Ratios of heavy hadron semileptonic decay rates

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Ratios of charmed meson and baryon semileptonic decay rates appear to be satisfactorily described by considering only the lowest-lying (S-wave) hadronic final states and assuming the kinematic factor describing phase space suppression is the same as that for free quarks. For example, the rate for D_s semileptonic decay is known to be $(17.0 \pm 5.3)\%$ lower than those for D^0 or D^+, and the model accounts for this difference. When applied to hadrons containing b quarks, this method implies that the B_s semileptonic decay rate is about 1% higher than that of the nonstrange B mesons. This small difference thus suggests surprisingly good local quark-hadron duality for B semileptonic decays, complementing the expectation based on inclusive quark-hadron duality that these differences in rates should not exceed a few tenths of a percent. For Λ_b semileptonic decay, however, the inclusive rate is predicted to be about 13% greater than that of the nonstrange B mesons. This value, representing a considerable departure from a calculation using a heavy-quark expansion, is close to the corresponding experimental ratio $\Gamma(\Lambda_b)/\bar{\Gamma}(B) = 1.13 \pm 0.03$ of total decay rates.

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I. INTRODUCTION

An early prediction for charmed meson decays [1], based on isospin symmetry, was the equality of Cabibbo-favored D^0 and D^+ semileptonic decay rates, borne out by experiment within errors [2]. On the other hand, the D_s semileptonic decay rate is now known to be about $(17.0 \pm 5.3)\%$ lower than the average of the D^0 and D^+ rates. This difference not only sheds light on strong-interaction dynamics, but can serve as a useful calibration when tagging D_s decays. The corresponding ratio of strange and nonstrange B meson semileptonic decay rates is much less well known, and there is only fragmentary information on Λ_b semileptonic decays.

In the present paper we briefly review what heavy-quark symmetry has to say about the ratios of semileptonic decay rates of various hadrons containing heavy (c and b) quarks (Sec. II). We then introduce an effective-quark method for comparing decays by means of the kinematic factor that characterizes $\mu^- \to e^- \bar{\nu}_e \nu_\mu$ when the electron mass is not neglected (Sec. III). This method is shown in Sec. IV to reproduce the relative suppression of the D_s semileptonic rate and the apparent enhancement of the Λ_c semileptonic rate (still not very precisely measured). When applied to B, B_s, and Λ_b decays (Sec. V), it leads to the prediction of relative *enhancements* of the B_s and Λ_b semileptonic rates by $\sim$1% and $\sim$13%, respectively, with respect to those of the nonstrange B mesons. Verification of these predictions would be surprisingly good evidence for local quark-hadron duality for mesons, complementing the expectation based on the operator-product expansion [3] that differences in semileptonic decay rates of mesons containing b quarks should not exceed a few tenths of a percent. For baryons the large deviation from unity is similar to that observed in total decay rates. Prospects for checking these predictions, and a summary, are contained in Sec. VI.

II. EXPECTATIONS FROM HEAVY-QUARK SYMMETRY

The relation between semileptonic decays of free quarks and those of hadrons is based on the notion of quark-hadron duality, whose origins and concepts are well described in the review of Ref. [3]. The corrections to a free-quark picture may be framed in terms of an operator-product expansion involving terms proportional to inverse powers of the mass m_Q of the decaying quark and to powers of the strong coupling constant α_S [4]. For an early discussion of the magnitude of such terms, see Ref. [5].

Corrections of $\mathcal{O}(1/m_Q)$ to the free-quark picture were proposed in Refs. [6,7]. The absence of such terms was shown in Ref. [3] to involve nontrivial cancellations. Terms of $\mathcal{O}(m_s\Lambda_{\rm QCD}/m_Q^2)$ can affect the semileptonic rates. One can scale the observed difference of $(17.0 \pm 5.3)\%$ between the strange and nonstrange D meson semileptonic decay rates by a factor of $(m_c/m_b)^2 \sim 0.1$ to estimate a difference of no more than a percent or two for strange and nonstrange B mesons.

Nonperturbative corrections which violate flavor symmetry have been estimated not to exceed half a percent for b semileptonic decays [3]. A more recent investigation [8] finds no evidence for violation of quark-hadron duality in inclusive $b \to c$ decays, and concludes that in the limit of $m_b \gg \Lambda_{\rm QCD}$ inclusive B decay rates are equal to the b

quark decay rates. A similar conclusion is reached by Kowalewski and Mannel in a mini-review of determinations of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements V_{cb} and V_{ub} in Ref. [2].

The above arguments apply to inclusive semileptonic decays. These decays populate the lowest-lying hadronic final states, consisting of the lowest S-wave $q\bar{q}$ or qqq levels, but such final states do not saturate the total semileptonic widths. The P-wave levels are excited to a degree, and there may also be contributions from nonresonant hadronic continuum. This leads to nontrivial form factors for excitation of the lowest levels.

The question then arises: To what degree are the conclusions of quark-hadron duality mirrored in the ratios of decay rates to the lowest-lying levels? In the present discussion we offer a simplified model predicting the ratios of semileptonic decay rates just on the basis of transitions to the lowest-lying levels. We are encouraged by the fact that this model reproduces the relative suppression of the D_s semileptonic decay rate and the (less-well-measured) apparent enhancement of the Λ_c semileptonic decay rate correctly. We find that such a model predicts effects of order 1% for mesons containing a b quark, in accord with the naive scaling arguments presented above. For baryons, however, we find roughly the same enhancement of semileptonic decay rate with respect to those of the nonstrange B mesons, roughly 13%, that is seen in total decay rates [2].

III. CALCULATIONS AT EFFECTIVE-QUARK LEVEL

Semileptonic decays of charm and beauty mesons involve final states consisting largely, though not exclusively, of the lowest-lying pseudoscalar and vector mesons. Cabibbo-favored charm decays and Cabibbo-Kobayashi-Maskawa (CKM)-favored beauty decays are noted in Table I, along with kinematic factors defined by

$$f(x) \equiv 1 - 8x + 8x^3 - x^4 + 12x^2 \ln(1/x), \qquad x \equiv (M_f/M_i)^2, \tag{1}$$

where M_i and M_f are masses of decay and daughter mesons. For decays related by isospin, we have quoted charge-averaged initial and final masses. The kinematic factor $f(x)$ was applied to semileptonic charm decays, for example, in Refs. [9,10].

We model the effects of initial- and final-state mass differences by assuming that decays are characterized by the kinematic factor $f(x)$. Strictly speaking, this factor applies to the decay of a fermion into three fermions, e.g., $\mu^- \to e^- \bar{\nu}_e \nu_\mu$, in which one of the final fermions (the e in this example) has nonzero mass. We then characterize the decay rate by assuming it is proportional to $2J_f + 1$, where $J_f = 0, 1$ is the spin of the final state, assumed to be either the ground-state pseudoscalar meson or the ground-state vector meson. These are the states listed in Table I. The effective kinematic factor $\bar{f}$ is then the spin-weighted average of that for a pseudoscalar final state (weight 1/4) and a vector final state (weight 3/4). Thus, for example,

$$\begin{aligned}\bar{f}(D) &= \frac{1}{4} f\left(\left[\frac{\bar{M}(K)}{\bar{M}(D)}\right]^2\right) + \frac{3}{4} f\left(\left[\frac{\bar{M}(K^*)}{\bar{M}(D)}\right]^2\right) \\ &= \frac{1}{4}(0.5971) + \frac{3}{4}(0.1887) = 0.2908. \end{aligned} \tag{2}$$

Here $\bar{M}$ refers to the charge-averaged masses quoted in Table I.

The lowest-lying baryons containing a heavy quark Q are of the form $\Lambda_Q = Q[ud]$, where the brackets denote a ud pair of spin zero and isospin zero. In the approximation we are making, the only final states considered when $Q \to Q'\ell\nu_\ell$ are those in which the final quarks are in an S-wave and the ud pair continues to have zero spin and isospin. Thus, we consider only $\Lambda_c \to \Lambda\ell^+\nu_\ell$ and $\Lambda_b \to \Lambda_c\ell^-\bar{\nu}_\ell$.

This model is an oversimplification when applied to the calculation of actual decay rates. For example, it neglects form factors for decays into the lowest-lying hadrons and important branching fractions to higher-lying hadronic final states, and does not accurately represent the vector-to-pseudoscalar ratio of semileptonic rates. Nevertheless, we may expect it to be useful for calculating *ratios* of semileptonic decay rates, as the kinematic differences between semileptonic decays of different hadrons containing the same heavy quark are mirrored to some extent in decays to higher-lying states. We expect the use of free-quark kinematics with quark masses replaced by hadron masses and the averaging over the spins of the final states (for meson decays) to mimic the effects of confinement. Our "cartoon" version of quark-hadron duality ultimately

TABLE I. Cabibbo-favored semileptonic decays of charmed hadrons and CKM-favored semileptonic decays of beauty hadrons to lowest-lying S-wave states. M_i and M_f are masses of initial and final hadronic states based on [2], $x \equiv (M_f/M_i)^2$, and $f(x)$ is defined in Eq. (1).

Decay	M_i (MeV/c^2)	M_f (MeV/c^2)	x	$f(x)$
$D \to \bar{K}\ell^+\nu_\ell$ [a]	1867.22	495.65	0.070 46	0.5971
$D \to \bar{K}^*\ell^+\nu_\ell$ [a]	1867.22	893.80	0.229 13	0.1887
$D_s^+ \to \eta\ell^+\nu_\ell$	1968.47	547.85	0.077 46	0.5682
$D_s^+ \to \eta'\ell^+\nu_\ell$	1968.47	957.78	0.236 74	0.1781
$D_s^+ \to \phi\ell^+\nu_\ell$	1968.47	1019.46	0.268 21	0.1395
$\Lambda_c^+ \to \Lambda\ell^+\nu_\ell$	2286.46	1115.68	0.238 10	0.1763
$\bar{B} \to D\ell^-\bar{\nu}_\ell$ [a]	5279.34	1867.22	0.125 09	0.4050
$\bar{B} \to D^*\ell^-\bar{\nu}_\ell$ [a]	5279.34	2008.61	0.144 75	0.3518
$\bar{B}_s \to D_s^+\ell^-\bar{\nu}_\ell$	5366.3	1968.47	0.134 56	0.3785
$\bar{B}_s \to D_s^{*+}\ell^-\bar{\nu}_\ell$	5366.3	2112.3	0.154 94	0.3268
$\Lambda_b \to \Lambda_c\ell^-\bar{\nu}_\ell$	5620.2	2286.46	0.165 51	0.3027

[a]Charge-averaged masses.

should be replaced by an approach based on the operator-product expansion and heavy-quark effective theory. We shall compare a few of our predictions with those of the latter approach.

IV. APPLICATION TO SEMILEPTONIC CHARM DECAYS

We first check the equality of semileptonic D^0 and D^+ decay rates as predicted for Cabibbo-favored decays. We assume (e, μ) universality and quote branching fractions for $D \to Xe^+\nu_e$, which are better known than the corresponding semimuonic values. With [2] $\mathcal{B}(D^0 \to Xe\nu) = (6.49 \pm 0.11)\%$ and $\tau(D^0) = (410.1 \pm 1.5)$ fs, we have $\Gamma(D^0 \to Xe\nu) = (1.583 \pm 0.027) \times 10^{11}\ \mathrm{s}^{-1}$, while with $\mathcal{B}(D^+ \to Xe^+\nu_e = (16.07 \pm 0.30)\%$ and $\tau(D^+) = (1040 \pm 7)$ fs, we have $\Gamma(D^+ \to Xe^+\nu_e) = (1.545 \pm 0.031) \times 10^{11}\ \mathrm{s}^{-1}$. The values for D^0 and D^+ are equal to better than 1σ. Averaging them we obtain $\bar{\Gamma}(D \to X\ell^+\nu_\ell) = (1.567 \pm 0.020) \times 10^{11}\ \mathrm{s}^{-1}$. We have ignored a possible difference between the small Cabibbo-suppressed semileptonic decay rates of D^0 and D^+.

The D_s semileptonic decay rate is significantly smaller than that of the nonstrange D mesons. With $\mathcal{B}(D_s^+ \to Xe^+\nu_e) = (6.5 \pm 0.4)\%$ and $\tau(D_s^+) = (500 \pm 7)$ fs, we have $\Gamma(D_s^+ \to Xe^+\nu_e) = (1.300 \pm 0.082) \times 10^{11}\ \mathrm{s}^{-1}$ [11], or

$$\frac{\Gamma(D_s^+ \to X\ell^+\nu_\ell)}{\bar{\Gamma}(D \to X\ell^+\nu_\ell)} = 0.830 \pm 0.053. \tag{3}$$

Thus, the semileptonic D_s decay rate is $(17.0 \pm 5.3)\%$ lower than the charge-averaged nonstrange D semileptonic rate. We now show that the model described in Sec. III reproduces this inequality. We will first study Cabibbo-favored decays and then calculate small corrections from Cabibbo-suppressed decays.

The semileptonic decays of D are assumed to be dominated by Cabibbo-favored $K\ell\nu$ and $K^*\ell\nu$ in the ratio of 1:3. For the D_s semileptonic decay we need to know the $s\bar{s}$ content of the η and η', the two mesons we are assuming dominate the pseudoscalar final state. We shall quote results for two extremes of octet-singlet mixing angles θ_η seen in the literature [12,13]. These are summarized in Table II, where

$$\begin{aligned} \eta &= c_n^{(\eta)}(u\bar{u} + d\bar{d}) + c_s^{(\eta)} s\bar{s}, \\ \eta' &= c_n^{(\eta')}(u\bar{u} + d\bar{d}) + c_s^{(\eta')} s\bar{s}. \end{aligned} \tag{4}$$

TABLE II. Octet-singlet mixing assumptions for η and η'.

θ_η	$c_n^{(\eta)}$	$c_s^{(\eta)}$	$c_n^{(\eta')}$	$c_s^{(\eta')}$
9.74°	$\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$
19.47°	$\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{6}}$	$\frac{2}{\sqrt{6}}$

The mixing angle $\theta_\eta = 9.74°$ was proposed by Isgur [12], while the quark composition corresponding to $\theta_\eta = 19.47°$ has been proposed by several authors [13] on phenomenological grounds.

With the mixing angle $\theta_\eta = 9.74°$, the η and η' each consist of half nonstrange and half strange quarks. The weighted average of f for D_s decays is then

$$\begin{aligned} \bar{f}(D_s) &= \tfrac{1}{4}[\tfrac{1}{2}(0.5682) + \tfrac{1}{2}(0.1781)] + \tfrac{3}{4}(0.1395) \\ &= 0.1979 \qquad (\theta_\eta = 9.74°). \end{aligned} \tag{5}$$

One then predicts

$$\begin{aligned} \frac{\Gamma(D_s \to X\ell\nu)}{\bar{\Gamma}(D \to X\ell\nu)} &= \left(\frac{M(D_s)}{\bar{M}(D)}\right)^5 \frac{\bar{f}(D_s)}{\bar{f}(D)} = (1.3022)(0.6805) \\ &= 0.886 \qquad (\theta_\eta = 9.74°). \end{aligned} \tag{6}$$

With the mixing angle $\theta_\eta = 19.47°$, the η is composed 1/3 of strange and 2/3 of nonstrange quarks, while the η' is 2/3 strange and 1/3 nonstrange. The corresponding values of $\bar{f}(D_s)$ and the semileptonic rate ratio are

$$\begin{aligned} \bar{f}(D_s) &= \tfrac{1}{4}[\tfrac{1}{3}(0.5682) + \tfrac{2}{3}(0.1781)] + \tfrac{3}{4}(0.1395) \\ &= 0.1817 \qquad (\theta_\eta = 19.47°), \end{aligned} \tag{7}$$

$$\begin{aligned} \frac{\Gamma(D_s \to X\ell\nu)}{\bar{\Gamma}(D \to X\ell\nu)} &= \left(\frac{M(D_s)}{\bar{M}(D)}\right)^5 \frac{\bar{f}(D_s)}{\bar{f}(D)} = (1.3022)(0.6246) \\ &= 0.813 \qquad (\theta_\eta = 19.47°). \end{aligned} \tag{8}$$

Both ratios are compatible with the experimental one.

The experimental semileptonic branching fractions of D_s are [2]

$$\begin{aligned} \mathcal{B}(\eta\ell\nu_\ell) &= (2.67 \pm 0.29)\%, \\ \mathcal{B}(\eta'\ell\nu_\ell) &= (0.99 \pm 0.23)\%, \\ \mathcal{B}(\phi\ell\nu_\ell) &= (2.49 \pm 0.14)\%, \end{aligned} \tag{9}$$

The η and η' branching fractions are compared with each other, with the ϕ branching fraction, and with predictions of the model, in Table III. The pseudoscalar-to-vector ratio is underestimated; nonetheless, the η'/η ratio is compatible with a value of θ_η toward the lower end of the range considered.

The equality between semileptonic widths of D^0 and D^+ may be violated in $\Delta I = 1/2$ Cabibbo-suppressed decays. We now study corrections from these decays to the total D^0, D^+ and D_s^+ semileptonic widths. Decay modes into the lowest-lying pseudoscalar and vector mesons are listed in Table IV for the three charmed mesons. The weighted averages of the function f for Cabibbo-suppressed decays of the three mesons are

$$\bar{f}_{\rm CS}(D^0) = \frac{1}{4}f(D^0 \to \pi^-) + \frac{3}{4}f(D^0 \to \rho^-) = 0.4546,$$

$$\bar{f}_{\rm CS}(D^+) = \begin{cases} \frac{1}{4}\left[\frac{1}{2}f(D^+ \to \pi^0) + \frac{1}{4}f(D^+ \to \eta) + \frac{1}{4}f(D^+ \to \eta')\right] \\ + \frac{3}{4}\left[\frac{1}{2}f(D^+ \to \rho^0) + \frac{1}{2}f(D^+ \to \omega)\right] = 0.3766 \qquad (\theta_\eta = 9.74°), \\ \frac{1}{4}\left[\frac{1}{2}f(D^+ \to \pi^0) + \frac{1}{3}f(D^+ \to \eta) + \frac{1}{6}f(D^+ \to \eta')\right] \\ + \frac{3}{4}\left[\frac{1}{2}f(D^+ \to \rho^0) + \frac{1}{2}f(D^+ \to \omega)\right] = 0.3847 \qquad (\theta_\eta = 19.47°), \end{cases} \tag{10}$$

$$\bar{f}_{\rm CS}(D_s^+) = \frac{1}{4}f(D_s^+ \to K^0) + \frac{3}{4}f(D_s^+ \to K^{*0}) = 0.3234.$$

In order to calculate the difference between the total D^0 and D^+ semileptonic widths, we include small differences between charged and neutral meson masses, which affect also rates for Cabibbo-favored semileptonic decays. Thus, instead of the charge-averaged value of $\bar{f}(D)$ in Eq. (2), we now use two slightly different kinematic factors for D^0 and D^+:

$$\begin{aligned} \bar{f}(D^0) &= \tfrac{1}{4}f(D^0 \to K^-) + \tfrac{3}{4}f(D^0 \to K^{*-}) \\ &= \tfrac{1}{4}(0.5987) + \tfrac{3}{4}(0.1895) = 0.2918, \\ \bar{f}(D^+) &= \tfrac{1}{4}f(D^+ \to \bar{K}^0) + \tfrac{3}{4}f(D^+ \to \bar{K}^{*0}) \\ &= \tfrac{1}{4}(0.5955) + \tfrac{3}{4}(0.1880) = 0.2899. \end{aligned} \tag{11}$$

TABLE III. Ratios R of D_s semileptonic branching fractions to $\mathcal{B}(D_s \to \phi\ell\nu_\ell)$.

	$R(\eta\ell\nu_\ell)$	$R(\eta'\ell\nu_\ell)$	$R(\eta')/R(\eta)$
$\theta_\eta = 19.47°$	0.453	0.284	0.627
$\theta_\eta = 9.74°$	0.679	0.213	0.313
Experiment	1.07 ± 0.13	0.40 ± 0.10	0.37 ± 0.09

TABLE IV. Cabibbo-suppressed semileptonic decays of charmed mesons to lowest-lying pseudoscalar and vector states. Notations are as in Table I.

Decay	M_i (MeV/c^2)	M_f (MeV/c^2)	x	$f(x)$
$D^0 \to \pi^-\ell^+\nu_\ell$	1864.83	139.57	0.005 60	0.9571
$D^0 \to \rho^-\ell^+\nu_\ell$	1864.83	775.11	0.172 76	0.2871
$D^+ \to \pi^0\ell^+\nu_\ell$	1869.60	134.98	0.005 21	0.9600
$D^+ \to \eta\ell^+\nu_\ell$	1869.60	547.85	0.085 87	0.5353
$D^+ \to \eta'\ell^+\nu_\ell$	1869.60	957.78	0.262 44	0.1460
$D^+ \to \rho^0\ell^+\nu_\ell$	1869.60	775.49	0.172 05	0.2886
$D^+ \to \omega\ell^+\nu_\ell$	1869.60	782.65	0.175 24	0.2820
$D_s^+ \to K^0\ell^+\nu_\ell$	1968.47	497.61	0.063 90	0.6256
$D_s^+ \to K^{*0}\ell^+\nu_\ell$	1968.47	895.94	0.207 16	0.2227

Using the values in Eqs. (10) and (11) we calculate

$$\frac{\Gamma(D^0 \to X\ell\nu)}{\Gamma(D^+ \to X\ell\nu)} = \left[\frac{M(D^0)}{M(D^+)}\right]^5 \frac{\bar{f}(D^0) + \tan^2\theta_c \bar{f}_{\rm CS}(D^0)}{\bar{f}(D^+) + \tan^2\theta_c \bar{f}_{\rm CS}(D^+)} = \begin{cases} 1.007 & (\theta_\eta = 9.74°) \\ 1.005 & (\theta_\eta = 19.47°), \end{cases} \tag{12}$$

$$\frac{\Gamma(D_s^+ \to X\ell\nu)}{\Gamma(D^0 \to X\ell\nu)} = \left[\frac{M(D_s^+)}{M(D^0)}\right]^5 \frac{\bar{f}(D_s^+) + \tan^2\theta_c \bar{f}_{\rm CS}(D_s^+)}{\bar{f}(D^0) + \tan^2\theta_c \bar{f}_{\rm CS}(D^0)} = \begin{cases} 0.892 & (\theta_\eta = 9.74°) \\ 0.825 & (\theta_\eta = 19.47°), \end{cases} \tag{13}$$

where [2] $\tan\theta_c = |V_{us}/V_{ud}| = 0.231$. Thus, in our model the D^0 and D^+ total semileptonic widths are predicted to be equal within less than 1% independent of the $\eta - \eta'$ mixing angle. The observed ratio [see the discussion before Eq. (3)] is

$$\frac{\Gamma(D^0 \to X\ell\nu)}{\Gamma(D^+ \to X\ell\nu)} = \frac{1.583 \pm 0.027}{1.545 \pm 0.031} = 1.025 \pm 0.027, \tag{14}$$

so the errors on branching fractions need to be improved considerably before the predicted deviation of this ratio from 1 can be identified. The predicted ratio of D_s and D total semileptonic widths, which depends somewhat on the mixing angle, is consistent with the experimental value (3) within about 1σ. An alternative interpretation of the semileptonic width difference observed for D_s and D mesons in terms of a weak annihilation amplitude contributing to D_s decays [14–16] has been shown to be disfavored by the measured lepton energy spectrum in these decays [17].

A similar discussion may be applied to Λ_c semileptonic decays. Here we will neglect Cabibbo-suppressed decays in view of the large experimental uncertainty in the observed semileptonic decay rate. Only one early value has been published [18]: $\mathcal{B}(\Lambda_c \to e^+X) = (4.5 \pm 1.7)\%$ (see also Ref. [19]), leading when combined with the Λ_c lifetime [2] of (200 ± 6) fs to

$$\Gamma(\Lambda_c \to e^+X) = (2.25 \pm 0.85) \times 10^{11}\ \mathrm{s}^{-1}. \tag{15}$$

Comparing this with the charge-averaged nonstrange D semileptonic decay rate $\bar{\Gamma}(D \to e^+X) = (1.567 \pm 0.020) \times 10^{11}\ \mathrm{s}^{-1}$, we have

$$\frac{\Gamma(\Lambda_c \to e^+X)}{\bar{\Gamma}(D \to e^+X)} = 1.44 \pm 0.54, \qquad (16)$$

poorly determined but with a central value considerably above 1. The model predicts this ratio to be

$$\frac{\Gamma(\Lambda_c \to e^+X)}{\bar{\Gamma}(D \to e^+X)} = \left[\frac{M(\Lambda_c)}{\bar{M}(D)}\right]^5 \left[\frac{f(\Lambda_c)}{\bar{f}(D)}\right] = (2.753)(0.606) = 1.67. \qquad (17)$$

This prediction should be compared with an estimate of about 1.2 based on a heavy-quark expansion including $1/m_c^2$ terms [20]. Reducing the experimental error in $\mathcal{B}(\Lambda_c \to e^+X)$ by a factor of 3 would be a useful first step in testing these predictions.

V. APPLICATION TO BEAUTY DECAYS

The experimental semileptonic branching fractions for beauty decays and the corresponding decay rates [2] are summarized in Table V. For B decays we quote the $X\ell\nu$ branching fractions based on assuming e-μ universality, while for B_s decays we quote the branching fraction to $D_s^-\ell^+\nu_\ell X$, which is the only one available in Ref. [2]. The semileptonic nonstrange B decay rates are consistent with one another within better than 1σ, while the 30% error on the B_s semileptonic decay rate prevents one from making any crisp statement about its ratio to the nonstrange rates. To predict this ratio, we proceed as we did for charm decays, evaluating the weighted averages of the function f for CKM-favored B and B_s decays:

$$\bar{f}(B) = \tfrac{1}{4}f(B \to D) + \tfrac{3}{4}f(B \to D^*) = \tfrac{1}{4}(0.4050) + \tfrac{3}{4}(0.3518) = 0.3651, \qquad (18)$$

$$\bar{f}(B_s) = \tfrac{1}{4}f(B_s \to D_s) + \tfrac{3}{4}f(B_s \to D_s^*) = \tfrac{1}{4}(0.3785) + \tfrac{3}{4}(0.3268) = 0.3398. \qquad (19)$$

TABLE V. Semileptonic branching fractions, total lifetimes, and semileptonic decay rates of B and B_s mesons.

Meson	$\mathcal{B}_{SL}$ (%)	Lifetime τ (fs)	Γ_{SL} (units of 10^{10} s)
B^0	10.33 ± 0.28	1525 ± 9	6.77 ± 0.19
B^+	10.99 ± 0.28	1638 ± 11	6.71 ± 0.18
B_s	7.9 ± 2.4	1472^{+24}_{-26}	5.4 ± 1.6

We then predict

$$\frac{\Gamma(B_s \to X\ell^+\nu_\ell)}{\bar{\Gamma}(B \to X\ell\nu_\ell)} = \left(\frac{M(B_s)}{\bar{M}(B)}\right)^5 \frac{\bar{f}(B_s)}{\bar{f}(B)} = (1.0851)(0.9306) = 1.010. \qquad (20)$$

We are thus led to expect an *enhancement* by 1% of the ratio of the strange to nonstrange B semileptonic decay rates. It will be interesting to see if this prediction can be tested in forthcoming experiments at lepton or hadron colliders.

Corrections from CKM-suppressed decays to the ratio (20) and to the equality of B^0 and B^+ semileptonic widths are expected to be very small as they are proportional to $(|V_{ub}|/|V_{cb}|)^2 \simeq 0.01$. The two ratios of total inclusive widths are given by

$$\frac{\Gamma(B^0 \to X\ell\nu)}{\Gamma(B^+ \to X\ell\nu)} = \left[\frac{M(B^0)}{M(B^+)}\right]^5 \frac{\bar{f}(B^0) + |V_{ub}/V_{cb}|^2\bar{f}_{\mathrm{CKMS}}(B^0)}{\bar{f}(B^+) + |V_{ub}/V_{cb}|^2\bar{f}_{\mathrm{CKMS}}(B^+)}, \qquad (21)$$

$$\frac{\Gamma(B_s \to X\ell\nu)}{\Gamma(B^0 \to X\ell\nu)} = \left[\frac{M(B_s)}{M(B^0)}\right]^5 \frac{\bar{f}(B_s) + |V_{ub}/V_{cb}|^2\bar{f}_{\mathrm{CKMS}}(B_s)}{\bar{f}(B^0) + |V_{ub}/V_{cb}|^2\bar{f}_{\mathrm{CKMS}}(B^0)}. \qquad (22)$$

The weighted averages of the functions f for CKM-suppressed decays are denoted $\bar{f}_{\mathrm{CKMS}}$. For completeness, we calculate these functions using decay modes listed in Table VI:

$$\bar{f}_{\mathrm{CKMS}}(B^0) = \frac{1}{4}f(B^0 \to \pi^-) + \frac{3}{4}f(B^0 \to \rho^-) = 0.8854,$$

$$\bar{f}_{\mathrm{CKMS}}(B^+) = \begin{cases} \frac{1}{4}\left[\frac{1}{2}f(B^+ \to \pi^0) + \frac{1}{4}f(B^+ \to \eta) + \frac{1}{4}f(B^+ \to \eta')\right] \\ \quad + \frac{3}{4}\left[\frac{1}{2}f(B^+ \to \rho^0) + \frac{1}{2}f(B^+ \to \omega)\right] = 0.8664 \qquad (\theta_\eta = 9.74^\circ), \\ \frac{1}{4}\left[\frac{1}{2}f(B^+ \to \pi^0) + \frac{1}{3}f(B^+ \to \eta) + \frac{1}{6}f(B^+ \to \eta')\right] \\ \quad + \frac{3}{4}\left[\frac{1}{2}f(B^+ \to \rho^0) + \frac{1}{2}f(B^+ \to \omega)\right] = 0.8693 \qquad (\theta_\eta = 19.47^\circ), \end{cases} \qquad (23)$$

$$\bar{f}_{\mathrm{CKMS}}(B_s) = \frac{1}{4}f(B_s \to K^-) + \frac{3}{4}f(B_s \to K^{*-}) = 0.8432.$$

TABLE VI. CKM-suppressed semileptonic decays of beauty mesons to lowest-lying pseudoscalar and vector states. Notations are as in Table I.

Decay	M_i (MeV/c^2)	M_f (MeV/c^2)	x	$f(x)$
$B^0 \to \pi^- \ell^+ \nu_\ell$	5279.50	139.57	0.000 699	0.9945
$B^0 \to \rho^- \ell^+ \nu_\ell$	5279.50	775.11	0.021 555	0.8490
$B^+ \to \pi^0 \ell^+ \nu_\ell$	5279.17	134.98	0.000 654	0.9948
$B^+ \to \eta \ell^+ \nu_\ell$	5279.17	547.85	0.010 769	0.9202
$B^+ \to \eta' \ell^+ \nu_\ell$	5279.17	957.78	0.032 915	0.7813
$B^+ \to \rho^0 \ell^+ \nu_\ell$	5279.17	775.49	0.021 579	0.8489
$B^+ \to \omega \ell^+ \nu_\ell$	5279.17	782.65	0.021 979	0.8464
$B_s \to K^- \ell^+ \nu_\ell$	5366.3	493.68	0.008 463	0.9364
$B_s \to K^{*-} \ell^+ \nu_\ell$	5366.3	891.66	0.027 609	0.8121

The approximate equality of the three values of $\bar{f}_{\rm CKMS}$ multiplying $|V_{ub}/V_{cb}|^2$ implies their negligible effect on the above two ratios of widths. This applies also to the factor $[M(B^0)/M(B^+)]^5 = 1.0003$ entering the first ratio. The largest correction to this ratio, a few parts in a thousand, comes from $\bar{f}(B^0)/\bar{f}(B^+)$. Using masses for charged and neutral B, D, and D^* mesons [2], we find

$$\frac{\Gamma(B^0 \to X\ell\nu)}{\Gamma(B^+ \to X\ell\nu)} \approx \frac{\bar{f}(B^0)}{\bar{f}(B^+)} = \frac{0.3644}{0.3657} = 0.996, \tag{24}$$

$$\frac{\Gamma(B_s \to X\ell\nu)}{\Gamma(B^0 \to X\ell\nu)} \approx \left[\frac{M(B_s)}{M(B^0)}\right]^5 \frac{\bar{f}(B_s)}{\bar{f}(B^0)} = 1.012. \tag{25}$$

We now turn to the semileptonic decay rate of Λ_b. No inclusive semileptonic decay branching fraction is quoted in Ref. [2]. A similar method to the one discussed for Λ_c lead to the prediction

$$\frac{\Gamma(\Lambda_b \to X\ell^-\bar{\nu}_\ell)}{\bar{\Gamma}(B \to X\ell^+\nu_\ell)} = \left[\frac{M(\Lambda_b)}{\bar{M}(B)}\right]^5 \frac{f(\Lambda_b)}{\bar{f}(B)}$$
$$= (1.367)(0.829) = 1.134, \tag{26}$$

where $\bar{\Gamma}$ is the average of charged and neutral B decay rates. This represents a considerable departure from the expectation of an approach using operator product and heavy-quark expansions which predicts this ratio to be 1.03 [20]. (See also Ref. [21] where this ratio is calculated to be around 1.05.) A departure from unity similar to (26) is seen when comparing total decay rates as quoted in Ref. [2],

$$\frac{\Gamma(\Lambda_b)}{\bar{\Gamma}(B)} = \frac{\bar{\tau}(B)}{\tau(\Lambda_b)} = \frac{(1.570 \pm 0.007)\ \text{ps}}{(1.391^{+0.038}_{-0.037})\ \text{ps}} = 1.129 \pm 0.031. \tag{27}$$

A somewhat smaller value [22], $\Gamma(\Lambda_b)/\bar{\Gamma}(B) = 1.024 \pm 0.032$, was reported while we were completing the writeup of this paper.

VI. SUMMARY

We have presented a simplified model for estimating ratios of semileptonic decay rates of hadrons containing charm and bottom quarks. The model uses kinematic factors appropriate to free-fermion decays, but endows the initial and final fermions with the physical masses of ground-state hadrons. This approach may be thought of as a cartoon version of local quark-hadron duality. It appears to reproduce known ratios of rates for charm decays, including the suppression of the D_s semileptonic rate by $(17.0 \pm 5.3)\%$ relative to those of the nonstrange D^0 and D^+ (equal within errors). For hadrons containing b quarks, it predicts an enhancement of about 1.2% for $\Gamma(B_s \to X\ell\nu)$ and about 13% for $\Gamma(\Lambda_b \to X\ell\nu)$ relative to $\bar{\Gamma}(B \to X\ell\nu)$. The latter result represents a significant deviation from expectations of the operator product and heavy-quark expansion, and is similar to the departure from unity exhibited by the ratio of total decay rates.

The prospects for testing differences in semileptonic decay rates for B and B_s at the 1%–2% level are challenging. The best chance we see would involve the use of tagged B_s decays, such as obtained by the Belle Collaboration in a large sample of B_s–$\bar{B}_s$ pairs. It may be more feasible to study the considerably larger deviation from unity predicted for the ratio of baryon to meson semileptonic decay rates. Given the large sample of charmed baryons produced at B factories and foreseen at LHCb, it would also be helpful to perform an improved measurement of $\mathcal{B}(\Lambda_c \to e^+X)$, to check our prediction that the corresponding inclusive semileptonic decay rate is 1.67 times the average for nonstrange D mesons. We look forward to such tests and to a first measurement of $\mathcal{B}(\Lambda_b \to e^-X)$.

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