Spontaneously broken topological $SL(5, R)$ gauge theory with standard gravity emerging

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A completely metric-free $sl(5, R)$ gauge framework is developed in four dimensions. After spontaneous symmetry breaking of the corresponding topological BF scheme, Einstein spaces with a *tiny* cosmological constant emerge, similarly as in (anti-)de Sitter gauge theories of gravity. The induced Λ is related to the scale of the symmetry breaking. A "background" metric surfaces from a Higgs-like mechanism. The finiteness of such a topological scheme converts into asymptotic safeness after quantization of the spontaneously broken model.

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I. INTRODUCTION

Topological ideas in the realm of gravity date back to Riemann, Clifford[1] and Weyl and found a more concrete realization in the wormholes [1,2] of Wheeler characterized by the Euler-Poincaré invariant. They result from attaching handles to black holes. Here we are going to analyze a primordial topological and scale-free gauge theory which induces standard gravity for the macroscopical world.

Observationally, Einstein's general relativity (GR) is rather well established for the solar system and double pulsars [3]. GR is based on the metric tensor $g_{\mu\nu}$ which, however, plays a double role: Measuring macroscopic distances ds in spacetime and serving as a gravitational potential for the Christoffel connection $\Gamma^{\{\}}_{\alpha\beta}$ of Riemannian geometry. This dichotomy seems to be one of the main obstacles for quantizing gravity. Eddington [4] already suggested 1921 to regard the *dimensionless* connection as the basic field and the metric merely as a derived concept.

Consequently, the primordial action in four dimensions should be constructed from a *metric-free* topological action such as the Pontrjagin invariant of the corresponding gauge connection [5]. Similarly as in the Yang-Mills theory, a quantization can be achieved by amending the definition of curvature and the Bianchi identities via topological ghosts [6]. In such a graded Cartan formalism, the nilpotency of the ghost operators is on par with the Poincaré lemma $dd \equiv 0$ for the exterior derivative. Using a BRST antifield formalism with a duality gauge fixing, a consistent quantization in spaces of double dual curvature is obtained [7]. For Euclidean signature, the constraint imposes instanton type solutions on the curvature-squared "Yang-Mielke theory" of gravity [8–10], proposed in its affine form already by Weyl 1919 and later by Higgs [11], Yang [12] and others. However, such exact configurations exhibit a "vacuum degeneracy". One needs to modify the double duality of the curvature via *scale breaking* terms, in order to retain Einstein's equations with an induced cosmological constant of partially topological origin as the unique macroscopic "background".

Such scale breaking terms arise more naturally in a constraint formalism of Plebanski [13], the so-called BF scheme [14,15], in which the gauge curvature is usually denoted by F and B is a kind of Lagrange multiplier. Topological actions appear to be rather prospective because they are renormalizable and free of anomalies.

In the case of gravity, we depart from the meta-linear group $SL(5, R)$ but stick to *four dimensions* [16]. Our approach is truly metric-independent, in contradistinction to the more common (Anti-)de Sitter gauge theories [17–23] which are from the outset "metric-contaminated" due to Cartan-Killing metric $\hat{g}_{AB}$ of the (pseudo–)orthogonal groups $O(5)$, $O(1, 4)$ or $O(2, 3)$, respectively. After applying spontaneous symmetry breaking to the corresponding topological BF theory, Einstein spaces emerge with a tiny cosmological constant related to the scale at which the symmetry breaking occurs. Here the "background" metric of spacetime is induced via a Higgs-like mechanism [24]. The renormalizability of such a topological scheme converts into the issue of asymptotic safeness after quantization of the spontaneously broken model.

The paper is organized as follows: In the next section, the Lie algebra of the meta-linear group $SL(5, R)$ is decomposed into pseudotranslations and linear transformation $GL(4, R)$ and identified as a graded affine algebra. The corresponding gauge formalism is setup and related to Cartan connections and curvature in Sec. III. The topological BF scheme is applied to a metric-free $SL(5, R)$ model of gravity in Sec. IV. A symmetry breaking via a quadratic term in $\hat{B}$ induces a nontrivial dynamics being effectively equivalent to the standard Einstein equations with cosmological constant. In Sec. V, a spontaneously symmetry

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[1]At the International Congress for Logic, Methodology, and Philosophy of Science in 1960 Wheeler began quoting William Kingdon Clifford's "Space theory of Matter" of 1870. He continued, "The vision of Clifford and Einstein can be summarized in a single phrase, "a geometrodynamical universe": a world whose properties are described by geometry, and a geometry whose curvature changes with time—a dynamical geometry".

breaking of the $SL(5,R)$ gauge group via a Higgs-like mechanism is proposed. Likewise, the metric of spacetime arises as an induced concept, as is explained in Sec. VI. The original BF theory is finite, after symmetry breaking asymptotic safeness of the resulting Hilbert-Einstein truncation may still survive, as indicated in Sec. VII. The section titled Outlook concludes the paper.

II. THE LIE ALGEBRA OF THE META-LINEAR GROUP

The structure group $SL(5,R)$ of our gauge model is generated by the trace-free generators

$$\not{L}^A{}_B := L^A{}_B - \frac{1}{5}\delta^A_B L^C{}_C \tag{2.1}$$

which span the Lie algebra $sl(5,R)$ of five–dimensional meta-linear transformations. Their commutation relations

$$[\not{L}^A{}_B, \not{L}^C{}_D] = \delta^A_D \not{L}^C{}_B - \delta^C_B \not{L}^A{}_D \tag{2.2}$$

comprise those of the $gl(4,R)$ subalgebra

$$[L^\alpha{}_\beta, L^\gamma{}_\delta] = \delta^\alpha_\delta L^\gamma{}_\beta - \delta^\gamma_\beta L^\alpha{}_\delta, \tag{2.3}$$

where $\alpha, \cdots, \delta = 0, \cdots, 3$, plus those for the generators $\ell P_\alpha := L^4{}_\alpha = L^4{}_\alpha$ and $\ell P^\beta_* := L^\beta{}_4 = L^\beta{}_4$ of translations:

$$[P_\alpha, P_\beta] = 0, \qquad [P^\alpha_*, P^\beta_*] = 0, \tag{2.4}$$

$$[L^\alpha{}_\beta, P_\gamma] = \delta^\alpha_\gamma P_\beta, \qquad [L^\alpha{}_\beta, P^\gamma_*] = -\delta^\gamma_\beta P^\alpha_*. \tag{2.5}$$

The latter reveal that P^α_* is a vector and P_β a covector with respect to the $GL(4,R)$ subgroup. The physical dimension of ℓ is [length], later on dynamically related via $\ell = \sqrt{3/\Lambda}$ to the cosmological constant (4.17). Similarly as for the de Sitter groups [25], they generate only *pseudo-translations*, since Eq. (2.2) implies

$$[P_\alpha, P^\beta_*] = \frac{1}{\ell^2}\not{L}^\beta{}_\alpha. \tag{2.6}$$

Here the normalization $\not{L}^4{}_4 = 0$ is employed which is equivalent to $L^4{}_4 = L^\gamma{}_\gamma/4$. Consequently, the commutation relations for the Abelian subalgebra generated by the dilations $\mathcal{D} := L^\gamma_\gamma$ are already included in (2.3), since $L^\alpha{}_\beta = \not{L}^\alpha{}_\beta + \frac{1}{4}\delta^\alpha_\beta \mathcal{D}$.

We normalize the trace to $\mathrm{Tr}\{L^A{}_B \not{L}^C{}_D\} = \delta^A_D \delta^C_B$ which implies that

$$\mathrm{Tr}\{P_\alpha P^\beta_*\} = \frac{1}{\ell^2}\delta^\beta_\alpha \tag{2.7}$$

holds for the pseudotranslational generators.

According to Kobayashi [26], the Lie algebra of meta-linear groups can be rewritten as that of the semisimple *graded affine* group $A^*(n,R)$ of the same rank:

$$SL(n+1,R) \approx A_*(n,R) := R^n \oplus GL(n,R) \oplus R^n_*. \tag{2.8}$$

Although this decomposition superficially looks like a generalization of the *conformal group*, the two translational subgroups R^n and R^n_* cannot be identified with translations and special conformal transformations, respectively. In the limit $\ell \to \infty$ of a Wigner-Inönü type group contraction, the generators P_α and P^β_* would commute.

III. GRADED AFFINE GAUGE APPROACH

Consequently, gauge transformations induced by the structure group $SL(n+1,R)$ are more easily described by its graded version $A_*(n,R)$. Using a matrix representation they read

$$A(x) := \left\{\begin{pmatrix} \Lambda(x) & \tau(x) \\ \tau_*(x) & 1 \end{pmatrix}\right\}, \tag{3.1}$$

where $A(x) \in \mathcal{A}_*(n,R)$, the $\Lambda(x) \in \mathcal{GL}(n,R)$ are linear gauge transformations, whereas $\tau(x) := \exp[\xi^\alpha P_\alpha] \in \mathcal{T}(n,R)$ as well as $\tau_*(x) := \exp[\xi^*_\beta P^\beta_*] \in \mathcal{T}_*(n,R)$ represent local (pseudo-) translations. Generalizing the affine gauge approach of Ref. [27] to this case, we introduce the Lie algebra valued one-form

$$\hat{\Gamma} = \begin{pmatrix} \Gamma^{(L)} & \Gamma^{(T)} \\ \Gamma^{(T)}_* & 0 \end{pmatrix} = \begin{pmatrix} \Gamma^{(L)\beta}_\alpha L^\alpha_\beta & \Gamma^{(T)\alpha}\ell P_\alpha \\ \Gamma^{*(T)}_\beta \ell P^\beta_* & 0 \end{pmatrix} \tag{3.2}$$

of a *pseudoaffine* connection [26] compatible with the grading. In accordance with Yang-Mills theory, it transforms inhomogeneously under graded affine gauge transformations:

$$\hat{\Gamma} \overset{A^{-1}(x)}{\longrightarrow} \hat{\Gamma}' = A^{-1}(x)\hat{\Gamma}A(x) + A^{-1}(x)dA(x). \tag{3.3}$$

In our conventions, the inverse gauge transformation

$$A^{-1}(x) = \begin{pmatrix} \Lambda^{-1}(x) & -\Lambda^{-1}(x)\tau(x) \\ -\tau_*(x)\Lambda^{-1}(x) & 1 \end{pmatrix}, \tag{3.4}$$

is regarded as an *active* one. For the two Abelian cosets of pseudotranslations we require that $\tau\tau_* = -\Lambda\tau_*\Lambda^{-1}\tau$. For $\ell \to \infty$ this holds automatically due to the degeneracy of the trace (2.7) in this limit.

The graded affine *curvature* 2-form decomposes into

$$\hat{R} := d\hat{\Gamma} + \hat{\Gamma}\wedge\hat{\Gamma} = \begin{pmatrix} R^{(L)} + \Gamma^{(T)}\wedge\Gamma^{(T)}_* & d\Gamma^{(T)} + \Gamma^{(L)}\wedge\Gamma^{(T)} \\ d\Gamma^{(T)}_* + \Gamma^{(T)}_*\wedge\Gamma^{(L)} & 0 \end{pmatrix}, \tag{3.5}$$

where the exterior product of Lie algebra-valued forms is understood in terms of the adjoint representation $AdA(B) = [A,B]$. The curvature $R^{(L)} := d\Gamma^{(L)} + \Gamma^{(L)} \wedge \Gamma^{(L)}$ is associated with the general linear subgroup $GL(n,R)$, whereas

$$R^{(T)} := d\Gamma^{(T)} + \Gamma^{(L)}\wedge\Gamma^{(T)} = (T^\beta - R_\alpha{}^\beta\xi^\alpha)\ell P_\beta \tag{3.6}$$

is the translational part related later on to torsion [28]. As required, the curvature transforms covariantly under the graded affine gauge group:

$$\hat{R} \overset{A^{-1}(x)}{\longrightarrow} \hat{R}' = A^{-1}(x)\hat{R}A(x). \tag{3.7}$$

The exterior covariant derivative $\hat{D} := d + \hat{\Gamma}\wedge$ acts on a Möbius type affine p-form $\hat{\Psi} = \begin{pmatrix} \Psi \\ 1 \end{pmatrix}$ as follows:

$$\hat{D}\,\hat{\Psi} = \begin{pmatrix} d\Psi + \Gamma^{(L)} \wedge \Psi + \Gamma^{(T)} \\ \Gamma_*^{(T)} \wedge \Psi \end{pmatrix} = \begin{pmatrix} D\Psi + \Gamma^{(T)} \\ \Gamma_*^{(T)} \wedge \Psi \end{pmatrix}, \tag{3.8}$$

such that the *Ricci identity* takes the form

$$\hat{D}\,\hat{D}\,\hat{\Psi} = \begin{pmatrix} DD\Psi + \Gamma^{(T)} \wedge \Gamma_*^{(T)} \wedge \Psi + D\Gamma^{(T)} \\ \\ D\Gamma_*^{(T)} \wedge \Psi \end{pmatrix} = \hat{R} \wedge \hat{\Psi}. \tag{3.9}$$

Similarly as in the de Sitter case [23,29], the inhomogeneous transformation law (3.3) for the meta-linear connection $\hat{\Gamma}$ can be split such that the linear connection $\Gamma^{(L)}$ acquires the conventional transformation rule

$$\Gamma^{(L)} \overset{A^{-1}(x)}{\longrightarrow} \Gamma^{(L)\prime} = \Lambda^{-1}(x)\Gamma^{(L)}\Lambda(x) + \Lambda^{-1}(x)d\Lambda(x) \tag{3.10}$$

of a Yang-Mills type connection for the gauge group $\mathcal{GL}(n, R)$. The remaining translational pieces of the connection (3.2) transform as

$$\Gamma^{(T)} \overset{A^{-1}(x)}{\longrightarrow} \Gamma^{(T)\prime} = \Lambda^{-1}(x)\Big[\Gamma^{(T)} + D\tau(x)\Big],$$
$$\Gamma_*^{(T)} \overset{A^{-1}(x)}{\longrightarrow} \Gamma_*^{(T)\prime} = \Big[\Gamma_*^{(T)} + D\tau_*(x)\Big]\Lambda(x). \tag{3.11}$$

Graded Cartan connection

In spite of the occurrence of the covariant exterior derivative $D\tau(x) := d\tau(x) + \Gamma^{(L)}\tau(x)$, the translational part $\Gamma^{(T)}$ does not transform as a covector as is required for the coframe $\vartheta := \vartheta^\alpha P_\alpha$, i.e. a one-form with values in the Lie algebra of R^n. A similar rule holds for $\Gamma_*^{(T)}$.

However, following Trautman [30] let us introduce the Möbius type zero-forms:

$$\hat{\xi} = \begin{pmatrix} \xi \\ 1 \end{pmatrix} = \begin{pmatrix} \xi^\alpha P_\alpha \\ 1 \end{pmatrix}, \qquad \hat{\xi}_* = (\xi_*, 1) = (\xi_\beta^* P_*^\beta, 1), \tag{3.12}$$

whose components transform as vector or covector

$$\xi \overset{A^{-1}(x)}{\longrightarrow} \xi' = \Lambda^{-1}(x)[\xi - \tau(x)] \tag{3.13}$$

$$\xi_* \overset{A^{-1}(x)}{\longrightarrow} \xi_*' = [\xi_* - \tau_*(x)]\Lambda^{-1}(x), \tag{3.14}$$

respectively, under active graded affine gauge transformations. Then one can introduce new one-forms:

$$\vartheta := \Gamma^{(T)} + D\xi, \qquad \theta := \Gamma_*^{(T)} + D\xi_*, \tag{3.15}$$

which transform as vector or covector, respectively, under the graded $\mathcal{A}_*(n, R)$, as required:

$$\vartheta \overset{A^{-1}(x)}{\longrightarrow} \vartheta' = \Lambda^{-1}(x)\vartheta, \qquad \theta \overset{A^{-1}(x)}{\longrightarrow} \theta' = \theta\Lambda^{-1}(x). \tag{3.16}$$

Since $\xi = \xi^\alpha P_\alpha$ and $\xi_* = \xi_\beta^* P_*^\beta$ acquire their values in the coset space $A_*(n, R)/GL(n, R) \approx R^n \otimes R_*^n$, they can be regarded as "generalized Higgs fields" which, according to (3.16), "hide" the action of the local pseudotranslations $\mathcal{T}(n, R) \otimes \mathcal{T}_*(n, R)$ on ϑ and θ.

Under the constraints

$$D\xi \overset{*}{=} 0, \qquad D\xi_* \overset{*}{=} 0 \tag{3.17}$$

the translational connections $\Gamma^{(T)}$ and $\Gamma_*^{(T)}$ become *soldered*[2] to the spacetime manifold and the translational parts of the graded affine gauge group get "spontaneously broken" with the gauge action of pseudotranslations partially "eaten-up", cf. Ref. [32].

If we postulate the even stronger constraints $\xi = \xi_* = 0$ of "zero section" vector fields, the pseudoaffine connection $\hat{\Gamma}$ reduces [33] to the graded *Cartan connection*

$$\overset{=}{\Gamma} = \begin{pmatrix} \Gamma^{(L)} & \vartheta \\ \theta & 0 \end{pmatrix} \tag{3.18}$$

which is not anymore a connection in the usual sense.

As an upshot of our Cartan type construction, for the physical interesting case of $n = 4$ the $sl(5, R)$-valued gauge connection decomposes into

$$\hat{\Gamma} := \hat{\Gamma}_A{}^B \not{L}^A{}_B = \Gamma_\alpha{}^\beta L^\alpha{}_\beta + \vartheta^\alpha P_\alpha + \theta_\beta P_*^\beta, \tag{3.19}$$

and the corresponding curvature takes the standard form

$$\hat{R} := R_A{}^B \not{L}^A{}_B := [d\Gamma_A{}^B - \Gamma_A{}^C \wedge \Gamma_C{}^B]\not{L}^A{}_B$$
$$= [R_\alpha{}^\beta - \frac{1}{\ell^2}\theta_\alpha \wedge \vartheta^\beta]L^\alpha{}_\beta + T^\beta P_\beta + D\theta_\alpha P_*^\alpha, \tag{3.20}$$

of Maurer-Cartan equations. In order to obtain the expansion, the commutator (2.6) and the dimensionality 1/[length] of the generators P_α and P_*^β of pseudotranslations have been employed. The 2-form $T^\beta := D\vartheta^\beta$ is the standard torsion and the $sl(5, R)$ Chern-Simons (CS) term decomposes [28] into

$$\hat{C} := -\frac{1}{2}\Big(\Gamma_A{}^B \wedge d\Gamma_B{}^A - \frac{2}{3}\Gamma_A{}^B \wedge \Gamma_B{}^C \wedge \Gamma_C{}^A\Big)$$
$$= C_{\rm RR} - 2C_{\rm TT}, \tag{3.21}$$

[2] A Cartan transport of a radius vector ξ^α occurs via "rolling without sliding" when $D\xi^\alpha =^* \vartheta^\alpha$, cf. Refs. [28,31].

where

$$C_{\rm TT} := \frac{1}{4\ell^2}\left[\vartheta^\beta \wedge D\theta_\beta + \theta_\alpha \wedge T^\alpha\right] \stackrel{*}{=} \frac{1}{2\ell^2} g_{\alpha\beta}\vartheta^\alpha \wedge T^\beta \tag{3.22}$$

denotes the *translational* CS term of Nieh and Yan (NY) [34,35]. In the noncommutative differential geometry, it resembles the Higgs-dependent term in Eq. (3.11) of Ref. [36].

Only in the reduction to the (pseudo-)orthogonal de Sitter groups, θ and ϑ become mutually transposed [23] one-forms.

IV. GRAVITY FROM SPONTANEOUSLY BROKEN *BF* THEORY

Following Ref. [16], let us devise a gravitational *BF* scheme by introducing the Lie algebra-valued 2-form:

$$\hat{B} := B_A{}^B \not{L}^A{}_B = (b_\alpha P^\alpha_* + \hat{b}^\alpha P_\alpha)\ell^2 + B_\alpha{}^\beta L^\alpha{}_\beta \tag{4.1}$$

and consider the $SL(5,R)$-invariant *BF* Lagrangian in a primordial *metric-independent* setting:

$$\tilde{L}_{{\rm SL}(5,R)} = -{\rm Tr}\{\hat{B}\wedge\hat{R}\} - d\hat{C}. \tag{4.2}$$

The pure *BF* system has been *modified* [14,15] via the topological Pontrjagin 4-form $d\hat{C}$, in order to provide us, after variation with respect to $\Gamma_A{}^B$, with the Bianchi identity

$$\hat{D}B_A{}^B \cong \hat{D}\hat{R}_A{}^B \equiv 0. \tag{4.3}$$

Note that multiplying the Pontrjagin term by a constant phase Θ would not change the topological framework, since it could be absorbed in a redefinition of $\hat{B}$.

The variation with respect to $\hat{B}$ would lead to $\hat{R} = 0$. In order to liberate us from this physically too strong constraint of constant curvature, we will consider a Lagrangian amended by a term quadratic in $\hat{B}$, i.e.:

$$\begin{aligned}\tilde{L}_{\rm SSB} &= \tilde{L}_{{\rm SL}(5,R)} + \frac{1}{2}\eta_{ABCDE}B^{AB}\wedge B^{CD}\Phi^E\\ &= \tilde{L}_{{\rm SL}(5,R)} + B^{AB}\wedge B^{(\star)}_{ABE}\Phi^E.\end{aligned} \tag{4.4}$$

The antisymmetric unit tensor $\eta_{ABCDE} := \sqrt{\hat{g}}\epsilon_{ABCDE}$ is constructed from the *metric-free* Levi-Civita symbol ϵ_{ABCDE} covariant under the five-dimensional structure group $SL(5,R)$ together with the determinant $\hat{g}$ of the Cartan-Killing metric $\hat{g}_{AB}$ needed to rise and lower the Lie algebra indices of $B_A{}^B$. Alternatively,

$$B^{(\star)}_{ABE} := \frac{1}{2}\eta_{ABCDE}B^{CD} \tag{4.5}$$

can regarded as the *Lie dual* of $B_A{}^B$, validating the equivalent expression in Eq. (4.4). Such a "semitopological" [37] or deformed *BF* scheme with the additional quadratic term induces a symmetry breaking (SB) of the $SL(5,R)$ to the special orthogonal group $SO(5)$ or, depending on the signature, to the de Sitter or Anti-de Sitter groups $SO(1,4)$ or $SO(2,3)$, respectively. Moreover, the five-index structure of η_{ABCDE} has forced us to complement our covariant construction by a vector-valued scalar field $\Phi^A = \{\phi^\alpha, \phi^4\}$ which is CP odd.

Let us assume that it satisfies the $SO(5)$ gauge-invariant constraints

$$\hat{g}_{AB}\Phi^A\Phi^B = \mu^2, \qquad \Phi_E\hat{D}\Phi^E = 0, \tag{4.6}$$

which correspond to Eq. (24) of Pagels [20]. The parameter μ will turn out to be rather small.

After a *Spontaneous Symmetry Breaking* (SSB), its vacuum expectation value is assumed to take the value

$$\langle\Phi^E\rangle = \Phi_0^E := (0,0,0,0,\mu)^t \tag{4.7}$$

as its ground state.

Then, as a consequence of both steps, the quadratic term in B of

$$\tilde{L}_{\rm SSB} = -{\rm Tr}\{\hat{B}\wedge\hat{R}\} + \frac{\mu}{2}\eta_{\alpha\beta\gamma\delta}B^{\alpha\beta}\wedge B^{\gamma\delta} - dC_{\rm RR} + 2dC_{\rm TT} \tag{4.8}$$

is invariant only under the Lorentz subgroup $SO(1,3)$ (or $SO(4)$ for Euclidean signature). Moreover, our $SL(5,R)$ decent (3.21) of the 4D topological invariants requires a fixed value $\Theta_{\rm T}/\Theta_{\rm L} = -2$ for the ratio of the individual $\Theta_{\rm L}$ and $\Theta_{\rm T}$-angles, thus leaving no room for the so-called Barbero-Immirizi parameter of Ref. [38,39] other than the singular case $\gamma = 1$. Even after field quantization, this is in contradistinction to Ref. [40] where the chiral anomaly in a spacetime with torsion is erroneously related to the divergent NY term, cf. Refs. [41–43].

Expanding the trace, the Lagrangian (4.8) is equivalent to

$$\begin{aligned}\tilde{L}_{\rm SSB} = &-b_\alpha\wedge T^\alpha - \hat{b}^\alpha\wedge D\theta_\alpha - B_\beta{}^\alpha\wedge\left[R_\alpha{}^\beta - \frac{1}{\ell^2}\theta_\alpha\wedge\vartheta^\beta\right]\\ &+\frac{\mu}{2}\eta_{\alpha\beta\gamma\delta}B^{\alpha\beta}\wedge B^{\gamma\delta} - dC_{\rm RR} + 2dC_{\rm TT}.\end{aligned} \tag{4.9}$$

Variation with respect to the translational two forms b_α and $\hat{b}^\alpha$ lead to vanishing torsion

$$T^\alpha = 0 \tag{4.10}$$

and to

$$D\theta_\alpha = D(g_{\alpha\beta}\vartheta^\beta) = Dg_{\alpha\beta}\wedge\vartheta^\beta + g_{\alpha\beta}T^\beta = 0, \tag{4.11}$$

provided that

$$\theta_\alpha = g_{\alpha\beta}\vartheta^\beta \tag{4.12}$$

holds, as the result of some spontaneous symmetry breaking. A possible generalization to nonvanishing torsion would present limitations [44] on such a *BF* scheme to the extent that a Belinfante-Rosenfeld symmetrization would be mandatory.

For a nondegenerate coframe ϑ^β and vanishing torsion, Eq. (4.11) is equivalent to

$$Dg_{\alpha\beta} = 0, \tag{4.13}$$

i.e. to the covariant constancy of the induced spacetime metric. The remaining Pontrjagin term merely serves to reproduce the Bianchi identity (4.3), now in terms of the Lorentz connection $\overset{\circ}{\Gamma}{}^{\alpha\beta} = -\overset{\circ}{\Gamma}{}^{\beta\alpha}$.

The variation with respect to the $gl(4,R)$-valued 2form $B_\alpha{}^\beta$ leads to

$$B_{\alpha\beta} \cong \frac{1}{\mu}\eta_{\alpha\beta\gamma\delta}\Big[R^{\gamma\delta} - \frac{1}{\ell^2}\theta^\gamma\wedge\vartheta^\delta\Big] = \frac{2}{\mu\ell^2}[\ell^2 R^{(\star)}_{\alpha\beta} - \eta_{\alpha\beta}] \tag{4.14}$$

as an equation[3] of motion. Thus for $\mu \neq 0$ our Lagrangian is the "on shell" equivalent to the *deformed* [45,46] Euler 4-form

$$\tilde{L}_{\rm SSB} + dC_{\rm RR} \cong \frac{1}{2\mu}\eta_{\alpha\beta\gamma\delta} \times\Big[R^{\alpha\beta} - \frac{1}{\ell^2}\vartheta^\alpha\wedge\vartheta^\beta\Big]\wedge\Big[R^{\gamma\delta} - \frac{1}{\ell^2}\vartheta^\gamma\wedge\vartheta^\delta\Big] \geq 0 \tag{4.15}$$

which is at most quadratic in the Riemannian curvature $R_{\alpha\beta} = R^{\{\}}_{\alpha\beta}$. Modulo the Pontrjagin term, it is positive [47] for Euclidean signature.

Hilbert-Einstein truncation

After expansion, the celebrated Hilbert-Einstein Lagrangian and an induced cosmological term remain, supplemented by the topological Euler[4] and Pontrjagin 4-forms, i.e.,

$$\tilde{L}_{\rm SSB} \cong -\frac{1}{2\kappa}R^{\alpha\beta}\wedge\eta_{\alpha\beta} + \frac{\Lambda}{\kappa}\eta + \frac{1}{\mu}R^{\alpha\beta}\wedge R^{(\star)}_{\alpha\beta} - \frac{1}{2}R^{\alpha\beta}\wedge R_{\alpha\beta} \tag{4.16}$$

emerges, where $\kappa = 8\pi G_{\rm N} = \ell^2_{\rm Planck}$ denotes Einstein's gravitational constant in natural units ($\hbar = c = 1$) and $\rho_\Lambda = \Lambda/\kappa = \Omega_\Lambda\rho_{\rm crit} < \rho_{\rm crit}$ is the density related to the cosmological constant Λ. These constants are related via

$$\kappa = \mu\ell^2/4, \qquad \Lambda = 12\kappa/\mu\ell^4 = 3/\ell^2 \tag{4.17}$$

to the symmetry breaking scale

$$\mu = \frac{4}{3}\kappa\Lambda = \frac{1}{3}(2\kappa)^2\rho_\Lambda. \tag{4.18}$$

The latter is itself independent of a rescaling of the fundamental length ℓ. Since μ is observationally an extremely small number of the order of 10^{-120} related to constant dark energy (DE) [52,53], in our approach GR with a small cosmological constant Λ turns out to be a tiny *deformation* of a topological BF theory via a symmetry breaking $B\wedge B$ term, as anticipated by Smolin [54] and Mikovic [55] in the restricted case of the de Sitter group. Thus the "worst fine tuning in physics" [45] converts into a "marble" of symmetry breaking.

In concordance with the "on shell" Lagrangian (4.16), the variation with respect to ϑ^β of the spontaneously broken one (4.9) amended by matter leads directly to

$$-\frac{2}{\ell^2}B_\beta{}^\alpha\wedge\theta_\alpha + \Sigma_\beta = \frac{4}{\mu\ell^2}\Big[R^{(\star)}_{\alpha\beta}\wedge\vartheta^\alpha - \Lambda\eta_\beta\Big] + \Sigma_\beta \cong 0, \tag{4.19}$$

i.e, the standard Einstein's equations

$$\frac{1}{2}R^{\{\}\beta\gamma}\wedge\eta_{\alpha\beta\gamma} + \Lambda\eta_\alpha = \kappa\Big(\Sigma_\alpha - D^{\{\}}\mu_\alpha\Big) \tag{4.20}$$

with the cosmological constant Λ given by (4.17). Quite generally, matter couplings will need the Belinfante-Rosenfeld symmetrization for the energy-momentum current [56]. Local degrees of freedom of gravity (like gravitational waves) arise only if the gauge breaking term controlled by the parameter μ is nonzero. In such a scenario, gravity is an *emergent* phenomenon, cf. Laughlin [57].

V. HIGGS-LIKE MECHANISM OF SYMMETRY BREAKING

So far, these symmetry reductions have been done by hand via the constraint (4.6). They are lacking the motivation from the *Higgs mechanism* [24], where the Lagrangian

$$L_{\rm Higgs} = \frac{1}{2}\hat{D}\Phi^A\wedge{}^*\hat{D}\Phi_A + U(\Phi^B\Phi_B) \tag{5.1}$$

[3] Observe that the Lie dual $\eta_{\alpha\beta} := \frac{1}{2}\eta_{\alpha\beta\gamma\delta}\vartheta^\gamma\wedge\vartheta^\delta = {}^*(\vartheta^\alpha\wedge\vartheta^\beta)$ of the "unit" 2-form $\vartheta^\alpha\wedge\vartheta^\beta$ is *equivalent* to its Hodge dual $*$ as a consequence of the soldering of the coframe ϑ^α, cf. Eq. (3.7.8) of Ref. [28]. Thus, the two 4-forms $R^{(\star)}_{\alpha\beta}\wedge\vartheta^\alpha\wedge\vartheta^\beta \equiv R^{\alpha\beta}\wedge\eta_{\alpha\beta}$ for the Einstein-Cartan (EC) Lagrangian are equivalent.

[4] The Lie dual $R^{(\star)}_{\alpha\beta} := \frac{1}{2}\eta_{\alpha\beta\gamma\delta}R^{\gamma\delta}$ of the curvature is featuring in the Euler invariant $dC_{{\rm RR}^{(\star)}} := \frac{1}{2}d(\Gamma_{\alpha\beta}\wedge R^{\alpha\beta(\star)} - \frac{1}{3}\Gamma_\alpha{}^{\beta(\star)}\wedge\Gamma_\beta{}^\gamma\wedge\Gamma_\gamma{}^\alpha) \equiv \frac{1}{2}R_{\alpha\beta}\wedge{}^*R^{\alpha\beta} - 2\,{\rm Ric}_{\alpha\beta}\wedge{}^*{\rm Ric}^{\alpha\beta} + \frac{1}{2}\,{\rm Ric}_\alpha{}^\alpha\wedge{}^*{\rm Ric}_\beta{}^\beta$. It has an equivalent representation in terms of Weyl's [48–50] Lagrangian $L_{\rm SKY} = -R_{\alpha\beta}\wedge^* R^{\alpha\beta}/2$, plus a Ricci-squared and a curvature scalar squared term. The second expression involving the symmetric Ricci tensor, i.e. the zero-form ${\rm Ric}_{\alpha\beta} := (-1)^{{\rm sig}*}(R_{(\alpha}{}^\delta\wedge\eta_{\delta|\beta)})$, is known as Gauss-Bonnet identity. Field redefinitions in such Lagrangians have GR as a stable fixed point [51].

commonly involves, besides a kinetic term for the quintet Φ^A of scalar fields, an invariant "Mexican hat" type quartic potential in Φ^B, cf. Ref. [22]. Let the ground state (4.7) be a minimum of the self-interaction $U(\Phi^B\Phi_B)$ such that $\delta U/\delta\Phi_0^E = 0$ and $\Phi_{E0}\hat{D}\Phi_0^E = 0$. The sign $\delta^2 U/\delta^2\Phi^E > 0$ of the second order variation determines the stability of the ground state at a local minimum.

Since the vacuum expectation value

$$\langle\hat{D}\Phi^E\rangle = \hat{D}\Phi_0^E = \mu({\Gamma_4}^\alpha, 0)^t = \frac{\mu}{\ell}(\vartheta^\alpha - D\xi^\alpha, 0)^t, \tag{5.2}$$

is related to pseudotranslations, the remaining kinetic term

$$\hat{D}\Phi_0^A \wedge {}^*\hat{D}\Phi_{A0} = \mu^2 {\Gamma_4}^\alpha \wedge {}^*{\Gamma_\alpha}^4 = \frac{\mu^2}{\ell^2}(\vartheta^\alpha - D\xi^\alpha)\wedge{}^*(\theta_\alpha - D\xi_\alpha^*) \overset{*}{=} \frac{1}{\kappa}\Lambda_{\rm H}\eta \tag{5.3}$$

would provide a small mass term to the pseudotranslational connections ${\Gamma_4}^\alpha$ and ${\Gamma_\alpha}^4$, leaving the linear $gl(4, R)$-valued connection ${\Gamma_\alpha}^\beta$ massless and thus possible gravitons in the metrical reduction. Since these gravitons remain *strictly* massless they do not suffer from a van Dam-Veltman-Zakharov discontinuity, cf. Ref. [58]. In the soldering gauge $D\xi^\alpha =^* 0$ and $D\xi_\beta^* =^* 0$, or even more for a zero affine section, Eq. (5.3) is equivalent to a cosmological term in addition to Λ, although the Higgs-induced constant

$$\Lambda_{\rm H} = \mu^3 \tag{5.4}$$

is diminutive.

An interesting variant arises when the quadratic term in (4.4) is generalized, similarly as in Ref. [59], to a 4-form potential

$$V = V(B, \Phi) = V(I), \tag{5.5}$$

where

$$I_1 := B^{AB} \wedge B_{ABE}^{(\star)}\Phi^E, \qquad I_2 := B^{AB} \wedge {}^*B_{ABE}^{(\star)}\Phi^E \tag{5.6}$$

are the only quadratic invariants in B. In the case of I_1, the variation with respect to ${B_A}^B$ yields the relation

$$R_{AB} \cong 2\frac{\partial V}{\partial I}B_{ABE}^{(\star)}\Phi^E \tag{5.7}$$

for the curvature. In order to invert the latter with respect to B_{AB}, a nondegenerate Hessian for $V(I)$ is mandatory. In the ground state (4.7), the relation (5.7) would replace (4.14) such that certain nonlinear curvature Lagrangians $L(R)$ arise, cf. Refs. [60,61].

The variation $\delta\Phi^E$ of the full Lagrangian yields

$$\hat{D}\,{}^*\hat{D}\Phi_E + \frac{\delta U}{\delta\Phi^E} + \frac{\partial V}{\partial I}B^{AB}\wedge B_{ABE}^{(\star)} = 0. \tag{5.8}$$

For a minimum, the last term needs to compensate the Higgs-induced cosmological constant $\Lambda_{\rm H}$. In principle one could dismiss the quartic potential in Φ^E in favor of a potential (5.5) involving B.

Other $SL(5, R)$ gauge-invariant 4-forms are linear in B and necessarily involve derivatives of the Higgs field:

$$\begin{aligned} I_3 &:= B_{AB}\wedge\hat{D}\Phi^A\wedge\hat{D}\Phi^B,\\ I_4 &:= B_{AB}\wedge{}^*(\hat{D}\Phi^A\wedge\hat{D}\Phi^B),\\ I_5 &:= \hat{D}\Phi^A\wedge\hat{D}\Phi^B\wedge B_{ABE}^{(\star)}\Phi^E,\\ I_6 &:= \hat{D}\Phi^A\wedge\hat{D}\Phi^B\wedge{}^*B_{ABE}^{(\star)}\Phi^E. \end{aligned} \tag{5.9}$$

They could be amended to the usual kinetic term in (5.1).

The term I_6 involves a kind of *double dual* of the $sl(5, R)$-valued B form, as exemplified by the curvature

$${}^*R_{AB}^{(\star)} := \frac{1}{2}\eta_{ABCDE}{}^*R^{CD}\Phi^E. \tag{5.10}$$

This definition can be generalized to five dimensions (5D) as in Eq. (50) of Ref. [45]. Accordingly, there exists the possibility to construct an imbedding of our four-dimensional $SL(5, R)$ gauge model into a surface term $dC_\star$ in 5D such that it dimensionally reduces to the deformed Euler characteristic (4.15) in the macroscopical 4D, cf. also Ref. [62].

VI. EMERGENCE OF A SPACETIME METRIC

As mentioned in the Introduction, already Eddington [4] was considering the gauge connection as primordial and the metric as a derived concept. A more recent proposal is that of 't Hooft [63] by constructing an "alternative" metric $ds^2 =^* o_{\alpha\beta}D\xi^\alpha\otimes_{\rm s}D\xi^\beta$ from a "quartet" ξ^α of scalar fields. In the Cartan formalism [33,64], a related coset field naturally arise in the affine gauge theory [27,32,65] after imposing locally the gauge condition of vanishing translational connection.

A rather concrete proposal is that of Pagels [20,45], according to which the metric should surface as a *composite* Higgs field via the vacuum expectation value

$$\begin{aligned} ds^2 &:= \frac{\ell^2}{\mu^2}\langle\hat{D}\Phi^A\otimes_{\rm s}\hat{D}\Phi_A\rangle = \frac{\ell^2}{\mu^2}\hat{D}\Phi_0^A\otimes_{\rm s}\hat{D}\Phi_{A0}\\ &= \ell^2{\Gamma_4}^\alpha\otimes_{\rm s}{\Gamma_\alpha}^4\\ &= (\vartheta^\alpha - D\xi^\alpha)\otimes_{\rm s}{}^*(\theta_\alpha - D\xi_\alpha^*) \overset{*}{=} \vartheta^\alpha\otimes_{\rm s}\theta_\alpha\\ &= g_{ij}dx^i\otimes_{\rm s}dx^j. \end{aligned} \tag{6.1}$$

Since our $SL(5, R)$ gauge model is broken in two steps from $SO(5)$ down to the Lorentz group $SO(1, 3)$ as an exact subgroup, a local holonomic spacetime metric g_{ij} is then induced.[5] This complies with our anticipation (4.12) that

[5]When considering a five-dimensional embedding [45], one could start from $ds_5^2 := g_{AB}\hat{D}\Phi^A\otimes_{\rm s}\hat{D}\Phi^B$ in 5D and then apply a spontaneously spacetime reduction such that the "premetric" via $ds^2_{\rm pre} := \ell^2{\Gamma_4}^\alpha\otimes_{\rm s}{\Gamma_\alpha}^4$ arises. Although such a construction is degenerate [27] or could even be zero in the gauge of vanishing translational connections, it could be perspective for quantization.

the two coframes turn out to be orthogonal after SSB, as well with the decomposition (3.21) of the topological $SL(5,R)$ CS term. Since the "premetric" $\Gamma_4{}^{\alpha} \otimes_s \Gamma_{\alpha}{}^{4}$ is dimensionless, we had to multiply the vacuum expectation (6.1) of the derivative of the Higgs with a *huge* dimensional factor $\ell^2/\mu^2 = (\kappa/4\Lambda_H)$. This is suggesting that the induced metric applies only for macroscopic distances, as is anticipated in the emerging Einstein Eq. (4.20). In the limit $\mu \to 0$ of $SL(5,R)$ symmetry restoration, however, macroscopic distances ds would loose their meaning.

Eventually, our choice (4.4) of the term quadratic in $\hat{B}$, provides a SSB directly down to the Lorentz group $SO(1,3) \in O(5)$, where the constraint (4.13) of vanishing nonmetricity $Q_{\alpha\beta} := -Dg_{\alpha\beta}$ surfaces automatically, without the need of postulating it as in Ref. [66].

On the other hand, upon symmetry breaking, the ground state of the modified BF Lagrangian (4.2) for gravity requires Einstein spaces (4.19) for its metrical "background", as we have already shown. Moreover, a canonical analysis [39] confirms the corresponding helicity states of gravitons. Thus, classically a consistent and physically viable scheme emerges.

VII. ASYMPTOTIC SAFENESS OF RUNNING COUPLING CONSTANTS

Primordially, all degrees of freedom of our gauge model are in the $sl(5,R)$-valued connection. Such topological BF theories are known to be finite [67]. Moreover, they are anomaly-free in 4D, as a consequence of the vector supersymmetry inherent in its BRST quantization. Typically, the chiral anomaly is proportional to the Pontrjagin term of the corresponding gauge group. In a modified BF scheme, such an invariant "counter-term" is already incorporated into the action (4.2) via the $SL(5,R)$ invariant $d\hat{C}$.

In our semitopological BF scheme, at first the deformed Euler term (4.15) is emerging as an effective quadratic curvature Lagrangian, inheriting a dimensionless coupling constant μ. When it is not only tiny but has $\mu_* = 0$ as its fixed point, such a Lagrangian (4.4) is asymptotic free [68].

After symmetry breaking, however, the emerging gravity [69] is, at low energies, described by the graviton degrees of freedom inherent in the metric g_{ij}. For manifolds with trivial topology, we may even neglect the Euler and Pontrjagin terms and define for the resulting Hilbert-Einstein *truncation* with cosmological term the two *dimensionless* running coupling constants

$$g_N := \kappa k^2, \qquad \lambda := \Lambda/k^2, \tag{7.1}$$

where k is the renormalization scale in momentum space. Asymptotic safeness amounts to the requirement that dimensionless coupling constants remain bounded in the ultraviolet (UV) limit $k \to \infty$. In our 4D case, this is controlled by the *renormalization group equations*

$$\begin{aligned} k\frac{\partial}{\partial k} g_N &= \beta_1(g_N, \lambda) = (2 + d_N g_N) g_N, \\ k\frac{\partial}{\partial k}\lambda &= \beta_2(g_N, \lambda), \end{aligned} \tag{7.2}$$

where $d_N := k\partial_k \ln g_N = -2a$ is the anomalous dimension of the Newtonian coupling constant g_N.

According to the *asymptotic safety scenario* [70,71], they run into some nontrivial fixed points g_{N*} and λ_*, depending on the truncation of the effective Lagrangian (4.16). For the Hilbert-Einstein action, the beta function β_1 has an infrared (IR) attractive fixed point at $g_{N*} = 0$ and an UV attractive nontrivial fixed point at $g_{N*} = 1/a$, where a is some finite constant.

Quite generally, the product with the universal bound

$$\mu_* \le \frac{4}{3} g_{N*} \lambda_* \simeq 0.2 \tag{7.3}$$

appears to be rather robust [72]. We view this as an indication of emergent relativity via a small symmetry breaking parameter μ. From such a perspective, standard gravity appears not any more [73] nonrenormalizable, as is usual surmised from perturbation theory.

VIII. OUTLOOK

In our primordial $SL(5,R)$ gauge theory of gravity, one cannot talk of distances, causal order or even black holes. Thus the nonrenormalizability argument in Ref. [74] that asymptotic density states are dominated by black holes, does not apply to the unbroken high-energy phase. Only in the low energy limit a line element ds emerges that is necessary for measuring macroscopic distances by clocks and rods. Rather surprising is that such a tiny SSB with $\mu \simeq 10^{-120}$ observationally is sufficient in order to generate the feeble gravity we are acquainted with.

The analogy with *self-dual* system suggests also that $SL(5,R)$ gauge-invariant BF theory is completely integrable, essentially due to the Bianchi identity (4.3). For gravity emerging as an effective theory after SSB, such a property only survives for subsystems such as the Ernst equation [75]. Moreover, a gauge unification of all fundamental interactions may be pursued along the lines of Ref. [76]. It needs to be seen, if the asymptotic safety scenario [72] really supports the speculation [77] that gravity emerges from an entropy force in a broken BF theory.

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