PHYSICAL REVIEW D **83**, 036002 (2011)

New results on pionic twist-3 distribution amplitudes within the QCD sum rules

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(Received 6 October 2010; published 7 February 2011)

We present an improved calculation on the pionic twist-3 distribution amplitudes ϕ_p^π and ϕ_σ^π, which are studied within the QCD sum rules. By adding all the uncertainties in quadrature, it is found that $\langle\xi_p^2\rangle = 0.248^{+0.076}_{-0.052}$, $\langle\xi_p^4\rangle = 0.262^{+0.080}_{-0.055}$, $\langle\xi_\sigma^2\rangle = 0.102^{+0.035}_{-0.025}$ and $\langle\xi_\sigma^4\rangle = 0.094^{+0.028}_{-0.020}$. Furthermore, with the help of these moments, we construct a model for the twist-3 wave functions $\psi^\pi_{p,\sigma}(x, \mathbf{k}_\perp)$, which have better end point behavior and are helpful for perturbative QCD approach. The obtained twist-3 distribution amplitudes are adopted to calculate the $B\to\pi$ transition form factor $f^+_{B\pi}$ within the QCD light cone sum rules up to next-to-leading order. By suitable choice of the parameters, we obtain a consistent $f^+_{B\pi}$ with those obtained in the literature.

DOI: 10.1103/PhysRevD.83.036002 PACS numbers: 11.55.Hx, 12.38.Aw

I. INTRODUCTION

Distribution amplitude (DA) shows the momentum fraction distributions of partons in hadron in a particular Fock state, which possess an important component for QCD factorization theory [1]. It is convenient to arrange DA by its different twist structures such that when the energy scale of the process is large enough, one can safely neglect those higher power suppressed contributions from higher twists. Because of its simpler structure, the leading-twist DA has attracted much attention in the literature [1,2]. Very recently, the new *BABAR* data on the pion-photon transition form factor [3] arouses people's new interests on the pion twist-2 DA [4].

However the higher-twist DAs are much more involved [5–7], which describes either contributions of the transverse motion of quarks (antiquarks) in the leading-twist components or contributions of higher Fock states with additional gluons and/or quark-antiquark pairs or etc. In certain cases, higher-twist structures, especially the twist-3 DAs, may provide sizable contributions and there are usually the main uncertainties for the theoretical estimation. For example, as for the case of pion electromagnetic form factor and $B\to\pi$ transition form factor, most calculations give large twist-3 contributions that are even dominant over that of the leading twist in the intermediate or even large Q^2 region. It should be pointed out that those large contributions of twist-3 DAs are usually based on the asymptotic behavior of ϕ_p^π, i.e., $\phi_p^{\rm AS}\equiv 1$. And with such a naive asymptotic behavior of twist-3 DA, the end point singularity can not be effectively suppressed, which inversely leads to a large twist-3 contribution [8–12]. While, by taking the k_T factorization approach [13] and by constructing a twist-3 wave function model based on the DA moments derived by Refs. [14,15], it has been found that the contributions from twist-3 DAs are really power suppressed to that of the leading-twist DA within large Q^2 region [16,17]. So, a twist-3 DA with a better end point behavior other than the asymptotic one shall lead to a better understanding of these form factors.

At present, the pionic twist-3 structures are far from affirmation [6,7,14,15,18,19], and it would be interesting to do further studies on the twist-3 DAs/wave functions. In the present paper, we shall first make a study on the pionic twist-3 DA moments within the QCD sum rules, and then make a discussion on its physical effects by constructing a reasonable wave function model and by taking $B\to\pi$ transition form factor as an explicit example.

The remaining parts of the paper are organized as follows: In Sec. II, we present the calculation technology for the twist-3 DAs ϕ_p^π and ϕ_σ^π within the framework of QCD sum rules. In Sec. III, we present our numerical results for ϕ_p^π and ϕ_σ^π, where the DA moments together with their uncertainties are discussed in detail. Based on the derived DA moments, we construct the wave function models for ψ_p^π and ψ_σ^π in Sec. IV, and then make a discussion on their rationality by further applying them to deal with the $B\to\pi$ transition form factor within the QCD light cone sum rules up to next-to-leading order (NLO). The final section is reserved for a summary.

II. CALCULATION TECHNOLOGY FOR THE TWIST-3 DAS ϕ_p^π AND ϕ_σ^π

The pionic twist-3 DAs ϕ_p^π and ϕ_σ^π are defined as [19]

$$\langle 0|\bar{u}(x) i\gamma_5 d(-x)|\pi_q^-\rangle = \mu_\pi f_\pi \int_0^1 du e^{i\xi(q\cdot x)}\phi_p^\pi(u) \quad (1)$$

and

$$\langle 0|\bar{u}(x)\sigma_{\mu\nu}\gamma_5 d(-x)|\pi_q^-\rangle = -\frac{i}{3}\mu_\pi f_\pi \int_0^1 du e^{i\xi(q\cdot x)} \times (q_\mu x_\nu - q_\nu x_\mu)\phi_\sigma^\pi(u), \quad (2)$$

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where $\xi = (2u - 1)$, and due to the equations of motion of the quarks inside pion [19], $\mu_\pi = \frac{m_\pi^2}{m_u + m_d}$, whose value is 1.6 ± 0.2 GeV at the scale 1 GeV [7]. Normally, these two DAs can be expanded by the conventional Gegenbauler expansions, $\phi_p^\pi(\xi) = \sum_n \langle \xi_p^{2n} \rangle C_{2n}^{1/2}(\xi)$ and $\phi_\sigma^\pi(\xi) = 3(1 - \xi^2)/4 \sum_n \langle \xi_\sigma^{2n} \rangle C_{2n}^{3/2}(\xi)$, where $C_{2n}^{1/2,3/2}(\xi)$ are Gegenbauler polynomials and $\langle \xi_{p,\sigma}^{2n} \rangle$ are the so-called Gegenbauler moments. As for the pionic case, because of the chiral symmetry, only even moments' terms are nonzero.

The Gegenbauler moments can be calculated under the QCD sum rules. For such purpose, one can define the two correlation functions as

$$(z \cdot q)^{2n} I_p^{(2n,0)}(q^2) \equiv -i \int d^4x e^{iq\cdot x} \langle 0 | T\{\bar{d}(x)\gamma_5 (iz \cdot \overleftrightarrow{D})^{2n} u(x), \bar{u}(0)\gamma_5 d(0)\}|0\rangle \tag{3}$$

and

$$-i(q_\mu z_\nu - q_\nu z_\mu)(z\cdot q)^{2n} I_\sigma^{(2n,0)}(q^2) \equiv -i \int d^4x e^{iq\cdot x} \langle 0 | T\{\bar{d}(x)\sigma_{\mu\nu}\gamma_5 (iz \cdot \overleftrightarrow{D})^{2n+1} \times u(x), \bar{u}(0)\gamma_5 d(0)\}|0\rangle. \tag{4}$$

Following the same calculation technology as described in detail in Refs. [14,15], i.e., the QCD sum rules under the background field approach [20–23], we obtain two sum rules for the moments of ϕ_p^π and ϕ_σ^π with the condensates up to dimension six, i.e.,

$$\langle \xi_p^{2n} \rangle = \frac{M^4 e^{m_\pi^2/M^2}}{f_\pi^2 \mu_\pi^2} \left\{ \frac{3}{8\pi^2} \frac{1}{2n+1} \left[1 - \left(1 + \frac{s_\pi^p}{M^2}\right) e^{-s_\pi^p/M^2} \right] + \frac{2n-1}{2} \frac{(m_u + m_d)\langle \bar{q}q \rangle}{M^4} + \frac{2n+3}{24} \frac{\langle \frac{\alpha_s}{\pi} G^2 \rangle}{M^4} + \frac{16\pi}{81}[21 + 8n(n+1)] \frac{\langle \sqrt{\alpha_s} \bar{q}q \rangle^2}{M^6} \right\} \tag{5}$$

and

$$\langle \xi_\sigma^{2n} \rangle = \frac{M^4 e^{m_\pi^2/M^2}}{f_\pi^2 \mu_\pi^2} \frac{3}{2n+1} \left\{ \frac{3}{8\pi^2} \frac{1}{2n+3} \times \left[1 - \left(1 + \frac{s_\pi^\sigma}{M^2}\right) e^{-s_\pi^\sigma/M^2} \right] + \frac{2n+1}{2} \frac{(m_u + m_d)\langle \bar{q}q \rangle}{M^4} + \frac{2n+1}{24} \frac{\langle \frac{\alpha_s}{\pi} G^2 \rangle}{M^4} + \frac{16\pi}{81}(8n^2 - 2) \frac{\langle \sqrt{\alpha_s} \bar{q}q \rangle^2}{M^6} \right\}, \tag{6}$$

where M is the Borel parameter, s_π^p and s_π^σ are continuum threshold, which are usually taken to be around the mass square of the first exciting state $\pi'(1300)$ of pion. For clarity, we will take $s_\pi^{p,\sigma} = 1.59$, 1.69 and $1.79\ \mathrm{GeV}^2$ to do our discussion. For the nonperturbative vacuum condensate, we take [24] $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle \simeq -(240\ \mathrm{MeV})^3$ and $\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.012\ \mathrm{GeV}^4$, which are at the renormalization scale 1 GeV and shall be evaluated up to the concerned scale by using the leading order anomalous dimensions of the condensates. The pion decay constant, the pion mass and the α_s are taken as the center values of Refs. [25,26], i.e., $f_\pi = 0.1304$ GeV and $m_\pi = 0.1396$ GeV and $\alpha_s(M_Z) = 0.1184$. The renormalization scale for the present case is taken as $\mu = M$.

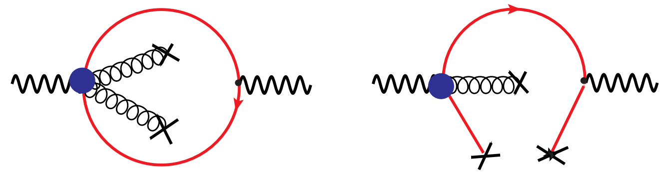

FIG. 1 (color online). Feynman diagrams that are missed in Ref. [15], where the gluon and the quark condensates are depicted as crosses, and the big dot stands for the currents $[\bar{d}(x)\gamma_5(iz \cdot \overleftrightarrow{D})^{2n} u(x)]$ and $[\bar{d}(x)\sigma_{\mu\nu}\gamma_5(iz \cdot \overleftrightarrow{D})^{2n+1} u(x)]$ for ϕ_p^π and ϕ_σ^π, respectively.

As a cross check, we make a comparison with the sum rules derived in Refs. [14,15]. First, our sum rules agree with the corresponding sum rules derived by Ref. [15], except for the two coefficients of the dimension-four $\langle \frac{\alpha_s}{\pi} G^2 \rangle$ and dimension six matrix element $\langle \sqrt{\alpha_s} \bar{q}q \rangle^2$. It is found that such a difference is caused by the two Feynman diagrams shown by FIG. 1, which are missed by the authors of Ref. [15]. According to the background field approach [20–23], the Feynman rules are derived under the Schwinger Gauge or "the fixed-point gauge" [27]. And because of the gluon's or quark's equation of motion, only those interactions that have at most one background field coupling can have contribution to the interaction vertex. As for the two Feynman diagrams shown by FIG. 1, the two background fields are attached to a big dot, which stands for the two complex currents $[\bar{d}(x)\gamma_5(iz \cdot \overleftrightarrow{D})^{2n} u(x)]$ and $[\bar{d}(x)\sigma_{\mu\nu}\gamma_5(iz \cdot \overleftrightarrow{D})^{2n+1} u(x)]$ for ϕ_p^π and ϕ_σ^π, respectively. Since the two background fields can attach to different places of the two complex currents, these two Feynman should be taken into consideration. Second, Ref. [14] only presents the sum rules for ϕ_p. And, one may find a typo error in Ref. [14], i.e., the term before the dimension-three quark condensate $<\bar{q}q>$ and dimension six quark condensate $<\bar{q}q>^2$ should be "+" other than "−".

III. NUMERICAL RESULTS FOR THE MOMENTS OF ϕ_p^π AND ϕ_σ^π

Basing on the QCD sum rules (5) and (6), we discuss the Gegenbauler moments of ϕ_p^π and ϕ_σ^π. For the purpose, we adopt the usual criteria to fix the Borel window for the first two moments $\langle \xi_{p,\sigma}^{2,4} \rangle$, i.e., the continuum contribution is less than 30% of the total dispersion integration and the dimension six condensate contribution is no more than 30%. It is found that the continued contribution increases with the *increment* of M^2 and the contribution from the

TABLE I. Uncertainties of the second moment $\langle\xi_p^2\rangle_{\mu_\pi}$, where μ_π(1 GeV) = 1.4, 1.6, 1.8 GeV, respectively.

s_π^p (GeV^2)	1.59	1.69	1.79
M^2 (GeV^2)	[0.696, 0.874]	[0.696, 0.905]	[0.696, 0.937]
$\langle\xi_p^2\rangle_{(1.4)}$	0.315 ± 0.002	0.324 ± 0.005	0.334 ± 0.009
$\langle\xi_p^2\rangle_{(1.6)}$	0.242 ± 0.002	0.248 ± 0.004	0.255 ± 0.007
$\langle\xi_p^2\rangle_{(1.8)}$	0.191 ± 0.001	0.196 ± 0.003	0.202 ± 0.005

TABLE III. Uncertainties of the second moment $\langle\xi_\sigma^2\rangle_{\mu_\pi}$, where μ_π(1 GeV) = 1.4, 1.6, 1.8 GeV, respectively.

s_π^σ (GeV^2)	1.59	1.69	1.79
M^2 (GeV^2)	[0.439, 0.798]	[0.439, 0.832]	[0.439, 0.867]
$\langle\xi_\sigma^2\rangle_{(1.4)}$	0.129 ± 0.013	0.134 ± 0.017	0.139 ± 0.021
$\langle\xi_\sigma^2\rangle_{(1.6)}$	0.099 ± 0.010	0.102 ± 0.013	0.106 ± 0.016
$\langle\xi_\sigma^2\rangle_{(1.8)}$	0.078 ± 0.008	0.081 ± 0.010	0.084 ± 0.013

dimension-six condensate term increases with the *decrement* of M^2, then possible Borel windows can be obtained as required.

All the obtained Borel windows and the corresponding second and fourth moments of ϕ_p^π and ϕ_σ^π are collected in Tables I, II, III, and IV), respectively. Tables I, II, III, and IV) indicate that

(i) The second and fourth moments of ϕ_p^π are slightly affected by the Borel parameter M^2. This shows that the present sum rules (5) is reasonable. More explicitly, it can be found that the second and forth moments of ϕ_p^π, the uncertainty caused by M^2 is less than 2%. For sum rules (6), it is noted that for the second moment of ϕ_σ^π, the uncertainty caused by M^2 reaches up to a somewhat larger value $\sim$13%. While for higher moments, it is found that the uncertainty caused by M^2 is back to being less than 2%. If we require the flatness of the moments versus M^2 as an extra condition to constrain the Borel window then the large uncertainty for the second moments can be reduced. Here, to make a consistent analysis over all the moments of ϕ_p^π and ϕ_σ^π, we do not imply any further constraint for ϕ_σ^π. To show these points more clearly, we draw the curves of the second and forth moments of $\phi_{p,\sigma}^\pi$ versus M^2 in Figs. 2 and 3), which shows that the moments (except for the second moments of ϕ_σ^π) are almost flat in the allowable Borel window.

(ii) The Borel windows will be broadened with the increment of the continued threshold $s_\pi^{p,\sigma}$. And accordingly, the second and the fourth moments of $\phi_{p,\sigma}^\pi$ shall increase with increment of $s_\pi^{p,\sigma}$. When increasing $s_\pi^{p,\sigma}$ by 0.10 GeV, the second and fourth moments of $\phi_{p,\sigma}^\pi$ shall be increased by 3%–5%.

(iii) The twist-3 DA moments are greatly affected by the value of μ_π, which provides the dominant uncertainty for the present sum rules. So, a better determination of μ_π shall greatly improve our knowledge on twist-3 DA. All the twist-3 DA moments are decreased with the increment of μ_π. When increasing μ_π by 0.2 GeV, the second and fourth moments of $\phi_{p,\sigma}^\pi$ shall be decreased by 20%–25%.

(iv) By taking all uncertainty sources into consideration, we obtain

$$\langle\xi_p^2\rangle = 0.248 \pm 0.004|_{M^2} \begin{matrix}+0.007\\-0.006\end{matrix}\Big|_{s_\pi^p} \begin{matrix}+0.076\\-0.052\end{matrix}\Big|_{\mu_\pi},$$
$$\langle\xi_p^4\rangle = 0.262 \pm 0.003|_{M^2} \begin{matrix}+0.005\\-0.005\end{matrix}\Big|_{s_\pi^p} \begin{matrix}+0.080\\-0.055\end{matrix}\Big|_{\mu_\pi},$$
$$\langle\xi_\sigma^2\rangle = 0.102 \pm 0.013|_{M^2} \begin{matrix}+0.004\\-0.003\end{matrix}\Big|_{s_\pi^\sigma} \begin{matrix}+0.032\\-0.021\end{matrix}\Big|_{\mu_\pi} \tag{7}$$

and

$$\langle\xi_\sigma^4\rangle = 0.094 \pm 0.001|_{M^2} \begin{matrix}+0.002\\-0.002\end{matrix}|_{s_\pi^\sigma} \begin{matrix}+0.028\\-0.020\end{matrix}|_{\mu_\pi}, \tag{8}$$

where the center value is obtained by taking all the input parameters to be their center values, i.e., $s_\pi^{p,\sigma} = 1.69$ GeV^2 and $\mu_\pi(1 \text{ GeV}) = 1.6$ GeV^2. And the uncertainty of a particular parameter is obtained by fixing other parameters to be their center accordingly.

(v) As a summary, by adding the uncertainties in quadrature, we obtain $\langle\xi_p^2\rangle = 0.248^{+0.076}_{-0.052}$, $\langle\xi_p^4\rangle = 0.262^{+0.080}_{-0.055}$, $\langle\xi_\sigma^2\rangle = 0.102^{+0.035}_{-0.025}$ and $\langle\xi_\sigma^4\rangle = 0.094^{+0.028}_{-0.020}$.

TABLE II. Uncertainties of the fourth moment $\langle\xi_p^4\rangle_{\mu_\pi}$, where μ_π(1 GeV) = 1.4, 1.6, 1.8 GeV, respectively.

s_π^p (GeV^2)	1.59	1.69	1.79
M^2 (GeV^2)	[0.958, 1.052]	[0.958, 1.080]	[0.958, 1.108]
$\langle\xi_p^4\rangle_{(1.4)}$	0.335 ± 0.004	0.342 ± 0.004	0.349 ± 0.004
$\langle\xi_p^4\rangle_{(1.6)}$	0.257 ± 0.003	0.262 ± 0.003	0.267 ± 0.003
$\langle\xi_p^4\rangle_{(1.8)}$	0.203 ± 0.002	0.207 ± 0.002	0.211 ± 0.002

TABLE IV. Uncertainties of the fourth moment $\langle\xi_\sigma^4\rangle_{\mu_\pi}$, where μ_π(1 GeV) = 1.4, 1.6, 1.8 GeV, respectively.

s_π^σ (GeV^2)	1.59	1.69	1.79
M^2 (GeV^2)	[0.807, 0.988]	[0.807, 1.017]	[0.807, 1.047]
$\langle\xi_\sigma^4\rangle_{(1.4)}$	0.120 ± 0.001	0.122 ± 0.001	0.125 ± 0.001
$\langle\xi_\sigma^4\rangle_{(1.6)}$	0.092 ± 0.001	0.094 ± 0.001	0.096 ± 0.001
$\langle\xi_\sigma^4\rangle_{(1.8)}$	0.072 ± 0.001	0.074 ± 0.001	0.076 ± 0.001

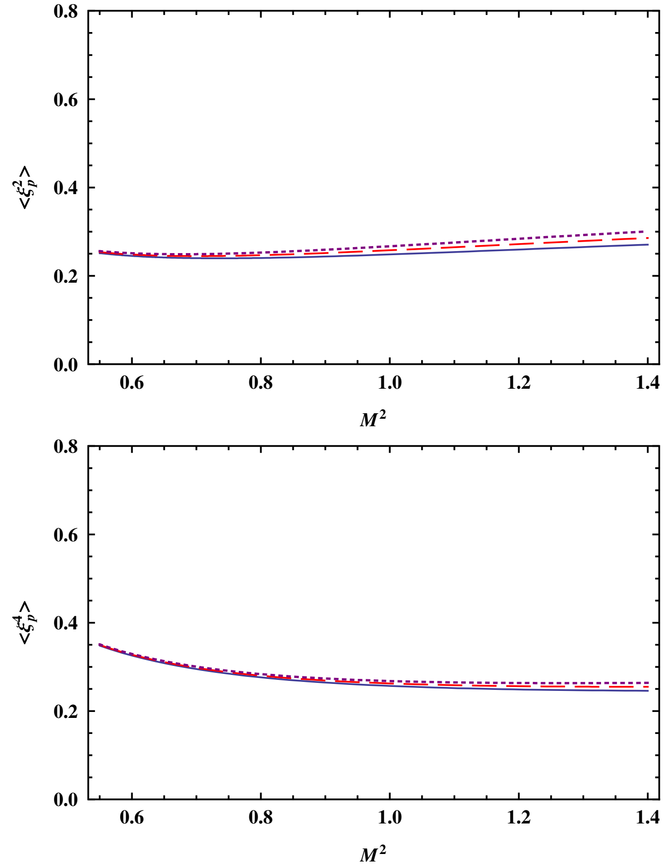

FIG. 2 (color online). The second and forth moments of twist-3 DA ϕ_p^π versus the Borel parameter M^2 corresponding to $\mu_\pi = 1.6$ GeV. The solid line, dashed line and dotted line correspond to $s_\pi^p = 1.59$, 1.69, 1.79 GeV^2, respectively.

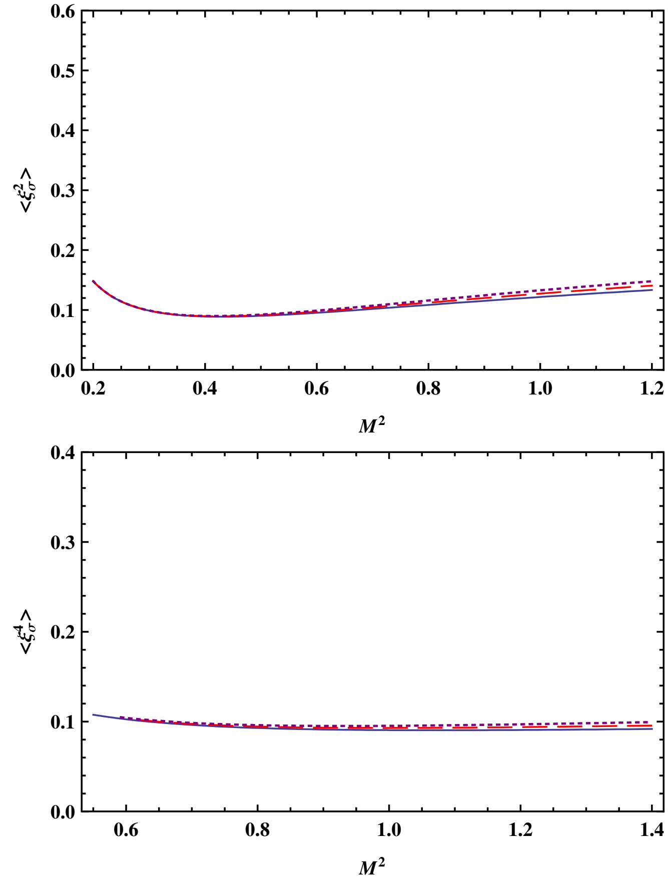

FIG. 3 (color online). The second and forth moments of twist-3 DAs ϕ_σ^π versus the Borel parameter M^2 corresponding to $\mu_\pi = 1.6$ GeV. The solid line, dashed line and dotted line correspond to $s_\pi^\sigma = 1.59$, 1.69, 1.79 GeV^2, respectively.

IV. MODELS FOR ψ_p^π AND ψ_σ^π AND THEIR APPLICATION FOR $B \to \pi$ FORM FACTOR

A. Models for ψ_p^π and ψ_σ^π

The wave function is the key component for the k_T factorization approach [13], which is helpful to suppress the end point singularity in certain processes. And, it would be useful to know the properties of the twist-3 wave functions ψ_p^π and ψ_σ^π.

The wave function is purely nonperturbative, so it can hardly be determined from the first principle of QCD. Generally, the pion wave function and its corresponding DA is related through the relation, $\phi_{p,\sigma}^\pi(x, \mu_f) = \int_{|k_\perp|<\mu_f} \frac{d^2\mathbf{k}_\perp}{16\pi^3} \psi_{p,\sigma}^\pi(x, \mathbf{k}_\perp)$, where μ_f is the factorization scale that is around $\mathcal{O}(1\text{ GeV})$. So, on the other hand, if we have known the DA moments well, then we can inversely obtain some valuable properties of the wave function .

Our model for the pionic twist-3 wave functions is based on the assumptions that I) its longitudinal distributions is determined by its DA moments as derived by the sum rules (5) and (6) and its longitudinal behavior is dominated by the first two Gegenbauler moments; II) its transverse momentum dependence is determined by the so-called BHL-prescription [28,29], which is obtained by using the connection between the equal-time wave function $\psi_{\rm c.m.}(\mathbf{q}_\perp)$ in the rest frame and the light cone wave function $\psi_{\rm LC}(x, \mathbf{k}_\perp)$ in the infinite momentum frame, i.e., $\psi_{\rm c.m.}(\mathbf{q}_\perp) \leftrightarrow \psi_{\rm LC}(\frac{\mathbf{k}_\perp^2+m^2}{4x(1-x)} - m^2)$, where m stands for the light constitute quark mass. It is found that the transverse momentum dependence is just in the exponential form of the off shell energy of the constitute quarks, which agrees with Brodsky and Teramond's holographic model that is obtained by using the anti-de Sitter/conformal field theory correspondence [30,31]. And, the pionic twist-3 wave functions $\psi_p^\pi(x, \mathbf{k}_\perp)$ and $\psi_\sigma^\pi(x, \mathbf{k}_\perp)$ take the following form:

$$\psi_p^\pi(x, \mathbf{k}_\perp) = [1 + B_p C_2^{1/2}(2x-1) + C_p C_4^{1/2}(2x-1)] \times \frac{A_p}{x(1-x)} \exp\left[-\frac{m^2+\mathbf{k}_\perp^2}{8\beta_p^2 x(1-x)}\right] \tag{9}$$

and

TABLE V. Wave function parameters for $\psi_p^\pi(x, \mathbf{k}_\perp)$ with varying DA moments.

$\langle\xi_p^2\rangle$		0.196			0.248			0.324	
$\langle\xi_p^4\rangle$	0.207	0.262	0.342	0.207	0.262	0.342	0.207	0.262	0.342
A_p (GeV^{-2})	93.4987	92.4429	90.9983	89.7356	88.7233	87.2936	84.7201	83.7330	82.3809
B_p	1.2835	1.2926	1.3056	1.3177	1.3261	1.3385	1.3679	1.3756	1.3868
C_p	1.3579	1.4165	1.5005	1.3635	1.4210	1.5045	1.3710	1.4277	1.5097
β_p (GeV)	0.5849	0.5882	0.5930	0.5950	0.5984	0.6033	0.6095	0.6131	0.6181

$$\psi_\sigma^\pi(x, \mathbf{k}_\perp) = [1 + B_\sigma C_2^{3/2}(2x-1) + C_\sigma C_4^{3/2}(2x-1)] \times \frac{A_\sigma}{x(1-x)} \exp\left[-\left(\frac{m^2 + \mathbf{k}_\perp^2}{8\beta_\sigma^2 x(1-x)}\right)\right]. \tag{10}$$

The parameters $A_{p,\sigma}$, $B_{p,\sigma}$, $C_{p,\sigma}$ and $\beta_{p,\sigma}$ can be determined by the average value of the transverse momentum $\langle\mathbf{k}_\perp^2\rangle_{p,\sigma}$, the wave function normalization $\int_0^1 dx \int_{|\mathbf{k}_\perp|<\mu_f} \times \frac{d^2\mathbf{k}_\perp}{16\pi^3} \psi_{p,\sigma}^\pi(x, \mathbf{k}_\perp) = 1$, and the first two DA moments determined by the last section. The average value of the transverse momentum square is defined as

$$\langle\mathbf{k}_\perp^2\rangle_{p,\sigma} = \frac{\int dx d^2\mathbf{k}_\perp |\mathbf{k}_\perp^2| |\psi_{p,\sigma}^\pi(x, \mathbf{k}_\perp)|^2}{\int dx d^2\mathbf{k}_\perp |\psi_{p,\sigma}^\pi(x, \mathbf{k}_\perp)|^2} \tag{11}$$

and $\langle\mathbf{k}_\perp^2\rangle_{p,\sigma}^{1/2} = 0.350$ GeV [32]. Using the relation between the DA and the wave function, we obtain

$$\phi_p^\pi(x, \mu_f) = [1 + B_p C_2^{1/2}(2x-1) + C_p C_4^{1/2}(2x-1)] \times \frac{A_p \beta_p^2}{2\pi^2} \exp\left[-\frac{m^2}{8\beta_p^2 x(1-x)}\right] \times \left\{1 - \exp\left[-\frac{\mu_f^2}{8\beta_p^2 x(1-x)}\right]\right\} \tag{12}$$

and

$$\phi_\sigma^\pi(x, \mu_f) = [1 + B_\sigma C_2^{3/2}(2x-1) + C_\sigma C_4^{3/2}(2x-1)] \times \frac{A_\sigma \beta_\sigma^2}{2\pi^2} \exp\left[-\frac{m^2}{8\beta_\sigma^2 x(1-x)}\right] \times \left\{1 - \exp\left[-\frac{\mu_f^2}{8\beta_\sigma^2 x(1-x)}\right]\right\}. \tag{13}$$

The Gegenbauer moments $\langle\xi_{p,\sigma}^{2n}\rangle$ for the DAs defined by Eqs. (12) and (13) at the scale μ_f can be defined as

$$\langle\xi_p^{2n}\rangle|_{\mu_f} = \frac{\int_0^1 dx \phi_p^\pi(x, \mu_f) C_{2n}^{1/2}(2x-1)}{\int_0^1 dx [C_{2n}^{1/2}(2x-1)]^2} \tag{14}$$

and

$$\langle\xi_\sigma^{2n}\rangle|_{\mu_f} = \frac{\int_0^1 dx \phi_\sigma^\pi(x, \mu_f) C_{2n}^{3/2}(2x-1)}{\int_0^1 dx 6x(1-x)[C_{2n}^{3/2}(2x-1)]^2}. \tag{15}$$

With the help of the above formulas, we can determine the wave function parameters for varying DA moments, which are presented in Table V and VI.

We present ϕ_p^π and ϕ_σ^π in Figs. 4 and 5. Our DAs, i.e., Eqs. (12) and (13), are drawn by solid lines and are derived by setting the second and fourth moments to be their center values shown in Tables I, II, III, and IV). As a comparison, we also present the DAs of Ref. [6,7,19] in Figs. 4 and 5), which are shown by the dashed lines and the dotted lines, respectively. As for ϕ_p^π, in difference to its asymptotic behavior, it will provide great suppression in the end point region. Such a behavior will be helpful to derive a reasonable power behavior for twist-3 DA [17]. It is noted that to study the right behavior of the DA itself, the full form of the twist-3 DAs (12) and (13) is more useful and more accurate than its truncated Gegenbauer form.

B. Reanalysis of the $B \to \pi$ transition form factor

The $B \to \pi$ transition form factor provides a good platform to study the pion properties. Especially within the QCD light cone sum rules [33], one can concentrate on different twist structures of pion DA by properly choosing the correlator. By choosing the corrector with the proper current, because of the cancellation among the different γ-structures, different twist DAs will remain in the final formulas.

Three typical correlators are suggested in the literature to calculate $f_{B\pi}^+(q^2)$. The first one is $\Pi_\mu^+(p,q) = i\int d^4x e^{iq\cdot x}\langle\pi(p)|T\{\bar{q}(x)\gamma_\mu b(x), \bar{b}(0) i m_b \gamma_5 q(0)\}|0\rangle$, which

TABLE VI. Wave function parameters for $\psi_\sigma^\pi(x, \mathbf{k}_\perp)$ with varying DA moments.

$\langle\xi_\sigma^2\rangle$		0.077			0.102			0.127	
$\langle\xi_\sigma^4\rangle$	0.074	0.094	0.122	0.074	0.094	0.122	0.074	0.094	0.122
A_σ (GeV^{-2})	196.486	194.804	192.128	187.336	185.532	182.777	178.403	176.532	173.584
B_σ	−0.0284	−0.0309	−0.0349	−0.0115	−0.0144	−0.0191	0.0053	0.0019	−0.0032
C_σ	0.0922	0.1108	0.1364	0.0885	0.1069	0.1322	0.0845	0.1026	0.1276
β_σ (GeV)	0.4107	0.4123	0.4147	0.4169	0.4187	0.4213	0.4234	0.4253	0.4283

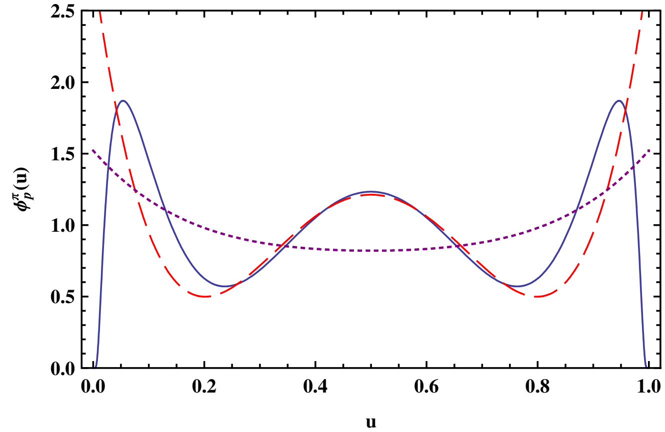

FIG. 4 (color online). Pion twist-3 DA ϕ_p^π. The solid line, the dashed line and the dotted line are for our DA defined by Eq. (12), DA of Ref. [19] and DA of Refs. [6,7], respectively.

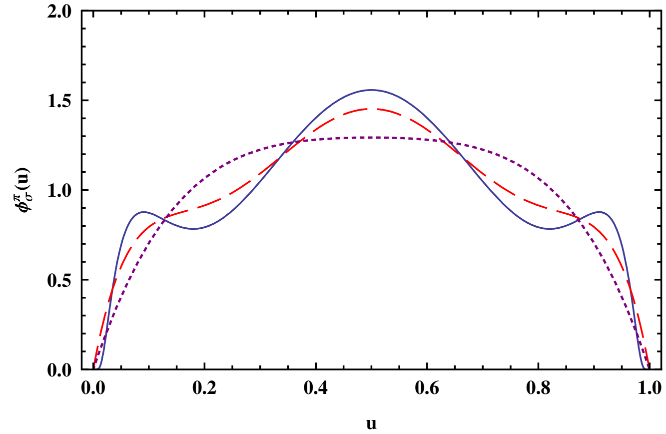

FIG. 5 (color online). Pion twist-3 DA ϕ_σ^π. The solid line, the dashed line and the dotted line are for our DA defined by Eq. (13), DA of Ref. [19] and DA of Refs. [6,7], respectively.

is usually adopted in the literature. After doing the simplification, it can be found that all the twist-2, twist-3 and twist-4 terms remain, all of which can provide sizable contributions [34–36]. The second one is $\Pi_\mu^+(p,q) = i\int d^4x e^{iq\cdot x}\langle\pi(p)|T\{\bar{q}(x)\gamma_\mu(1+\gamma_5)b(x), \bar{b}(0)im_b(1+\gamma_5)\times q(0)\}|0\rangle$, which is constructed by using the chiral current. After doing the simplification, it can be found that the twist-3 terms can rightly be canceled, so one only needs to consider the twist-2 and twist-4 DAs [37,38]. The third one is $\Pi_\mu^+(p,q) = i\int d^4x e^{iq\cdot x}\langle\pi(p)|T\{\bar{q}(x)\gamma_\mu(1+\gamma_5)b(x), \bar{b}(0)im_b(1-\gamma_5)q(0)\}|0\rangle$. After doing the simplification, it can be found that the terms involving the twist-2 DA are exactly be canceled, so one only needs to consider the dominant twist-3 DAs [39].

In the present paper, our purpose is to test the properties of the obtained twist-3 DAs in the last section, so we adopt the third correlator to do our analysis. Following the same procedure of Refs. [36,39], we can obtain the light cone sum rules for the $f_{B\pi}^+(q^2)$ up to NLO:

$$f_{B\pi}^+(q^2) = \frac{f_\pi \mu_\pi}{m_B^2 f_B}\exp\left[\frac{m_B^2}{M^2}\right]\left[F_0(q^2, M^2, s_0) + \frac{\alpha_s C_F}{4\pi}F_1(q^2, M^2, s_0)\right], \tag{16}$$

where the leading order $F_0(q^2, M^2, s_0)$ takes the form

$$F_0(q^2, M^2, s_0) = m_b\left\{\int_\Delta^1 \frac{du}{u}\exp\left[-\frac{m_b^2 - q^2(1-u)}{uM^2}\right]\left[u\phi_p^\pi(u) + \frac{1}{6}\left(2 + \frac{m_b^2+q^2}{uM^2}\right)\phi_\sigma^\pi(u)\right] - \frac{2f_{3\pi}}{f_\pi\mu_\pi}\right.$$
$$\left.\times \int_0^1 v dv \int D\alpha_i \frac{\theta(\alpha_1 + v\alpha_3 - \Delta)}{(\alpha_1 + v\alpha_3)^2}\exp\left[-\frac{m_b^2 - q^2(1-\alpha_1 - v\alpha_3)}{(\alpha_1 + v\alpha_3)M^2}\right]\left[1 - \frac{m_b^2 - q^2}{(\alpha_1+v\alpha_3)M^2}\right]\phi_{3\pi}(\alpha_i)\right\} \tag{17}$$

and the NLO $F_1(q^2, M^2, s_0)$ takes the form

$$F_1(q^2, M^2, s_0) = \frac{1}{\pi m_b}\int_{m_b^2}^{s_0} ds e^{-s/M^2} \times \int_0^1 du[\mathrm{Im}_s T_1^p(q^2, s, u)\phi_p^\pi(u) + \mathrm{Im}_s T_1^\sigma(q^2, s, u)\phi_\sigma^\pi(u)], \tag{18}$$

where $D\alpha_i = d\alpha_1 d\alpha_2 d\alpha_3 \delta(1-\alpha_1-\alpha_2-\alpha_3)$, m_b stands for the running b-quark mass under the $\overline{MS}$ scheme, $\triangle = \frac{m_b^2-q^2}{s_0-q^2}$, $\phi_{3\pi}(\alpha_i)$ is the pionic three-particle twist-3 DA with the normalization parameter $f_{3\pi}$. The leading order result agrees well with that of Ref. [39]. And it has been found that our present NLO result is different to that of Ref. [36] only by an overall factor "2", which is caused by the use of different correlators. To shorten the paper, we do not present them here (interested readers may turn to Ref. [36] for detailed formulas). Moreover, the three-particle twist-3 DA can be derived from the exact relation between the two-particle and three-particle twist-3 DAs, which can be derived by the equations of motion [19]. It is noted that the contributions from $\phi_{3\pi}(\alpha_i)$ to the present sum rules (16) is quite small, which is less than 0.5%, so we simply take its form as suggested in Ref. [19] to do our calculation.

We present our results for $f_{B\pi}^+$ in Fig. 6, where the light cone sum rule with certain extrapolation [38], and the quenched lattice QCD result [40] and the unquenched

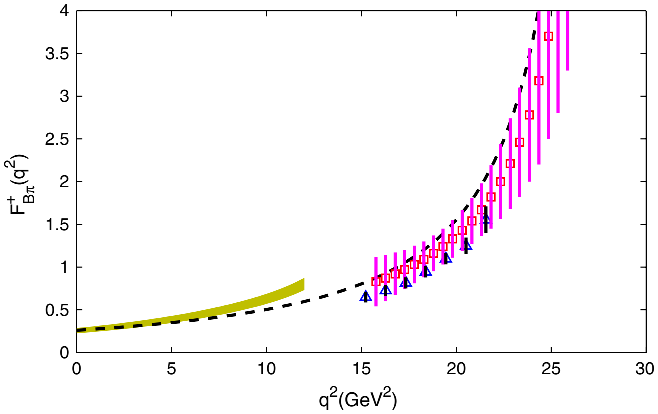

FIG. 6 (color online). $f^+_{B\pi}(q^2)$. The shaded band is our result with uncertainties caused by the input parameters, and as a comparison, the extrapolated light cone sum rules of Ref. [38] is shown by a dashed line. The quenched [40] and unquenched [41] lattice QCD results are presented by rectangles and triangles, respectively.

lattice QCD result [41] are also presented for comparison. The light cone sum rule for $B \to \pi$ is valid up to momentum transfer $q^2 \sim m_b^2 - 2m_b\bar{\Lambda}$, typically at $0 < q^2 < 14\ \mathrm{GeV}^2$, and to be on safe side, we take the maximal allowed q^2 to be 12 GeV^2. Here, in doing the numerical calculation, the center value of $f^+_{B\pi}$ is obtained by taking $f_B = 169$ MeV, $s_0 = 32.6\ \mathrm{GeV}^2$ [38] and $\bar{m}_b(\bar{m}_b) = 4.20$ GeV, $m_B = 5.279$ GeV [25]. The effective threshold for different correlators should be different from each other so as to derive a reliable estimation [42]. As for the present adopted correlator with a chiral current, the value of s_0 should be slightly lower than that of the conventional corrector (i.e., the first correlator) so as to include the unwanted scalar state with $J^P = 0^+$ into the continuum contribution [43].

It is found that our present result agrees with those of Refs. [6,7,19] under the same parameter values. The shaded band is obtained by varying the pionic parameters within their reasonable regions. As for the Borel parameter, its reasonable region is $[14, 20]$ GeV^2 and we fix its value to be $M^2 = 16\ \mathrm{GeV}^2$ to do our calculation. The typical energy scale for $B \to \pi$ is $\mu_b = (m_B^2 - m_b^2)^{1/2} \simeq 2.2$ GeV, and by running μ_π from 1 GeV to μ_b, we obtain $\mu_\pi(\mu_b) = 1.93 \pm 0.24$ GeV. At $q^2 = 0$, we obtain $f^+_{B\pi}(0) = 0.253 \pm 0.031$. It is found that the main uncertainty is caused by μ_π, and the twist-3 DAs models (12) and (13) lead to small uncertainties. As for the NLO corrections, it will provide sizable contributions and its value will increase with the increment of q^2, e.g., for $q^2 = 0$ its contribution is $\sim$6% and for $q^2 = 10\ \mathrm{GeV}^2$ its contribution changes to $\sim$16%, so it should be included to provide a sound estimation. As a cross check, it is found that our present result of $f^+_{B\pi}(0)$ is consistent with those of Refs. [35–38] within reasonable parameter regions.

V. SUMMARY

We have studied the twist-3 DAs ϕ_p^π and ϕ_σ^π within the QCD sum rules. And by adding all the uncertainties in quadrature, the uncertainties for the moments are about 20%–35%. Furthermore, these moments can provide helpful constraints on the twist-3 DAs and hence the wave functions. The presently constructed twist-3 wave functions shall be useful for the k_T factorization approach [13], where the transverse momentum dependence both in the wave function and the hard scattering part should be treated on equal footing. A consistency check of our twist-3 DAs is done by studying the $B \to \pi$ transition form factor $f^+_{B\pi}$ with the QCD light cone sum rules up to NLO.

Recently, Ref. [44] points out a non-negligible contribution of higher-twist processes in large p_t hadron production in hadronic collisions, where the hadron is produced directly in the hard subprocesses rather than by gluon or quark jet fragmentation. So a better understanding of higher-twist wave functions or DAs shall be helpful for further studies on higher-twist contributions. Especially, the forthcoming RHIC and LHC measurements will provide further tests of the dynamics of large-p_t hadron production beyond the leading twist.

ACKNOWLEDGMENTS

This work was supported in part by the Fundamental Research Funds for the Central Universities under Grant No. CDJXS10100035, by Natural Science Foundation Project of CQ CSTC under Grant No. 2008BB0298 and by Natural Science Foundation of China under Grant No. 10805082 and No. 11075225.

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