositive signs, showing, then, that at e^2 order the Σ and Λ^μ functions are finite. Although, here, we restrict ourselves to the case of $\xi = 1$, these results can be generalized. It is possible to show that, for $\xi \neq 1$, the divergences associated with the electron self-energy function and the vertex part of the QED_4 are also canceled by the Podolsky contribution. And, as an immediate consequence of the finiteness of Σ and Λ^μ, we verified that the main WFT identity (59) keeps being satisfied.

As a final comment, the Podolsky parameter a, which appears in all the expressions evaluated here as a free parameter (as the inverse of the photon mass), can have its range of values limited through applications of the Podolsky theory. For example, we can evaluate now the physical quantity $\bar{u}(p')\Lambda^\mu u(p)$ that is related to the form factors $F_1(q^2)$ and $F_2(q^2)$ of electric charge e, and to the anomalous magnetic moment of the electron, respectively. We expect to set a bound limit to the Podolsky parameter a through the use of precise experimental data from the electron magnetic moment, by calculating the form factor $F_2(q^2)$ for the GQED_4. This study is now under development. We can also express the quantum theory in a more formal and constructive method, through dispersion relations [23], which can give more transparent results and, also, a direct evaluation of electron anomalous momentum. Another interesting issue is the study of the gauge properties of the propagators for the GQED_4, constructing and analyzing the Landau-Khalatnikov-Fradkin transformation [24] for the theory. As mentioned before, a renormalization process for the photon propagator is necessary, due to the divergence present in the self-energy; although the divergence is the same as that of the QED_4, the renormalization constant and, also, the running coupling constant may differ from the results for the QED_4 due to the poles from the photon propagator expression (38).

Going beyond $T = 0$, we can study the GQED_4 at finite temperature and derive all the thermodynamical quantities of the theory, including the energy-density distribution. And, following the idea of a recent study of the Podolsky electromagnetism at finite temperature [20], where a bound value was set to the Podolsky parameter a through the energy distribution using the cosmic microwave background radiation temperature, we can also use the cosmic microwave background radiation temperature to set a value to a through the thermodynamical quantities of the GQED_4. These issues and others will be further elaborated, the subject of deeper investigations, and reported elsewhere.

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APPENDIX A: *d*-DIMENSIONAL IDENTITIES

As we made use of dimensional regularization in the evaluation of the radiative correction expressions, we present here some useful d-dimensional identities associated with integrals, properties of gamma functions, and Dirac matrices.

1. Integration in *d* dimensions

The useful results of integrals that appear throughout the paper are

$$\int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - m^2)^\alpha} = \frac{i(-1)^{d/2}}{(4\pi)^{d/2}} \frac{\Gamma(\alpha - \frac{d}{2})}{\Gamma(\alpha)[-m^2]^{\alpha-(d/2)}}, \tag{A1}$$

$$\int \frac{d^d k}{(2\pi)^d} \frac{k_\mu k_\nu}{(k^2 - m^2)^\alpha} = \frac{i(-1)^{d/2}}{2(4\pi)^{d/2}} \frac{\eta_{\mu\nu}\Gamma(\alpha - 1 - \frac{d}{2})}{\Gamma(\alpha)[-m^2]^{\alpha-1-d/2}}, \tag{A2}$$

$$\int \frac{d^d k}{(2\pi)^d} \frac{k_\mu k_\nu k_\sigma k_\rho}{(k^2 - m^2)^\alpha} = \frac{i(-1)^{d/2}}{4(4\pi)^{-d/2}} \Gamma\left(\alpha - 2 - \frac{d}{2}\right) \times \frac{[\eta_{\mu\nu}\eta_{\sigma\rho} + \eta_{\nu\rho}\eta_{\mu\sigma} + \eta_{\rho\mu}\eta_{\nu\sigma}]}{\Gamma(\alpha)[-m^2]^{\alpha-2-(d/2)}}. \tag{A3}$$

2. The gamma function

An important property of the gamma function, with small ϵ, is given by the following relation:

$$\Gamma(-n + \epsilon) = \frac{(-1)^n}{n!}\left[\frac{1}{\epsilon} + \psi_1(n + 1) + O(\epsilon)\right], \tag{A4}$$

where

$$\psi_1(n + 1) = 1 + \frac{1}{2} + \ldots\ldots + \frac{1}{n} - \gamma, \tag{A5}$$

and γ is the Euler-Mascheroni constant. We needed the formulae

$$z\Gamma(z) = \Gamma(z + 1), \qquad \mathcal{X}^{-(\epsilon/2)} \simeq 1 - \frac{\epsilon}{2}\ln\mathcal{X}, \tag{A6}$$

as well.

3. Dirac matrices

The algebra of Dirac matrices in d dimensions is

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}, \tag{A7}$$

where $\eta^{\mu\nu}$ is the metric tensor in d-dimensional Minkowski space (with signature $+ - - \ldots$), so that $\delta^\mu_\mu = d$; hence,

$$\begin{aligned}
\gamma^\mu\gamma_\mu &= d,\\
\gamma^\sigma\gamma^\mu\gamma_\sigma &= (2-d)\gamma^\mu,\\
\gamma^\sigma\gamma^\lambda\gamma^\mu\gamma_\sigma &= 2(\gamma^\mu\gamma^\lambda - \gamma^\lambda\gamma^\mu) + d\gamma^\lambda\gamma^\mu,\\
\gamma^\alpha\gamma^\sigma\gamma^\mu\gamma_\sigma\gamma_\alpha &= (2-d)^2\gamma^\mu,\\
\gamma^\alpha\gamma^\sigma\gamma^\mu\gamma_\alpha\gamma_\sigma &= [2d-(2-d)^2]\gamma^\mu,\\
\gamma^\sigma\gamma^\lambda\gamma^\mu\gamma^\eta\gamma_\sigma &= 2(\gamma^\eta\gamma^\lambda\gamma^\mu - \gamma^\mu\gamma^\lambda\gamma^\eta + \gamma^\lambda\gamma^\mu\gamma^\eta)\\
&\quad - d\gamma^\lambda\gamma^\mu\gamma^\eta.
\end{aligned} \tag{A8}$$

In addition,

$$\begin{aligned}
&\mathrm{Tr}(\text{odd no. of } \gamma \text{ matrices}) = 0,\\
&\mathrm{Tr}I = f(d), \qquad \mathrm{Tr}\gamma_\sigma\gamma_\alpha = f(d)\eta_{\sigma\alpha},\\
&\mathrm{Tr}\gamma^\sigma\gamma^\lambda\gamma^\mu\gamma^\eta = f(d)[\eta^{\sigma\lambda}\eta^{\mu\eta} - \eta^{\sigma\mu}\eta^{\lambda\eta} + \eta^{\sigma\eta}\eta^{\lambda\mu}],
\end{aligned} \tag{A9}$$

where $f(d)$ is an arbitrary well-behaved function, with $f(4) = 4$.

APPENDIX B: CALCULUS OF $\Sigma^{(2)}_{\mathrm{Pod}}(p)$ AND $\Sigma^{(3)}_{\mathrm{Pod}}(p)$

In order to evaluate the terms $\Sigma^{(2)}_{\mathrm{Pod}}$ and $\Sigma^{(3)}_{\mathrm{Pod}}$, we will follow the same steps presented in the calculation of $\Sigma^{(1)}_{\mathrm{Pod}}$ in Subsec. VA. First, we recall the Feynman parametrization and the dimensional regularization. Thus, from (70), we obtain

$$\Sigma^{(2)}_{\mathrm{Pod}}(p) = -2i\mu^{4-d}\int d\varsigma \int \frac{d^dk}{(2\pi)^d} \times \frac{(\hat{k}+\hat{p}y)m[(1-y)\hat{p}-\hat{k}+m](\hat{k}+\hat{p}y)}{[k^2+b_x^2]^3}, \tag{B1}$$

where we have changed $k \to k - py$, introduced $b_x^2 = (1-y)yp^2 - m^2 + x\frac{1}{a^2}$, and used Eq. (96) for the measure $d\varsigma$. Since the integral of the odd powers of k in the numerator is zero, it is enough to evaluate the contribution of even powers. Then, carrying out the k integration, Eq. (B1) is written as

$$\begin{aligned}
\Sigma^{(2)}_{\mathrm{Pod}}(p) &= \frac{(-1)^{d/2}\mu^{4-d}}{2(4\pi)^{d/2}}\Gamma\left(2-\frac{d}{2}\right)\int d\varsigma\{[2(1-y)\\
&\quad -(1+y)d]\hat{p} + md\}[b_x^2]^{(d/2)-2}\\
&\quad + (-1)^{d/2}\frac{\mu^{4-d}}{(4\pi)^{d/2}}\Gamma\left(3-\frac{d}{2}\right)\\
&\quad \times \int d\varsigma[(1-y)\hat{p}+m]m^2y^2[b_x^2]^{(d/2)-3}.
\end{aligned} \tag{B2}$$

Indeed, expanding (B2) for $d \to 4$, we find that $\Sigma^{(2)}_{\mathrm{Pod}}$ can be expressed as

$$\Sigma^{(2)}_{\mathrm{Pod}}(p) = \frac{1}{\epsilon}\frac{1}{8\pi^2}(m-\hat{p}) + \Sigma^{(2)}_{\mathrm{Pod\ Finite}}(p), \tag{B3}$$

where

$$\begin{aligned}
\Sigma^{(2)}_{\mathrm{Pod\ Finite}}(p) &= \frac{1}{16\pi^2}\left[\left(\gamma+\frac{2}{3}\right)\hat{p} - \left(\gamma+\frac{1}{2}\right)m\right]\\
&\quad + \frac{1}{16\pi^2}\int d\varsigma\left\{\frac{[(1-y)\hat{p}+m]p^2y^2}{b_x^2}\right.\\
&\quad \left. - [2m-(1+3y)\hat{p}]\ln\left|\frac{b_x^2}{4\pi\mu^2}\right|\right\}.
\end{aligned} \tag{B4}$$

The term $\Sigma^{(3)}_{\mathrm{Pod}}$ (71) is evaluated following the same steps as in the previous ones through the Feynman parametrization and the dimensional regularization, and also replacing $k \to k - pz$:

$$\Sigma^{(3)}_{\mathrm{Pod}}(p) = 2i\mu^{4-d}\int d\varsigma\int\frac{d^dk}{(2\pi)^d} \times \frac{(\hat{k}+\hat{p}y)[(1-y)\hat{p}-\hat{k}+m](\hat{k}+\hat{p}y)}{[k^2+b^2]^3}, \tag{B5}$$

where $b^2 = (1-y)(yp^2 - \frac{1}{a^2}) - m^2y$, as we have defined in Subsec. VA. Carrying out the momentum integration now, we find for (B5) the expression

$$\begin{aligned}
\Sigma^{(3)}_{\mathrm{Pod}}(p) &= (-1)^{(d/2)-1}\frac{\mu^{4-d}}{2(4\pi)^{d/2}}\Gamma\left(2-\frac{d}{2}\right)\int d\varsigma\{[2(1-y)\\
&\quad -(1+y)d]\hat{p}+md\}[b^2]^{(d/2)-2}\\
&\quad -(-1)^{d/2}\frac{\mu^{4-d}}{(4\pi)^{d/2}}\Gamma\left(3-\frac{d}{2}\right)\\
&\quad \times\int d\varsigma[(1-y)\hat{p}+m]p^2y^2[b^2]^{(d/2)-3},
\end{aligned} \tag{B6}$$

which in the limit $d \to 4$ is written as

$$\Sigma^{(3)}_{\mathrm{Pod}}(p) = -\frac{1}{\epsilon}\frac{1}{8\pi^2}(m-\hat{p}) + \Sigma^{(3)}_{\mathrm{Pod\ Finite}}, \tag{B7}$$

with

$$\begin{aligned}
\Sigma^{(3)}_{\mathrm{Pod\ Finite}}(p) &= \frac{1}{16\pi^2}\left[\left(\gamma+\frac{1}{2}\right)m - \left(\gamma+\frac{2}{3}\right)\hat{p}\right]\\
&\quad - \frac{1}{16\pi^2}\int d\varsigma\left\{\frac{[(1-y)\hat{p}+m]p^2y^2}{b^2}\right.\\
&\quad \left. - [2m-(1+3y)\hat{p}]\ln\left|\frac{b^2}{4\pi\mu^2}\right|\right\}.
\end{aligned} \tag{B8}$$

Therefore, from the results of Eqs. (76), (B4), and (B8) we obtain the following expression of the finite part of the Podolsky contribution $\Sigma_{\text{Pod Finite}}$:

$$\Sigma_{\text{Pod Finite}} = \frac{1}{16\pi^2}[(\gamma+1)\hat{p} - 2m(2\gamma+1)] + \frac{1}{16\pi^2}\int d\varsigma\bigg\{[2m - (1+3y)\hat{p}]\ln\bigg|\frac{(1-y)(yp^2 - \frac{1}{a^2}) - ym^2}{(1-y)yp^2 - \frac{x}{a^2} - ym^2}\bigg| + \hat{p}[(1-y)\hat{p} + m]\hat{p}y^2\bigg[\frac{1}{(1-y)(yp^2 - \frac{1}{a^2}) - ym^2} - \frac{1}{(1-y)yp^2 - \frac{x}{a^2} - ym^2}\bigg]\bigg\} + \frac{1}{8\pi^2}\int_0^1 dz[(1-z)\hat{p} - 2m]\ln\bigg|\frac{(1-z)(zp^2 - \frac{1}{a^2}) - zm^2}{4\pi\mu^2}\bigg|. \tag{B9}$$

APPENDIX C: CALCULUS OF $\Lambda^{(2)\mu}_{\text{Pod}}(p', p)$ AND $\Lambda^{(3)\mu}_{\text{Pod}}(p', p)$

In the same way as we did in Subsec. V B, we will proceed here into the calculation of the terms $\Lambda^{\mu(2)}_{\text{Pod}}$ and $\Lambda^{\mu(3)}_{\text{Pod}}$ of the Podolsky contribution to the vertex part at e^2 order. Now, recalling the Feynman parametrization and the dimensional regularization, Eq. (90) is expressed as

$$\Lambda^{\mu(2)}_{\text{Pod}}(p', p) = 3i\mu^{4-d}(4\pi)^{d/2}\int d\xi\int\frac{d^dk}{(2\pi)^d} \times \frac{\boldsymbol{O}^\mu(k, p', p, x, y)}{[k^2 + \Delta^2 - \frac{1}{a^2}z]^4}, \tag{C1}$$

where we have replaced k by $k - xp - yp'$ and defined conveniently the function

$$\boldsymbol{O}^\mu(k, p', p, x, y) = (\hat{k} - x\hat{p} - y\hat{p}')[(1-y)\hat{p}' - \hat{k} - x\hat{p} + m]\gamma^\mu[(1-x)\hat{p} - \hat{k} - y\hat{p}' + m]\gamma^\nu(\hat{k} - x\hat{p} - y\hat{p}').$$

After a manipulation of γ matrices in (C1) and evaluating the momentum integration, one gets

$$\Lambda^{\mu(2)}_{\text{Pod}}(p', p) = -\frac{(-1)^{d/2}}{4}\frac{\mu^{4-d}}{(4\pi)^{d/2}}(d^2 + 2d)\gamma^\mu\Gamma\Big(2 - \frac{d}{2}\Big)\int d\xi\frac{1}{[\Delta^2 - \frac{1}{a^2}z]^{2-(d/2)}} - (-1)^{d/2}\frac{\mu^{4-d}}{(4\pi)^{d/2}}\Gamma\Big(4 - \frac{d}{2}\Big) \times \int d\xi\frac{\boldsymbol{\Sigma_1}(x, y, \hat{p}', \hat{p})\gamma^\mu\boldsymbol{\Sigma_2}(x, y, \hat{p}', \hat{p})}{[\Delta^2 - \frac{1}{a^2}z]^{4-(d/2)}} + \frac{(-1)^{d/2}}{2}\frac{\mu^{4-d}}{(4\pi)^{d/2}}\Gamma\Big(3 - \frac{d}{2}\Big)\int d\xi\frac{1}{[\Delta^2 - \frac{1}{a^2}z]^{3-(d/2)}} \times [d\boldsymbol{\Sigma_1}(x, y, \hat{p}', \hat{p})\gamma^\mu + d\gamma^\mu\boldsymbol{\Sigma_2}(x, y, \hat{p}', \hat{p}) - \boldsymbol{\Sigma}^\mu_{\boldsymbol{3}}(x, y, \hat{p}', \hat{p}) - (2-d)(\hat{p}' - m)\gamma^\mu(\hat{p} - m)], \tag{C2}$$

where we have defined the functions $\boldsymbol{\Sigma_1}$, $\boldsymbol{\Sigma_2}$, and $\boldsymbol{\Sigma}^\mu_{\boldsymbol{3}}$ and the measure $d\xi$ in (105) and (106), respectively.

Now, Eq. (C2) in the limit $d \longrightarrow 4$ assumes the form

$$\Lambda^{\mu(2)}_{\text{Pod}}(p', p) = -\frac{1}{\epsilon}\frac{1}{8\pi^2}\gamma^\mu + \Lambda^{\mu(2)}_{\text{Pod Finite}}(p', p), \tag{C3}$$

with

$$\Lambda^{\mu(2)}_{\text{Pod Finite}}(p', p) = \frac{1}{16\pi^2}\gamma^\mu\bigg[\frac{5}{6} + \gamma + 6\int d\xi\ln\bigg|\frac{\Delta^2 - \frac{1}{a^2}z}{4\pi\mu^2}\bigg|\bigg] - \frac{1}{16\pi^2}\int d\xi\frac{\boldsymbol{\Sigma_1}(x, y, \hat{p}', \hat{p})\gamma^\mu\boldsymbol{\Sigma_2}(x, y, \hat{p}', \hat{p})}{[\Delta^2 - \frac{1}{a^2}z]^2} + \frac{1}{8\pi^2}\int d\xi\frac{1}{[\Delta^2 - \frac{1}{a^2}z]}\bigg[\boldsymbol{\Sigma_1}(x, y, \hat{p}', \hat{p})\gamma^\mu + \gamma^\mu\boldsymbol{\Sigma_2}(x, y, \hat{p}', \hat{p}) - \frac{1}{4}\boldsymbol{\Sigma}^\mu_{\boldsymbol{3}}(x, y, \hat{p}', \hat{p}) + \frac{1}{2}(\hat{p}' - m)\gamma^\mu(\hat{p} - m)\bigg]. \tag{C4}$$

Following the same steps as before, we find for $\Lambda^{\mu(3)}_{\rm Pod}(p', p)$, Eq. (91), the following expression:

$$\Lambda^{\mu(3)}_{\rm Pod}(p', p) = -3i\mu^{4-d}\int d\xi \int \frac{d^dk}{(2\pi)^d}\frac{\boldsymbol{O}^\mu(k, p', p, x, y)}{[k^2 + \Delta^2 - \frac{1}{a^2}(1-x-y)]^4}. \tag{C5}$$

Now, when we evaluate the momentum integration in (C5), we get

$$\begin{aligned}\Lambda^{\mu(3)}_{\rm Pod}(p', p) = {}& \frac{(-1)^{d/2}}{4}\frac{\mu^{4-d}}{(4\pi)^{-(d/2)}}(d^2+2d)\gamma^\mu\Gamma\Big(2-\frac{d}{2}\Big)\int d\xi\frac{1}{[\Delta^2 - \frac{1}{a^2}(1-x-y)]^{2-(d/2)}} + (-1)^{d/2}\frac{\mu^{4-d}}{(4\pi)^{d/2}}\Gamma\Big(4-\frac{d}{2}\Big)\\ &\times\int d\xi\frac{\boldsymbol{\Sigma_1}(x, y, \hat{p}', \hat{p})\gamma^\mu\boldsymbol{\Sigma_2}(x, y, \hat{p}', \hat{p})}{[\Delta^2 - \frac{1}{a^2}(1-x-y)]^{4-(d/2)}} - \frac{(-1)^{d/2}}{2}\frac{\mu^{4-d}}{(4\pi)^{d/2}}\Gamma\Big(3-\frac{d}{2}\Big)\int d\xi\frac{1}{[\Delta^2 - \frac{1}{a^2}(1-x-y)]^{3-(d/2)}}\\ &\times[d\boldsymbol{\Sigma_1}(x, y, \hat{p}', \hat{p})\gamma^\mu + d\gamma^\mu\boldsymbol{\Sigma_2}(x, y, \hat{p}', \hat{p}) - \boldsymbol{\Sigma}^\mu_{\mathbf{3}}(x, y, \hat{p}', \hat{p}) - (2-d)(\hat{p}'-m)\gamma^\mu(\hat{p}-m)],\end{aligned} \tag{C6}$$

which in the limit $d \longrightarrow 4$ is expressed as

$$\Lambda^{\mu(3)}_{\rm Pod}(p', p) = \frac{1}{\epsilon}\frac{1}{8\pi^2}\gamma^\mu + \Lambda^{\mu(3)}_{\rm Pod\,Finite}(p', p), \tag{C7}$$

with the finite part written as

$$\begin{aligned}\Lambda^{\mu(3)}_{\rm Pod\,Finite}(p', p) = {}& -\frac{1}{16\pi^2}\gamma^\mu\Big[\frac{5}{6} + \gamma + 6\int d\xi\ln\Big|\frac{\Delta^2 - \frac{1}{a^2}(1-x-y)}{4\pi\mu^2}\Big|\Big] + \frac{1}{16\pi^2}\int d\xi\frac{\boldsymbol{\Sigma_1}(x, y, \hat{p}', \hat{p})\gamma^\mu\boldsymbol{\Sigma_2}(x, y, \hat{p}', \hat{p})}{[\Delta^2 - \frac{1}{a^2}(1-x-y)]^2}\\ &-\frac{1}{8\pi^2}\int d\xi\frac{1}{[\Delta^2 - \frac{1}{a^2}(1-x-y)]}\Big[\boldsymbol{\Sigma_1}(x, y, \hat{p}', \hat{p})\gamma^\mu + \gamma^\mu\boldsymbol{\Sigma_2}(x, y, \hat{p}', \hat{p})\\ &-\frac{1}{4}\boldsymbol{\Sigma}^\mu_{\mathbf{3}}(x, y, \hat{p}', \hat{p}) + \frac{1}{2}(\hat{p}'-m)\gamma^\mu(\hat{p}-m)\Big].\end{aligned} \tag{C8}$$

After a rearrangement of Eqs. (98), (C4), and (C8), we find that the expression of the finite part of the Podolsky contribution $\Lambda^\mu_{\rm Pod\,Finite}$ is written as follows:

$$\begin{aligned}\Lambda^{\mu}_{\rm Pod\,Finite}(p', p) = {}& \frac{1}{8\pi^2}\gamma^\mu\Big[1 + \frac{\gamma}{2} + \int d\varsigma\ln\Big|\frac{\Delta^2 - \frac{1}{a^2}(1-x-y)}{4\pi\mu^2}\Big|\Big] + \frac{1}{16\pi^2}\int d\varsigma\frac{\boldsymbol{\Xi}^\mu(p', p, x, y)}{\Delta^2 - \frac{1}{a^2}(1-x-y)}\\ &+\frac{3}{8\pi^2}\gamma^\mu\int d\xi\ln\Big|\frac{\Delta^2 - \frac{1}{a^2}z}{\Delta^2 - \frac{1}{a^2}(1-x-y)}\Big| + \frac{1}{16\pi^2}\int d\xi\boldsymbol{\Sigma_1}(x, y, \hat{p}', \hat{p})\gamma^\mu\boldsymbol{\Sigma_2}(x, y, \hat{p}', \hat{p})\\ &\times\Big\{\frac{\mathbf{1}}{[\Delta^2 - \frac{1}{a^2}(1-x-y)]^2} - \frac{\mathbf{1}}{[\Delta^2 - \frac{1}{a^2}z]^2}\Big\} + \frac{1}{8\pi^2}\int d\xi\Big[\boldsymbol{\Sigma_1}(x, y, \hat{p}', \hat{p})\gamma^\mu + \gamma^\mu\boldsymbol{\Sigma_2}(x, y, \hat{p}', \hat{p})\\ &-\frac{1}{4}\boldsymbol{\Sigma}^\mu_{\mathbf{3}}(x, y, \hat{p}', \hat{p}) + \frac{1}{2}(\hat{p}'-m)\gamma^\mu(\hat{p}-m)\Big]\Big[\frac{1}{\Delta^2 - \frac{1}{a^2}z} - \frac{\mathbf{1}}{\Delta^2 - \frac{1}{a^2}(1-x-y)}\Big].\end{aligned} \tag{C9}$$

[1] S. Weinberg, Physica (Amsterdam) **96A**, 327 (1979).

[2] M. Ostrogradski, Mem. Ac. St. Petersburg VI **4**, 385 (1850); P. Weiss, Proc. R. Soc. A **169**, 102 (1938); T. S. Chang, Math. Proc. Cambridge Philos. Soc. **44**, 76 (1948).

[3] A. A. Slavnov, Theor. Math. Phys. **13**, 1064 (1972); **33**, 977 (1977).

[4] V. V. Nesterenko, J. Phys. A **22**, 1673 (1989); L. Alvares-Gaume, L. Labastida, and J. M. F. Ramalho, Nucl. Phys. **B334**, 103 (1990).

[5] M. Baker, L. Carson, J. S. Ball, and F. Zachariasen, Nucl. Phys. **B229**, 456 (1983); A. I. Alekseev, B. A. Arbuzov, and V. A. Baikov, Theor. Math. Phys. **52**, 739 (1982); **59**, 372 (1984).

[6] K. S. Stelle, Phys. Rev. D **16**, 953 (1977); Gen. Relativ. Gravit. **9**, 353 (1978).
[7] R. R. Cuzinatto, C. A. M. de Melo, and P. J. Pompeia, Ann. Phys. (N.Y.) **322**, 1211 (2007).
[8] F. Bopp, Ann. Phys. (Leipzig) **430**, 345 (1940).
[9] B. Podolsky, Phys. Rev. **62**, 68 (1942); B. Podolsky and C. Kikuchy, Phys. Rev. **65**, 228 (1944); B. Podolsky and P. Schwed, Rev. Mod. Phys. **20**, 40 (1948).
[10] J. Frenkel, Phys. Rev. E **54**, 5859 (1996).
[11] E. R. Williams, J. E. Faller, and H. A. Hill, Phys. Rev. Lett. **26**, 721 (1971); L. Davis, A. S. Goldhaber, and M. M. Nieto, Phys. Rev. Lett. **35**, 1402 (1975); J. Luo, L. C. Tu, Z. K. Hu, and E. J. Luan, Phys. Rev. Lett. **90**, 081801 (2003); L.-C. Tu, J. Luo, and G. T. Gillies, Rep. Prog. Phys. **68**, 77 (2005).
[12] A. E. S. Green, Ph.D. thesis, University of Cincinnati, (Institution Report No. AAT 0151695, 1948); R. E. Martin, Ph.D. thesis, University of Cincinnati, 1960.
[13] C. A. P. Galvão and B. M. Pimentel, Can. J. Phys. **66**, 460 (1988).
[14] P. A. M. Dirac, *Lectures on Quantum Mechanics* (Yeshiva University, New York, 1964); A. Hanson, T. Regge, and C. Teitelboim, *Constrained Hamiltonian Systems*, (Accademia Nazionale dei Lincei, Rome, 1976); K. Sundermeyer, *Constrained Dynamics*, Lecture Notes in Physics Vol. 169 (Springer, New York, 1982).
[15] E. S. Fradkin, U. Esposito, and S. Termini, Riv. Nuovo Cimento Soc. Ital. Fis. **2**, 498 (1970).
[16] L. D. Faddeev, P. Senjanovic, and G. Miao, Teor. Mat. Fiz. **1**, 3 (1969) [Theor. Math. Phys. **1**, 1 (1969)]; P. Senjanovic, Ann. Phys. (N.Y.) **100**, 227 (1976); Y.-G. Miao, Ann. Phys. (N.Y.) **209**, 248(E) (1991).
[17] L. D. Faddeev and V. N. Popov, Phys. Lett. **25B**, 29 (1967); B. S. DeWitt, Phys. Rev. **160**, 1113 (1967).
[18] J. Schwinger, Proc. Natl. Acad. Sci. U.S.A. **37**, 452 (1951); K. Symanzik, Z. Naturforsch. **9A**, 809 (1954); E. S. Fradkin, Sov. Phys. JETP **2**, 148 (1956).
[19] F. J. Dyson, Phys. Rev. **75**, 1736 (1949).
[20] C. A. Bonin, R. Bufalo, B. M. Pimentel, and G. E. R. Zambrano, Phys. Rev. D **81**, 025003 (2010).
[21] N. N. Bogoliubov and D. V. Shirkov, *Introduction to the Theory of Quantized Fields* (John Wiley and Sons, New York, 1980), 3rd ed.; A. I. Akhiezer and V. B. Berestetskii, *Quantum Electrodynmics* (Interscience, New York, 1965), 2nd ed.; P. H. Frampton, *Gauge Field Theories* (Wiley-VCH, Verlag, 2008), 3rd ed..
[22] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965).
[23] K. Nishijima, *Fields and Particles: Field Theory and Dispersion Relations* (Benjamin Cummings, New York, 1969).
[24] L. D. Landau and I. M. Khalatnikov, Zh. Eksp. Teor. Fiz. **29**, 89 (1955) [Sov. Phys. JETP **2**, 69 (1956)]; E. S. Fradkin, Sov. Phys. JETP **2**, 361 (1956).