

**Fermion resonances on a thick brane with a piecewise warp factor**Hai-Tao Li,<sup>\*</sup> Yu-Xiao Liu,<sup>†</sup> Zhen-Hua Zhao,<sup>‡</sup> and Heng Guo<sup>§</sup>*Institute of Theoretical Physics, Lanzhou University, Lanzhou 730000, People's Republic of China*  
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In this paper, we mainly investigate the problems of resonances of massive Kaluza-Klein (KK) fermions on a single scalar constructed thick brane with a piecewise warp factor matching smoothly. The distance between two boundaries and the other parameters are determined by one free parameter through three junction conditions. For the generalized Yukawa coupling  $\eta\bar{\Psi}\phi^k\Psi$  with odd  $k = 1, 3, 5, \dots$ , the mass eigenvalue  $m$ , width  $\Gamma$ , lifetime  $\tau$ , and maximal probability  $P_{\max}$  of fermion resonances are obtained. Our numerical calculations show that the brane without internal structure also favors the appearance of resonant states for both left- and right-handed fermions. The scalar-fermion coupling and the thickness of the brane influence the resonant behaviors of the massive KK fermions.

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**I. INTRODUCTION**

As we know, in order to unify electromagnetism with Einstein gravity, Kaluza and Klein first proposed that space-times have more than three spatial dimensions [1]. Later, higher dimensional space-time with large extra dimensions [2–16] had been paid more attention; furthermore, the emphasis shifted to a “brane world” picture which opened up a rich and interesting route towards solving the long-standing problems like the mass hierarchy and cosmological constant problems in high-energy physics. Subsequently, the idea had been applied in the standard model [2,3,10]. In particular, inspired also in [17,18], it was put forward that at the energy scales of standard model the matter fields cannot propagate into extra dimensions while gravity on the other hand can permeate through all dimensions [9,11,12]. A new wave of research in the field of extra dimensions came with the framework of Arkani-Hamed, Dimopoulos, and Dvali who proposed the large extra dimensions model (ADD model) [9], which lowers the energy scale of quantum gravity to 1 TeV by localizing the standard model fields to a 4-brane in a higher dimensional space-time so that the hierarchy problem can be addressed within the framework of ADD model. In Ref. [11], the Randall-Sundrum (RS) model provides an alternative explanation for the gauge hierarchy problem in particle physics based on small extra dimensions and a nonfactorizable bulk geometry (AdS<sub>5</sub>). Subsequent progress about brane world and extra dimension suggested that the warped metric could even provide an alternative to compactification for the extra dimensions [12,13].

In RS warped brane world scenarios, the brane is the so-called thin brane, which is an infinitely thin object, in which the energy density of the brane is a deltalike

function with respect to a fifth dimension coordinate. So this model is a very idealized brane world model. Recently, more realistic thick brane models were investigated in higher dimensional space-time [19–31]. In thick brane scenarios, the coupling between gravity and scalars should be introduced. By introducing scalar fields in the bulk [32], the modulus can be stabilized in a RS warped brane world scenario. In the thick brane scenario, the known solutions can be classified into topologically nontrivial solutions and trivial ones. A comprehensive review on the thick brane solutions and related topics is given in Ref. [33] and the definition of thick brane was given to avoid the problems related to possible different understandings of this term.

Then, it is possible to describe the matter and interactions localized on the brane by a natural mechanism in higher dimensional space-time. One can consider various bulk fields on the brane, such as spin 0 scalars, spin 1/2 fermions, and so forth [34–42]. However, the localization of spin 1/2 fermions shows very interesting and important properties. In addition, the localization problem of fermions on a kind of topological defect called domain wall has been extensively investigated [35–42]. For localizing fermions on the thick branes or domain walls, the other kind of interactions were introduced in addition to gravity. The common interactions include the generalized Yukawa coupling between the fermions and the background scalar field. Besides, one can consider the gauge field [43,44] localized in our universe. The localization problem can also be discussed in the context of supersymmetry and supergravity [45–47]. In different backgrounds, such as vortex background [48–51] and general space-time background [52], the localization of various fields was investigated.

The discontinuous Kaluza-Klein (KK) modes with gap called bound states and the continuous gapless states with  $m^2 > 0$  can be derived [53–56]. Besides, the metastable KK states of the graviton or fermion with finite lifetime can also be obtained in many brane world models

<sup>\*</sup>liht07@lzu.cn<sup>†</sup>liuyx@lzu.edu.cn; corresponding author.<sup>‡</sup>zhaozh02@gmail.com<sup>§</sup>guoh06@lzu.cn

[22,35,57–68]. Recently, the problem of the fermion resonances has been extensively studied in many thick brane models [59,63–66,68,69]. In Ref. [59], the authors investigated the general properties of localization of fermions and scalars in smoothed field-theoretical versions of the type II RS brane world model. They reached an important conclusion that, if discrete bound states are present in the gravity-free case, those become resonant states in the continuum, while off-resonant modes are highly suppressed on the brane. In Ref. [63], the authors investigated fermion resonances on the Bloch brane constructed with two background scalar fields [70]. The problem of the massive resonant KK modes with even-parity for both chiralities was studied in depth and their appearance is related to branes with internal structure. In Ref. [64], the fermion resonances were studied in the context of a de Sitter thick brane; the authors found that there exist resonant KK modes in a single brane without internal structure. The two conclusions are different because in Ref. [64] the coupling parameter  $\eta$  was varied but in Ref. [70]  $\eta$  was fixed. Particularly, in Ref. [66], the authors extended the fermion resonances to multiscalar generated thick branes and found the Kaluza-Klein chiral decompositions of massive fermion resonances are the parity-chirality decompositions. In this paper, we consider the fermion resonances on the thick brane model with a piecewise warp factor [62]. In addition to the conventional analyses for the mass eigenvalue  $m$ , width  $\Gamma$ , lifetime  $\tau$ , maximal probability  $P_{\max}$ , and mass spectra of the massive KK fermions, we give an analysis about the effects of a free parameter  $V_0$  of the brane on the fermion resonances. Some new and interesting results are obtained and discussed.

The organization of this paper is as follows. In Sec. II, we review the general aspects of the thick brane model with a piecewise warp factor. The equations of motion and the solution of the background scalar field are obtained. Besides, taking into account the interaction between the fermion and the scalar field by a generalized Yukawa coupling, the Schrödinger-like equations are derived with the KK decomposition. In Sec. III, we provide a complete analysis of the fermion resonances on the thick brane in detail. Finally, our discussions and conclusions are presented in Sec. IV.

## II. REVIEW OF THE THICK BRANE WITH A PIECEWISE WARP FACTOR

In this section, we will review the construction of the thick brane with a piecewise warp factor. With the actions as the starting points, the field equations for the scalar field and the Dirac field are obtained, respectively. Especially, inspired by the warp factor in [62], we construct a new warp factor and the solutions of the scalar field and the scalar potential are obtained.

### A. The setup of the thick brane with a piecewise warp factor

Let us consider the single scalar field thick brane in  $(4 + 1)$ -dimensional space-time with a scalar potential  $V(\phi)$ . Specifically, the action describing such a system is given by

$$S = \int d^5x \sqrt{-g} \left[ \frac{1}{2\kappa_5^2} R - \frac{1}{2} g^{MN} \partial_M \phi \partial_N \phi - V(\phi) \right], \quad (1)$$

where  $R$  is the scalar curvature,  $g = \det(g_{MN})$ ,  $M, N = 0, 1, 2, 3, 4$ , and  $\kappa_5^2 = 8\pi G_5$  with  $G_5$  the five-dimensional Newtonian constant of gravitation. We choose the conformally flat metric and the line element is assumed as

$$ds^2 = g_{MN} dx^M dx^N = e^{2A(z)} (\eta_{\mu\nu} dx^\mu dx^\nu + dz^2), \quad (2)$$

where  $e^{2A}$  is the warp factor, the metric tensor presents signature in the form  $(-, +, +, +, +)$ , and  $z$  stands for the coordinate of the extra space dimension. In the ansatz, the warp factor and the scalar field are considered to be functions of  $z$  only, i.e.,  $A = A(z)$ ,  $\phi = \phi(z)$ . It is straightforward to obtain the Ricci tensor and the curvature scalar:

$$R_{\mu\nu} = -(3A'^2 + A'')\eta_{\mu\nu}, \quad R_{44} = -4A'', \quad (3)$$

$$R = -4(3A'^2 + 2A'')e^{-2A}, \quad (4)$$

where the prime denotes derivative with respect to the fifth coordinate  $z$ .

The Euler-Lagrange equations can be obtained from the action (1) with the metric ansatz (2). After some simple variational calculations, the following coupled nonlinear differential equations are obtained:

$$\phi'^2 = 3(A'^2 - A''), \quad (5)$$

$$V(\phi) = \frac{3}{2}(-3A'^2 - A'')e^{-2A}, \quad (6)$$

$$\frac{dV(\phi)}{d\phi} = (3A'\phi' + \phi'')e^{-2A}. \quad (7)$$

In order to obtain a thick brane, we modify the warp factor in [62] as follows:

$$A(z) = \begin{cases} 2 \ln \left[ \frac{\cos(\sqrt{V_0}|z|) + 2}{3} \right], & |z| \leq \frac{d}{2} \\ -\frac{1}{2} \ln[k_0^2(|z| + \beta)^2], & |z| > \frac{d}{2}, \end{cases} \quad (8)$$

where  $V_0$ ,  $d$ , and  $\beta$  are constants,  $k_0 = \sqrt{-\Lambda/6}$  [71] and the warp factor is a piecewise defined function. For various values of  $V_0$ , the corresponding warp factor is plotted in Fig. 1(a). We note that the shape of the warp factor here is a normal distribution configuration like in many other thick brane models. It is worth noting that the continuity of the scalar field  $\phi$  and its first order derivative  $\phi'$  closely depends on the continuity of the warp factor  $A$ , its first

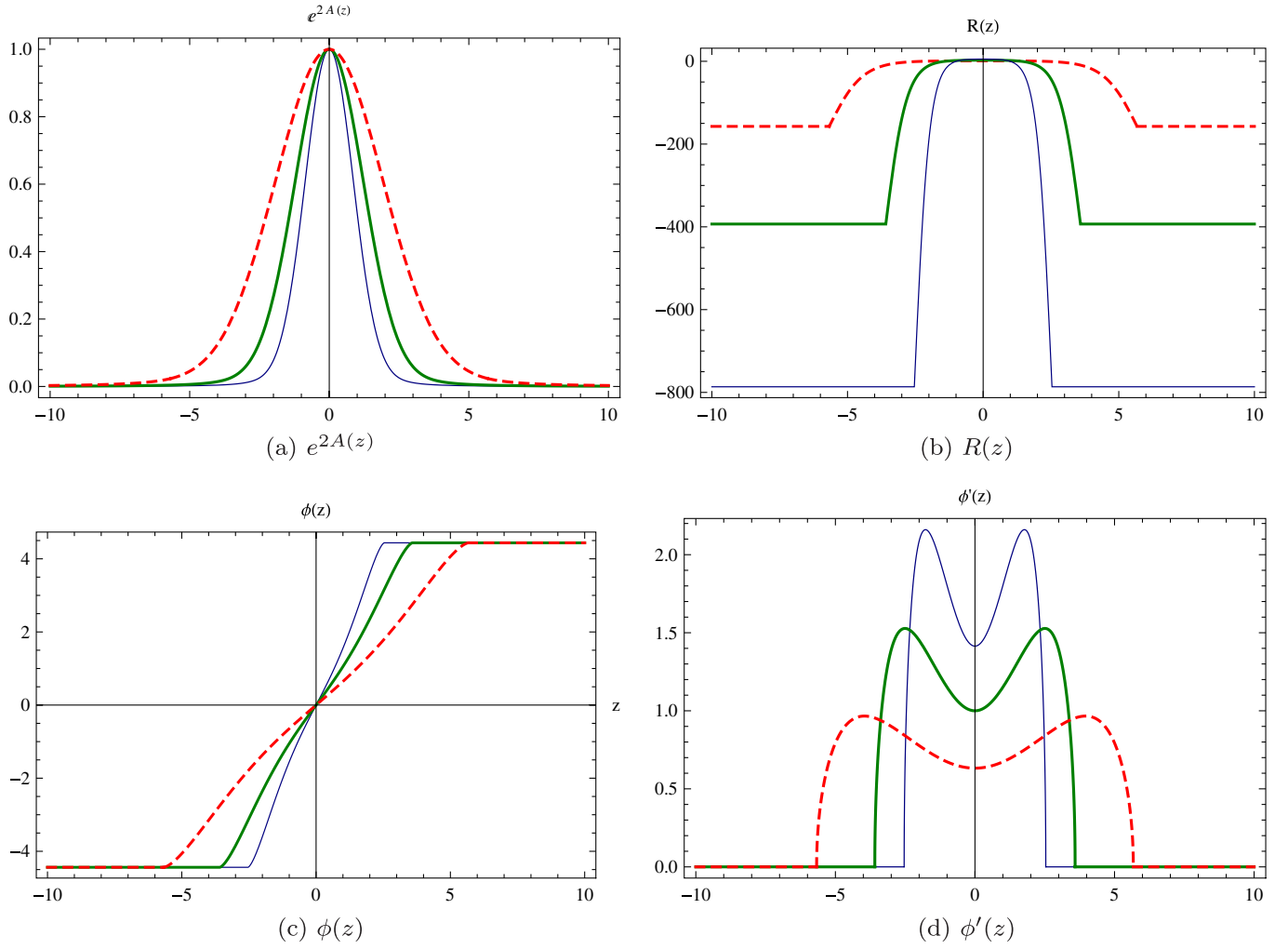


FIG. 1 (color online). The warp factor  $e^{2A(z)}$ , curvature scalar  $R(z)$ , scalar field  $\phi(z)$ , and its first derivative  $\phi'(z)$ . The parameters are set to  $V_0 = 0.2$  (dashed thicker), 0.5 (solid thicker), and 1 (solid thinner).

order derivative  $A'$ , and its second order derivative  $A''$  at the junctions  $z = \pm d/2$ , which impose the following three junction conditions among the parameters  $\beta$ ,  $k_0$ ,  $V_0$ , and  $d$ :

$$k_0 \left( \frac{d}{2} + \beta \right) = 9 \left[ \cos \left( \frac{\sqrt{V_0} d}{2} \right) + 2 \right]^{-2}, \quad (9)$$

$$\left( \frac{d}{2} + \beta \right)^{-1} = \frac{2\sqrt{V_0} \sin \left( \frac{\sqrt{V_0} d}{2} \right)}{2 + \cos \left( \frac{\sqrt{V_0} d}{2} \right)}, \quad (10)$$

$$\left( \frac{d}{2} + \beta \right)^{-2} = -\frac{2V_0 [1 + 2 \cos \left( \frac{\sqrt{V_0} d}{2} \right)]}{[2 + \cos \left( \frac{\sqrt{V_0} d}{2} \right)]^2}. \quad (11)$$

From Eqs. (9)–(11), the parameters  $\beta$ ,  $k_0$ , and  $d$  can be derived in terms of  $V_0$ , namely,

$$d = \frac{2 \arccos \left( \frac{1-\sqrt{7}}{2} \right)}{\sqrt{V_0}}, \quad \beta = -\frac{1.50145}{\sqrt{V_0}}, \quad (12)$$

$$k_0 = 6.2708 \sqrt{V_0}.$$

The above three conditions can ensure the correct Israel junction conditions. The similar situation was explained in the work [62]. This means that, in order to get a continuous scalar field without derivative jumps, the parameters must satisfy the three junction conditions. Thus, with the three junction conditions, we obtain a thick brane, and the thickness of the brane  $d$  is characterized by a free parameter  $V_0$ . In the thick brane scenario, the warp factor is generally expressed in terms of a “smooth function”  $e^{2A(z)}$  of the fifth coordinate  $z$ . Here, the second order derivatives of the warp factor are continuous at two junctions  $z = \pm d/2$  but the warp factor has third order derivative jumps. In a sense, we can interpret  $e^{2A(z)}$  as a smooth curve [72]. So, we call this kind of thick brane the thick brane with a piecewise warp factor matching smoothly.

Notice that the junction points  $z = \pm d/2$  are located within the first period of the periodic function  $[(\cos(\sqrt{V_0}|z|) + 2)/3]^4$ . In this paper, we focus on the problem of the fermion resonances of the massive chiral KK modes in the background of the thick brane with a piecewise warp factor matching smoothly.

From the field equations (5)–(7) and the warp factor (8), together with the junction conditions, the scalar field and the scalar potential are given by

$$\phi(z) = \begin{cases} 2i\sqrt{2q}\Phi(z), & |z| \leq \frac{d}{2} \\ \text{sign}(z)2i\sqrt{2q}\Phi(d/2), & |z| > \frac{d}{2}, \end{cases} \quad (13)$$

$$\Phi(z) = F(\varphi(z), p) - 6\Pi(q/3, \varphi(z), p) + 4\Pi(q, \varphi(z), p), \quad (14)$$

$$\varphi(z) = i \operatorname{arcsinh} \left[ \frac{1}{\sqrt{q}} \tan \left( \frac{\sqrt{V_0}z}{2} \right) \right], \quad (15)$$

$$p = (-53 + 20\sqrt{7})/3, \quad q = 2\sqrt{7} - 5,$$

$$V(\phi(z)) = \begin{cases} \frac{243V_0[3\cos(2\sqrt{V_0}z) + 2\cos(\sqrt{V_0}z) - 2]}{[\cos(\sqrt{V_0}z) + 2]^6}, & |z| \leq \frac{d}{2} \\ -6k_0^2, & |z| > \frac{d}{2}, \end{cases} \quad (16)$$

where  $i$  is the imaginary unit,  $F(\varphi(z), p)$  is the elliptic integral of the first kind, and  $\Pi(q, \varphi(z), p)$  is the incomplete elliptic integral. The shape of the scalar field is shown in Fig. 1(c). From the figure, we can see that the scalar field has a kinklike structure. Within the intervals  $|z| > d/2$ , the scalar field tends to two constants. It is worth noting that we can ensure that the first order derivative  $\phi'$  is zero when  $|z| \geq d/2$  [see Fig. 1(d)]. In other words, the extradimensional field profile  $\phi(z)$  has no derivative jumps at  $z = \pm d/2$ . It is easy to check that the curvature scalar  $R(z)$  and the scalar potential  $V(\phi(z))$  are continuous.<sup>1</sup>

Here, the energy density of the thick brane is

$$T_{00} = g_{00}\mathcal{L} - 2\frac{\partial\mathcal{L}}{\partial g^{00}} = e^{2A(z)}\left(\frac{1}{2}\phi'^2 + V(\phi)\right), \quad (17)$$

where  $\mathcal{L}$  is the Lagrangian density of the matter field. The distribution of the energy density  $T_{00}$  along the extra dimension  $z$  is illustrated in Fig. 2. The energy density has no double-peaked structure which is seen as characterizing a brane with internal structure [28,63].

We now want to argue that the massless mode of gravity (i.e., the four-dimensional graviton) can be localized on the brane. We consider the gravitational perturbation with the metric fluctuations given by  $ds^2 = e^{2A(z)}((\eta_{\mu\nu} + h_{\mu\nu}(x, z))dx^\mu dx^\nu + dz^2)$ . Furthermore, we can decompose  $h_{\mu\nu}(x, z)$  in the form  $h_{\mu\nu} = e^{(3/2)A}\hat{h}_{\mu\nu}(x)\hat{\psi}_m(z)$ , and re-

<sup>1</sup>We thank the referee for pointing out the problem of the discontinuity of the scalar potential in the first version of the paper.

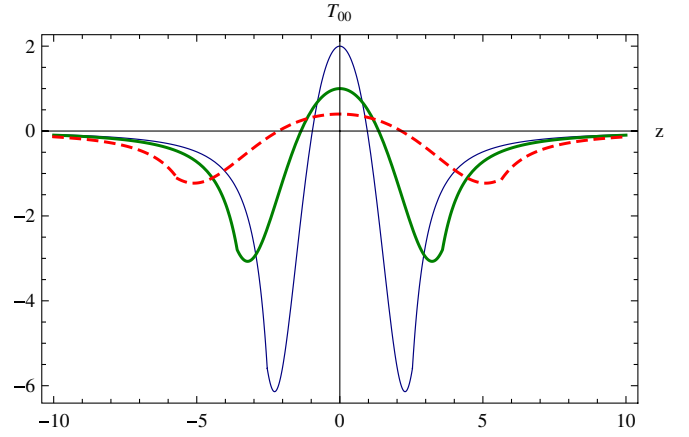


FIG. 2 (color online). The energy density  $T_{00}(z)$ . The parameters are set to  $V_0 = 0.2$  (dashed thicker),  $0.5$  (solid thicker), and  $1$  (solid thinner).

quire that  $\hat{h}$  be a four-dimensional mass eigenstate mode  $\square\hat{h}_{\mu\nu} = m^2\hat{h}_{\mu\nu}$ , where  $\square = \eta^{\mu\nu}\partial_\mu\partial_\nu$ . The Schrödinger-like equation under the transverse-traceless gauge can be obtained [12,19,22,71,73]:

$$-\hat{\psi}_m''(z) + V_G(z)\hat{\psi}_m(z) = m^2\hat{\psi}_m(z), \quad (18)$$

with the effective potential

$$V_G(z) = \frac{3}{2}A'' + \frac{9}{4}A'^2. \quad (19)$$

With the warp factor (8), it is easy to check that this is a volcano potential like in the RS model. Furthermore, there exists a zero mode solution  $\hat{\psi}_0 \propto e^{(3/2)A(z)}$ . It can be shown that the graviton zero mode actually is normalizable, namely,  $\int_{-\infty}^{+\infty} dz |\hat{\psi}_0|^2 \propto c_0 + k_0^{-3}(d/2 + \beta)^{-2}$  with  $c_0 = \frac{2}{729} \int_0^{d/2} dz [\cos(\sqrt{V_0}|z|) + 2]^6$  a constant. So the massless four-dimensional graviton can be produced by a zero mode [12,19,20,22,24].

Here, the observed four-dimensional effective Planck scale  $M_{\text{Pl}}$  can be derived. As we know, the five-dimensional Einstein-Hilbert action of gravity can be reduced to the four-dimensional one, namely,

$$\begin{aligned} S &\sim M_*^3 \int dx^5 \sqrt{g^{(5)}} R^{(5)} \supset M_*^3 \int e^{3A} dz \int dx^4 \sqrt{g^{(4)}} R^{(4)} \\ &= M_{\text{Pl}}^2 \int dx^4 \sqrt{g^{(4)}} R^{(4)}, \end{aligned} \quad (20)$$

where  $M_*$  is the five-dimensional fundamental scale. Thus the four-dimensional effective Planck scale can be derived:

$$M_{\text{Pl}}^2 = M_*^3 \int_{-\infty}^{+\infty} e^{3A} dz = M_*^3 [c_0 + k_0^{-3}(d/2 + \beta)^{-2}]. \quad (21)$$

The four-dimensional effective Planck scale can be expressed through the fundamental scale and a finite one-dimensional warped volume.

### B. Spin 1/2 fermion fields on the brane

In this subsection, we will review the general aspects of the spin half fermions on the brane. Let us consider a massless bulk fermion coupled to the background scalar by the generalized Yukawa coupling in five-dimensional space-time. The action reads

$$S_{1/2} = \int d^5x \sqrt{-g} [\bar{\Psi} \Gamma^M D_M \Psi - \eta \bar{\Psi} F(\phi) \Psi], \quad (22)$$

where the spin connection  $\omega_M$  in the covariant derivative

$$D_M \Psi = (\partial_M + \omega_M) \Psi \quad (23)$$

is defined as

$$\omega_M = \frac{1}{4} \omega_M^{\bar{M}\bar{N}} \Gamma_{\bar{M}} \Gamma_{\bar{N}}. \quad (24)$$

In five-dimensional space-time, the spinor structure of the four component fermions is determined by  $\Gamma^M = E_M^{\bar{M}} \Gamma^{\bar{M}}$  with the  $E_M^{\bar{M}}$  being the vielbein and  $\{\Gamma^M, \Gamma^N\} = 2g^{MN}$ . Here we use the capital Latin letters  $M, N, \dots$  and  $\bar{M}, \bar{N}, \dots$  to label the indices of five-dimensional space-time coordinates and the local Lorentz ones, respectively. And  $\Gamma^M$  are the curved gamma matrices and  $\gamma^{\bar{M}}$  are the flat ones.

With the conformally flat metric (2), the nonvanishing components of the spin connection  $\omega_M$  are given by

$$\omega_\mu = \frac{1}{2} (\partial_z A) \gamma_\mu \gamma_5. \quad (25)$$

Then the five-dimensional Dirac equation can be derived from the action (22)

$$[\gamma^\mu \partial_\mu + \gamma^5 (\partial_z + 2\partial_z A) - \eta e^A F(\phi)] \Psi = 0, \quad (26)$$

where  $\gamma^\mu \partial_\mu$  is the four-dimensional Dirac operator on the brane.

In what follows, we focus on the fermion field equations of the five-dimensional fluctuations, and reduce the Dirac fermion field  $\Psi$  to the four-dimensional effective field on the brane. Following the routine in Ref. [40], we carry out the general KK decomposition according to the chiralities of the fermions,

$$\Psi(x, z) = e^{-2A} \sum_n [\psi_{L_n}(x) f_{L_n}(z) + \psi_{R_n}(x) f_{R_n}(z)], \quad (27)$$

where  $\psi_{L_n}(x) = -\gamma^5 \psi_{L_n}(x)$  and  $\psi_{R_n}(x) = \gamma^5 \psi_{R_n}(x)$  are the left-handed and right-handed components of the Dirac fermion fields on the brane, respectively. They satisfy the four-dimensional massive Dirac equations in the form of  $\gamma^\mu \partial_\mu \psi_{L_n}(x) = m_n \psi_{R_n}(x)$  and  $\gamma^\mu \partial_\mu \psi_{R_n}(x) = m_n \psi_{L_n}(x)$ . At the same time, the KK modes  $f_{L_n}(z)$  and  $f_{R_n}(z)$  of the chiral decomposition about the Dirac fields meet the form of the following coupled equations:

$$[\partial_z + \eta e^A F(\phi)] f_{L_n}(z) = m_n f_{R_n}(z), \quad (28)$$

$$[\partial_z - \eta e^A F(\phi)] f_{R_n}(z) = -m_n f_{L_n}(z). \quad (29)$$

With the purpose of obtaining the standard four-dimensional effective action for the massive chiral fermions, the following orthonormality conditions for  $f_{L_n}(z)$  and  $f_{R_n}(z)$  are needed:

$$\begin{aligned} \int_{-\infty}^{+\infty} f_{L_m} f_{L_n} dz &= \int_{-\infty}^{+\infty} f_{R_m} f_{R_n} dz = \delta_{mn}, \\ \int_{-\infty}^{+\infty} f_{L_m} f_{R_n} dz &= 0. \end{aligned} \quad (30)$$

Inspecting Eqs. (28) and (29), we have

$$\left( \frac{d}{dz} - \eta e^A F(\phi) \right) \left( \frac{d}{dz} + \eta e^A F(\phi) \right) f_L = -m_n^2 f_L, \quad (31)$$

$$\left( \frac{d}{dz} + \eta e^A F(\phi) \right) \left( \frac{d}{dz} - \eta e^A F(\phi) \right) f_R = -m_n^2 f_R. \quad (32)$$

We have thus obtained the Schrödinger-like equations for the left and right chiral fermions [63,64]

$$H_L f_L(z) = m^2 f_L(z), \quad (33)$$

$$H_R f_R(z) = m^2 f_R(z), \quad (34)$$

where the corresponding Hamiltonians are defined as

$$H_L = \left( -\frac{d}{dz} + \eta e^A F(\phi) \right) \left( \frac{d}{dz} + \eta e^A F(\phi) \right), \quad (35)$$

$$H_R = \left( -\frac{d}{dz} - \eta e^A F(\phi) \right) \left( \frac{d}{dz} - \eta e^A F(\phi) \right). \quad (36)$$

In view of (31) and (32), it may seem obvious that the Schrödinger equations can be rewritten as

$$[-\partial_z^2 + V_L(z)] f_L = m^2 f_L, \quad (37a)$$

$$[-\partial_z^2 + V_R(z)] f_R = m^2 f_R, \quad (37b)$$

where the effective potentials of the KK modes have the explicit expressions

$$V_L(z) = [\eta e^A F(\phi)]^2 - \partial_z [\eta e^A F(\phi)], \quad (38a)$$

$$V_R(z) = [\eta e^A F(\phi)]^2 + \partial_z [\eta e^A F(\phi)]. \quad (38b)$$

### III. FERMION RESONANCES ON THE THICK BRANE

In this section, we mainly discuss the problem of the fermion resonances on the thick brane with a piecewise warp factor. Specifically, we investigate the effects of two parameters  $k$  and  $V_0$  on the resonant states of the massive KK modes, respectively. Here  $k$  is a parameter in the generalized Yukawa coupling  $\eta \bar{\Psi} \phi^k \Psi$ ,  $k = 1, 3, 5, \dots$ . Our discussions focus on the properties, i.e., the mass spectra, the lifetimes, and the number of the massive resonant KK modes, related to the two parameters.

In Ref. [62], the parameter  $x$  which effectively parametrizes the thickness of the domain wall has a very interesting impact on the gravitation resonances. In this paper, we want to know the impacts of the parameters on the behaviors of the fermion resonances.

In this section, some new natures of the resonant states are obtained. We will show that the number of the resonant states increases when  $k$  becomes larger, however, it decreases as  $V_0$  increases. The height of the potential function of the resonant fermions with left-handed chirality is the same as that of the one with right-handed chirality. The parameter  $V_0$  determines the depth of the potential well of the resonant fermions, but it has no influence on the height of the potential function.

### A. The fermion zero modes

As we know, in order to study the localization and resonance (quasilocalization) problems, the coupling between the fermion field and the background scalar field should be introduced. Here, we choose the generalized Yukawa coupling  $F(\phi) = \phi^k$  with odd  $k = 1, 3, 5, \dots$ . In this subsection, we mainly study the localization of the zero modes of left- and right-handed fermions on the brane.

With the given  $k = 1$  and brane solution  $\phi(z)$  in Eq. (13) obtained in the previous section, the potential functions can be rewritten in the following specific forms:

$$V_L(z) = \begin{cases} \frac{\eta l}{81} \left[ \eta l^3 \phi^2(z) + 18 \phi(z) \sqrt{V_0} \sin(\sqrt{V_0} z) + \frac{9q\sqrt{3V_0} \left[ \sec^2\left(\frac{\sqrt{V_0} z}{2}\right) + 4l - 16 \right]}{\sqrt{\left[ q + \tan^2\left(\frac{\sqrt{V_0} z}{2}\right) \right] \left( 19 - 7\sqrt{7} + \frac{3m}{l-1} \right)}} \right], & |z| \leq \frac{d}{2} \\ \frac{\eta \phi(z)(\eta \phi(z)(z \pm \beta) + \sqrt{k_0^2(z \pm \beta)^2})}{k_0^2(z \pm \beta)^3}, & |z| > \frac{d}{2}, \end{cases} \quad (39a)$$

$$V_R(z) = V_L(z)|_{\eta \rightarrow -\eta}, \quad (39b)$$

where  $l = 2 + \cos(\sqrt{V_0} z)$ .

In what follows, we analyze the asymptotic behavior of the potential functions for the left- and right-handed fermions. After simple calculation and analysis, we find that both the potential functions have a very good asymptotic behavior. Specifically, when  $z$  tends to zero and infinity, their asymptotic values are constants. It can be expressed as follows:

$$V_L(0) = -V_R(0) = -\sqrt{2V_0}\eta, \quad (40)$$

$$V_{L,R}(z \rightarrow \pm\infty) \rightarrow 0. \quad (41)$$

It can be seen that, for the same set of parameters, the potential function of left-handed fermion KK modes  $V_L(z)$  has an opposite value compared to the one of right-handed fermion KK modes  $V_R(z)$  at the coordinate origin  $z = 0$ . When  $z$  tends to infinity, both potential functions vanish. The behaviors of the two potential functions are illustrated in Fig. 3 for given  $V_0$  and various values of  $\eta$ . From Eq. (38) and the continuity of the first derivatives of the scalar field and warp factor, we know that the potentials  $V_L(z)$  and  $V_R(z)$  are continuous. However, they are not smooth because the first order derivative of the scalar field is not smooth, this can be seen from Fig. 1(d). The insets in Fig. 3 show the enlarged view of the continuity of the potentials for clarity. From the left panel of Fig. 3, we can see that  $V_L(z)$  is a modified volcano-type potential like in most of the other contexts. So there is no discrete mass spectrum of bound states, and there exists no mass gap to

separate the massless mode from the massive KK modes for  $V_L(z)$ .

In Fig. 3, from the right panel, we see that there is a potential barrier around the brane location for  $V_R(z)$ . Nevertheless, for stronger coupling (bigger  $\eta$ ), the potential well emerges at the top of the barrier. This means that the behavior of the potential function is related to the scalar-fermion coupling. In order to clearly examine the impact of  $V_0$  on the behavior of  $V_R(z)$ , we fix  $k$  and  $\eta$  in the following. The shapes of  $V_L(z)$  and  $V_R(z)$  for various values of  $V_0$  can be depicted easily. As we know, for  $V_R(z)$ , in general, the potential barrier cannot trap a massive fermion. However, when  $V_0$  becomes smaller, the potential well located at the top of  $V_R(z)$  is deep enough, and a massive fermion with a finite lifetime would appear, however, the height of the potential remains unchanged. This means that the phenomenon of massive resonant states will also occur.

If we recast the scalar-fermion coupling  $\eta \bar{\Psi} \phi \Psi$  ( $k = 1$ ) to  $\eta \bar{\Psi} \phi^3 \Psi$  ( $k = 3$ ), the configurations of the two potential functions  $V_L(z)$  and  $V_R(z)$  will change. In this situation,  $V_L(z)$  becomes a double-well potential, but the right-handed one is still a single-well potential. However, for the two potential functions, if the other parameters are fixed, when the parameter  $V_0$  becomes smaller, the potential well will become deeper. Furthermore, at the center of the brane along the direction of the extra dimension, the values of the potential functions  $V_{L,R}(z)$  are zero, i.e.,  $V_L(0) = V_R(0) = 0$ . This result is similar to that obtained in Refs. [64,66].

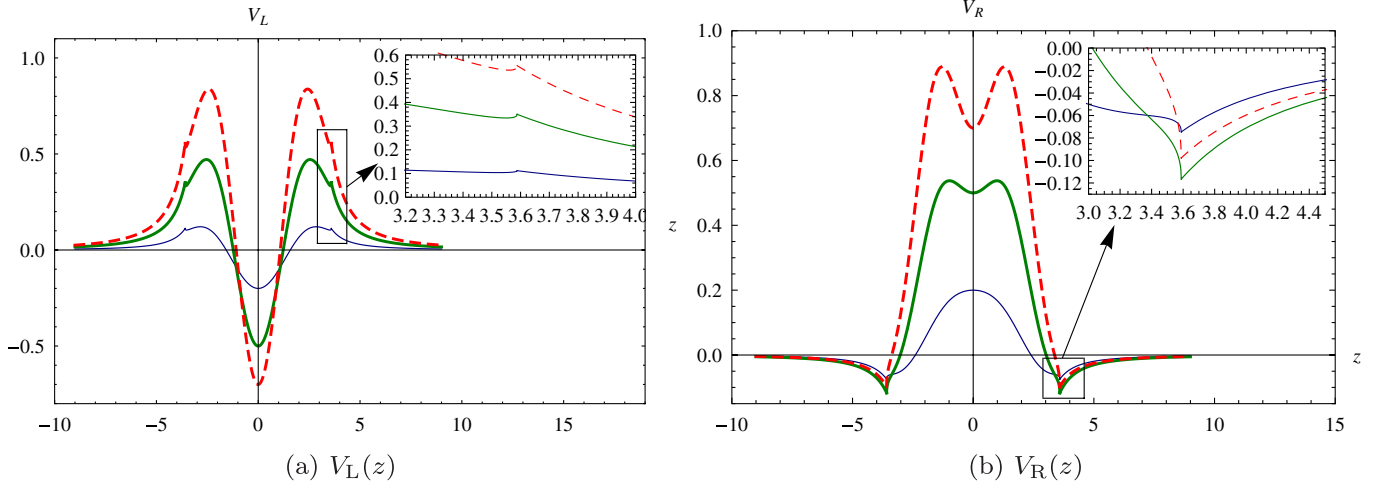


FIG. 3 (color online). The potential functions  $V_L(z)$  and  $V_R(z)$  for  $F(\phi) = \phi$ . The insets show the enlarged view of the junction points for clarity. The parameters are set to  $V_0 = 0.5$ ,  $\eta = 0.7$  (dashed thicker),  $0.5$  (solid thicker), and  $0.2$  (solid thinner).

As discussed above,  $V_L(z)$  is always a modified volcano type of potential. It is well known that the potential of the left-handed fermions does not provide mass gap between the zero mode and KK excitation modes, so there is a

continuous gapless spectrum of KK excitation modes. From Eqs. (28) and (29), the zero mode of the left-handed fermions is derived as

$$f_{L0}(z) \propto \exp\left(-\eta \int_0^z d\bar{z} e^{A(\bar{z})} \phi(\bar{z})\right) = \begin{cases} \exp\left(-\frac{\eta}{9} \int_0^z d\bar{z} [\cos(\sqrt{V_0}|\bar{z}|) + 2]^2 \phi(\bar{z})\right), & |z| \leq \frac{d}{2} \\ \exp\left(-\frac{\eta}{9} \int_0^{d/2} d\bar{z} [\cos(\sqrt{V_0}|\bar{z}|) + 2]^2 \phi(\bar{z})\right) \exp\left(-\frac{\eta}{k_0} \int_{d/2}^z d\bar{z} \frac{\phi(\bar{z})}{|\bar{z}| + \beta}\right), & |z| \geq \frac{d}{2}. \end{cases}$$

When  $|z| \geq d/2$ , in the first factor, because the integrand and the integrating range are finite, the integration result must be a constant:

$$c_1 = \exp\left(-\frac{\eta}{9} \int_0^{d/2} d\bar{z} [\cos(\sqrt{V_0}|\bar{z}|) + 2]^2 \phi(\bar{z})\right). \quad (42)$$

In the second factor, we know that  $\phi(\bar{z}) = c_2$  (a constant) when  $\bar{z} \geq d/2$ . So, we obtain

$$f_{L0}(z) \propto \exp\left(-\frac{\eta c_2}{k_0} \int_{d/2}^z d\bar{z} \frac{1}{|\bar{z}| + \beta}\right) = c_3 (|z| + \beta)^{-(\eta c_2)/k_0}, \quad (43)$$

where  $c_3 = (d/2 + \beta)^{\eta c_2/k_0}$ . Then the normalization condition

$$\int_{-\infty}^{+\infty} f_{L0}^2(z) dz = 1 \quad (44)$$

is equivalent to

$$\int_{d/2}^{+\infty} dz (z + \beta)^{-(2\eta c_2)/k_0} < \infty. \quad (45)$$

It is clear that the normalization condition is turned out to be  $2\eta c_2/k_0 > 1$ . Hence, the zero mode (42) is normalizable only if  $\eta c_2/k_0 > 1/2$ . Besides, although there is no explicit expression for  $f_{L0}(z)$ , we can numerically integrate (42). The plot of the normalizable zero mode  $f_{L0}(z)$  via numerical integration is given in Fig. 4. From Fig. 4, we can better understand the process of the calculation. On the other hand, the potential  $V_R(z)$  around the brane location is always positive, and it gradually becomes zero when  $z \rightarrow \infty$ . We know that this type of potential cannot trap any bound state fermions with right chirality and there exists no zero mode of right-handed fermions. This result is consistent with the previous well-known conclusion that massless fermions must be single chirality in the brane world models [35,64].

Nevertheless, for a given  $V_0$ , the structure of the potential  $V_R$  is determined by the parameters  $k$  and the coupling constant  $\eta$  jointly. For a given  $k$ , when  $\eta$  increases, a potential well around the location of the brane would emerge and the well would be deeper and deeper. In this case, there may exist massive resonant KK modes. In order to make a potential well appear, we need very large  $\eta$ , i.e., the coupling between the scalar and the fermion is very

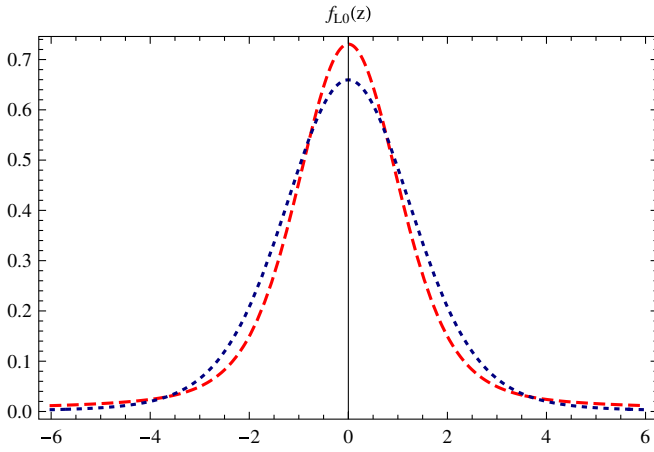


FIG. 4 (color online). The normalizable fermion zero mode  $f_{L0}(z)$ . The parameters are set to  $\eta = 1.0$ ,  $V_0 = 0.2$  for the dotted line and  $V_0 = 0.5$  for the dashed line.

strong [64,66]. In this paper, we would not consider this case and our main task is focusing on the impacts of parameters  $k$  and  $V_0$  on the massive resonant states.

## B. The massive KK modes

As mentioned above, in this subsection, we mainly discuss the problem of the fermion resonances on the thick brane. Some interesting properties and results will be obtained. Following the routine in Refs. [64,66], with the potential functions (39), we get the numerical solutions of the Schrödinger equations (37) with the Numerov algorithm. Furthermore, the resonant probabilities, lifetimes, and mass spectra of the resonances will be obtained.

When the parameter  $k$  in scalar-fermion coupling is treated as a variable, the maxima of potential functions  $V_L(z)$  and  $V_R(z)$  are the same magnitude and the properties of resonances are same to the results obtained in Refs. [64,66]. When we treat the parameter  $V_0$  as a variable, the maxima of potential functions  $V_L(z)$  and  $V_R(z)$  are also the same magnitude but the resonant behaviors of the massive chiral fermions with the left- and right-handed chiralities are different from the case in that  $k$  is treated as a variable. But the resonant states satisfy the Kaluza-Klein parity-chirality decompositions of massive fermion resonances obtained in Ref. [66], because the resonant peaks have a one-to-one correspondence between left- and right-handed fermions at some resonant mass eigenvalue.

### 1. Case I: $k$ as a variable

In this subsection, we mainly study the impacts of the parameter  $k$ , which is related to the coupling type between the scalar field and the fermion, on the massive resonant KK modes. From the context of numerical analysis, we know that one should impose the initial or boundary conditions for the second order differential equations (37).

Here, we attach two types of initial conditions for Eqs. (37). They are

$$f(0) = d_0, \quad f'(0) = 0, \quad (46)$$

and

$$f(0) = 0, \quad f'(0) = d_1. \quad (47)$$

Because the potential functions have a  $Z_2$  symmetry, the solutions of the Schrödinger equations (37) with even parity and odd parity will be obtained for the above two conditions, respectively. In this paper, the constants  $d_0$  and  $d_1$  are set to  $d_0 = 1$ ,  $d_1 = 5$  for the sake of convenience. From the point of view of quantum mechanics, we know that the massive KK modes will feel a combined effect of the potential barrier and the potential well around the location of the brane. Therefore, the massive KK modes will show different natures when the depth of the potential well changes.

For the simplest generalized Yukawa coupling type  $F(\phi) = \phi^k$ ,  $k = 1, 3$ , when we choose a  $V_0$ , some resonant states would be obtained. When the eigenvalues deviate from the mass eigenvalues of the resonances, the resonant states will become unclear and even disappear. The graphics of massive resonant wave functions, potential functions, and off-resonant wave functions are shown in Fig. 5. From the figure, we can obviously see the impacts of the potential functions  $V_L(z)$  and  $V_R(z)$  on the left- and right-handed massive KK modes with even parity or odd parity. From the comparison between the massive resonant wave functions and massive off-resonant ones, we find that the resonances occur only at some particular mass eigenvalues, e.g.,  $m^2 = 0.072$  (even),  $0.072$  (odd), etc. for  $k = 3$ ,  $\eta = 0.05$ , and  $V_0 = 0.05$ . These mass eigenvalues show that they follow the Kaluza-Klein parity-chirality decompositions of the massive fermion resonances obtained [66]:

$$\Psi(x, z) = e^{-2A} \sum_n [\psi_{Ln}(x) f_{Ln}^{(E)}(z) + \psi_{Rn}(x) f_{Rn}^{(O)}(z)], \quad (48a)$$

$$\Psi(x, z) = e^{-2A} \sum_n [\psi_{Ln}(x) f_{Ln}^{(O)}(z) + \psi_{Rn}(x) f_{Rn}^{(E)}(z)], \quad (48b)$$

where the superscripts ‘‘E’’ and ‘‘O’’ stand for even parity and odd parity, respectively. As we know, this demonstrates that Dirac fermions can be composed of the left-handed fermions with odd parity and the right-handed ones with even parity, and vice versa [64,66]. Specifically, the parities and chiralities of the resonant massive KK modes are conserved for left- and right-handed resonant massive fermions.

Besides, we note that the potential of left-handed fermions for  $k = 3$  becomes a double-well potential as discussed in Refs. [64,66], and this can be seen from Fig. 5(b). After simple calculations, we find that the potential of left-handed fermions is always a double-well one for  $F(\phi) = \phi^k$  with odd  $k \geq 3$ , whereas the potential of right-handed fermions is a single-well potential all the



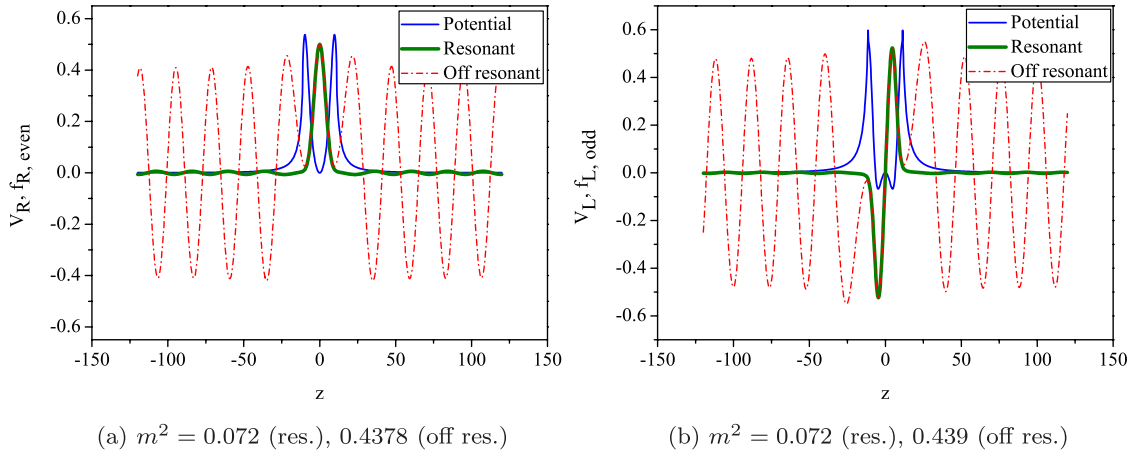


FIG. 5 (color online). The shapes of massive resonant KK modes, potential functions, off-resonant wave functions of left-handed and right-handed fermions with even parity and odd parity for  $F(\phi) = \phi^3$ . The parameters are set to  $z_{\max} = 120$ ,  $V_0 = 0.05$  and  $\eta = 0.05$ . The terms “res.” and “off res.” stand for resonant states and off-resonant states, respectively.

time. However, there exist fermion resonances for both kinds of potential wells.

According to the knowledge of quantum mechanics, we know that, when a microscopic particle encounters a potential well, the microscopic particle will stay in the potential well with a limited lifetime. Next, we will carefully study the resonant processes. Since the Schrödinger equations (37) can be recast into the form of  $\mathcal{O}_{L,R}^\dagger \mathcal{O}_{L,R} f_{L,R}(z) = m^2 f_{L,R}(z)$ , the probability for finding the massive KK modes around the brane location along extra dimension is  $|f_{L,R}(z)|^2$ . From the previous section, we know that the energy density of the thick brane concentrates in a finite range along the direction of the extra dimension. Strictly speaking, we can interpret  $|f_{L,R}(0)|^2$  as the probability for finding the massive KK modes at the center of the brane. However, for the massive KK modes with odd parity, the wave functions at the center of the brane are  $f_{L,R}^{(O)}(0) = 0$ . It means that the probability is poorly defined, and we cannot understand the resonances very well. Therefore, without loss of generality, we define a relative probability to express the probability of massive KK modes with even and odd parities around the brane location. The relative probability is defined as [64]

$$P_{L,R}(m^2) = \frac{\int_{-z_b}^{z_b} |f_{L,R}(z)|^2 dz}{\int_{-z_{\max}}^{z_{\max}} |f_{L,R}(z)|^2 dz}, \quad (49)$$

where the relationship between  $z_b$  and  $z_{\max}$  is chosen as  $z_{\max} = 10z_b$ , so the probability for the plane wave modes with the eigenvalue  $m^2$  is  $1/10$ . When we set the same parameters  $V_0 = 0.05$  and  $\eta = 0.05$  as in Fig. 5, for  $F(\phi) = \phi^k$  with  $k = 3$ , the probabilities  $P_{L,R}$  (as a function of  $m^2$ ) for finding massive KK modes of left- and right-chirality fermions around the brane location are depicted in Fig. 6. We find that the probability is maximized at some mass eigenvalue (a series of resonant peaks) and in

the figure the plateau corresponds to  $z_b/z_{\max} = 0.1$ . The number of resonant peaks for left-handed KK modes is equal to that of the ones for right-handed KK modes. In a sense, this shows that a massive Dirac fermion can be composed of the left- and right-handed massive KK modes.

Furthermore, in the numerical calculations, we find that the magnitude of the probability for finding a fermion around the brane is related to the step size in the numerical experiments. Accordingly, in order to analyze the fermion resonances, the step size of the coordinate used in the numerical calculations must be small enough to reflect the true face of resonances. In the subsequent discussions, we take a small enough step size to study the probability of the resonant states. In this case, more detailed graphs of resonant probabilities with respect to the resonant mass  $m$  can be obtained. These resonant peaks correspond to the massive resonant KK modes with even and odd parity obtained in Fig. 5. We can clearly see that more notable details (such as the maximum  $P_{\max}$  of resonant probabilities) of resonant peaks are shown compared with the resonant peaks in Fig. 6. At the same time, from Fig. 6, we can see that when the mass eigenvalue increases, the resonant peaks become thicker and thicker. On the other hand, the resonant peaks with relatively smaller mass eigenvalues are narrower, and the resonant peak with the smallest mass eigenvalue (the first peak) is the thinnest one. Later, we will know that the narrowest resonant peak has the maximum resonant lifetime.

To further analyze the lifetime of a fermion resonance, first, we define the width  $\Gamma = \Delta m$  of a resonant state as the width at the half maximum of a resonant peak [57]. In this case, a massive fermion will disappear into the fifth dimension after staying on the brane for some time  $\tau \sim \Gamma^{-1}$ . Thus,  $\tau$  is called the lifetime of a fermion resonance mentioned above. After numerical calculations, we can get a lifetime from each peak of the fermion resonance. In Table I, we list the eigenvalue  $m^2$ , mass  $m$ , width  $\Gamma$ ,

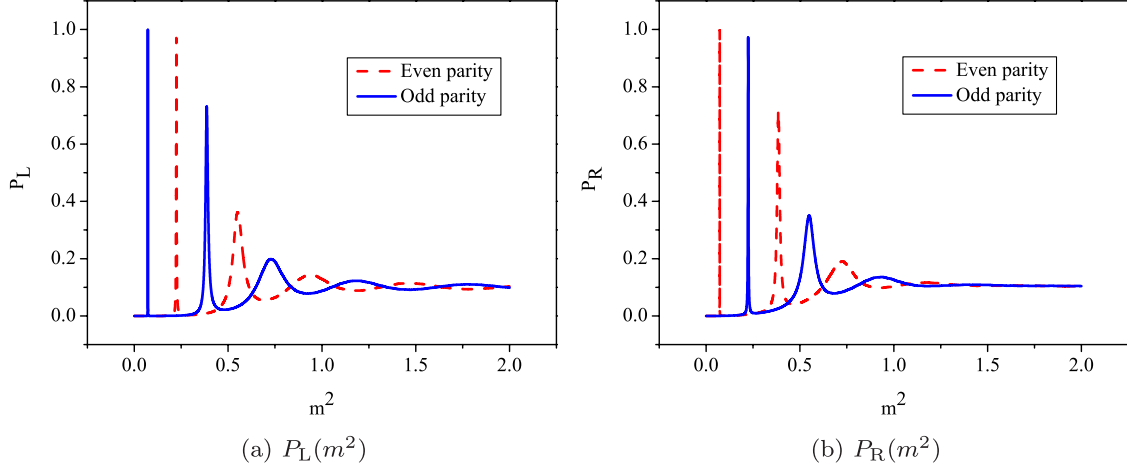


FIG. 6 (color online). The probability  $P_{L,R}$  (as a function of  $m^2$ ) for finding massive KK modes of left- and right-chirality fermions with mass  $m^2$  around the brane location for  $F(\phi) = \phi^3$ . The solid lines and dashed lines are plotted for the odd-parity and even-parity massive fermions, respectively. The parameters are set to  $z_{\max} = 120$ ,  $V_0 = 0.05$ , and  $\eta = 0.05$ .

lifetime  $\tau$ , and maximum probability  $P_{\max}$  for resonances of left and right chiral fermions with odd-parity and even-parity solutions for  $F(\phi) = \phi^3$ . It is interesting that the potential wells of the massive chiral fermions are not deep enough for  $k = 1$ ,  $V_0 = 0.05$ , and  $\eta = 0.05$ , consequently, there is no resonant state in this situation. However, if we choose a larger  $\eta$ , the height of the potential of the massive chiral fermions will become larger, then a series of massive fermion resonant states will appear.

Note that the data in Table I only reflect the information of the fermion resonances for  $k = 3$ . After further calculations, we find that, for a given set of parameters, the number of resonant states increases with  $k$ . The resonant mass eigenvalue  $m$  of a left-handed fermion with odd parity is equal to that of a right-handed fermion with even parity. On the other hand, the resonant mass eigenvalue  $m$  of a right-handed fermion with odd parity is also equal to that of a left-handed fermion with even parity. This

means that, in this case, the massive KK modes satisfy the equations of the Kaluza-Klein parity-chirality decompositions (48). In other words, a massive Dirac fermion can be composed of the left- and right-handed massive KK modes, as mentioned above. Indeed, this result can be understood in the context of supersymmetric quantum mechanics. The Hamiltonians in (35) and (36) can be factorized as  $H_L = A^\dagger A$  and  $H_R = AA^\dagger$ . Hence  $H_L$  and  $H_R$  are actually conjugated supersymmetric partner Hamiltonians with the superpartner potentials  $V_L(z)$  and  $V_R(z)$ . This leads to the correspondence between the spectra of left- and right-handed fermions except a normalized zero mode of  $f_{L0}(z)$ , i.e. the spectra of  $H_L$  and  $H_R$  are degenerate. The underlying reason for the degeneracy of the spectra of  $H_L$  and  $H_R$  can be understood from the properties of the supersymmetric algebra [74].

For the lifetimes of the massive resonant states, there exists a following rule. For a relatively smaller resonant mass eigenvalue  $m$ , the resonant state has a relatively larger lifetime  $\tau$ . Actually, this rule can be seen obviously from the profiles of the peaks of the resonant probabilities. From Fig. 6, we can clearly see that, when the magnitude of a mass eigenvalue increases, the resonant peak would become thicker. The resonant peaks with relatively huger mass eigenvalues would be broader, and the resonant peak with the biggest mass eigenvalue (the last peak) is the thickest one. When the resonant peaks become wide enough, the massive KK modes will not stay on the brane, and the time scale  $\tau \rightarrow 0$ . Therefore, the massive KK modes will tunnel through the brane at a moment, consequently, they will become free both in the bulk and on the brane. On the contrary, for the resonant state with the smallest mass eigenvalue, it will have a very long time to stay on the brane. The massive KK modes with a huge lifetime  $\tau$  will gradually pass through the brane, and slowly fade away into the bulk. The extreme situation is that, when

TABLE I. The eigenvalue  $m^2$ , mass  $m$ , width  $\Gamma$ , lifetime  $\tau$ , and maximum probability  $P_{\max}$  for resonances of left and right chiral fermions with odd-parity and even-parity solutions for  $F(\phi) = \phi^3$ .  $\mathcal{C}$  and  $\mathcal{P}$  stand for chirality and parity, respectively.  $\mathcal{L}$  and  $\mathcal{R}$  are short for left handed and right handed, respectively. The parameters are set to  $z_{\max} = 120$ ,  $V_0 = 0.05$ , and  $\eta = 0.05$ .

$k$	$\mathcal{C}$	$\mathcal{P}$	$m_n^2$	$m_n$	$\Gamma$	$\tau$	$P_{\max}$
3	$\mathcal{L}$	Odd	0.072	0.269	0.000 3	3 505.3	0.999
		Even	0.224	0.474	0.003	343.0	0.971
		Odd	0.386	0.621	0.013	78.97	0.731
	$\mathcal{R}$	eEven	0.551	0.742	0.043	22.99	0.362
		Even	0.072	0.269	0.000 3	3 422.2	0.998
		Odd	0.224	0.473	0.003	324.0	0.972
	Even	0.385	0.621	0.014	71.21	0.716	
	Odd	0.549	0.741	0.051	19.63	0.350	

$\tau$  tends to infinity, i.e.,  $\tau \rightarrow \infty$ , the resonant state will become a bound state.

In addition, we can plot the mass spectrum of the massive resonant KK modes. For  $F(\phi) = \phi^k$  with  $k = 1, 3, 5, \dots$ , the mass spectrum of the massive fermion resonances is depicted in Fig. 7 for  $k = 3$ . We can see that the number of the resonant states of the massive fermions with left- and right-handed chiralities increases with  $k$ . However, in the later discussions, we will find that the situation is slightly different. The resonant states here are the ones whose resonant mass eigenvalues are below the maxima  $V_{L,R,\max}$  of the potentials (note that  $V_{L,\max}$  and  $V_{R,\max}$  are the same magnitude). In general, the number of the right-handed resonant states of the massive KK modes is equal to that of the ones with left-handed chirality.

## 2. Case II: $V_0$ as a variable

In this subsection, we mainly discuss the impacts of the brane parameter  $V_0$  (it decides the thickness of the brane by the relation  $d = 5.0745/\sqrt{V_0}$ ) on the resonant states of the massive KK fermions, especially on the resonant probabilities, resonant masses, and the number of resonant states. We find that the impacts of  $V_0$  and  $k$  on the resonances are different.

For given  $\eta$ ,  $k$ , and various values of  $V_0$ , we study the resonances of massive KK modes. As investigated in the previous subsection, using the Numerov method, we numerically solve the Schrödinger equations (37) with the potential functions (39). Following the same routine used above, for  $F(\phi) = \phi$ , the eigenvalue  $m^2$ , mass  $m$ , width  $\Gamma$ , lifetime  $\tau$ , and maximal probability  $P_{\max}$  are listed in Table II for massive resonances of left and right chiral fermions with odd and even parities.

From Table II, we can clearly see that all the values of the eigenvalue  $m^2$ , mass  $m$ , width  $\Gamma$ , lifetime  $\tau$ , and maximal probability  $P_{\max}$  follow the same rules obtained in the previous subsection. However, when  $V_0$  becomes larger, the results are quite different from those with smaller  $V_0$ . In this case, we find that the maximum values  $V_{L,\max}$  and  $V_{R,\max}$  of the potential functions of the left- and right-handed KK modes are clearly the same magnitude. The

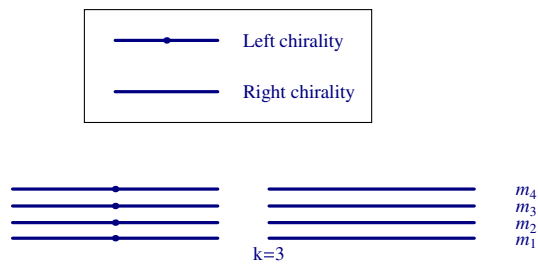


FIG. 7 (color online). The mass spectrum of resonances for  $F(\phi) = \phi^3$ . The parameters are set to  $z_{\max} = 120$ ,  $V_0 = 0.05$ , and  $\eta = 0.05$ .

TABLE II. The eigenvalues  $m^2$ , mass  $m$ , width  $\Gamma$ , lifetime  $\tau$ , and maximum probability  $P_{\max}$  for resonances of left and right chiral fermions with odd parity and even parity for  $F(\phi) = \phi$ .  $\mathcal{C}$  and  $\mathcal{P}$  stand for chirality and parity, respectively.  $\mathcal{L}$  and  $\mathcal{R}$  are short for left- handed and right handed, respectively. The parameters are set to  $\eta = 1$ ,  $z_{\max} = 30$  for  $V_0 = 0.1$ , and  $z_{\max} = 60$  for  $V_0 = 0.5$ .

$V_0$	$\mathcal{C}$	$\mathcal{P}$	$m_n^2$	$m_n$	$\Gamma$	$\tau$	$P_{\max}$
0.1	$\mathcal{L}$	Odd	0.815	0.902	0.004	266.1	0.987
		Even	1.417	1.191	0.047	21.34	0.548
	$\mathcal{R}$	Even	0.814	0.902	0.004	261.6	0.989
		Odd	1.416	1.190	0.049	20.44	0.562
0.5	$\mathcal{L}$	Odd	1.547	1.244	0.152	6.569	0.408
	$\mathcal{R}$	Even	1.522	1.234	0.166	6.021	0.418

potential for the smaller  $V_0$  is deeper than that of ones for bigger  $V_0$ , while the height of potential remains unchanged for various values of  $V_0$ . Consequently, as  $V_0$  increases, the number of resonant states decreases.

We find that the probability of the first peak of resonant states with right-handed chirality is the maximum, and it is equal to that of the one with left-handed chirality. For both chiralities, as the resonant mass eigenvalue increases, the resonant probability decreases. The resonant probabilities of left- and right-handed massive KK fermions have a good match at the same resonant mass eigenvalue. Accordingly, there is a one-to-one correspondence between the peaks of left- and right-handed fermions at the same resonant mass eigenvalue. As already discussed, the mass spectrum can be plotted from the data in Table II. Until now, we have seen that there exists a good match of the probabilities  $P_{\mathcal{L}}$  and  $P_{\mathcal{R}}$  for all of the excited states and its origin is the consistency of the behavior of the potentials  $V_L$  and  $V_R$ . We can see that, for each  $n$ , which labels the  $n$ th resonance, within some given numerical error, the resonant mass  $m_n$  of the resonant KK modes with left-handed chirality is equal to that of ones with right-handed chirality. In other words, a massive Dirac fermion could also be composed of the left- and right-handed massive KK modes.

In a word, in this section we mainly investigate the resonant problem of the massive chiral KK modes with odd and even parities. For various values of  $k$  and  $V_0$ , we obtain the eigenvalue  $m^2$ , mass  $m$ , width  $\Gamma$ , lifetime  $\tau$ , and maximum probability  $P_{\max}$  for the resonances of left and right chiral fermions with odd and even parities for  $F(\phi) = \phi^k$  with odd  $k = 1, 3$ . The results can be extended to  $F(\phi) = \phi^k$  with odd  $k = 5, 7, \dots$ . Besides, we obtained the graphs of the mass spectra of the massive KK resonant states. For the two cases, as discussed in Refs. [64,66], the resonant states of the massive KK modes satisfy the Kaluza-Klein parity-chirality decompositions. As the parameter  $V_0$  becomes larger, the depth of the potential wells  $V_L$  and  $V_R$  becomes shallower, while the height of the potentials remains unchanged. Consequently, as  $V_0$

increases, the number of resonant states decreases. In other words, when the thickness of the brane becomes thicker, the number of resonant states increases. The resonant probabilities  $P_L(m_n)$  and  $P_R(m_n)$  of the resonant massive KK modes have a good match at some resonant mass eigenvalue for all of the excited states.

#### IV. DISCUSSIONS AND CONCLUSIONS

In this paper, we have investigated the fermion resonances in the background of a single scalar field generated thick brane with a piecewise warp factor. Using the warp factor inspired by the one in Ref. [62], we obtained the kinklike background scalar field without derivative jumps under three junction conditions. The distance  $d$  between the two boundaries, which is just the thickness of brane, can be obtained in terms of one free parameter  $V_0$ . Next, we introduced the interaction between a massless bulk fermion and the background real scalar field by means of a general Yukawa coupling  $\eta\bar{\Psi}F(\phi)\Psi$  in five-dimensional space-time, where  $F(\phi) = \phi^k$  with odd  $k = 1, 3, 5, \dots$

When the parameter  $k$  is treated as a variable, we found that the coupling between the scalar and fermion influences the resonances. The number of resonant states of the massive KK fermions with the left- and right-handed chiralities increases with  $k$ . Here, the number of resonant states is defined as the number of those resonant states with eigenvalue  $m^2$  lower than the highest point of the potential. We found that the number of the resonant KK modes with left-handed chirality is equal to that of the ones with right-handed chirality. This means that the fermion resonant states of massive KK modes satisfy the Kaluza-Klein parity-chirality decompositions [66]. For every  $k = \{1, 3, 5, \dots\}$ , the resonant KK fermions with smaller mass eigenvalues have larger lifetimes than those with bigger mass eigenvalues. On the other hand, we can see that the wider resonant peaks have smaller lifetimes. Furthermore, the first resonant peak is the narrowest one, and it has the longest life expectancy. Accordingly, the KK fermion resonances with smaller mass eigenvalues can stay on the thick brane for a longer time. In other words, these quasibound states (resonant states) are more stable. Conversely, the KK fermion resonances with huger mass eigenvalues can only stay on the brane for a little time, i.e., the resonances become too unstable and cease to appear [63].

When the brane parameter  $V_0$  is treated as a variable, we found that the thickness of the brane also influences the fermion resonances. The number of the peaks of resonant probability of the massive KK fermions with left- and right-handed chiralities decreases with  $V_0$ , namely, the number of resonant states decreases with  $V_0$ . When  $V_0$  varies, the maximum values  $V_{L,\max}$  and  $V_{R,\max}$  of the potential functions of the left- and right-handed KK modes are the same magnitude. Accordingly, the probabilities of

the resonant KK modes of left-handed fermions are equal to that of the ones of right-handed fermions. Further, the masses and lifetimes of left and right chiral resonances are almost the same, which demonstrates that it is possible to compose a massive Dirac fermion from the left and right chiral resonances. In this case, the resonant probabilities of the left- and right-handed resonant KK modes with the same mass are the same for all the massive resonant KK modes. There is a good match for the resonant probabilities of the fermion resonances with left and right chiralities.

Indeed, if a resonance is supposed to represent a metastable four-dimensional massive fermion field, it must contain both left and right parts, which are mixed by the four-dimensional Dirac equations [see Eqs. (28) and (29)]. We can see that there is no mismatch for peaks of the probabilities  $P_L$  and  $P_R$  (as a function of  $m^2$ ) for finding massive KK modes of left- and right-chirality fermions. The numbers of peaks of  $P_L$  and  $P_R$  are equal to each other. So the resonant masses of the left- and right-handed fermions must be equal to each other. So our computations are consistent with the coupled four-dimensional Dirac equations. Actually, the definition of the number of resonant states in Ref. [66] can be used because there is a good match for the maximum values  $V_{L,\max}$  and  $V_{R,\max}$  of the potential functions of the left- and right-handed KK modes. From the definition, we can see that only the resonant states with eigenvalue  $m^2$  lower than the highest point of the potential are considered as resonant states because they have obvious resonant peaks, namely, they have relatively larger resonant probabilities. In other words, the resonant states with unobvious resonant peaks were neglected. So the resonant states with larger mass eigenvalues (the ones with quite small resonant probabilities) were not taken into account. The prior condition is that the maxima of the potential functions  $V_L$  and  $V_R$  are equal to each other, or at least they have the same magnitude. Actually, the numbers of resonant states can be considered as numbers of peaks of  $P_L$  or  $P_R$ . Although the resonant probabilities (whose resonant masses is larger than the threshold value of  $V_{R,\max}$ ) are quite small, we should take them into account. Thus, from the physical point of view, all the resonant peaks describe the resonant states. Essentially, the resonant behavior coincides with the qualitative change of the energy density of the brane.

With respect to resonances for chiral fermions on the thick brane with a piecewise warp factor, we can resume our conclusions with the following points: (i) The thick brane with a piecewise warp factor was derived under three junction conditions which ensure the correct Israel junction conditions. The kinklike scalar field without derivative jumps can be given. Especially, the continuous scalar potential and energy density were also obtained. (ii) The thick brane without internal structure also favors the appearance of resonant states for both left- and right-handed fermions. When the brane becomes thicker ( $V_0$  becomes

smaller), the number of resonant states increases, and vice versa. (iii) The scalar-fermion coupling influences the resonant behaviors of the KK fermions. The resonant properties (such as resonant probability, lifetime, mass, and number) of the left-handed fermions are the same as the ones of the corresponding right-handed fermions. (iv) Strictly speaking, all of the resonant peaks describe the resonant states. The definition of the number of resonances given in Ref. [66] should be redefined. Indeed, the number of resonant states can be considered as number of resonant peaks. The fermion resonant states of the massive KK modes satisfy the Kaluza-Klein parity-chirality decompositions. (v) It is worth noting that many systems with boundaries cannot be investigated in the thick brane scenario, instead the thin branes as the boundary terms should be added into the action. Such systems can be studied with the method in Ref. [19]. The properties of the fermion resonances in thick brane models and the

problems in the brane with boundaries are rich and interesting. We leave these issues for future works.

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