

**Vector and axial-vector charmoniumlike states**

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After constructing all the tetraquark interpolating currents with  $J^{PC} = 1^{--}, 1^{+-}, 1^{++}$  and  $1^{+-}$  in a systematic way, we investigate the two-point correlation functions to extract the masses of the charmoniumlike states with QCD sum rule. For the  $1^{--} qc\bar{q}\bar{c}$  charmoniumlike state,  $m_X = 4.6 \sim 4.7$  GeV, which implies a possible tetraquark interpretation for the state  $Y(4660)$ . The masses for both the  $1^{++} qc\bar{q}\bar{c}$  and  $sc\bar{s}\bar{c}$  charmoniumlike states are around  $4.0 \sim 4.2$  GeV, which are slightly above the mass of  $X(3872)$ . For the  $1^{+-}$  and  $1^{+-} qc\bar{q}\bar{c}$  charmoniumlike states, the extracted masses are around  $4.5 \sim 4.7$  GeV and  $4.0 \sim 4.2$  GeV, respectively. As a by-product, the bottomoniumlike states are also studied. We also discuss the possible decay modes and experimental search of the charmoniumlike states.

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**I. INTRODUCTION**

In the past several years, many unexpected charmoniumlike states have been discovered at  $B$  factories, some of which lie above the open charm threshold and decay into final states that contain a  $c\bar{c}$  pair. Some of them do not fit in the conventional quark model easily and are considered as the candidates of the exotic states beyond the quark model, such as the molecular states, tetraquark states, the charmonium hybrid mesons, baryonium states and so on. For experimental reviews of these new states, one can consult Refs. [1–5].

The underlying structure of these new states inspired the extensive study of the hadron spectroscopy.  $X(3872)$  is the best studied charmoniumlike state since its discovery by the Belle Collaboration [6]. Although the analysis of angular distributions favors the assignment  $J^{PC} = 1^{++}$  [7,8], the  $2^{-+}$  possibility is not ruled out [9]. The mass and decay mode of  $X(3872)$  are very different from that of the  $2^3P_1 c\bar{c}$  state. Up to now, the possible interpretations of  $X(3872)$  include the molecular state [10–16], tetraquark state [17–19], cusp [20] and hybrid charmonium [21]. In Ref. [17], the authors studied the  $J^{PC} = 1^{++}$  state using a tetraquark current in the framework of the QCD sum rule approach.

The initial state radiation (ISR) process played an important role in the search of the  $1^{--}$  charmoniumlike states at  $B$  factories. BABAR Collaboration first observed  $Y(4260)$  in the  $e^+e^- \rightarrow \gamma_{\text{ISR}} J/\psi \pi^+ \pi^-$  process [22], which was confirmed by Belle Collaboration [23]. Then, BABAR studied the  $\gamma_{\text{ISR}} \psi(2S) \pi^+ \pi^-$  channel and observed a broad enhancement around 4.32 GeV. Using the same technique,

Belle observed two distinct resonances  $Y(4360)$  and  $Y(4660)$  in the  $\psi(2S) \pi^+ \pi^-$  mass distribution [24]. The masses of these new charmoniumlike states are higher than the open charm threshold. In the potential model, the  $D^{(*)} \bar{D}^{(*)}$  channels are predicted to be the dominant decay modes of the charmonium above the open charm threshold. However, they have not been observed yet [25] for the  $Y$  mesons. In Refs. [26,27], the authors studied the  $1^{--}$  charmoniumlike  $Y$  mesons using the QCD sum rule approach. Maiani *et al.* tried to assign  $Y(4260)$  as the  $sc\bar{s}\bar{c}$  tetraquark in a  $P$ -wave state [28].  $Y(4260)$  was also interpreted as the interesting charmonium hybrid state [29–31].  $Y(4660)$  was considered as a  $\psi(2S) f_0(980)$  bound state [32].

The exotic state with  $J^{PC} = 1^{-+}$  cannot be a  $q\bar{q}$  state in the simple quark model. Neither can the  $1^{-+}$  state be formed by two gluons due to the Landau-Yang selection rule [33,34]. States with such a quantum number are good candidates of the hybrid meson, which has been studied with the MIT Bag model [35,36], the flux tube model [37,38] and the QCD sum rule formalism [39,40]. Recently there have been some efforts on the  $1^{-+}$  charmoniumlike exotic states. For example, the structure of  $X(4350)$  was studied using a  $D_s^* D_{s0}^*$  current with  $J^{PC} = 1^{-+}$  [41]. Moreover, the newly observed state  $Y(4140)$  was argued as a  $1^{-+}$  exotic charmonium hybrid state [42].

In Ref. [43], Nielsen *et al.* reviewed the charmoniumlike states from the perspective of the QCD sum rule approach. They studied new resonances such as  $X(3872)$ ,  $Z^+(4430)$  and  $Y(4660)$  etc. using one single molecular-type or tetraquark-type interpolating current. In this paper, we first construct all the local charmoniumlike tetraquark currents with  $J^{PC} = 1^{--}, 1^{+-}, 1^{++}$  and  $1^{+-}$  in a systematic way. With these independent currents, we study the two-point correlation functions and extract the masses of the possible

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$1^{-+}, 1^{--}, 1^{++}, 1^{+-}$  states. We study both the  $qQ\bar{q}\bar{Q}$  and  $sQ\bar{s}\bar{Q}$  systems where  $Q = c, b$ .

The paper is organized as follows. In Sec. II, we construct the tetraquark interpolating currents with  $J^{\text{PC}} = 1^{-+}, 1^{--}, 1^{++}$  and  $1^{+-}$  using the diquark and antidiquark fields. In Sec. III, we calculate the correlation functions and spectral densities of the interpolating currents and collect them in the Appendix. We perform the numerical analysis and extract the masses in Sec. IV and discuss the possible decay modes and experimental search of the charmoniumlike states in Sec. V. The last section is a brief summary.

## II. TETRAQUARK INTERPOLATING CURRENTS

In this section, we construct the diquark-antidiquark type of currents using the technique developed in our previous works [44–46]. Considering the Lorentz structures, there are five independent diquark fields (antidiquark fields):  $q_a^T C \gamma_5 c_b$ ,  $q_a^T C c_b$ ,  $q_a^T C \gamma_\mu \gamma_5 c_b$ ,  $q_a^T C \gamma_\mu c_b$  and  $q_a^T C \sigma_{\mu\nu} c_b$  ( $\bar{q}_a \gamma_5 C \bar{c}_b^T$ ,  $\bar{q}_a C \bar{c}_b^T$ ,  $\bar{q}_a \gamma_\mu \gamma_5 C \bar{c}_b^T$ ,  $\bar{q}_a \gamma_\mu C \bar{c}_b^T$  and  $\bar{q}_a \sigma_{\mu\nu} C \bar{c}_b^T$ ), where  $a, b$  are color indices and  $q$  denotes an *up* or *down* quark. We will also take account of  $q_a^T C \sigma_{\mu\nu} \gamma_5 c_b$  although it is equivalent to  $q_a^T C \sigma_{\mu\nu} c_b$ . In fact, they have different parities. Using these diquarks and antidiquarks as the basis, we can compose a six-order

matrix  $O$ . The elements of  $O$  are the tetraquark operators without color structures. We show the spins and parities of the matrix elements except for the tensor operators in Table I.

Under the charge-conjugation transformation

$$\mathbb{C} O \mathbb{C}^{-1} = O^T, \quad (1)$$

where  $O^T$  is the transpose of  $O$ . With this relation, we can define the symmetric matrix  $S$  and antisymmetric matrix  $A$ :

$$S = O + O^T, \quad A = O - O^T. \quad (2)$$

The elements of these two matrices are the tetraquark operators that have definite  $C$  parity: they have even and odd  $C$  parities for the elements of  $S$  and  $A$ , respectively.  $A_{ii} = 0$  indicates that the  $J^{\text{PC}} = 0^{+-}$  tetraquark currents without derivatives do not exist, which has been proven in Ref. [45]. We do not use  $O_{66}$  in the construction of the currents since it is equal to  $O_{55}$ . One also notes that  $O_{56}$  is equal to  $O_{65}$ , which results in  $A_{56} = 0$ .

The diquark and antidiquark should have the same color symmetries to compose a color singlet tetraquark current. So the color structure of the tetraquark is either  $\mathbf{6} \otimes \bar{\mathbf{6}}$  or  $\mathbf{3} \otimes \mathbf{3}$ , which is denoted by  $\mathbf{6}$  and  $\mathbf{3}$ , respectively. Finally, we can obtain the tetraquark interpolating currents with  $J^{\text{PC}} = 1^{-+}, 1^{--}, 1^{++}$  and  $1^{+-}$  from the matrices  $S$  and  $A$ :

(i) The interpolating currents with  $J^{\text{PC}} = 1^{-+}$  and  $1^{--}$  are

$$\begin{aligned} J_{1\mu} &= S_{13}^6(A_{13}^6) = q_a^T C \gamma_5 c_b (\bar{q}_a \gamma_\mu \gamma_5 C \bar{c}_b^T + \bar{q}_b \gamma_\mu \gamma_5 C \bar{c}_a^T) \pm q_a^T C \gamma_\mu \gamma_5 c_b (\bar{q}_a \gamma_5 C \bar{c}_b^T + \bar{q}_b \gamma_5 C \bar{c}_a^T), \\ J_{2\mu} &= S_{45}^3(A_{45}^3) = q_a^T C \gamma^\nu c_b (\bar{q}_a \sigma_{\mu\nu} C \bar{c}_b^T - \bar{q}_b \sigma_{\mu\nu} C \bar{c}_a^T) \pm q_a^T C \sigma_{\mu\nu} c_b (\bar{q}_a \gamma^\nu C \bar{c}_b^T - \bar{q}_b \gamma^\nu C \bar{c}_a^T), \\ J_{3\mu} &= S_{13}^3(A_{13}^3) = q_a^T C \gamma_5 c_b (\bar{q}_a \gamma_\mu \gamma_5 C \bar{c}_b^T - \bar{q}_b \gamma_\mu \gamma_5 C \bar{c}_a^T) \pm q_a^T C \gamma_\mu \gamma_5 c_b (\bar{q}_a \gamma_5 C \bar{c}_b^T - \bar{q}_b \gamma_5 C \bar{c}_a^T), \\ J_{4\mu} &= S_{45}^6(A_{45}^6) = q_a^T C \gamma^\nu c_b (\bar{q}_a \sigma_{\mu\nu} C \bar{c}_b^T + \bar{q}_b \sigma_{\mu\nu} C \bar{c}_a^T) \pm q_a^T C \sigma_{\mu\nu} c_b (\bar{q}_a \gamma^\nu C \bar{c}_b^T + \bar{q}_b \gamma^\nu C \bar{c}_a^T), \\ J_{5\mu} &= S_{24}^6(A_{24}^6) = q_a^T C c_b (\bar{q}_a \gamma_\mu C \bar{c}_b^T + \bar{q}_b \gamma_\mu C \bar{c}_a^T) \pm q_a^T C \gamma_\mu c_b (\bar{q}_a C \bar{c}_b^T + \bar{q}_b C \bar{c}_a^T), \\ J_{6\mu} &= S_{36}^6(A_{36}^6) = q_a^T C \gamma^\nu \gamma_5 c_b (\bar{q}_a \sigma_{\mu\nu} \gamma_5 C \bar{c}_b^T + \bar{q}_b \sigma_{\mu\nu} \gamma_5 C \bar{c}_a^T) \pm q_a^T C \sigma_{\mu\nu} \gamma_5 c_b (\bar{q}_a \gamma^\nu \gamma_5 C \bar{c}_b^T + \bar{q}_b \gamma^\nu \gamma_5 C \bar{c}_a^T), \\ J_{7\mu} &= S_{24}^3(A_{24}^3) = q_a^T C c_b (\bar{q}_a \gamma_\mu C \bar{c}_b^T - \bar{q}_b \gamma_\mu C \bar{c}_a^T) \pm q_a^T C \gamma_\mu c_b (\bar{q}_a C \bar{c}_b^T - \bar{q}_b C \bar{c}_a^T), \\ J_{8\mu} &= S_{36}^3(A_{36}^3) = q_a^T C \gamma^\nu \gamma_5 c_b (\bar{q}_a \sigma_{\mu\nu} \gamma_5 C \bar{c}_b^T - \bar{q}_b \sigma_{\mu\nu} \gamma_5 C \bar{c}_a^T) \pm q_a^T C \sigma_{\mu\nu} \gamma_5 c_b (\bar{q}_a \gamma^\nu \gamma_5 C \bar{c}_b^T - \bar{q}_b \gamma^\nu \gamma_5 C \bar{c}_a^T), \end{aligned} \quad (3)$$

where “ $S$ ” and “ $+$ ” correspond to  $J^{\text{PC}} = 1^{-+}$ , “ $A$ ” and “ $-$ ” correspond to  $J^{\text{PC}} = 1^{--}$ .

TABLE I. The spins and parities of the elements of the matrix  $O$ .

Operators	$J^P$	$\bar{q}_a \gamma_5 C \bar{c}_b^T$	$\bar{q}_a C \bar{c}_b^T$	$\bar{q}_a \gamma_\mu \gamma_5 C \bar{c}_b^T$	$\bar{q}_a \gamma_\mu C \bar{c}_b^T$	$\bar{q}_a \sigma_{\mu\nu} C \bar{c}_b^T$	$\bar{q}_a \sigma_{\mu\nu} \gamma_5 C \bar{c}_b^T$
		$0^+$	$0^-$	$1^-$	$1^+$	$1^-$	$1^+$
$q_a^T C \gamma_5 c_b$	$0^+$	$0^+$	$0^-$	$1^-$	$1^+$	—	—
$q_a^T C c_b$	$0^-$	$0^-$	$0^+$	$1^+$	$1^-$	—	—
$q_a^T C \gamma_\mu \gamma_5 c_b$	$1^-$	$1^-$	$1^+$	$0^+$	$0^-$	$1^+$	$1^-$
$q_a^T C \gamma_\mu c_b$	$1^+$	$1^+$	$1^-$	$0^-$	$0^+$	$1^-$	$1^+$
$q_a^T C \sigma_{\mu\nu} c_b$	$1^-$	—	—	$1^+$	$1^-$	$0^+$	$0^-$
$q_a^T C \sigma_{\mu\nu} \gamma_5 c_b$	$1^+$	—	—	$1^-$	$1^+$	$0^-$	$0^+$

(ii) The interpolating currents with  $J^{\text{PC}} = 1^{++}$  and  $1^{+-}$  are

$$\begin{aligned}
J_{1\mu} &= S_{23}^6(A_{23}^6) = q_a^T C c_b (\bar{q}_a \gamma_\mu \gamma_5 C \bar{c}_b^T + \bar{q}_b \gamma_\mu \gamma_5 C \bar{c}_a^T) \pm q_a^T C \gamma_\mu \gamma_5 c_b (\bar{q}_a C \bar{c}_b^T + \bar{q}_b C \bar{c}_a^T), \\
J_{2\mu} &= S_{23}^3(A_{23}^3) = q_a^T C c_b (\bar{q}_a \gamma_\mu \gamma_5 C \bar{c}_b^T - \bar{q}_b \gamma_\mu \gamma_5 C \bar{c}_a^T) \pm q_a^T C \gamma_\mu \gamma_5 c_b (\bar{q}_a C \bar{c}_b^T - \bar{q}_b C \bar{c}_a^T), \\
J_{3\mu} &= S_{14}^6(A_{14}^6) = q_a^T C \gamma_5 c_b (\bar{q}_a \gamma_\mu C \bar{c}_b^T + \bar{q}_b \gamma_\mu C \bar{c}_a^T) \pm q_a^T C \gamma_\mu c_b (\bar{q}_a \gamma_5 C \bar{c}_b^T + \bar{q}_b \gamma_5 C \bar{c}_a^T), \\
J_{4\mu} &= S_{14}^3(A_{14}^3) = q_a^T C \gamma_5 c_b (\bar{q}_a \gamma_\mu C \bar{c}_b^T - \bar{q}_b \gamma_\mu C \bar{c}_a^T) \pm q_a^T C \gamma_\mu c_b (\bar{q}_a \gamma_5 C \bar{c}_b^T - \bar{q}_b \gamma_5 C \bar{c}_a^T), \\
J_{5\mu} &= S_{46}^6(A_{46}^6) = q_a^T C \gamma^\nu c_b (\bar{q}_a \sigma_{\mu\nu} \gamma_5 C \bar{c}_b^T + \bar{q}_b \sigma_{\mu\nu} \gamma_5 C \bar{c}_a^T) \pm q_a^T C \sigma_{\mu\nu} \gamma_5 c_b (\bar{q}_a \gamma^\nu C \bar{c}_b^T + \bar{q}_b \gamma^\nu C \bar{c}_a^T), \\
J_{6\mu} &= S_{46}^3(A_{46}^3) = q_a^T C \gamma^\nu c_b (\bar{q}_a \sigma_{\mu\nu} \gamma_5 C \bar{c}_b^T - \bar{q}_b \sigma_{\mu\nu} \gamma_5 C \bar{c}_a^T) \pm q_a^T C \sigma_{\mu\nu} \gamma_5 c_b (\bar{q}_a \gamma^\nu C \bar{c}_b^T - \bar{q}_b \gamma^\nu C \bar{c}_a^T), \\
J_{7\mu} &= S_{35}^6(A_{35}^6) = q_a^T C \gamma^\nu \gamma_5 c_b (\bar{q}_a \sigma_{\mu\nu} C \bar{c}_b^T + \bar{q}_b \sigma_{\mu\nu} C \bar{c}_a^T) \pm q_a^T C \sigma_{\mu\nu} c_b (\bar{q}_a \gamma^\nu \gamma_5 C \bar{c}_b^T + \bar{q}_b \gamma^\nu \gamma_5 C \bar{c}_a^T), \\
J_{8\mu} &= S_{35}^3(A_{35}^3) = q_a^T C \gamma^\nu \gamma_5 c_b (\bar{q}_a \sigma_{\mu\nu} C \bar{c}_b^T - \bar{q}_b \sigma_{\mu\nu} C \bar{c}_a^T) \pm q_a^T C \sigma_{\mu\nu} c_b (\bar{q}_a \gamma^\nu \gamma_5 C \bar{c}_b^T - \bar{q}_b \gamma^\nu \gamma_5 C \bar{c}_a^T),
\end{aligned} \tag{4}$$

where “S” and “+” correspond to  $J^{\text{PC}} = 1^{++}$ , “A” and “-” correspond to  $J^{\text{PC}} = 1^{+-}$ .

In order to have definite isospin and  $G$  parity, all the currents in Eqs. (3) and (4) should contain  $(u\bar{c}\bar{u}\bar{c} + d\bar{c}\bar{d}\bar{c})$ . However, we do not differentiate the *up* and *down* quarks in our analysis due to the isospin symmetry and denote them by  $q$ .

### III. SPECTRAL DENSITY

In the past several decades, QCD sum rule has been widely used to study the hadron structures and proven to be a very powerful nonperturbative method [47–49]. We consider the two-point correlation function:

$$\begin{aligned}
\Pi_{\mu\nu}(q^2) &= i \int d^4x e^{iqx} \langle 0 | T [J_\mu(x) J_\nu^\dagger(0)] | 0 \rangle \\
&= -\Pi_1(q^2) (g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) + \Pi_0(q^2) \frac{q_\mu q_\nu}{q^2}. \tag{5}
\end{aligned}$$

There are two independent parts of  $\Pi_{\mu\nu}$  with different Lorentz structures because  $J_\mu$  is not a conserved current.  $\Pi_1(q^2)$  is related to the vector meson while  $\Pi_0(q^2)$  is the scalar current polarization function.

$$\begin{aligned}
iS_q^{ab}(x) &= \frac{i\delta^{ab}}{2\pi^2 x^4} \hat{x} + \frac{i}{32\pi^2} \frac{\lambda_{ab}^n}{2} g_s G_{\mu\nu}^n \frac{1}{x^2} (\sigma^{\mu\nu} \hat{x} + \hat{x} \sigma^{\mu\nu}) - \frac{\delta^{ab}}{12} \langle \bar{q}q \rangle + \frac{\delta^{ab} x^2}{192} \langle \bar{q} g_s \sigma G q \rangle, \\
iS_c^{ab}(p) &= \frac{i\delta^{ab}}{\hat{p} - m_c} + \frac{i}{4} g_s \frac{\lambda_{ab}^n}{2} G_{\mu\nu}^n \frac{\sigma^{\mu\nu} (\hat{p} + m_c) + (\hat{p} + m_c) \sigma^{\mu\nu}}{(p^2 - m_c^2)^2} + \frac{i\delta^{ab}}{12} \langle g_s^2 GG \rangle m_c \frac{p^2 + m_c \hat{p}}{(p^2 - m_c^2)^4},
\end{aligned} \tag{8}$$

where  $\hat{x} \equiv \gamma_\mu x^\mu$ ,  $\hat{p} \equiv \gamma_\mu p^\mu$ ,  $\langle \bar{q} g_s \sigma G q \rangle = \langle g_s \bar{q} \sigma^{\mu\nu} G_{\mu\nu} q \rangle$ ,  $\langle g_s^2 GG \rangle = \langle g_s^2 G_{\mu\nu}^a G^{\mu\nu a} \rangle$ ,  $a$  and  $b$  are color indices. The dimensional regularization is used throughout our calculation. The spectral density is then obtained with  $\rho(s) = \frac{1}{\pi} \text{Im} \Pi(q^2)$ .

One of the important assumptions in the QCD sum rule approach is the quark-hadron duality, which ensures the equivalence of the correlation functions obtained at the hadron level and the quark gluon level. To improve the convergence of the OPE series, the Borel transformation is

performed to the correlation functions at both levels. Considering the spectral function defined in Eq. (7), we arrive at

$$\Pi_1(q^2) = \int_{4m_c^2}^{\infty} \frac{\rho(s)}{s - q^2 - i\epsilon}, \tag{6}$$

where the lower limit of integration is the square of the sum of the mass of all current quarks (omitting the light quark mass). The spectral function is defined as

$$\begin{aligned}
\rho(s) &\equiv \sum_n \delta(s - m_n^2) \langle 0 | \eta | n \rangle \langle n | \eta^\dagger | 0 \rangle \\
&= f_X^2 \delta(s - m_X^2) + \text{continuum}, \tag{7}
\end{aligned}$$

where the usual pole plus continuum parametrization of the hadronic spectral density is adopted.

On the other hand, the correlation function can also be calculated at the quark gluon level via the operator product expansion (OPE) method. Using the same technique as in Refs. [17,41,46,50], we evaluate the Wilson coefficient up to dimension eight while the coordinate space expression for the light quark propagator and the momentum space expression for the charm quark propagator are adopted:

performed to the correlation functions at both levels. Considering the spectral function defined in Eq. (7), we arrive at

$$f_X^2 e^{-m_X^2/M_B^2} = \int_{4m_c^2}^{s_0} ds e^{-s/M_B^2} \rho(s), \tag{9}$$

where  $s_0$  is the threshold parameter. Then we can extract the mass  $m_X$ :

$$m_X^2 = \frac{\int_{4m_c^2}^{s_0} ds e^{-s/M_B^2} s \rho(s)}{\int_{4m_c^2}^{s_0} ds e^{-s/M_B^2} \rho(s)}. \quad (10)$$

The extracted spectral densities of all the tetraquark currents in Eqs. (3) and (4) are listed in the Appendix. For each quantum number, we just list the expressions of four currents. Others could be obtained conveniently by the simple replacement  $m_c \rightarrow -m_c$ . We collect the terms that are proportional to the light quark mass  $m_q$  into the expressions of the spectral densities. These terms give small contributions to the correlation functions involving the  $qc\bar{q}\bar{c}$  system and hence can be ignored. In the case of the  $sc\bar{s}\bar{c}$  system, however, they give important corrections because the strange quark mass  $m_s$  is much larger than  $m_u$  and  $m_d$ . Both the quark condensate  $\langle\bar{q}q\rangle$  and quark gluon mixed condensate  $\langle\bar{q}g_s\sigma Gq\rangle$  appear with both  $m_q$  and  $m_c$ , which is very different from the case in the pseudoscalar channel [46], where only the  $m_q$  related terms contribute. We neglect the three gluon condensate  $g_s^3\langle fGGG\rangle$  because they are strongly suppressed and negligible [46].

#### IV. QCD SUM RULE ANALYSIS

We perform the QCD sum rule analysis using the following parameter values of the quark masses and various condensates [47,51–54]:

$$\begin{aligned} m_u(2 \text{ GeV}) &= (2.9 \pm 0.6) \text{ MeV}, \\ m_d(2 \text{ GeV}) &= (5.2 \pm 0.9) \text{ MeV}, \\ m_q(2 \text{ GeV}) &= (4.0 \pm 0.7) \text{ MeV}, \\ m_s(2 \text{ GeV}) &= (101_{-21}^{+29}) \text{ MeV}, \\ m_c(m_c) &= (1.23 \pm 0.09) \text{ GeV}, \\ m_b(m_b) &= (4.20 \pm 0.07) \text{ GeV}, \\ \langle\bar{q}q\rangle &= -(0.23 \pm 0.03)^3 \text{ GeV}^3, \\ \langle\bar{q}g_s\sigma Gq\rangle &= -M_0^2\langle\bar{q}q\rangle, \\ M_0^2 &= (0.8 \pm 0.2) \text{ GeV}^2, \\ \langle\bar{s}s\rangle/\langle\bar{q}q\rangle &= 0.8 \pm 0.2, \\ \langle g_s^2 GG\rangle &= 0.88 \text{ GeV}^4, \end{aligned} \quad (11)$$

where the *up*, *down* and *strange* quark masses are the current quark masses in a mass-independent subtraction scheme such as  $\overline{MS}$  at a scale  $\mu = 2 \text{ GeV}$ . The *charm* and *bottom* quark masses are the running masses in the  $\overline{MS}$  scheme.

##### A. The tetraquark systems with $J^{\text{PC}} = 1^{-+}$ and $1^{--}$

The stability of the QCD sum rule requires a suitable working region of the threshold value  $s_0$  and the Borel mass  $M_B$ . In our analysis, we choose the value of  $s_0$  around which the variation of the extracted mass  $m_X$  with  $M_B^2$  is

minimum. The working region of the Borel mass is determined by the convergence of the OPE series and the pole contribution. The requirement of the convergence of the OPE series leads to the lower bound  $M_{\text{min}}^2$  of the Borel parameter while the constraint of the pole contribution yields the upper bound of  $M_B^2$ .

We first study the interpolating currents with  $J^{\text{PC}} = 1^{-+}$  and  $1^{--}$  in the  $qc\bar{q}\bar{c}$  systems. For these currents, the contribution of the four quark condensate  $\langle\bar{q}q\rangle^2$  is negative and its absolute value is bigger than other condensates in the region of  $M_B^2 < 3.1 \text{ GeV}^2$ . It is the dominant power contribution to the correlation function in this region. In fact, the quark condensate  $\langle\bar{q}q\rangle$  in the spectral density of the  $1^{--}$  currents is proportional to the light quark mass  $m_q$  and vanishes if we take  $m_q = 0$ . For the  $1^{-+}$  currents, we require that the perturbative term be larger than 5 times of the four quark condensate to ensure the convergence of the OPE series, which results in the lower bound of the Borel parameter,  $M_{\text{min}}^2 \sim 2.9 \text{ GeV}^2$ . The constraint for the  $1^{--}$  currents is stricter due to the vanishing  $\langle\bar{q}q\rangle$  condensate. The pole contribution (PC) is defined as

$$\text{PC} = \frac{\int_{4m_c^2}^{s_0} ds e^{-s/M_B^2} \rho(s)}{\int_{4m_c^2}^{\infty} ds e^{-s/M_B^2} \rho(s)}. \quad (12)$$

It is clear that the PC depends on the threshold value  $s_0$ . As mentioned above,  $s_0$  is chosen to ensure the minimum variation of  $m_X$  with  $M_B^2$ . For example, we choose  $s_0 \sim 26 \text{ GeV}^2$  for  $J_{6\mu}$  with  $J^{\text{PC}} = 1^{-+}$  in Fig. 1(a). By requiring that the pole contribution be larger than 40%, we obtain the upper limit  $M_{\text{max}}^2$  of the Borel parameter.

After performing the QCD sum rule analysis in the working range of the parameters obtained above, only the currents  $J_{6\mu}$ ,  $J_{7\mu}$  and  $J_{8\mu}$  with  $J^{\text{PC}} = 1^{-+}$  have the stable mass sum rules. In Fig. 1, we show the variation of  $m_X$  with the threshold value  $s_0$  and Borel parameter  $M_B^2$  for the current  $J_{6\mu}$  in the  $qc\bar{q}\bar{c}$  system. The plateau in the region of  $10 \sim 14 \text{ GeV}^2$  in Fig. 1(a) is just an unphysical artifact because both the numerator and denominator in Eq. (10) are negative within this region. The situation also occurs in the pseudoscalar channel in Ref. [46]. The variation of  $m_X$  with the Borel parameter  $M_B^2$  is weak around the region  $s_0 \sim 5.1^2 \text{ GeV}^2$ , as shown in Fig. 1(b). For  $J_{1\mu}$ ,  $J_{2\mu}$ ,  $J_{3\mu}$ ,  $J_{4\mu}$  and  $J_{5\mu}$ , the stability is so bad that the extracted mass  $m_X$  grows monotonically with the threshold value  $s_0$  and the Borel parameter  $M_B$ . These currents may couple to the  $1^{-+}$  states very weakly and the continuum contribution may be quite large, leading to the above unstable mass sum rules.

We show the Borel window, the threshold value, the extracted mass and the pole contribution corresponding to the tetraquark currents  $J_{6\mu} \sim J_{8\mu}$  with  $J^{\text{PC}} = 1^{-+}$  in the  $qc\bar{q}\bar{c}$  system in Table II. The results of the  $1^{--}$  system are listed in Table III. We present the numerical results for the currents which lead to the stable mass sum rules in the

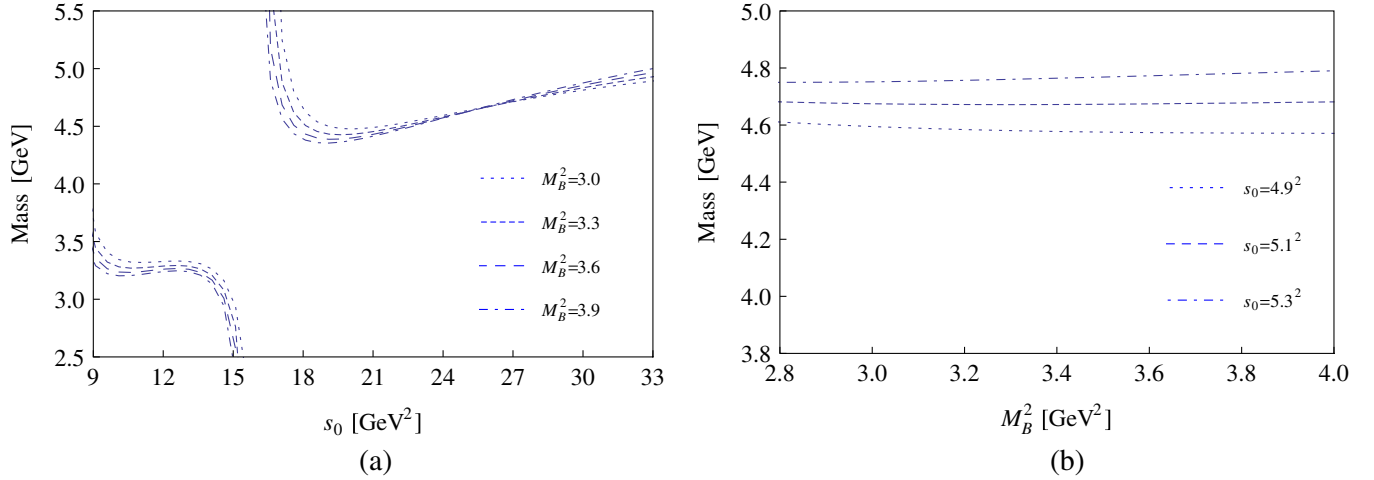


FIG. 1 (color online). The variation of  $m_X$  with  $s_0$  (a) and  $M_B^2$  (b) corresponding to the current  $J_{6\mu}$  for the  $1^{-+} qc\bar{q}\bar{c}$  system.

working range of the Borel parameter. Only the errors from the uncertainty of the threshold values and variation of the Borel parameter are taken into account. Other possible error sources include the truncation of the OPE series and the uncertainty of the quark masses, condensate values and so on.

The analysis can easily be extended to the  $sc\bar{s}\bar{c}$  systems. We keep the  $m_s$  related terms in the spectral densities. These terms give important corrections to the OPE series and are propitious to enhance the stability of the sum rule. Especially for the currents with  $J^{\text{PC}} = 1^{-+}$ ,  $\langle\bar{s}s\rangle$  is now proportional to the strange quark  $m_s$  and larger than  $\langle\bar{s}s\rangle^2$  in the Borel window, which is very different from the  $qc\bar{q}\bar{c}$

system. Using the parameters in Eq. (11), we also collect the numerical results for the  $1^{-+}$  and  $1^{--} sc\bar{s}\bar{c}$  systems in Table II and III, respectively. Obviously, the  $sc\bar{s}\bar{c}$  systems have better stabilities than the  $qc\bar{q}\bar{c}$  systems. We show the variation of  $m_{X'}$  with  $s_0$  and  $M_B^2$  for the current  $J_{6\mu}$  with  $J^{\text{PC}} = 1^{-+}$  in Fig. 2, which is very similar to Fig. 1 except the value of  $s_0$  around which the variation of  $m_{X'}$  with  $M_B^2$  is minimum. The extracted mass of the  $sc\bar{s}\bar{c}$  state is a little higher than that of the  $qc\bar{q}\bar{c}$  state. The mass difference  $m_{X'} - m_X$  for the same interpolating current is about 0.2 GeV, which is about  $2(m_s - m_q)$  within the errors.

The extracted mass of the  $qc\bar{q}\bar{c}$  state with  $J^{\text{PC}} = 1^{--}$  in Table III is about 4.6 ~ 4.7 GeV, which is consistent with

TABLE II. The threshold value, Borel window, mass and pole contribution corresponding to the currents with  $J^{\text{PC}} = 1^{-+}$  in the  $qc\bar{q}\bar{c}$ ,  $sc\bar{s}\bar{c}$ ,  $qb\bar{q}\bar{b}$  and  $sb\bar{s}\bar{b}$  systems.

	Currents	$s_0$ (GeV <sup>2</sup> )	$[M_{\min}^2, M_{\max}^2]$ (GeV <sup>2</sup> )	$m_X$ (GeV)	PC(%)
$qc\bar{q}\bar{c}$ system	$J_{6\mu}$	5.1 <sup>2</sup>	2.9 ~ 3.9	4.67 ± 0.10	50.2
	$J_{7\mu}$	5.2 <sup>2</sup>	2.9 ~ 4.2	4.77 ± 0.10	47.4
	$J_{8\mu}$	4.9 <sup>2</sup>	2.9 ~ 3.4	4.53 ± 0.10	46.3
$sc\bar{s}\bar{c}$ system	$J_{1\mu}$	5.0 <sup>2</sup>	2.9 ~ 3.4	4.67 ± 0.10	44.3
	$J_{2\mu}$	5.0 <sup>2</sup>	2.9 ~ 3.4	4.65 ± 0.09	45.6
	$J_{3\mu}$	4.9 <sup>2</sup>	2.9 ~ 3.3	4.54 ± 0.10	44.4
	$J_{4\mu}$	5.1 <sup>2</sup>	2.9 ~ 3.7	4.72 ± 0.09	44.8
	$J_{5\mu}$	5.0 <sup>2</sup>	2.9 ~ 3.6	4.62 ± 0.10	42.8
	$J_{6\mu}$	5.3 <sup>2</sup>	2.9 ~ 4.3	4.84 ± 0.10	47.3
	$J_{7\mu}$	5.3 <sup>2</sup>	2.9 ~ 4.3	4.87 ± 0.10	46.2
	$J_{8\mu}$	5.2 <sup>2</sup>	2.9 ~ 4.1	4.77 ± 0.10	44.1
$qb\bar{q}\bar{b}$ system	$J_{6\mu}$	11.0 <sup>2</sup>	7.2 ~ 8.6	10.53 ± 0.11	44.2
	$J_{7\mu}$	11.0 <sup>2</sup>	7.2 ~ 8.6	10.53 ± 0.10	44.1
	$J_{8\mu}$	11.0 <sup>2</sup>	7.2 ~ 8.6	10.49 ± 0.11	44.7
$qb\bar{q}\bar{b}$ system	$J_{4\mu}$	11.0 <sup>2</sup>	7.2 ~ 8.1	10.62 ± 0.10	41.2
	$J_{5\mu}$	11.0 <sup>2</sup>	7.2 ~ 8.4	10.56 ± 0.10	43.8
	$J_{6\mu}$	11.0 <sup>2</sup>	7.2 ~ 8.3	10.63 ± 0.10	42.4
	$J_{7\mu}$	11.0 <sup>2</sup>	7.2 ~ 8.3	10.62 ± 0.09	42.5
	$J_{8\mu}$	11.0 <sup>2</sup>	7.2 ~ 8.3	10.59 ± 0.10	43.1



TABLE III. The threshold value, Borel window, mass and pole contribution corresponding to the currents with  $J^{PC} = 1^{--}$  in the  $qc\bar{q}\bar{c}$ ,  $sc\bar{s}\bar{c}$ ,  $qb\bar{q}\bar{b}$  and  $sb\bar{s}\bar{b}$  systems.

	Currents	$s_0$ (GeV <sup>2</sup> )	$[M_{\min}^2, M_{\max}^2]$ (GeV <sup>2</sup> )	$m_X$ (GeV)	PC(%)
$qc\bar{q}\bar{c}$ system	$J_{1\mu}$	$5.0^2$	$2.9 \sim 3.6$	$4.64 \pm 0.09$	44.1
	$J_{4\mu}$	$5.0^2$	$2.9 \sim 3.6$	$4.61 \pm 0.10$	46.4
	$J_{7\mu}$	$5.2^2$	$2.9 \sim 4.1$	$4.74 \pm 0.10$	47.3
$sc\bar{s}\bar{c}$ system	$J_{1\mu}$	$5.4^2$	$2.8 \sim 4.5$	$4.92 \pm 0.10$	50.3
	$J_{2\mu}$	$5.0^2$	$2.8 \sim 3.5$	$4.64 \pm 0.09$	48.6
	$J_{3\mu}$	$4.9^2$	$2.8 \sim 3.4$	$4.52 \pm 0.10$	45.6
	$J_{4\mu}$	$5.4^2$	$2.8 \sim 4.5$	$4.88 \pm 0.10$	51.7
	$J_{7\mu}$	$5.3^2$	$2.8 \sim 4.3$	$4.86 \pm 0.10$	46.0
	$J_{8\mu}$	$4.8^2$	$2.8 \sim 3.1$	$4.48 \pm 0.10$	43.2
$qb\bar{q}\bar{b}$ system	$J_{7\mu}$	$11.0^2$	$7.2 \sim 8.5$	$10.51 \pm 0.10$	45.8
$sb\bar{s}\bar{b}$ system	$J_{1\mu}$	$11.0^2$	$7.2 \sim 8.3$	$10.60 \pm 0.10$	47.0
	$J_{2\mu}$	$11.0^2$	$7.2 \sim 8.4$	$10.55 \pm 0.11$	43.6
	$J_{3\mu}$	$11.0^2$	$7.2 \sim 8.4$	$10.55 \pm 0.10$	43.7
	$J_{4\mu}$	$11.0^2$	$7.2 \sim 8.4$	$10.53 \pm 0.11$	44.3
	$J_{7\mu}$	$11.0^2$	$7.2 \sim 8.2$	$10.62 \pm 0.10$	42.0
	$J_{8\mu}$	$11.0^2$	$7.2 \sim 8.4$	$10.53 \pm 0.10$	44.1

the mass of the meson  $Y(4660)$ . One may wonder whether  $Y(4660)$  could be a tetraquark state.

Properties of the bottomoniumlike analogues tend to be very similar because of the heavy quark symmetry. Replacing  $m_c$  with  $m_b$  in the correlation functions and repeating the same analysis procedures done above, we collect the relevant results of the  $qb\bar{q}\bar{b}$  and  $sb\bar{s}\bar{b}$  systems in Table II and III. One notes that the bottomoniumlike systems are less stable than the corresponding charmoniumlike systems.

### B. The tetraquark systems with $J^{PC} = 1^{++}$ and $1^{+-}$

We study the currents with  $J^{PC} = 1^{++}$  and  $1^{+-}$  in this subsection. The spectral densities of these currents are very similar to that of the  $1^{--}$  and  $1^{+-}$  currents, as shown in the Appendix. The analysis shows that the OPE convergence

becomes worse than that in the vector channel. In this channel, the quark condensate  $\langle \bar{q}q \rangle$  is larger than any other condensates for all the currents. The OPE convergence of the currents  $J_{5\mu}, J_{6\mu}, J_{7\mu}, J_{8\mu}$  is a little better than that of  $J_{1\mu}, J_{2\mu}, J_{3\mu}, J_{4\mu}$ .

Only the currents  $J_{3\mu}$  and  $J_{4\mu}$  in the  $1^{++}$   $qc\bar{q}\bar{c}$  system display stable mass sum rules. We obtain the working region of the Borel parameter in  $3.0 \leq M_B^2 \leq 3.4$  GeV<sup>2</sup> while taking  $s_0 = 4.6^2$  GeV<sup>2</sup> for current  $J_{4\mu}$ . The variations of  $m_X$  with the threshold value  $s_0$  and Borel parameter  $M_B^2$  are shown in Fig. 3, from which the  $M_B^2$  dependence is very weak around the chosen threshold values. Taking into account only the errors from the variation of  $M_B$  and  $s_0$ , the extracted mass is  $m_X = 4.03$  GeV, which is slightly above the mass of  $X(3872)$  within the errors.

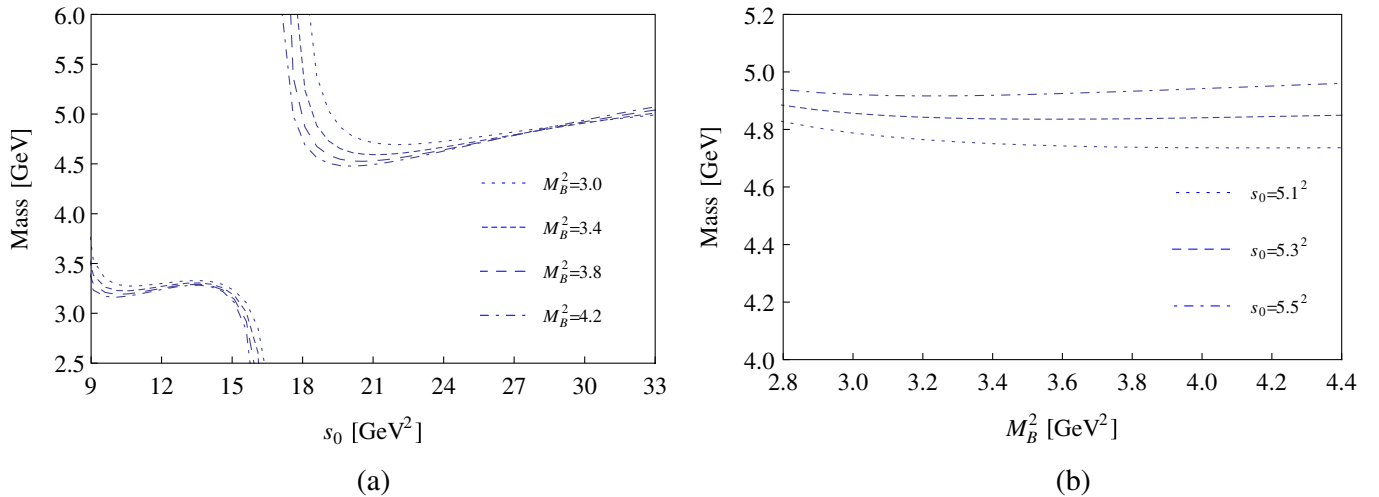


FIG. 2 (color online). The variation of  $m_X^s$  with  $s_0$  (a) and  $M_B^2$  (b) corresponding to the current  $J_{6\mu}$  for the  $1^{+-}$   $sc\bar{s}\bar{c}$  system.

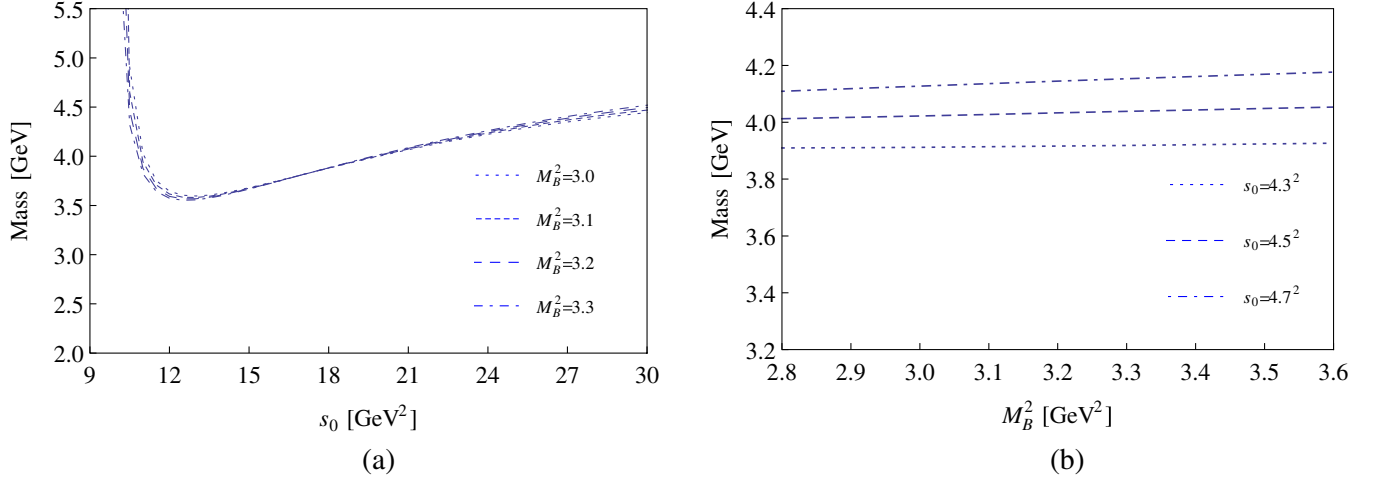


FIG. 3 (color online). The variation of  $m_X$  with  $s_0$  (a) and  $M_B^2$  (b) corresponding to the current  $J_{4\mu}$  for the  $1^{++}$   $qc\bar{q}\bar{c}$  system.

The  $sc\bar{s}\bar{c}$ ,  $qb\bar{q}\bar{b}$  and  $sb\bar{s}\bar{b}$  systems can be studied conveniently by replacement of the parameters, including the quark masses and the various condensates. The numerical results are listed in Table IV for the  $1^{++}$  systems and Table V for the  $1^{+-}$  systems. Now the bottomoniumlike systems are more stable than the corresponding charmoniumlike systems.

## V. DECAY PATTERNS OF THE CHARMONIUMLIKE STATES

The decay properties of the charmoniumlike states are important for the study of their structures and detections at experiments. In this section, we study the decay patterns of the charmoniumlike states with  $J^{PC} = 1^{-+}, 1^{-}, 1^{++}$  and  $1^{+-}$ . Only the two-body hadronic decay is considered. Replacing the  $D$  meson and charmonium state by the  $B$  meson and bottomonium state, respectively, one gets the decay patterns of the bottomoniumlike states so long as the kinematics allows.

The  $G$  parity of a charmoniumlike state is defined as  $G = C \cdot (-1)^I$ , where  $I$  is the isospin. By considering the

conservation of the angular momentum,  $P$  parity,  $C$  parity, isospin and  $G$  parity, we collect the  $S$ -wave and  $P$ -wave decay modes of these charmoniumlike states in Table VI and VII. For the vector channel, one notes that the  $S$ -wave decay modes are dominant and the final states always contain a  $S$ -wave and  $P$ -wave meson pair. Such a decay pattern is also speculated to be characteristic of the hybrid meson. In fact, the tetraquark state mixes easily with the hybrid state  $cG\bar{c}$ . In quantum field theory the charmoniumlike tetraquark operator and the  $cG\bar{c}$  hybrid operator with the same quantum numbers probably couple to the same physical state. In Table VII, the  $S$ -wave decay modes  $J/\psi\omega$  and  $J/\psi\rho$  are listed in the  $1^{++}$  channel, which is consistent with the decay properties of  $X(3872)$  [6,55]. Up to now, no experimental signals are observed for the charmoniumlike  $1^{-+}$  and  $1^{+-}$  states.  $Y(4360)$  and  $Y(4660)$  are only observed in the  $\psi(2S)\pi^+\pi^-$  channel [24,51]. The possible decay modes listed in Table VI and VII may be useful to the future search of these interesting charmoniumlike and bottomoniumlike states at the experimental facilities such as Super-B factories, PANDA, LHC and RHIC.

TABLE IV. The threshold value, Borel window, mass and pole contribution corresponding to the currents with  $J^{PC} = 1^{++}$  in the  $qc\bar{q}\bar{c}$ ,  $sc\bar{s}\bar{c}$ ,  $qb\bar{q}\bar{b}$  and  $sb\bar{s}\bar{b}$  systems.

	Currents	$s_0$ ( $\text{GeV}^2$ )	$[M_{\min}^2, M_{\max}^2]$ ( $\text{GeV}^2$ )	$m_X$ (GeV)	PC(%)
$qc\bar{q}\bar{c}$ system	$J_{3\mu}$	$4.6^2$	$3.0 \sim 3.4$	$4.19 \pm 0.10$	47.3
	$J_{4\mu}$	$4.5^2$	$3.0 \sim 3.3$	$4.03 \pm 0.11$	46.8
$sc\bar{s}\bar{c}$ system	$J_{3\mu}$	$4.6^2$	$3.0 \sim 3.4$	$4.22 \pm 0.10$	45.7
	$J_{4\mu}$	$4.5^2$	$3.0 \sim 3.3$	$4.07 \pm 0.10$	44.4
$qb\bar{q}\bar{b}$ system	$J_{3\mu}$	$10.9^2$	$8.5 \sim 9.5$	$10.32 \pm 0.09$	47.0
	$J_{4\mu}$	$10.8^2$	$8.5 \sim 9.2$	$10.22 \pm 0.11$	44.6
	$J_{7\mu}$	$10.7^2$	$7.8 \sim 8.4$	$10.14 \pm 0.10$	44.8
	$J_{8\mu}$	$10.7^2$	$7.8 \sim 8.4$	$10.14 \pm 0.09$	44.8
$sb\bar{s}\bar{b}$ system	$J_{3\mu}$	$10.9^2$	$8.5 \sim 9.5$	$10.34 \pm 0.09$	46.1
	$J_{4\mu}$	$10.8^2$	$8.5 \sim 9.1$	$10.25 \pm 0.10$	43.3
	$J_{7\mu}$	$10.8^2$	$7.5 \sim 8.6$	$10.24 \pm 0.11$	47.1
	$J_{8\mu}$	$10.8^2$	$7.5 \sim 8.6$	$10.24 \pm 0.10$	47.1

TABLE V. The threshold value, Borel window, mass and pole contribution corresponding to the currents with  $J^{PC} = 1^{+-}$  in the  $qc\bar{q}\bar{c}$ ,  $sc\bar{s}\bar{c}$ ,  $qb\bar{q}\bar{b}$  and  $sb\bar{s}\bar{b}$  systems.

	Currents	$s_0$ (GeV <sup>2</sup> )	$[M_{\min}^2, M_{\max}^2]$ (GeV <sup>2</sup> )	$m_X$ (GeV)	PC(%)
$qc\bar{q}\bar{c}$ system	$J_{3\mu}$	4.6 <sup>2</sup>	3.0 ~ 3.4	4.16 ± 0.10	46.2
	$J_{4\mu}$	4.5 <sup>2</sup>	3.0 ~ 3.3	4.02 ± 0.09	44.6
	$J_{5\mu}$	4.5 <sup>2</sup>	3.0 ~ 3.4	4.00 ± 0.11	46.0
	$J_{6\mu}$	4.6 <sup>2</sup>	3.0 ~ 3.4	4.14 ± 0.09	47.0
$sc\bar{s}\bar{c}$ system	$J_{3\mu}$	4.7 <sup>2</sup>	3.0 ~ 3.6	4.24 ± 0.10	49.6
	$J_{4\mu}$	4.6 <sup>2</sup>	3.0 ~ 3.5	4.12 ± 0.11	47.3
	$J_{5\mu}$	4.5 <sup>2</sup>	3.0 ~ 3.3	4.03 ± 0.11	44.2
	$J_{6\mu}$	4.6 <sup>2</sup>	3.0 ~ 3.4	4.16 ± 0.11	46.0
$qb\bar{q}\bar{b}$ system	$J_{3\mu}$	10.6 <sup>2</sup>	7.5 ~ 8.5	10.08 ± 0.10	45.9
	$J_{4\mu}$	10.6 <sup>2</sup>	7.5 ~ 8.5	10.07 ± 0.10	46.2
	$J_{5\mu}$	10.6 <sup>2</sup>	7.5 ~ 8.4	10.05 ± 0.10	45.3
	$J_{6\mu}$	10.7 <sup>2</sup>	7.5 ~ 8.7	10.15 ± 0.10	47.6
$sb\bar{s}\bar{b}$ system	$J_{3\mu}$	10.6 <sup>2</sup>	7.5 ~ 8.3	10.11 ± 0.10	43.8
	$J_{4\mu}$	10.6 <sup>2</sup>	7.5 ~ 8.4	10.10 ± 0.10	44.1
	$J_{5\mu}$	10.6 <sup>2</sup>	7.5 ~ 8.3	10.08 ± 0.10	43.7
	$J_{6\mu}$	10.7 <sup>2</sup>	7.5 ~ 8.5	10.18 ± 0.10	46.5

TABLE VI. The possible decay modes of the  $1^{--}$  and  $1^{+-}$  charmoniumlike states.

$I^G J^{PC}$	S wave	P wave
$0^- 1^{--}$	$D^*(2007)^0 \bar{D}_0^*(2400)^0 + c.c., J/\psi f_0(600),$ $D_1(2420)^0 \bar{D}^0(1865) + c.c., J/\psi f_0(980),$ $D_1(2420)^0 \bar{D}^*(2007)^0 + c.c., \psi(2S) f_0(600),$ $\eta_c(1S) h_1(1170), \chi_{c0}(1P) \omega, \chi_{c1}(1P) \omega$	$D^0(1865) \bar{D}^0(1865), D^*(2007)^0 \bar{D}^*(2007)^0,$ $D^*(2010)^+ D^*(2010)^-, J/\psi \eta,$ $J/\psi \eta', \psi(2S) \eta, \eta_c(1S) \omega, \eta_c(2S) \omega$ $\chi_{c0}(1P) h_1(1170), \chi_{c1}(1P) h_1(1170)$
$1^+ 1^{--}$	$D^*(2007)^0 \bar{D}_0^*(2400)^0 + c.c.,$ $D_1(2420)^0 \bar{D}^0(1865) + c.c., J/\psi a_0(980),$ $D_1(2420)^0 \bar{D}^*(2007)^0 + c.c., \chi_{c0}(1P) \rho,$ $\chi_{c1}(1P) \rho, J/\psi a_1(1260), \eta_c(1S) b_1(1235),$	$D^0(1865) \bar{D}^0(1865), D^*(2007)^0 \bar{D}^*(2007)^0,$ $D^*(2010)^+ D^*(2010)^-, J/\psi \pi, \psi(2S) \pi,$ $\eta_c(1S) \rho, \eta_c(2S) \rho, \chi_{c0}(1P) b_1(1235),$
$0^+ 1^{+-}$	$D^*(2007)^0 \bar{D}_0^*(2400)^0 + c.c.,$ $D_1(2420)^0 \bar{D}^0(1865) + c.c.,$ $D_1(2420)^0 \bar{D}^*(2007)^0 + c.c., \chi_{c1}(1P) \eta,$ $\eta_c(1S) f_1(1285), J/\psi h_1(1170), \chi_{c1}(1P) \eta'$	$D^0(1865) \bar{D}^0(1865), D^*(2007)^0 \bar{D}^*(2007)^0,$ $D^*(2010)^+ D^*(2010)^-, \chi_{c0}(1P) f_0(600),$ $\chi_{c0}(1P) f_0(980), \eta_c(1S) \eta, \eta_c(1S) \eta', J/\psi \omega,$ $\psi(2S) \omega, \chi_{c1}(1P) f_0(600), \chi_{c1}(1P) f_0(980),$
$1^- 1^{+-}$	$D^*(2007)^0 \bar{D}_0^*(2400)^0 + c.c., \eta_c(1S) a_1(1260),$ $D_1(2420)^0 \bar{D}^0(1865) + c.c., J/\psi b_1(1235),$ $D_1(2420)^0 \bar{D}^*(2007)^0 + c.c., \chi_{c1}(1P) \pi$	$D^0(1865) \bar{D}^0(1865), D^*(2007)^0 \bar{D}^*(2007)^0,$ $D^*(2010)^+ D^*(2010)^-, \chi_{c0}(1P) a_0(980),$ $\eta_c(1S) \pi, \eta_c(2S) \pi, J/\psi \rho, \psi(2S) \rho$

TABLE VII. The possible decay modes of the  $1^{++}$  and  $1^{+-}$  charmoniumlike states.

$I^G J^{PC}$	S wave	P wave
$0^+ 1^{++}$	$D^0(1865) \bar{D}^*(2007)^0 + c.c., \chi_{c1}(1P) f_0(600),$ $D(1870)^+ D^*(2010)^- + c.c., J/\psi \omega$	$\chi_{c0}(1P) \eta, \chi_{c1}(1P) \eta,$ $\eta_c(1S) f_0(600), \eta_c(1S) f_0(980)$
$1^- 1^{++}$	$D^0(1865) \bar{D}^*(2007)^0 + c.c.,$ $D(1870)^+ D^*(2010)^- + c.c., J/\psi \rho$	$\chi_{c0}(1P) \pi,$ $\chi_{c1}(1P) \pi, \eta_c(1S) a_0(980)$
$0^- 1^{+-}$	$D^0(1865) \bar{D}^*(2007)^0 + c.c., J/\psi \eta, J/\psi \eta',$ $D(1870)^+ D^*(2010)^- + c.c., \eta_c(1S) \omega$	$J/\psi f_0(980), \chi_{c0}(1P) \omega,$ $\eta_c(1S) h_1(1170), J/\psi f_0(600)$
$1^+ 1^{+-}$	$D^0(1865) \bar{D}^*(2007)^0 + c.c., J/\psi \pi, \psi(2S) \pi,$ $D(1870)^+ D^*(2010)^- + c.c., \eta_c(1S) \rho$	$J/\psi a_0(980), \chi_{c0}(1P) \rho$



## VI. SUMMARY

We have constructed a matrix  $O$  which is composed of the tetraquark operators with different Lorentz structures. The charge-conjugation transformation of the matrix is equal to its transpose. With this relation, we can define the symmetric matrix  $S$  and antisymmetric matrix  $A$ . Considering the color structure, the elements of  $S$  and  $A$  are the tetraquark operators with definite  $C$  parities. Then we can obtain all the tetraquark interpolating currents with  $J^{\text{PC}} = 1^{-+}$ ,  $1^{--}$ ,  $1^{++}$  and  $1^{+-}$ , as shown in Eqs. (3) and (4).

At the hadronic level, there exists a big difference between the molecular states and tetraquark states. The molecular states are commonly assumed to be bound states of two hadrons formed by the exchange of the color singlet light mesons while the tetraquark states are generally bound by the QCD force at the quark gluon level. However, within the framework of the QCD sum rule approach, the only difference lies in the interpolating current used in the study of the molecular and tetraquark states. Other procedures including the operator product expansion, the calculation of the Wilson coefficient and numerical analysis are the same. In principle, if we exhaust all the possible molecular-type currents and all the possible tetraquark-type currents, we can rigorously show that these two sets of interpolating currents are equivalent by using the Fierz rearrangement.

However, there exists an important difference between one single molecular-type current and one single tetraquark-type current. For example, every single tetraquark-type current is a linear combination of several independent molecular-type currents. In this respect, one well-known example is the light scalar-isoscalar sigma meson. The tetraquark-type current (or their combination/mixing) leads to a better result than the simple pion-pion molecular current. It is possible to distinguish the tetraquark and molecular structures after exhaustive and comprehensive hard work, which is one of the motivations of our present systematical investigation.

By studying the two-point correlation functions, we have calculated the spectral densities of these currents at the quark gluon level. The four quark condensate  $\langle \bar{q}q \rangle^2$  is dominant for all the currents with  $J^{\text{PC}} = 1^{-+}$  and  $1^{--}$  in the  $qc\bar{q}\bar{c}$  systems. For the currents with  $J^{\text{PC}} = 1^{++}$  and  $1^{+-}$ , however, the quark condensate  $\langle \bar{q}q \rangle$  becomes the most important corrections to the correlation functions. These properties of the spectral densities lead to a better OPE convergence for the currents in the vector channel than that in the axial-vector channel. The  $m_s$  related terms are kept in the spectral densities in order to study the contribution of the strange quark in the  $sc\bar{s}\bar{c}$  system. Actually, they give important corrections and enhance the stabilities of the QCD sum rules.

The tetraquark assignments of  $X(3872)$  and  $Y(4660)$  have been studied within the framework of the QCD sum rule approach, as mentioned in the introduction. In

Ref. [17], the current  $J_{4\mu}$  was used to study the  $1^{++} qc\bar{q}\bar{c}$  system with the extracted mass around 3.92 GeV. The current  $J_{3\mu}$  was used to study the  $1^{--} sc\bar{s}\bar{c}$  system with the extracted mass around 4.65 GeV in Ref. [27]. One difference of our present analysis and the previous ones lies in the criteria of fixing the Borel window and the value of the threshold parameter  $s_0$ , which leads to the slightly different extracted masses of the states. We have imposed a strict requirement that (1) the pole contribution be larger than 40% and (2) dual stability, i.e., the variation of the extracted mass with both  $s_0$  and the Borel parameter be minimum. The other difference is that we have exhausted the tetraquark interpolating currents.

In the working range of the Borel parameter, only the currents  $J_{1\mu}$ ,  $J_{4\mu}$  and  $J_{7\mu}$  with  $J^{\text{PC}} = 1^{--}$  display stable QCD sum rules in the  $qc\bar{q}\bar{c}$  system. The extracted mass is around 4.6 ~ 4.7 GeV from these currents, which is consistent with the mass of the meson  $Y(4660)$ . This result implies a possible tetraquark interpretation for  $Y(4660)$ . In the  $sc\bar{s}\bar{c}$  system, all currents except  $J_{5\mu}$ ,  $J_{6\mu}$  have stable QCD sum rules. The mass difference  $m_{X^s} - m_X \approx 0.2$  GeV for the same type of the interpolating current, which is roughly  $2(m_s - m_q)$ . For the  $1^{--}$  bottomonium-like states, the masses lie around 10.5 GeV and 10.6 GeV for the  $qb\bar{q}\bar{b}$  and  $sb\bar{s}\bar{b}$  systems, respectively. The Borel window for the currents in the axial-vector channel is very small because of the bad OPE convergence. For the currents with  $J^{\text{PC}} = 1^{++}$  in the  $qc\bar{q}\bar{c}$  system, only  $J_{3\mu}$  and  $J_{4\mu}$  lead to stable QCD sum rules. The same situation occurs in the  $sc\bar{s}\bar{c}$  system. Both the extracted masses are about 4.0 ~ 4.2 GeV, which is 0.1 ~ 0.2 GeV higher than the mass of  $X(3872)$ . The masses of the  $1^{++}$  bottomonium-like states are about 10.2 GeV for both the  $qb\bar{q}\bar{b}$  and  $sb\bar{s}\bar{b}$  systems.

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## APPENDIX A: THE SPECTRAL DENSITIES

In this Appendix we show the spectral densities of the tetraquark interpolating currents defined in Eqs. (3) and (4). Various power corrections include the four quark condensate  $\langle \bar{q}q \rangle^2$ , quark gluon mixed condensate  $\langle \bar{q}g_s\sigma Gq \rangle$  and dimension eight condensate  $\langle \bar{q}g_s\sigma Gq \rangle \langle \bar{q}q \rangle$ :

$$\begin{aligned} \rho(s) = & \rho^{\text{pert}}(s) + \rho^{\langle \bar{q}q \rangle}(s) + \rho^{\langle GG \rangle}(s) + \rho^{\langle \bar{q}q \rangle^2}(s) \\ & + \rho^{\langle \bar{q}g_s\sigma Gq \rangle}(s). \end{aligned} \quad (\text{A1})$$

The integration limits in the expressions are

$$\alpha_{\max} = \frac{1 + \sqrt{1 - 4m_c^2/s}}{2}, \quad \alpha_{\min} = \frac{1 - \sqrt{1 - 4m_c^2/s}}{2}, \quad \beta_{\max} = 1 - \alpha, \quad \beta_{\min} = \frac{\alpha m_c^2}{\alpha s - m_c^2}.$$

### 1. The spectral densities for the currents with $J^{\text{PC}} = 1^{-+}$

For the interpolating current  $J_{1\mu}$ :

$$\begin{aligned} \rho_1^{\text{pert}}(s) &= \frac{1}{3 \times 2^8 \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{(1 - \alpha - \beta)[(\alpha + \beta)m_c^2 - \alpha\beta s]^3}{\alpha^2 \beta^3} \left\{ \frac{3(1 + \alpha + \beta)[(\alpha + \beta)m_c^2 - \alpha\beta s]}{\alpha} \right. \\ &\quad \left. + \frac{2m_c^2(1 - \alpha - \beta)^2}{\alpha} + 24m_c m_q(1 - \alpha - \beta) \right\}, \\ \rho_1^{\langle \bar{q}q \rangle}(s) &= \frac{\langle \bar{q}q \rangle}{8\pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{[(\alpha + \beta)m_c^2 - \alpha\beta s]}{\alpha\beta} \\ &\quad \times \left\{ \frac{2m_c(1 - \alpha - \beta)[(\alpha + \beta)m_c^2 - \alpha\beta s]}{\beta} + m_q[(\alpha + \beta - 3)m_c^2 - 4\alpha\beta s] \right\}, \\ \rho_1^{\langle GG \rangle}(s) &= -\frac{\langle g_s^2 GG \rangle}{3^2 \times 2^{12} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \left\{ \frac{(1 - \alpha - \beta)^2 m_c^2}{\alpha^2} \right. \\ &\quad \times \left[ \frac{[96\beta^2 + 36\alpha\beta - (1 - \alpha - \beta)(5\alpha + 48\beta)][(\alpha + \beta)m_c^2 - \alpha\beta s]}{\alpha\beta^2} + \frac{30[(\alpha + \beta)m_c^2 - \alpha\beta s]}{\beta^2} \right. \\ &\quad \left. \left. - \frac{96[2(\alpha + \beta)m_c^2 - 3\alpha\beta s] + 16(1 - \alpha - \beta)m_c^2}{\alpha} \right] - \frac{48(1 - \alpha - \beta)[(\alpha + \beta)m_c^2 - \alpha\beta s]s}{\alpha} \right\}, \\ \rho_1^{\langle \bar{q}g_s \sigma Gq \rangle}(s) &= \frac{m_q \langle \bar{q}g_s \sigma Gq \rangle (8m_c^2 + s)}{96\pi^4} \sqrt{1 - 4m_c^2/s} \\ &\quad - \frac{\langle \bar{q}g_s \sigma Gq \rangle}{3 \times 2^7 \pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \left\{ \frac{m_c[48\alpha\beta + (1 - \alpha - \beta)(7\alpha + 6\beta)][(\alpha + \beta)m_c^2 - \alpha\beta s]}{\alpha\beta^2} \right. \\ &\quad \left. - \frac{2m_c(1 - \alpha - \beta)[2(\alpha + \beta)m_c^2 - 3\alpha\beta s]}{\beta^2} + \frac{6m_q[(\alpha + \beta - 2)m_c^2 - \alpha\beta s]}{\alpha} \right\}, \\ \rho_1^{\langle \bar{q}q \rangle^2}(s) &= -\frac{\langle \bar{q}q \rangle^2 (8m_c^2 - 6m_c m_q + s)}{36\pi^2} \sqrt{1 - 4m_c^2/s}, \\ \Pi_1^{\langle \bar{q}g_s \sigma Gq \rangle \langle \bar{q}q \rangle}(M_B^2) &= -\frac{\langle \bar{q}g_s \sigma Gq \rangle \langle \bar{q}q \rangle}{12\pi^2} \int_0^1 d\alpha \left\{ (M_B^2 + \frac{m_c^2}{\alpha})(1 - \alpha) + \frac{2m_c^4}{\alpha^2 M_B^2} + \frac{m_c^2}{2\alpha} + \frac{M_B^2 \alpha}{4} \right\} e^{-(m_c^2/\alpha(1-\alpha)M_B^2)}. \end{aligned} \quad (\text{A2})$$

For the interpolating current  $J_{2\mu}$ :

$$\begin{aligned} \rho_2^{\text{pert}}(s) &= \frac{3}{2} \rho_1^{\text{pert}}(s), \quad \rho_2^{\langle \bar{q}q \rangle}(s) = \frac{3}{2} \rho_1^{\langle \bar{q}q \rangle}(s), \quad \rho_2^{\langle \bar{q}q \rangle^2}(s) = \frac{3}{2} \rho_1^{\langle \bar{q}q \rangle^2}(s), \\ \rho_2^{\langle GG \rangle}(s) &= \frac{\langle g_s^2 GG \rangle}{3^2 \times 2^{12} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\quad \times \left\{ \frac{(1 - \alpha - \beta)^2 m_c^2}{\alpha^2} \left[ \frac{(1 - \alpha - \beta)(13\alpha + 72\beta)[(\alpha + \beta)m_c^2 - \alpha\beta s]}{\alpha\beta^2} - \frac{42[(\alpha + \beta)m_c^2 - \alpha\beta s]}{\beta^2} \right. \right. \\ &\quad \left. \left. + \frac{24[(1 + 5\alpha + 5\beta)m_c^2 - 12\alpha\beta s]}{\alpha} \right] + \frac{12[(\alpha + \beta)m_c^2 - \alpha\beta s]}{\alpha\beta} [(5 - \alpha - \beta)m_c^2 - (1 - \alpha - \beta)^2 s - 2\alpha\beta s] \right\}, \\ \rho_2^{\langle \bar{q}g_s \sigma Gq \rangle}(s) &= \frac{m_q \langle \bar{q}g_s \sigma Gq \rangle (8m_c^2 + s)}{64\pi^4} \sqrt{1 - 4m_c^2/s} \\ &\quad - \frac{m_c \langle \bar{q}g_s \sigma Gq \rangle}{2^7 \pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \left\{ \frac{3(1 - \alpha - \beta)[3(\alpha + \beta)m_c^2 - 5\alpha\beta s]}{\beta^2} + \frac{20[(\alpha + \beta)m_c^2 - \alpha\beta s]}{\beta} \right\}, \\ \Pi_2^{\langle \bar{q}g_s \sigma Gq \rangle \langle \bar{q}q \rangle}(M_B^2) &= -\frac{\langle \bar{q}g_s \sigma Gq \rangle \langle \bar{q}q \rangle}{8\pi^2} \int_0^1 d\alpha \left\{ M_B^2(1 - \alpha) + \frac{1 - \alpha}{\alpha} m_c^2 + \frac{2m_c^4}{\alpha^2 M_B^2} \right\} e^{-(m_c^2/\alpha(1-\alpha)M_B^2)}. \end{aligned} \quad (\text{A3})$$

For the interpolating current  $J_{3\mu}$ :

$$\begin{aligned}
\rho_3^{\text{pert}}(s) &= \frac{1}{2}\rho_1^{\text{pert}}(s), & \rho_3^{\langle\bar{q}q\rangle}(s) &= \frac{1}{2}\rho_1^{\langle\bar{q}q\rangle}(s), & \rho_3^{\langle\bar{q}q\rangle^2}(s) &= \frac{1}{2}\rho_1^{\langle\bar{q}q\rangle^2}(s), \\
\rho_3^{\langle GG\rangle}(s) &= \frac{\langle g_s^2 GG\rangle}{3^2 \times 2^{12} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\
&\times \left\{ \frac{(1-\alpha-\beta)^2 m_c^2}{\alpha^2} \left[ \frac{[36\alpha\beta + 24\beta(1-\alpha-\beta) - \alpha(5+\alpha+\beta)][(\alpha+\beta)m_c^2 - \alpha\beta s]}{\alpha\beta^2} \right. \right. \\
&+ \left. \frac{48[(\alpha+\beta)m_c^2 - 2\alpha\beta s]}{\alpha} + \frac{8(1-\alpha-\beta)m_c^2}{\alpha} \right] - \frac{48(1-\alpha-\beta)[(\alpha+\beta)m_c^2 - \alpha\beta s]}{\alpha} \left. \right\}, \\
\rho_3^{\langle\bar{q}g_s\sigma Gq\rangle}(s) &= \frac{m_q \langle\bar{q}g_s\sigma Gq\rangle (8m_c^2 + s)}{192\pi^4} \sqrt{1 - 4m_c^2/s} \\
&+ \frac{\langle\bar{q}g_s\sigma Gq\rangle}{3 \times 2^7 \pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \left\{ \frac{6m_c(1-5\alpha-\beta)[(\alpha+\beta)m_c^2 - \alpha\beta s]}{\alpha\beta} \right. \\
&- \left. \frac{m_c(1-\alpha-\beta)[15(\alpha+\beta)m_c^2 - 29\alpha\beta s]}{\beta^2} + \frac{6m_q[(\alpha+\beta-2)m_c^2 - \alpha\beta s]}{\alpha} \right\}, \\
\Pi_3^{\langle\bar{q}g_s\sigma Gq\rangle\langle\bar{q}q\rangle}(M_B^2) &= -\frac{\langle\bar{q}g_s\sigma Gq\rangle\langle\bar{q}q\rangle}{48\pi^2} \int_0^1 d\alpha \left\{ (2-3\alpha)M_B^2 - 2m_c^2 + \frac{4m_c^4}{\alpha^2 M_B^2} \right\} e^{-(m_c^2/\alpha(1-\alpha)M_B^2)}. \tag{A4}
\end{aligned}$$

For the interpolating current  $J_{4\mu}$ :

$$\begin{aligned}
\rho_4^{\text{pert}}(s) &= 3\rho_1^{\text{pert}}(s), & \rho_4^{\langle\bar{q}q\rangle}(s) &= 3\rho_1^{\langle\bar{q}q\rangle}(s), & \rho_4^{\langle\bar{q}q\rangle^2}(s) &= 3\rho_1^{\langle\bar{q}q\rangle^2}(s), \\
\rho_4^{\langle GG\rangle}(s) &= \frac{\langle g_s^2 GG\rangle}{3^2 \times 2^{12} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \left\{ \frac{(1-\alpha-\beta)^2 m_c^2}{\alpha^2} \left[ \frac{48[m_c^2(1+5\alpha+5\beta) - 12\alpha\beta s]}{\alpha} \right. \right. \\
&+ \left. \frac{[216\alpha\beta + (1-\alpha-\beta)(65\alpha+144\beta)][(\alpha+\beta)m_c^2 - \alpha\beta s]}{\alpha\beta^2} - \frac{210[(\alpha+\beta)m_c^2 - \alpha\beta s]}{\beta^2} \right] \\
&+ \left. \frac{12[(\alpha+\beta)m_c^2 - \alpha\beta s]}{\alpha\beta} [5(5-\alpha-\beta)m_c^2 - 5(1-\alpha-\beta)^2 s - 24(1-\alpha-\beta)\beta s - 10\alpha\beta s] \right\}, \\
\rho_4^{\langle\bar{q}g_s\sigma Gq\rangle}(s) &= \frac{m_q \langle\bar{q}g_s\sigma Gq\rangle (8m_c^2 + s)}{32\pi^4} \sqrt{1 - 4m_c^2/s} \\
&- \frac{\langle\bar{q}g_s\sigma Gq\rangle}{2^7 \pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \left\{ \frac{m_c[28\alpha\beta - (23\alpha+12\beta)(1-\alpha-\beta)][(\alpha+\beta)m_c^2 - \alpha\beta s]}{\alpha\beta^2} \right. \\
&+ \left. \frac{22m_c(1-\alpha-\beta)[2(\alpha+\beta)m_c^2 - 3\alpha\beta s]}{\beta^2} - \frac{12m_q[(\alpha+\beta-2)m_c^2 - \alpha\beta s]}{\alpha} \right\}, \\
\Pi_4^{\langle\bar{q}g_s\sigma Gq\rangle\langle\bar{q}q\rangle}(M_B^2) &= -\frac{\langle\bar{q}g_s\sigma Gq\rangle\langle\bar{q}q\rangle}{8\pi^2} \int_0^1 d\alpha \left\{ M_B^2(2-3\alpha) - 2m_c^2 + \frac{4m_c^4}{\alpha^2 M_B^2} \right\} e^{-(m_c^2/\alpha(1-\alpha)M_B^2)}. \tag{A5}
\end{aligned}$$

From these results the expressions for the currents  $J_{5\mu}$ ,  $J_{6\mu}$ ,  $J_{7\mu}$  and  $J_{8\mu}$  can then be obtained conveniently by the replacement  $m_c \rightarrow -m_c$ :

$$\begin{aligned}
\rho_1(s)^{m_c \rightarrow -m_c} &= \rho_5(s), & \rho_2(s)^{m_c \rightarrow -m_c} &= \rho_8(s), \\
\rho_3(s)^{m_c \rightarrow -m_c} &= \rho_7(s), & \rho_4(s)^{m_c \rightarrow -m_c} &= \rho_6(s). \tag{A6}
\end{aligned}$$

## 2. The spectral densities for the currents with $J^{\text{PC}} = 1^{--}$

For the interpolating current  $J_{1\mu}$ :

$$\begin{aligned}
\rho_1^{\text{pert}}(s) &= \frac{1}{3 \times 2^8 \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{(1 - \alpha - \beta)[(\alpha + \beta)m_c^2 - \alpha\beta s]^3}{\alpha^3 \beta^3} \\
&\quad \times \left\{ 3(1 + \alpha + \beta)[(\alpha + \beta)m_c^2 - \alpha\beta s] - 2m_c^2(1 - \alpha - \beta)^2 \right\}, \\
\rho_1^{\langle \bar{q}q \rangle}(s) &= -\frac{m_q \langle \bar{q}q \rangle}{8\pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{[(\alpha + \beta)m_c^2 - \alpha\beta s][(5 - \alpha - \beta)m_c^2 + 2\alpha\beta s]}{\alpha\beta}, \\
\rho_1^{\langle GG \rangle}(s) &= -\frac{\langle g_s^2 GG \rangle}{3^2 \times 2^{12} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\
&\quad \times \left\{ \frac{(1 - \alpha - \beta)^2 [(\alpha + \beta)m_c^2 - \alpha\beta s] m_c^2}{\alpha^2 \beta} \left[ \frac{48(1 - \alpha - \beta)}{\alpha} - \frac{5(5 + \alpha + \beta)}{\beta} - 36 \right] \right. \\
&\quad \left. + \frac{16(1 - \alpha - \beta)^2 [(1 - 7\alpha - 7\beta)m_c^2 + 12\alpha\beta s] m_c^2}{\alpha^3} - \frac{48(1 - \alpha - \beta)[(\alpha + \beta)m_c^2 - \alpha\beta s] s}{\alpha} \right\}, \\
\rho_1^{\langle \bar{q}g_s \sigma Gq \rangle}(s) &= \frac{m_q \langle \bar{q}g_s \sigma Gq \rangle (16m_c^2 - s)}{3 \times 2^5 \pi^4} \sqrt{1 - 4m_c^2/s} + \frac{\langle \bar{q}g_s \sigma Gq \rangle}{3 \times 2^7 \pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\
&\quad \times \left\{ \frac{(1 - \alpha - \beta)m_c}{\beta} \left[ \frac{6[(\alpha + \beta)m_c^2 - \alpha\beta s]}{\alpha} + \frac{27(\alpha + \beta)m_c^2 - 49\alpha\beta s}{\beta} \right] + \frac{6m_q((2 + \alpha + \beta)m_c^2 - \alpha\beta s)}{\alpha} \right\}, \\
\rho_1^{\langle \bar{q}q \rangle^2}(s) &= -\frac{\langle \bar{q}q \rangle^2 (16m_c^2 - s)}{36\pi^2} \sqrt{1 - 4m_c^2/s}, \\
\Pi_1^{\langle \bar{q}g_s \sigma Gq \rangle \langle \bar{q}q \rangle}(M_B^2) &= \frac{\langle \bar{q}g_s \sigma Gq \rangle \langle \bar{q}q \rangle}{48\pi^2} \int_0^1 d\alpha \left\{ (4 - 3\alpha)M_B^2 + \frac{2(1 - 2\alpha)m_c^2}{\alpha} - \frac{8m_c^4}{\alpha^2 M_B^2} \right\} e^{-(m_c^2/\alpha(1-\alpha)M_B^2)}. \tag{A7}
\end{aligned}$$

For the interpolating current  $J_{2\mu}$ :

$$\begin{aligned}
\rho_2^{\text{pert}}(s) &= \frac{3}{2} \rho_1^{\text{pert}}(s), \quad \rho_2^{\langle \bar{q}q \rangle}(s) = \frac{3}{2} \rho_1^{\langle \bar{q}q \rangle}(s), \quad \rho_2^{\langle \bar{q}q \rangle^2}(s) = \frac{3}{2} \rho_1^{\langle \bar{q}q \rangle^2}(s), \\
\rho_2^{\langle GG \rangle}(s) &= -\frac{\langle g_s^2 GG \rangle}{3^2 \times 2^{12} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\
&\quad \times \left\{ \frac{(1 - \alpha - \beta)^2 [(\alpha + \beta)m_c^2 - \alpha\beta s]}{\alpha^2 \beta} \left[ \frac{72(1 - \alpha - \beta)m_c^2}{\alpha} - \frac{(29 + 13\alpha + 13\beta)m_c^2 - 12\alpha\beta s}{\beta} \right] \right. \\
&\quad \left. + \frac{12[(\alpha + \beta)m_c^2 - \alpha\beta s][(5 - \alpha - \beta)m_c^2 + 2\alpha\beta s]}{\alpha\beta} \right. \\
&\quad \left. + \frac{24(1 - \alpha - \beta)^2 [(1 - 7\alpha - 7\beta)m_c^2 + 12\alpha\beta s] m_c^2}{\alpha^3} \right\}, \\
\rho_2^{\langle \bar{q}g_s \sigma Gq \rangle}(s) &= \frac{m_q \langle \bar{q}g_s \sigma Gq \rangle (16m_c^2 - s)}{64\pi^4} \sqrt{1 - 4m_c^2/s} + \frac{m_c \langle \bar{q}g_s \sigma Gq \rangle}{3 \times 2^7 \pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\
&\quad \times \frac{(1 - \alpha - \beta)}{\beta} \left[ \frac{12[(\alpha + \beta)m_c^2 - \alpha\beta s]}{\alpha} + \frac{3(\alpha + \beta)m_c^2 - 13\alpha\beta s}{\beta} \right], \\
\Pi_2^{\langle \bar{q}g_s \sigma Gq \rangle \langle \bar{q}q \rangle}(M_B^2) &= \frac{\langle \bar{q}g_s \sigma Gq \rangle \langle \bar{q}q \rangle}{8\pi^2} \int_0^1 d\alpha \left\{ (1 - \alpha)M_B^2 + \frac{(1 - \alpha)m_c^2}{\alpha} - \frac{2m_c^4}{\alpha^2 M_B^2} \right\} e^{-(m_c^2/\alpha(1-\alpha)M_B^2)}. \tag{A8}
\end{aligned}$$

For the interpolating current  $J_{3\mu}$ :

$$\begin{aligned}
\rho_3^{\text{pert}}(s) &= \frac{1}{2}\rho_1^{\text{pert}}(s), & \rho_3^{\langle\bar{q}q\rangle}(s) &= \frac{1}{2}\rho_1^{\langle\bar{q}q\rangle}(s), & \rho_3^{\langle\bar{q}q\rangle^2}(s) &= \frac{1}{2}\rho_1^{\langle\bar{q}q\rangle^2}(s), \\
\rho_3^{\langle GG\rangle}(s) &= -\frac{\langle g_s^2 GG\rangle}{3^2 \times 2^{12} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\
&\times \left\{ \frac{(1-\alpha-\beta)^2[(\alpha+\beta)m_c^2 - \alpha\beta s]m_c^2}{\alpha^2\beta} \left[ \frac{24(1-\alpha-\beta)}{\alpha} - \frac{5+\alpha+\beta}{\beta} + 36 \right] \right. \\
&+ \left. \frac{8(1-\alpha-\beta)^2[(1-7\alpha-7\beta)m_c^2 + 12\alpha\beta s]m_c^2}{\alpha^3} + \frac{48(1-\alpha-\beta)[(\alpha+\beta)m_c^2 - \alpha\beta s]s}{\alpha} \right\}, \\
\rho_3^{\langle\bar{q}g_s\sigma Gq\rangle}(s) &= \frac{m_q\langle\bar{q}g_s\sigma Gq\rangle(16m_c^2 - s)}{3 \times 2^6 \pi^4} \sqrt{1 - 4m_c^2/s} - \frac{\langle\bar{q}g_s\sigma Gq\rangle}{3 \times 2^7 \pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\
&\times \left\{ \frac{(1-\alpha-\beta)m_c}{\beta} \left[ \frac{6[(\alpha+\beta)m_c^2 - \alpha\beta s]}{\alpha} + \frac{9(\alpha+\beta)m_c^2 - 19\alpha\beta s}{\beta} \right] + \frac{6m_q((2+\alpha+\beta)m_c^2 - \alpha\beta s)}{\alpha} \right\}, \\
\Pi_3^{\langle\bar{q}g_s\sigma Gq\rangle\langle\bar{q}q\rangle}(M_B^2) &= \frac{\langle\bar{q}g_s\sigma Gq\rangle\langle\bar{q}q\rangle}{48\pi^2} \int_0^1 d\alpha \left\{ (2-3\alpha)M_B^2 + \frac{2(2-\alpha)m_c^2}{\alpha} - \frac{4m_c^4}{\alpha^2 M_B^2} \right\} e^{-(m_c^2/\alpha(1-\alpha)M_B^2)}. \tag{A9}
\end{aligned}$$

For the interpolating current  $J_{4\mu}$ :

$$\begin{aligned}
\rho_4^{\text{pert}}(s) &= 3\rho_1^{\text{pert}}(s), & \rho_4^{\langle\bar{q}q\rangle}(s) &= 3\rho_1^{\langle\bar{q}q\rangle}(s), & \rho_4^{\langle\bar{q}q\rangle^2}(s) &= 3\rho_1^{\langle\bar{q}q\rangle^2}(s), \\
\rho_4^{\langle GG\rangle}(s) &= -\frac{\langle g_s^2 GG\rangle}{3^2 \times 2^{12} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\
&\times \left\{ \frac{(1-\alpha-\beta)^2[(\alpha+\beta)m_c^2 - \alpha\beta s]m_c^2}{\alpha^2\beta} \left[ \frac{144(1-\alpha-\beta)}{\alpha} + \frac{65(1-\alpha-\beta)}{\beta} - \frac{210}{\beta} + 216 \right] \right. \\
&+ \frac{60(5-\alpha-\beta)[(\alpha+\beta)m_c^2 - \alpha\beta s]m_c^2}{\alpha\beta} + \frac{48(1-\alpha-\beta)^2[(1-7\alpha-7\beta)m_c^2 + 12\alpha\beta s]m_c^2}{\alpha^3} \\
&+ \left. \left[ \frac{288(1-\alpha-\beta)}{\alpha} + \frac{60(1-\alpha-\beta)^2}{\alpha\beta} + 120 \right] [(\alpha+\beta)m_c^2 - \alpha\beta s]s \right\}, \\
\rho_4^{\langle\bar{q}g_s\sigma Gq\rangle}(s) &= \frac{m_q\langle\bar{q}g_s\sigma Gq\rangle(16m_c^2 - s)}{32\pi^4} \sqrt{1 - 4m_c^2/s} + \frac{\langle\bar{q}g_s\sigma Gq\rangle}{3 \times 2^7 \pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\
&\times \left\{ \frac{(1-\alpha-\beta)m_c}{\beta} \left[ \frac{24[(\alpha+\beta)m_c^2 - \alpha\beta s]}{\alpha} + \frac{87(\alpha+\beta)m_c^2 - 161\alpha\beta s}{\beta} \right] - \frac{36m_q((2+\alpha+\beta)m_c^2 - \alpha\beta s)}{\alpha} \right\}, \\
\Pi_4^{\langle\bar{q}g_s\sigma Gq\rangle\langle\bar{q}q\rangle}(M_B^2) &= \frac{\langle\bar{q}g_s\sigma Gq\rangle\langle\bar{q}q\rangle}{8\pi^2} \int_0^1 d\alpha \left\{ (2-3\alpha)M_B^2 + \frac{2(2-\alpha)m_c^2}{\alpha} - \frac{4m_c^4}{\alpha^2 M_B^2} \right\} e^{-(m_c^2/\alpha(1-\alpha)M_B^2)}. \tag{A10}
\end{aligned}$$

From these results the expressions for the currents  $J_{5\mu}$ ,  $J_{6\mu}$ ,  $J_{7\mu}$  and  $J_{8\mu}$  can then be obtained conveniently by the replacement  $m_c \rightarrow -m_c$ :

$$\begin{aligned}
\rho_1(s) \xrightarrow{m_c \rightarrow -m_c} \rho_5(s), & & \rho_2(s) \xrightarrow{m_c \rightarrow -m_c} \rho_8(s), \\
\rho_3(s) \xrightarrow{m_c \rightarrow -m_c} \rho_7(s), & & \rho_4(s) \xrightarrow{m_c \rightarrow -m_c} \rho_6(s).
\end{aligned} \tag{A11}$$



### 3. The spectral densities for the currents with $J^{\text{PC}} = 1^{++}$

For the interpolating current  $J_{1\mu}$ :

$$\begin{aligned}
\rho_1^{\text{pert}}(s) &= \frac{1}{2^8 \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{(1 - \alpha - \beta)[(\alpha + \beta)m_c^2 - \alpha\beta s]^2}{\alpha^2 \beta^3} \\
&\quad \times \left\{ \frac{(1 + \alpha + \beta)[(\alpha + \beta)m_c^2 - \alpha\beta s]^2}{\alpha} + 12m_c m_q (1 - \alpha - \beta)[(\alpha + \beta)m_c^2 - 3\alpha\beta s] \right\}, \\
\rho_1^{\langle \bar{q}q \rangle}(s) &= \frac{\langle \bar{q}q \rangle}{8\pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{[(\alpha + \beta)m_c^2 - \alpha\beta s]}{\alpha\beta} \\
&\quad \times \left\{ \frac{m_c(1 - \alpha - \beta)[3(\alpha + \beta)m_c^2 - 7\alpha\beta s]}{\beta} + m_q[(4 + \alpha + \beta)m_c^2 - 3\alpha\beta s] \right\}, \\
\rho_1^{\langle GG \rangle}(s) &= \frac{\langle g_s^2 GG \rangle}{3^2 \times 2^{12} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\
&\quad \times \left\{ \frac{(1 - \alpha - \beta)^2 m_c^2}{\alpha^2} \left[ \frac{96[(\alpha + \beta)m_c^2 - 2\alpha\beta s]}{\alpha} + \frac{5(5 + \alpha + 7\beta)[(\alpha + \beta)m_c^2 - \alpha\beta s]}{\beta^2} \right] \right. \\
&\quad \left. - \frac{6[(\alpha + \beta)m_c^2 - \alpha\beta s]}{\alpha\beta} \left[ \frac{4(1 - \alpha - \beta)(5m_c^2 - 2\alpha\beta s)}{\alpha} - 5m_c^2(1 + \alpha + \beta) \right] \right\}, \\
\rho_1^{\langle \bar{q}g_s \sigma Gq \rangle}(s) &= -\frac{m_c^2 m_q \langle \bar{q}g_s \sigma Gq \rangle}{8\pi^4} \sqrt{1 - 4m_c^2/s} + \frac{\langle \bar{q}g_s \sigma Gq \rangle}{3 \times 2^7 \pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\
&\quad \times \left\{ \frac{m_c(1 - \alpha - \beta)[3(\alpha + \beta)m_c^2 - \alpha\beta s]}{\beta^2} - \frac{12m_q m_c^2}{\beta} - \frac{(29m_c + 5m_q)[3(\alpha + \beta)m_c^2 - 5\alpha\beta s]}{\beta} \right\}, \\
\rho_1^{\langle \bar{q}q \rangle^2}(s) &= \frac{m_c \langle \bar{q}q \rangle^2}{12\pi^2} \left( 4m_c + 2m_q - \frac{m_q s}{4m_c^2 - s} \right) \sqrt{1 - 4m_c^2/s}, \\
\Pi_1^{\langle \bar{q}g_s \sigma Gq \rangle \langle \bar{q}q \rangle}(M_B^2) &= -\frac{\langle \bar{q}g_s \sigma Gq \rangle \langle \bar{q}q \rangle}{3^2 \times 2^5 \pi^2} \int_0^1 d\alpha \left\{ 15\alpha M_B^2 - \frac{2(6 - 11\alpha)m_c^2}{\alpha(1 - \alpha)} - \frac{48m_c^4}{\alpha^2 M_B^2} \right\} e^{-(m_c^2/\alpha(1-\alpha))M_B^2}. \tag{A12}
\end{aligned}$$

For the interpolating current  $J_{2\mu}$ :

$$\begin{aligned}
\rho_2^{\text{pert}}(s) &= \frac{1}{2} \rho_1^{\text{pert}}(s), \quad \rho_2^{\langle \bar{q}q \rangle}(s) = \frac{1}{2} \rho_1^{\langle \bar{q}q \rangle}(s), \quad \rho_2^{\langle \bar{q}q \rangle^2}(s) = \frac{1}{2} \rho_1^{\langle \bar{q}q \rangle^2}(s), \\
\rho_2^{\langle GG \rangle}(s) &= \frac{\langle g_s^2 GG \rangle}{3^2 \times 2^{12} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\
&\quad \times \left\{ \frac{(1 - \alpha - \beta)^2 m_c^2}{\alpha^2} \left[ \frac{48[(\alpha + \beta)m_c^2 - 2\alpha\beta s]}{\alpha} + \frac{(5 + \alpha + 7\beta)[(\alpha + \beta)m_c^2 - \alpha\beta s]}{\beta^2} \right] \right. \\
&\quad \left. - \frac{6[(\alpha + \beta)m_c^2 - \alpha\beta s]}{\alpha\beta} \left[ \frac{4(1 - \alpha - \beta)(m_c^2 + 2\alpha\beta s)}{\alpha} - m_c^2(1 + \alpha + \beta) \right] \right\}, \\
\rho_2^{\langle \bar{q}g_s \sigma Gq \rangle}(s) &= -\frac{m_c^2 m_q \langle \bar{q}g_s \sigma Gq \rangle}{16\pi^4} \sqrt{1 - 4m_c^2/s} + \frac{\langle \bar{q}g_s \sigma Gq \rangle}{3 \times 2^7 \pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\
&\quad \times \left\{ \frac{m_c(1 - \alpha - \beta)[15(\alpha + \beta)m_c^2 - 29\alpha\beta s]}{\beta^2} + \frac{12m_q m_c^2}{\beta} - \frac{(13m_c + m_q)[3(\alpha + \beta)m_c^2 - 5\alpha\beta s]}{\beta} \right\}, \\
\Pi_2^{\langle \bar{q}g_s \sigma Gq \rangle \langle \bar{q}q \rangle}(M_B^2) &= -\frac{\langle \bar{q}g_s \sigma Gq \rangle \langle \bar{q}q \rangle}{3^2 \times 2^5 \pi^2} \int_0^1 d\alpha \left\{ 3\alpha M_B^2 + \frac{2(6 - 5\alpha)m_c^2}{\alpha(1 - \alpha)} - \frac{24m_c^4}{\alpha^2 M_B^2} \right\} e^{-(m_c^2/\alpha(1-\alpha))M_B^2}. \tag{A13}
\end{aligned}$$

For the interpolating current  $J_{5\mu}$ :

$$\begin{aligned}
\rho_5^{\text{pert}}(s) &= \frac{1}{3 \times 2^8 \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{(1-\alpha-\beta)[(\alpha+\beta)m_c^2 - \alpha\beta s]^2}{\alpha^2 \beta^3} \left\{ \frac{9(1+\alpha+\beta)[(\alpha+\beta)m_c^2 - \alpha\beta s]^2}{\alpha} \right. \\
&\quad \left. + \frac{4m_c^2(1-\alpha-\beta)(5+\alpha+\beta)[(\alpha+\beta)m_c^2 - \alpha\beta s]}{\alpha} + 36m_c m_q(1-\alpha-\beta)[(\alpha+\beta)m_c^2 - 3\alpha\beta s] \right\}, \\
\rho_5^{\langle \bar{q}q \rangle}(s) &= \frac{\langle \bar{q}q \rangle}{8\pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\
&\quad \times \left[ \frac{(\alpha+\beta)m_c^2 - \alpha\beta s}{\alpha\beta} \left\{ \frac{m_c(1-\alpha-\beta)[3(\alpha+\beta)m_c^2 - 7\alpha\beta s]}{\beta} + m_q[(14+11\alpha+11\beta)m_c^2 - 23\alpha\beta s] \right\} \right], \\
\rho_5^{\langle GG \rangle}(s) &= \frac{\langle g_s^2 GG \rangle}{3^2 \times 2^{12} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \left\{ \frac{[(\alpha+\beta)m_c^2 - \alpha\beta s]m_c^2}{\alpha\beta} \left[ \frac{96(1-\alpha-\beta)^2(5+\alpha+\beta)}{\alpha^2} \right. \right. \\
&\quad \left. \left. + \frac{54(1-\alpha-\beta)(3+\alpha+\beta)}{\alpha} + \frac{5(1-\alpha-\beta)^2(29+13\alpha+13\beta)}{\alpha\beta} + 90(1+\alpha+\beta) \right] \right. \\
&\quad \left. - \frac{[(\alpha+\beta)m_c^2 - \alpha\beta s]s}{\alpha\beta} \left[ 288(1-\alpha-\beta)\alpha + 60(1-\alpha-\beta)^2 + 120\alpha\beta \right] \right. \\
&\quad \left. + \frac{32(1-\alpha-\beta)^2[5(1+2\alpha+2\beta)m_c^2 - 18\alpha\beta s]m_c^2}{\alpha^3} \right\}, \\
\rho_5^{\langle \bar{q}g_s \sigma Gq \rangle}(s) &= \frac{m_q \langle \bar{q}g_s \sigma Gq \rangle (s - 10m_c^2)}{48\pi^4} \sqrt{1 - 4m_c^2/s} - \frac{\langle \bar{q}g_s \sigma Gq \rangle}{3 \times 2^7 \pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\
&\quad \times \left\{ \frac{(29m_c - 9m_q)[3(\alpha+\beta)m_c^2 - 5\alpha\beta s]}{\beta} + \frac{6m_c(1-\alpha-\beta)[3(\alpha+\beta)m_c^2 - 5\alpha\beta s]}{\alpha\beta} \right. \\
&\quad \left. - \frac{(1-\alpha-\beta)[63(\alpha+\beta)m_c^2 - 129\alpha\beta s]}{\beta^2} - \frac{72m_q m_c^2}{\beta} \right\}, \\
\rho_5^{\langle \bar{q}q \rangle^2}(s) &= \frac{\langle \bar{q}q \rangle^2}{36\pi^2} \left[ 20m_c^2 - 2s + \frac{3m_q m_c(8m_c^2 - 3s)}{4m_c^2 - s} \right] \sqrt{1 - 4m_c^2/s}, \\
\Pi_5^{\langle \bar{q}g_s \sigma Gq \rangle \langle \bar{q}q \rangle}(M_B^2) &= -\frac{\langle \bar{q}g_s \sigma Gq \rangle \langle \bar{q}q \rangle}{96\pi^2} \int_0^1 d\alpha \left\{ (7\alpha + 16)M_B^2 - \frac{2(8\alpha^2 - \alpha - 20)m_c^2}{\alpha(1-\alpha)} - \frac{16(3-5\alpha)m_c^4}{\alpha^2(1-\alpha)M_B^2} \right\} e^{-(m_c^2/\alpha(1-\alpha)M_B^2)}.
\end{aligned} \tag{A14}$$

For the interpolating current  $J_{6\mu}$ :

$$\begin{aligned}
\rho_6^{\text{pert}}(s) &= \frac{1}{2} \rho_5^{\text{pert}}(s), \quad \rho_6^{\langle \bar{q}q \rangle}(s) = \frac{1}{2} \rho_5^{\langle \bar{q}q \rangle}(s), \quad \rho_6^{\langle \bar{q}q \rangle^2}(s) = \frac{1}{2} \rho_5^{\langle \bar{q}q \rangle^2}(s), \\
\rho_6^{\langle GG \rangle}(s) &= \frac{\langle g_s^2 GG \rangle}{3^2 \times 2^{12} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \left\{ \frac{[(\alpha+\beta)m_c^2 - \alpha\beta s]m_c^2}{\alpha\beta} \left[ \frac{48(1-\alpha-\beta)^2(5+\alpha+\beta)}{\alpha^2} \right. \right. \\
&\quad \left. \left. - \frac{18(1-\alpha-\beta)(3+\alpha+\beta)}{\alpha} + \frac{(1-\alpha-\beta)^2(29+13\alpha+13\beta)}{\alpha\beta} + 18(1+\alpha+\beta) \right] \right. \\
&\quad \left. - \frac{12[(1-\alpha-\beta)^2 + 2\alpha\beta][(\alpha+\beta)m_c^2 - \alpha\beta s]s}{\alpha\beta} + \frac{16(1-\alpha-\beta)^2[5(1+2\alpha+2\beta)m_c^2 - 18\alpha\beta s]m_c^2}{\alpha^3} \right\}, \\
\rho_6^{\langle \bar{q}g_s \sigma Gq \rangle}(s) &= \frac{m_q \langle \bar{q}g_s \sigma Gq \rangle (s - 10m_c^2)}{96\pi^4} \sqrt{1 - 4m_c^2/s} - \frac{\langle \bar{q}g_s \sigma Gq \rangle}{3 \times 2^7 \pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\
&\quad \times \left\{ \frac{(13m_c + 3m_q)[3(\alpha+\beta)m_c^2 - 5\alpha\beta s]}{\beta} + \frac{(1-\alpha-\beta)(6\beta - 9\alpha)[3(\alpha+\beta)m_c^2 - 5\alpha\beta s]m_c}{\alpha\beta^2} \right\}, \\
\Pi_6^{\langle \bar{q}g_s \sigma Gq \rangle \langle \bar{q}q \rangle}(M_B^2) &= -\frac{\langle \bar{q}g_s \sigma Gq \rangle \langle \bar{q}q \rangle}{96\pi^2} \int_0^1 d\alpha \left\{ (8\alpha + 11)M_B^2 - \frac{2(4\alpha^2 - 9\alpha - 4)m_c^2}{\alpha(1-\alpha)} - \frac{8(3-5\alpha)m_c^4}{\alpha^2(1-\alpha)M_B^2} \right\} e^{-(m_c^2/\alpha(1-\alpha)M_B^2)}.
\end{aligned} \tag{A15}$$

From these results the expressions for the currents  $J_{3\mu}$ ,  $J_{4\mu}$ ,  $J_{7\mu}$  and  $J_{8\mu}$  can then be obtained conveniently by the replacement  $m_c \rightarrow -m_c$ :

$$\rho_1(s) \xrightarrow{m_c \rightarrow -m_c} \rho_3(s), \quad \rho_2(s) \xrightarrow{m_c \rightarrow -m_c} \rho_4(s), \quad \rho_5(s) \xrightarrow{m_c \rightarrow -m_c} \rho_7(s), \quad \rho_6(s) \xrightarrow{m_c \rightarrow -m_c} \rho_8(s). \quad (\text{A16})$$

#### 4. The spectral densities for the currents with $J^{\text{PC}} = 1^{+-}$

For the interpolating current  $J_{1\mu}$ :

$$\begin{aligned} \rho_1^{\text{pert}}(s) &= \frac{1}{2^8 \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{(1-\alpha-\beta)[(\alpha+\beta)m_c^2 - \alpha\beta s]^2}{\alpha^2 \beta^3} \\ &\quad \times \left\{ \frac{(1+\alpha+\beta)[(\alpha+\beta)m_c^2 - \alpha\beta s]^2}{\alpha} + 12m_c m_q (1-\alpha-\beta)[(\alpha+\beta)m_c^2 - 3\alpha\beta s] \right\}, \\ \rho_1^{\langle \bar{q}q \rangle}(s) &= \frac{\langle \bar{q}q \rangle}{8\pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\quad \times \left[ \frac{(\alpha+\beta)m_c^2 - \alpha\beta s}{\alpha\beta} \left\{ \frac{m_c(1-\alpha-\beta)[3(\alpha+\beta)m_c^2 - 7\alpha\beta s]}{\beta} + m_q[(4+\alpha+\beta)m_c^2 - 3\alpha\beta s] \right\} \right], \\ \rho_1^{\langle GG \rangle}(s) &= \frac{\langle g_s^2 GG \rangle}{3^2 \times 2^{12} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\quad \times \left\{ \frac{(1-\alpha-\beta)^2 m_c^2}{\alpha^2} \left[ \frac{96[(\alpha+\beta)m_c^2 - 2\alpha\beta s]}{\alpha} - \frac{5(5+\alpha+7\beta)[(\alpha+\beta)m_c^2 - \alpha\beta s]}{\beta^2} \right] \right. \\ &\quad \left. + \frac{6[(\alpha+\beta)m_c^2 - \alpha\beta s]}{\alpha\beta} \left[ \frac{4(1-\alpha-\beta)(5m_c^2 + 2\alpha\beta s)}{\alpha} - 5m_c^2(1+\alpha+\beta) \right] \right\}, \\ \rho_1^{\langle \bar{q}g_s \sigma Gq \rangle}(s) &= -\frac{m_c^2 m_q \langle \bar{q}g_s \sigma Gq \rangle}{8\pi^4} \sqrt{1-4m_c^2/s} - \frac{\langle \bar{q}g_s \sigma Gq \rangle}{3 \times 2^7 \pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\quad \times \left\{ \frac{m_c(1-\alpha-\beta)[27(\alpha+\beta)m_c^2 - 49\alpha\beta s]}{\beta^2} + \frac{12m_q m_c^2}{\beta} + \frac{(19m_c - 5m_q)[3(\alpha+\beta)m_c^2 - 5\alpha\beta s]}{\beta} \right\}, \\ \rho_1^{\langle \bar{q}q \rangle^2}(s) &= \frac{m_c \langle \bar{q}q \rangle^2}{12\pi^2} \left( 4m_c + 2m_q - \frac{m_q s}{4m_c^2 - s} \right) \sqrt{1-4m_c^2/s}, \\ \Pi_1^{\langle \bar{q}g_s \sigma Gq \rangle \langle \bar{q}q \rangle}(M_B^2) &= \frac{\langle \bar{q}g_s \sigma Gq \rangle \langle \bar{q}q \rangle}{3^2 \times 2^5 \pi^2} \int_0^1 d\alpha \left\{ 15\alpha M_B^2 + \frac{2(6-\alpha)m_c^2}{\alpha(1-\alpha)} + \frac{48m_c^4}{\alpha^2 M_B^2} \right\} e^{-(m_c^2/\alpha(1-\alpha)M_B^2)}. \end{aligned} \quad (\text{A17})$$

For the interpolating current  $J_{2\mu}$ :

$$\begin{aligned} \rho_2^{\text{pert}}(s) &= \frac{1}{2} \rho_1^{\text{pert}}(s), \quad \rho_2^{\langle \bar{q}q \rangle}(s) = \frac{1}{2} \rho_1^{\langle \bar{q}q \rangle}(s), \quad \rho_2^{\langle \bar{q}q \rangle^2}(s) = \frac{1}{2} \rho_1^{\langle \bar{q}q \rangle^2}(s), \\ \rho_2^{\langle GG \rangle}(s) &= \frac{\langle g_s^2 GG \rangle}{3^2 \times 2^{12} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\quad \times \left\{ \frac{(1-\alpha-\beta)^2 m_c^2}{\alpha^2} \left[ \frac{48[(\alpha+\beta)m_c^2 - 2\alpha\beta s]}{\alpha} - \frac{(5+\alpha+7\beta)[(\alpha+\beta)m_c^2 - \alpha\beta s]}{\beta^2} \right] \right. \\ &\quad \left. + \frac{6[(\alpha+\beta)m_c^2 - \alpha\beta s]}{\alpha\beta} \left[ \frac{4(1-\alpha-\beta)(m_c^2 - 2\alpha\beta s)}{\alpha} - m_c^2(1+\alpha+\beta) \right] \right\}, \\ \rho_2^{\langle \bar{q}g_s \sigma Gq \rangle}(s) &= -\frac{m_c^2 m_q \langle \bar{q}g_s \sigma Gq \rangle}{16\pi^4} \sqrt{1-4m_c^2/s} + \frac{\langle \bar{q}g_s \sigma Gq \rangle}{3 \times 2^7 \pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\quad \times \left\{ \frac{m_c(1-\alpha-\beta)[9(\alpha+\beta)m_c^2 - 19\alpha\beta s]}{\beta^2} + \frac{12m_q m_c^2}{\beta} - \frac{(11m_c - m_q)[3(\alpha+\beta)m_c^2 - 5\alpha\beta s]}{\beta} \right\}, \\ \Pi_2^{\langle \bar{q}g_s \sigma Gq \rangle \langle \bar{q}q \rangle}(M_B^2) &= \frac{\langle \bar{q}g_s \sigma Gq \rangle \langle \bar{q}q \rangle}{3^2 \times 2^5 \pi^2} \int_0^1 d\alpha \left\{ 3\alpha M_B^2 - \frac{2(6-7\alpha)m_c^2}{\alpha(1-\alpha)} + \frac{24m_c^4}{\alpha^2 M_B^2} \right\} e^{-(m_c^2/\alpha(1-\alpha)M_B^2)}. \end{aligned} \quad (\text{A18})$$

For the interpolating current  $J_{5\mu}$ :

$$\begin{aligned}
\rho_5^{\text{pert}}(s) &= \frac{1}{3 \times 2^8 \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{(1-\alpha-\beta)[(\alpha+\beta)m_c^2 - \alpha\beta s]^2}{\alpha^2 \beta^3} \left\{ \frac{9(1+\alpha+\beta)[(\alpha+\beta)m_c^2 - \alpha\beta s]^2}{\alpha} \right. \\
&\quad \left. - \frac{4m_c^2(1-\alpha-\beta)(5+\alpha+\beta)[(\alpha+\beta)m_c^2 - \alpha\beta s]}{\alpha} - 108m_c m_q (1-\alpha-\beta)[(\alpha+\beta)m_c^2 - 3\alpha\beta s] \right\}, \\
\rho_5^{\langle \bar{q}q \rangle}(s) &= -\frac{\langle \bar{q}q \rangle}{8\pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\
&\quad \times \frac{[(\alpha+\beta)m_c^2 - \alpha\beta s]}{\alpha\beta} \left\{ \frac{3m_c(1-\alpha-\beta)[3(\alpha+\beta)m_c^2 - 7\alpha\beta s]}{\beta} - 5m_q[(2-\alpha-\beta)m_c^2 + \alpha\beta s] \right\}, \\
\rho_5^{\langle GG \rangle}(s) &= -\frac{\langle g_s^2 GG \rangle}{3^2 \times 2^{12} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \left\{ \frac{[(\alpha+\beta)m_c^2 - \alpha\beta s]m_c^2}{\alpha\beta} \left[ \frac{96(1-\alpha-\beta)^2(5+\alpha+\beta)}{\alpha^2} \right. \right. \\
&\quad \left. \left. + \frac{54(1-\alpha-\beta)(3+\alpha+\beta)}{\alpha} + \frac{5(1-\alpha-\beta)^2(29+13\alpha+13\beta)}{\alpha\beta} + 90(1+\alpha+\beta) \right] \right. \\
&\quad \left. + \frac{[(\alpha+\beta)m_c^2 - \alpha\beta s]s}{\alpha\beta} \left[ 288(1-\alpha-\beta)\alpha + 60(1-\alpha-\beta)^2 + 120\alpha\beta \right] \right. \\
&\quad \left. + \frac{32(1-\alpha-\beta)^2[(5-8\alpha-8\beta)m_c^2 + 18\alpha\beta s]m_c^2}{\alpha^3} \right\}, \\
\rho_5^{\langle \bar{q}g_s \sigma Gq \rangle}(s) &= -\frac{m_q \langle \bar{q}g_s \sigma Gq \rangle (s + 26m_c^2)}{48\pi^4} \sqrt{1-4m_c^2/s} - \frac{\langle \bar{q}g_s \sigma Gq \rangle}{3 \times 2^7 \pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\
&\quad \times \left\{ \frac{(1-\alpha-\beta)[87(\alpha+\beta)m_c^2 - 161\alpha\beta s]m_c}{\beta^2} + \frac{(14-71\alpha-14\beta)[3(\alpha+\beta)m_c^2 - 5\alpha\beta s]m_c}{\alpha\beta} \right. \\
&\quad \left. - \frac{9m_q[(8-3\alpha-3\beta)m_c^2 + 5\alpha\beta s]}{\beta} \right\}, \\
\rho_5^{\langle \bar{q}q \rangle^2}(s) &= \frac{\langle \bar{q}q \rangle^2}{36\pi^2} \left[ 52m_c^2 + 2s - \frac{9m_q m_c (8m_c^2 - 3s)}{4m_c^2 - s} \right] \sqrt{1-4m_c^2/s}, \\
\Pi_5^{\langle \bar{q}g_s \sigma Gq \rangle \langle \bar{q}q \rangle}(M_B^2) &= \frac{\langle \bar{q}g_s \sigma Gq \rangle \langle \bar{q}q \rangle}{96\pi^2} \int_0^1 d\alpha \left\{ (7\alpha + 16)M_B^2 - \frac{2(8\alpha^2 - 25\alpha + 4)m_c^2}{\alpha(1-\alpha)} + \frac{16(3-\alpha)m_c^4}{\alpha^2(1-\alpha)M_B^2} \right\} e^{-(m_c^2/\alpha(1-\alpha))M_B^2}. \quad (\text{A19})
\end{aligned}$$

For the interpolating current  $J_{6\mu}$ :

$$\begin{aligned}
\rho_6^{\text{pert}}(s) &= \frac{1}{2} \rho_5^{\text{pert}}(s), \quad \rho_6^{\langle \bar{q}q \rangle}(s) = \frac{1}{2} \rho_5^{\langle \bar{q}q \rangle}(s), \quad \rho_6^{\langle \bar{q}q \rangle^2}(s) = \frac{1}{2} \rho_5^{\langle \bar{q}q \rangle^2}(s), \\
\rho_6^{\langle GG \rangle}(s) &= -\frac{\langle g_s^2 GG \rangle}{3^2 \times 2^{12} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \left\{ \frac{[(\alpha+\beta)m_c^2 - \alpha\beta s]m_c^2}{\alpha\beta} \left[ \frac{48(1-\alpha-\beta)^2(5+\alpha+\beta)}{\alpha^2} \right. \right. \\
&\quad \left. \left. - \frac{18(1-\alpha-\beta)(3+\alpha+\beta)}{\alpha} + \frac{(1-\alpha-\beta)^2(29+13\alpha+13\beta)}{\alpha\beta} + 18(1+\alpha+\beta) \right] \right. \\
&\quad \left. + \frac{12[(1-\alpha-\beta)^2 + 2\alpha\beta][(\alpha+\beta)m_c^2 - \alpha\beta s]s}{\alpha\beta} + \frac{16(1-\alpha-\beta)^2[(5-8\alpha-8\beta)m_c^2 + 18\alpha\beta s]m_c^2}{\alpha^3} \right\}, \\
\rho_6^{\langle \bar{q}g_s \sigma Gq \rangle}(s) &= -\frac{m_q \langle \bar{q}g_s \sigma Gq \rangle (s + 26m_c^2)}{96\pi^4} \sqrt{1-4m_c^2/s} - \frac{\langle \bar{q}g_s \sigma Gq \rangle}{3 \times 2^7 \pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\
&\quad \times \left\{ \frac{(1-\alpha-\beta)[3(\alpha+\beta)m_c^2 - 13\alpha\beta s]m_c}{\beta^2} - \frac{[(2+31\alpha-2\beta)m_c + 3\alpha m_q][3(\alpha+\beta)m_c^2 - 5\alpha\beta s]}{\alpha\beta} \right\}, \\
\Pi_6^{\langle \bar{q}g_s \sigma Gq \rangle \langle \bar{q}q \rangle}(M_B^2) &= \frac{\langle \bar{q}g_s \sigma Gq \rangle \langle \bar{q}q \rangle}{96\pi^2} \int_0^1 d\alpha \left\{ (8\alpha + 11)M_B^2 - \frac{2(4\alpha^2 - 9\alpha - 4)m_c^2}{\alpha(1-\alpha)} + \frac{8(3-\alpha)m_c^4}{\alpha^2(1-\alpha)M_B^2} \right\} e^{-(m_c^2/\alpha(1-\alpha))M_B^2}. \quad (\text{A20})
\end{aligned}$$

From these results the expressions for the currents  $J_{3\mu}$ ,  $J_{4\mu}$ ,  $J_{7\mu}$  and  $J_{8\mu}$  can then be obtained conveniently by the replacement  $m_c \rightarrow -m_c$ :

$$\rho_1(s)^{m_c \rightarrow -m_c} \rho_3(s), \quad \rho_2(s)^{m_c \rightarrow -m_c} \rho_4(s), \quad \rho_5(s)^{m_c \rightarrow -m_c} \rho_7(s), \quad \rho_6(s)^{m_c \rightarrow -m_c} \rho_8(s). \quad (\text{A21})$$

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