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Fate of *R* parity

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The possible origin of the R-parity–violating interactions in the minimal supersymmetric standard model and its connection to the radiative symmetry-breaking mechanism is investigated in the context of the simplest model where the radiative symmetry-breaking mechanism can be implemented. We find that, in the majority of the parameter space, R parity is spontaneously broken at the low scale. These results hint that R-parity–violating processes could be observed at the Large Hadron Collider, if supersymmetry is realized in nature.

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I. INTRODUCTION

The minimal supersymmetric standard model (MSSM) is considered as one of the most appealing extensions of the standard model of strong and electroweak interactions. This theory has a variety of appealing characteristics, including solutions to the hierarchy problem and a dark-matter candidate. However, at the renormalizable level, the MSSM Lagrangian contains flagrant baryon- and lepton-number–violating operators, the most infamous of which lead to rapid proton decay (see Ref. [1] for a review on supersymmetry (SUSY) and Ref. [2] for the study of the proton decay issue in SUSY).

The most common approach to this problem is the introduction of a discrete symmetry, R parity, defined as $R = (-1)^{3(B-L)+2S}$, where B, L, and S are baryon number, lepton number, and spin, respectively (see Ref. [3] for a review on R-parity violation). The conservation of R parity also ensures that the lightest SUSY particle is stable and, therefore, a cold dark-matter candidate. While R parity is closely linked to $B-L$, they are not synonymous. Specifically, R parity allows for terms that break $B-L$ by an even amount. For general arguments on R-parity conservation, see Refs. [4,5].

Theories with local $B-L$ symmetries help shed light on R parity. R parity is an exact symmetry, as long as the same is true for $B-L$. Breaking $B-L$ by a field with even charge (the canonical $B-L$ model) guarantees automatic R-parity conservation even below the symmetry scale, since only $B-L$ violation by an even amount is allowed. An alternative is $B-L$ breaking through the right-handed sneutrino, a field which must always be included due to anomaly cancellation. Since the right-handed sneutrino has a charge of one, its vacuum expectation value (VEV) results in spontaneous R-parity violation. Phenomenologically, this is a viable scenario that does not induce tree-level rapid proton decay, and dark matter is still possible if the gravitino is the lightest SUSY particle.

Recently, spontaneous R-parity violation has been studied in the case of minimal $B-L$ models [6–10]. However, the following question is still relevant: *Does the canonical $B-L$ model favor R-parity conservation or violation?*. In this paper, we study this question in the simplest local $U(1)_{B-L}$ extension of the MSSM, assuming, for simplicity, minimal supergravity (mSUGRA) boundary conditions for the soft terms. We investigate the fate of R parity using the radiative symmetry-breaking mechanism (RSBM) and show that, for the majority of the parameter space, R parity is broken; namely, it is the right-handed sneutrino that acquires a negative mass squared and, therefore, a VEV. This is a surprising result that, at the very least, questions the feasibility of conserving R parity in such a framework. These results are quite general and apply to any SUSY theory where $B-L$ is part of the gauge symmetry.

II. THEORETICAL FRAMEWORK

We investigate the possible connection between the RSBM and the fate of R parity in the simplest $B-L$ model, based on the gauge group

$$SU(3) \bigotimes SU(2)_L \bigotimes U(1)_Y \bigotimes U(1)_{B-L},$$

with particle content listed in Table I.

The most general superpotential is given by

$$\mathcal{W} = \mathcal{W}_{\rm MSSM} + \mathcal{W}_{B-L}, \tag{1}$$

TABLE I. $SU(2)_L \otimes U(1)_Y \otimes U(1)_{B-L}$ charges for the particle content.

Field	$SU(2)_L$	$U(1)_Y$	$U(1)_{B-L}$
$\hat{Q} = (\hat{u}, \hat{d})$	2	1/6	1/3
$\hat{u}^c$	1	−2/3	−1/3
$\hat{d}^c$	1	1/3	−1/3
$\hat{L} = (\hat{\nu}, \hat{e})$	2	−1/2	−1
$\hat{e}^c$	1	1	1
$\hat{\nu}^c$	1	0	1
$\hat{H}_u = (\hat{H}_u^+, \hat{H}_u^0)$	2	1/2	0
$\hat{H}_d = (\hat{H}_d^0, \hat{H}_d^-)$	2	−1/2	0
$\hat{X}$	1	0	−2
$\hat{\bar{X}}$	1	0	2

$$\mathcal{W}_{\rm MSSM} = Y_u \hat{Q}\hat{H}_u\hat{u}^c + Y_d\hat{Q}\hat{H}_d\hat{d}^c + Y_e\hat{L}\hat{H}_d\hat{e}^c + \mu\hat{H}_u\hat{H}_d, \qquad (2)$$

$$\mathcal{W}_{B-L} = Y_\nu\hat{L}\hat{H}_u\hat{\nu}^c + f\hat{\nu}^c\hat{\nu}^c\hat{X} - \mu_X\hat{X}\hat{\bar{X}}, \qquad (3)$$

and the corresponding soft SUSY-breaking Lagrangian is

$$-\mathcal{L}_{\rm Soft} \supset \left(a_\nu\tilde{L}H_u\tilde{\nu}^c - a_X\tilde{\nu}^c\tilde{\nu}^c X - b_X X\bar{X} + \tfrac{1}{2}M_{BL}\tilde{B}'\tilde{B}' + h.c.\right) + m_X^2|X|^2 + m_{\bar{X}}^2|\bar{X}|^2 + m_{\tilde{\nu}^c}^2|\tilde{\nu}^c|^2, \qquad (4)$$

where we have suppressed flavor and group indices, and $\tilde{B}'$ is the $B-L$ gaugino.

Spontaneous $B-L$ violation requires either the VEV of X, $\bar{X}$, or $\tilde{\nu}^c$ to be nonzero; however, the fate of R parity lies solely in the VEV of $\tilde{\nu}^c$: $\langle\tilde{\nu}^c\rangle = 0$ corresponds to R-parity conservation, while $\langle\tilde{\nu}^c\rangle \neq 0$ indicates spontaneous R-parity violation. Addressing the values of these VEVs requires the minimization conditions that can be derived from the full potential, where $(\langle X\rangle, \langle\bar{X}\rangle, \langle\tilde{\nu}^c\rangle) = 1/\sqrt{2}(x, \bar{x}, n)$[1]:

$$\langle V\rangle = \langle V_F\rangle + \langle V_D\rangle + \langle V_{\rm Soft}\rangle, \qquad (5)$$

$$\langle V_F\rangle = \frac{1}{4}f^2n^4 + f^2n^2x^2 + \frac{1}{2}\mu_X^2(x^2+\bar{x}^2) - \frac{1}{\sqrt{2}}f\mu_X n^2\bar{x}, \qquad (6)$$

$$\langle V_D\rangle = \tfrac{1}{32}g_{BL}^2(2\bar{x}^2 - 2x^2 + n^2)^2, \qquad (7)$$

$$\langle V_{\rm Soft}\rangle = -\frac{1}{\sqrt{2}}a_X n^2 x - b_X x\bar{x} + \frac{1}{2}m_X^2x^2 + \frac{1}{2}m_{\bar{X}}^2\bar{x}^2 + \frac{1}{2}m_{\tilde{\nu}^c}^2n^2. \qquad (8)$$

Only two cases exist for spontaneous $B-L$ symmetry breaking: Case (1) $n=0$; $x, \bar{x} \neq 0$, implying R-parity conservation; or Case (2) $x, \bar{x}, n \neq 0$, implying spontaneous R-parity violation. Note that a third case, $n \neq 0$; $x, \bar{x} = 0$, cannot exist due to the linear term for x in Eq. (8) and for $\bar{x}$ in Eq. (6), which always induce a VEV for these fields.

(1) Case (1): *R-Parity Conservation*
This is the traditional case studied in the literature. The minimization conditions for x and $\bar{x}$ are very similar in form to those of v_u and v_d in the MSSM:

$$\frac{1}{2}M_{Z'}^2 = -|\mu_X|^2 + \frac{m_X^2\tan^2 z - m_{\bar{X}}^2}{1-\tan^2 z}, \qquad (9)$$

where $\tan z \equiv x/\bar{x}$, and $M_{Z'}^2 \equiv g_{BL}^2(x^2+\bar{x}^2)$, which is the mass for the Z' boson associated with broken $B-L$.

To attain a better understanding of the situation, let us examine Eq. (9) in the limit $x \gg \bar{x}$, with $m_X^2 < 0$ and $m_{\bar{X}}^2 > 0$, so that it reduces to

$$\tfrac{1}{2}M_{Z'}^2 = -|\mu_X|^2 - m_X^2. \qquad (10)$$

Since the left-hand side is positive definite, *the relationship* $-m_X^2 > |\mu_X|^2$ *must be obeyed for spontaneous* $B-L$ *violation*; a tachyonic m_X^2 is not enough. This relationship between μ_X and m_X is similar to the relationship in the MSSM between μ and m_{H_u}, a relationship typically referred to as the μ problem; i.e., why is μ of the order of the SUSY mass scale? Then, in Case (1), in addition to the MSSM μ problem, we have introduced a new μ problem for μ_X.
As can be seen from Eq. (10), x is of the order of the SUSY mass scale or about a TeV. Replacing X by its VEV in the term $f\nu^c\nu^c X$ in the superpotential leads to the heavy Majorana mass term for the right-handed neutrinos and, ultimately, to the Type I seesaw mechanism [11] for neutrino masses:

$$m_\nu = v_u^2 Y_\nu^T(fx)^{-1}Y_\nu. \qquad (11)$$

Since the masses of the right-handed neutrinos are of order TeV, realistic neutrino masses require $Y_\nu \sim 10^{-6-7}$. The rest of the spectrum is given in Appendix B.

(2) Case (2): *R-Parity Violation*
Evaluation of the minimization conditions, in this case, is illuminating in the limits $n \gg x, \bar{x}, a_X$ and $g_{BL}^2 \ll 1$, which will prove to be the case of interest in the numerical section:

$$n^2 = \frac{(-m_{\tilde{\nu}^c}^2)\Lambda_{\bar{X}}^2}{f^2m_{\bar{X}}^2 + \frac{1}{8}g_{BL}^2\Lambda_{\bar{X}}^2}, \qquad (12)$$

$$\bar{x} = \frac{(-m_{\tilde{\nu}^c}^2)f\mu_X}{\sqrt{2}(f^2m_{\bar{X}}^2 + \frac{1}{8}g_{BL}^2\Lambda_{\bar{X}}^2)}, \qquad (13)$$

$$x = \frac{(-m_{\tilde{\nu}^c}^2)[a_X\Lambda_{\bar{X}}^2 + fb_X\mu_X]}{(2f^2 - \frac{1}{4}g_{BL}^2)(-m_{\tilde{\nu}^c}^2)\Lambda_{\bar{X}}^2 + f^2m_{\bar{X}}^2\Lambda_X^2 + \frac{1}{8}g_{BL}^2\Lambda_{\bar{X}}^2\Lambda_X^2}, \qquad (14)$$

where $\Lambda_X^2 \equiv \mu_X^2 + m_X^2$, and $\Lambda_{\bar{X}}^2 \equiv \mu_X^2 + m_{\bar{X}}^2$.

[1] Technically, the left-handed sneutrino has a VEV, as well, but in order to generate the correct neutrino masses, this VEV must be quite small compared to the others and, so, can safely be ignored here [7].

These equations indicate several things: *spontaneous $B-L$ symmetry breaking in the R-parity–violating case only requires* $m^2_{\tilde{\nu}^c} < 0$ and does not introduce a new μ problem, so that μ_X can be larger than the TeV scale; that x and $\bar{x}$ are triggered by linear terms, since they go as these linear terms suppressed by the effective mass squared; and all VEVs increase with μ_X up to a point, after which n asymptotes, while x and $\bar{x}$ decrease as $1/\mu_x$. The $\mu_X \to \infty$ serves as a decoupling limit, since $x, \bar{x} \to 0$, and $n^2 \to -8m_{\tilde{\nu}^c}/g^2_{BL}$, as in the minimal model [7]. Neutrino masses, in this case, will have a more complicated form that will depend both on the Type I seesaw contribution and an R-parity contribution, although the bounds on Y_ν are similar to Case (1). The Z' mass, in this case, is

$$M^2_{Z'} = \frac{1}{4} g^2_{BL}(n^2 + 4x^2 + 4\bar{x}^2), \tag{15}$$

and the rest of the spectrum is given in Appendix B.

The important question now becomes: are either of these cases possible from the perspective of the RSBM? Specifically, will running from some SUSY-breaking boundary conditions drive either X or $\tilde{\nu}^c$ tachyonic, or neither? To answer this, we adopt the mSUGRA ansatz motivated by the fact that gravity is one of the simplest ways to transmit SUSY breaking [12]. The following boundary conditions are valid at the grand unified theory (GUT) scale:

$$m^2_X = m^2_{\bar{X}} = m^2_{\tilde{\nu}^c_i} = \ldots = m^2_0, \tag{16}$$

$$A_X = f A_0; A_\nu = Y_\nu A_0; \ldots, \tag{17}$$

$$M_{BL} = \ldots = M_{1/2}, \tag{18}$$

where ... indicates MSSM parameters.

The necessary renormalization group equations (RGEs), derived using [13], will only be functions of the couplings beyond the MSSM, since Y_ν is small enough to be neglected. We assume that g_{BL} unifies with the other gauge couplings at the GUT scale and use the SO(10) GUT renormalization factor, $\sqrt{3/8}$. In the approximation $f_3 = f \gg f_1, f_2$, and the RGEs are given by[2]

$$16\pi^2 \frac{dm^2_{\tilde{\nu}^c}}{dt} = [8f^2 X_X - 3g^2_{BL}M^2_{BL}], \tag{19}$$

$$16\pi^2 \frac{dm^2_X}{dt} = [4f^2 X_X - 12g^2_{BL}M^2_{BL}], \tag{20}$$

$$16\pi^2 \frac{dm^2_{\bar{X}}}{dt} = -12g^2_{BL}M^2_{BL}, \tag{21}$$

where $t = \ln\mu$, and $X_X \equiv m^2_X + 2m^2_{\tilde{\nu}^c} + 4a^2_X$. See Appendix A for the full set of RGEs, including the contributions from three families of right-handed neutrinos.

Experience from radiative electroweak symmetry breaking in the MSSM [15] indicates that Yukawa terms in the beta functions tend to drive the masses squared negative, while gaugino terms do the opposite. Because of its smaller $B-L$ charge, $\tilde{\nu}^c$ has the smallest gaugino factor while also having the largest Yukawa factor. Since, in mSUGRA, all of these fields have the same mass at the GUT scale, it is clear that $m^2_{\tilde{\nu}^c}$ will evolve to the smallest value in the simple one-family approximation. When including all three families, m^2_X gets an enhancement from a trace of f, Eq. (A10), which could lead to it being tachyonic and, therefore, to R-parity conservation. The question of whether the RSBM is possible, as well as the fate of R parity throughout the parameter space, will be addressed numerically in the next section. It is important to mention that one gets only bilinear interactions, which violate R parity after symmetry breaking. For details, see Refs. [6–10].

III. R PARITY: CONSERVATION OR VIOLATION?

In addition to addressing the feasibility of the RSBM, in general, and the fate of R parity, specifically, it would also be prudent to identify the part of parameter space that leads to a realistic spectrum. One strong experimental constraint is the bound on the Z' mass: $M_{Z'}/g_{BL} > 5$ TeV [16], indicating the need for a large mass scale, independent of the fate of R parity, and translates into a large value for m_0 at the GUT scale.

The hyperbolic branch/focus point region of mSUGRA [17] allows for such large m_0 without too much fine-tuning in the MSSM Higgs sector and naturally leads to a slight hierarchy between the electroweak scale and the $B-L$

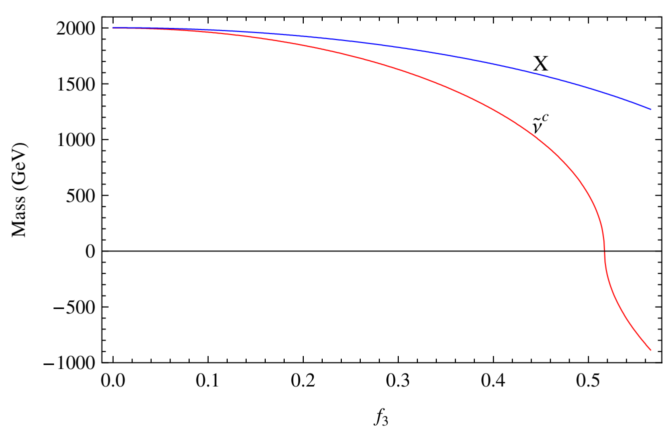

FIG. 1 (color online). Soft masses in the form $\mathrm{sign}(m^2_\phi)|m_\phi|$ for X (blue line) and $\tilde{\nu}^c$ (red line) versus f_3, for $m_0 = 2000$ GeV, $M_{1/2} = 200$ GeV, $A_0 = 0$, and negligible f_1 and f_2. The RSBM is possible for $f_3 \gtrsim 0.51$ and spontaneous R-parity violation.

[2] We would like to note that our results are in disagreement with the results in Ref. [14].

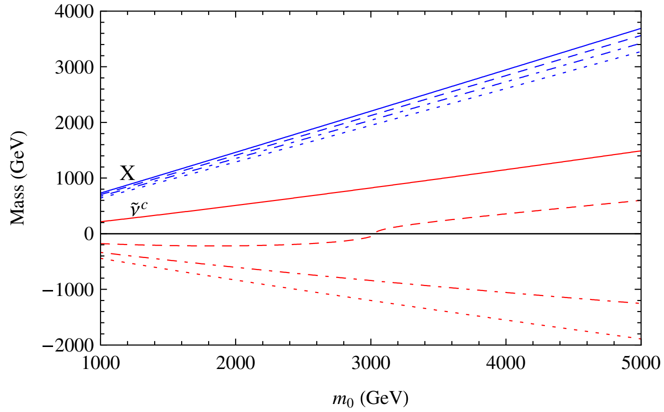

FIG. 2 (color online). Soft masses in the form $\mathrm{sign}(m_\phi^2)|m_\phi|$ for X (blue lines) and $\tilde{\nu}^c$ (all red lines that appear below $\tilde{\nu}^c$) versus m_0 for $f_3 = 0.5$ (solid lines), 0.52 (dashed lines), 0.54 (dot-dashed lines), and 0.56 (dotted lines); all other parameters are the same as in Fig. 1.

scale. Also, the approximations made in the previous section for Case (2) are valid. Values for $\tan\beta$ and f are inputted at the SUSY scale, and we assume that g_{BL} unifies with the other gauge couplings at the GUT scale. We find that A_0 has very little effect and, therefore, set it to zero. The elecrtoweak symmetry-breaking minimization conditions are solved for μ and B, and we assumed that $B_X = B$ at the GUT scale, where $b_X = B_X \mu_X$. Specifying μ_X then determines the spectrum.

Calculating the soft masses of X and $\tilde{\nu}^c$ with increasing f_3 yields Fig. 1, for $m_0 = 2000$ GeV, $M_{1/2} = 200$ GeV, $A_0 = 0$, and negligible f_1 and f_2. As expected, in the $f_1, f_2 \ll f_3$ limit, only the $\tilde{\nu}^c$ mass becomes tachyonic, so, while the RSBM can be successful, it leads to spontaneous R-parity breaking. Note that f_3 exhibits fixed-point-like behavior (as discussed in a similar scenario in [18]). This means that its range, allowing for the RSBM, corresponds to a larger range of values at the GUT scale. In Fig. 2, the X and $\tilde{\nu}^c$ soft masses for different values f_3 versus m_0, with all other parameters, are the same as in Fig. 1. It indicates that the m_0 parameter also plays an important role in determining the overall size of the tachyonic mass and, therefore, the Z' mass, and can even derail the RSBM for lower values of f_3.

For $f_1 \sim f_2 \sim f_3$, the Yukawa term in the RGE for m_X^2 is effectively enhanced by a factor of 3; see Eq. (A10) as compared to Eq. (20), which can lead to an R-parity–conserving minimum, since no such factor appears for $m_{\tilde{\nu}^c}^2$. We show these effects in Fig. 3, where red dots indicate spontaneous R-parity violation; blue squares show the region of R-parity conservation in the $f_2 - f_1$ plane for $f_3 = 0.4$ (panel a) and $f_3 = 0.55$ (panel b); and $m_0 = 2000$ GeV, $M_{1/2} = 200$ GeV, and $A_0 = 0$. In Fig. 3(a), f_1 or $f_2 \sim 0.52$ is needed for the RSBM, while only $f_1 \sim f_2 \gtrsim 0.4$ allows for R-parity conservation (there is about a 50-50 split between R-parity conservation and violation). If f_1 or $f_2 > 0.52$, these couplings are no longer perturbative at the GUT scale. As one increases the value of f_3, the R-parity–conserving points disappear, as reflected in Fig. 3(b). In this case, f_1 or $f_2 \gtrsim 0.4$ leads to nonperturbative values at the GUT scale, due to the larger value of f_3.

The graphs in Fig. 3 are a bit misleading, since they are just slices of the three-dimensional space $f_1 - f_2 - f_3$, which is displayed in Figs. 4 and 5, with the same legend as the former figure. The points sit on a shell that roughly composes one-eighth of a cube, with sides of about length one. Below the shell, the RSBM is not possible, due to the small values of f, while those outside are not perturbative up to the GUT scale. The majority of the parameter space that allows for the RSBM is dominated by

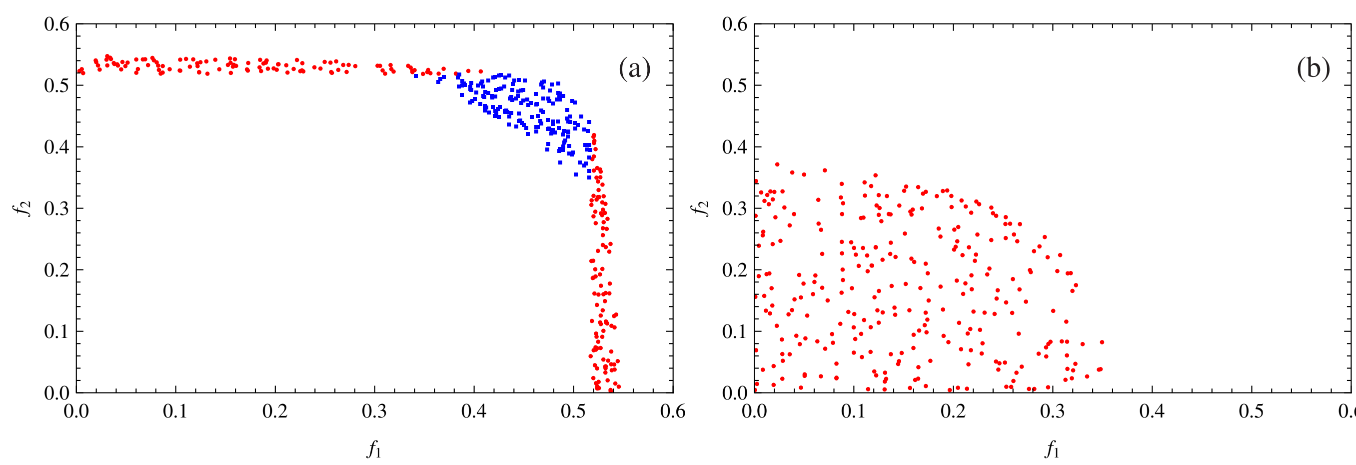

FIG. 3 (color online). The state of the $B - L$ breaking vacuum in the $f_2 - f_1$ plane, with $m_0 = 2000$ GeV, $M_{1/2} = 200$ GeV, and $A_0 = 0$ for $f_3 = 0.4$ (panel a) and $f_3 = 0.55$ (panel b). Blue squares indicate R-parity conservation, while red dots indicate R-parity violation. In (a), the empty space below the curve indicates no RSBM, while, in both graphs, in the space above the curves, the f's are no longer perturbative at the GUT scale. In (a), there is about an even number of R-parity–conserving and violating vacua, but increasing f_3 tips the favor towards R-parity violation and eventually only allows for R-parity violation, as in (b).

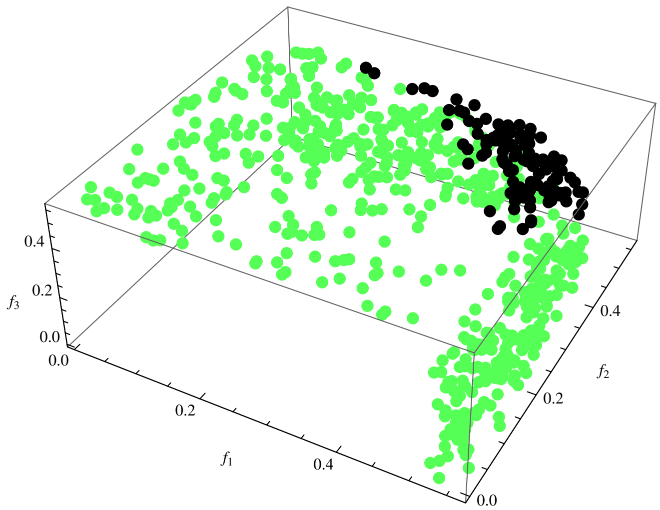

FIG. 4 (color online). The state of the $B-L$ breaking vacuum in the $f_1-f_2-f_3$ space, with $m_0 = 2000$ GeV, $M_{1/2} = 200$ GeV, and $A_0 = 0$. Black dots indicate *R*-parity conservation, while green (light gray) dots indicate *R*-parity violation; the latter appears 5 times more often. The key point is that only fairly degenerate values of f (and, therefore, the right-handed neutrinos) allow for *R*-parity conservation. We have checked that all physical masses are positive in these cases.

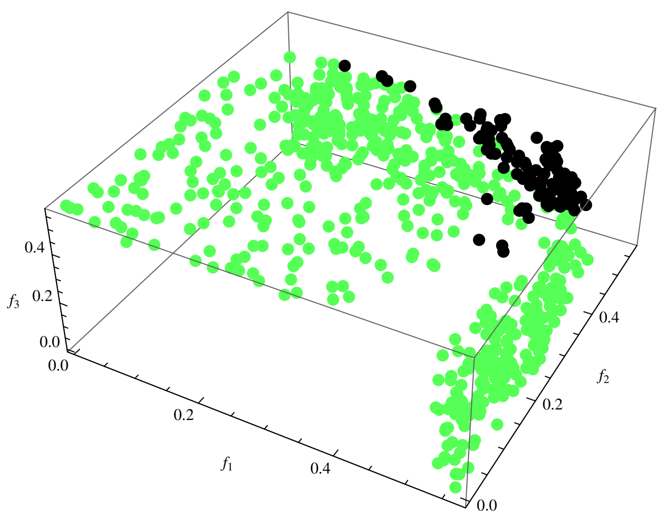

FIG. 5 (color online). The state of the $B-L$ breaking vacuum in the $f_1-f_2-f_3$ space, with $m_0 = 5000$ GeV, $M_{1/2} = 500$ GeV, and $A_0 = 0$. The legend and results are the same as those in Fig. 4.

R-parity violation (5 times more prevalent), while only $f_1 \sim f_2 \sim f_3$ allows for *R*-parity conservation. These last figures summarize the findings of this paper: when the RSBM is realized, the *R*-parity–breaking vacuum is more probable than the *R*-parity–conserving one, especially when a hierarchy exists within the f matrix. Only when this matrix is fairly degenerate (degenerate right-handed neutrinos) does the running allow for *R*-parity conservation.

IV. SUMMARY

The possible origin of the *R*-parity–violating interactions in the MSSM and its connection to the radiative symmetry-breaking mechanism has been investigated in the simplest possible model. We have found that, in the majority of the parameter space, *R* parity is spontaneously broken at the low scale, and the soft SUSY mass scale defines the $B-L$ and *R*-parity–breaking scales. These results can be achieved in any extension of the MSSM where $B-L$ is part of the gauge symmetry and only bilinear *R*-parity–violating interactions are generated. The main result of this paper hints at the possibility that *R*-parity–violating processes will be observed at the Large Hadron Collider, if supersymmetry is discovered.

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APPENDIX A: RENORMALIZATION GROUP EQUATIONS

We present first the gamma functions, which are useful for deriving the RGEs. Here, $i = 1, 2, 3$.

$$\gamma_X = \frac{1}{16\pi^2}(2\,\mathrm{Tr} f^2 - 3g_{BL}^2), \tag{A1}$$

$$\gamma_{\bar{X}} = \frac{1}{16\pi^2}(-3g_{BL}^2), \tag{A2}$$

$$\gamma_{\nu_i^c} = \frac{1}{16\pi^2}\left(4f_i^2 - \frac{3}{4}g_{BL}^2\right), \tag{A3}$$

where repeated indices are not summed, and $f = \mathrm{diag}(f_1, f_2, f_3)$, since f can always be diagonalized by rotating the right-handed neutrino fields. The same holds true here for a_X, due to the mSUGRA ansatz.

The RGEs are given by

$$16\pi^2 \frac{dg_{BL}}{dt} = 9g_{BL}^3, \tag{A4}$$

$$16\pi^2 \frac{df_i}{dt} = f_3\left(8f_i^2 + 2\,\mathrm{Tr} f^2 - \frac{9}{2}g_{BL}^2\right), \tag{A5}$$

$$16\pi^2 \frac{dM_{BL}}{dt} = 18g_{BL}^2 M_{BL}, \tag{A6}$$

$$16\pi^2 \frac{da_{X_i}}{dt} = f_X[16 f_i a_{X_i} + 4\mathrm{Tr}(f a_X) - 9 g_{BL}^2 M_{BL}] \quad \text{(A7)}$$

$$+ a_{X_i}\left(8 f_i^2 + 2\,\mathrm{Tr} f^2 - \frac{9}{2} g_{BL}^2\right), \quad \text{(A8)}$$

$$16\pi^2 \frac{dm_{\bar{X}}^2}{dt} = -12 g_{BL}^2 M_{BL}^2, \quad \text{(A9)}$$

$$16\pi^2 \frac{dm_X^2}{dt} = [4\,\mathrm{Tr} f^2 m_X^2 + 8\,\mathrm{Tr}(f^2 m_{\tilde{\nu}^c}^2) + 4\,\mathrm{Tr} a_X^2 - 12 g_{BL}^2 M_{BL}^2], \quad \text{(A10)}$$

$$16\pi^2 \frac{dm_{\tilde{\nu}_i^c}^2}{dt} = [8 f_i^2 (m_X^2 + 2 m_{\tilde{\nu}_i^c}^2) + 8 a_{X_i}^2 - 3 g_{BL}^2 M_{BL}^2]. \quad \text{(A11)}$$

APPENDIX B: SPECTRUM

In calculating the following spectrum, we assume that $\langle \tilde{\nu}_3^c, X, \bar{X} \rangle = \frac{1}{\sqrt{2}}(n, x, \bar{x})$, and all others are zero. The pseudoscalar mass matrix in the basis $\mathrm{Im}(\tilde{\nu}_3^c, X, \bar{X})$ is

$$\mathcal{M}_P = \begin{pmatrix} 2\sqrt{2}(a_X x + f_3 \mu_X \bar{x}) & \sqrt{2} a_X n & -\sqrt{2} f_3 \mu_X n \\ \sqrt{2} a_X n & \frac{a_X n^2 + \sqrt{2} b_X \bar{x}}{\sqrt{2} x} & b_X \\ -\sqrt{2} f_3 n \mu_X & b_X & \frac{f_3 \mu_X n^2 + \sqrt{2} b_X x}{\sqrt{2} \bar{x}} \end{pmatrix}. \quad \text{(B1)}$$

The scalar mass matrix in the basis $\mathrm{Re}(\tilde{\nu}_3^c, X, \bar{X})$ is

$$\mathcal{M}_S = \begin{pmatrix} \left(2 f_3^2 + \frac{1}{4} g_{BL}^2\right) n^2 & \left(4 f_3^2 - \frac{1}{2} g_{BL}^2\right) n x - \sqrt{2} a_X n & -\sqrt{2} f_3 \mu_x n + \frac{1}{2} g_{BL}^2 n \bar{x} \\ \left(4 f_3^2 - \frac{1}{2} g_{BL}^2\right) n x - \sqrt{2} a_X n & \frac{a_3 n^2 + \sqrt{2} b_X \bar{x}}{\sqrt{2} x} + g_{BL}^2 x^2 & -b_X - g_{BL}^2 x \bar{x} \\ -\sqrt{2} f_3 \mu_x n + \frac{1}{2} g_{BL}^2 n \bar{x} & -b_X - g_{BL}^2 x \bar{x} & \frac{f_3 \mu_X n^2 + \sqrt{2} b_X x}{\sqrt{2} \bar{x}} + g_{BL}^2 \bar{x}^2 \end{pmatrix}. \quad \text{(B2)}$$

The neutralino mass matrix in the basis $(B', \nu^c, \tilde{X}, \tilde{\bar{X}})$ is

$$\mathcal{M}_{\chi^0} = \begin{pmatrix} M_{BL} & \frac{1}{2} g_{BL} n & -g_{BL} x & g_{BL} \bar{x} \\ \frac{1}{2} g_{BL} n & \sqrt{2} f_3 x & \sqrt{2} f_3 n & 0 \\ -g_{BL} x & \sqrt{2} f_3 n & 0 & -\mu_X \\ g_{BL} \bar{x} & 0 & -\mu_X & 0 \end{pmatrix}. \quad \text{(B3)}$$

The sfermion mass, with matrices in the basis $(\tilde{f}_L, \tilde{f}_R)$, is

$$\mathcal{M}_{\tilde{u}}^2 = \begin{pmatrix} m_{\tilde{Q}}^2 + m_u^2 - \frac{1}{8}(g_2^2 - \frac{1}{3} g_1^2)(v_u^2 - v_d^2) + \frac{1}{3} D_{BL} & \frac{1}{\sqrt{2}}(a_u v_u - Y_u \mu v_d) \\ \frac{1}{\sqrt{2}}(a_u v_u - Y_u \mu v_d) & m_{\tilde{u}^c}^2 + m_u^2 - \frac{1}{6} g_1^2 (v_u^2 - v_d^2) - \frac{1}{3} D_{BL} \end{pmatrix}, \quad \text{(B4)}$$

$$\mathcal{M}_{\tilde{d}}^2 = \begin{pmatrix} m_{\tilde{Q}}^2 + m_d^2 + \frac{1}{8}\left(g_2^2 + \frac{1}{3} g_1^2\right)(v_u^2 - v_d^2) + \frac{1}{3} D_{BL} & \frac{1}{\sqrt{2}}(Y_d \mu v_u - a_d v_d) \\ \frac{1}{\sqrt{2}}(Y_d \mu v_u - a_d v_d) & m_{\tilde{d}^c}^2 + m_d^2 + \frac{1}{12} g_1^2 (v_u^2 - v_d^2) - \frac{1}{3} D_{BL} \end{pmatrix}, \quad \text{(B5)}$$

$$\mathcal{M}_{\tilde{e}}^2 = \begin{pmatrix} m_{\tilde{L}}^2 + m_e^2 + \frac{1}{8}(g_2^2 - g_1^2)(v_u^2 - v_d^2) - D_{BL} & \frac{1}{\sqrt{2}}(Y_e \mu v_u - a_e v_d) \\ \frac{1}{\sqrt{2}}(Y_e \mu v_u - a_e v_d) & m_{\tilde{e}^c}^2 + m_e^2 + \frac{1}{4} g_1^2 (v_u^2 - v_d^2) + D_{BL} \end{pmatrix}, \quad \text{(B6)}$$

$$m_{\tilde{\nu}_L}^2 = m_{\tilde{L}}^2 - \frac{1}{8}(g_2^2 + g_1^2)(v_u^2 - v_d^2) - D_{BL}, \quad \text{(B7)}$$

$$m_{\tilde{N}_{I_i}}^2 = m_{\tilde{\nu}_i^c}^2 + 2 f_i^2 x^2 - f_i f_3 n^2 + \sqrt{2} a_{X_i} x + \sqrt{2} f_i \mu_X \bar{x} + D_{BL}, \quad \text{(B8)}$$

$$m_{\tilde{N}_{R_i}}^2 = m_{\tilde{\nu}_i^c}^2 + 2 f_i^2 x^2 + f_i f_3 n^2 - \sqrt{2} a_{X_i} x - \sqrt{2} f_i \mu_X \bar{x} + D_{BL}, \quad \text{(B9)}$$

where $D_{BL} \equiv \frac{1}{8} g_{BL}^2 (2\bar{x}^2 - 2x^2 + n^2)$; m_u, m_d, and m_e are the respective fermion masses; and a_u, a_d, and a_e are the trilinear a terms corresponding to the Yukawa couplings

Y_u, Y_d, and Y_e. The right-handed sneutrino eigenstates are the scalars $\tilde{N}_{R_i}$, and the pseudoscalars are $\tilde{N}_{I_i}$, where i runs only over the first two generations, and repeated indices are not summed. The third generation mixes with the Higgses, Eqs. (B1) and (B2). The above masses are for *R*-parity violation, Case (2) from the text. For the *R*-parity–conserving case, Case (1), take the limit $n \to 0$; the $B - L$ Higgs masses are given by the lower two-by-two block matrices of Eqs. (B1) and (B2); and i in Eqs. (B8) and (B9) runs over all three generations.

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