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TeV scale left-right symmetry with spontaneous *D*-parity breaking

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The different scenarios of spontaneous breaking of D parity have been studied in both the nonsupersymmetric and the supersymmetric version of the left-right symmetric models (LRSM). We explore the possibility of a TeV scale $SU(2)_R$ breaking scale M_R and hence TeV scale right-handed neutrinos from both minimization of the scalar potential as well as the coupling constant unification point of view. We show that, although minimization of the scalar potential allows the possibility of a TeV scale M_R and tiny neutrino masses in LRSM with spontaneous D-parity breaking, the gauge coupling unification at a high scale $\sim 10^{16}$ GeV does not favor a TeV scale symmetry breaking except in the supersymmetric left-right model with Higgs doublet and bidoublet. The phenomenology of neutrino mass is also discussed.

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I. INTRODUCTION

The left-right symmetric model (LRSM) is a novel extension of the standard model of particle physics [1–5]. In such models the parity is spontaneously broken and the smallness of neutrino masses [6–9] arises in a natural way via a seesaw mechanism [10–13]. Incorporating supersymmetry (susy) into such models comes with a couple of other advantages in terms of the gauge hierarchy problem, coupling constant unification among many others. Another advantage in such susy models is that they provide a natural candidate for dark matter in terms of the lightest superparticle (LSP). In the minimal supersymmetric standard model (MSSM), this LSP is stable only if we incorporate an extra symmetry called R parity $R_p = (-1)^{3(B-L)+2s}$. However, in supersymmetric left-right (SUSYLR) models [14–17] based on the gauge group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, this R parity is a part of the gauge symmetry and, hence, need not be put by hand. Since $U(1)_{B-L}$ symmetry is broken by a Higgs triplet with even $B - L$ quantum number, R parity is still preserved at low energy.

In the usual LRSM, the scale of parity breaking and $SU(2)_R$ gauge symmetry breaking are identical which is not necessary. There have been lots of studies on left-right symmetric models where the parity symmetry gets broken much before the $SU(2)_R$ gauge symmetry breaks by so-called spontaneous D-parity breaking [18,19]. In this paper we analyze various types of susy and non-susy left-right models with spontaneous D-parity breaking and check analytically whether the minimization of the scalar potential allows a TeV scale $SU(2)_R$ breaking scale (provided parity breaks at a much higher scale) as well as tiny neutrino masses. We then check whether such a choice of intermediate symmetry breaking scales unifies the gauge coupling constants in the SUSYLR framework. We discuss the possible phenomenology of neutrino mass in each cases separately.

Motivation and outlook.—Since many papers exist in the literature studying these aspects of the left-right symmetric models, we summarize here our motivation for this study and how our analysis differs from earlier works. Before the precision measurements of the weak mixing angle and the strong coupling constants, the evolution of the gauge coupling constants could allow low-scale left-right symmetry breaking [20]. This could be achieved with a single stage symmetry breaking. Later it was found that, by invoking more intermediate scales, it is possible to have more freedom to adjust the different symmetry breaking scales. However, after the precision electroweak measurements at LEP, it was found that the simplest left-right symmetric models would not allow a left-right symmetry breaking below 10^{12} GeV, in both single stage symmetry breaking as well as multistage symmetry breaking [21–23]. $SO(10)$ based models also got constrained with the allowed intermediate scale in the range of 10^9–10^{10} GeV [24,25]. Introducing the Pati-Salam symmetry breaking scale would not allow lowering the left-right symmetry breaking scale both in the supersymmetric as well as the nonsupersymmetric models. It would be possible to break the $SU(2)_R$ to $U(1)_R$ at a higher scale and then break the group $U(1)_R$ at a lower scale, but the breaking scale of $SU(2)_R$ could not be lowered, keeping the theory consistent with the potential minimization and gauge coupling evolution.

In a recent paper, it has been demonstrated that, by introducing additional scalars it is possible to lower the scale of left-right symmetry breaking, i.e., break the symmetry group $SU(2)_R$ [26]. In this paper we studied the different symmetry breaking patterns to check the consistency with the potential minimization and gauge coupling evolution and see which of these models could allow TeV scale left-right symmetry breaking. We restricted our analysis to only a single stage symmetry breaking, because

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by introducing the additional symmetry breaking scales it was not found to help lowering the left-right symmetry breaking scales. Of course, our analysis does not rule out other possibilities of lowering the left-right breaking scale by introducing newer symmetry breaking scales and new physics. However, this analysis demonstrates, within the simplest framework of single stage symmetry breaking, which models are consistent with potential minimization, gauge coupling unification, and allow a TeV scale left-right symmetry breaking,

This paper is organized as follows. In the Sec. II, we will study the potential minimization of the non-susy and the susy version of various left-right symmetric models and check the minimization of the scalar potential. Then in Sec. III we study the gauge coupling unification in all the SUSYLR models we have considered and discuss the neutrino mass in Secs. IV and V. We discuss the results and conclusion in Sec. VI and finally conclude in Sec. VII.

II. LR MODELS WITH SPONTANEOUS *D*-PARITY BREAKING

In left-right symmetric models with spontaneous *D*-parity breaking, the discrete parity symmetry gets broken (by the vev of a parity odd singlet scalar field) much before the $SU(2)_R$ gauge symmetry breaks. The gauge group is effectively $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P$, where P is the discrete left-right symmetry which we call D parity. This D-parity symmetry is different from the Lorentz parity in the sense that Lorentz parity interchanges left-handed fermions with the right-handed ones but the bosonic fields remain the same. Whereas, the D parity also interchanges the $SU(2)_L$ Higgs fields with the $SU(2)_R$ Higgs fields. The parity odd singlet field breaks this gauge symmetry at high scale $\sim(10^{16}$–$10^{19})$ GeV to $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ which further breaks down to the standard model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ at a lower scale. The D-parity breaking introduces an asymmetry between left and right-handed Higgs fields and makes the coupling constants of $SU(2)_R$ and $SU(2)_L$ evolve separately under the renormalization group. It should be noted that this D-parity breaking is different from the low energy parity breaking observed in the weak interactions which arises as a result of $SU(2)_R$ gauge symmetry breaking at a scale higher than the electroweak scale. In such a D-parity breaking scenario, the seesaw relation also gets modified from usual LRSM. Although the type I seesaw term still remains sensitive to the $SU(2)_R$ breaking scale M_R, the other seesaw terms, namely, type II and type III [27], become sensitive to the D-parity breaking scale. A very high value of parity breaking scale therefore leads to type I seesaw dominance. In this section we are going to discuss various such models with different particle contents.

A. LRSM with Higgs doublets

We first study the non-susy left-right symmetric extension of the standard model with only Higgs doublets. In addition to the usual fermions of the standard model, we require the right-handed neutrinos to complete the representations. One of the important features of the model is that it allows spontaneous parity violation. The Higgs representations then requires a bidoublet field, which breaks the electroweak symmetry and gives masses to the fermions. But the neutrinos can have a Dirac mass only, which is then expected to be of the order of other fermion masses. To implement the seesaw mechanism and obtain the observed tiny mass of the left-handed neutrinos naturally, one also introduces a singlet fermion plus fermion triplet. However, we shall restrict ourselves to the scalar sector and shall not discuss the implications of the singlet neutrinos and the neutrino masses.

The particle content of the left-right symmetric model with Higgs doublet is

$$\begin{aligned} &\text{fermions}: Q_L \equiv (3,2,1,1/3), \quad Q_R \equiv (3,1,2,1/3),\\ &\Psi_L \equiv (1,2,1,-1), \quad \Psi_R \equiv (1,1,2,-1)\\ &\text{scalars: } \Phi \equiv (1,2,2,0), \quad H_L \equiv (1,2,1,1),\\ &H_R \equiv (1,1,2,1) \quad \rho \equiv (1,1,1,0), \end{aligned}$$

where the numbers in the brackets are the quantum numbers corresponding to the gauge group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. In addition to the bidoublet scalar field Φ, we also introduced two doublet fields H_L and H_R to break the left-right symmetry and contribute to the neutrino masses. The scalar singlet ρ is a D-parity odd field and changes sign under the exchange of $SU(2)_L$ with $SU(2)_R$. Thus, the symmetry breaking pattern becomes

$$\begin{aligned} &SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P \underset{\rightarrow}{\langle\rho\rangle} SU(2)_L \times SU(2)_R\\ &\times U(1)_{B-L} \underset{\rightarrow}{\langle H_R\rangle} SU(2)_L \times U(1)_Y \underset{\rightarrow}{\langle\Phi\rangle} U(1)_{\text{em}}. \end{aligned}$$

We denoted the vacuum expectation values of the neutral components of the Higgs fields as

$$\begin{aligned} \langle\Phi_1\rangle &= v_1, v_2, \qquad \langle H_L\rangle = v_L,\\ \langle H_R\rangle &= v_R, \qquad \langle\rho\rangle = s. \end{aligned}$$

The scalar potential with all these fields can then be written as

$$\begin{aligned}V = {}& \mu_1^2 \mathrm{Tr}[\Phi_1^\dagger \Phi_1] + \mu_2^2(\mathrm{Tr}[\Phi_2\Phi_1^\dagger] + \mathrm{Tr}[\Phi_2^\dagger \Phi_1]) + \lambda_1(\mathrm{Tr}[\Phi_1^\dagger\Phi_1])^2 + \lambda_2[(\mathrm{Tr}[\Phi_2\Phi_1^\dagger])^2 + (\mathrm{Tr}[\Phi_2^\dagger\Phi_1])^2] \\ & + \lambda_3 \mathrm{Tr}[\Phi_2\Phi_1^\dagger]\mathrm{Tr}[\Phi_2^\dagger\Phi_1] + \lambda_4 \mathrm{Tr}[\Phi_1^\dagger\Phi_1](\mathrm{Tr}[\Phi_2\Phi_1^\dagger] + \mathrm{Tr}[\Phi_2^\dagger\Phi_1]) + \mu_h^2(H_L^\dagger H_L + H_R^\dagger H_R) \\ & + \lambda_5[(H_L^\dagger H_L)^2 + (H_R^\dagger H_R)^2] + \lambda_6(H_L^\dagger H_L)(H_R^\dagger H_R) + \alpha_1 \mathrm{Tr}[\Phi_1^\dagger\Phi_1](H_L^\dagger H_L + H_R^\dagger H_R) \\ & + \alpha_2(H_L^\dagger\Phi_1\Phi_1^\dagger H_L + H_R^\dagger\Phi_1^\dagger\Phi_1 H_R) + \alpha_3(H_L^\dagger\Phi_2\Phi_2^\dagger H_L + H_R^\dagger\Phi_2^\dagger\Phi_2 H_R) + \alpha_4(H_L^\dagger\Phi_1\Phi_2^\dagger H_L + H_R^\dagger\Phi_1^\dagger\Phi_2 H_R) \\ & + \alpha_4^*(H_L^\dagger\Phi_2\Phi_1^\dagger H_L + H_R^\dagger\Phi_2^\dagger\Phi_2 H_R) + \mu_{h\phi 1}(H_L^\dagger\Phi_1 H_R + H_R^\dagger\Phi_1^\dagger H_L) + \mu_{h\phi 2}(H_L^\dagger\Phi_2 H_R + H_R^\dagger\Phi_2^\dagger H_L) \\ & - \mu_\rho^2\rho^2 + \lambda_7\rho^4 + M\rho(H_L^\dagger H_L - H_R^\dagger H_R) + \lambda_8\rho^2(H_L^\dagger H_L + H_R^\dagger H_R)\lambda_9\rho^2 \mathrm{Tr}[\Phi_1^\dagger\Phi_1] + \lambda_{10}\rho^2[\mathrm{Det}[\Phi_1] + \mathrm{Det}[\Phi_1^\dagger]],\end{aligned}$$

where $\Phi_2 = \tau_2\Phi_1^*\tau_2$.

To find a consistent solution we now minimize the scalar potential and obtain

$$\frac{\partial V}{\partial v_L} = \mu_L^2 v_L + \lambda_5 v_L^3 + \frac{\lambda_6}{2} v_L v_R^2 + \mu_{h\phi}(v_1 + v_2)v_R = 0 \quad (1)$$

$$\frac{\partial V}{\partial v_R} = \mu_R^2 v_R + \lambda_5 v_R^3 + \frac{\lambda_6}{2} v_R v_L^2 + \mu_{h\phi}(v_1 + v_2)v_L = 0, \quad (2)$$

where μ_L^2 and μ_R^2 are effective mass terms of H_L and H_R given by

$$\begin{aligned}\mu_L^2 &= \mu_h^2 + Ms + \lambda_8 s^2 + (\alpha_4 + \alpha_4^*)v_1v_2 + \alpha_1(v_1^2 + v_2^2) + \alpha_2 v_2^2 + \alpha_3 v_1^2 \\ \mu_R^2 &= \mu_h^2 - Ms + \lambda_8 s^2 + (\alpha_4 + \alpha_4^*)v_1v_2 + \alpha_1(v_1^2 + v_2^2) + \alpha_2 v_2^2 + \alpha_3 v_1^2. \quad (3)\end{aligned}$$

Thus, after the singlet field η gets a vev the left-handed Higgs doublet becomes heavy and decouples whereas the right-handed Higgs can be much lighter by appropriate fine-tuning of the parameters in (3). From Eqs. (1) and (2) we get

$$v_Lv_R(2Ms) + \left(\lambda_5 - \frac{\lambda_6}{2}\right)(v_L^2 - v_R^2)v_Lv_R + \mu_{h\phi}(v_1 + v_2)(v_R^2 - v_L^2) = 0.$$

Thus, a nonzero value of $\langle\rho\rangle = s$ does not allow a solution with $v_L = v_R$. The seesaw relation from the above equation is

$$v_Lv_R = \frac{\mu_{h\phi}(v_1 + v_2)(v_L^2 - v_R^2)}{2Ms + (\lambda_5 - \frac{\lambda_6}{2})(v_L^2 - v_R^2)}.$$

Assuming $v_L \ll v_R \ll s, M$ will give

$$v_L = \frac{-\mu_{h\phi}(v_1 + v_2)v_R}{2Ms}. \quad (4)$$

Thus, we can have small v_L/v_R by appropriately choosing the scales of $M, s, \mu_{h\phi}$ which will account for tiny neutrino masses. In contrast LRSM without D-parity breaking where the right-handed scale v_R has to be very high to account for small v_L/v_R, here we can have v_R of TeV scale also. For example, if we set $\mu_{h\phi} = M = s = 10^8$ GeV, and $v_{1,2} \sim M_Z$ then $\frac{v_L}{v_R}$ comes out to be of the order 10^{-6} which is desired for the type III seesaw to dominate as we will see when we discuss neutrino masses. The gauge coupling unification has been studied extensively in this model, so we shall not repeat them here. In the absence of D-parity breaking, the left-right symmetry breaking scale comes out to be very high, but in D-parity violating models it is possible to lower the scale of left-right symmetry breaking with some amount of fine-tuning of parameters. However, for the supersymmetric models restrictions are more stringent, so we shall study them in detail.

B. LRSM with Higgs triplets

In this section we shall study the left-right symmetric models with different particle contents. The usual fermions, including the right-handed neutrinos, belong to the similar representations as in the previous section. However, the scalar sector now contains triplet Higgs scalars in addition to the bidoublet Higgs scalar to break the left-right symmetry. The triplet Higgs scalars can then give Majorana masses to the neutrinos and allow the seesaw mechanism without the need for any additional singlet fermions. The parity odd singlet scalar was originally introduced in this model, so we shall include them in our discussions.

The particle content of LRSM with Higgs triplets is

$$\begin{aligned}&\text{fermions}: Q_L \equiv (3, 2, 1, 1/3), \quad Q_R \equiv (3, 1, 2, 1/3), \\ &\Psi_L \equiv (1, 2, 1, -1), \quad \Psi_R \equiv (1, 1, 2, -1) \\ &\text{scalars}: \Phi \equiv (1, 2, 2, 0), \quad \triangle_L \equiv (1, 3, 1, 2), \\ &\triangle_R \equiv (1, 1, 3, 2) \quad \rho \equiv (1, 1, 1, 0).\end{aligned}$$

The symmetry breaking pattern in this model remains the same as in the previous model although the structure of neutrino masses changes. In the symmetry breaking pattern, the scalar Δ_c now replaces the role of H_R, but otherwise there is no change. The vacuum expectation values of the neutral components of the Higgs fields are denoted by $\Phi_1, \triangle_L, \triangle_R, \rho$ as

$$\langle\Phi_1\rangle = v_1, v_2, \quad \langle\triangle_L\rangle = v_L, \quad \langle\triangle_R\rangle = v_R, \quad \langle\rho\rangle = s.$$

The scalar potential can then be written as

$$V = \mu_1^2 \mathrm{Tr}[\Phi_1^\dagger \Phi_1] + \mu_2^2(\mathrm{Tr}[\Phi_2\Phi_1^\dagger] + \mathrm{Tr}[\Phi_2^\dagger\Phi_1]) + \lambda_1(\mathrm{Tr}[\Phi_1^\dagger\Phi_1])^2 + \lambda_2[(\mathrm{Tr}[\Phi_2\Phi_1^\dagger])^2 + (\mathrm{Tr}[\Phi_2^\dagger\Phi_1])^2] + \lambda_3 \mathrm{Tr}[\Phi_2\Phi_1^\dagger]\mathrm{Tr}[\Phi_2^\dagger\Phi_1] + \lambda_4 \mathrm{Tr}[\Phi_1^\dagger\Phi_1](\mathrm{Tr}[\Phi_2\Phi_1^\dagger] + \mathrm{Tr}[\Phi_2^\dagger\Phi_1]) + \mu_\triangle^2(\mathrm{Tr}[\triangle_L^\dagger\triangle_L] + \mathrm{Tr}[\triangle_R^\dagger\triangle_R]) + f_1[(\mathrm{Tr}[\triangle_L^\dagger\triangle_L])^2 + (\mathrm{Tr}[\triangle_R^\dagger\triangle_R])^2] + f_2(\mathrm{Tr}[\triangle_L\triangle_L]\mathrm{Tr}[\triangle_L^\dagger\triangle_L^\dagger] + \mathrm{Tr}[\triangle_R\triangle_R]\mathrm{Tr}[\triangle_R^\dagger\triangle_R^\dagger]) + f_3 \mathrm{Tr}[\triangle_L^\dagger\triangle_L]\mathrm{Tr}[\triangle_R^\dagger\triangle_R] + f_4(\mathrm{Tr}[\triangle_L\triangle_L]\mathrm{Tr}[\triangle_R^\dagger\triangle_R^\dagger] + \mathrm{Tr}[\triangle_R\triangle_R]\mathrm{Tr}[\triangle_L^\dagger\triangle_L^\dagger]) + \alpha_1 \mathrm{Tr}[\Phi_1^\dagger\Phi_1] \times (\mathrm{Tr}[\triangle_L^\dagger\triangle_L] + \mathrm{Tr}[\triangle_R^\dagger\triangle_R]) + \alpha_2(\mathrm{Tr}[\Phi_2^\dagger\Phi_1]\mathrm{Tr}[\triangle_R^\dagger\triangle_R] + \mathrm{Tr}[\Phi_1^\dagger\Phi_2]\mathrm{Tr}[\triangle_L^\dagger\triangle_L]) + \alpha_2^*(\mathrm{Tr}[\Phi_1^\dagger\Phi_2]\mathrm{Tr}[\triangle_R^\dagger\triangle_R] + \mathrm{Tr}[\Phi_2^\dagger\Phi_1]\mathrm{Tr}[\triangle_L^\dagger\triangle_L]) + \alpha_3(\mathrm{Tr}[\Phi_1\Phi_1^\dagger \triangle_L \triangle_L^\dagger] + \mathrm{Tr}[\Phi_1^\dagger\Phi_1 \triangle_R \triangle_R^\dagger]) + \beta_1(\mathrm{Tr}[\Phi_1 \triangle_R \Phi_1^\dagger\triangle_L^\dagger] + \mathrm{Tr}[\Phi_1^\dagger \triangle_L \Phi_1\triangle_R^\dagger]) + \beta_2(\mathrm{Tr}[\Phi_2 \triangle_R \Phi_1^\dagger\triangle_L^\dagger] + \mathrm{Tr}[\Phi_2^\dagger \triangle_L \Phi_1\triangle_R^\dagger]) + \beta_3(\mathrm{Tr}[\Phi_1 \triangle_R \Phi_2^\dagger\triangle_L^\dagger] + \mathrm{Tr}[\Phi_1^\dagger \triangle_L \Phi_2\triangle_R^\dagger]) - \mu_\rho^2\rho^2 + \lambda_5\rho^4 + M\rho(\mathrm{Tr}[\triangle_L^\dagger\triangle_L] - \mathrm{Tr}[\triangle_R^\dagger\triangle_R])\lambda_6\rho^2(\mathrm{Tr}[\triangle_L^\dagger\triangle_L] + \mathrm{Tr}[\triangle_R^\dagger\triangle_R]) + \lambda_7\rho^2 \mathrm{Tr}[\Phi_1^\dagger\Phi_1] + \lambda_8\rho^2[\mathrm{Det}[\Phi_1] + \mathrm{Det}[\Phi_1^\dagger]],$$

where $\Phi_2 = \tau_2\Phi_1^*\tau_2$. Minimizing the scalar potential we now obtain various conditions

$$\frac{\partial V}{\partial v_L} = \mu_L^2 v_L + 2f_1 v_L^3 + f_3 v_L v_R^2 + (\beta_1 v_1 v_2 + \beta_2 v_1^2 + \beta_3 v_2^2)v_R = 0 \qquad (5)$$

$$\frac{\partial V}{\partial v_R} = \mu_R^2 v_R + 2f_1 v_R^3 + f_3 v_R v_L^2 + (\beta_1 v_1 v_2 + \beta_2 v_1^2 + \beta_3 v_2^2)v_L = 0, \qquad (6)$$

where μ_L^2 and μ_R^2 are effective mass terms of $\triangle_L$ and $\triangle_R$ given by

$$\mu_L^2 = \mu_\triangle^2 + Ms + \lambda_6 s^2 + 2(\alpha_2 + \alpha_2^*)v_1 v_2 + \alpha_1(v_1^2 + v_2^2) + \alpha_3 v_2^2$$
$$\mu_R^2 = \mu_\triangle^2 - Ms + \lambda_6 s^2 + 2(\alpha_2 + \alpha_2^*)v_1 v_2 + \alpha_1(v_1^2 + v_2^2) + \alpha_3 v_2^2.$$

Thus, like in the previous case, here also the Higgs triplets $\triangle_L$ become heavier than $\triangle_R$ after the singlet η acquires a vev at the high scale. Equations (5) and (6) gives

$$(2Ms + (v_R^2 - v_L^2)(f_3 - 2f_1))v_L v_R = (v_L^2 - v_R^2)(\beta_1 v_1 v_2 + \beta_2 v_1^2 + \beta_3 v_2^2).$$

Thus, a nonzero vev of ρ disallows those solutions for which $v_L = v_R$. Assuming $v_L \ll v_R \ll s$, M will give

$$v_L = \frac{-v_R(\beta_1 v_1 v_2 + \beta_2 v_1^2 + \beta_3 v_2^2)}{2Ms}. \qquad (7)$$

Thus, we can have a small $v_L \sim$ eV by appropriately choosing v_R and M, s. Here if we take v_R of TeV scale; then the scale of parity breaking M, s should be low ($\sim 10^8$–10^9 GeV) so as to give $v_L \sim$ eV needed to account for neutrino masses as we will see later.

C. SUSYLR model with Higgs doublets

We shall now study various supersymmetric left-right models. These models are much more restrictive compared to the non-susy models. Although the spontaneous parity violation is one of the most important features of the non-susy versions of the left-right symmetric models, in the susy left-right models with triplet Higgs scalars breaking parity becomes very difficult and one has to extend the model to incorporate any natural mechanism of parity violation. In this section we shall discuss the model where the left-right symmetry is broken by a Higgs doublet scalar.

In the particle contents, the fermions belong to the fermion superfields and we denote all the fermions and scalars by their corresponding superfields. We can then write the particle contents of the supersymmetric left-right model with Higgs doublet in terms of their superfields as

matter superfield: $Q_L = (3, 2, 1, 1/3)$,
$Q_R = (3, 1, 2, 1/3)$ $\quad \Psi_L = (1, 2, 1, -1)$,
$\Psi_R = (1, 1, 2, -1)$
Higgs superfield: $\Phi_1 = (1, 2, 2, 0)$, $\quad \Phi_2 = (1, 2, 2, 0)$
$H_L = (1, 2, 1, 1)$, $\quad \bar{H}_L = (1, 2, 1, -1)$,
$H_R = (1, 1, 2, -1)$, $\quad \bar{H}_R = (1, 1, 2, 1)$,
$\rho = (1, 1, 1, 0)$,

where Higgs particles with "bar" in the notation helps in anomaly cancellation of the model.

The minimal Higgs doublet model without the singlet Higgs ρ was discussed in [28]. Here, a singlet scalar field ρ is introduced, which has the special property that it is even under the usual parity of the Lorentz group, but it is odd under the parity that relates the gauge groups $SU(2)_L$ and $SU(2)_R$. This field ρ is thus a scalar and not a pseudoscalar field, but under the D-parity transformation that interchanges $SU(2)_L$ with $SU(2)_R$, it is odd. This kind of work is proposed in [29,30]. Although all the scalar fields are even under the parity of the Lorentz group, under the D parity the Higgs sector transforms as

$$H_L \leftrightarrow H_R, \qquad \bar{H}_L \leftrightarrow \bar{H}_R, \qquad \Phi \leftrightarrow \Phi^\dagger, \qquad \rho \leftrightarrow -\rho.$$

The Higgs part of the superpotential relevant in our case is

$$\begin{aligned} W = {} & \mu_{ij}\,\mathrm{Tr}[\tau_2\Phi_i^T\tau_2\Phi_j] + M\rho\rho \\ & + f_1(H_L^T\Phi_i H_R + \bar{H}_L^T\Phi_i\bar{H}_R) \\ & + m_h(H_L^T\tau_2\bar{H}_L + H_R^T\tau_2\bar{H}_R) \\ & + \lambda_1\rho(H_L^T\tau_2\bar{H}_L - H_R^T\tau_2\bar{H}_R). \end{aligned}$$

The scalar potential is $V = V_F + V_D + V_{\mathrm{soft}}$, where $V_F = |F_i^2|$, $F_i = -\frac{\partial W}{\partial \phi}$ is the F-term scalar potential, $V_D = D^a D^a/2$, $D^a = -g(\phi_i^* T_{ij}^a \phi_j)$ is the D term of the scalar potential, and V_{soft} is the soft supersymmetry breaking scalar potential. We introduce the soft susy breaking terms to check if they alter relations between various mass scales in the model. The soft susy breaking superpotential in this case is given by

$$\begin{aligned} V_{\mathrm{soft}} = {} & m_H^2 H_L^\dagger H_L + m_H^2\bar{H}_L^\dagger\bar{H}_L + m_H^2 H_R^\dagger H_R + m_H^2\bar{H}_R^\dagger\bar{H}_R \\ & + m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 + m_\rho^2\rho^\dagger\rho \\ & + (B_1 H_L^T\tau_2\bar{H}_L + B_2 H_R^T\tau_2\bar{H}_R \\ & + B\mu_{ij}\,\mathrm{Tr}[\tau_2\Phi_i\tau_2\Phi_j] + \mathrm{H.c.}) \\ & + (A_1 H_L^T\Phi_i H_R + A_2\bar{H}_L\Phi_i\bar{H}_R \\ & + A_3(\rho H_L^T\tau_2\bar{H}_L - \rho H_R^T\tau_2\bar{H}_R) + \mathrm{H.c.}), \end{aligned} \tag{8}$$

where all the parameters m_H, m_{11}, m_{22}, B, A are of the order of susy breaking scale $M_{\mathrm{susy}} \sim$ TeV. We denote the vev of the neutral components of Φ_1, Φ_2, H_L, $\bar{H}_L$, H_R, $\bar{H}_R$, and ρ as $\langle(\Phi_1)_{11}\rangle = v_1$, $\langle(\Phi_2)_{22}\rangle = v_2$, $\langle H_L, \bar{H}_L\rangle = v_L$, $\langle H_R, \bar{H}_R\rangle = v_R$, $\langle\rho\rangle = s$.

Minimizing the potential with respect to v_L, v_R, we get the relations

$$\begin{aligned} \frac{\partial V}{\partial v_L} = {} & -\mu_L^2(2v_L) + 2v_L v_R^2 f_1^2 + f_1 v_R(m_h + 4\mu)(v_1 \\ & + v_2) + (2m_H^2 - m_h^2)v_L + A_1 s v_L + \frac{A_2 v_1 v_R}{2} \\ & + \lambda_1^2 v_L(v_R^2 - v_L^2) \\ = {} & 0 \Rightarrow \frac{v_L}{v_R} = \frac{f_1(m_h + 4\mu)(v_1 + v_2) + \frac{A_1 v_1}{2}}{2\mu_L^2 - 2f_1^2 v_R^2 - \lambda_1^2(v_R^2 - v_L^2) - A_2 s} \end{aligned} \tag{9}$$

$$\begin{aligned} \frac{\partial V}{\partial v_R} = {} & -\mu_R^2(2v_R) + 2v_R v_L^2 f_1^2 + f_1 v_R(m_h + 4\mu)(v_1 + v_2) \\ & + (2m_H^2 - m_h^2)v_R - A_2 s v_R + \frac{A_1 v_1 v_L}{2} \\ & - \lambda_1^2 v_L(v_R^2 - v_L^2) = 0, \end{aligned} \tag{10}$$

where μ_L^2, μ_R^2 are given by

$$\begin{aligned} \mu_L^2 &= \tfrac{1}{4}[2(m_h + \lambda_1 s)^2 - 4Ms\lambda_1 - f_1^2(v_1^2 + v_2^2)] \\ \mu_R^2 &= \tfrac{1}{4}[2(m_h - \lambda_1 s)^2 + 4Ms\lambda_1 - f_1^2(v_1^2 + v_2^2)]. \end{aligned}$$

From Eqs. (9) and (10) we get

$$\begin{aligned} & (A_1 v_1 + 4(f_1^2 + \lambda_1^2)v_L v_R + 2f_1(v_1 + v_2)(m_h + 4\mu)) \\ & \quad \times (v_R^2 - v_L^2) + (4sA_2 + 8\lambda_1 s(M - m_h))v_L v_R = 0 \end{aligned}$$

which shows that the minimization disallows the solutions where $v_L = v_R$. Assuming $v_L \ll v_{1,2}, \mu, A \ll s, M, m_h$ and $v_L \ll v_R$, the above expression gives rise to

$$v_L = \frac{v_R(2f_1 m_h(v_1 + v_2) + 4(f_1^2 + \lambda_1^2)v_L v_R + A_1 v_1)}{8(m_h - M)s\lambda_1 + 4sA_2}. \tag{11}$$

Thus, by appropriate choice of m_h, M, s we can have TeV scale $SU(2)_R$ breaking scale v_R as well as $v_L/v_R \sim (10^{-6}\text{–}10^{-9})$ which is necessary to account for small neutrino masses as we will see later. For example, if we set

$$m_h \sim M \sim s \sim 10^{16}\ \mathrm{GeV} \qquad D\text{-parity breaking scale}$$

and allow $2m_h - M \sim 10^8$ GeV by appropriate fine-tuning, then the above relation will give rise to the desired ratio $v_L/v_R \sim 10^{-6}$. For such a choice of scales we can fine-tune the parameters to get a light H_R having mass $\mu_R \sim v_R \sim$ TeV and a heavy H_L having mass $\mu_L \sim s$, $M \sim 10^{16}$ GeV. This will be important in the renormalization group running of the couplings as we will see later.

D. SUSYLR model with Higgs triplets

The particle contents of supersymmetric left-right model with Higgs triplets in terms of their superfields are

$$\begin{aligned} & \text{matter superfield}: Q = (3, 2, 1, 1/3), \\ & Q^c = (3, 1, 2, 1/3), \quad L = (1, 2, 1, -1), \\ & L^c = (1, 1, 2, -1) \\ & \text{Higgs superfield: } \Phi_1 = (1, 2, 2, 0), \quad \Phi_2 = (1, 2, 2, 0), \\ & \Delta = (1, 3, 1, 2), \quad \bar{\Delta} = (1, 3, 1, -2), \\ & \Delta^c = (1, 1, 3, -2), \quad \bar{\Delta}^c = (1, 1, 3, 2), \\ & \rho = (1, 1, 1, 0). \end{aligned}$$

The left-right symmetry could be broken by either doublet Higgs scalars or triplet Higgs scalar. We will show that for a minimal choice of parameters, it is convenient to break the group with a triplet Higgs scalar. As pointed out in [14], the bidoublets are doubled to achieve a nonvanishing Cabibbo-Kobayashi-Maskawa quark mixing and the number of triplets is doubled for the sake of anomaly cancellation.

The superpotential for this theory is given by

$$W = Y^{(i)_q} Q^T \tau_2 \Phi_i \tau_2 Q^c + Y^{(i)_l} L^T \tau_2 \Phi_i \tau_2 L^c + i(f L^T \tau_2 \Delta L + f^* L^{cT} \tau_2 \Delta^c L^c) + M\rho^2 + m_\Delta \,\mathrm{Tr}(\Delta \bar{\Delta}) + m_\Delta^* \,\mathrm{Tr}(\Delta^c \bar{\Delta}^c) + \mu_{ij} \,\mathrm{Tr}(\tau_2 \Phi_i^T \tau_2 \Phi_j). \tag{12}$$

All couplings $Y^{(i)_{q,l}}$, μ_{ij}, μ_Δ, f in the above potential are complex with the additional constraint that μ_{ij}, f, and f^* are symmetric matrices. The scalar potential is $V = V_F + V_D + V_{\mathrm{soft}}$, where $V_F = |F_i^2|$, $F_i = -\frac{\partial W}{\partial \phi}$ is the F-term scalar potential, $V_D = D^a D^a/2$, $D^a = -g(\phi_i^* T_{ij}^a \phi_j)$ is the D term of the scalar potential, and V_{soft} is the soft supersymmetry breaking terms in the scalar potential. In the particular model, the soft-susy breaking terms are given by

$$V_{\mathrm{soft}} = m_\delta^2 \,\mathrm{Tr}[\Delta^\dagger \triangle] + m_\delta^2 \,\mathrm{Tr}[\bar{\Delta}^\dagger \bar{\Delta}] + m_\delta^2 \,\mathrm{Tr}[(\Delta^c)^\dagger \Delta^c] + m_\delta^2 \,\mathrm{Tr}[\bar{\Delta}^{\dagger c} \bar{\Delta}^c] + m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 + m_\rho^2 \rho^\dagger \rho + (B\mu_{ij} \,\mathrm{Tr}[\tau_2 \Phi_i \tau_2 \Phi_j] + A\rho(\mathrm{Tr}[\Delta \bar{\Delta}] - \mathrm{Tr}[\Delta^c \bar{\Delta}^c] + \mathrm{H.c.}), \tag{13}$$

where all the parameters in the soft supersymmetry breaking scalar potential are of the order of supersymmetry breaking scale $M_{\mathrm{susy}} \sim$ TeV. We denote the vev of the neutral components of Φ_1, Φ_2, Δ, $\bar{\Delta}$, Δ^c, $\bar{\Delta}^c$, and ρ as

$$\langle (\Phi_1)_{11} \rangle = v_1, \quad \langle (\Phi_2)_{22} \rangle = v_2, \quad \langle \Delta, \bar{\Delta} \rangle = v_L, \quad \langle \Delta^c, \bar{\Delta}^c \rangle = v_R, \quad \langle \rho \rangle = s.$$

Minimizing the scalar potential with respect to v_L, v_R we get

$$\frac{\partial V}{\partial v_L} = v_L[2(m_\Delta + \lambda_1 s)^2 + 2\lambda_1^2(v_L^2 - v_R^2) + As + 2m_\delta^2] = 0 \Rightarrow v_R^2 - v_L^2 = \frac{2m_\delta^2 + (A + 2\lambda_1 M)s + 2(m_\Delta + \lambda_1 s)^2}{2\lambda_1^2} \tag{14}$$

$$\frac{\partial V}{\partial v_R} = v_R[2(m_\Delta - \lambda_1 s)^2 - 2\lambda_1^2(v_L^2 - v_R^2) - As + 2m_\delta^2] = 0 \Rightarrow v_R^2 - v_L^2 = \frac{-2m_\delta^2 + (A + 2\lambda_1 M)s - 2(m_\Delta - \lambda_1 s)^2}{2\lambda_1^2}. \tag{15}$$

Also

$$v_R \frac{\partial V}{\partial v_L} - v_L \frac{\partial V}{\partial v_R} = 4 v_L v_R [2(Ms + 2m_\Delta s)\lambda_1 + 2\lambda_1^2(v_L^2 - v_R^2) + As] = 0 \Rightarrow v_R^2 - v_L^2 = \frac{2\lambda_1(Ms + 2m_\Delta) + As}{2\lambda_1^2}. \tag{16}$$

Thus, the minimization conditions disallow solutions with $v_L = v_R$. But from Eqs. (14)–(16) it can be seen that it is difficult to adjust the various scales M, s, $m_\triangle$ so as to satisfy them simultaneously and give rise to a TeV scale v_R and an eV scale v_L. Thus, we need to add more particles to the above particle content which can give rise to spontaneous D-parity breaking with a TeV scale v_R. This scenario of the minimal SUSYLR model with parity odd singlet was studied long ago and was shown [31] that the charge-breaking vacua have a lower potential than the charge-preserving vacua and as such the ground state does not conserve electric charge.

E. SUSYLR model with Higgs triplets and bitriplet

The minimal left-right supersymmetric model with triplet Higgs bosons leads to several nettlesome obstructions which may be considered to be a guidance towards a unique consistent theory. One of the most important problems is the spontaneous breaking of left-right symmetry and a substantial amount of work has been done to cure this problem. This can be cured either by adding some extra fields to the minimal particle content [31] or with the help of a nonrenormalization operator [16]. There is another solution to the problem, which resembles the nonsupersymmetric solution, relating the vacuum expectation values (vevs) of the left-handed and right-handed triplet Higgs scalars to the Higgs bidoublet vev through a seesaw relation. The novel feature consists in the introduction of a bitriplet Higgs and another Higgs singlet under the left-right group [32]. We will try to extremize the full potential of this particular model and see what are the mass scales, different vevs coming out from the extremization.

We now present our model, where we include a bitriplet $\eta = (1, 3, 3, 0)$ and a parity odd singlet field $\rho = (1, 1, 1, 0)$ in the minimal supersymmetric left-right model with triplet Higgs discussed in the previous subsection. These fields are vectorlike and hence do not contribute to anomaly, so we consider only one of these fields. Under parity, these fields transform as $\eta \leftrightarrow \eta$ and $\rho \leftrightarrow -\rho$. The superpotential for the model is written in the more general tensorial notation [32] as follows:

$$W = f\eta_{\alpha i}\Delta_\alpha\Delta_i^c + f^*\eta_{\alpha i}\bar{\Delta}_\alpha\bar{\Delta}_i^c + \lambda_1\eta_{\alpha i}\Phi_{am}\Phi_{bn}(\tau^\alpha\epsilon)_{ab} \times(\tau^i\epsilon)_{mn} + m_\eta\eta_{\alpha i}\eta_{\alpha i} + M_\Delta(\Delta_\alpha\bar{\Delta}_\alpha + \Delta_i^c\bar{\Delta}_i^c) + \mu\epsilon_{ab}\Phi_{bm}\epsilon_{mn}\Phi_{an} + m_\rho\rho^2 + \lambda_2\rho(\Delta_\alpha\bar{\Delta}_\alpha - \Delta_i^c\bar{\Delta}_i^c), \tag{17}$$

where α, a, b are $SU(2)_L$ and i, m, n are $SU(2)_R$ indices. The symmetry breaking pattern in this model is

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P\underset{\rightarrow}{\langle\rho\rangle}SU(2)_L \times SU(2)_R \times U(1)_{B-L}\underset{\rightarrow}{\langle\triangle_c\rangle}SU(2)_L \times U(1)_Y\underset{\rightarrow}{\langle\Phi\rangle}U(1)_{\rm em}.$$

Denoting the vev's as $\langle\Delta_-\rangle = \langle\bar{\Delta}_+\rangle = v_L$, $\langle\Delta_+^c\rangle = \langle\bar{\Delta}_-^c\rangle = v_R$, $\langle\Phi_{+-}\rangle = v$, $\langle\Phi_{-+}\rangle = v'$, $\langle\eta_{+-}\rangle = u_1$, $\langle\eta_{-+}\rangle = u_2$, $\langle\eta_{00}\rangle = u_0$, and $\langle\rho\rangle = s$.

The scalar potential is $V = V_F + V_D + V_{\rm soft}$, where $V_F = |F_i^2|$, $F_i = -\frac{\partial W}{\partial\phi}$ is the F-term scalar potential, $V_D = D^aD^a/2$, $D^a = -g(\phi_i^* T_{ij}^a\phi_j)$ is the D term of the scalar potential, and $V_{\rm soft}$ is the soft supersymmetry breaking terms in the scalar potential. In the particular model, the soft-susy breaking terms are given by

$$V_{\rm soft} = V_{\rm soft}(\text{containing } \triangle \text{ and } \Phi) + m_{\eta(\rm soft)}\eta_{\alpha i}^\dagger\eta_{\alpha i} + (A_2\eta_{\alpha i}\Phi_{\rm am}\Phi_{\rm bn}(\tau^\alpha\epsilon)_{\rm ab}(\tau^i\epsilon)_{\rm mn} + A_3(\eta_{\alpha i}\Delta_\alpha\Delta_i^c) + \text{H.c.}), \tag{18}$$

where $V_{\rm soft}$(containing $\triangle$ and Φ) is given by Eq. (13) in the Sec. II D.

Minimizing the scalar potential with respect to v_L, v_R we get

$$\frac{\partial V}{\partial v_L} = \mu_L^2(2v_L) + 2\lambda_2^2 v_L(v_L^2 - v_R^2) + 2(fu_1 + f^*u_2)M_\Delta v_R + v_R(f + f^*) \times[2m_\eta(u_1 + u_2 + u_3) + \lambda_1 v^2 + v_L v_R(f + f^*)] + 4v_L m_\delta^2 + 2Av_L s + A_3 v_R(u_1 + u_2 + u_3) = 0 \tag{19}$$

$$\frac{\partial V}{\partial v_R} = \mu_R^2(2v_R) - 2\lambda_2^2 v_R(v_L^2 - v_R^2) + 2(fu_1 + f^*u_2)M_\Delta v_L + v_L(f + f^*) \times[2m_\eta(u_1 + u_2 + u_3) + \lambda_1 v^2 + v_L v_R(f + f^*)] + 4v_R m_\delta^2 - 2Av_R s + A_3 v_L(u_1 + u_2 + u_3) = 0, \tag{20}$$

where the effective mass terms μ_L^2, μ_R^2 are given by

$$\mu_L^2 = (M_\Delta + \lambda_2 s)^2 + \lambda_2 m_\rho s + \frac{1}{2}(f^2u_1^2 + f^{*2}u_2^2) \tag{21}$$

$$\mu_R^2 = (M_\Delta - \lambda_2 s)^2 - \lambda_2 m_\rho s + \tfrac{1}{2}(f^2u_1^2 + f^{*2}u_2^2). \tag{22}$$

Thus, after the singlet field ρ acquires a vev, the degeneracy of the Higgs triplets goes away and the left-handed triplets being very heavy get decoupled, whereas the right-handed triplets can be as light as 1 TeV by appropriate fine-tuning in the above two expressions. Assuming $v_L \ll v, v', \mu, A \ll m_\rho, s$, and $v_L \ll v_R$ we get from Eqs. (19) and (20)

$$v_L = \frac{-v_R[M_\Delta u_2 f^* + m_\eta(u_2 + u_3)(f + f^*) + u_1(fM_\Delta + m_\eta(f + f^*)]}{2m_\rho s\lambda_2 + 4M_\Delta s\lambda_2 + 2As}. \tag{23}$$

Thus, we can get a small v_L($\sim$eV) and a TeV scale v_R by appropriate choice of M_Δ, m_η, m_ρ, s. We take the vev of the bitriplet $u \ll M_Z$. Thus, if we want $v_R \sim 1$ TeV then the above relation will give us an eV scale v_L only if the scale of parity breaking is kept low; that is, $s \sim m_\rho \sim M_\Delta \sim 10^{10}$ GeV. Thus, in such a type II seesaw dominated case, the right-handed triplets Δ^c will be as light as $\mu_R \sim v_R \sim 1$ TeV and the left-handed triplets Δ as heavy as $\mu_L \sim 10^{10}$ GeV by appropriate fine-tuning of the parameters. However, as we will see later, such a light Higgs triplet with $B - L$ charge 2 spoils the gauge coupling unification. Hence, we are forced to keep the intermediate symmetry breaking scale M_R close to the unification scale.

III. GAUGE COUPLING UNIFICATION

Grand unified theories (GUTs) offer the possibility of unifying the three gauge groups viz., $SU(3)$, $SU(2)$, and $U(1)$ of the standard model into one large group at a high energy scale M_U. This scale is determined as the intersection point of the $SU(3)$, $SU(2)$, and $U(1)$ couplings. The particle content of the theory completely determines the variation of the couplings with energy. It is hard to achieve low intermediate scale without taking into account the effect of D-parity breaking in the renormalization group equations (RGEs). We have seen in the previous section that, in spontaneous D-parity breaking models, the minimization of the scalar potential simultaneously allows us to have right-handed scale v_R of the order of TeV and tiny neutrino masses from seesaw mechanisms. However, the evolution of gauge couplings will be very different in models with Higgs triplets and with Higgs doublets. In this section we study the renormalization group evolution of the gauge couplings and see if unification at a high scale ($\sim 10^{16}$ GeV) allows us to have a TeV scale v_R. Similar analyses were done in [26,33] for the Higgs doublet case. Here we use the $U(1)$ normalization constant $\sqrt{\frac{3}{8}}$ as in [34].

We restrict our study to the supersymmetric case only. The gauge coupling unification in the nonsupersymmetric versions of such models were studied before and can be found in [29,35].

A. Unification in SUSYLR model with Higgs doublets

We will study the evolution of couplings according to their respective beta functions with the account of spontaneous D-parity breaking. The renormalization group equations (RGEs) for this model can be written as

$$\frac{d\alpha_i}{dt} = \alpha_i^2[b_i + \alpha_j b_{ij} + O(\alpha^2)], \tag{24}$$

where $t = 2\pi ln(M)$ (M is the varying energy scale), $\alpha_i = \frac{g_i^2}{4\pi}$ is the coupling strength. Also b_i and b_{ij} are the one loop and two loop beta coefficients and we will study only the one loop contributions to RGEs [34]. The indices $i, j = 1, 2, 3$ refer to the gauge group $U(1)$, $SU(2)$, and $SU(3)$, respectively.

The particle content of the SUSYLR model with Higgs doublets is shown in Sec. II C. It turns out that the minimal particle content is not enough for proper gauge coupling unification. For required unification purposes we add two copies of $\delta(1, 1, 1, 2)$, $\bar{\delta}(1, 1, 1, -2)$ at the $SU(2)_R$ breaking scale. The beta functions are given as

(i) Below the susy breaking scale $M_{\rm susy}$ the beta functions are the same as those of the standard model:

$$b_s = -7, \qquad b_{2L} = -\frac{19}{6} \qquad b_Y = \frac{41}{10}.$$

(ii) For $M_{\rm susy} < M < M_R$, the beta functions are the same as those of the MSSM:

$$b_s = -9 + 2n_g, \qquad b_{2L} = -6 + 2n_g + \frac{n_b}{2},$$
$$b_Y = 2n_g + \frac{3}{10}n_b.$$

(iii) For $M_R < M < \langle\rho\rangle$ the beta functions are

$$b_s = -9 + 2n_g, \qquad b_{2L} = -6 + 2n_g + n_b$$
$$b_{2R} = -6 + 2n_g + n_b + \frac{n_{\rm HR}}{2},$$
$$b_{B-L} = 2n_g + 3n_\delta + \frac{3}{4}n_{\rm HR}.$$

(iv) For $\langle\rho\rangle < M < M_{\rm GUT}$ the beta functions are

$$b_s = -9 + 2n_g,$$
$$b_{2L} = -6 + 2n_g + n_b + \frac{n_{\rm HL}}{2}$$
$$b_{2R} = -6 + 2n_g + n_b + \frac{n_{\rm HR}}{2},$$
$$b_{B-L} = 2n_g + 3n_\delta + \frac{3}{4}(n_{\rm HL} + n_{\rm HR}),$$

where n_g is the number of fermion generations and number of Higgs bidoublets $n_b = 2$, number of Higgs doublets $n_{\rm HL} = n_{\rm HR} = 2$, and number of extra Higgs singlets $n_\delta = 2$. The experimental initial values for the couplings at the electroweak scale $M = M_Z$ [36] are

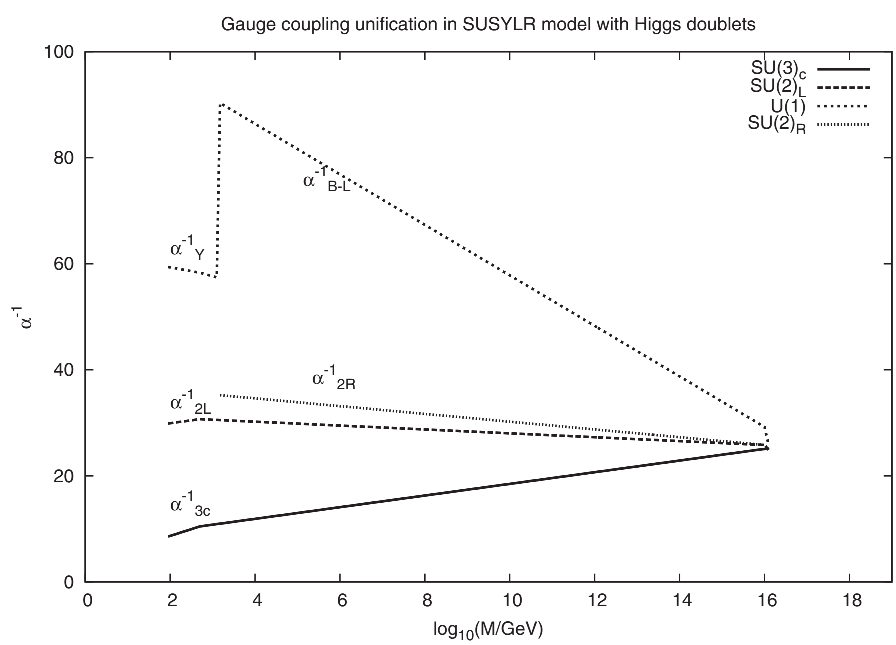

FIG. 1. Gauge coupling unification in the SUSYLR model with Higgs doublets and $M_{\rm susy} = 500$ GeV, $M_R = 1.5$ TeV, $M_\rho = 10^{16}$ GeV.

$$\begin{pmatrix} \alpha_s(M_Z) \\ \alpha_{2L}(M_Z) \\ \alpha_{1Y}(M_Z) \end{pmatrix} = \begin{pmatrix} 0.118 \pm 0.003 \\ 0.033\,493^{+0.000\,042}_{-0.000\,038} \\ 0.016\,829 \pm 0.000\,017 \end{pmatrix}. \quad (25)$$

The normalization condition at $M = M_R$, where the $U(1)_Y$ gauge coupling merge with $SU(2)_R \times U(1)_{B-L}$ is $\alpha_{B-L}^{-1} = \frac{5}{2}\alpha_Y^{-1} - \frac{3}{2}\alpha_L^{-1}$. Using all these we arrive at the gauge coupling unification as shown in Fig. 1. Here we have taken $M_{\rm susy} = 500$ GeV, $M_R = 1.5$ TeV, and $M_\rho = 10^{16}$ GeV. The couplings seems to unify at a scale slightly above the D-parity breaking scale. Thus, the D-parity breaking scale need not be the same as the GUT scale, but can be lower also. However, if we make the D-parity breaking scale arbitrarily lower, the unification will not be possible as can be seen from Fig. 1. Since both the left-handed and right-handed Higgs doublets will contribute to the $U(1)_{B-L}$ couplings after the D-parity breaking scale, the α_{BL}^{-1} will come down sharply and meet the other couplings at some energy below the expected GUT scale.

B. Unification in SUSYLR model with Higgs triplets

The particle content of the SUSYLR model with Higgs triplets is shown in Sec. II D. It is very difficult to achieve unification with low M_R with the minimal particle content. We add a parity odd singlet $\rho(1, 1, 1, 0)$ to achieve spontaneous D-parity breaking. This may change the scale of M_R, but it is found that the M_R remains higher than 10^{10} GeV. For unification purposes, we need in the recent model, one heavy bidoublet $\chi(1, 2, 2, 0)$ has been added which gets mass at the $SU(2)_R$ breaking scale. Below the $SU(2)_R$ breaking scale the beta functions are similar to the MSSM as written above. The beta functions above this scale are as follows:

(i) For $M_R < M < M_\rho$ the beta functions are

$$b_s = -9 + 2n_g, \quad b_{2L} = -6 + 2n_g + n_b + \frac{n_\chi}{2}$$
$$b_{2R} = -6 + 2n_g + n_b + 2n_\triangle + \frac{n_\chi}{2},$$
$$b_{B-L} = 2n_g + \frac{9}{2}n_\triangle.$$

(ii) For $\langle\rho\rangle < M < M_{\rm GUT}$ the beta functions are

$$b_s = -9 + 2n_g,$$
$$b_{2L} = -6 + 2n_g + n_b + 2n_\triangle + \frac{n_\chi}{2}$$
$$b_{2R} = -6 + 2n_g + n_b + 2n_\triangle + \frac{n_\chi}{2},$$
$$b_{B-L} = 2n_g + 9n_\triangle,$$

where number of Higgs triplets $n_\triangle = 2$, number of additional Higgs field added for unification $n_\chi = 1$, number of generations $n_g = 3$, and number of Higgs bidoublets $n_b = 2$. Using the same initial values and normalization relations like before, we arrive at the gauge coupling unification as shown in Fig. 2. Here the unification scale $M_{\rm GUT}$ coincides with the D-parity breaking scale M_ρ. Lower values of M_R will make the unification worse because of the large contributions of triplets to the $U(1)_{B-L}$ beta functions compared to the doublets in the previous case. Thus, in the minimal triplet case, both the minimization conditions as well as unification disallow a TeV scale v_R. Although after adding a bitriplet, the minimization conditions allow a TeV scale v_R, it will not make the unification better as we discuss in the next subsection.

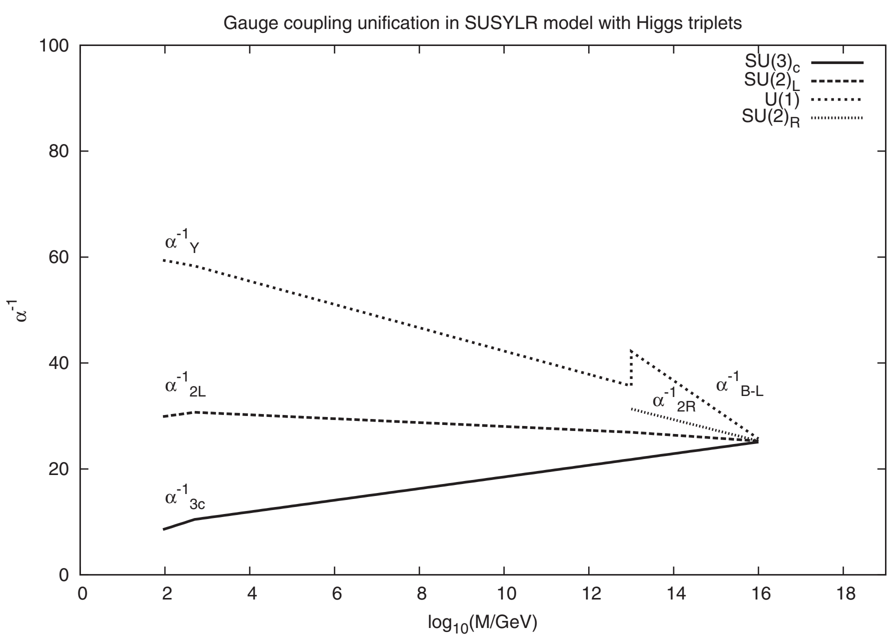

FIG. 2. Gauge coupling unification in the SUSYLR model with Higgs triplets and $M_R = 10^{13}$ GeV, $M_\rho = 10^{16}$ GeV.

C. Unification in SUSYLR model with Higgs triplets and bitriplet

As we saw before, the minimization of the scalar potential in a SUSYLR model with Higgs triplets with spontaneous D-parity breaking does not allow a TeV scale M_R. The same thing is true from a gauge coupling unification point of view as shown in the previous subsection. Now we consider the SUSYLR model with Higgs triplet as well as bitriplet [32]. For unification purposes we include two pairs of heavy colored superfields $\chi(3,1,1,0)$, $\bar{\chi}(\bar{3},1,1,0)$ which decouple after the $SU(2)_R$ breaking scale M_R. The beta functions above M_R are as follows:

(i) For $M_R < M < M_\rho$ the beta functions are

$$\begin{aligned} b_s &= -9 + 2n_g + \frac{n_\chi}{2}, \\ b_{2L} &= -6 + 2n_g + n_b + 2n_\eta \\ b_{2R} &= -6 + 2n_g + n_b + 2n_\triangle + 2n_\eta, \\ b_{B-L} &= 2n_g + \frac{9}{2}n_\triangle. \end{aligned}$$

(ii) For $\langle\rho\rangle < M < M_{\rm GUT}$ the beta functions are

$$\begin{aligned} b_s &= -9 + 2n_g + \frac{n_\chi}{2}, \\ b_{2L} &= -6 + 2n_g + n_b + 2n_\triangle + 2n_\eta \\ b_{2R} &= -6 + 2n_g + n_b + 2n_\triangle + 2n_\eta, \\ b_{B-L} &= 2n_g + 9n_\triangle, \end{aligned}$$

where number of Higgs triplets $n_\triangle = 2$, number of colored Higgs $n_\chi = 3$, number of generations $n_g = 3$, number of Higgs bidoublets $n_b = 2$, and number of Higgs bitriplets $n_\eta = 1$. Using the same initial values and normalization relations like before, we arrive at the gauge coupling unification as shown in Fig. 3. Here the unification scale is the same as the D-parity breaking scale. Similar to the case with just Higgs triplets, here also a lower value of M_R makes the unification look worse. Thus, although minimization of the scalar potential allows the possibility of a TeV scale M_R in this model, the gauge coupling unification criteria rules out such a possibility.

IV. NEUTRINO MASS IN SUSYLR MODEL WITH HIGGS DOUBLETS

In left-right symmetric models with only doublet scalar fields, the question of neutrino masses has been discussed in detail. We shall try to restrict ourselves as close as possible to these existing nonsupersymmetric models, and check the consistency of these solutions when D parity is broken spontaneously in the present SUSYLR model.

We introduced a singlet fermionic superfield S to the particle content of the model discussed in Sec. II C. This kind of model has been discussed without the D-parity breaking effect and from the neutrino mass prospective [28]. The effect of this singlet field has been accounted in the RGEs shown in Sec. III A. With the addition of this singlet fermion, the superpotential and resulting neutrino mass matrix become

$$W = \mathcal{M}_{ij}S_iS_j + F_{ij}\Psi_{Li}S_jH_L + F'_{ij}\Psi_{Ri}S_jH_R, \qquad (26)$$

and

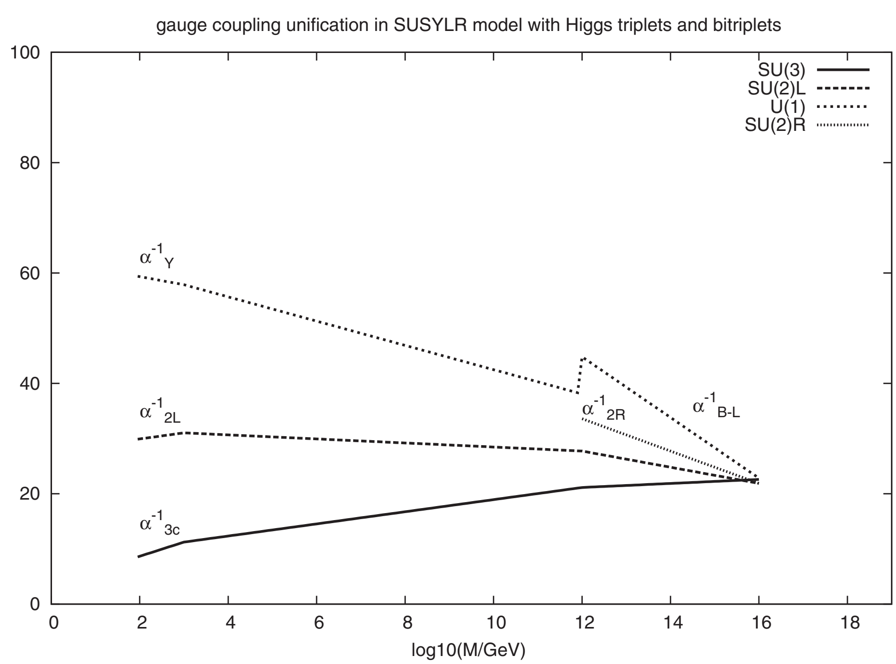

FIG. 3. Gauge coupling unification in the bitriplet model with two extra pairs of colored superfields $\chi(3,1,1,0)$, $\bar{\chi}(\bar{3},1,1,0)$, $M_{\rm susy} = 1$ TeV, $M_R = 10^{12}$ GeV, $M_\rho = 10^{16}$ GeV. The extra superfields decouple below the scale M_R.

$$W_{\rm neut} = (\nu_i \quad N_i^c \quad S_i) \begin{pmatrix} 0 & (M_N)_{ij} & F_{ij}v_L \\ (M_N)_{ji} & 0 & F'_{ij}v_R \\ F_{ji}v_L & F'_{ji}v_R & \mathcal{M}_{ij} \end{pmatrix} \times \begin{pmatrix} \nu_j \\ N_j^c \\ S_j \end{pmatrix}, \tag{27}$$

where M_N is the general Dirac term coming from the term $(M_N)_{ij}\nu_i N_j^c$. In the above mass matrix, the mass of the singlet $\mathcal{M}_{ij}$ and the vev of the right-handed Higgs doublet v_R are heavy, while M_N and the vev of the left-handed Higgs doublet v_L are of low scale.

The resulting light neutrino mass matrix after diagonalizing the above mass matrix is

$$M_\nu = -M_N M_R^{-1} M_N^T - (M_N H + H^T M_N^T)\left(\frac{v_L}{v_R}\right), \tag{28}$$

where

$$H \equiv (F' \cdot F^{-1})^T, \tag{29}$$

$$M_R = (F v_R)\mathcal{M}^{-1}(F^T v_R). \tag{30}$$

Here we can see that the first term in Eq. (28) is the usual type I seesaw contribution and the second term is another seesaw term giving rise to a double seesaw mechanism. This second term contribution to ν mass will dominate over type I if the elements of the matrix $\mathcal{M}_{ij}$ are small compared to the contribution of H term. It is clear from Eq. (30) that the scale of M_R found to be TeV for $\mathcal{M}_{ij} = 1$ TeV, $v_R = 1$ TeV which automatically comes from the minimization of the potential and is consistent with the renormalization-group evolutions which have already been studied in Sec. III A and F of the order of unity. With the mass scales and M_N of the order of MeV, we can foind neutrino mass to be eV.

A. Neutrino mass in case of fermionic triplet

Let us introduce fermionic triplets (one for each family) order to realize the type III seesaw mechanism:

$$\Sigma_L = \frac{1}{2}\begin{pmatrix} \Sigma_L^0 & \sqrt{2}\Sigma_L^+ \\ \sqrt{2}\Sigma_L^- & -\Sigma_L^0 \end{pmatrix} \equiv (3, 1, 1, 0),$$

and

$$\Sigma_R = \frac{1}{2}\begin{pmatrix} \Sigma_R^0 & \sqrt{2}\Sigma_R^+ \\ \sqrt{2}\Sigma_R^- & -\Sigma_R^0 \end{pmatrix} \equiv (1, 3, 1, 0).$$

Under left-right parity transformation, one has the following relations:

$$\Sigma_L \leftrightarrow \Sigma_R.$$

In the context of lepton masses, the relevant term in the Lagrangian is

$$\mathcal{L}_\ell = \bar{\ell}_L(Y_1\Phi + Y_2\tilde{\Phi})\ell_R + \text{H.c.},$$

where $\tilde{\Phi} = \tau_2\Phi\tau_2$. Once the bidoublet Φ takes vev. i.e., $v_1 = \langle\phi_1^0\rangle$ and $v_2 = \langle\phi_2^0\rangle$, the Dirac mass matrix for the neutrinos is

$$m_\nu^D = Y_1 v_1 + Y_2 v_2.$$

The relevant Yukawa terms that give masses (for the type III seesaw mass matrix) to the three generations of leptons are given by

$$\mathcal{L}_\nu^{III} = h_{ij}\ell_{iL}^T C i\sigma_2 \Sigma_{jL} H_L + g_{ij}\ell_{iR}^T C i\sigma_2 \Sigma_{jR} H_R + M_\Sigma\,\text{Tr}(\Sigma_L^T C\Sigma_L + \Sigma_R^T C\Sigma_R) + \text{H.c} \tag{31}$$

Once the Higgs doublets get vev i.e.,$v_L = \langle H_L^0\rangle$ and $v_R = \langle H_R^0\rangle$, $SU(2)_L \otimes SU(2)_R$ is broken spontaneously. Now the mass matrix in the basis $(\nu_L, \nu_R, \Sigma_R^0, \Sigma_L^0)$ reads as

$$M_\nu^{III} = \begin{pmatrix} 0 & m_\nu^D & 0 & hv_L \\ (m_\nu^D)^T & 0 & gv_R & 0 \\ 0 & g^T v_R & M_\Sigma & 0 \\ h^T v_L & 0 & 0 & M_\Sigma \end{pmatrix}. \tag{32}$$

As one expects the neutrino masses are generated through the type I + type III seesaw mechanisms and one has a *double* seesaw mechanism since the mass of the right-handed neutrinos are generated through the type III seesaw once we integrate out Σ_R^0.

The neutrino mass formula derived from the above mass matrix is given by

$$m_{\nu_L} = \frac{1}{v_R^2(g^T g)}[m_\nu^D M_\Sigma (m_\nu^D)^T - v_R v_L m_\nu^D (gh)^T - v_R v_L (gh)(m_\nu^D)^T] \tag{33}$$

with right-handed neutrino masses

$$M_R = v_R^2 g(M_\Sigma)^{-1} g^T. \tag{34}$$

We take the Dirac mass of all three neutrinos to be of MeV order. This fixes the scale of the M_Σ and M_R so as to give rise to eV scale neutrino masses on the left-hand side of the above relation [33]. If we assume that the first term of [33] will dominate then the seesaw relations will become $m_\nu = \frac{m_e^2}{M_R}$. As $m_e = 0.5$ MeV, we need the values of the right-handed Majorana neutrino as $M_R = 10^3$ GeV to have 0.1 eV light neutrino mass. We can arrive at the appropriate value of M_R by choosing g and M_Σ. Since we are taking $v_R \sim 1$ TeV, hence to get $M_R \geq 1$ TeV we must have $M_\Sigma \leq 1$ TeV. Once the scale of the right-handed Majorana neutrino gets fixed by the light neutrino mass, we can find the values of M_Σ and v_R. We have taken the Yukawa couplings as $g, h < 1$, $v_R = 10^3$ GeV in

Eq. (34) and these lead to triplet fermion masses: $M_\Sigma \sim 10^3$ GeV.

If $M_\Sigma \ll 1$ TeV and $v_R \sim 1$ TeV, then the first term of the above neutrino mass formula becomes too small to give rise to neutrino masses. In that case the second and the third terms in Eq. (33) can contribute to the neutrino masses if $v_L/v_R \sim 10^{-6}$. Such a ratio can naturally be achieved (even if we have a TeV scale v_R) by choosing various symmetry breaking scales and mass parameters as we discussed in Sec. II.

B. Role of Σ_L, Σ_R in unification

The fermion triplets with $U(1)_{B-L}$ charge zero contribute to the $SU(2)_L$ and $SU(2)_R$ gauge coupling running. As discussed above, for the seesaw purposes we have to take low values of M_Σ<; $= v_R$ which will ruin the gauge coupling unification for a TeV scale $SU(2)_R$ breaking scale v_R. Unification and small neutrino mass are possible only if $SU(2)_R$ breaking scale as well as mass of the triplet fermions are close to the unification scale. However, if we add fermion singlet in place of triplets then there are no constraints from a unification point of view on v_R and M_Σ. The mass matrix becomes 3×3 in this case. Thus, in the supersymmetric left-right model with Higgs doublets, we can achieve unification with TeV scale $SU(2)_R$ breaking scale only if the fermion singlet is added in place of triplets as in the conventional type III seesaw.

V. NEUTRINO MASS IN SUSYLR MODEL WITH HIGGS TRIPLETS AND BITRIPLETS

The relevant Yukawa couplings which lead to small nonzero neutrino mass is given by

$$\mathcal{L}_\nu^{II} = y_{ij}\ell_{iL}\Phi\ell_{jR} + y'_{ij}\ell_{iL}\tilde{\Phi}\ell_{jR} + \text{H.c.} + f'_{ij}(\ell_{iR}^T Ci\sigma_2\Delta_R\ell_{jR} + (R \leftrightarrow L)) + \text{H.c.} \quad (35)$$

The Majorana Yukawa couplings f are the same for both left- and right-handed neutrinos because of left-right symmetry. After symmetry breaking, the effective mass matrix of the neutrinos is

$$m_\nu = \frac{-fv^2v_R}{2m_\sigma s} - \frac{v^2}{v_R}yf^{-1}y^T = m_\nu^{II} + m_\nu^I.$$

Consider the values of y, f are of the order of unity, then the relative magnitude of m_ν^{II} and m_ν^I depends on the parameters like v_R, m_σ, s. As discussed in Sec. II, the type II term can become dominant (even if $v_R \sim 1$ TeV) if we take $m_\sigma \sim s \sim 10^8$–$10^{10}$ GeV.

VI. RESULTS AND DISCUSSIONS

(i) Spontaneous breaking of Lorentz parity occurs via the Higgs doublet in the SUSYLR model with doublet Higgs only and via Higgs triplets/bitriplet in the SUSYLR model with Higgs triplets and bitriplet. After taking into account spontaneous D-parity breaking, the minimization of the scalar potential also allows the possibility of $M_R \sim$ TeV, $v_L \sim$ eV in LRSM with Higgs triplets and SUSYLR models with Higgs triplets and Higgs bitriplet. It also allows $M_R \sim$ TeV, $v_L/v_R \sim 10^{-6}$ in both susy and non-susy LR models with Higgs doublets.

(ii) In the SUSYLR model with Higgs doublets we can have a TeV scale M_R as well as $v_L/v_R \sim 10^{-6}$ by keeping the D-parity breaking scale very high $\sim 10^{16}$ GeV. The gauge couplings also unify for the same choice of scales although at the cost of adding extra particles which contribute to the beta functions at high energy. However, if we add fermion triplets for seesaw, then unification is not possible with TeV scale $SU(2)_R$ breaking scale. Adding fermion singlet for seesaw purposes can evade this difficulty.

(iii) In the SUSYLR model with Higgs triplet, the minimization conditions do not allow the possibility of a TeV scale M_R and eV scale v_L simultaneously although gauge couplings unify if we take M_R as high as 10^{13} GeV. Thus, we cannot have TeV scale M_R, type II seesaw dominance, and gauge coupling unification simultaneously.

(iv) In the SUSYLR model with Higgs triplets and bitriplet, we can have TeV scale M_R and eV scale v_L only if we keep the D-parity breaking scale as low as 10^{10} GeV. However, such a choice of parity breaking scale spoils the gauge coupling unification. The gauge couplings unify if we take $M_R = 10^{12}$ GeV and the D-parity breaking scale as 10^{16} GeV with inclusion of two extra pairs of colored particles. Thus, we cannot have a TeV scale M_R and unification simultaneously.

VII. CONCLUSION

In this work we have analyzed the different scenarios of spontaneous breaking of D parity in both the non-susy and the susy version of left-right symmetric models. We have discussed the possibility of obtaining a TeV scale M_R, gauge coupling unification, and type II/type III seesaw dominance of neutrino mass within the framework of different SUSYLR models. In all the models where we explore the possibility of a TeV scale M_R, it is difficult to achieve unification with the minimal particle content. We have added some extra scalar particles as well as their superpartners with suitable transformation properties under the gauge group to achieve unification. We have shown that, except for the SUSYLR model with Higgs doublets, we can not have a TeV scale M_R and gauge coupling unification. In the SUSYLR model with Higgs doublet, the type III seesaw can dominate even if the D-parity breaking scale is as high as the GUT scale whereas in the SUSYLR model with Higgs triplets and bitriplet, the D-parity breaking scale has to be kept as low as 10^{10} GeV for type II seesaw to dominate. However, adding fermion triplets to give rise to seesaw spoils the unification with a TeV scale M_R in the SUSYLR model with Higgs doublet. Adding fermion singlets instead of triplets does not give rise to this problem and can reproduce the necessary seesaw without affecting the RG evolution of the couplings.

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