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Minimal flipped $SO(10) \otimes U(1)$ supersymmetric Higgs model

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We investigate the conditions on the Higgs sector that allow supersymmetric $SO(10)$ grand unified theories to break spontaneously to the standard electroweak model at the renormalizable level. If one considers Higgs representations of dimension up to the adjoint, a supersymmetric standard model vacuum requires, in most cases, the presence of nonrenormalizable operators. The active role of Planck-induced nonrenormalizable operators in the breaking of the gauge symmetry introduces a hierarchy in the mass spectrum at the grand unified theory scale that may be an issue for gauge unification and proton decay. We show that the minimal Higgs scenario that allows for a renormalizable breaking to the standard model is obtained by considering flipped $SO(10) \otimes U(1)$ with one adjoint (45_H) and two pairs of $16_H \oplus \overline{16}_H$ Higgs representations. We consider a nonanomalous matter content and discuss the embedding of the model in an E_6 grand unified scenario just above the flipped $SO(10)$ scale.

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I. INTRODUCTION

It has been shown recently [1,2] that quantum effects solve the long-standing issue [3] of the incompatibility between the dynamics of the simplest Higgs sectors in the renormalizable nonsupersymmetric $SO(10)$ grand unified theory (GUT) and the gauge unification constraints. In particular, such minimal grand unified scenarios not only support viable $SO(10)$ breaking patterns passing through intermediate $SU(4)_C \otimes SU(2)_L \otimes U(1)_R$ or $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ gauge symmetries [or their $SU(3)_c \otimes SU(2)_L \otimes U(1)_R \otimes U(1)_{B-L}$ intersection], but they also include all the ingredients necessary for a potentially realistic description of the standard model (SM) flavor structure.

On the other hand, the simplest scenario featuring the Higgs scalars in $10_H \oplus 16_H \oplus 45_H$ of $SO(10)$ fails when addressing the neutrino spectrum: in nonsupersymmetric models, the $B-L$ breaking scale M_{B-L} turns out to be generally a few orders of magnitude below the GUT scale M_G. Thus, the scale of the right-handed (RH) neutrino masses $M_N \sim M_{B-L}^2/M_P$ emerging first at the $d=5$ level from an operator of the form $16_F^2(16_H^*)^2/M_P$ (with M_P typically identified with the Planck scale) undershoots by orders of magnitude the range of about 10^{12} to 10^{14} GeV naturally suggested by the seesaw mechanism. The same effective result is obtained in the nonsupersymmetric case within the radiative seesaw scheme [4].

This issue can be somewhat alleviated by considering 126_H in place of 16_H in the Higgs sector, since in such a case the neutrino masses can be generated at the renormalizable level by the term $16_F^2 126_H^*$. This lifts the problematic M_{B-L}/M_P suppression factor inherent to the $d=5$ effective mass and yields $M_N \sim M_{B-L}$, which might be, at least in principle, acceptable. This scenario, though conceptually simple, cf. [2], involves a detailed one-loop analysis of the scalar potential governing the dynamics of the $10_H \oplus 126_H \oplus 45_H$ Higgs sector that, to our knowledge, still remains to be done.

Invoking TeV-scale supersymmetry (SUSY), the qualitative picture changes dramatically. Indeed, the gauge running within the MSSM prefers M_{B-L} in the proximity of M_G and, hence, the Planck-suppressed $d=5$ RH neutrino mass operator $16_F^2\overline{16}_H^2/M_P$, available whenever $16_H \oplus \overline{16}_H$ is present in the Higgs sector, can naturally reproduce the desired range for M_N. Let us recall that both 16_H and $\overline{16}_H$ are required in order to retain SUSY below the GUT scale.

On the other hand, it is well known [5–7] that the relevant superpotential does not support, at the renormalizable level, a supersymmetric breaking of the $SO(10)$ gauge group to the SM. This is due to the constraints on the vacuum manifold imposed by the F- and D-flatness conditions which, apart from linking the magnitudes of the $SU(5)$-singlet 16_H and $\overline{16}_H$ vacuum expectation values (VEVs), make the adjoint VEV $\langle 45_H\rangle$ aligned with $\langle 16_H\overline{16}_H\rangle$. As a consequence, an $SU(5)$ subgroup of the initial $SO(10)$ gauge symmetry remains unbroken. In this respect, a renormalizable Higgs sector with $126_H \oplus \overline{126}_H$ in place of $16_H \oplus \overline{16}_H$ suffers from the same "$SU(5)$ lock," because also in $\overline{126}_H$ the SM-singlet direction is $SU(5)$ invariant.

This issue can be addressed by giving up renormalizability. However, this option may be rather problematic since it introduces a delicate interplay between physics

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at two different scales, $M_G \ll M_P$, with the consequence of splitting the GUT-scale thresholds over several orders of magnitude around M_G. This may affect proton decay as well as the SUSY gauge unification, and may force the $B - L$ scale below the GUT scale. The latter is harmful for the setting with $16_H \oplus \overline{16}_H$ relying on a $d = 5$ RH neutrino mass operator. The models with $126_H \oplus \overline{126}_H$ are also prone to trouble with gauge unification, due to the number of large Higgs multiplets spread around the GUT scale.

Thus, in none of the cases above does the simplest conceivable $SO(10)$ Higgs sector, spanned over the smallest irreducible representations (up to the adjoint), offer a natural scenario for realistic model building. Since the option of a simple GUT-scale Higgs dynamics involving small representations governed by a simple renormalizable superpotential is particularly attractive, we aimed at studying the conditions under which the seemingly ubiquitous $SU(5)$ lock can be overcome, while keeping only spinorial and adjoint $SO(10)$ representations.

Let us emphasize that the assumption that the gauge symmetry breaking is driven by the renormalizable part of the Higgs superpotential does not clash with the fact that, in models with $16_H \oplus \overline{16}_H$, the neutrino masses are generated at the nonrenormalizable level, and other fermions may be sensitive to physics beyond the GUT scale. As far as symmetry breaking is concerned, Planck-induced $d \geq 5$ effective interactions are irrelevant perturbations in this picture.

The simplest attempt to break the $SU(5)$ lock by doubling either $16_H \oplus \overline{16}_H$ or 45_H in order to relax the F-flatness constraints is easily shown not to work. In the former case, there is only one SM-singlet field direction associated with each of the $16_H \oplus \overline{16}_H$ pairs. Thus, F flatness makes the VEVs in 45_H align along this direction regardless of the number of $16_H \oplus \overline{16}_H$'s contributing to the relevant F term, $\partial W/\partial 45_H$ (see, for instance, Eq. (6) in Ref. [7]). Doubling the number of 45_H's does not help either. Since there is no mixing among the 45's besides the mass term, F flatness aligns both $\langle 45_H \rangle$'s in the $SU(5)$ direction of $16_H \oplus \overline{16}_H$. For three (and more) adjoints a mixing term of the form $45_1 45_2 45_3$ is allowed, but it turns out to be irrelevant to the minimization so that the alignment is maintained.

From this brief excursus one might conclude that, as far as the Higgs content is considered, the price for tractability and predictivity is high on SUSY $SO(10)$ models, as the desired group-theoretical simplicity of the Higgs sector, with representations up to the adjoint, appears to be nonviable.

In this paper, we point out that all these issues are alleviated if one considers a flipped variant of the SUSY $SO(10)$ unification. In particular, we shall show that the flipped $SO(10) \otimes U(1)$ scenario [8–10] offers an attractive option to break the gauge symmetry to the SM at the renormalizable level by means of a quite simple Higgs sector, namely, a couple of $SO(10)$ spinors $16_{1,2} \oplus \overline{16}_{1,2}$ and one adjoint 45_H.

Within the extended $SO(10) \otimes U(1)$ gauge algebra, one finds, in general, *three* inequivalent embeddings of the SM hypercharge. In addition to the two solutions with the hypercharge stretching over the $SU(5)$ or the $SU(5) \otimes U(1)$ subgroups of $SO(10)$ [respectively dubbed as the "standard" and "flipped" $SU(5)$ embeddings], there is a third, flipped $SO(10)$, solution inherent to the $SO(10) \otimes U(1)$ case, with a nontrivial projection of the SM hypercharge onto the $U(1)$ factor.

While the difference between the standard and the flipped $SU(5)$ embeddings is semantical from the $SO(10)$ point of view, the flipped $SO(10)$ case is qualitatively different. In particular, the symmetry-breaking "power" of the $SO(10)$ spinor and adjoint representations is boosted with respect to the standard $SO(10)$ case, increasing the number of SM-singlet fields that may acquire nonvanishing VEVs. Technically, flipping allows for a *pair* of SM singlets in each of the 16_H and $\overline{16}_H$ "Weyl" spinors, together with *four* SM singlets within 45_H. This is at the root of the possibility of implementing the gauge symmetry breaking by means of a simple renormalizable Higgs sector. Let us just remark that, if renormalizability is not required, the breaking can be realized without the adjoint Higgs field; see, for instance, the flipped $SO(10)$ model with an additional anomalous $U(1)$ of Ref. [11].

Nevertheless, flipping is not *per se* sufficient to cure the $SU(5)$ lock of standard $SO(10)$ with $16_H \oplus \overline{16}_H \oplus 45_H$ in the Higgs sector. Indeed, the adjoint does not reduce the rank, and the bispinor, in spite of the two qualitatively different SM singlets involved, can lower it only by a single unit, leaving a residual $SU(5) \otimes U(1)$ symmetry [the two SM-singlet directions in the 16_H still retain an $SU(5)$ algebra as a little group]. Only when two sets of $16_H \oplus \overline{16}_H$ (interacting via 45_H) are introduced, the two pairs of SM-singlet VEVs in the spinor multiplets are not generally aligned and the little group is reduced to the SM.

Thus, the simplest renormalizable SUSY Higgs model that can provide the spontaneous breaking of the $SO(10)$ GUT symmetry to the SM by means of Higgs representations not larger than the adjoint is the flipped $SO(10) \otimes U(1)$ scenario with *two copies* of the $16 \oplus \overline{16}$ bi-spinor supplemented by the adjoint 45. Notice further that in the flipped embedding the spinor representations also include weak doublets that may trigger the electroweak symmetry breaking and allow for renormalizable Yukawa interactions with the chiral matter fields distributed in the flipped embedding over $16 \oplus 10 \oplus 1$.

Remarkably, the basics of the mechanism we advocate can be embedded in an underlying *nonrenormalizable* E_6 Higgs model featuring a pair of $27_H \oplus \overline{27}_H$ and the adjoint 78_H.

Technical similarities apart, there is, however, a crucial difference between the $SO(10) \otimes U(1)$ and E_6 scenarios, which is related to the fact that the Lie algebra of E_6 is larger than that of $SO(10) \otimes U(1)$. It has been shown long ago [12] that the renormalizable SUSY E_6 Higgs model spanned on a single copy of $27_H \oplus \overline{27}_H \oplus 78_H$ leaves an $SO(10)$ symmetry unbroken. Two pairs of $27_H \oplus \overline{27}_H$ are needed to reduce the rank by two units. In spite of the fact that the two SM-singlet directions in the 27_H are exactly those of the flipped 16_H, the little group of the SM-singlet directions $\langle 27_H \oplus \overline{27}_H \rangle$ and $\langle 78_H \rangle$ remains at the renormalizable level $SU(5)$, as we will explicitly show.

Adding NR adjoint interactions allows for a disentanglement of the $\langle 78_H \rangle$, such that the little group is reduced to the SM. Since a one-step E_6 breaking is phenomenologically problematic as mentioned earlier, we argue for a two-step breaking, via flipped $SO(10) \otimes U(1)$, with the E_6 scale near the Planck scale.

In summary, we make the case for an anomaly-free flipped $SO(10) \otimes U(1)$ partial unification scenario. We provide a detailed discussion of the symmetry-breaking pattern obtained within the minimal flipped $SO(10)$ SUSY Higgs model and consider its possible E_6 embedding. We finally present an elementary discussion of the flavor structure offered by these settings.

II. THE GUT-SCALE LITTLE HIERARCHY

In supersymmetric $SO(10)$ models with just $45_H \oplus 16_H \oplus \overline{16}_H$ governing the GUT breaking, one way to obtain the misalignment between the adjoint and the spinors is by invoking new physics at the Planck scale, parametrized in a model-independent way by a tower of effective operators suppressed by powers of M_P.

What we call the "GUT-scale little hierarchy" is the hierarchy induced in the GUT spectrum by M_G/M_P suppressed effective operators, which may split the GUT-scale thresholds over several orders of magnitude. In turn, this may be highly problematic for proton stability and the gauge unification in low-energy SUSY scenarios (as discussed, for instance, in Ref. [13]). It may also jeopardize the neutrino mass generation in the seesaw scheme. We briefly review the relevant issues here.

A. Proton decay and effective neutrino masses

In Ref. [14] the emphasis is set on a class of neutrino-mass-related operators which turns out to be particularly dangerous for proton stability in scenarios with a nonrenormalizable GUT-breaking sector. The relevant interactions can be schematically written as

$$W_Y \supset \frac{1}{M_P} 16_F g 16_F 16_H 16_H + \frac{1}{M_P} 16_F f 16_F \overline{16}_H \overline{16}_H \supset \frac{v_R}{M_P}(QgL\bar{T} + QfQT), \quad (1)$$

where g and f are matrices in the family space, $v_R \equiv |\langle 16_H \rangle| = |\langle \overline{16}_H \rangle|$, and T ($\bar{T}$) is the color triplet (antitriplet) contained in the $\overline{16}_H$ (16_H). Integrating out the color triplets, whose mass term is labeled M_T, one obtains the following effective superpotential involving fields belonging to $SU(2)_L$ doublets,

$$W_{\rm eff}^L = \frac{v_R^2}{M_P^2 M_T}(u^T F d')(u^T G V' \ell - d'^T G V' \nu'), \quad (2)$$

where u and ℓ denote the physical left-handed up-quarks and charged lepton superfields in the basis in which neutral gaugino interactions are flavor diagonal. The d' and ν' fields are related to the physical down-quark and light neutrino fields d and ν by $d' = V_{\rm CKM} d$ and $\nu' = V_{\rm PMNS}\nu$. In turn, $V' = V_u^\dagger V_\ell$, where V_u and V_ℓ diagonalize the left-handed up-quark and charged lepton mass matrices, respectively. The 3×3 matrices (G, F) are given by $(G, F) = V_u^T (g, f) V_u$.

By exploiting the correlations between the g and f matrices and the matter masses and mixings and by taking into account the uncertainties related to the low-energy SUSY spectrum, the GUT thresholds, and the hadronic matrix elements, the authors of Ref. [14] argue that the effective operators in Eq. (2) lead to a proton lifetime

$$\Gamma^{-1}(\bar{\nu} K^+) \sim (0.6\text{–}3) \times 10^{33}\ \text{yrs}, \quad (3)$$

at the verge of the current experimental lower bound of 0.67×10^{33} yrs [15]. In obtaining Eq. (3) the authors assume that the color triplet masses cluster about the GUT scale, $M_T \approx \langle 16_H \rangle \sim \langle 45_H \rangle \equiv M_G$. On the other hand, in scenarios where at the renormalizable level $SO(10)$ is broken to $SU(5)$ and the residual $SU(5)$ symmetry is broken to SM by means of nonrenormalizable operators, the effective scale of the $SU(5)$ breaking physics is typically suppressed by $\langle 16_H \rangle / M_P$ or $\langle 45_H \rangle / M_P$ with respect to M_G. As a consequence, the $SU(5)$ part of the colored triplet Higgsino spectrum is effectively pulled down to the M_G^2/M_P scale, clashing with proton stability.

B. GUT-scale thresholds and one-step unification

The "delayed" residual $SU(5)$ breakdown has obvious implications for the shape of the gauge coupling unification pattern. Indeed, the gauge bosons associated with the $SU(5)$/SM coset, together with the relevant part of the Higgs spectrum, tend to be uniformly shifted [6] by a factor $M_G/M_P \sim 10^{-2}$ below the scale of the $SO(10)/SU(5)$ gauge spectrum, which sets the unification scale M_G. These thresholds may jeopardize the successful one-step gauge unification pattern favored by the TeV-scale SUSY extension of the SM (MSSM).

C. GUT-scale thresholds and neutrino masses

With a nontrivial interplay among several GUT-scale thresholds [6], one may, in principle, end up with a viable gauge unification pattern. Namely, the threshold effects in different SM gauge sectors may be such that unification is preserved at a larger scale. In such a case the M_G/M_P suppression is at least partially undone. This, in turn, is unwelcome for the neutrino mass scale because the VEVs

TABLE I. Comparative summary of supersymmetric vacua left invariant by the SM-singlet VEVs in various combinations of spinorial and adjoint Higgs representations of standard $SO(10)$ and flipped $SO(10) \otimes U(1)$. The results for a renormalizable and a nonrenormalizable Higgs superpotential are, respectively, listed.

	Standard $SO(10)$		Flipped $SO(10) \otimes U(1)$	
Higgs superfields	R	NR	R	NR
$16 \oplus \overline{16}$	$SO(10)$	$SU(5)$	$SO(10) \otimes U(1)$	$SU(5) \otimes U(1)$
$2 \times (16 \oplus \overline{16})$	$SO(10)$	$SU(5)$	$SO(10) \otimes U(1)$	SM
$45 \oplus 16 \oplus \overline{16}$	$SU(5)$ [5]	SM [6]	$SU(5) \otimes U(1)$	SM $\otimes U(1)$
$45 \oplus 2 \times (16 \oplus \overline{16})$	$SU(5)$	SM	**SM**	SM

entering the $d = 5$ effective operator responsible for the RH neutrino Majorana mass term $16_F^2 \overline{16}_H^2 / M_P$ are raised accordingly, and thus $M_R \sim M_G^2/M_P$ tends to overshoot the upper limit $M_R \lesssim 10^{14}$ GeV implied by the light neutrino masses generated by the seesaw mechanism.

Thus, although the Planck-induced operators can provide a key to overcoming the $SU(5)$ lock of the minimal SUSY $SO(10) \to SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ Higgs model with $16_H \oplus \overline{16}_H \oplus 45_H$, such an effective scenario is prone to failure when addressing the measured proton stability and light neutrino phenomenology.

III. MINIMAL FLIPPED $SO(10)$ HIGGS MODEL

As already anticipated in the previous sections, in a standard $SO(10)$ framework with a Higgs sector built off the lowest-dimensional representations (up to the adjoint), it is rather difficult to achieve a phenomenologically viable symmetry-breaking pattern even when admitting multiple copies of each type of multiplet. First, with a single 45_H at play, at the renormalizable level the little group of all SM-singlet VEVs is $SU(5)$ regardless of the number of $16_H \oplus \overline{16}_H$ pairs. The reason is that one cannot get anything more than an $SU(5)$ singlet out of a number of $SU(5)$ singlets. The same is true with a second 45_H added into the Higgs sector because there is no renormalizable mixing among the two 45_H's apart from the mass term that, without loss of generality, can be taken diagonal. With a third adjoint Higgs representation at play, a cubic $45_1 45_2 45_3$ interaction is allowed. However, due to the total antisymmetry of the invariant and to the fact that the adjoints commute on the SM vacuum, the cubic term does not contribute to the F-term equations [16]. This makes the simple flipped $SO(10) \otimes U(1)$ model proposed in this work a framework worth considering. For the sake of completeness, let us also recall that by admitting Higgs representations larger than the adjoint, a renormalizable $SO(10) \to$ SM breaking can be devised with the Higgs sector of the form $54_H \oplus 45_H \oplus 16_H \oplus \overline{16}_H$ [17], or $54_H \oplus 45_H \oplus 126_H \oplus \overline{126}_H$ [7] for a renormalizable seesaw mechanism.

In Tables I and II we collect a list of the supersymmetric vacua that are obtained in the basic $SO(10)$ Higgs models and their E_6 embeddings by considering a set of Higgs representations of the dimension of the adjoint and smaller, with all SM-singlet VEVs turned on. The cases of a renormalizable (R) or nonrenormalizable (NR) Higgs potential are compared. We quote reference papers where results relevant for the present study were obtained without any aim of exhausting the available literature. The results without a reference are either verified by us or follow by comparison with other cases and rank counting. The main results of this study are shown in boldface.

We are going to show that by considering a nonstandard hypercharge embedding in $SO(10) \otimes U(1)$ [flipped $SO(10)$], the breaking to the SM is achievable at the renormalizable level with $45_H \oplus 2 \times (16_H \oplus \overline{16}_H)$ Higgs fields. Let us stress that what we require is that the GUT symmetry breaking is driven by the renormalizable part of the superpotential, while Planck-suppressed interactions may be relevant for the fermion mass spectrum, in particular, for the neutrino sector.

A. Introducing the model

1. Hypercharge embeddings in $SO(10) \otimes U(1)$

The so-called flipped realization of the $SO(10)$ gauge symmetry requires an additional $U(1)_X$ gauge factor in order to provide an extra degree of freedom for the SM hypercharge identification. For a fixed embedding of the $SU(3)_c \otimes SU(2)_L$ subgroup within $SO(10)$, the SM hypercharge can be generally spanned over the three remaining Cartans generating the Abelian $U(1)^3$ subgroup of the $SO(10) \otimes U(1)_X/(SU(3)_c \otimes SU(2)_L)$ coset. There are two consistent implementations of the SM hypercharge within the $SO(10)$ algebra [commonly denoted by standard and flipped $SU(5)$], while a third one becomes available due to the presence of $U(1)_X$.

TABLE II. Same as in Table I for the E_6 gauge group with fundamental and adjoint Higgs representations.

Higgs superfields	R	NR
$27 \oplus \overline{27}$	E_6	$SO(10)$
$2 \times (27 \oplus \overline{27})$	E_6	$SU(5)$
$78 \oplus 27 \oplus \overline{27}$	$SO(10)$ [12]	SM $\otimes U(1)$
$78 \oplus 2 \times (27 \oplus \overline{27})$	$\mathbf{SU(5)}$	SM

In order to discuss the different embeddings, we find it useful to consider two bases for the $U(1)^3$ subgroup. Adopting the traditional left-right (LR) basis corresponding to the $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ subalgebra of $SO(10)$, one can span the SM hypercharge on the generators of $U(1)_R \otimes U(1)_{B-L} \otimes U(1)_X$:

$$Y = \alpha T_R^{(3)} + \beta(B-L) + \gamma X. \tag{4}$$

The normalization of the $T_R^{(3)}$ and $B-L$ charges is chosen so that the decompositions of the spinorial and vector representations of $SO(10)$ with respect to $SU(3)_c \otimes SU(2)_L \otimes U(1)_R \otimes U(1)_{B-L}$ read

$$\begin{aligned} 16 &= (3,2;0,+\tfrac{1}{3}) \oplus (\bar{3},1;+\tfrac{1}{2},-\tfrac{1}{3}) \oplus (\bar{3},1;-\tfrac{1}{2},-\tfrac{1}{3}) \\ &\quad \oplus (1,2;0,-1) \oplus (1,1;+\tfrac{1}{2},+1) \oplus (1,1;-\tfrac{1}{2},+1), \\ 10 &= (3,1;0,-\tfrac{2}{3}) \oplus (\bar{3},1;0,+\tfrac{2}{3}) \oplus (1,2;+\tfrac{1}{2},0) \\ &\quad \oplus (1,2;-\tfrac{1}{2},0), \end{aligned} \tag{5}$$

which account for the standard $B-L$ and $T_R^{(3)}$ assignments.

Alternatively, considering the $SU(5) \otimes U(1)_Z$ subalgebra of $SO(10)$, we identify the $U(1)_{Y'} \otimes U(1)_Z \otimes U(1)_X$ subgroup of $SO(10) \otimes U(1)_X$, and equivalently write

$$Y = \tilde{\alpha} Y' + \tilde{\beta} Z + \tilde{\gamma} X, \tag{6}$$

where Y' and Z are normalized so that the $SU(3)_c \otimes SU(2)_L \otimes U(1)_{Y'} \otimes U(1)_Z$ analogue of Eqs. (5) reads

$$\begin{aligned} 16 &= (3,2;+\tfrac{1}{6},+1) \oplus (\bar{3},1;+\tfrac{1}{3},-3) \oplus (\bar{3},1;-\tfrac{2}{3},+1) \\ &\quad \oplus (1,2;-\tfrac{1}{2},-3) \oplus (1,1;+1,+1) \oplus (1,1;0,+5), \\ 10 &= (3,1;-\tfrac{1}{3},-2) \oplus (\bar{3},1;+\tfrac{1}{3},+2) \oplus (1,2;+\tfrac{1}{2},-2) \\ &\quad \oplus (1,2;-\tfrac{1}{2},+2). \end{aligned} \tag{7}$$

In both cases, the $U(1)_X$ charge has been conveniently fixed to $X_{16} = +1$ for the spinorial representation [and thus $X_{10} = -2$ and also $X_1 = +4$ for the $SO(10)$ vector and singlet, respectively; this is also the minimal way to obtain an anomaly-free $U(1)_X$ that allows $SO(10) \otimes U(1)_X$ to be naturally embedded into E_6].

It is a straightforward exercise to show that in order to accommodate the SM quark multiplets with quantum numbers $Q = (3, 2, +\frac{1}{6})$, $u^c = (\bar{3}, 1, -\frac{2}{3})$, and $d^c = (\bar{3}, 1, +\frac{1}{3})$, there are only three solutions.

On the $U(1)^3$ bases of Eqs. (4) and (6) (respectively) one obtains

$$\alpha = 1, \qquad \beta = \tfrac{1}{2}, \qquad \gamma = 0, \qquad (\tilde{\alpha} = 1, \tilde{\beta} = 0, \tilde{\gamma} = 0), \tag{8}$$

which is nothing but the standard embedding of the SM matter into $SO(10)$. Explicitly, $Y = T_R^{(3)} + \frac{1}{2}(B-L)$ in the LR basis [while $Y = Y'$ in the $SU(5)$ picture].

The second option is characterized by

$$\alpha = -1, \qquad \beta = \tfrac{1}{2}, \qquad \gamma = 0, \qquad (\tilde{\alpha} = -\tfrac{1}{5}, \tilde{\beta} = \tfrac{1}{5}, \tilde{\gamma} = 0), \tag{9}$$

which is usually denoted as a "flipped $SU(5)$" [18,19] embedding because the SM hypercharge is spanned nontrivially on the $SU(5) \otimes U(1)_Z$ subgroup[1] of $SO(10)$, $Y = \frac{1}{5}(Z - Y')$. Remarkably, from the $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ perspective this setting corresponds to a sign flip of the $SU(2)_R$ Cartan operator $T_R^{(3)}$, namely $Y = -T_R^{(3)} + \frac{1}{2}(B-L)$, which can be viewed as a π rotation in the $SU(2)_R$ algebra.

A third solution corresponds to

$$\alpha = 0, \qquad \beta = -\frac{1}{4}, \qquad \gamma = \frac{1}{4}, \qquad \left(\tilde{\alpha} = -\frac{1}{5}, \tilde{\beta} = -\frac{1}{20}, \tilde{\gamma} = \frac{1}{4}\right), \tag{10}$$

denoted as a "flipped $SO(10)$" [8–10] embedding of the SM hypercharge. Notice, in particular, the fundamental difference between the setting (10) with $\gamma = \tilde{\gamma} = \frac{1}{4}$ and the two previous cases (8) and (9) where $U(1)_X$ does not play any role.

Analogously to what is found for Y, once we consider the additional anomaly-free $U(1)_X$ gauge factor, there are three SM-compatible ways of embedding the physical $(B-L)$ into $SO(10) \otimes U(1)_X$. Using the $SU(5)$ compatible description they are, respectively, given by (see Ref. [20] for a complete set of relations)

$$(B-L) = \tfrac{1}{5}(4Y' + Z), \tag{11}$$

$$(B-L) = \frac{1}{20}(16Y' - Z + 5X), \tag{12}$$

$$(B-L) = -\frac{1}{20}(8Y' - 3Z - 5X), \tag{13}$$

where the first assignment is the standard $B-L$ embedding in Eq. (4). Out of 3×3 possible pairs of Y and $(B-L)$ charges, only six correspond to the quantum numbers of the SM matter [20]. By focusing on the flipped $SO(10)$ hypercharge embedding in Eq. (10), the two SM-compatible $(B-L)$ assignments are those in

[1] By definition, a flipped variant of a specific GUT model based on a simple gauge group G is obtained by embedding the SM hypercharge nontrivially into the $G \otimes U(1)$ tensor product.

Eqs. (12) and (13) (they are related by a sign flip in $T_R^{(3)}$). In what follows we shall employ the $(B-L)$ assignment in Eq. (13).

2. Spinor and adjoint SM singlets in flipped $SO(10)$

The active role of the $U(1)_X$ generator in the SM hypercharge (and $B-L$) identification within the flipped $SO(10)$ scenario has relevant consequences for model building. In particular, the SM decomposition of the $SO(10)$ representations changes so that there are additional SM singlets both in $16_H \oplus \overline{16}_H$ and in 45_H.

The pattern of SM-singlet components in flipped $SO(10)$ has a simple and intuitive interpretation from the $SO(10) \otimes U(1)_X \subset E_6$ perspective, where $16_{+1} \oplus \overline{16}_{-1}$ [with the subscript indicating the $U(1)_X$ charge] are contained in $27 \oplus \overline{27}$ while 45_0 is part of the E_6 adjoint 78. The point is that the flipped SM hypercharge assignment makes the various SM singlets within the complete E_6 representations "migrate" among their different $SO(10)$ submultiplets; namely, the two SM singlets in the 27 of E_6 that in the standard embedding (8) reside in the $SO(10)$ singlet 1 and spinorial 16 components both happen to fall into just the single $16 \subset 27$ in the flipped $SO(10)$ case.

Similarly, there are two additional SM-singlet directions in 45_0 in the flipped $SO(10)$ scenario, that, in the standard $SO(10)$ embedding, belong to the $16_{-3} \oplus \overline{16}_{+3}$ components of the 78 of E_6, thus accounting for a total of four adjoint SM singlets.

In Tables III, IV, and V we summarize the decomposition of the 10_{-2}, 16_{+1}, and 45_0 representations of $SO(10) \otimes U(1)_X$ under the SM subgroup, in both the standard and the flipped $SO(10)$ cases [and in both the LR and $SU(5)$ descriptions]. The pattern of the SM-singlet components is emphasized in boldface.

3. The supersymmetric flipped $SO(10)$ *model*

The presence of additional SM singlets [some of them transforming nontrivially under $SU(5)$] in the lowest-dimensional representations of the flipped realization of the $SO(10)$ gauge symmetry provides the ground for obtaining a viable symmetry breaking with a significantly simplified renormalizable Higgs sector. Naively, one may guess that the pair of VEVs in 16_H (plus another conjugated pair in $\overline{16}_H$ to maintain the required D flatness) might be enough to break the GUT symmetry entirely, since one component transforms as a 10 of $SU(5) \subset SO(10)$, while the other one is identified with the $SU(5)$ singlet (cf. Table IV). Notice that even in the presence of an additional four-dimensional vacuum manifold of the adjoint Higgs multiplet, the little group is determined by the 16_H VEVs since, due to the simple form of the renormalizable superpotential, F flatness makes the VEVs of 45_H align with those of $16_H\overline{16}_H$, providing just enough freedom for them to develop nonzero values.

TABLE III. Decomposition of the fundamental ten-dimensional representation under $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$, for standard $SO(10)$ and flipped $SO(10) \otimes U(1)_X$ ($SO(10)_f$), respectively. In the first two columns (LR) the subscripts keep track of the $SU(4)_C$ origin of the multiplets (the extra symbols $\pm$ correspond to the eigenvalues of the $T_R^{(3)}$ Cartan generator), while in the last two columns the $SU(5)$ content is shown.

LR		$SU(5)$	
$SO(10)$	$SO(10)_f$	$SO(10)$	$SO(10)_f$
$(3,1;-\frac{1}{3})_6$	$(3,1;-\frac{1}{3})_6$	$(3,1;-\frac{1}{3})_5$	$(3,1;-\frac{1}{3})_5$
$(\bar{3},1;+\frac{1}{3})_6$	$(\bar{3},1;-\frac{2}{3})_6$	$(1,2;+\frac{1}{2})_5$	$(1,2;-\frac{1}{2})_5$
$(1,2;+\frac{1}{2})_{1^+}$	$(1,2;-\frac{1}{2})_{1^+}$	$(\bar{3},1;+\frac{1}{3})_{\bar{5}}$	$(\bar{3},1;-\frac{2}{3})_{\bar{5}}$
$(1,2;-\frac{1}{2})_{1^-}$	$(1,2;-\frac{1}{2})_{1^-}$	$(1,2;-\frac{1}{2})_{\bar{5}}$	$(1,2;-\frac{1}{2})_{\bar{5}}$

TABLE IV. The same as in Table III for the spinor 16-dimensional representation. The SM singlets are emphasized in boldface and shall be denoted, in the $SU(5)$ description, as $e \equiv (1,1;0)_{10}$ and $\nu \equiv (1,1;0)_1$. The LR decomposition shows that e and ν belong to an $SU(2)_R$ doublet.

LR		$SU(5)$	
$SO(10)$	$SO(10)_f$	$SO(10)$	$SO(10)_f$
$(3,2;+\frac{1}{6})_4$	$(3,2;+\frac{1}{6})_4$	$(\bar{3},1;+\frac{1}{3})_{\bar{5}}$	$(\bar{3},1;+\frac{1}{3})_{\bar{5}}$
$(1,2;-\frac{1}{2})_4$	$(1,2;+\frac{1}{2})_4$	$(1,2;-\frac{1}{2})_{\bar{5}}$	$(1,2;+\frac{1}{2})_{\bar{5}}$
$(\bar{3},1;+\frac{1}{3})_{\bar{4}^+}$	$(\bar{3},1;+\frac{1}{3})_{\bar{4}^+}$	$(3,2;+\frac{1}{6})_{10}$	$(3,2;+\frac{1}{6})_{10}$
$(\bar{3},1;-\frac{2}{3})_{\bar{4}^-}$	$(\bar{3},1;+\frac{1}{3})_{\bar{4}^-}$	$(\bar{3},1;-\frac{2}{3})_{10}$	$(\bar{3},1;+\frac{1}{3})_{10}$
$(1,1;+1)_{\bar{4}^+}$	$\mathbf{(1,1;0)_{\bar{4}^+}}$	$(1,1;+1)_{10}$	$\mathbf{(1,1;0)_{10}}$
$\mathbf{(1,1;0)_{\bar{4}^-}}$	$\mathbf{(1,1;0)_{\bar{4}^-}}$	$\mathbf{(1,1;0)_{1}}$	$\mathbf{(1,1;0)_{1}}$

TABLE V. The same as in Table III for the 45 representation. The SM singlets are given in boldface and labeled throughout the text as $\omega_Y \equiv (1,1;0)_{15}$, $\omega^+ \equiv (1,1;0)_{1^+}$, $\omega_R \equiv (1,1;0)_{1^0}$, and $\omega^- \equiv (1,1;0)_{1^-}$, where again the LR notation has been used. The LR decomposition also shows that ω^+, ω_R, and ω^- belong to an $SU(2)_R$ triplet, while ω_Y is a $B-L$ singlet.

LR		$SU(5)$	
$SO(10)$	$SO(10)_f$	$SO(10)$	$SO(10)_f$
$\mathbf{(1,1;0)_{1^0}}$	$\mathbf{(1,1;0)_{1^0}}$	$\mathbf{(1,1;0)_{1}}$	$\mathbf{(1,1;0)_{1}}$
$\mathbf{(1,1;0)_{15}}$	$\mathbf{(1,1;0)_{15}}$	$\mathbf{(1,1;0)_{24}}$	$\mathbf{(1,1;0)_{24}}$
$(8,1;0)_{15}$	$(8,1;0)_{15}$	$(8,1;0)_{24}$	$(8,1;0)_{24}$
$(3,1;+\frac{2}{3})_{15}$	$(3,1;-\frac{1}{3})_{15}$	$(3,2;-\frac{5}{6})_{24}$	$(3,2;+\frac{1}{6})_{24}$
$(\bar{3},1;-\frac{2}{3})_{15}$	$(\bar{3},1;+\frac{1}{3})_{15}$	$(\bar{3},2;+\frac{5}{6})_{24}$	$(\bar{3},2;-\frac{1}{6})_{24}$
$(1,3;0)_1$	$(1,3;0)_1$	$(1,3;0)_{24}$	$(1,3;0)_{24}$
$(3,2;+\frac{1}{6})_{6^+}$	$(3,2;+\frac{1}{6})_{6^+}$	$(3,2;+\frac{1}{6})_{10}$	$(3,2;+\frac{1}{6})_{10}$
$(\bar{3},2;+\frac{5}{6})_{6^+}$	$(\bar{3},2;-\frac{1}{6})_{6^+}$	$(\bar{3},1;-\frac{2}{3})_{10}$	$(\bar{3},1;+\frac{1}{3})_{10}$
$(1,1;+1)_{1^+}$	$\mathbf{(1,1;0)_{1^+}}$	$(1,1;+1)_{10}$	$\mathbf{(1,1;0)_{10}}$
$(\bar{3},2;-\frac{1}{6})_{6^-}$	$(\bar{3},2;-\frac{1}{6})_{6^-}$	$(\bar{3},2;-\frac{1}{6})_{\overline{10}}$	$(\bar{3},2;-\frac{1}{6})_{\overline{10}}$
$(3,2;-\frac{5}{6})_{6^-}$	$(3,2;+\frac{1}{6})_{6^-}$	$(3,1;+\frac{2}{3})_{\overline{10}}$	$(3,1;-\frac{1}{3})_{\overline{10}}$
$(1,1;-1)_{1^-}$	$\mathbf{(1,1;0)_{1^-}}$	$(1,1;-1)_{\overline{10}}$	$\mathbf{(1,1;0)_{\overline{10}}}$

Unfortunately, this is still not enough to support the desired symmetry-breaking pattern. The two VEV directions in 16_H are equivalent to only one, and a residual $SU(5) \otimes U(1)$ symmetry is always preserved by $\langle 16 \rangle_H$ [21]. Thus, even in the flipped $SO(10) \otimes U(1)$ setting the Higgs model spanned on $16_H \oplus \overline{16}_H \oplus 45_H$ suffers from an $SU(5) \otimes U(1)$ lock analogous to the one of the standard SUSY $SO(10)$ models with the same Higgs sector. This can be understood by taking into account the freedom in choosing the basis in the $SO(10)$ algebra so that the pair of VEVs within 16 can be "rotated" onto a single component, which can then be viewed as the direction of the singlet in the decomposition of $16 = \bar{5} \oplus 10 \oplus 1$ with respect to an $SU(5)$ subgroup of the original $SO(10)$ gauge symmetry.

On the other hand, with a pair of interacting $16_H \oplus \overline{16}_H$'s the vacuum directions in the two 16_H's need not be aligned and the intersection of the two different invariant subalgebras [e.g., standard and flipped $SU(5)$ for a specific VEV configuration] leaves as a little group the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ of the SM. F flatness then makes the adjoint VEVs (45_H is the needed carrier of the 16_H interaction at the renormalizable level) aligned to the SM vacuum. Hence, as we will show in the next section, $2 \times (16_H + \overline{16}_H) \oplus 45_H$ defines the minimal renormalizable Higgs setting for the SUSY flipped $SO(10) \otimes U(1)_X$ model. For comparison, let us reiterate that in the standard renormalizable $SO(10)$ setting the SUSY vacuum is always $SU(5)$ regardless of how many copies of $16_H \oplus \overline{16}_H$ are employed together with, at most, a pair of adjoints.

4. The matter sector

Because of the flipped hypercharge assignment, the SM matter can no longer be fully embedded into the 16-dimensional $SO(10)$ spinor, as in the standard case. By inspecting Table IV one can see that in the flipped setting the pair of the SM submultiplets of 16 transforming as u^c and e^c is traded for an extra d^c-like state and an extra SM singlet. The former pair is instead found in the $SO(10)$ vector and the singlet (the lepton doublet appears in the vector multiplet as well). Thus, flipping spreads each of the SM matter generations across $16 \oplus 10 \oplus 1$ of $SO(10)$, which, by construction, can be viewed as the complete 27-dimensional fundamental representation of $E_6 \supset SO(10) \otimes U(1)_X$. This brings in a set of additional degrees of freedom, in particular, $(1, 1, 0)_{16}$, $(\bar{3}, 1, +\frac{1}{3})_{16}$, $(1, 2, +\frac{1}{2})_{16}$, $(3, 1, -\frac{1}{3})_{10}$, and $(1, 2, -\frac{1}{2})_{10}$, where the subscript indicates their $SO(10)$ origin. Notice, however, that these SM "exotics" can be grouped into superheavy vectorlike pairs, and thus no extra states appear in the low-energy spectrum. Furthermore, the $U(1)_X$ anomalies associated with each of the $SO(10) \otimes U(1)_X$ matter multiplets cancel when summed over the entire reducible representation $16_1 \oplus 10_{-2} \oplus 1_4$. An elementary discussion of the matter spectrum in this scenario is deferred to Sec. V.

B. Supersymmetric vacuum

The most general renormalizable Higgs superpotential, made of the representations $45 \oplus 16_1 \oplus \overline{16}_1 \oplus 16_2 \oplus \overline{16}_2$, is given by

$$W_H = \frac{\mu}{2} \mathrm{Tr} 45^2 + \rho_{ij} 16_i \overline{16}_j + \tau_{ij} 16_i 45 \overline{16}_j, \tag{14}$$

where i, $j = 1$, 2 and the notation is explained in Appendix A 1. Without loss of generality, we can take μ real by a global phase redefinition, while τ (or ρ) can be diagonalized by a bi-unitary transformation acting on the flavor indices of the 16 and the $\overline{16}$. Let us choose, for instance, $\tau_{ij} = \tau_i \delta_{ij}$, with τ_i real. We label the SM singlets contained in the 16's in the following way: $e \equiv (1, 1; 0)_{10}$ [only for flipped $SO(10)$] and $\nu \equiv (1, 1; 0)_1$ (for all embeddings).

By plugging in the SM-singlet VEVs ω_R, ω_Y, ω^+, ω^-, $e_{1,2}$, $\bar{e}_{1,2}$, $\nu_{1,2}$, and $\bar{\nu}_{1,2}$ (cf. Appendix A 1), the superpotential on the vacuum reads

$$\begin{aligned}
\langle W_H \rangle = {} & \mu(2\omega_R^2 + 3\omega_Y^2 + 4\omega^-\omega^+) + \rho_{11}(e_1\bar{e}_1 + \nu_1\bar{\nu}_1) + \rho_{21}(e_2\bar{e}_1 + \nu_2\bar{\nu}_1) + \rho_{12}(e_1\bar{e}_2 + \nu_1\bar{\nu}_2) + \rho_{22}(e_2\bar{e}_2 + \nu_2\bar{\nu}_2) \\
& + \tau_1\Big[-\omega^- e_1\bar{\nu}_1 - \omega^+\nu_1\bar{e}_1 - \frac{\omega_R}{\sqrt{2}}(e_1\bar{e}_1 - \nu_1\bar{\nu}_1) + \frac{3}{2}\frac{\omega_Y}{\sqrt{2}}(e_1\bar{e}_1 + \nu_1\bar{\nu}_1)\Big] \\
& + \tau_2\Big[-\omega^- e_2\bar{\nu}_2 - \omega^+\nu_2\bar{e}_2 - \frac{\omega_R}{\sqrt{2}}(e_2\bar{e}_2 - \nu_2\bar{\nu}_2) + \frac{3}{2}\frac{\omega_Y}{\sqrt{2}}(e_2\bar{e}_2 + \nu_2\bar{\nu}_2)\Big].
\end{aligned} \tag{15}$$

In order to retain SUSY down to the TeV scale we must require that the GUT gauge symmetry breaking preserves supersymmetry. In Appendix A 2 we work out the relevant D- and F-term equations. We find that the existence of a nontrivial vacuum requires ρ (and τ for consistency) to be Hermitian matrices. This is a consequence of the fact that D-term flatness for the flipped $SO(10)$ embedding implies $\langle 16_i \rangle = \langle \overline{16}_i \rangle^*$ [see Eq. (A30) and the discussion next to it]. With this restriction the vacuum manifold is given by

$$\begin{aligned}
8\mu\omega^+ &= \tau_1 r_1^2 \sin 2\alpha_1 e^{i(\phi_{e_1}-\phi_{\nu_1})} + \tau_2 r_2^2 \sin 2\alpha_2 e^{i(\phi_{e_2}-\phi_{\nu_2})},\\
8\mu\omega^- &= \tau_1 r_1^2 \sin 2\alpha_1 e^{-i(\phi_{e_1}-\phi_{\nu_1})}\\
&\quad + \tau_2 r_2^2 \sin 2\alpha_2 e^{-i(\phi_{e_2}-\phi_{\nu_2})},\\
4\sqrt{2}\mu\omega_R &= \tau_1 r_1^2 \cos 2\alpha_1 + \tau_2 r_2^2 \cos 2\alpha_2,\\
4\sqrt{2}\mu\omega_Y &= -\tau_1 r_1^2 - \tau_2 r_2^2,\\
e_{1,2} &= r_{1,2}\cos\alpha_{1,2} e^{i\phi_{e_{1,2}}},\\
\nu_{1,2} &= r_{1,2}\sin\alpha_{1,2} e^{i\phi_{\nu_{1,2}}},\\
\bar{e}_{1,2} &= r_{1,2}\cos\alpha_{1,2} e^{-i\phi_{e_{1,2}}},\\
\bar{\nu}_{1,2} &= r_{1,2}\sin\alpha_{1,2} e^{-i\phi_{\nu_{1,2}}}, \qquad (16)
\end{aligned}$$

where $r_{1,2}$ and $\alpha^\pm \equiv \alpha_1 \pm \alpha_2$ are fixed in terms of the superpotential parameters,

$$r_1^2 = -\frac{2\mu(\rho_{22}\tau_1 - 5\rho_{11}\tau_2)}{3\tau_1^2\tau_2}, \qquad (17)$$

$$r_2^2 = -\frac{2\mu(\rho_{11}\tau_2 - 5\rho_{22}\tau_1)}{3\tau_1\tau_2^2}, \qquad (18)$$

$$\cos\alpha^- = \xi\frac{\sin\Phi_\nu - \sin\Phi_e}{\sin(\Phi_\nu - \Phi_e)}, \qquad (19)$$

$$\cos\alpha^+ = \xi\frac{\sin\Phi_\nu + \sin\Phi_e}{\sin(\Phi_\nu - \Phi_e)}, \qquad (20)$$

with

$$\xi = \frac{6|\rho_{12}|}{\sqrt{-\frac{5\rho_{11}^2\tau_2}{\tau_1} - \frac{5\rho_{22}^2\tau_1}{\tau_2} + 26\rho_{22}\rho_{11}}}. \qquad (21)$$

The phase factors Φ_ν and Φ_e are defined as

$$\Phi_\nu \equiv \phi_{\nu_1} - \phi_{\nu_2} + \phi_{\rho_{12}}, \qquad \Phi_e \equiv \phi_{e_1} - \phi_{e_2} + \phi_{\rho_{12}}, \qquad (22)$$

in terms of the relevant phases $\phi_{\nu_{1,2}}$, $\phi_{e_{1,2}}$, and $\phi_{\rho_{12}}$. Equations (19) and (20) imply that for $\Phi_\nu = \Phi_e = \Phi$, Eq. (19) reduces to $\cos\alpha^- \to \xi\cos\Phi$ while α^+ is undetermined (thus parametrizing an orbit of isomorphic vacua).

In order to determine the little group of the vacuum manifold, we explicitly compute the corresponding gauge boson spectrum in Appendix A 3. We find that, for $\alpha^- \neq 0$ and/or $\Phi_\nu \neq \Phi_e$, the vacuum in Eq. (16) does preserve the SM algebra.

As already mentioned in the Introduction, this result is a consequence of the misalignment of the spinor VEVs, which is made possible at the renormalizable level by the interaction with the 45_H. If we choose to align the $16_1 \oplus \overline{16}_1$ and $16_2 \oplus \overline{16}_2$ VEVs ($\alpha^- = 0$ and $\Phi_\nu = \Phi_e$) or, equivalently, to decouple one of the Higgs spinors from the vacuum ($r_2 = 0$, for instance), the little group is $SU(5) \otimes U(1)$.

This result can be easily understood by observing that in the case with just one pair of $16_H \oplus \overline{16}_H$ (or with two pairs of $16_H \oplus \overline{16}_H$ aligned) the two SM-singlet directions, e_H and ν_H, are connected by an $SU(2)_R$ transformation. This freedom can be used to rotate one of the VEVs to zero, so that the little group is standard or flipped $SU(5) \otimes U(1)$, depending on which of the two VEVs is zero.

In this respect, the Higgs adjoint plays the role of a renormalizable agent that prevents the two pairs of spinor vacua from aligning with each other along the $SU(5) \otimes U(1)$ direction. Actually, by decoupling the adjoint Higgs, F flatness makes the (aligned) $16_i \oplus \overline{16}_i$ vacuum trivial, as one verifies by inspecting the F terms in Eq. (A14) of Appendix A 2 for $\langle 45_H \rangle = 0$ and $\det\rho \neq 0$.

The same result with just two pairs of $16_H \oplus \overline{16}_H$ Higgs multiplets is obtained by adding NR spinor interactions, at the cost of introducing a potentially critical GUT-scale threshold hierarchy. In the flipped $SO(10)$ setup proposed here, the GUT symmetry breaking is driven by the renormalizable part of the Higgs superpotential, thus naturally allowing for a one-step matching with the minimal supersymmetric extension of the SM (MSSM).

Before addressing the possible embedding of the model in a unified E_6 scenario, we comment in brief on the naturalness of the doublet-triplet mass splitting in flipped embeddings.

C. Doublet-triplet splitting in flipped models

Flipped embeddings offer a rather economical way to implement the doublet-triplet (DT) splitting through the so-called missing partner (MP) mechanism [22,23]. In order to show the relevant features, let us consider first the flipped $SU(5) \otimes U(1)_Z$.

In order to implement the MP mechanism in the flipped $SU(5) \otimes U(1)_Z$, the Higgs superpotential is required to have the couplings

$$W_H \supset 10_{+1}10_{+1}5_{-2} + \overline{10}_{-1}\overline{10}_{-1}\bar{5}_{+2}, \qquad (23)$$

where the subscripts correspond to the $U(1)_Z$ quantum numbers, but not the $5_{-2}\bar{5}_{+2}$ mass term. From Eq. (23) we extract the relevant terms that lead to a mass for the Higgs triplets,

$$\begin{aligned}W_H \supset\ & \langle(1,1;0)_{10}\rangle(\bar{3},1;+\tfrac{1}{3})_{10}(3,1;-\tfrac{1}{3})_5 + \langle(1,1;0)_{\overline{10}}\rangle\\ & \times(3,1;-\tfrac{1}{3})_{\overline{10}}(\bar{3},1;+\tfrac{1}{3})_{\bar{5}}. \qquad (24)\end{aligned}$$

On the other hand, the Higgs doublets, contained in the $5_{-2} \oplus \bar{5}_{+2}$, remain massless since they have no partner in the $10_{+1} \oplus \overline{10}_{-1}$ to couple with.

The MP mechanism cannot be implemented in standard $SO(10)$. The relevant interactions, the analogue of Eq. (23), are contained in the $SO(10)$ invariant term

$$W_H \supset 16\,16\,10 + \overline{16}\,\overline{16}\,10, \qquad (25)$$

which, however, gives a mass to the doublets as well, via the superpotential terms

$$W_H \supset \langle(1,1;0)_{1_{16}}\rangle(1,2;-\tfrac{1}{2})_{\bar{5}_{16}}(1,2;+\tfrac{1}{2})_{5_{10}} + \langle(1,1;0)_{1_{\overline{16}}}\rangle \times (1,2;+\tfrac{1}{2})_{5_{\overline{16}}}(1,2;-\tfrac{1}{2})_{\bar{5}_{10}}. \qquad (26)$$

Flipped $SO(10)\otimes U(1)_X$, on the other hand, offers again the possibility of implementing the MP mechanism. The price to pay is the necessity of avoiding a large number of terms, both bilinear and trilinear, in the Higgs superpotential. In particular, the analogue of Eq. (23) is given by the NR term [11]

$$W_H \supset \frac{1}{M_P}\overline{16}_1 16_2 16_2 \overline{16}_1 + \frac{1}{M_P} 16_1 \overline{16}_2 \overline{16}_2 16_1. \qquad (27)$$

By requiring that 16_1 ($\overline{16}_1$) takes a VEV in the 1_{16} ($1_{\overline{16}}$) direction while 16_2 ($\overline{16}_2$) in the 10_{16} ($\overline{10}_{\overline{16}}$) component, one gets

$$W_H \supset \frac{1}{M_P}\langle 1_{\overline{16}_1}\rangle\langle 10_{16_2}\rangle 10_{16_2} 5_{\overline{16}_1} + \frac{1}{M_P}\langle 1_{16_1}\rangle \times \langle\overline{10}_{\overline{16}_2}\rangle \overline{10}_{\overline{16}_2}\bar{5}_{16_1}, \qquad (28)$$

which closely resembles Eq. (23), leading to massive triplets and massless doublets. In order to have, minimally, one pair of electroweak doublets, one must further require that the 2×2 mass matrix of the 16's has rank equal to 1. Because of the active role of NR operators, the Higgs triplets turn out to be 2 orders of magnitude below the flipped $SO(10)\otimes U(1)_X$ scale, reintroducing the issues discussed in Sec. II.

An alternative possibility for naturally implementing the DT splitting in $SO(10)$ is the Dimopoulos-Wilczek (DW) (or the missing VEV) mechanism [24]. In order to explain the key features, it is convenient to decompose the relevant $SO(10)$ representations in terms of the $SU(4)_C \otimes SU(2)_L \otimes SU(2)_R$ group,

$$\begin{aligned} 45 &\equiv (1,1,3)\oplus(15,1,1)\oplus\ldots,\\ 16 &\equiv (4,2,1)\oplus(\bar{4},1,2),\\ \overline{16} &\equiv (\bar{4},2,1)\oplus(4,1,2),\\ 10 &\equiv (6,1,1)\oplus(1,2,2), \end{aligned} \qquad (29)$$

where $\omega_R \equiv \langle(1,1,3)\rangle$ and $\omega_Y \equiv \langle(15,1,1)\rangle$. In the standard $SO(10)$ case (see [25–27] for a recent discussion) one assumes that the $SU(2)_L$ doublets are contained in two vector multiplets (10_1 and 10_2). From the decompositions in Eq. (29) it is easy to see that the interaction $10_1 45\, 10_2$ (where the antisymmetry of 45 requires the presence of two 10's) leaves the $SU(2)_L$ doublets massless provided that $\omega_R = 0$. Naturalness requires the absence of other superpotential terms, such as a direct mass term for one of the 10's and the interaction term $16\,45\,\overline{16}$. The latter aligns the SUSY vacuum in the $SU(5)$ direction ($\omega_R = \omega_Y$), thus destabilizing the DW solution.

On the other hand, the absence of the $16\,45\,\overline{16}$ interaction enlarges the global symmetries of the scalar potential with the consequent appearance of a set of light pseudo-Goldstone bosons in the spectrum. To avoid this, the adjoint and the spinor sector must be coupled in an indirect way by adding extra fields and symmetries (see, for instance, [25–27]).

Our flipped $SO(10)\otimes U(1)_X$ setting offers the rather economical possibility of embedding the electroweak doublets directly into the spinors without the need of 10_H (see Sec. V). As a matter of fact, there exists a variant of the DW mechanism where the $SU(2)_L$ doublets, contained in the $16_H \oplus \overline{16}_H$, are kept massless by the condition $\omega_Y = 0$ (see e.g. [28]). However, in order to satisfy in a natural way the F flatness for the configuration $\omega_Y = 0$, again a contrived superpotential is required, when compared to that in Eq. (14). In conclusion, we cannot implement in our simple setup any of the natural mechanisms proposed so far (see also [29]), and we have to resort to the standard minimal fine-tuning.

IV. MINIMAL E_6 EMBEDDING

The natural and minimal unified embedding of the flipped $SO(10)\otimes U(1)$ model is E_6 with one 78_H and two pairs of $27_H \oplus \overline{27}_H$ in the Higgs sector. The three matter families are contained in three 27_F chiral superfields. The decomposition of the 27 and 78 representations under the SM quantum numbers is detailed in Tables VI, VII, VIII, and IX, according to the different hypercharge embeddings.

TABLE VI. Decomposition of the fundamental representation 27 of E_6 under $SU(3)_c\otimes SU(2)_L\otimes U(1)_Y$, according to the three SM-compatible different embeddings of the hypercharge (f stands for flipped). The numerical subscripts keep track of the $SU(5)$ and $SO(10)$ origin.

$SU(5)$	$SU(5)_f$	$SO(10)_f$
$(\bar{3},1;+\frac{1}{3})_{\bar{5}_{16}}$	$(\bar{3},1;-\frac{2}{3})_{\bar{5}_{16}}$	$(\bar{3},1;+\frac{1}{3})_{\bar{5}_{16}}$
$(1,2;-\frac{1}{2})_{\bar{5}_{16}}$	$(1,2;-\frac{1}{2})_{\bar{5}_{16}}$	$(1,2;+\frac{1}{2})_{\bar{5}_{16}}$
$(3,2;+\frac{1}{6})_{10_{16}}$	$(3,2;+\frac{1}{6})_{10_{16}}$	$(3,2;+\frac{1}{6})_{10_{16}}$
$(\bar{3},1;-\frac{2}{3})_{10_{16}}$	$(\bar{3},1;+\frac{1}{3})_{10_{16}}$	$(\bar{3},1;+\frac{1}{3})_{10_{16}}$
$(1,1;+1)_{10_{16}}$	$(1,1;0)_{10_{16}}$	$(1,1;0)_{10_{16}}$
$(1,1;0)_{1_{16}}$	$(1,1;+1)_{1_{16}}$	$(1,1;0)_{1_{16}}$
$(3,1;-\frac{1}{3})_{5_{10}}$	$(3,1;-\frac{1}{3})_{5_{10}}$	$(3,1;-\frac{1}{3})_{5_{10}}$
$(1,2;+\frac{1}{2})_{5_{10}}$	$(1,2;-\frac{1}{2})_{5_{10}}$	$(1,2;-\frac{1}{2})_{5_{10}}$
$(\bar{3},1;+\frac{1}{3})_{\bar{5}_{10}}$	$(\bar{3},1;+\frac{1}{3})_{\bar{5}_{10}}$	$(\bar{3},1;-\frac{2}{3})_{\bar{5}_{10}}$
$(1,2;-\frac{1}{2})_{\bar{5}_{10}}$	$(1,2;+\frac{1}{2})_{\bar{5}_{10}}$	$(1,2;-\frac{1}{2})_{\bar{5}_{10}}$
$(1,1;0)_{1_1}$	$(1,1;0)_{1_1}$	$(1,1;+1)_{1_1}$

TABLE VII. The same as in Table VI, where the subscripts keep track of the $SU(4)_C$ and $SO(10)$ origins. The symbols $\pm$ refer to the eigenvalues of $T_R^{(3)}$.

$SU(5)$	$SU(5)_f$	$SO(10)_f$
$(3, 2; +\frac{1}{6})_{4_{16}}$	$(3, 2; +\frac{1}{6})_{4_{16}}$	$(3, 2; +\frac{1}{6})_{4_{16}}$
$(1, 2; -\frac{1}{2})_{4_{16}}$	$(1, 2; -\frac{1}{2})_{4_{16}}$	$(1, 2; +\frac{1}{2})_{4_{16}}$
$(\bar{3}, 1; +\frac{1}{3})_{\bar{4}^+_{16}}$	$(\bar{3}, 1; -\frac{2}{3})_{\bar{4}^+_{16}}$	$(\bar{3}, 1; +\frac{1}{3})_{\bar{4}^+_{16}}$
$(\bar{3}, 1; -\frac{2}{3})_{\bar{4}^-_{16}}$	$(\bar{3}, 1; +\frac{1}{3})_{\bar{4}^-_{16}}$	$(\bar{3}, 1; +\frac{1}{3})_{\bar{4}^-_{16}}$
$(1, 1; +1)_{\bar{4}^+_{16}}$	$(1, 1; 0)_{\bar{4}^+_{16}}$	$(1, 1; 0)_{\bar{4}^+_{16}}$
$(1, 1; 0)_{\bar{4}^-_{16}}$	$(1, 1; +1)_{\bar{4}^-_{16}}$	$(1, 1; 0)_{\bar{4}^-_{16}}$
$(3, 1; -\frac{1}{3})_{6_{10}}$	$(3, 1; -\frac{1}{3})_{6_{10}}$	$(3, 1; -\frac{1}{3})_{6_{10}}$
$(\bar{3}, 1; +\frac{1}{3})_{6_{10}}$	$(\bar{3}, 1; +\frac{1}{3})_{6_{10}}$	$(\bar{3}, 1; -\frac{2}{3})_{6_{10}}$
$(1, 2; +\frac{1}{2})_{1^+_{10}}$	$(1, 2; -\frac{1}{2})_{1^+_{10}}$	$(1, 2; -\frac{1}{2})_{1^+_{10}}$
$(1, 2; -\frac{1}{2})_{1^-_{10}}$	$(1, 2; +\frac{1}{2})_{1^-_{10}}$	$(1, 2; -\frac{1}{2})_{1^-_{10}}$
$(1, 1; 0)_{1_1}$	$(1, 1; 0)_{1_1}$	$(1, 1; +1)_{1_1}$

In analogy with the flipped $SO(10)$ discussion, we shall label the SM singlets contained in the 27 as $e \equiv (1, 1; 0)_{1_1}$ and $\nu \equiv (1, 1; 0)_{1_{16}}$.

As we are going to show, the little group of a supersymmetric $\langle 78 \oplus 27_1 \oplus 27_2 \oplus \overline{27}_1 \oplus \overline{27}_2 \rangle$ vacuum is $SU(5)$ in the renormalizable case. This is just a consequence of the larger E_6 algebra. In order to obtain a SM vacuum, we need to resort to a NR scenario that allows for a disentanglement of the $\langle 78_H \rangle$ directions and, consistently, for a flipped $SO(10) \otimes U(1)$ intermediate stage. We shall make the case for an E_6 gauge symmetry broken near the Planck scale, leaving an effective flipped $SO(10)$ scenario down to the 10^{16} GeV.

TABLE VIII. The same as in Table VI for the 78 representation.

$SU(5)$	$SU(5)_f$	$SO(10)_f$
$(1, 1; 0)_{1_1}$	$(1, 1; 0)_{1_1}$	$(1, 1; 0)_{1_1}$
$(1, 1; 0)_{1_{45}}$	$(1, 1; 0)_{1_{45}}$	$(1, 1; 0)_{1_{45}}$
$(8, 1; 0)_{24_{45}}$	$(8, 1; 0)_{24_{45}}$	$(8, 1; 0)_{24_{45}}$
$(3, 2; -\frac{5}{6})_{24_{45}}$	$(3, 2; +\frac{1}{6})_{24_{45}}$	$(3, 2; +\frac{1}{6})_{24_{45}}$
$(\bar{3}, 2; +\frac{5}{6})_{24_{45}}$	$(\bar{3}, 2; -\frac{1}{6})_{24_{45}}$	$(\bar{3}, 2; -\frac{1}{6})_{24_{45}}$
$(1, 3; 0)_{24_{45}}$	$(1, 3; 0)_{24_{45}}$	$(1, 3; 0)_{24_{45}}$
$(1, 1; 0)_{24_{45}}$	$(1, 1; 0)_{24_{45}}$	$(1, 1; 0)_{24_{45}}$
$(3, 2; +\frac{1}{6})_{10_{45}}$	$(3, 2; -\frac{5}{6})_{10_{45}}$	$(3, 2; +\frac{1}{6})_{10_{45}}$
$(\bar{3}, 1; -\frac{2}{3})_{10_{45}}$	$(\bar{3}, 1; -\frac{2}{3})_{10_{45}}$	$(\bar{3}, 1; +\frac{1}{3})_{10_{45}}$
$(1, 1; +1)_{10_{45}}$	$(1, 1; -1)_{10_{45}}$	$(1, 1; 0)_{10_{45}}$
$(\bar{3}, 2; -\frac{1}{6})_{\overline{10}_{45}}$	$(\bar{3}, 2; +\frac{5}{6})_{\overline{10}_{45}}$	$(\bar{3}, 2; -\frac{1}{6})_{\overline{10}_{45}}$
$(3, 1; +\frac{2}{3})_{\overline{10}_{45}}$	$(3, 1; +\frac{2}{3})_{\overline{10}_{45}}$	$(3, 1; -\frac{1}{3})_{\overline{10}_{45}}$
$(1, 1; -1)_{\overline{10}_{45}}$	$(1, 1; +1)_{\overline{10}_{45}}$	$(1, 1; 0)_{\overline{10}_{45}}$
$(\bar{3}, 1; +\frac{1}{3})_{\bar{5}_{16}}$	$(\bar{3}, 1; -\frac{2}{3})_{\bar{5}_{16}}$	$(\bar{3}, 1; -\frac{2}{3})_{\bar{5}_{16}}$
$(1, 2; -\frac{1}{2})_{\bar{5}_{16}}$	$(1, 2; -\frac{1}{2})_{\bar{5}_{16}}$	$(1, 2; -\frac{1}{2})_{\bar{5}_{16}}$
$(3, 2; +\frac{1}{6})_{10_{16}}$	$(3, 2; +\frac{1}{6})_{10_{16}}$	$(3, 2; -\frac{5}{6})_{10_{16}}$
$(\bar{3}, 1; -\frac{2}{3})_{10_{16}}$	$(\bar{3}, 1; +\frac{1}{3})_{10_{16}}$	$(\bar{3}, 1; -\frac{2}{3})_{10_{16}}$
$(1, 1; +1)_{10_{16}}$	$(1, 1; 0)_{10_{16}}$	$(1, 1; -1)_{10_{16}}$
$(1, 1; 0)_{1_{16}}$	$(1, 1; +1)_{1_{16}}$	$(1, 1; -1)_{1_{16}}$
$(3, 1; -\frac{1}{3})_{5_{\overline{16}}}$	$(3, 1; +\frac{2}{3})_{5_{\overline{16}}}$	$(3, 1; +\frac{2}{3})_{5_{\overline{16}}}$
$(1, 2; +\frac{1}{2})_{5_{\overline{16}}}$	$(1, 2; +\frac{1}{2})_{5_{\overline{16}}}$	$(1, 2; +\frac{1}{2})_{5_{\overline{16}}}$
$(\bar{3}, 2; -\frac{1}{6})_{\overline{10}_{\overline{16}}}$	$(\bar{3}, 2; -\frac{1}{6})_{\overline{10}_{\overline{16}}}$	$(\bar{3}, 2; +\frac{5}{6})_{\overline{10}_{\overline{16}}}$
$(3, 1; +\frac{2}{3})_{\overline{10}_{\overline{16}}}$	$(3, 1; -\frac{1}{3})_{\overline{10}_{\overline{16}}}$	$(3, 1; +\frac{2}{3})_{\overline{10}_{\overline{16}}}$
$(1, 1; -1)_{\overline{10}_{\overline{16}}}$	$(1, 1; 0)_{\overline{10}_{\overline{16}}}$	$(1, 1; +1)_{\overline{10}_{\overline{16}}}$
$(1, 1; 0)_{1_{\overline{16}}}$	$(1, 1; -1)_{1_{\overline{16}}}$	$(1, 1; +1)_{1_{\overline{16}}}$

TABLE IX. The same as in Table VII for the 78 representation.

$SU(5)$	$SU(5)_f$	$SO(10)_f$
$(1, 1; 0)_{1_1}$	$(1, 1; 0)_{1_1}$	$(1, 1; 0)_{1_1}$
$(1, 1; 0)_{15_{45}}$	$(1, 1; 0)_{15_{45}}$	$(1, 1; 0)_{15_{45}}$
$(8, 1; 0)_{15_{45}}$	$(8, 1; 0)_{15_{45}}$	$(8, 1; 0)_{15_{45}}$
$(3, 1; +\frac{2}{3})_{15_{45}}$	$(3, 1; +\frac{2}{3})_{15_{45}}$	$(3, 1; -\frac{1}{3})_{15_{45}}$
$(\bar{3}, 1; -\frac{2}{3})_{15_{45}}$	$(\bar{3}, 1; -\frac{2}{3})_{15_{45}}$	$(\bar{3}, 1; +\frac{1}{3})_{15_{45}}$
$(1, 3; 0)_{1_{45}}$	$(1, 3; 0)_{1_{45}}$	$(1, 3; 0)_{1_{45}}$
$(1, 1; +1)_{1^+_{45}}$	$(1, 1; -1)_{1^+_{45}}$	$(1, 1; 0)_{1^+_{45}}$
$(1, 1; 0)_{1^0_{45}}$	$(1, 1; 0)_{1^0_{45}}$	$(1, 1; 0)_{1^0_{45}}$
$(1, 1; -1)_{1^-_{45}}$	$(1, 1; +1)_{1^-_{45}}$	$(1, 1; 0)_{1^-_{45}}$
$(3, 2; +\frac{1}{6})_{6^+_{45}}$	$(3, 2; -\frac{5}{6})_{6^+_{45}}$	$(3, 2; +\frac{1}{6})_{6^+_{45}}$
$(3, 2; -\frac{5}{6})_{6^-_{45}}$	$(3, 2; +\frac{1}{6})_{6^-_{45}}$	$(3, 2; +\frac{1}{6})_{6^-_{45}}$
$(\bar{3}, 2; +\frac{5}{6})_{6^+_{45}}$	$(\bar{3}, 2; -\frac{1}{6})_{6^+_{45}}$	$(\bar{3}, 2; -\frac{1}{6})_{6^+_{45}}$
$(\bar{3}, 2; -\frac{1}{6})_{6^-_{45}}$	$(\bar{3}, 2; +\frac{5}{6})_{6^-_{45}}$	$(\bar{3}, 2; -\frac{1}{6})_{6^-_{45}}$
$(3, 2; +\frac{1}{6})_{4_{16}}$	$(3, 2; +\frac{1}{6})_{4_{16}}$	$(3, 2; -\frac{5}{6})_{4_{16}}$
$(1, 2; -\frac{1}{2})_{4_{16}}$	$(1, 2; -\frac{1}{2})_{4_{16}}$	$(1, 2; -\frac{1}{2})_{4_{16}}$
$(\bar{3}, 1; +\frac{1}{3})_{\bar{4}^+_{16}}$	$(\bar{3}, 1; -\frac{2}{3})_{\bar{4}^+_{16}}$	$(\bar{3}, 1; -\frac{2}{3})_{\bar{4}^+_{16}}$
$(\bar{3}, 1; -\frac{2}{3})_{\bar{4}^-_{16}}$	$(\bar{3}, 1; +\frac{1}{3})_{\bar{4}^-_{16}}$	$(\bar{3}, 1; -\frac{2}{3})_{\bar{4}^-_{16}}$
$(1, 1; +1)_{\bar{4}^+_{16}}$	$(1, 1; 0)_{\bar{4}^+_{16}}$	$(1, 1; -1)_{\bar{4}^+_{16}}$
$(1, 1; 0)_{\bar{4}^-_{16}}$	$(1, 1; +1)_{\bar{4}^-_{16}}$	$(1, 1; -1)_{\bar{4}^-_{16}}$
$(\bar{3}, 2; -\frac{1}{6})_{\bar{4}_{\overline{16}}}$	$(\bar{3}, 2; -\frac{1}{6})_{\bar{4}_{\overline{16}}}$	$(\bar{3}, 2; +\frac{5}{6})_{\bar{4}_{\overline{16}}}$
$(1, 2; +\frac{1}{2})_{\bar{4}_{\overline{16}}}$	$(1, 2; +\frac{1}{2})_{\bar{4}_{\overline{16}}}$	$(1, 2; +\frac{1}{2})_{\bar{4}_{\overline{16}}}$
$(3, 1; -\frac{1}{3})_{4^-_{\overline{16}}}$	$(3, 1; +\frac{2}{3})_{4^-_{\overline{16}}}$	$(3, 1; +\frac{2}{3})_{4^-_{\overline{16}}}$
$(3, 1; +\frac{2}{3})_{4^+_{\overline{16}}}$	$(3, 1; -\frac{1}{3})_{4^+_{\overline{16}}}$	$(3, 1; +\frac{2}{3})_{4^+_{\overline{16}}}$
$(1, 1; -1)_{4^-_{\overline{16}}}$	$(1, 1; 0)_{4^-_{\overline{16}}}$	$(1, 1; +1)_{4^-_{\overline{16}}}$
$(1, 1; 0)_{4^+_{\overline{16}}}$	$(1, 1; -1)_{4^+_{\overline{16}}}$	$(1, 1; +1)_{4^+_{\overline{16}}}$

A. Y and $B - L$ into E_6

Interpreting the different possible definitions of the SM hypercharge in terms of the E_6 maximal subalgebra $SU(3)_c \otimes SU(3)_L \otimes SU(3)_R$, one finds that the three assignments in Eqs. (8)–(10) are each orthogonal to the three possible ways of embedding $SU(2)_I$ (with $I = R, R', E$) into $SU(3)_R$ [20]. Working in the Gell-Mann basis (cf. Appendix B 1) the $SU(3)_R$ Cartan generators read

$$T_R^{(3)} = \tfrac{1}{2}(T^{1'}_{1'} - T^{2'}_{2'}), \tag{30}$$

$$T_R^{(8)} = \frac{1}{2\sqrt{3}}(T^{1'}_{1'} + T^{2'}_{2'} - 2T^{3'}_{3'}), \tag{31}$$

which defines the $SU(2)_R$ embedding. The $SU(2)_{R'}$ and $SU(2)_E$ embeddings are obtained from Eqs. (30) and (31) by flipping, respectively, $2'\leftrightarrow 3'$ and $3'\leftrightarrow 1'$. Considering the standard and flipped $SO(10)$ embeddings of the hypercharge in Eqs. (8) and (10), in the $SU(3)^3$ notation they respectively read

$$Y=\frac{1}{\sqrt{3}}T_L^{(8)}+T_R^{(3)}+\frac{1}{\sqrt{3}}T_R^{(8)}=\frac{1}{\sqrt{3}}T_L^{(8)}-\frac{2}{\sqrt{3}}T_E^{(8)} \quad (32)$$

and

$$Y=\frac{1}{\sqrt{3}}T_L^{(8)}-\frac{2}{\sqrt{3}}T_R^{(8)}=\frac{1}{\sqrt{3}}T_L^{(8)}+T_E^{(3)}+\frac{1}{\sqrt{3}}T_E^{(8)}. \quad (33)$$

The three SM-compatible assignments of $B-L$ in Eqs. (11)–(13) are also orthogonal to the three possible ways of embedding $SU(2)_I$ into $SU(3)_R$. However, once we fix the embedding of the hypercharge, we have only two consistent choices for $B-L$ available. They correspond to the pairs where Y and $B-L$ are not orthogonal to the same $SU(2)_I$ [20].

For the standard hypercharge embedding, the $B-L$ assignment in Eq. (11) reads

$$B-L=\frac{2}{\sqrt{3}}(T_L^{(8)}+T_R^{(8)})=\frac{2}{\sqrt{3}}T_L^{(8)}-T_E^{(3)}-\frac{1}{\sqrt{3}}T_E^{(8)}, \quad (34)$$

while the $B-L$ assignment in Eq. (13), consistent with the flipped $SO(10)$ embedding of the hypercharge, reads

$$B-L=\frac{2}{\sqrt{3}}T_L^{(8)}-T_R^{(3)}-\frac{1}{\sqrt{3}}T_R^{(8)}=\frac{2}{\sqrt{3}}(T_L^{(8)}+T_E^{(8)}). \quad (35)$$

B. The E_6 vacuum manifold

The most general renormalizable Higgs superpotential, made of the representations $78\oplus 27_1\oplus 27_2\oplus \overline{27}_1\oplus\overline{27}_2$, is given by

$$W_H=\frac{\mu}{2}\,\mathrm{Tr}78^2+\rho_{ij}27_i\overline{27}_j+\tau_{ij}27_i78\overline{27}_j+\alpha_{ijk}27_i27_j27_k+\beta_{ijk}\overline{27}_i\overline{27}_j\overline{27}_k, \quad (36)$$

where $i,j=1,2$. The couplings α_{ijk} and β_{ijk} are totally symmetric in ijk, so that each one of them contains four complex parameters. Without loss of generality, we can take μ real by a phase redefinition of the superpotential, while τ can be diagonalized by a bi-unitary transformation acting on the indices of the 27 and the $\overline{27}$. We take $\tau_{ij}=\tau_i\delta_{ij}$, with τ_i real. Notice that α and β are not relevant for the present study, since the corresponding invariants vanish on the SM orbit.

In the standard hypercharge embedding of Eq. (32), the SM-preserving vacuum directions are parametrized by

$$\langle 78\rangle=a_1T_{2'}^{3'}+a_2T_{3'}^{2'}+\frac{a_3}{\sqrt{6}}(T_{1'}^{1'}+T_{2'}^{2'}-2T_{3'}^{3'})+\frac{a_4}{\sqrt{2}}(T_{1'}^{1'}-T_{2'}^{2'})+\frac{b_3}{\sqrt{6}}(T_1^1+T_2^2-2T_3^3), \quad (37)$$

and

$$\langle 27_i\rangle=(e_i)v_{3'}^3+(\nu_i)v_{2'}^3, \quad (38)$$

$$\langle\overline{27}_i\rangle=(\bar e_i)u_3^{3'}+(\bar\nu_i)u_3^{2'}, \quad (39)$$

where a_1, a_2, a_3, a_4, b_3, $e_{1,2}$, $\bar e_{1,2}$, $\nu_{1,2}$, and $\bar\nu_{1,2}$ are 13 SM-singlet VEVs (see Appendix B 1 for the notation). Given the $B-L$ expression in Eq. (34) and the fact that we can rewrite the Cartan part of $\langle 78\rangle$ as

$$\sqrt{2}a_4T_R^{(3)}+\frac{1}{\sqrt{2}}(a_3+b_3)(T_R^{(8)}+T_L^{(8)})+\frac{1}{\sqrt{2}}(a_3-b_3)(T_R^{(8)}-T_L^{(8)}), \quad (40)$$

we readily identify the standard $SO(10)$ VEVs used in the previous section with the present E_6 notation as $\omega_R\propto a_4$, $\omega_Y\propto a_3+b_3$, while $\Omega\propto a_3-b_3$ is the $SO(10)\otimes U(1)_X$ singlet VEV in E_6 ($T_X\propto T_R^{(8)}-T_L^{(8)}$).

We can also write the vacuum manifold in such a way that it is manifestly invariant under the flipped $SO(10)$ hypercharge in Eq. (33). This can be obtained by flipping $1'\leftrightarrow 3'$ in Eqs. (37)–(39), yielding

$$\langle 78\rangle=a_1T_{2'}^{1'}+a_2T_{1'}^{2'}+\sqrt{2}a_4'T_E^{(3)}+\frac{1}{\sqrt{2}}(a_3'+b_3)(T_E^{(8)}+T_L^{(8)})+\frac{1}{\sqrt{2}}(a_3'-b_3)(T_E^{(8)}-T_L^{(8)}), \quad (41)$$

$$\langle 27_i\rangle=(e_i)v_{1'}^3+(\nu_i)v_{2'}^3, \quad (42)$$

$$\langle\overline{27}_i\rangle=(\bar e_i)u_3^{1'}+(\bar\nu_i)u_3^{2'}, \quad (43)$$

where we recognize the $B-L$ generator defined in Eq. (35). Notice that the Cartan subalgebra is actually invariant under both the standard and the flipped $SO(10)$ form of Y. We have

$$a_3'T_E^{(8)}+a_4'T_E^{(3)}=a_3T_R^{(8)}+a_4T_R^{(3)}, \quad (44)$$

with

$$2a_3'=-a_3-\sqrt{3}a_4, \quad (45)$$

$$2a_4'=-\sqrt{3}a_3+a_4, \quad (46)$$

thus making the use of the $a_{3,4}$ or $a_{3,4}'$ directions in the flipped or standard vacuum manifold completely equivalent. We can now complete the identification of the notation used for E_6 with that of the flipped $SO(10) \otimes U(1)_X$ model studied in Sec. III, by $\omega^\pm \propto a_{1,2}$.

From the E_6 standpoint, the analyses of the standard and flipped vacuum manifolds given, respectively, in Eqs. (37)–(39) and Eqs. (41)–(43) lead, as expected, to the same results, with the roles of the standard and the flipped hypercharge interchanged (see Appendix B). In order to determine the vacuum little group, we may therefore proceed with the explicit discussion of the standard setting.

By writing the superpotential in Eq. (36) on the SM-preserving vacuum in Eqs. (37)–(39), we find

$$\begin{aligned}\langle W_H\rangle = {}& \mu\left(a_1a_2 + \frac{a_3^2}{2} + \frac{a_4^2}{2} + \frac{b_3^2}{2}\right) + \rho_{11}(e_1\bar{e}_1 + \nu_1\bar{\nu}_1) + \rho_{21}(e_2\bar{e}_1 + \nu_2\bar{\nu}_1) + \rho_{12}(e_1\bar{e}_2 + \nu_1\bar{\nu}_2) + \rho_{22}(e_2\bar{e}_2 + \nu_2\bar{\nu}_2) \\ & + \tau_1\left[-a_1e_1\bar{\nu}_1 - a_2\nu_1\bar{e}_1 + \sqrt{\frac{2}{3}}a_3\left(e_1\bar{e}_1 - \frac{1}{2}\nu_1\bar{\nu}_1\right) + \frac{a_4\nu_1\bar{\nu}_1}{\sqrt{2}} - \sqrt{\frac{2}{3}}b_3(e_1\bar{e}_1 + \nu_1\bar{\nu}_1)\right] \\ & + \tau_2\left[-a_1e_2\bar{\nu}_2 - a_2\nu_2\bar{e}_2 + \sqrt{\frac{2}{3}}a_3\left(e_2\bar{e}_2 - \frac{1}{2}\nu_2\bar{\nu}_2\right) + \frac{a_4\nu_2\bar{\nu}_2}{\sqrt{2}} - \sqrt{\frac{2}{3}}b_3(e_2\bar{e}_2 + \nu_2\bar{\nu}_2)\right]. \end{aligned} \tag{47}$$

When applying the constraints coming from D- and F-term equations, a nontrivial vacuum exists if ρ and τ are Hermitian, as in the flipped $SO(10)$ case. This is a consequence of the fact that D flatness implies $\langle 27_i\rangle = \langle\overline{27}_i\rangle^*$ (see Appendix B 2 for details).

After imposing all the constraints due to D and F flatness, the E_6 vacuum manifold can be finally written as

$$\begin{aligned} 2\mu a_1 &= \tau_1 r_1^2 \sin 2\alpha_1 e^{i(\phi_{\nu_1}-\phi_{e_1})} + \tau_2 r_2^2 \sin 2\alpha_2 e^{i(\phi_{\nu_2}-\phi_{e_2})}, \\ 2\mu a_2 &= \tau_1 r_1^2 \sin 2\alpha_1 e^{-i(\phi_{\nu_1}-\phi_{e_1})} \\ &\quad + \tau_2 r_2^2 \sin 2\alpha_2 e^{-i(\phi_{\nu_2}-\phi_{e_2})}, \\ 2\sqrt{6}\mu a_3 &= -\tau_1 r_1^2(3\cos 2\alpha_1 + 1) - \tau_2 r_2^2(3\cos 2\alpha_2 + 1), \\ \sqrt{2}\mu a_4 &= -\tau_1 r_1^2 \sin^2\alpha_1 - \tau_2 r_2^2 \sin^2\alpha_2, \\ \sqrt{3}\mu b_3 &= \sqrt{2}\tau_1 r_1^2 + \sqrt{2}\tau_2 r_2^2, \\ e_{1,2} &= r_{1,2}\cos\alpha_{1,2} e^{i\phi_{e_{1,2}}}, \\ \nu_{1,2} &= r_{1,2}\sin\alpha_{1,2} e^{i\phi_{\nu_{1,2}}}, \\ \bar{e}_{1,2} &= r_{1,2}\cos\alpha_{1,2} e^{-i\phi_{e_{1,2}}}, \\ \bar{\nu}_{1,2} &= r_{1,2}\sin\alpha_{1,2} e^{-i\phi_{\nu_{1,2}}}, \end{aligned} \tag{48}$$

where $r_{1,2}$ and $\alpha^\pm \equiv \alpha_1 \pm \alpha_2$ are fixed in terms of superpotential parameters as follows:

$$r_1^2 = -\frac{\mu(\rho_{22}\tau_1 - 4\rho_{11}\tau_2)}{5\tau_1^2\tau_2}, \tag{49}$$

$$r_2^2 = -\frac{\mu(\rho_{11}\tau_2 - 4\rho_{22}\tau_1)}{5\tau_1\tau_2^2}, \tag{50}$$

$$\cos\alpha^- = \xi\frac{\sin\Phi_\nu - \sin\Phi_e}{\sin(\Phi_\nu - \Phi_e)}, \tag{51}$$

$$\cos\alpha^+ = \xi\frac{\sin\Phi_\nu + \sin\Phi_e}{\sin(\Phi_\nu - \Phi_e)}, \tag{52}$$

with

$$\xi = \frac{5|\rho_{12}|}{\sqrt{-\frac{4\rho_{11}^2\tau_2}{\tau_1} - \frac{4\rho_{22}^2\tau_1}{\tau_2} + 17\rho_{22}\rho_{11}}}. \tag{53}$$

The phase factors Φ_ν and Φ_e are defined as

$$\Phi_\nu \equiv \phi_{\nu_1} - \phi_{\nu_2} + \phi_{\rho_{12}}, \qquad \Phi_e \equiv \phi_{e_1} - \phi_{e_2} + \phi_{\rho_{12}}. \tag{54}$$

In Appendix B 3 we show that the little group of the vacuum manifold in Eq. (48) is $SU(5)$.

It is instructive to look at the configuration in which one pair of 27_H, let us say $27_2 \oplus \overline{27}_2$, is decoupled. This case can be obtained by setting $\tau_2 = \rho_{12} = \rho_{22} = 0$ in the relevant equations. In agreement with Ref. [12], we find that α_1 turns out to be undetermined by the F-term constraints, thus parametrizing a set of isomorphic solutions. We may therefore take in Eq. (48) $\alpha_1 = \alpha_2 = 0$ and show that the little group corresponds in this case to $SO(10)$ (see Appendix B 3), thus recovering the result of Ref. [12].

The same result is obtained in the case in which the vacua of the two copies of $27_H \oplus \overline{27}_H$ are aligned, i.e. $\alpha^- = 0$ and $\Phi_\nu = \Phi_e$. Analogously to the discussion in Sec. III B, α^+ is, in this case, undetermined and it can be set to zero, which leads us again to the one $27_H \oplus \overline{27}_H$ case, with $SO(10)$ as the preserved algebra.

These results are intuitively understood by considering that, in case there is just one pair of $27_H \oplus \overline{27}_H$ (or the vacua of the two pairs of $27_i \oplus \overline{27}_i$ are aligned), the SM-singlet directions e and ν are connected by an $SU(2)_R$ transformation which can be used to rotate one of the VEVs to zero, so that the little group is locked to an $SO(10)$ configuration. On the other hand, two misaligned $27_H \oplus \overline{27}_H$ VEVs in the $e - \nu$ plane lead (just by inspection of the VEV quantum numbers) to an $SU(5)$ little group.

In analogy with the flipped $SO(10)$ case, the Higgs adjoint plays the role of a renormalizable agent that prevents the two pairs of $\langle 27_i \oplus \overline{27}_i \rangle$ from aligning within each other along the $SO(10)$ vacuum. Actually, by decoupling the adjoint Higgs, F flatness makes the (aligned) $27_i \oplus 27_i$ vacuum trivial, as one verifies by inspecting the F terms in Eq. (B18) of Appendix B 2 for $\langle 78_H \rangle = 0$ and $\det\rho \neq 0$.

In conclusion, due to the larger E_6 algebra, the vacuum little group remains $SU(5)$. In this respect we guess that the authors of Ref. [30], who advocate a $78_H \oplus 2 \times (27_H \oplus \overline{27}_H)$ Higgs sector, implicitly refer to a NR setting.

C. Breaking the residual $SU(5)$ via effective interactions

In this section we consider the possibility of breaking the residual $SU(5)$ symmetry of the renormalizable E_6 vacuum through the inclusion of effective adjoint Higgs interactions near the Planck scale M_P. We argue that an effective flipped $SO(10) \otimes U(1)_X \equiv SO(10)_f$ may survive down to the $M_f \approx 10^{16}$ GeV scale, with thresholds spread in between M_P and M_f in such a way that they do not affect the proton stability but lead to realistic neutrino masses.

The relevant part of the nonrenormalizable superpotential at the E_6 scale $M_E < M_P$ can be written as

$$W_H^{\rm NR} = \frac{1}{M_P}[\lambda_1 ({\rm Tr}78^2)^2 + \lambda_2 \,{\rm Tr}78^4 + \ldots], \qquad (55)$$

where the ellipsis stands for terms which include powers of the 27's representations and $D \geq 5$ operators. Projecting Eq. (55) along the SM-singlet vacuum directions in Eqs. (37)–(39), we obtain

$$\langle W_H^{\rm NR} \rangle = \frac{1}{M_P}\Big\{\lambda_1(2a_1a_2 + a_3^2 + a_4^2 + b_3^2)^2 + \lambda_2\Big[2a_1a_2\Big(a_1^2a_2^2 + a_3^2 + a_4^2 + \frac{1}{\sqrt{3}}a_3a_4\Big) + \frac{1}{2}(a_3^2 + a_4^2)^2 + \frac{1}{2}b_3^4\Big] + \ldots\Big\}. \qquad (56)$$

One verifies that including the NR contribution in the F-term equations allows for a disentanglement of the $\langle 78 \rangle$ and $\langle 27_1 \oplus \overline{27}_1 \oplus 27_2 \oplus \overline{27}_2 \rangle$ VEVs, so that the breaking to the SM is achieved. In particular, the SUSY vacuum allows for an intermediate $SO(10)_f$ stage [that is prevented by the simple renormalizable vacuum manifold in Eq. (48)]. By including Eq. (56) in the F-term equations, we can consistently neglect all VEVs except for the $SO(10) \otimes U(1)$ singlet Ω, which reads

$$\Omega^2 = -\frac{\mu M_P}{5\lambda_1 + \frac{1}{2}\lambda_2}. \qquad (57)$$

It is therefore possible to envisage a scenario where the E_6 symmetry is broken at a scale $M_E < M_P$, leaving an effective flipped $SO(10) \otimes U(1)_X$ scenario down to the 10^{16} GeV, as discussed in Sec. III. All remaining SM-singlet VEVs are contained in $45 \oplus 16_1 \oplus \overline{16}_1 \oplus 16_2 \oplus \overline{16}_2$ which are the only Higgs multiplets required to survive at the $M_f \ll M_E$ scale. It is clear that this is a plausibility argument and that a detailed study of the E_6 vacuum and related thresholds is needed to ascertain the feasibility of the scenario.

The NR breaking of E_6 through an intermediate $SO(10)_f$ stage driven by $\Omega \gg M_f$, while allowing (as we shall discuss next) for a consistent unification pattern, avoids the issues arising within a one-step breaking. As a matter of fact, the colored triplets responsible for $D = 5$ proton decay live naturally at the $\Omega^2/M_P > M_f$ scale, while the masses of the SM-singlet neutrino states which enter the "extended" type-I seesaw formula are governed by the $\langle 27 \rangle \sim M_f$ (see the discussion in Sec. V).

D. A unified E_6 scenario

Let us examine the plausibility of the two-step gauge unification scenario discussed in the previous subsection. We consider here just a simplified description that neglects threshold effects. As a first quantitative estimate of the running effects on the $SO(10)_f$ couplings, let us introduce the quantity

$$\Delta(M_f) \equiv \frac{\alpha_{\hat{X}}^{-1}(M_f) - \alpha_{10}^{-1}(M_f)}{\alpha_E^{-1}} = \frac{1}{\alpha_E^{-1}}\,\frac{b_{\hat{X}} - b_{10}}{2\pi}\log\frac{M_E}{M_f}, \qquad (58)$$

where M_E is the E_6 unification scale and α_E is the E_6 gauge coupling. The $U(1)_X$ charge has been properly normalized to $\hat{X} = X/\sqrt{24}$. The one-loop beta coefficients for the superfield content $45_H \oplus 2 \times (16_H \oplus \overline{16}_H) \oplus 3 \times (16_F \oplus 10_F \oplus 1_F) \oplus 45_G$ are found to be $b_{10} = 1$ and $b_{\hat{X}} = 67/24$.

Taking, for the sake of an estimate, a typical MSSM value for the GUT coupling $\alpha_E^{-1} \approx 25$, for $M_E/M_f < 10^2$, one finds $\Delta(M_f) < 5\%$.

In order to match the $SO(10)_f$ couplings with the measured SM couplings, we consider as a typical setup the two-loop MSSM gauge running with a 1 TeV SUSY scale. The (one-loop) matching of the non-Abelian gauge couplings (in dimensional reduction) at the scale M_f reads

$$\alpha_{10}^{-1}(M_f) = \alpha_2^{-1}(M_f) = \alpha_3^{-1}(M_f), \qquad (59)$$

while for the properly normalized hypercharge $\hat{Y}$ one obtains

$$\alpha_{\hat{Y}}^{-1}(M_f) = (\hat{\alpha}^2 + \hat{\beta}^2)\alpha_{10}^{-1}(M_f) + \hat{\gamma}^2\alpha_{\hat{X}}^{-1}(M_f). \qquad (60)$$

Here we have implemented the relation among the properly normalized $U(1)$ generators [see Eq. (10)]

$$\hat{Y} = \hat{\alpha}\hat{Y}' + \hat{\beta}\hat{Z} + \hat{\gamma}\hat{X}, \qquad (61)$$

with $\{\hat{\alpha}, \hat{\beta}, \hat{\gamma}\} = \{-\frac{1}{5}, -\frac{1}{5}\sqrt{\frac{3}{2}}, \frac{3}{\sqrt{10}}\}$.

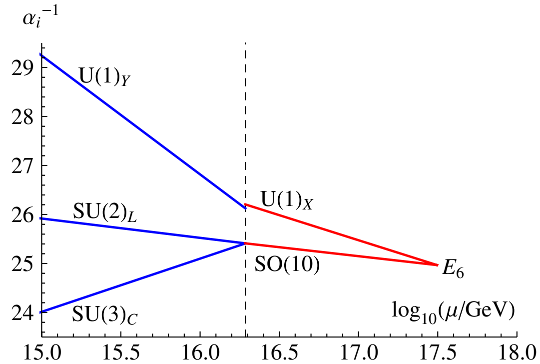

FIG. 1 (color online). Sample picture of the gauge coupling unification in the E_6-embedded $SO(10) \otimes U(1)_X$ model.

The result of this simple exercise is depicted in Fig. 1. Barring detailed threshold effects, it is interesting to see that the qualitative behavior of the relevant gauge couplings is, indeed, consistent with the basic picture of the flipped $SO(10) \otimes U(1)_X$ embedded into a genuine E_6 GUT emerging below the Planck scale.

V. TOWARDS A REALISTIC FLAVOR

The aim of this section is to provide an elementary discussion of the main features and of the possible issues arising in the Yukawa sector of the flipped $SO(10) \otimes U(1)_X$ model under consideration. In order to keep the discussion simple, we shall consider a basic Higgs content with just one pair of $16_H \oplus \overline{16}_H$. As a complement to the tables given in Sec. III, we summarize the SM decomposition of the representations relevant to the Yukawa sector in Table X.

For what follows, we refer to [31–34] and references therein, where the basic features of models with extended matter sector are discussed in the E_6 and the standard $SO(10)$ context. For a scenario employing flipped $SO(10) \otimes U(1)$ [with an additional anomalous $U(1)$] see Ref. [11].

A. Yukawa sector of the flipped $SO(10)$ model

Considering, for simplicity, just one pair of spinor Higgs multiplets, and imposing a Z_2 matter parity (negative for matter and positive for Higgs superfields), the Yukawa superpotential (up to $d = 5$ operators) reads

$$W_Y = Y_U 16_F 10_F 16_H + \frac{1}{M_P}[Y_E 10_F 1_F \overline{16}_H \overline{16}_H + Y_D 16_F 16_F \overline{16}_H \overline{16}_H], \tag{62}$$

where family indices are understood. Notice (cf. Table XI) that due to the flipped embedding the up quarks receive mass at the renormalizable level, while all the other fermion masses need Planck-suppressed effective contributions in order to achieve a realistic texture.

1. Mass matrices

In order to avoid the recursive $1/M_P$ factors, we introduce the following notation for the relevant VEVs (see Table X): $\hat{v}_d \equiv v_d/M_P$, $\hat{\nu}_H \equiv \nu_H/M_P$, and $\hat{s}_H \equiv s_H/M_P$. The M_f-scale mass matrices for the matter fields sharing the same unbroken $SU(3)_c \otimes U(1)_Q$ quantum numbers can be extracted readily by inspecting the SM decomposition of the relevant $1 + 10 + 16$ matter multiplets in the flipped $SO(10)$ setting:

$$M_u = Y_U v_u, \tag{63}$$

$$M_d = \begin{pmatrix} Y_D \hat{\nu}_H v_d & Y_D \hat{s}_H v_d \\ Y_U s_H & Y_U \nu_H \end{pmatrix}, \tag{64}$$

$$M_e = \begin{pmatrix} Y_E \hat{\nu}_H v_d & Y_U s_H \\ Y_E \hat{s}_H v_d & Y_U \nu_H \end{pmatrix}, \tag{65}$$

$$M_\nu = \begin{pmatrix} 0 & 0 & Y_U s_H & 0 & Y_U v_u \\ 0 & 0 & Y_U \nu_H & Y_U v_u & 0 \\ Y_U s_H & Y_U \nu_H & Y_D \hat{v}_d v_d & 2Y_D \hat{v}_d \nu_H & 2Y_D \hat{v}_d s_H \\ 0 & Y_U v_u & 2Y_D \hat{\nu}_H v_d & Y_D \hat{\nu}_H \nu_H & 2Y_D \hat{\nu}_H s_H \\ Y_U v_u & 0 & 2Y_D \hat{s}_H v_d & 2Y_D \hat{s}_H \nu_H & Y_D \hat{s}_H s_H \end{pmatrix}, \tag{66}$$

TABLE X. SM decomposition of $SO(10)$ representations relevant for the Yukawa sector in the standard and flipped hypercharge embedding. In the $SO(10)_f$ case $B - L$ is assigned according to Eq. (13). A self-explanatory SM notation is used, with the outer subscripts labeling the $SU(5)$ origin. The $SU(2)_L$ doublets decompose as $Q = (U, D)$, $L = (N, E)$, $\Lambda = (\Lambda^0, \Lambda^-)$, and $\Lambda^c = (\Lambda^{c+}, \Lambda^{c0})$. Accordingly, $\langle H_u \rangle = (0, v_u)$ and $\langle H_d \rangle = (v_d, 0)$. The D-flatness constraint on the SM-singlet VEVs, s_H and ν_H, is taken into account.

	$SO(10)$	$SO(10)_f$
16_F	$(D^c \oplus L)_{\bar{5}} \oplus (U^c \oplus Q \oplus E^c)_{10} \oplus (N^c)_1$	$(D^c \oplus \Lambda^c)_{\bar{5}} \oplus (\Delta^c \oplus Q \oplus S)_{10} \oplus (N^c)_1$
10_F	$(\Delta \oplus \Lambda^c)_5 \oplus (\Delta^c \oplus \Lambda)_{\bar{5}}$	$(\Delta \oplus L)_5 \oplus (U^c \oplus \Lambda)_{\bar{5}}$
1_F	$(S)_1$	$(E^c)_1$
$\langle 16_H \rangle$	$(0 \oplus \langle H_d \rangle)_{\bar{5}} \oplus (0 \oplus 0 \oplus 0)_{10} \oplus (\nu_H)_1$	$(0 \oplus \langle H_u \rangle)_{\bar{5}} \oplus (0 \oplus 0 \oplus s_H)_{10} \oplus (\nu_H)_1$
$\langle \overline{16}_H \rangle$	$(0 \oplus \langle H_u \rangle)_5 \oplus (0 \oplus 0 \oplus 0)_{\overline{10}} \oplus (\nu_H)_1$	$(0 \oplus \langle H_d \rangle)_5 \oplus (0 \oplus 0 \oplus s_H)_{\overline{10}} \oplus (\nu_H)_1$

TABLE XI. Decomposition of the invariants in Eq. (62) according to flipped $SU(5)$ and the SM. The number in the round brackets stands for the multiplicity of the invariant. The contractions $\bar{5}_{10_F}1_{1_F}\langle\overline{10}_H\rangle\langle\overline{10}_H\rangle$ and $\bar{5}_{16_F}1_{16_F}\langle\overline{10}_H\rangle\langle\overline{10}_H\rangle$ yield no SM invariant.

$16_F 10_F\langle 16_H\rangle$	$10_F 1_F\langle\overline{16}_H\rangle\langle\overline{16}_H\rangle$	$16_F 16_F\langle\overline{16}_H\rangle\langle\overline{16}_H\rangle$
(1) $10_F\bar{5}_F\langle\bar{5}_H\rangle \supset (QU^c + S\Lambda)\langle H_u\rangle$	(2) $\bar{5}_F 1_F\langle 5_H\rangle\langle\bar{1}_H\rangle \supset \Lambda E^c\langle H_d\rangle\nu_H$	(1) $1_F 1_F\langle\bar{1}_H\rangle\langle\bar{1}_H\rangle \supset N^c N^c \nu_H^2$
(1) $1_F 5_F\langle\bar{5}_H\rangle \supset N^c L\langle H_u\rangle$	(2) $5_F 1_F\langle\overline{10}_H\rangle\langle 5_H\rangle \supset LE^c\langle H_d\rangle s_H$	(1) $10_F 10_F\langle\overline{10}_H\rangle\langle\overline{10}_H\rangle \supset SSs_H^2$
(1) $\bar{5}_F 5_F\langle 1_H\rangle \supset (D^c\Delta + \Lambda^c L)\nu_H$		(4) $10_F 1_F\langle\overline{10}_H\rangle\langle\bar{1}_H\rangle \supset SN^c s_H \nu_H$
(1) $\bar{5}_F\bar{5}_F\langle 10_H\rangle \supset \Lambda^c\Lambda s_H$		(1) $\bar{5}_F\bar{5}_F\langle 5_H\rangle\langle 5_H\rangle \supset \Lambda^c\Lambda^c\langle H_d\rangle\langle H_d\rangle$
(1) $10_F 5_F\langle 10_H\rangle \supset \Delta^c\Delta s_H$		(4) $10_F\bar{5}_F\langle\overline{10}_H\rangle\langle 5_H\rangle \supset (\Lambda^c S + QD^c)\langle H_d\rangle s_H$
		(2) $10_F 10_F\langle 5_H\rangle\langle\bar{1}_H\rangle \supset Q\Delta^c\langle H_d\rangle\nu_H$
		(4) $\bar{5}_F 1_F\langle 5_H\rangle\langle\bar{1}_H\rangle \supset \Lambda^c N^c\langle H_d\rangle\nu_H$

where, for convenience, we redefined $Y_D \to Y_D/2$ and $Y_E \to Y_E/2$. The basis $(U)(U^c)$ is used for M_u, $(D, \Delta)\times(\Delta^c, D^c)$ for M_d, and $(\Lambda^-, E)(E^c, \Lambda^{c+})$ for M_e. The Majorana mass matrix M_ν is written in the basis $(\Lambda^0, N, \Lambda^{c0}, N^c, S)$.

2. Effective mass matrices

Below the $M_f \sim s_H \sim \nu_H$ scale, the exotic (vector) part of the matter spectrum decouples, and one is left with the three standard MSSM families. In what follows, we shall use the calligraphic symbol $\mathcal{M}$ for the 3×3 effective MSSM fermion mass matrices in order to distinguish them from the mass matrices in Eqs. (63)–(66).

(i) *Up-type quarks:* The effective up-quark mass matrix coincides with the mass matrix in Eq. (63),

$$\mathcal{M}_u = Y_U v_u. \tag{67}$$

(ii) *Down-type quarks and charged leptons:* The 6×6 mass matrices in Eqs. (64) and (65) can be brought into a convenient form by means of the transformations

$$M_d \to M_d U_d^\dagger \equiv M_d', \qquad M_e \to U_e^* M_e \equiv M_e', \tag{68}$$

where $U_{d,e}$ are 6×6 unitary matrices such that M_d' and M_e' are block-triangular,

$$M_d' = \mathcal{O}\begin{pmatrix} v & v \\ 0 & M_f \end{pmatrix}, \quad M_e' = \mathcal{O}\begin{pmatrix} v & 0 \\ v & M_f \end{pmatrix}. \tag{69}$$

Here v denotes weak scale entries. This corresponds to the change of basis

$$\begin{pmatrix} d^c \\ \tilde{\Delta}^c \end{pmatrix} \equiv U_d \begin{pmatrix} \Delta^c \\ D^c \end{pmatrix}, \qquad \begin{pmatrix} e \\ \tilde{\Lambda}^- \end{pmatrix} \equiv U_e \begin{pmatrix} \Lambda^- \\ E \end{pmatrix}, \tag{70}$$

in the RH down-quark and left-handed charged lepton sectors, respectively. The upper components of the rotated vectors (d^c and e) correspond to the light MSSM degrees of freedom. Since the residual rotations acting on the left-handed down-quark and RH charged lepton components, which transform the $M_{d,e}'$ matrices into fully block-diagonal forms, are extremely tiny [of $\mathcal{O}(v/M_f)$], the 3×3 upper-left blocks (ULB) in Eq. (69) can be identified with the effective light down-type quark and charged lepton mass matrices, i.e., $\mathcal{M}_d \equiv (M_d')_{\rm ULB}$ and $\mathcal{M}_e \equiv (M_e')_{\rm ULB}$.

It is instructive to work out the explicit form of the unitary matrices U_d and U_e. For the sake of simplicity, in what follows we shall stick to the single family case and assume the reality of all the relevant parameters. Dropping same order Yukawa factors as well, one writes Eqs. (64) and (65) as

$$M_d = \begin{pmatrix} v_\nu & v_s \\ s_H & \nu_H \end{pmatrix}, \qquad M_e = \begin{pmatrix} v_\nu & s_H \\ v_s & \nu_H \end{pmatrix}, \tag{71}$$

and the matrices U_d and U_e are explicitly given by

$$U_{d,e} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}. \tag{72}$$

By applying Eq. (68) we get that M_d' and M_e' have the form in Eq. (69) provided that $\tan\alpha = s_H/\nu_H$. In particular, with a specific choice of the global phase, we can write

$$\cos\alpha = \frac{\nu_H}{\sqrt{s_H^2 + \nu_H^2}}, \quad \sin\alpha = \frac{s_H}{\sqrt{s_H^2 + \nu_H^2}}, \tag{73}$$

so that the mass eigenstates [up to $\mathcal{O}(v/M_f)$ effects] are finally given by [see Eq. (70)]

$$\begin{pmatrix} d^c \\ \tilde{\Delta}^c \end{pmatrix} = \frac{1}{\sqrt{s_H^2 + \nu_H^2}} \begin{pmatrix} \nu_H\Delta^c - s_H D^c \\ s_H\Delta^c + \nu_H D^c \end{pmatrix} \tag{74}$$

and

$$\begin{pmatrix} e \\ \tilde{\Lambda}^- \end{pmatrix} = \frac{1}{\sqrt{s_H^2 + \nu_H^2}} \begin{pmatrix} \nu_H\Lambda^- - s_H E \\ s_H\Lambda^- + \nu_H E \end{pmatrix}, \tag{75}$$

where the upper (SM) components have mass of $\mathcal{O}(v_{\nu,s})$ and the lower (exotic) ones have mass of $\mathcal{O}(M_f)$.

(iii) *Neutrinos:* Working again in the same approximation, the lightest eigenvalue of M_ν in Eq. (66) is given by

$$m_\nu \sim \frac{(\nu_H^2 + s_H^2)^2 + 2s_H^2\nu_H^2}{3s_H^2\nu_H^2(s_H^2 + \nu_H^2)} M_P v_u^2. \tag{76}$$

For $s_H \sim \nu_H \sim M_f \sim 10^{16}$ GeV, $M_P \sim 10^{18}$ GeV, and $v_u \sim 10^2$ GeV, one obtains

$$m_\nu \sim \frac{v_u^2}{M_f^2/M_P} \sim 0.1 \text{ eV}, \tag{77}$$

which is within the ballpark of the current lower bounds on the light neutrino masses set by the oscillation experiments.

It is also useful to examine the composition of the lightest neutrino eigenstate ν. At the leading order, the light neutrino eigenvector obeys the equation $M_\nu \nu = 0$ which, in the components $\nu = (x_1, x_2, x_3, x_4, x_5)$, reads

$$s_H x_3 = 0, \tag{78}$$

$$\nu_H x_3 = 0, \tag{79}$$

$$s_H x_1 + \nu_H x_2 = 0, \tag{80}$$

$$\hat{\nu}_H \nu_H x_4 + 2\hat{\nu}_H s_H x_5 = 0, \tag{81}$$

$$2\hat{s}_H \nu_H x_4 + \hat{s}_H s_H x_5 = 0. \tag{82}$$

By inspection, Eqs. (81) and (82) are compatible only if $x_4 = x_5 = 0$, while Eqs. (78) and (79) imply $x_3 = 0$. Thus, the nonvanishing components of the neutrino eigenvector are just x_1 and x_2. From Eq. (80), up to a phase factor, we obtain

$$\nu = \frac{\nu_H}{\sqrt{\nu_H^2 + s_H^2}} \Lambda^0 + \frac{-s_H}{\sqrt{\nu_H^2 + s_H^2}} N. \tag{83}$$

Notice that the lightest neutrino eigenstate ν and the lightest charged lepton show the same admixtures of the corresponding electroweak doublet components. Actually, this can be easily understood by taking the limit $v_u = v_d = 0$ in which the preserved $SU(2)_L$ gauge symmetry imposes the same U_e transformation on the (Λ^0, N) components. Explicitly, given the form of U_e in Eq. (72), one obtains in the rotated basis

$$M'_\nu = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & M_f & 0 & 0 \\ 0 & M_f & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{M_f^2}{M_P} & 2\frac{M_f^2}{M_P} \\ 0 & 0 & 0 & 2\frac{M_f^2}{M_P} & \frac{M_f^2}{M_P} \end{pmatrix}, \tag{84}$$

where we have taken $s_H \sim \nu_H \sim M_f$. M'_ν is defined on the basis $(\nu, \tilde{\Lambda}^0, \Lambda^{c0}, N^c, S)$, where

$$\begin{pmatrix} \nu \\ \tilde{\Lambda}^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \Lambda^0 - N \\ \Lambda^0 + N \end{pmatrix}. \tag{85}$$

In conclusion, we see that the "light" eigenstate ν decouples from the heavy spectrum,

$$m_{\nu_{M_1}} \sim -M_f^2/M_P, \quad \nu_{M_1} \sim \frac{1}{\sqrt{2}}(N^c - S), \tag{86}$$

$$m_{\nu_{M_2}} \sim 3\cdot M_f^2/M_P, \quad \nu_{M_2} \sim \frac{1}{\sqrt{2}}(N^c + S), \tag{87}$$

$$m_{\nu_{\mathrm{PD}_1}} \sim -M_f, \qquad \nu_{\mathrm{PD}_1} \sim \frac{1}{\sqrt{2}}(\tilde{\Lambda}^0 - \Lambda^{c0}), \tag{88}$$

$$m_{\nu_{\mathrm{PD}_2}} \sim M_f, \qquad \nu_{\mathrm{PD}_2} \sim \frac{1}{\sqrt{2}}(\tilde{\Lambda}^0 + \Lambda^{c0}), \tag{89}$$

where ν_{M_1} and ν_{M_2} are two Majorana neutrinos of intermediate mass, $O(10^{14})$ GeV, while the states ν_{PD_1} and ν_{PD_2} form a pseudo-Dirac neutrino of mass $O(10^{16})$ GeV.

Notice finally that the charged current $W_L \bar{\nu}_L e_L$ coupling is unaffected [cf. Eq. (83) with Eq. (75)], contrary to the claim in Refs. [31,32] that are based on the unjustified assumption that the physical electron e is predominantly made of E.

VI. CONCLUSIONS

In this paper we attempted to pin down the minimal Higgs setting within the framework of the supersymmetric $SO(10)$ and E_6 unifications, consistent with a breaking of the unified gauge symmetry down to the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ of the standard model driven by renormalizable interactions.

The breaking of the GUT symmetries down to the SM at the renormalizable level is a very interesting option which, simplicity apart, is supported by the success of the single-step gauge unification inherent to the TeV-scale minimal SUSY extension of the SM. Indeed, if any part of the GUT → SM symmetry breakdown were due to nonrenormalizable (Planck-induced) operators, one has to face a plethora of thresholds spread below the GUT scale, which may dramatically affect the gauge running and also the proton lifetime.

On top of that, the $B - L$ breaking scale in the vicinity of $M_G \sim 10^{16}$ GeV is particularly favored by the experimental lower limit on the light neutrino mass scale ($\sqrt{\Delta m_A^2} \sim 0.05$ eV) in models in which the RH neutrinos, driving the singlet (type-I) variant of the seesaw mechanism, receive their masses from Planck-suppressed operators, as in the scenarios discussed in this work.

We argued that the simplest SUSY $SO(10)$ Higgs model that can support a full breaking of the unified symmetry down to the SM at the renormalizable level corresponds to the flipped $SO(10) \otimes U(1)$ scenario with a $2 \times (16_H \oplus \overline{16}_H) \oplus 45_H$ Higgs sector. The enhanced breaking power of the spinorial pairs $16_H \oplus \overline{16}_H$ and the adjoint 45_H in the flipped case, each with twice as many SM singlets as the same multiplet in the standard $SO(10)$ context, does make room for the desired single-step breaking of the rank $= 6$ $SO(10) \otimes U(1)$ gauge symmetry down to the rank $= 4$ SM. These results follow from a detailed analysis of the relevant F- and D-flatness constraints on the gauge boson spectrum.

We also considered the natural embedding of the flipped $SO(10) \otimes U(1)$ model into the exceptional group E_6. With an extra copy of the fundamental conjugated pair of $27_H \oplus \overline{27}_H$ of E_6 [comprising $16_H \oplus \overline{16}_H$ of its $SO(10)$ subgroup] on top of the simplest nontrivial renormalizable SUSY E_6 Higgs sector spanned over $27_H \oplus \overline{27}_H \oplus 78_H$, the original symmetry is reduced to rank $= 4$. However, due to the rich structure of E_6 as compared to its $SO(10) \otimes U(1)$ subgroup, the breaking chain stops at the $SU(5)$ level and nonrenormalizable operators are still needed for a full $E_6 \to$ SM breaking.

We made the case for a two-step breaking of an E_6 GUT realized in the vicinity of the Planck scale via an intermediate flipped $SO(10) \otimes U(1)$ stage. Remarkably enough, even in the simplest picture, the few percent mismatch observed within the two-loop MSSM gauge coupling evolution at the scale of the "one-step" grand unification is naturally accommodated in this scheme, and it is understood as an artefact of a delayed E_6 unification superseding the flipped $SO(10) \otimes U(1)$ partial unification. A study of GUT threshold effects and a detailed discussion of the matter spectrum will be part of future work.

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APPENDIX A: FLIPPED SO(10) VACUUM

1. Flipped SO(10) notation

We work in the basis of Ref. [35], where the adjoint is projected along the positive-chirality spinorial generators

$$45 \equiv 45_{ij} \Sigma^+_{ij}, \tag{A1}$$

with $i, j = 1, \ldots, 10$. Here

$$\begin{pmatrix} \Sigma^+ \\ \Sigma^- \end{pmatrix} \equiv \frac{1}{2}(I_{32} \pm \Gamma_\chi)\Sigma, \tag{A2}$$

where I_{32} is the 32-dimensional identity matrix and Γ_χ is the ten-dimensional analogue of the Dirac γ_5 matrix defined as

$$\Gamma_\chi \equiv -i\Gamma_1\Gamma_2\Gamma_3\Gamma_4\Gamma_5\Gamma_6\Gamma_7\Gamma_8\Gamma_9\Gamma_{10}. \tag{A3}$$

The Γ_i factors are given by the following tensor products of ordinary Pauli matrices σ_i and the two-dimensional identity I_2:

$$\begin{aligned}
\Gamma_1 &\equiv \sigma_1 \otimes \sigma_1 \otimes I_2 \otimes I_2 \otimes \sigma_2,\\
\Gamma_2 &\equiv \sigma_1 \otimes \sigma_2 \otimes I_2 \otimes \sigma_3 \otimes \sigma_2,\\
\Gamma_3 &\equiv \sigma_1 \otimes \sigma_1 \otimes I_2 \otimes \sigma_2 \otimes \sigma_3,\\
\Gamma_4 &\equiv \sigma_1 \otimes \sigma_2 \otimes I_2 \otimes \sigma_2 \otimes I_2,\\
\Gamma_5 &\equiv \sigma_1 \otimes \sigma_1 \otimes I_2 \otimes \sigma_2 \otimes \sigma_1,\\
\Gamma_6 &\equiv \sigma_1 \otimes \sigma_2 \otimes I_2 \otimes \sigma_1 \otimes \sigma_2,\\
\Gamma_7 &\equiv \sigma_1 \otimes \sigma_3 \otimes \sigma_1 \otimes I_2 \otimes I_2,\\
\Gamma_8 &\equiv \sigma_1 \otimes \sigma_3 \otimes \sigma_2 \otimes I_2 \otimes I_2,\\
\Gamma_9 &\equiv \sigma_1 \otimes \sigma_3 \otimes \sigma_3 \otimes I_2 \otimes I_2,\\
\Gamma_{10} &\equiv \sigma_2 \otimes I_2 \otimes I_2 \otimes I_2 \otimes I_2,
\end{aligned} \tag{A4}$$

which satisfy the Clifford algebra

$$\{\Gamma_i, \Gamma_j\} = 2\delta_{ij} I_{32}. \tag{A5}$$

The spinorial generators Σ_{ij} are then defined as

$$\Sigma_{ij} \equiv \frac{i}{4}[\Gamma_i, \Gamma_j]. \tag{A6}$$

On the flipped $SO(10)$ vacuum the adjoint representation reads

$$\langle 45 \rangle = \begin{pmatrix} \langle 45 \rangle_L & \cdot \\ \cdot & \langle 45 \rangle_R \end{pmatrix}, \tag{A7}$$

where

$$\langle 45 \rangle_L = \mathrm{diag}(\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8), \tag{A8}$$

and

$$\langle 45\rangle_R = \begin{pmatrix} \lambda_9 & \cdot & \cdot & \cdot & \omega^+ & \cdot & \cdot & \cdot \\ \cdot & \lambda_{10} & \cdot & \cdot & \cdot & \omega^+ & \cdot & \cdot \\ \cdot & \cdot & \lambda_{11} & \cdot & \cdot & \cdot & \omega^+ & \cdot \\ \cdot & \cdot & \cdot & \lambda_{12} & \cdot & \cdot & \cdot & \omega^+ \\ \omega^- & \cdot & \cdot & \cdot & \lambda_{13} & \cdot & \cdot & \cdot \\ \cdot & \omega^- & \cdot & \cdot & \cdot & \lambda_{14} & \cdot & \cdot \\ \cdot & \cdot & \omega^- & \cdot & \cdot & \cdot & \lambda_{15} & \cdot \\ \cdot & \cdot & \cdot & \omega^- & \cdot & \cdot & \cdot & \lambda_{16} \end{pmatrix}. \tag{A9}$$

In the convention defined in Sec. III B (cf. also the caption of Table V), the diagonal entries are given by

$$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_5 = \lambda_6 = \lambda_7 = \frac{\omega_Y}{2\sqrt{2}},$$
$$\lambda_4 = \lambda_8 = -\frac{3\omega_Y}{2\sqrt{2}},$$
$$\lambda_9 = \lambda_{10} = \lambda_{11} = -\frac{\omega_Y}{2\sqrt{2}} - \frac{\omega_R}{\sqrt{2}}, \qquad \lambda_{12} = \frac{3\omega_Y}{2\sqrt{2}} - \frac{\omega_R}{\sqrt{2}},$$
$$\lambda_{13} = \lambda_{14} = \lambda_{15} = -\frac{\omega_Y}{2\sqrt{2}} + \frac{\omega_R}{\sqrt{2}}, \qquad \lambda_{16} = \frac{3\omega_Y}{2\sqrt{2}} + \frac{\omega_R}{\sqrt{2}}, \tag{A10}$$

where ω_Y and ω_R are real, while $\omega^+ = \omega^{-*}$.

Analogously, the spinor and the antispinor SM-obedient vacuum directions are given by

$$\langle 16\rangle^T = (\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\, e \cdot\cdot\cdot - \nu), \tag{A11}$$

$$\langle\overline{16}\rangle^T = (\cdot\cdot\cdot\, \bar{\nu} \cdot\cdot\cdot\, \bar{e} \cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot), \tag{A12}$$

where the dots stand for zeros, and the nonvanishing VEVs are generally complex.

It is worth reminding the reader that the shorthand notation $16\overline{16}$ and $16\,45\,\overline{16}$ in Eq. (14) stands for $16^T\mathcal{C}\overline{16}$ and $16^T 45^T \mathcal{C}\overline{16}$, where $\mathcal{C}$ is the "charge conjugation" matrix obeying $(\Sigma^+)^T\mathcal{C} + \mathcal{C}\Sigma^- = 0$. In the current convention, $\mathcal{C}$ is given by

$$\mathcal{C} = \begin{pmatrix} \cdot & \cdot & \cdot & -I_4 \\ \cdot & \cdot & I_4 & \cdot \\ \cdot & I_4 & \cdot & \cdot \\ -I_4 & \cdot & \cdot & \cdot \end{pmatrix}, \tag{A13}$$

where I_4 is the four-dimensional identity matrix.

2. Supersymmetric vacuum manifold

In order for SUSY to survive the spontaneous GUT symmetry breakdown at M_G, the vacuum manifold must be D and F flat at the GUT scale. The relevant superpotential W_H given in Eq. (14), with the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$-preserving vacuum parametrized by Eqs. (A7), (A11), and (A12), yields the following F-flatness equations:

$$F_{\omega_R} = -4\mu\omega_R + \frac{\tau_1}{\sqrt{2}}(e_1\bar{e}_1 - \nu_1\bar{\nu}_1) + \frac{\tau_2}{\sqrt{2}}(e_2\bar{e}_2 - \nu_2\bar{\nu}_2) = 0,$$
$$\frac{2}{3}F_{\omega_Y} = 4\mu\omega_Y + \frac{\tau_1}{\sqrt{2}}(e_1\bar{e}_1 + \nu_1\bar{\nu}_1) + \frac{\tau_2}{\sqrt{2}}(e_2\bar{e}_2 + \nu_2\bar{\nu}_2) = 0,$$
$$F_{\omega^+} = 4\mu\omega^- - \tau_1\nu_1\bar{e}_1 - \tau_2\nu_2\bar{e}_2 = 0,$$
$$F_{\omega^-} = 4\mu\omega^+ - \tau_1 e_1\bar{\nu}_1 - \tau_2 e_2\bar{\nu}_2 = 0,$$
$$F_{e_1} = \tau_1\left(-\omega^-\bar{\nu}_1 - \frac{\bar{e}_1\omega_R}{\sqrt{2}} + \frac{3\bar{e}_1\omega_Y}{2\sqrt{2}}\right) + \rho_{11}\bar{e}_1 + \rho_{12}\bar{e}_2 = 0,$$
$$F_{e_2} = \tau_2\left(-\omega^-\bar{\nu}_2 - \frac{\bar{e}_2\omega_R}{\sqrt{2}} + \frac{3\bar{e}_2\omega_Y}{2\sqrt{2}}\right) + \rho_{21}\bar{e}_1 + \rho_{22}\bar{e}_2 = 0,$$
$$F_{\nu_1} = \tau_1\left(-\omega^+\bar{e}_1 + \frac{\bar{\nu}_1\omega_R}{\sqrt{2}} + \frac{3\bar{\nu}_1\omega_Y}{2\sqrt{2}}\right) + \rho_{11}\bar{\nu}_1 + \rho_{12}\bar{\nu}_2 = 0,$$
$$F_{\nu_2} = \tau_2\left(-\omega^+\bar{e}_2 + \frac{\bar{\nu}_2\omega_R}{\sqrt{2}} + \frac{3\bar{\nu}_2\omega_Y}{2\sqrt{2}}\right) + \rho_{21}\bar{\nu}_1 + \rho_{22}\bar{\nu}_2 = 0,$$
$$F_{\bar{e}_1} = \tau_1\left(-\omega^+\nu_1 - \frac{e_1\omega_R}{\sqrt{2}} + \frac{3e_1\omega_Y}{2\sqrt{2}}\right) + \rho_{11}e_1 + \rho_{21}e_2 = 0,$$
$$F_{\bar{e}_2} = \tau_2\left(-\omega^+\nu_2 - \frac{e_2\omega_R}{\sqrt{2}} + \frac{3e_2\omega_Y}{2\sqrt{2}}\right) + \rho_{12}e_1 + \rho_{22}e_2 = 0,$$
$$F_{\bar{\nu}_1} = \tau_1\left(-\omega^- e_1 + \frac{\nu_1\omega_R}{\sqrt{2}} + \frac{3\nu_1\omega_Y}{2\sqrt{2}}\right) + \rho_{11}\nu_1 + \rho_{21}\nu_2 = 0,$$
$$F_{\bar{\nu}_2} = \tau_2\left(-\omega^- e_2 + \frac{\nu_2\omega_R}{\sqrt{2}} + \frac{3\nu_2\omega_Y}{2\sqrt{2}}\right) + \rho_{12}\nu_1 + \rho_{22}\nu_2 = 0. \tag{A14}$$

One can use the first four equations above to replace ω_R, ω_Y, ω^+, and ω^- in the remaining eight (complex) relations, which can be rewritten in the form

$$\begin{aligned} 16\mu F^{\omega}_{e_1} &= 16\mu(\rho_{11}\bar{e}_1 + \rho_{12}\bar{e}_2) - 5\tau_1^2(\nu_1\bar{\nu}_1 + e_1\bar{e}_1)\bar{e}_1 \\ &\quad - \tau_1\tau_2(\nu_2\bar{\nu}_2\bar{e}_1 + (4\nu_2\bar{\nu}_1 + 5e_2\bar{e}_1)\bar{e}_2) = 0, \\ 16\mu F^{\omega}_{\bar{e}_1} &= 16\mu(\rho_{11}e_1 + \rho_{21}e_2) - 5\tau_1^2(\bar{\nu}_1\nu_1 + \bar{e}_1e_1)e_1 \\ &\quad - \tau_1\tau_2(\bar{\nu}_2\nu_2e_1 + (4\bar{\nu}_2\nu_1 + 5\bar{e}_2e_1)e_2) = 0, \\ 16\mu F^{\omega}_{\nu_1} &= 16\mu(\rho_{11}\bar{\nu}_1 + \rho_{12}\bar{\nu}_2) - 5\tau_1^2(e_1\bar{e}_1 + \nu_1\bar{\nu}_1)\bar{\nu}_1 \\ &\quad - \tau_1\tau_2(e_2\bar{e}_2\bar{\nu}_1 + (4e_2\bar{e}_1 + 5\nu_2\bar{\nu}_1)\bar{\nu}_2) = 0, \\ 16\mu F^{\omega}_{\bar{\nu}_1} &= 16\mu(\rho_{11}\nu_1 + \rho_{21}\nu_2) - 5\tau_1^2(\bar{e}_1e_1 + \bar{\nu}_1\nu_1)\nu_1 \\ &\quad - \tau_1\tau_2(\bar{e}_2e_2\nu_1 + (4\bar{e}_2e_1 + 5\bar{\nu}_2\nu_1)\nu_2) = 0, \end{aligned} \tag{A15}$$

where the other four equations are obtained from these by exchanging $1 \leftrightarrow 2$.

There are two classes of D-flatness conditions corresponding, respectively, to the VEVs of the $U(1)_X$ and the $SO(10)$ generators. For the X charge one finds

$$\begin{aligned}D_X &= \langle 45\rangle^\dagger X\langle 45\rangle + \langle 16_1\rangle^\dagger X\langle 16_1\rangle + \langle\overline{16}_1\rangle^\dagger X\langle\overline{16}_1\rangle \\ &\quad + \langle 16_2\rangle^\dagger X\langle 16_2\rangle + \langle\overline{16}_2\rangle^\dagger X\langle\overline{16}_2\rangle \\ &= |e_1|^2 + |\nu_1|^2 - |\bar e_1|^2 - |\bar\nu_1|^2 + |e_2|^2 + |\nu_2|^2 \\ &\quad - |\bar e_2|^2 - |\bar\nu_2|^2 = 0,\end{aligned} \tag{A16}$$

while for the $SO(10)$ generators one has

$$D_{ij} \equiv D_{ij}^{45} + D_{ij}^{16\oplus\overline{16}} = 0, \tag{A17}$$

where

$$D_{ij}^{45} = \mathrm{Tr}\langle 45\rangle^\dagger[\Sigma_{ij}^+, \langle 45\rangle] \tag{A18}$$

and

$$\begin{aligned}D_{ij}^{16\oplus\overline{16}} &= \langle 16_1\rangle^\dagger\Sigma_{ij}^+\langle 16_1\rangle + \langle\overline{16}_1\rangle^\dagger\Sigma_{ij}^-\langle\overline{16}_1\rangle \\ &\quad + \langle 16_2\rangle^\dagger\Sigma_{ij}^+\langle 16_2\rangle + \langle\overline{16}_2\rangle^\dagger\Sigma_{ij}^-\langle\overline{16}_2\rangle.\end{aligned} \tag{A19}$$

Given that

$$\mathrm{Tr}\,\langle 45\rangle^\dagger[\Sigma_{ij}^+, \langle 45\rangle] = \mathrm{Tr}\Sigma_{ij}^+[\langle 45\rangle, \langle 45\rangle^\dagger], \tag{A20}$$

we obtain

$$[\langle 45\rangle, \langle 45\rangle^\dagger] = \begin{pmatrix} \cdot & \cdot \\ \cdot & D_R\end{pmatrix}, \tag{A21}$$

where

$$D_R = \begin{pmatrix} A & \cdot & \cdot & \cdot & \sqrt{2}B^* & \cdot & \cdot & \cdot \\ \cdot & A & \cdot & \cdot & \cdot & \sqrt{2}B^* & \cdot & \cdot \\ \cdot & \cdot & A & \cdot & \cdot & \cdot & \sqrt{2}B^* & \cdot \\ \cdot & \cdot & \cdot & A & \cdot & \cdot & \cdot & \sqrt{2}B^* \\ \sqrt{2}B & \cdot & \cdot & \cdot & -A & \cdot & \cdot & \cdot \\ \cdot & \sqrt{2}B & \cdot & \cdot & \cdot & -A & \cdot & \cdot \\ \cdot & \cdot & \sqrt{2}B & \cdot & \cdot & \cdot & -A & \cdot \\ \cdot & \cdot & \cdot & \sqrt{2}B & \cdot & \cdot & \cdot & -A \end{pmatrix}, \tag{A22}$$

and

$$A = |\omega^+|^2 - |\omega^-|^2, \qquad B = (\omega^+)^*\omega_R - (\omega_R)^*\omega^-. \tag{A23}$$

Since ω_R is real and $\omega^+ = (\omega^-)^*$, $D_{ij}^{45} = 0$ as it should be. Notice that $F_{\omega^\pm}$ flatness implies

$$\tau_1 e_1\bar\nu_1 + \tau_2 e_2\bar\nu_2 = \tau_1(\nu_1\bar e_1)^* + \tau_2(\nu_2\bar e_2)^*, \tag{A24}$$

where the reality of $\tau_{1,2}$ has been taken into account.

For the spinorial contribution in (A17) we find

$$\begin{aligned}D_{ij}^{16\oplus\overline{16}} &= (\Sigma_{ij}^+)_{12,12}(|e_1|^2 + |e_2|^2) + (\Sigma_{ij}^+)_{16,16} \\ &\quad\times(|\nu_1|^2 + |\nu_2|^2) + (\Sigma_{ij}^-)_{4,4}(|\bar\nu_1|^2 + |\bar\nu_2|^2) \\ &\quad + (\Sigma_{ij}^-)_{8,8}(|\bar e_1|^2 + |\bar e_2|^2) - (\Sigma_{ij}^+)_{12,16} \\ &\quad\times(e_1^*\nu_1 + e_2^*\nu_2) - (\Sigma_{ij}^+)_{16,12}(\nu_1^* e_1 + \nu_2^* e_2) \\ &\quad + (\Sigma_{ij}^-)_{4,8}(\bar\nu_1^*\bar e_1 + \bar\nu_2^*\bar e_2) + (\Sigma_{ij}^-)_{8,4}(\bar e_1^*\bar\nu_1 + \bar e_2^*\bar\nu_2).\end{aligned} \tag{A25}$$

Given $\Sigma^- = -\mathcal{C}^{-1}(\Sigma^+)^T\mathcal{C}$ and the explicit form of $\mathcal{C}$ in Eq. (A13), one can verify readily that

$$\begin{aligned}(\Sigma_{ij}^-)_{4,4} &= -(\Sigma_{ij}^+)_{16,16}, \\ (\Sigma_{ij}^-)_{8,8} &= -(\Sigma_{ij}^+)_{12,12}, \\ (\Sigma_{ij}^-)_{4,8} &= +(\Sigma_{ij}^+)_{12,16}.\end{aligned} \tag{A26}$$

Thus, $D_{ij}^{16\oplus\overline{16}}$ can be simplified to

$$\begin{aligned}&(\Sigma_{ij}^+)_{12,12}(|e_1|^2 + |e_2|^2 - |\bar e_1|^2 - |\bar e_2|^2) \\ &\quad + (\Sigma_{ij}^+)_{16,16}(|\nu_1|^2 + |\nu_2|^2 - |\bar\nu_1|^2 - |\bar\nu_2|^2) \\ &\quad - [(\Sigma_{ij}^+)_{12,16}(e_1^*\nu_1 + e_2^*\nu_2 - \bar\nu_1^*\bar e_1 - \bar\nu_2^*\bar e_2) + \text{c.c.}] = 0,\end{aligned} \tag{A27}$$

or, with Eq. (A16) at hand, to

$$\begin{aligned}&[(\Sigma_{ij}^+)_{16,16} - (\Sigma_{ij}^+)_{12,12}](|\nu_1|^2 + |\nu_2|^2 - |\bar\nu_1|^2 - |\bar\nu_2|^2) \\ &\quad - [(\Sigma_{ij}^+)_{12,16}(e_1^*\nu_1 + e_2^*\nu_2 - \bar\nu_1^*\bar e_1 - \bar\nu_2^*\bar e_2) + \text{c.c.}] = 0.\end{aligned} \tag{A28}$$

Taking into account the basic features of the spinorial generators Σ_{ij}^+ {e.g., the bracket $[(\Sigma_{ij}^+)_{16,16} - (\Sigma_{ij}^+)_{12,12}]$ and $(\Sigma_{ij}^+)_{12,16}$ can never act against each other because at least one of them always vanishes, or the fact that $(\Sigma_{ij}^+)_{12,16}$ is complex}, Eq. (A28) can be satisfied for all ij if and only if

$$\begin{aligned}|e_1|^2 + |e_2|^2 - |\bar e_1|^2 - |\bar e_2|^2 &= 0, \\ |\nu_1|^2 + |\nu_2|^2 - |\bar\nu_1|^2 - |\bar\nu_2|^2 &= 0, \\ e_1^*\nu_1 + e_2^*\nu_2 - \bar\nu_1^*\bar e_1 - \bar\nu_2^*\bar e_2 &= 0.\end{aligned} \tag{A29}$$

Combining this with Eq. (A24), the required D and F flatness can be, in general, maintained only if $e_{1,2}^* = \bar e_{1,2}$ and $\nu_{1,2}^* = \bar\nu_{1,2}$. Hence, we can write

$$e_{1,2} \equiv |e_{1,2}|e^{i\phi_{e_{1,2}}}, \qquad \bar{e}_{1,2} \equiv |e_{1,2}|e^{-i\phi_{e_{1,2}}},$$
$$\nu_{1,2} \equiv |\nu_{1,2}|e^{i\phi_{\nu_{1,2}}}, \qquad \bar{\nu}_{1,2} \equiv |\nu_{1,2}|e^{-i\phi_{\nu_{1,2}}}. \tag{A30}$$

With this at hand, one can further simplify the F-flatness conditions, Eq. (A15). To this end, it is convenient to define the following linear combinations,

$$L_V^- \equiv C_1^V \cos\phi_V - C_2^V \sin\phi_V, \tag{A31}$$

$$L_V^+ \equiv C_1^V \sin\phi_V + C_2^V \cos\phi_V, \tag{A32}$$

where

$$C_1^V \equiv \frac{1}{2i}(F_{\bar{V}}^{\omega} - F_V^{\omega}), \qquad C_2^V \equiv \frac{1}{2}(F_{\bar{V}}^{\omega} + F_V^{\omega}),$$

with V running over the spinorial VEVs e_1, e_2, ν_1, and ν_2. For μ, τ_1, and τ_2 real by definition, the requirement of $L_V^{\pm} = 0$ for all V is equivalent to

$$\begin{aligned} 4\mu\,\mathrm{Re}L_{e_1}^- = {} & |e_2|(\tau_1\tau_2|\nu_1||\nu_2|\sin(\phi_{e_1} - \phi_{e_2} - \phi_{\nu_1} \\ & + \phi_{\nu_2}) - 2\mu(|\rho_{21}|\sin(\phi_{e_1} - \phi_{e_2} - \phi_{\rho_{21}}) \\ & + |\rho_{12}|\sin(\phi_{e_1} - \phi_{e_2} + \phi_{\rho_{12}}))) \\ = {} & 0, \end{aligned}$$

$$\begin{aligned} 4\mu\,\mathrm{Re}L_{\nu_1}^- = {} & |\nu_2|(\tau_1\tau_2|e_1||e_2|\sin(\phi_{\nu_1} - \phi_{\nu_2} - \phi_{e_1} + \phi_{e_2}) \\ & - 2\mu(|\rho_{21}|\sin(\phi_{\nu_1} - \phi_{\nu_2} - \phi_{\rho_{21}}) \\ & + |\rho_{12}|\sin(\phi_{\nu_1} - \phi_{\nu_2} + \phi_{\rho_{12}}))) = 0, \end{aligned} \tag{A33}$$

$$\begin{aligned} -2\,\mathrm{Im}L_{e_1}^- = {} & |e_2|(|\rho_{21}|\cos(\phi_{e_1} - \phi_{e_2} - \phi_{\rho_{21}}) \\ & - |\rho_{12}|\cos(\phi_{e_1} - \phi_{e_2} + \phi_{\rho_{12}})) = 0, \end{aligned}$$
$$\begin{aligned} -2\,\mathrm{Im}L_{\nu_1}^- = {} & |\nu_2|(|\rho_{21}|\cos(\phi_{\nu_1} - \phi_{\nu_2} - \phi_{\rho_{21}}) \\ & - |\rho_{12}|\cos(\phi_{\nu_1} - \phi_{\nu_2} + \phi_{\rho_{12}})) = 0, \end{aligned} \tag{A34}$$

and

$$\begin{aligned} -16\mu\,\mathrm{Re}L_{e_1}^+ = {} & -16\mu|e_1||\rho_{11}|\cos(\phi_{\rho_{11}}) - 8\mu|e_2|(|\rho_{21}|\cos(\phi_{e_1} - \phi_{e_2} - \phi_{\rho_{21}}) + |\rho_{12}|\cos(\phi_{e_1} - \phi_{e_2} + \phi_{\rho_{12}})) \\ & + 5\tau_1^2(|e_1|^2 + |\nu_1|^2)|e_1| + \tau_1\tau_2((5|e_2|^2 + |\nu_2|^2)|e_1| + 4|\nu_1||\nu_2||e_2|\cos(\phi_{e_1} - \phi_{e_2} - \phi_{\nu_1} + \phi_{\nu_2})) \\ = {} & 0, \end{aligned}$$

$$\begin{aligned} -16\mu\,\mathrm{Re}L_{\nu_1}^+ = {} & -16\mu|\nu_1||\rho_{11}|\cos(\phi_{\rho_{11}}) - 8\mu|\nu_2|(|\rho_{21}|\cos(\phi_{\nu_1} - \phi_{\nu_2} - \phi_{\rho_{21}}) + |\rho_{12}|\cos(\phi_{\nu_1} - \phi_{\nu_2} + \phi_{\rho_{12}})) \\ & + 5\tau_1^2(|\nu_1|^2 + |e_1|^2)|\nu_1| + \tau_1\tau_2((5|\nu_2|^2 + |e_2|^2)|\nu_1| + 4|e_1||e_2||\nu_2|\cos(\phi_{\nu_1} - \phi_{\nu_2} - \phi_{e_1} + \phi_{e_2})) = 0, \end{aligned} \tag{A35}$$

$$2\,\mathrm{Im}L_{e_1}^+ = 2|e_1||\rho_{11}|\sin(\phi_{\rho_{11}}) + |e_2|(|\rho_{12}|\sin(\phi_{e_1} - \phi_{e_2} + \phi_{\rho_{12}}) - |\rho_{21}|\sin(\phi_{e_1} - \phi_{e_2} - \phi_{\rho_{21}})) = 0,$$

$$2\,\mathrm{Im}L_{\nu_1}^+ = 2|\nu_1||\rho_{11}|\sin(\phi_{\rho_{11}}) + |\nu_2|(|\rho_{12}|\sin(\phi_{\nu_1} - \phi_{\nu_2} + \phi_{\rho_{12}}) - |\rho_{21}|\sin(\phi_{\nu_1} - \phi_{\nu_2} - \phi_{\rho_{21}})) = 0, \tag{A36}$$

where, as before, the remaining eight real equations for $V = e_2, \nu_2$ are obtained by swapping $1 \leftrightarrow 2$.

Focusing first on L^-, one finds that $|e_1|L_{e_1}^- + |e_2|L_{e_2}^- = 0$ and $|\nu_1|L_{\nu_1}^- + |\nu_2|L_{\nu_2}^- = 0$. Thus, we can consider just $L_{e_1}^-$ and $L_{\nu_1}^-$ as independent equations. For instance, from $\mathrm{Im}L_{e_1}^- = 0$ one readily gets

$$\frac{|\rho_{21}|}{|\rho_{12}|} = \frac{\cos(\phi_{e_1} - \phi_{e_2} + \phi_{\rho_{12}})}{\cos(\phi_{e_1} - \phi_{e_2} - \phi_{\rho_{21}})}. \tag{A37}$$

On top of that, the remaining $\mathrm{Re}L_V^- = \mathrm{Im}L_V^- = 0$ equations can be solved only for $\phi_{\rho_{12}} = -\phi_{\rho_{21}}$, which, plugged into Eq. (A37), gives $|\rho_{12}| = |\rho_{21}|$. Thus, we end up with the following condition for the off-diagonal entries of the ρ matrix:

$$\rho_{21} = \rho_{12}^*. \tag{A38}$$

Inserting this into the $\mathrm{Re}L_{e_1}^- = 0$ and $\mathrm{Re}L_{\nu_1}^- = 0$ equations, they simplify to

$$-4\mu|\rho_{12}| = \tau_1\tau_2|\nu_1||\nu_2|\sin(\Phi_\nu - \Phi_e)\csc\Phi_e, \tag{A39}$$

$$4\mu|\rho_{12}| = \tau_1\tau_2|e_1||e_2|\sin(\Phi_\nu - \Phi_e)\csc\Phi_\nu, \tag{A40}$$

where we have denoted

$$\Phi_\nu \equiv \phi_{\nu_1} - \phi_{\nu_2} + \phi_{\rho_{12}}, \qquad \Phi_e \equiv \phi_{e_1} - \phi_{e_2} + \phi_{\rho_{12}}. \tag{A41}$$

These, taken together, yield

$$|e_1||e_2|\sin\Phi_e = -|\nu_1||\nu_2|\sin\Phi_\nu \tag{A42}$$

and

$$|\nu_1||\nu_2| + |e_1||e_2| = \frac{4\mu|\rho_{12}|}{\tau_1\tau_2}\frac{\sin\Phi_\nu - \sin\Phi_e}{\sin(\Phi_\nu - \Phi_e)}. \tag{A43}$$

Notice that in the zero phase limit the constraint (A42) is trivially relaxed, while $\frac{\sin\Phi_\nu-\sin\Phi_e}{\sin(\Phi_\nu-\Phi_e)}\longrightarrow 1$.

Returning to the $L_V^+=0$ equations, the constraint (A38) implies, e.g.,

$$\begin{aligned}\mathrm{Im}\,L^+_{e_1}&=|e_1||\rho_{11}|\sin(\phi_{\rho_{11}})=0,\\ \mathrm{Im}L^+_{e_2}&=|e_2||\rho_{22}|\sin(\phi_{\rho_{22}})=0,\\ \mathrm{Im}L^+_{\nu_1}&=|\nu_1||\rho_{11}|\sin(\phi_{\rho_{11}})=0,\\ \mathrm{Im}L^+_{\nu_2}&=|\nu_2||\rho_{22}|\sin(\phi_{\rho_{22}})=0.\end{aligned}\tag{A44}$$

For generic VEVs, these relations require $\phi_{\rho_{11}}$ and $\phi_{\rho_{22}}$ to vanish. In conclusion, a nontrivial vacuum requires ρ (and hence τ for consistency) to be Hermitian. This is a consequence of the fact that D flatness for the flipped $SO(10)$ embedding implies $\langle 16_i\rangle=\langle\overline{16}_i\rangle^*$, cf. Eq. (A30). Let us also note that such a setting is preserved by supersymmetric wave-function renormalization.

Taking $\rho=\rho^\dagger$ in the remaining $\mathrm{Re}L_V^+=0$ equations and trading $|\rho_{12}|$ for $|\nu_1||\nu_2|$ in $\mathrm{Re}L^+_{e_{1,2}}=0$ by means of Eq. (A39) and for $|e_1||e_2|$ in $\mathrm{Re}L^+_{\nu_{1,2}}=0$ via Eq. (A40), one obtains

$$\begin{aligned}-16\mu\,\mathrm{Re}L^+_{e_1}&=|e_1|[-16\mu\rho_{11}+5\tau_1^2(|\nu_1|^2+|e_1|^2)+\tau_1\tau_2(|\nu_2|^2+5|e_2|^2)]+4\tau_1\tau_2|\nu_1||\nu_2||e_2|\sin\Phi_\nu\csc\Phi_e=0,\\ -16\mu\,\mathrm{Re}L^+_{e_2}&=|e_2|[-16\mu\rho_{22}+5\tau_2^2(|\nu_2|^2+|e_2|^2)+\tau_1\tau_2(|\nu_1|^2+5|e_1|^2)]+4\tau_1\tau_2|\nu_1||\nu_2||e_1|\sin\Phi_\nu\csc\Phi_e=0,\\ -16\mu\,\mathrm{Re}L^+_{\nu_1}&=|\nu_1|[-16\mu\rho_{11}+5\tau_1^2(|e_1|^2+|\nu_1|^2)+\tau_1\tau_2(|e_2|^2+5|\nu_2|^2)]+4\tau_1\tau_2|\nu_2||e_1||e_2|\csc\Phi_\nu\sin\Phi_e=0,\\ -16\mu\,\mathrm{Re}L^+_{\nu_2}&=|\nu_2|[-16\mu\rho_{22}+5\tau_2^2(|e_2|^2+|\nu_2|^2)+\tau_1\tau_2(|e_1|^2+5|\nu_1|^2)]+4\tau_1\tau_2|\nu_1||e_1||e_2|\csc\Phi_\nu\sin\Phi_e=0.\end{aligned}\tag{A45}$$

Since only two out of these four equations are independent constraints, it is convenient to consider the following linear combinations,

$$\begin{aligned}C_3\equiv{}&|\nu_1|^2(|e_1|\mathrm{Re}L^+_{e_1}-|e_2|\mathrm{Re}L^+_{e_2})\\&-|e_1|^2(|\nu_1|\mathrm{Re}L^+_{\nu_1}-|\nu_2|\mathrm{Re}L^+_{\nu_2}),\end{aligned}\tag{A46}$$

$$\begin{aligned}C_4\equiv{}&|\nu_2|^2(|e_1|\mathrm{Re}L^+_{e_1}-|e_2|\mathrm{Re}L^+_{e_2})\\&-|e_2|^2(|\nu_1|\mathrm{Re}L^+_{\nu_1}-|\nu_2|\mathrm{Re}L^+_{\nu_2}),\end{aligned}\tag{A47}$$

which admit for a simple factorized form

$$\begin{aligned}16\mu C_3={}&(|\nu_2|^2|e_1|^2-|\nu_1|^2|e_2|^2)[5\tau_2^2(|\nu_2|^2+|e_2|^2)\\&+\tau_1\tau_2(|\nu_1|^2+|e_1|^2)-16\mu\rho_{22}]=0,\end{aligned}\tag{A48}$$

$$\begin{aligned}16\mu C_4={}&(|\nu_2|^2|e_1|^2-|\nu_1|^2|e_2|^2)[5\tau_1^2(|\nu_1|^2+|e_1|^2)\\&+\tau_1\tau_2(|\nu_2|^2+|e_2|^2)-16\mu\rho_{11}]=0.\end{aligned}\tag{A49}$$

These relations can be generically satisfied only if the square brackets are zero, providing

$$\begin{aligned}16\mu\rho_{11}&=5\tau_1^2(|\nu_1|^2+|e_1|^2)+\tau_1\tau_2(|\nu_2|^2+|e_2|^2),\\ 16\mu\rho_{22}&=5\tau_2^2(|\nu_2|^2+|e_2|^2)+\tau_1\tau_2(|\nu_1|^2+|e_1|^2).\end{aligned}\tag{A50}$$

By introducing a pair of symbolic two-dimensional vectors $\vec{r}_1=(|\nu_1|,|e_1|)$ and $\vec{r}_2=(|\nu_2|,|e_2|)$, one can write

$$\begin{aligned}&r_1^2=|\nu_1|^2+|e_1|^2,\qquad r_2^2=|\nu_2|^2+|e_2|^2,\\&\vec{r}_1.\vec{r}_2=|\nu_1||\nu_2|+|e_1||e_2|,\end{aligned}\tag{A51}$$

which, in combination with Eqs. (A43) and (A50), yields

$$\begin{aligned}r_1^2&=-\frac{2\mu(\rho_{22}\tau_1-5\rho_{11}\tau_2)}{3\tau_1^2\tau_2},\\ r_2^2&=-\frac{2\mu(\rho_{11}\tau_2-5\rho_{22}\tau_1)}{3\tau_1\tau_2^2},\\ \vec{r}_1.\vec{r}_2&=\frac{4\mu|\rho_{12}|}{\tau_1\tau_2}\frac{\sin\Phi_\nu-\sin\Phi_e}{\sin(\Phi_\nu-\Phi_e)}.\end{aligned}\tag{A52}$$

With this at hand, the vacuum manifold can be conveniently parametrized by means of two angles α_1 and α_2,

$$\begin{aligned}|\nu_1|&=r_1\sin\alpha_1,\qquad |e_1|=r_1\cos\alpha_1,\\ |\nu_2|&=r_2\sin\alpha_2,\qquad |e_2|=r_2\cos\alpha_2,\end{aligned}\tag{A53}$$

which are fixed in terms of the superpotential parameters. By defining $\alpha^\pm\equiv\alpha_1\pm\alpha_2$, Eqs. (A51)–(A53) give

$$\cos\alpha^-=\frac{\vec{r}_1.\vec{r}_2}{r_1r_2}=\xi\frac{\sin\Phi_\nu-\sin\Phi_e}{\sin(\Phi_\nu-\Phi_e)},\tag{A54}$$

where

$$\xi=\frac{6|\rho_{12}|}{\sqrt{-\frac{5\rho_{11}^2\tau_2}{\tau_1}-\frac{5\rho_{22}^2\tau_1}{\tau_2}+26\rho_{22}\rho_{11}}}.\tag{A55}$$

Analogously, Eq. (A42) can be rewritten as

$$\cos\alpha_1\cos\alpha_2\sin\Phi_e=-\sin\alpha_1\sin\alpha_2\sin\Phi_\nu,\tag{A56}$$

which gives

$$\frac{\sin\Phi_e}{\sin\Phi_\nu} = \frac{\cos\alpha^+ - \cos\alpha^-}{\cos\alpha^- + \cos\alpha^+}, \tag{A57}$$

and thus, using Eq. (A54), we obtain

$$\cos\alpha^+ = \xi \frac{\sin\Phi_\nu + \sin\Phi_e}{\sin(\Phi_\nu - \Phi_e)}. \tag{A58}$$

Notice also that in the real case (i.e., $\Phi_\nu = \Phi_e = 0$) α^+ is undetermined, while $\cos\alpha^- = \xi$.

This justifies the shape of the vacuum manifold given in Eq. (16) of Sec. III B.

3. Gauge boson spectrum

In order to determine the residual symmetry corresponding to a specific vacuum configuration, we compute explicitly the gauge spectrum. Given the $SO(10) \otimes U(1)_X$ covariant derivatives for the scalar components of the Higgs chiral superfields

$$\begin{aligned} D_\mu 16 &= \partial_\mu 16 - ig(A_\mu)_{(ij)}\Sigma^+_{(ij)}16 - ig_X X_\mu 16, \\ D_\mu \overline{16} &= \partial_\mu \overline{16} - ig(A_\mu)_{(ij)}\Sigma^-_{(ij)}\overline{16} + ig_X X_\mu \overline{16}, \\ D_\mu 45 &= \partial_\mu 45 - ig(A_\mu)_{(ij)}[\Sigma^+_{(ij)}, 45], \end{aligned} \tag{A59}$$

where the indices in brackets (ij) stand for ordered pairs, and the properly normalized kinetic terms

$$D_\mu 16^\dagger D_\mu 16, \qquad D_\mu \overline{16}^\dagger D_\mu \overline{16}, \qquad \tfrac{1}{4}\mathrm{Tr} D_\mu 45^\dagger D_\mu 45, \tag{A60}$$

one can write the 46-dimensional gauge boson mass matrix governing the mass bilinear of the form

$$\tfrac{1}{2}((A_\mu)_{(ij)}, X_\mu)\mathcal{M}^2(A, X)((A^\mu)_{(kl)}, X^\mu)^T \tag{A61}$$

as

$$\mathcal{M}^2(A, X) = \begin{pmatrix} \mathcal{M}^2_{(ij)(kl)} & \mathcal{M}^2_{(ij)X} \\ \mathcal{M}^2_{X(kl)} & \mathcal{M}^2_{XX} \end{pmatrix}. \tag{A62}$$

The relevant matrix elements are given by

$$\begin{aligned} \mathcal{M}^2_{(ij)(kl)} &= g^2(\langle 16\rangle^\dagger\{\Sigma^+_{(ij)}, \Sigma^+_{(kl)}\}\langle 16\rangle + \langle\overline{16}\rangle^\dagger\{\Sigma^-_{(ij)}, \Sigma^-_{(kl)}\} \\ &\quad \times \langle\overline{16}\rangle + \tfrac{1}{2}\mathrm{Tr}[\Sigma^+_{(ij)}, \langle 45\rangle]^\dagger[\Sigma^+_{(kl)}, \langle 45\rangle]), \\ \mathcal{M}^2_{(ij)X} &= 2gg_X(\langle 16\rangle^\dagger\Sigma^+_{(ij)}\langle 16\rangle - \langle\overline{16}\rangle^\dagger\Sigma^-_{(ij)}\langle\overline{16}\rangle), \\ \mathcal{M}^2_{X(kl)} &= 2gg_X(\langle 16\rangle^\dagger\Sigma^+_{(kl)}\langle 16\rangle - \langle\overline{16}\rangle^\dagger\Sigma^-_{(kl)}\langle\overline{16}\rangle), \\ \mathcal{M}^2_{XX} &= 2g_X^2(\langle 16\rangle^\dagger\langle 16\rangle + \langle\overline{16}\rangle^\dagger\langle\overline{16}\rangle). \end{aligned} \tag{A63}$$

a. Spinorial contribution

Considering first the contribution of the reducible representation $\langle 16_1 \oplus 16_2 \oplus \overline{16}_1 \oplus \overline{16}_2\rangle$ to the gauge boson mass matrix, we find

$$\mathcal{M}^2_{16}(1, 3, 0)_{1_{45}} = 0, \tag{A64}$$

$$\mathcal{M}^2_{16}(8, 1, 0)_{15_{45}} = 0, \tag{A65}$$

$$\begin{aligned} \mathcal{M}^2_{16}(3, 1, -\tfrac{1}{3})_{15_{45}} &= g^2(|e_1|^2 + |\nu_1|^2 + |e_2|^2 + |\nu_2|^2 \\ &\quad + |\bar{e}_1|^2 + |\bar{\nu}_1|^2 + |\bar{e}_2|^2 + |\bar{\nu}_2|^2). \end{aligned} \tag{A66}$$

In the $(6^-_{45}, 6^+_{45})$ basis (see Table V for the labeling of the states) we obtain

$$\mathcal{M}^2_{16}\left(3, 2, +\frac{1}{6}\right) = \begin{pmatrix} g^2(|\nu_1|^2 + |\nu_2|^2 + |\bar{\nu}_1|^2 + |\bar{\nu}_2|^2) & -ig^2(e_1^*\nu_1 + e_2^*\nu_2 + \bar{\nu}_1^*\bar{e}_1 + \bar{\nu}_2^*\bar{e}_2) \\ ig^2(e_1\nu_1^* + e_2\nu_2^* + \bar{\nu}_1\bar{e}_1^* + \bar{\nu}_2\bar{e}_2^*) & g^2(|e_1|^2 + |e_2|^2 + |\bar{e}_1|^2 + |\bar{e}_2|^2) \end{pmatrix}. \tag{A67}$$

The five-dimensional SM-singlet mass matrix in the $(15_{45}, 1^-_{45}, 1^0_{45}, 1^+_{45}, 1_1)$ basis reads

$$\mathcal{M}^2_{16}(1, 1, 0) = \begin{pmatrix} \frac{3}{2}g^2S_1 & i\sqrt{3}g^2S_3 & -\sqrt{\frac{3}{2}}g^2S_2 & -i\sqrt{3}g^2S_3^* & -\sqrt{3}gg_XS_1 \\ -i\sqrt{3}g^2S_3^* & g^2S_1 & 0 & 0 & 2igg_XS_3 \\ -\sqrt{\frac{3}{2}}g^2S_2 & 0 & g^2S_1 & 0 & \sqrt{2}gg_XS_2 \\ i\sqrt{3}g^2S_3 & 0 & 0 & g^2S_1 & -2igg_XS_3^* \\ -\sqrt{3}gg_XS_1 & -2igg_XS_3^* & \sqrt{2}gg_XS_2 & 2igg_XS_3 & 2g_X^2S_1 \end{pmatrix}, \tag{A68}$$

where $S_1 \equiv |e_1|^2 + |e_2|^2 + |\nu_1|^2 + |\nu_2|^2 + |\bar{e}_1|^2 + |\bar{e}_2|^2 + |\bar{\nu}_1|^2 + |\bar{\nu}_2|^2$, $S_2 \equiv |e_1|^2 + |e_2|^2 - |\nu_1|^2 - |\nu_2|^2 + |\bar{e}_1|^2 + |\bar{e}_2|^2 - |\bar{\nu}_1|^2 - |\bar{\nu}_2|^2$, and $S_3 \equiv e_1\nu_1^* + e_2\nu_2^* + \bar{e}_1^*\bar{\nu}_1 + \bar{e}_2^*\bar{\nu}_2$.

For generic VEVs Rank $\mathcal{M}^2_{16}(1, 1, 0) = 4$, and we recover 12 massless gauge bosons with the quantum numbers of the standard model algebra.

We verified that this result is maintained when implementing the constraints of the flipped vacuum manifold in Eq. (16). Since it is, by construction, the smallest algebra that can be preserved by the whole vacuum manifold, it must be maintained when adding the $\langle 45_H\rangle$ contribution. We can therefore claim that the invariant algebra on the generic vacuum is the SM. On the other hand, the 45_H already plays an active role in this result since it allows for a misalignment of the VEV directions in the two $16_H \oplus \overline{16}_H$ spinors such that the spinor vacuum preserves SM and not $SU(5) \otimes U(1)$. More details shall be given in the next section.

b. Adjoint contribution

Considering the contribution of $\langle 45_H \rangle$ to the gauge spectrum, we find

$$\mathcal{M}^2_{45}(1,3,0)_{1_{45}} = 0, \tag{A69}$$

$$\mathcal{M}^2_{45}(8,1,0)_{15_{45}} = 0, \tag{A70}$$

$$\mathcal{M}^2_{45}(3,1,-\tfrac{1}{3})_{15_{45}} = 4g^2\omega_Y^2. \tag{A71}$$

Analogously, in the $(6^-_{45}, 6^+_{45})$ basis, we have

$$\mathcal{M}^2_{45}\left(3,2,+\frac{1}{6}\right) = \begin{pmatrix} g^2((\omega_R+\omega_Y)^2 + 2\omega^-\omega^+) & i2\sqrt{2}g^2\omega_Y\omega^- \\ -i2\sqrt{2}g^2\omega_Y\omega^+ & g^2((\omega_R-\omega_Y)^2+2\omega^-\omega^+) \end{pmatrix}. \tag{A72}$$

The SM-singlet mass matrix in the $(15_{45}, 1^-_{45}, 1^0_{45}, 1^+_{45}, 1_1)$ basis reads

$$\mathcal{M}^2_{45}(1,1,0) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 4g^2(\omega_R^2+\omega^-\omega^+) & -i4g^2\omega_R\omega^- & 4g^2(\omega^-)^2 & 0 \\ 0 & i4g^2\omega_R\omega^+ & 8g^2\omega^-\omega^+ & -i4g^2\omega_R\omega^- & 0 \\ 0 & 4g^2(\omega^+)^2 & i4g^2\omega_R\omega^+ & 4g^2(\omega_R^2+\omega^-\omega^+) & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{A73}$$

For generic VEVs we find Rank $\mathcal{M}^2_{45}(1,1,0) = 2$, leading globally to the 14 massless gauge bosons of the $SU(3)_c \otimes SU(2)_L \otimes U(1)^3$ algebra.

As a consistency check, by switching on just the ω_R and ω_Y VEVs, we recover the results of [2] for standard $SO(10)$.

c. Vacuum little group

With the results of Appendixes A 3 a and A 3 b at hand, the residual gauge symmetry can be readily identified from the properties of the complete gauge boson mass matrix. For the sake of simplicity, here we shall present the results in the real VEV approximation.

Trading the VEVs for the superpotential parameters, one can immediately identify the strong and weak gauge bosons of the SM that, as expected, remain massless:

$$\mathcal{M}^2(8,1,0)_{15_{45}} = 0, \qquad \mathcal{M}^2(1,3,0)_{1_{45}} = 0. \tag{A74}$$

Similarly, it is straightforward to obtain

$$\mathcal{M}^2\left(3,1,-\frac{1}{3}\right)_{15_{45}} = \frac{4g^2}{9\tau_1^2\tau_2^2}(3\mu(\rho_{22}\tau_1(5\tau_1-\tau_2) + \rho_{11}\tau_2(5\tau_2-\tau_1)) + 2(\rho_{22}\tau_1+\rho_{11}\tau_2)^2). \tag{A75}$$

On the other hand, the complete matrices $\mathcal{M}^2(3,2,+\frac{1}{6})$ and $\mathcal{M}^2(1,1,0)$ turn out to be quite involved once the vacuum constraints are imposed, and we do not show them here explicitly. Nevertheless, it is sufficient to consider

$$\mathrm{Tr}\mathcal{M}^2\left(3,2,+\frac{1}{6}\right) = \frac{g^2}{8\mu^2}[16\mu^2(r_1^2+r_2^2) + \tau_1^2r_1^4 + \tau_2^2r_2^4 + \tau_1\tau_2r_1^2r_2^2(1+\cos2\alpha^-)] \tag{A76}$$

and

$$\det\mathcal{M}^2\left(3,2,+\frac{1}{6}\right) = \frac{g^4r_1^2r_2^2}{128\mu^4}[512\mu^4 + 32\mu^2(\tau_1^2r_1^2+\tau_2^2r_2^2) + \tau_1^2\tau_2^2r_1^2r_2^2(1-\cos2\alpha^-)]\sin^2\alpha^- \tag{A77}$$

to see that for a generic nonzero value of $\sin\alpha^-$ one gets Rank $\mathcal{M}^2(3,2,+\frac{1}{6}) = 2$. On the other hand, when $\alpha^- = 0$ (i.e., $\langle 16_1 \rangle \propto \langle 16_2 \rangle$) or $r_2 = 0$ (i.e., $\langle 16_2 \rangle = 0$), Rank $\mathcal{M}^2(3,2,+\frac{1}{6}) = 1$ and one is left with an additional massless $(3,2,+\frac{1}{6}) \oplus (\bar{3},2,-\frac{1}{6})$ gauge boson, corresponding to an enhanced residual symmetry.

In the case of the five-dimensional matrix $\mathcal{M}^2(1,1,0)$ it is sufficient to notice that for a generic nonzero $\sin\alpha^-$,

$$\text{Rank } \mathcal{M}^2(1,1,0) = 4, \tag{A78}$$

on the vacuum manifold, which leaves a massless $U(1)_Y$ gauge boson, thus completing the SM algebra. As before, for $\alpha^- = 0$ or for $r_2 = 0$, we find Rank $\mathcal{M}^2(1,1,0) = 3$. Taking into account the massless states in the $(3,2,+\frac{1}{6}) \oplus (\bar{3},2,-\frac{1}{6})$ sector, we recover, as expected, the flipped $SU(5) \otimes U(1)$ algebra.

APPENDIX B: E_6 VACUUM

1. The $SU(3)^3$ formalism

Following closely the notation of Ref. [12], we decompose the adjoint and fundamental representations of E_6

under its $SU(3)_C \otimes SU(3)_L \otimes SU(3)_R$ maximal subalgebra as

$$78 \equiv (8,1,1) \oplus (1,8,1) \oplus (1,1,8) \oplus (\bar{3},3,3) \oplus (3,\bar{3},\bar{3}) \subset T^\alpha_\beta \oplus T^i_j \oplus T^{i'}_{j'} \oplus Q^\alpha_{ij'} \oplus Q^{ij'}_\alpha, \tag{B1}$$

$$27 \equiv (3,3,1) \oplus (1,\bar{3},3) \oplus (\bar{3},1,\bar{3}) \equiv v_{\alpha i} \oplus v^i_{j'} \oplus v^{\alpha j'}, \tag{B2}$$

$$\overline{27} \equiv (\bar{3},\bar{3},1) \oplus (1,3,\bar{3}) \oplus (3,1,3) \equiv u^{\alpha i} \oplus u^{j'}_i \oplus u_{\alpha j'}, \tag{B3}$$

where the Greek, Latin, and primed-Latin indices, corresponding to $SU(3)_c$, $SU(3)_L$, and $SU(3)_R$, respectively, run from 1 to 3. As far as the $SU(3)$ algebras in Eq. (B1) are concerned, the generators follow the standard Gell-Mann convention

$$\begin{aligned}
T^{(1)} &= \frac{1}{2}(T^1_2 + T^2_1), & T^{(2)} &= \frac{i}{2}(T^1_2 - T^2_1),\\
T^{(3)} &= \frac{1}{2}(T^1_1 - T^2_2), & T^{(4)} &= \frac{1}{2}(T^1_3 + T^3_1),\\
T^{(5)} &= \frac{i}{2}(T^1_3 - T^3_1), & T^{(6)} &= \frac{1}{2}(T^2_3 + T^3_2),\\
T^{(7)} &= \frac{i}{2}(T^2_3 - T^3_2), & T^{(8)} &= \frac{1}{2\sqrt{3}}(T^1_1 + T^2_2 - 2T^3_3),
\end{aligned} \tag{B4}$$

with $(T^a_b)^k_l = \delta^k_b \delta^a_l$, so they are all normalized so that $\mathrm{Tr} T^{(a)} T^{(b)} = \frac{1}{2}\delta^{ab}$.

Taking into account Eqs. (B1)–(B4), the E_6 algebra can be written as

$$\begin{aligned}
[T^\alpha_\beta, T^\gamma_\eta] &= \delta^\alpha_\eta T^\gamma_\beta - \delta^\gamma_\beta T^\alpha_\eta,\\
[T^i_j, T^k_l] &= \delta^i_l T^k_j - \delta^k_j T^i_l,\\
[T^{i'}_{j'}, T^{k'}_{l'}] &= \delta^{i'}_{l'} T^{k'}_{j'} - \delta^{k'}_{j'} T^{i'}_{l'},\\
[T^\alpha_\beta, T^i_j] &= [T^\alpha_\beta, T^{i'}_{j'}] = [T^i_j, T^{i'}_{j'}] = 0,
\end{aligned} \tag{B5}$$

$$\begin{aligned}
[Q^\gamma_{ij'}, T^\alpha_\beta] &= \delta^\gamma_\beta Q^\alpha_{ij'},\\
[Q^{ij'}_\gamma, T^\alpha_\beta] &= -\delta^\alpha_\gamma Q^{ij'}_\beta,\\
[Q^\gamma_{ij'}, T^k_l] &= -\delta^k_i Q^\gamma_{lj'},\\
[Q^{ij'}_\gamma, T^k_l] &= \delta^i_l Q^{kj'}_\gamma,\\
[Q^\gamma_{ij'}, T^{k'}_{l'}] &= -\delta^{k'}_{j'} Q^\gamma_{il'},\\
[Q^{ij'}_\gamma, T^{k'}_{l'}] &= \delta^{j'}_{l'} Q^{ik'}_\gamma,
\end{aligned} \tag{B6}$$

$$\begin{aligned}
[Q^\alpha_{ij'}, Q^{kl'}_\beta] &= -\delta^\alpha_\beta \delta^k_i T^{l'}_{j'} - \delta^\alpha_\beta \delta^{l'}_{j'} T^k_i + \delta^k_i \delta^{l'}_{j'} T^\alpha_\beta,\\
[Q^\alpha_{ij'}, Q^\beta_{kl'}] &= \epsilon^{\alpha\beta\gamma} \epsilon_{ikp} \epsilon_{j'l'q'} Q^{pq'}_\gamma,\\
[Q^{ij'}_\alpha, Q^{kl'}_\beta] &= -\epsilon_{\alpha\beta\gamma} \epsilon^{ikp} \epsilon^{j'l'q'} Q^\gamma_{pq'}.
\end{aligned} \tag{B7}$$

The action of the algebra on the fundamental 27 representation reads

$$\begin{aligned}
T^\beta_\gamma v_{\alpha i} &= \delta^\beta_\alpha v_{\gamma i},\\
T^k_l v_{\alpha i} &= \delta^k_i v_{\alpha l},\\
T^{k'}_{l'} v_{\alpha i} &= 0,\\
Q^\beta_{pq'} v_{\alpha i} &= \delta^\beta_\alpha \epsilon_{pik} v^k_{q'},\\
Q^{pq'}_\beta v_{\alpha i} &= \delta^p_i \epsilon_{\beta\alpha\gamma} v^{\gamma q'},
\end{aligned} \tag{B8}$$

$$\begin{aligned}
T^\beta_\gamma v^i_{j'} &= 0,\\
T^k_l v^i_{j'} &= -\delta^i_l v^k_{j'},\\
T^{k'}_{l'} v^i_{j'} &= \delta^{k'}_{j'} v^i_{l'},\\
Q^\beta_{pq'} v^i_{j'} &= -\delta^i_p \epsilon_{q'j'k'} v^{\beta k'},\\
Q^{pq'}_\beta v^i_{j'} &= \delta^{q'}_{j'} \epsilon^{pik} v_{\beta k},
\end{aligned} \tag{B9}$$

$$\begin{aligned}
T^\beta_\gamma v^{\alpha j'} &= -\delta^\alpha_\gamma v^{\beta j'},\\
T^k_l v^{\alpha j'} &= 0,\\
T^{k'}_{l'} v^{\alpha j'} &= -\delta^{j'}_{l'} v^{\alpha k'},\\
Q^\beta_{pq'} v^{\alpha j'} &= -\delta^{j'}_{q'} \epsilon^{\beta\alpha\gamma} v_{\gamma p},\\
Q^{pq'}_\beta v^{\alpha j'} &= -\delta^\alpha_\beta \epsilon^{q'j'k'} v^p_{k'},
\end{aligned} \tag{B10}$$

and accordingly on $\overline{27}$,

$$\begin{aligned}
T^\beta_\gamma u^{\alpha i} &= -\delta^\alpha_\gamma u^{\beta i},\\
T^k_l u^{\alpha i} &= -\delta^i_l u^{\alpha k},\\
T^{k'}_{l'} u^{\alpha i} &= 0,\\
Q^\beta_{pq'} u^{\alpha i} &= -\delta^i_p \epsilon^{\beta\alpha\gamma} u_{\gamma q'},\\
Q^{pq'}_\beta u^{\alpha i} &= -\delta^\alpha_\beta \epsilon^{pik} u^{q'}_k,
\end{aligned} \tag{B11}$$

$$\begin{aligned}
T^\beta_\gamma u^{j'}_i &= 0,\\
T^k_l u^{j'}_i &= \delta^k_i u^{j'}_l,\\
T^{k'}_{l'} u^{j'}_i &= -\delta^{j'}_{l'} u^{k'}_i,\\
Q^\beta_{pq'} u^{j'}_i &= -\delta^{j'}_{q'} \epsilon_{pik} u^{\beta k},\\
Q^{pq'}_\beta u^{j'}_i &= \delta^p_i \epsilon^{q'j'k'} u_{\beta k'},
\end{aligned} \tag{B12}$$

$$\begin{aligned}
T^\beta_\gamma u_{\alpha j'} &= \delta^\beta_\alpha u_{\gamma j'},\\
T^k_l u_{\alpha j'} &= 0,\\
T^{k'}_{l'} u_{\alpha j'} &= \delta^{k'}_{j'} u_{\alpha l'},\\
Q^\beta_{pq'} u_{\alpha j'} &= \delta^\beta_\alpha \epsilon_{q'j'k'} u^{k'}_p,\\
Q^{pq'}_\beta u_{\alpha j'} &= \delta^{q'}_{j'} \epsilon_{\beta\alpha\gamma} u^{\gamma p}.
\end{aligned} \tag{B13}$$

Given the SM hypercharge definition

$$Y = \frac{1}{\sqrt{3}} T_L^{(8)} + T_R^{(3)} + \frac{1}{\sqrt{3}} T_R^{(8)}, \tag{B14}$$

the SM-preserving vacuum direction corresponds to [12]

$$\langle 78 \rangle = a_1 T_{2'}^{3'} + a_2 T_{3'}^{2'} + \frac{a_3}{\sqrt{6}}(T_{1'}^{1'} + T_{2'}^{2'} - 2T_{3'}^{3'}) + \frac{a_4}{\sqrt{2}}(T_{1'}^{1'} - T_{2'}^{2'}) + \frac{b_3}{\sqrt{6}}(T_1^1 + T_2^2 - 2T_3^3), \tag{B15}$$

$$\langle 27 \rangle = e v_{3'}^3 + \nu v_{2'}^3, \qquad \langle \overline{27} \rangle = \bar{e} u_3^{3'} + \bar{\nu} u_3^{2'}, \tag{B16}$$

where a_1, a_2, a_3, a_4, b_3, e, $\bar{e}$, ν, and $\bar{\nu}$ are SM-singlet VEVs. This can be checked by means of Eqs. (B5)–(B13). Notice that the adjoint VEVs a_3, a_4, and b_3 are real, while $a_1 = a_2^*$. The VEVs of $27 \oplus \overline{27}$ are generally complex.

2. E_6 vacuum manifold

Working out the D-flatness equations, one finds that the nontrivial constraints are given by

$$\begin{aligned} D_{E_\alpha} &= \left(\frac{3a_3}{\sqrt{6}} - \frac{a_4}{\sqrt{2}}\right)a_2^* - a_1\left(\frac{3a_3^*}{\sqrt{6}} - \frac{a_4^*}{\sqrt{2}}\right) + e_1^*\nu_1 - \bar{e}_1\bar{\nu}_1^* + e_2^*\nu_2 - \bar{e}_2\bar{\nu}_2^* = 0, \\ D_{T_R^{(8)}} &= 3(|a_1|^2 - |a_2|^2) + 2(|\bar{e}_1|^2 - |e_1|^2) + 2(|\bar{e}_2|^2 - |e_2|^2) + |\nu_1|^2 - |\bar{\nu}_1|^2 + |\nu_2|^2 - |\bar{\nu}_2|^2 = 0, \\ D_{T_R^{(3)}} &= |a_2|^2 - |a_1|^2 + |\bar{\nu}_1|^2 - |\nu_1|^2 + |\bar{\nu}_2|^2 - |\nu_2|^2 = 0, \\ D_{T_L^{(8)}} &= |e_1|^2 + |\nu_1|^2 + |e_2|^2 + |\nu_2|^2 - |\bar{e}_1|^2 - |\bar{\nu}_1|^2 - |\bar{e}_2|^2 - |\bar{\nu}_2|^2 = 0, \end{aligned} \tag{B17}$$

where D_{E_α} is the ladder operator from the (1, 1, 8) submultiplet of 78. Notice that the relations corresponding to $D_{T_R^{(8)}}$, $D_{T_R^{(3)}}$, and $D_{T_L^{(8)}}$ are linearly dependent, since the linear combination associated with the SM hypercharge in Eq. (B14) vanishes.

The superpotential W_H in Eq. (36) evaluated on the vacuum manifold (B15) and (B16) yields Eq. (47). Accordingly, one finds the following F-flatness equations:

$$\begin{aligned} F_{a_1} &= \mu a_2 - \tau_1 e_1\bar{\nu}_1 - \tau_2 e_2\bar{\nu}_2 = 0, \\ F_{a_2} &= \mu a_1 - \tau_1 \nu_1\bar{e}_1 - \tau_2 \nu_2\bar{e}_2 = 0, \\ F_{a_3} &= \mu a_3 - \frac{1}{\sqrt{6}}(\tau_1(\nu_1\bar{\nu}_1 - 2e_1\bar{e}_1) + \tau_2(\nu_2\bar{\nu}_2 - 2e_2\bar{e}_2)) = 0, \\ F_{a_4} &= \mu a_4 + \frac{1}{\sqrt{2}}(\tau_1\nu_1\bar{\nu}_1 + \tau_2\nu_2\bar{\nu}_2) = 0, \\ F_{b_3} &= \mu b_3 - \sqrt{\frac{2}{3}}(\tau_1(\nu_1\bar{\nu}_1 + e_1\bar{e}_1) + \tau_2(\nu_2\bar{\nu}_2 + e_2\bar{e}_2)) = 0, \\ 3F_{e_1} &= 3(\rho_{11}\bar{e}_1 + \rho_{12}\bar{e}_2) - \tau_1(\sqrt{6}(b_3 - a_3)\bar{e}_1 + 3a_1\bar{\nu}_1) = 0, \\ 3F_{e_2} &= 3(\rho_{21}\bar{e}_1 + \rho_{22}\bar{e}_2) - \tau_2(\sqrt{6}(b_3 - a_3)\bar{e}_2 + 3a_1\bar{\nu}_2) = 0, \\ 6F_{\nu_1} &= 6(\rho_{11}\bar{\nu}_1 + \rho_{12}\bar{\nu}_2) - \tau_1(\sqrt{2}(\sqrt{3}a_3 - 3a_4 + 2\sqrt{3}b_3)\bar{\nu}_1 + 6a_2\bar{e}_1) = 0, \\ 6F_{\nu_2} &= 6(\rho_{21}\bar{\nu}_1 + \rho_{22}\bar{\nu}_2) - \tau_2(\sqrt{2}(\sqrt{3}a_3 - 3a_4 + 2\sqrt{3}b_3)\bar{\nu}_2 + 6a_2\bar{e}_2) = 0, \\ 3F_{\bar{e}_1} &= 3(\rho_{11}e_1 + \rho_{21}e_2) - \tau_1(\sqrt{6}(b_3 - a_3)e_1 + 3a_2\nu_1) = 0, \\ 3F_{\bar{e}_2} &= 3(\rho_{12}e_1 + \rho_{22}e_2) - \tau_2(\sqrt{6}(b_3 - a_3)e_2 + 3a_2\nu_2) = 0, \\ 6F_{\bar{\nu}_1} &= 6(\rho_{11}\nu_1 + \rho_{21}\nu_2) - \tau_1(\sqrt{2}(\sqrt{3}a_3 - 3a_4 + 2\sqrt{3}b_3)\nu_1 + 6a_1e_1) = 0, \\ 6F_{\bar{\nu}_2} &= 6(\rho_{12}\nu_1 + \rho_{22}\nu_2) - \tau_2(\sqrt{2}(\sqrt{3}a_3 - 3a_4 + 2\sqrt{3}b_3)\nu_2 + 6a_1e_2) = 0. \end{aligned} \tag{B18}$$

Following the strategy of Appendix A 2 one can solve the first five equations above for a_1, a_2, a_3, a_4, and b_3:

$$\begin{aligned} \mu a_1 &= \tau_1\nu_1\bar{e}_1 + \tau_2\nu_2\bar{e}_2, \\ \mu a_2 &= \tau_1 e_1\bar{\nu}_1 + \tau_2 e_2\bar{\nu}_2, \\ \sqrt{6}\mu a_3 &= \tau_1(\nu_1\bar{\nu}_1 - 2e_1\bar{e}_1) + \tau_2(\nu_2\bar{\nu}_2 - 2e_2\bar{e}_2), \\ \sqrt{2}\mu a_4 &= -\tau_1\nu_1\bar{\nu}_1 - \tau_2\nu_2\bar{\nu}_2, \\ \sqrt{3}\mu b_3 &= \sqrt{2}(\tau_1(\nu_1\bar{\nu}_1 + e_1\bar{e}_1) + \tau_2(\nu_2\bar{\nu}_2 + e_2\bar{e}_2)). \end{aligned} \tag{B19}$$

Since $a_1 = a_2^*$ and τ_1 and τ_2 can be taken real without loss of generality (see Sec. IV B), the first two equations above imply

$$\tau_1\nu_1\bar{e}_1 + \tau_2\nu_2\bar{e}_2 = \tau_1(e_1\bar{\nu}_1)^* + \tau_2(e_2\bar{\nu}_2)^*. \tag{B20}$$

Using (B19) the remaining F-flatness conditions in Eq. (B18) can be rewritten in the form

$$\begin{aligned} 3\mu F_{e_1}^a &= 3\mu(\rho_{11}\bar{e}_1 + \rho_{12}\bar{e}_2) - 4\tau_1^2(\nu_1\bar{\nu}_1 + e_1\bar{e}_1)\bar{e}_1 - \tau_1\tau_2(3\nu_2\bar{\nu}_1\bar{e}_2 + (\nu_2\bar{\nu}_2 + 4e_2\bar{e}_2)\bar{e}_1) = 0, \\ 3\mu F_{\bar{e}_1}^a &= 3\mu(\rho_{11}e_1 + \rho_{21}e_2) - 4\tau_1^2(\bar{\nu}_1\nu_1 + \bar{e}_1e_1)e_1 - \tau_1\tau_2(3\bar{\nu}_2\nu_1e_2 + (\bar{\nu}_2\nu_2 + 4\bar{e}_2e_2)e_1) = 0, \\ 3\mu F_{\nu_1}^a &= 3\mu(\rho_{11}\bar{\nu}_1 + \rho_{12}\bar{\nu}_2) - 4\tau_1^2(e_1\bar{e}_1 + \nu_1\bar{\nu}_1)\bar{\nu}_1 - \tau_1\tau_2(3e_2\bar{e}_1\bar{\nu}_2 + (e_2\bar{e}_2 + 4\nu_2\bar{\nu}_2)\bar{\nu}_1) = 0, \\ 3\mu F_{\bar{\nu}_1}^a &= 3\mu(\rho_{11}\nu_1 + \rho_{21}\nu_2) - 4\tau_1^2(\bar{e}_1e_1 + \bar{\nu}_1\nu_1)\nu_1 - \tau_1\tau_2(3\bar{e}_2e_1\nu_2 + (\bar{e}_2e_2 + 4\bar{\nu}_2\nu_2)\nu_1) = 0, \end{aligned} \tag{B21}$$

and the additional four relations can again be obtained by exchanging $1 \leftrightarrow 2$. Similarly, the triplet of linearly independent D-flatness conditions in Eq. (B17) can be brought to the form

$$
\begin{aligned}
D_{E_\alpha} &= e_1^*\nu_1 - \bar{e}_1\bar{\nu}_1^* + e_2^*\nu_2 - \bar{e}_2\bar{\nu}_2^* = 0,\\
D_{T_R^{(3)}} &= |\bar{\nu}_1|^2 - |\nu_1|^2 + |\bar{\nu}_2|^2 - |\nu_2|^2 = 0,\\
D_{T_L^{(8)}} &= |e_1|^2 + |\nu_1|^2 + |e_2|^2 + |\nu_2|^2 - |\bar{e}_1|^2 \\
&\quad - |\bar{\nu}_1|^2 - |\bar{e}_2|^2 - |\bar{\nu}_2|^2 = 0.
\end{aligned} \tag{B22}
$$

Combining these with Eq. (B20), the D-flatness condition is ensured if and only if $e_{1,2}^* = \bar{e}_{1,2}$ and $\nu_{1,2}^* = \bar{\nu}_{1,2}$. Hence, in complete analogy with the flipped $SO(10)$ case, Eq. (A30), one can write

$$
\begin{aligned}
e_{1,2} &\equiv |e_{1,2}|e^{i\phi_{e_{1,2}}}, \qquad \bar{e}_{1,2} \equiv |e_{1,2}|e^{-i\phi_{e_{1,2}}},\\
\nu_{1,2} &\equiv |\nu_{1,2}|e^{i\phi_{\nu_{1,2}}}, \qquad \bar{\nu}_{1,2} \equiv |\nu_{1,2}|e^{-i\phi_{\nu_{1,2}}}.
\end{aligned} \tag{B23}
$$

From now on, the discussion of the vacuum manifold very closely follows that for the flipped $SO(10)$ in Sec. II A, and we shall not repeat it here. In particular, the existence of a nontrivial vacuum requires the Hermiticity of the ρ and τ couplings. This is related to the fact that D- and F-flatness conditions require $\langle 27_i\rangle = \langle\overline{27}_i\rangle^*$. The detailed shape of the resulting vacuum manifold so obtained is given in Eq. (48) of Sec. IV B.

3. Vacuum little group

In order to find the algebra left invariant by the vacuum configurations in Eq. (48), we need to compute the action of the E_6 generators on the $\langle 78 \oplus 27_1 \oplus 27_2 \oplus \overline{27}_1 \oplus \overline{27}_2\rangle$ VEV. From Eqs. (B5) and (B6) one obtains

$$
\begin{aligned}
T^\alpha_\beta\langle 78\rangle &= 0,\\
T^i_j\langle 78\rangle &= \frac{b_3}{\sqrt{6}}(\delta^i_1 T^1_j - \delta^1_j T^i_1 + \delta^i_2 T^2_j - \delta^2_j T^i_2 - 2\delta^i_3 T^3_j + 2\delta^3_j T^i_3),\\
T^{i'}_{j'}\langle 78\rangle &= a_1(\delta^{i'}_{2'} T^{3'}_{j'} - \delta^{3'}_{j'} T^{i'}_{2'}) + a_2(\delta^{i'}_{3'} T^{2'}_{j'} - \delta^{2'}_{j'} T^{i'}_{3'}) + \frac{a_3}{\sqrt{6}}(\delta^{i'}_{1'} T^{1'}_{j'} - \delta^{1'}_{j'} T^{i'}_{1'} + \delta^{i'}_{2'} T^{2'}_{j'} - \delta^{2'}_{j'} T^{i'}_{2'} - 2\delta^{i'}_{3'} T^{3'}_{j'} + 2\delta^{3'}_{j'} T^{i'}_{3'})\\
&\quad + \frac{a_4}{\sqrt{2}}(\delta^{i'}_{1'} T^{1'}_{j'} - \delta^{1'}_{j'} T^{i'}_{1'} - \delta^{i'}_{2'} T^{2'}_{j'} + \delta^{2'}_{j'} T^{i'}_{2'}),\\
Q^\alpha_{ij'}\langle 78\rangle &= -a_1(\delta^{3'}_{j'} Q^\alpha_{i2'}) - a_2(\delta^{2'}_{j'} Q^\alpha_{i3'}) - \frac{a_3}{\sqrt{6}}(\delta^{1'}_{j'} Q^\alpha_{i1'} + \delta^{2'}_{j'} Q^\alpha_{i2'} - 2\delta^{3'}_{j'} Q^\alpha_{i3'}) - \frac{a_4}{\sqrt{2}}(\delta^{1'}_{j'} Q^\alpha_{i1'} - \delta^{2'}_{j'} Q^\alpha_{i2'})\\
&\quad - \frac{b_3}{\sqrt{6}}(\delta^1_i Q^\alpha_{1j'} + \delta^2_i Q^\alpha_{2j'} - 2\delta^3_i Q^\alpha_{3j'}),\\
Q_\alpha^{ij'}\langle 78\rangle &= a_1(\delta^{j'}_{2'} Q_\alpha^{i3'}) + a_2(\delta^{j'}_{3'} Q_\alpha^{i2'}) + \frac{a_3}{\sqrt{6}}(\delta^{j'}_{1'} Q_\alpha^{i1'} + \delta^{j'}_{2'} Q_\alpha^{i2'} - 2\delta^{j'}_{3'} Q_\alpha^{i3'}) + \frac{a_4}{\sqrt{2}}(\delta^{j'}_{1'} Q_\alpha^{i1'} - \delta^{j'}_{2'} Q_\alpha^{i2'})\\
&\quad + \frac{b_3}{\sqrt{6}}(\delta^i_1 Q_\alpha^{1j'} + \delta^i_2 Q_\alpha^{2j'} - 2\delta^i_3 Q_\alpha^{3j'}),
\end{aligned} \tag{B24}
$$

on the adjoint vacuum. For $\langle 27_1 \oplus 27_2\rangle$ one finds

$$
\begin{aligned}
T^\alpha_\beta\langle 27_1 \oplus 27_2\rangle &= 0,\\
T^i_j\langle 27_1 \oplus 27_2\rangle &= -(e_1 + e_2)[\delta^3_j v^i_{3'}] - (\nu_1 + \nu_2)[\delta^3_j v^i_{2'}],\\
T^{i'}_{j'}\langle 27_1 \oplus 27_2\rangle &= (e_1 + e_2)[\delta^{i'}_{3'} v^3_{j'}] + (\nu_1 + \nu_2)[\delta^{i'}_{2'} v^3_{j'}],\\
Q^\alpha_{ij'}\langle 27_1 \oplus 27_2\rangle &= -(e_1 + e_2)[\delta^3_i \epsilon_{j'3'k'} v^{\alpha k'}] - (\nu_1 + \nu_2)[\delta^3_i \epsilon_{j'2'k'} v^{\alpha k'}],\\
Q_\alpha^{ij'}\langle 27_1 \oplus 27_2\rangle &= (e_1 + e_2)[\delta^{j'}_{3'} \epsilon^{i3k} v_{\alpha k}] + (\nu_1 + \nu_2)[\delta^{j'}_{2'} \epsilon^{i3k} v_{\alpha k}],
\end{aligned} \tag{B25}
$$

and, accordingly, for $\langle \overline{27}_1 \oplus \overline{27}_2\rangle$,

$$
\begin{aligned}
T^\alpha_\beta\langle \overline{27}_1 \oplus \overline{27}_2\rangle &= 0,\\
T^i_j\langle \overline{27}_1 \oplus \overline{27}_2\rangle &= (\bar{e}_1 + \bar{e}_2)[\delta^i_3 u^{3'}_j] + (\bar{\nu}_1 + \bar{\nu}_2)[\delta^i_3 u^{2'}_j],\\
T^{i'}_{j'}\langle \overline{27}_1 \oplus \overline{27}_2\rangle &= -(\bar{e}_1 + \bar{e}_2)[\delta^{3'}_{j'} u^{i'}_3] - (\bar{\nu}_1 + \bar{\nu}_2)[\delta^{2'}_{j'} u^{i'}_3],\\
Q^\alpha_{ij'}\langle \overline{27}_1 \oplus \overline{27}_2\rangle &= -(\bar{e}_1 + \bar{e}_2)[\delta^{3'}_{j'} \epsilon_{i3k} u^{\alpha k}] - (\bar{\nu}_1 + \bar{\nu}_2)[\delta^{2'}_{j'} \epsilon_{i3k} u^{\alpha k}],\\
Q_\alpha^{ij'}\langle \overline{27}_1 \oplus \overline{27}_2\rangle &= (\bar{e}_1 + \bar{e}_2)[\delta^i_3 \epsilon^{j'3'k'} u_{\alpha k'}] + (\bar{\nu}_1 + \bar{\nu}_2)[\delta^i_3 \epsilon^{j'2'k'} u_{\alpha k'}].
\end{aligned} \tag{B26}
$$

On the vacuum manifold in Eq. (48) one finds that the generators generally preserved by the VEVs of $78 \oplus 27_1 \oplus 27_2 \oplus \overline{27}_1 \oplus \overline{27}_2$ are

$$\begin{aligned} &T_c^{(1)}T_c^{(2)}T_c^{(3)}T_c^{(4)}T_c^{(5)}T_c^{(6)}T_c^{(7)}T_c^{(8)}\colon (8,1,0),\\ &T_L^{(1)}T_L^{(2)}T_L^{(3)}\colon (1,3,0),\\ &Y\colon (1,1,0),\\ &Q^\alpha_{11'}Q^\alpha_{21'}Q_\alpha^{11'}Q_\alpha^{21'}\colon (\bar{3},2,+\tfrac{5}{6}) \oplus (3,2,-\tfrac{5}{6}), \end{aligned} \tag{B27}$$

which generate an $SU(5)$ algebra. As an example showing the nontrivial constraints enforced by the vacuum manifold in Eq. (48), let us inspect the action of one of the leptoquark generators, say $Q^\alpha_{11'}$:

$$\begin{aligned} Q^\alpha_{11'}\langle 78\rangle &= -\frac{1}{\sqrt{6}}(a_3+\sqrt{3}a_4+b_3)Q^\alpha_{11'},\\ Q^\alpha_{11'}\langle 27_1 \oplus 27_2\rangle &= 0,\\ Q^\alpha_{11'}\langle \overline{27}_1 \oplus \overline{27}_2\rangle &= 0. \end{aligned} \tag{B28}$$

It is easy to check that $a_3+\sqrt{3}a_4+b_3$ vanishes on the whole vacuum manifold in Eq. (48) and, thus, $Q^\alpha_{11'}$ is preserved. Let us also remark that the $U(1)_Y$ charges above correspond to the standard $SO(10)$ embedding (see the discussion in Sec. IV B). In the flipped $SO(10)$ embedding, the $(\bar{3},2)\oplus(3,2)$ generators in Eq. (B27) carry hypercharges $\mp\frac{1}{6}$, respectively.

Considering instead the vacuum manifold invariant with respect to the flipped $SO(10)$ hypercharge [see Eqs. (41)–(43)], the preserved generators, in addition to those of the SM, are $Q^\alpha_{13'}Q^\alpha_{23'}Q_\alpha^{13'}Q_\alpha^{23'}$. These, for the standard hypercharge embedding of Eq. (32), transform as $(\bar{3},2,-\frac{1}{6})\oplus(3,2,+\frac{1}{6})$, whereas with the flipped hypercharge assignment in Eq. (33), the same transform as $(\bar{3},2,+\frac{5}{6})\oplus(3,2,-\frac{5}{6})$. Needless to say, one finds again $SU(5)$ as the vacuum little group.

It is interesting to consider the configuration $\alpha_1=\alpha_2=0$, which can be chosen without loss of generality once a pair, let us say $27_2\oplus\overline{27}_2$, is decoupled or when the two copies of $27_H\oplus\overline{27}_H$ are aligned. According to Eq. (48) this implies that all VEVs are equal to zero but $a_3=-b_3$ and e_1 (e_2). Then, from Eqs. (B24)–(B26), one verifies that the preserved generators are [see Eq. (B4) for notation]

$$\begin{aligned} &T_c^{(1)}T_c^{(2)}T_c^{(3)}T_c^{(4)}T_c^{(5)}T_c^{(6)}T_c^{(7)}T_c^{(8)}\colon (8,1,0),\\ &T_L^{(1)}T_L^{(2)}T_L^{(3)}\colon (1,3,0),\\ &T_R^{(1)}T_R^{(2)}T_R^{(3)}\colon (1,1,-1)\oplus(1,1,0)\oplus(1,1,+1),\\ &T_L^{(8)}+T_R^{(8)}\colon (1,1,0),\\ &Q^\alpha_{11'}Q^\alpha_{21'}Q_\alpha^{11'}Q_\alpha^{21'}\colon (\bar{3},2,+\tfrac{5}{6})\oplus(3,2,-\tfrac{5}{6}),\\ &Q^\alpha_{12'}Q^\alpha_{22'}Q_\alpha^{12'}Q_\alpha^{22'}\colon (\bar{3},2,-\tfrac{1}{6})\oplus(3,2,+\tfrac{1}{6}),\\ &Q^\alpha_{33'}Q_\alpha^{33'}\colon (\bar{3},1,-\tfrac{2}{3})\oplus(3,1,+\tfrac{2}{3}), \end{aligned} \tag{B29}$$

which support an $SO(10)$ algebra. In particular, $a_3=-b_3$ preserves $SO(10)\otimes U(1)$, where the extra $U(1)$ generator, which commutes with all $SO(10)$ generators, is proportional to $T_L^{(8)}-T_R^{(8)}$. On the other hand, the VEV e_1 breaks $T_L^{(8)}-T_R^{(8)}$ (while preserving the sum). We therefore recover the result of Ref. [12] for the E_6 setting with $78_H\oplus 27_H\oplus\overline{27}_H$.

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