

Pseudoscalar meson and decuplet baryon scattering lengths

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We have systematically calculated the S-wave pseudoscalar meson and decuplet baryon scattering lengths to the third order in the small-scale expansion scheme of the heavy baryon chiral perturbation theory. We hope that future study may test the framework and the present predictions.

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I. INTRODUCTION

The excited states of the nucleon and their strong interactions with various hadrons play an important role in our understanding of nonperturbative QCD. Generally speaking, their coupling with the pseudoscalar pion, kaon, and eta is governed by chiral dynamics. The first excited state of the nucleon, $J^P = 3/2^+$ $\Delta(1232)$, belongs to the lightest decuplet. It is degenerate with the nucleon in the large N_c limit [1,2]. From the strong decay widths, one can extract the coupling constants of the pseudoscalar meson decuplet baryon interaction. Although it may be quite difficult to measure their scattering lengths experimentally, unlike in the pion-nucleon and kaon-nucleon cases [3–5], it is still interesting to study the scattering processes theoretically.

As two well-known nonperturbative approaches of QCD at low energy, chiral perturbation theory and lattice QCD have been widely used in studying meson-baryon interactions [6–14]. With the explicitly consistent power counting scheme, the heavy baryon chiral perturbation theory (HB χ PT) is a practical and useful tool in the study of the meson-baryon scattering. The expansion is organized with p/Λ_χ , where p represents the momentum (mass) of the meson or the small residue momentum of the baryon in the nonrelativistic limit, and $\Lambda_\chi = 4\pi f_\pi$ is the scale of chiral symmetry breaking. Numerically it is around the baryon mass in the chiral limit. Because of the large mass of the strange quark, the chiral corrections from the kaon and eta meson loops are significantly larger than that from the pion loop, which leads to the worse convergence of the chiral expansion in the SU(3) case.

In HB χ PT, the effects of excited baryons are encoded in the unknown low-energy constants (LECs) in the Lagrangians. In the large N_c limit, one expects that the

inclusion of the decuplet baryon contribution may partially cancel the octet contribution and be helpful to the chiral expansion. In order to investigate the contributions of the Δ , HB χ PT with explicit spin-3/2 baryons and the small-scale expansion scheme have been developed in Refs. [15,16]. Now the expansion is organized with ϵ , where ϵ means p in HB χ PT or the mass difference δ between Δ and the nucleon in the chiral limit, which is widely used in the exploration of the decuplet related processes [17–21]. There exists another power counting scheme “ δ expansion” in the two-flavor case, where m_π scales as δ^2 [22–25].

In our previous work [21], we studied the decuplet contributions to the pseudoscalar meson octet baryon scattering lengths, which are important observables reflecting their strong interaction near the threshold. The inclusion of the decuplet baryons does not improve the convergence significantly. In this paper, we will study the scattering lengths of the pseudoscalar mesons and the light decuplet baryons in the framework of the SU(3) heavy baryon chiral perturbation theory with explicit decuplet baryons. We will explore whether the small-scale expansion works well in this case.

This paper is organized as follows. We present the basic notations and definitions in Sec. II. In Sec. III, we list the chiral corrections to the threshold T -matrices of the meson decuplet baryon scattering, which contains our main results. The LECs are estimated in Sec. IV. Finally, we give the numerical results and our discussions in Sec. V.

II. LAGRANGIANS

In the heavy baryon formalism, the chiral Lagrangians at the leading order are

$$\mathcal{L}_{\phi\phi}^{(2)} = f^2 \text{Tr} \left(u_\mu u^\mu + \frac{\chi_+}{4} \right), \quad (1)$$

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$$\begin{aligned} \mathcal{L}_{\phi B}^{(1)} = & \text{Tr}(\bar{B}(i\mathbf{v} \cdot \partial B + [\mathbf{v} \cdot \Gamma, B])) + 2D \text{Tr}(B\{S \cdot u, B\}) \\ & + 2F \text{Tr}(\bar{B}[S \cdot u, B]), \end{aligned} \quad (2)$$

$$\begin{aligned} \mathcal{L}_{\phi BT}^{(1)} = & -\bar{T}^\mu(i\mathbf{v} \cdot D - \delta)T_\mu + \mathcal{C}(\bar{T}^\mu u_\mu B + \bar{B}u_\mu T^\mu) \\ & + 2\mathcal{H}\bar{T}^\mu S \cdot u T_\mu. \end{aligned} \quad (3)$$

Here f is the decay constant of the pseudoscalar meson in the chiral limit, \mathbf{v} is the velocity of the baryon, S_μ is the spin operator, and δ is the mass difference between Δ and the nucleon. For the coupling constants, $D + F = g_A = 1.26$ where g_A is the axial vector coupling constant, $|\mathcal{C}| = 1.5$, and $|\mathcal{H}| = 1.9$. The D and F terms do not contribute to the threshold amplitudes we calculate. The superscript in the Lagrangians represents the corresponding chiral order. The notations for the fields read

$$\begin{aligned} \Gamma_\mu &= \frac{i}{2}[\xi^\dagger, \partial_\mu \xi], \\ u_\mu &= \frac{i}{2}\{\xi^\dagger, \partial_\mu \xi\}, \\ \xi &= \exp(i\phi/2f), \end{aligned} \quad (4)$$

$$\begin{aligned} \chi_\pm &= \xi^\dagger \chi \xi^\dagger \pm \xi \chi \xi, \\ \chi &= \text{diag}(m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2), \end{aligned} \quad (5)$$

$$\begin{aligned} \phi &= \sqrt{2} \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}, \\ B &= \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}, \end{aligned} \quad (6)$$

$$\begin{aligned} \mathcal{L}_{\phi BT}^{(3)} = & -\frac{\mathcal{H}^2}{6M^2} \bar{T}_\mu^{ijk} \mathbf{v} \cdot u_i^s (i\mathbf{v} \cdot D + \delta) \mathbf{v} \cdot u_j^l T_{slk}^\mu - \frac{\mathcal{H}^2}{12M^2} \bar{T}_\mu^{ijk} \mathbf{v} \cdot u_i^s (i\mathbf{v} \cdot D + \delta) \mathbf{v} \cdot u_s^l T_{ljk}^\mu \\ & + \{ (h_1 \bar{T}_\mu^{ijk} u_{\nu i}^s u_{\rho j}^l \mathbf{v}^\nu iD^\rho T_{slk}^\mu + \text{H.c.}) + (h_2 \bar{T}_\mu^{ijk} u_{\nu i}^s u_{\rho s}^l \mathbf{v}^\nu iD^\rho T_{ljk}^\mu + \text{H.c.}) + h_3 \text{Tr}(u_\nu u_\rho) \bar{T}^\mu \mathbf{v}^\nu iD^\rho T_\mu \\ & + (h_4 \bar{T}_\mu \tilde{\chi}_+ i\mathbf{v} \cdot D T^\mu + \text{H.c.}) + (h_5 \bar{T}_\mu^{ijk} \tilde{\chi}_{-i}^s \mathbf{v} \cdot u_s^l T_{ljk}^\mu + \text{H.c.}) \}, \end{aligned} \quad (10)$$

where $\tilde{\chi}_\pm = \chi_\pm - \frac{1}{3} \text{Tr}(\chi_\pm)$ and M is the mass of the nucleon. In principle, the unknown coefficients in the Lagrangians could be determined from either QCD or the experimental measurements. Because of the difficulty in the nonperturbative region of QCD and the lack of experimental data, we will resort to the resonance saturation method when estimating LECs [27].

III. THE T -MATRICES AT THRESHOLDS

The total S-wave scattering cross section near the threshold can be expressed in terms of the scattering length. It

$$\begin{aligned} iD_\mu T_{abc}^\nu = & i\partial_\mu T_{abc}^\nu + (\Gamma_\mu)_a^d T_{dbc}^\nu + (\Gamma_\mu)_b^d T_{adc}^\nu \\ & + (\Gamma_\mu)_c^d T_{abd}^\nu, \end{aligned} \quad (7)$$

and

$$\begin{aligned} T_{111} &= \Delta^{++}, & T_{112} &= \frac{\Delta^+}{\sqrt{3}}, & T_{122} &= \frac{\Delta^0}{\sqrt{3}}, \\ T_{222} &= \Delta^-, & T_{113} &= \frac{\Sigma^{*+}}{\sqrt{3}}, & T_{123} &= \frac{\Sigma^{*0}}{\sqrt{6}}, \\ T_{223} &= \frac{\Sigma^{*-}}{\sqrt{3}}, & T_{133} &= \frac{\Xi^{*0}}{\sqrt{3}}, & T_{233} &= \frac{\Xi^{*-}}{\sqrt{3}}, \\ T_{333} &= \Omega^-. \end{aligned} \quad (8)$$

The HB χ PT Lagrangians consist of two parts at higher order [26]: the recoil corrections and counter terms. The former part comes from the lower order relativistic chiral Lagrangians. The counter terms contain LECs and are constructed with respect to QCD symmetries. To $\mathcal{O}(\epsilon^3)$, the following high-order Lagrangians contribute in our calculation:

$$\begin{aligned} \mathcal{L}_{\phi BT}^{(2)} = & \frac{\mathcal{H}^2}{3M} \bar{T}_\mu^{ijk} \mathbf{v} \cdot u_i^s \mathbf{v} \cdot u_j^l T_{slk}^\mu + \frac{\mathcal{H}^2}{6M} \bar{T}_\mu^{ijk} \mathbf{v} \cdot u_i^s \\ & \times \mathbf{v} \cdot u_s^l T_{ljk}^\mu + \{d_1 \bar{T}_\mu^{ijk} \mathbf{v} \cdot u_i^s \mathbf{v} \cdot u_j^l T_{slk}^\mu \\ & + d_2 \bar{T}_\mu^{ijk} \mathbf{v} \cdot u_i^s \mathbf{v} \cdot u_s^l T_{ljk}^\mu + d_3 \text{Tr}(\mathbf{v} \cdot u \mathbf{v} \cdot u) \bar{T}_\mu T^\mu \\ & + d_4 \bar{T}_\mu \tilde{\chi}_+ T^\mu\}, \end{aligned} \quad (9)$$

can also be calculated directly using the threshold T -matrix T_{PT} . To ensure the same total cross section in quantum mechanics, the scattering length a_{PT} is related with the threshold T -matrix T_{PT} : $T_{PT} = 4\pi(1 + \frac{m_P}{M_T})a_{PT}$, where m_P and M_T are the masses of the pseudoscalar meson and the light decuplet baryon respectively. We use a convention that $a_{PT} > 0$ ($a_{PT} < 0$) if the interaction is attractive (repulsive). There are 27 independent isospin invariant amplitudes for the octet meson-decuplet baryon interactions. We list their explicit expressions in the following subsections. One may get any other amplitudes with the help of the isospin symmetry.

The first two order T -matrices are easy to derive from the Lagrangians (3) and (9). At the third order, loop corrections enter. There are 18 nonvanishing diagrams contributing to the threshold amplitudes, which are shown in Fig. 1. We calculate the loop integrations with the dimensional regularization and modified minimal subtraction. One may find useful integrals in the appendixes of Refs. [7,28]. The divergences from the loops can be absorbed through the LECs h_i ,

$$\begin{aligned} h_1 &= C^2 L + h_1', & h_2 &= \frac{1}{2} C^2 L + h_2', \\ h_3 &= -\frac{1}{3} C^2 L + h_3', & h_4 &= \frac{1}{8} C^2 L + h_4', \\ h_5 &= -\frac{9}{4} L + h_5', \end{aligned} \quad (11)$$

where

$$L = \frac{\lambda^{D-4}}{16\pi^2} \left\{ \frac{1}{D-4} + \frac{1}{2} (\gamma_E - 1 - \ln 4\pi) \right\}, \quad (12)$$

(Euler constant $\gamma_E = 0.5772157$),

and λ is the scale from the dimensional regularization.

To make the final expressions short, we introduce a constant and two functions,

$$\begin{aligned} J &= \frac{C^2 m_K^2}{12\pi^2 f^4 (m_\eta^2 - m_\pi^2)} \left\{ \left(\delta^3 - \frac{3}{2} m_\eta^2 \delta \right) \log \frac{m_\eta}{\lambda} \right. \\ &+ \left(\frac{3}{2} m_\pi^2 \delta - \delta^3 \right) \log \frac{m_\pi}{\lambda} - (\delta^2 - m_\pi^2)^{3/2} \\ &\times \left(\log \frac{\sqrt{\delta^2 - m_\pi^2} + \delta}{m_\pi} - i\pi \right) - (m_\eta^2 - \delta^2)^{3/2} \cos^{-1} \\ &\left. \times \left(-\frac{\delta}{m_\eta} \right) + (m_\eta^2 - m_\pi^2) \delta \right\}, \quad (13) \end{aligned}$$

$$W(m) = \frac{C^2}{16\pi^2 f^4} \begin{cases} -2\sqrt{m^2 - \delta^2} \cos^{-1} \left(-\frac{\delta}{m} \right) - 2\delta \log \frac{m}{\lambda} + \delta & m > \delta \\ -2\delta \log \frac{m}{\lambda} - 2\sqrt{\delta^2 - m^2} \log \frac{\sqrt{\delta^2 - m^2} + \delta}{m} + \delta + 2\pi i \sqrt{\delta^2 - m^2} & m \leq \delta \end{cases}, \quad (14)$$

$$V(m^2, \omega) = \frac{\omega^3 \log \frac{m}{\lambda}}{\pi^2 f^4} - \frac{\omega^3}{2\pi^2 f^4} - \frac{\omega^2}{\pi^2 f^4} \begin{cases} -\sqrt{m^2 - \omega^2} \cos^{-1} \left(-\frac{\omega}{m} \right) & m^2 \geq \omega^2 \\ \sqrt{\omega^2 - m^2} \log \frac{\sqrt{\omega^2 - m^2} - \omega}{m} & m^2 < \omega^2, \omega < 0 \\ \sqrt{\omega^2 - m^2} \left(-\log \frac{\sqrt{\omega^2 - m^2} + \omega}{m} + i\pi \right) & m^2 < \omega^2, \omega \geq 0 \end{cases} \quad (15)$$

In deriving the final T -matrices, we have used the Gell-Mann–Okubo mass relation $m_\eta^2 = (4m_K^2 - m_\pi^2)/3$, which makes the expressions more concise.

A. Pion-decuplet scattering

There are 9 independent T -matrices for the pion scattering due to the isospin symmetry. At the leading chiral order $O(\epsilon)$, only the simplest tree diagram from the Weinberg-Tomozawa terms is involved,

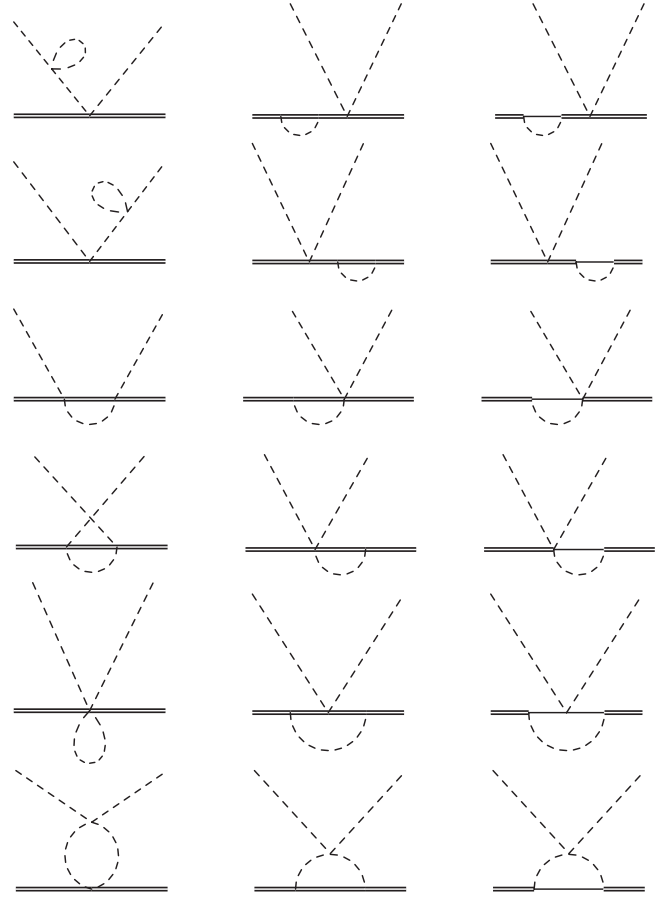


FIG. 1. Nonvanishing loop diagrams for the pseudoscalar meson-light decuplet baryon scattering lengths to $O(\epsilon^3)$ in HB χ PT with explicit Δ . Dashed lines, thin solid lines, and thick solid lines represent Goldstone bosons, octet baryons, and decuplet baryons, respectively.

$$\begin{aligned}
T_{\pi\Delta}^{(5/2)} &= -\frac{3m_\pi}{2f_\pi^2}, & T_{\pi\Delta}^{(3/2)} &= \frac{m_\pi}{f_\pi^2}, & T_{\pi\Delta}^{(1/2)} &= \frac{5m_\pi}{2f_\pi^2}, & T_{\pi\Sigma^*}^{(2)} &= -\frac{m_\pi}{f_\pi^2}, & T_{\pi\Sigma^*}^{(1)} &= \frac{m_\pi}{f_\pi^2}, \\
T_{\pi\Sigma^*}^{(0)} &= \frac{2m_\pi}{f_\pi^2}, & T_{\pi\Xi^*}^{(3/2)} &= -\frac{m_\pi}{2f_\pi^2}, & T_{\pi\Xi^*}^{(1/2)} &= \frac{m_\pi}{f_\pi^2}, & T_{\pi\Omega}^{(1)} &= 0,
\end{aligned} \tag{16}$$

where the superscript in the bracket represents the total isospin of the channel and f_π is the renormalized pion decay constant (similar to f_K, f_η in the following text). Since we express the T -matrices with f_π rather than f , the difference is of order $O(\epsilon^3)$. Explicitly, the renormalized decay constants to the next leading order read [29]

$$\begin{aligned}
f_\pi &= f \left\{ 1 - \frac{3}{2} \left[\frac{m_\pi^2}{12\pi^2 f^2} \ln \frac{m_\pi}{\lambda} + \frac{m_K^2}{24\pi^2 f^2} \ln \frac{m_K}{\lambda} \right] - \frac{1}{2} \left[-8L_4^r \frac{2m_K^2 + m_\pi^2}{f^2} - 8L_5^r \frac{m_\pi^2}{f^2} \right] \right. \\
&\quad \left. + \mathcal{O}(p^4) \right\}, \\
f_K &= f \left\{ 1 - \frac{3}{2} \left[\frac{m_K^2}{16\pi^2 f^2} \ln \frac{m_K}{\lambda} + \frac{m_\eta^2}{32\pi^2 f^2} \ln \frac{m_\eta}{\lambda} + \frac{m_\pi^2}{32\pi^2 f^2} \ln \frac{m_\pi}{\lambda} \right] \right. \\
&\quad \left. - \frac{1}{2} \left[-8L_4^r \frac{2m_K^2 + m_\pi^2}{f^2} - 8L_5^r \frac{m_K^2}{f^2} \right] + \mathcal{O}(p^4) \right\}, \\
f_\eta &= f \left\{ 1 - \frac{3}{2} \left[\frac{m_K^2}{8\pi^2 f^2} \ln \frac{m_K}{\lambda} \right] - \frac{1}{2} \left[-8L_4^r \frac{2m_K^2 + m_\pi^2}{f^2} - \frac{8}{3} L_5^r \frac{4m_K^2 - m_\pi^2}{f^2} \right] \right. \\
&\quad \left. + \mathcal{O}(p^4) \right\}.
\end{aligned} \tag{17}$$

In the above formulas, L_4^r and L_5^r are the renormalized LECs from the pseudoscalar meson Lagrangian [29]. However, one does not care about relevant terms. They will not appear in the loop results because the wave function renormalization of the mesons gives the same terms and they finally cancel out.

The results at $O(\epsilon^2)$ also come from the tree diagram,

$$\begin{aligned}
T_{\pi\Delta}^{(5/2)} &= -\frac{m_\pi^2(3\tilde{d}_2 + 6d_3 - 4d_4)}{6f_\pi^2}, & T_{\pi\Delta}^{(3/2)} &= -\frac{m_\pi^2(5\tilde{d}_1 + 3\tilde{d}_2 + 6d_3 - 4d_4)}{6f_\pi^2}, \\
T_{\pi\Delta}^{(1/2)} &= \frac{m_\pi^2(4\tilde{d}_1 - 3\tilde{d}_2 - 6d_3 + 4d_4)}{6f_\pi^2}, & T_{\pi\Sigma^*}^{(2)} &= -\frac{m_\pi^2(\tilde{d}_2 + 3d_3)}{3f_\pi^2}, \\
T_{\pi\Sigma^*}^{(1)} &= -\frac{m_\pi^2(\tilde{d}_1 + \tilde{d}_2 + 3d_3)}{3f_\pi^2}, & T_{\pi\Sigma^*}^{(0)} &= \frac{m_\pi^2(3\tilde{d}_1 - 2\tilde{d}_2 - 6d_3)}{6f_\pi^2}, \\
T_{\pi\Xi^*}^{(3/2)} &= -\frac{m_\pi^2(\tilde{d}_2 + 6d_3 + 4d_4)}{6f_\pi^2}, & T_{\pi\Xi^*}^{(1/2)} &= -\frac{m_\pi^2(\tilde{d}_2 + 6d_3 + 4d_4)}{6f_\pi^2}, \\
T_{\pi\Omega}^{(1)} &= -\frac{m_\pi^2(3d_3 + 4d_4)}{3f_\pi^2},
\end{aligned} \tag{18}$$

where

$$\tilde{d}_1 = d_1 + \frac{\mathcal{H}^2}{3M}, \quad \tilde{d}_2 = d_2 + \frac{\mathcal{H}^2}{6M}. \tag{19}$$

At $O(\epsilon^3)$, the T -matrices contain both loop and tree diagram contributions. Moreover, they receive contributions from the replacement of f with f_π in the $O(\epsilon^1)$ magnitude, that is, the log terms in Eq. (17) should be incorporated. Just for the simplicity of the writing and the reuse of the expressions at $O(\epsilon^2)$, we divide T into two parts at this order,

$$T = \tilde{T} + T_2. \quad (20)$$

One obtains T_2 from the $O(\epsilon^2)$ T -matrices (18) with the following replacements:

$$\tilde{d}_1 \rightarrow 2h'_1 \delta - \frac{\mathcal{H}^2 \delta}{3M^2}, \quad \tilde{d}_2 \rightarrow 2h'_2 - \frac{\mathcal{H}^2 \delta}{6M^2}, \quad d_3 \rightarrow h'_3 \delta, \quad d_4 \rightarrow 2h'_4 \delta. \quad (21)$$

\tilde{T} are listed below,

$$\begin{aligned} \tilde{T}_{\pi\Delta}^{(5/2)} &= -\frac{3}{16}V(m_K^2, -m_\pi) - \frac{15}{16}V(m_\pi^2, -m_\pi) - \frac{9}{16}V(m_\pi^2, m_\pi) - \frac{5\mathcal{H}^2 m_\eta m_\pi^2}{1728\pi f_\pi^4} \\ &\quad + \frac{5\mathcal{H}^2 m_\pi^3}{192\pi f_\pi^4} + \frac{4\tilde{h}_5 m_\pi^3}{f_\pi^2}, \\ \tilde{T}_{\pi\Delta}^{(3/2)} &= -\frac{1}{32}V(m_K^2, -m_\pi) - \frac{5}{32}V(m_K^2, m_\pi) - \frac{1}{4}V(m_\pi^2, m_\pi) - \frac{5\mathcal{H}^2 m_\eta m_\pi^2}{1728\pi f_\pi^4} - \frac{55\mathcal{H}^2 m_\pi^3}{1728\pi f_\pi^4} \\ &\quad - \frac{5}{6}m_\pi^2 W(m_\pi) - \frac{8\tilde{h}_5 m_\pi^3}{3f_\pi^2}, \\ \tilde{T}_{\pi\Delta}^{(1/2)} &= \frac{1}{16}V(m_K^2, -m_\pi) - \frac{1}{4}V(m_K^2, m_\pi) - \frac{15}{16}V(m_\pi^2, -m_\pi) - \frac{25}{16}V(m_\pi^2, m_\pi) - \frac{5\mathcal{H}^2 m_\eta m_\pi^2}{1728\pi f_\pi^4} + \frac{125\mathcal{H}^2 m_\pi^3}{1728\pi f_\pi^4} \\ &\quad + \frac{2}{3}m_\pi^2 W(m_\pi) - \frac{20\tilde{h}_5 m_\pi^3}{3f_\pi^2}, \\ \tilde{T}_{\pi\Sigma^*}^{(2)} &= -\frac{5}{16}V(m_K^2, -m_\pi) - \frac{3}{16}V(m_K^2, m_\pi) - \frac{1}{2}V(m_\pi^2, -m_\pi) - \frac{1}{4}V(m_\pi^2, m_\pi) + \frac{5\mathcal{H}^2 m_\pi^3}{432\pi f_\pi^4} + \frac{1}{12}m_\pi^2 W(m_\eta) \\ &\quad - \frac{1}{12}m_\pi^2 W(m_\pi) + \frac{8\tilde{h}_5 m_\pi^3}{3f_\pi^2}, \\ \tilde{T}_{\pi\Sigma^*}^{(1)} &= -\frac{3}{16}V(m_K^2, -m_\pi) - \frac{5}{16}V(m_K^2, m_\pi) - \frac{1}{4}V(m_\pi^2, m_\pi) - \frac{5\mathcal{H}^2 m_\pi^3}{432\pi f_\pi^4} + \frac{1}{12}m_\pi^2 W(m_\eta) \\ &\quad - \frac{5}{12}m_\pi^2 W(m_\pi) - \frac{8\tilde{h}_5 m_\pi^3}{3f_\pi^2}, \\ \tilde{T}_{\pi\Sigma^*}^{(0)} &= -\frac{1}{8}V(m_K^2, -m_\pi) - \frac{3}{8}V(m_K^2, m_\pi) - \frac{1}{2}V(m_\pi^2, -m_\pi) - V(m_\pi^2, m_\pi) + \frac{5\mathcal{H}^2 m_\pi^3}{108\pi f_\pi^4} + \frac{1}{12}m_\pi^2 W(m_\eta) \\ &\quad + \frac{5}{12}m_\pi^2 W(m_\pi) - \frac{16\tilde{h}_5 m_\pi^3}{3f_\pi^2}, \\ \tilde{T}_{\pi\Xi^*}^{(3/2)} &= -\frac{5}{16}V(m_K^2, -m_\pi) - \frac{1}{4}V(m_K^2, m_\pi) - \frac{3}{16}V(m_\pi^2, -m_\pi) - \frac{1}{16}V(m_\pi^2, m_\pi) - \frac{5\mathcal{H}^2 m_\eta m_\pi^2}{1728\pi f_\pi^4} + \frac{5\mathcal{H}^2 m_\pi^3}{1728\pi f_\pi^4} \\ &\quad + \frac{1}{12}m_\pi^2 W(m_\eta) - \frac{1}{12}m_\pi^2 W(m_\pi) + \frac{4\tilde{h}_5 m_\pi^3}{3f_\pi^2}, \\ \tilde{T}_{\pi\Xi^*}^{(1/2)} &= -\frac{7}{32}V(m_K^2, -m_\pi) - \frac{11}{32}V(m_K^2, m_\pi) - \frac{1}{4}V(m_\pi^2, m_\pi) - \frac{5\mathcal{H}^2 m_\eta m_\pi^2}{1728\pi f_\pi^4} + \frac{5\mathcal{H}^2 m_\pi^3}{1728\pi f_\pi^4} + \frac{1}{12}m_\pi^2 W(m_\eta) \\ &\quad - \frac{1}{12}m_\pi^2 W(m_\pi) - \frac{8\tilde{h}_5 m_\pi^3}{3f_\pi^2}, \\ \tilde{T}_{\pi\Omega}^{(1)} &= -\frac{3}{16}V(m_K^2, -m_\pi) - \frac{3}{16}V(m_K^2, m_\pi) - \frac{5\mathcal{H}^2 m_\eta m_\pi^2}{432\pi f_\pi^4}, \end{aligned} \quad (22)$$

where

$$\tilde{h}_5 = h_5^r - \frac{\mathcal{H}^2}{96M^2}. \quad (23)$$

B. Kaon-decuplet scattering

At the leading order, there are 14 independent T -matrices, which are

$$\begin{aligned} T_{K\Delta}^{(2)} &= -\frac{3m_K}{2f_K^2}, & T_{K\Delta}^{(1)} &= \frac{m_K}{2f_K^2}, & T_{K\Sigma^*}^{(3/2)} &= -\frac{m_K}{2f_K^2}, \\ T_{K\Sigma^*}^{(1/2)} &= \frac{m_K}{f_K^2}, & T_{K\Xi^*}^{(1)} &= \frac{m_K}{2f_K^2}, & T_{K\Xi^*}^{(0)} &= \frac{3m_K}{2f_K^2}, \\ T_{K\Omega}^{(1/2)} &= \frac{3m_K}{2f_K^2}, & T_{\bar{K}\Delta}^{(2)} &= 0, & T_{\bar{K}\Delta}^{(1)} &= \frac{2m_K}{f_K^2}, \\ T_{\bar{K}\Sigma^*}^{(3/2)} &= -\frac{m_K}{2f_K^2}, & T_{\bar{K}\Sigma^*}^{(1/2)} &= \frac{m_K}{f_K^2}, & T_{\bar{K}\Xi^*}^{(1)} &= -\frac{m_K}{f_K^2}, \\ T_{\bar{K}\Xi^*}^{(0)} &= 0, & T_{\bar{K}\Omega}^{(1/2)} &= -\frac{3m_K}{2f_K^2}. \end{aligned} \quad (24)$$

At the next leading order, we have

$$\begin{aligned} T_{K\Delta}^{(2)} &= -\frac{m_K^2(3\tilde{d}_2 + 6d_3 - 4d_4)}{6f_K^2}, & T_{K\Delta}^{(1)} &= \frac{m_K^2(\tilde{d}_2 - 6d_3 - 12d_4)}{6f_K^2}, \\ T_{K\Sigma^*}^{(3/2)} &= -\frac{m_K^2(2\tilde{d}_1 + 3\tilde{d}_2 + 6d_3 - 4d_4)}{6f_K^2}, & T_{K\Sigma^*}^{(1/2)} &= \frac{m_K^2(\tilde{d}_1 - 6d_3 - 8d_4)}{6f_K^2}, \\ T_{K\Xi^*}^{(1)} &= -\frac{m_K^2(2\tilde{d}_1 + 3\tilde{d}_2 + 6d_3 - 4d_4)}{6f_K^2}, & T_{K\Xi^*}^{(0)} &= \frac{m_K^2(2\tilde{d}_1 - \tilde{d}_2 - 6d_3 - 4d_4)}{6f_K^2}, \\ T_{K\Omega}^{(1/2)} &= -\frac{m_K^2(3\tilde{d}_2 + 6d_3 - 4d_4)}{6f_K^2}, & T_{\bar{K}\Delta}^{(2)} &= -\frac{m_K^2(3d_3 + 4d_4)}{3f_K^2}, \\ T_{\bar{K}\Delta}^{(1)} &= -\frac{m_K^2(2\tilde{d}_2 + 3d_3 - 4d_4)}{3f_K^2}, & T_{\bar{K}\Sigma^*}^{(3/2)} &= -\frac{m_K^2(\tilde{d}_2 + 6d_3 + 4d_4)}{6f_K^2}, \\ T_{\bar{K}\Sigma^*}^{(1/2)} &= -\frac{m_K^2(3\tilde{d}_1 + 4\tilde{d}_2 + 6d_3 - 8d_4)}{6f_K^2}, & T_{\bar{K}\Xi^*}^{(1)} &= -\frac{m_K^2(\tilde{d}_2 + 3d_3)}{3f_K^2}, \\ T_{\bar{K}\Xi^*}^{(0)} &= -\frac{m_K^2(2\tilde{d}_1 + 2\tilde{d}_2 + 3d_3 - 4d_4)}{3f_K^2}, & T_{\bar{K}\Omega}^{(1/2)} &= -\frac{m_K^2(3\tilde{d}_2 + 6d_3 - 4d_4)}{6f_K^2}. \end{aligned} \quad (25)$$

At the next-next leading order, we just list \tilde{T} here. One may get the total T -matrices from Eqs. (20) and (21):

$$\begin{aligned}
\tilde{T}_{K\Delta}^{(2)} &= -\frac{3}{4}V(m_K^2, -m_K) - \frac{9}{16}V(m_K^2, m_K) - \frac{9}{32}V(m_\eta^2, -m_K) - \frac{3}{32}V(m_\pi^2, -m_K) + \frac{5\mathcal{H}^2 m_K^2 m_\pi^2}{432\pi f_K^4(m_\eta + m_\pi)} \\
&\quad + \frac{5\mathcal{H}^2 m_K^2 m_\eta}{288\pi f_K^4} + \frac{4\tilde{h}_5 m_K^3}{f_K^2}, \\
\tilde{T}_{K\Delta}^{(1)} &= \frac{1}{4}V(m_K^2, -m_K) - \frac{1}{16}V(m_K^2, m_K) + \frac{3}{32}V(m_\eta^2, -m_K) - \frac{7}{32}V(m_\pi^2, -m_K) - \frac{25\mathcal{H}^2 m_K^2 m_\pi^2}{1296\pi f_K^4(m_\eta + m_\pi)} \\
&\quad - \frac{35\mathcal{H}^2 m_K^2 m_\eta}{2592\pi f_K^4} - \frac{4\tilde{h}_5 m_K^3}{3f_K^2}, \\
\tilde{T}_{K\Sigma^*}^{(3/2)} &= -\frac{3}{16}V(m_K^2, -m_K) - \frac{1}{16}V(m_K^2, m_K) - \frac{3}{8}V(m_\eta^2, -m_K) - \frac{9}{32}V(m_\eta^2, m_K) - \frac{1}{8}V(m_\pi^2, -m_K) - \frac{5}{32}V(m_\pi^2, m_K) \\
&\quad - \frac{J}{6} - \frac{1}{6}m_K^2 W(m_\eta) + \frac{4\tilde{h}_5 m_K^3}{3f_K^2}, \\
\tilde{T}_{K\Sigma^*}^{(1/2)} &= -\frac{1}{4}V(m_K^2, m_K) + \frac{3}{16}V(m_\eta^2, -m_K) - \frac{5}{16}V(m_\pi^2, -m_K) - \frac{1}{4}V(m_\pi^2, m_K) + \frac{J}{3} - \frac{1}{6}m_K^2 W(m_\eta) - \frac{8\tilde{h}_5 m_K^3}{3f_K^2}, \\
\tilde{T}_{K\Xi^*}^{(1)} &= -\frac{1}{8}V(m_K^2, -m_K) - \frac{1}{16}V(m_K^2, m_K) - \frac{9}{32}V(m_\eta^2, -m_K) - \frac{3}{8}V(m_\eta^2, m_K) - \frac{3}{32}V(m_\pi^2, -m_K) - \frac{1}{4}V(m_\pi^2, m_K) \\
&\quad - \frac{5\mathcal{H}^2 m_K^2 m_\pi^2}{1296\pi f_K^4(m_\eta + m_\pi)} + \frac{5\mathcal{H}^2 m_K^2 m_\eta}{2592\pi f_K^4} - \frac{J}{6} - \frac{1}{6}m_K^2 W(m_\eta) - \frac{4\tilde{h}_5 m_K^3}{3f_K^2}, \\
\tilde{T}_{K\Xi^*}^{(0)} &= -\frac{3}{8}V(m_K^2, -m_K) - \frac{9}{16}V(m_K^2, m_K) + \frac{9}{32}V(m_\eta^2, -m_K) - \frac{9}{32}V(m_\pi^2, -m_K) - \frac{3}{8}V(m_\pi^2, m_K) + \frac{5\mathcal{H}^2 m_K^2 m_\pi^2}{432\pi f_K^4(m_\eta + m_\pi)} \\
&\quad + \frac{5\mathcal{H}^2 m_K^2 m_\eta}{288\pi f_K^4} + \frac{J}{2} - \frac{1}{6}m_K^2 W(m_\eta) - \frac{4\tilde{h}_5 m_K^3}{f_K^2}, \\
\tilde{T}_{K\Omega}^{(1/2)} &= -\frac{9}{16}V(m_K^2, -m_K) - \frac{9}{16}V(m_K^2, m_K) - \frac{9}{32}V(m_\eta^2, m_K) - \frac{9}{32}V(m_\pi^2, m_K) + \frac{5\mathcal{H}^2 m_K^2 m_\eta}{216\pi f_K^4} - \frac{4\tilde{h}_5 m_K^3}{f_K^2}, \\
\tilde{T}_{K\Delta}^{(2)} &= -\frac{3}{16}V(m_K^2, -m_K) - \frac{3}{16}V(m_\pi^2, m_K) - \frac{5\mathcal{H}^2 m_K^2 m_\pi^2}{432\pi f_K^4(m_\eta + m_\pi)} - \frac{5\mathcal{H}^2 m_K^2 m_\eta}{864\pi f_K^4}, \\
\tilde{T}_{K\Delta}^{(1)} &= -\frac{11}{16}V(m_K^2, -m_K) - V(m_K^2, m_K) - \frac{3}{8}V(m_\eta^2, m_K) - \frac{1}{16}V(m_\pi^2, m_K) + \frac{25\mathcal{H}^2 m_K^2 m_\pi^2}{1296\pi f_K^4(m_\eta + m_\pi)} + \frac{65\mathcal{H}^2 m_K^2 m_\eta}{2592\pi f_K^4} - \frac{16\tilde{h}_5 m_K^3}{3f_K^2}, \\
\tilde{T}_{K\Sigma^*}^{(3/2)} &= -\frac{3}{16}V(m_K^2, -m_K) - \frac{1}{16}V(m_K^2, m_K) - \frac{3}{32}V(m_\eta^2, -m_K) - \frac{7}{32}V(m_\pi^2, -m_K) - \frac{1}{4}V(m_\pi^2, m_K) + \frac{J}{6} - \frac{1}{6}m_K^2 W(m_\eta) + \frac{4\tilde{h}_5 m_K^3}{3f_K^2}, \\
\tilde{T}_{K\Sigma^*}^{(1/2)} &= -\frac{1}{4}V(m_K^2, m_K) - \frac{3}{8}V(m_\eta^2, -m_K) - \frac{9}{16}V(m_\eta^2, m_K) - \frac{1}{8}V(m_\pi^2, -m_K) - \frac{1}{16}V(m_\pi^2, m_K) - \frac{J}{3} - \frac{1}{6}m_K^2 W(m_\eta) - \frac{8\tilde{h}_5 m_K^3}{3f_K^2}, \\
\tilde{T}_{K\Xi^*}^{(1)} &= -\frac{5}{16}V(m_K^2, -m_K) - \frac{1}{4}V(m_K^2, m_K) - \frac{3}{16}V(m_\eta^2, -m_K) - \frac{5}{16}V(m_\pi^2, -m_K) - \frac{3}{16}V(m_\pi^2, m_K) + \frac{5\mathcal{H}^2 m_K^2 m_\pi^2}{1296\pi f_K^4(m_\eta + m_\pi)} \\
&\quad + \frac{25\mathcal{H}^2 m_K^2 m_\eta}{2592\pi f_K^4} + \frac{J}{6} - \frac{1}{6}m_K^2 W(m_\eta) + \frac{8\tilde{h}_5 m_K^3}{3f_K^2}, \\
\tilde{T}_{K\Xi^*}^{(0)} &= \frac{3}{16}V(m_K^2, -m_K) - \frac{9}{16}V(m_\eta^2, -m_K) - \frac{9}{16}V(m_\eta^2, m_K) - \frac{3}{16}V(m_\pi^2, -m_K) \\
&\quad - \frac{5\mathcal{H}^2 m_K^2 m_\pi^2}{432\pi f_K^4(m_\eta + m_\pi)} - \frac{5\mathcal{H}^2 m_K^2 m_\eta}{864\pi f_K^4} - \frac{J}{2} - \frac{1}{6}m_K^2 W(m_\eta), \\
\tilde{T}_{K\Omega}^{(1/2)} &= -\frac{9}{16}V(m_K^2, -m_K) - \frac{9}{16}V(m_K^2, m_K) - \frac{9}{32}V(m_\eta^2, -m_K) - \frac{9}{32}V(m_\pi^2, -m_K) + \frac{5\mathcal{H}^2 m_K^2 m_\eta}{216\pi f_K^4} + \frac{4\tilde{h}_5 m_K^3}{f_K^2}. \tag{26}
\end{aligned}$$

C. Eta-decuplet scattering

Eta and decuplet baryon T -matrices start at the second chiral order,

$$\begin{aligned} T_{\eta\Delta}^{(3/2)} &= -\frac{(\tilde{d}_1 + \tilde{d}_2 + 6d_3 + 8d_4)m_\eta^2}{6f_\eta^2} + \frac{2d_4m_\pi^2}{3f_\eta^2}, & T_{\eta\Sigma^*}^{(1)} &= \frac{(\tilde{d}_1 - 2\tilde{d}_2 - 6d_3)m_\eta^2}{6f_\eta^2}, \\ T_{\eta\Xi^*}^{(1/2)} &= -\frac{(3\tilde{d}_2 + 6d_3 - 8d_4)m_\eta^2}{6f_\eta^2} - \frac{2d_4m_\pi^2}{3f_\eta^2}, & T_{\eta\Omega}^{(0)} &= \frac{(2\tilde{d}_1 + 2\tilde{d}_2 + 3d_3 - 8d_3 - 8d_4)m_\eta^2}{3f_\eta^2} - \frac{4d_4m_\pi^2}{3f_\eta^2}. \end{aligned} \quad (27)$$

At the third order, we have

$$\begin{aligned} \tilde{T}_{\eta\Delta}^{(3/2)} &= -\frac{9}{32}V(m_K^2, -m_\eta) - \frac{9}{32}V(m_K^2, m_\eta) + \frac{5\mathcal{H}^2m_K^3}{432\pi f_\eta^4} - \frac{5\mathcal{H}^2m_K^2m_\eta}{324\pi f_\eta^4} + \frac{35\mathcal{H}^2m_\eta m_\pi^2}{5184\pi f_\eta^4} - \frac{25\mathcal{H}^2m_\pi^3}{1728\pi f_\eta^4} \\ &\quad - \frac{1}{3}m_K^2W(m_K) + \frac{1}{6}m_\pi^2W(m_\pi), \\ \tilde{T}_{\eta\Sigma^*}^{(1)} &= -\frac{3}{4}V(m_K^2, -m_\eta) - \frac{3}{4}V(m_K^2, m_\eta) + \frac{5\mathcal{H}^2m_K^3}{162\pi f_\eta^4} - \frac{5\mathcal{H}^2m_\pi^3}{648\pi f_\eta^4} + \frac{4}{9}m_K^2W(m_\eta) - \frac{2}{9}m_K^2W(m_K) \\ &\quad - \frac{7}{36}m_\pi^2W(m_\eta) + \frac{5}{36}m_\pi^2W(m_\pi), \\ \tilde{T}_{\eta\Xi^*}^{(1/2)} &= -\frac{27}{32}V(m_K^2, -m_\eta) - \frac{27}{32}V(m_K^2, m_\eta) + \frac{5\mathcal{H}^2m_K^3}{144\pi f_\eta^4} - \frac{5\mathcal{H}^2m_K^2m_\eta}{324\pi f_\eta^4} + \frac{35\mathcal{H}^2m_\eta m_\pi^2}{5184\pi f_\eta^4} - \frac{5\mathcal{H}^2m_\pi^3}{1728\pi f_\eta^4} + \frac{4}{9}m_K^2W(m_\eta) \\ &\quad - \frac{1}{3}m_K^2W(m_K) - \frac{7}{36}m_\pi^2W(m_\eta) + \frac{1}{12}m_\pi^2W(m_\pi), \\ \tilde{T}_{\eta\Omega}^{(0)} &= -\frac{9}{16}V(m_K^2, -m_\eta) - \frac{9}{16}V(m_K^2, m_\eta) + \frac{5\mathcal{H}^2m_K^3}{216\pi f_\eta^4} - \frac{5\mathcal{H}^2m_K^2m_\eta}{81\pi f_\eta^4} + \frac{35\mathcal{H}^2m_\eta m_\pi^2}{1296\pi f_\eta^4} - \frac{2}{3}m_K^2W(m_K). \end{aligned} \quad (28)$$

We have checked that our results satisfy the $SU(3)$ symmetry and the crossing symmetry. Moreover, we have calculated the T -matrices of all 80 channels explicitly.

IV. LOW-ENERGY CONSTANTS

Before the numerical calculation, we have to determine the coupling constants in the Lagrangian. For those in the leading order, we use [26,30]

$$\begin{aligned} m_\pi &= 139.57 \text{ MeV}, & m_K &= 493.68 \text{ MeV}, \\ m_\eta &= 547.75 \text{ MeV}, & \delta &= 294 \text{ MeV}, \\ f_\pi &= 92.4 \text{ MeV}, & f_K &= 113 \text{ MeV}, & f_\eta &= 1.2f_K, \\ \mathcal{C} &= -1.5, & \mathcal{H} &= -1.9. \end{aligned} \quad (29)$$

At the second chiral order, d_4 could be determined from the mass splitting of the light decuplet,

$$d_4 = \frac{3(m_\Delta - m_{\Sigma^*})}{4(m_\pi^2 - m_K^2)} = 0.51 \text{ GeV}^{-1}. \quad (30)$$

Other LECs cannot be determined from the available experimental data. Thus we try to estimate them with the resonance saturation method, which was originally used in the meson case [27] and applied to the pion-nucleon interaction later [31]. According to this method, the effects from higher resonances are encoded in the LECs. Here

we consider contributions from baryon resonances below 1.6 GeV and ignore contributions from higher baryons.

The first baryon multiplet contributing to LECs is the $J^P = 3/2^-$ octet, which contains $N(1520)$, $\Lambda(1690)$, $\Sigma(1670)$, and $\Xi(1820)$. We simply denote it as N_{1520}^μ here. The relevant interacting Lagrangian is

$$\mathcal{L}_{N_{1520}} = iG_{N(1520)}(\bar{N}_{1520}^\mu \not{T}_\mu - \bar{T}_\mu \not{N}_{1520}^\mu). \quad (31)$$

For the complete relativistic Lagrangian, additional terms may be constructed according to the point transformation, such as $i(\bar{N}_{1520}^\mu \gamma_\mu \not{\gamma}^\nu T_\nu - \bar{T}_\mu \gamma^\mu \not{\gamma}_\nu N_{1520}^\nu)$ and $i(\bar{N}_{1520}^\mu (\gamma_\mu g_{\sigma\nu} + \gamma_\nu g_{\sigma\mu})u^\sigma T^\nu - \bar{T}^\mu (\gamma_\mu g_{\sigma\nu} + \gamma_\nu g_{\sigma\mu})u^\sigma N_{1520}^\nu)$ [32]. Because we are considering the case at threshold and the external baryons are on shell, such terms do not contribute and we ignore them here. By integrating out N_{1520} , one gets

$$\begin{aligned} \mathcal{L}_{\text{eff}}^{N(1520)} &\sim \frac{G_{N(1520)}^2}{m_{1520} - m_\Delta} (\bar{T}_\mu^{ijk} v \cdot u_i^s v \cdot u_j^l T_{slk}^\mu \\ &\quad - \bar{T}_\mu^{ijk} v \cdot u_i^s v \cdot u_s^l T_{ljk}^\mu). \end{aligned} \quad (32)$$

The $J^P = 3/2^+$ decuplet containing $\Delta(1600)$ is the lightest one contributing to LECs, and is denoted as T_{1600} here. The interacting Lagrangian reads

$$\mathcal{L}_{T_{1600}} = G_{\Delta(1600)}(\bar{T}_{1600}^\mu \not{\gamma}_5 T_\mu + \bar{T}^\mu \not{\gamma}_5 T_{1600\mu}). \quad (33)$$

The contribution to LECs may be derived from the resultant effective term,

$$\mathcal{L}_{\text{eff}}^{\Delta(1600)} \sim \frac{G_{\Delta(1600)}^2}{m_{1600} + m_{\Delta}} \left(\frac{2}{3} \bar{T}_{\mu}^{ijk} \mathbf{v} \cdot u_i^s \mathbf{v} \cdot u_j^l T_{slk}^{\mu} + \frac{1}{3} \bar{T}_{\mu}^{ijk} \mathbf{v} \cdot u_i^s \mathbf{v} \cdot u_s^l T_{ljk}^{\mu} \right). \quad (34)$$

Recall the fact that the time component of the on-shell Rarita-Schwinger field vanishes in the static limit; there is no chiral coupling with $J = 1/2$ baryon and pion at threshold. For the πN interaction [31], both the intermediate Δ and N^* baryons contribute to LECs because they are off-shell. In the present case, the external $J = 3/2$ baryon is on-shell and static and the intermediate excited $J = 1/2$ baryons do not contribute. As a result, we only need to consider the intermediate $J = 3/2$ baryons.

Besides the baryon resonances, the scalar meson σ would also contribute to LECs through t-channel. From the Lagrangians,

$$\begin{aligned} \mathcal{L}_{\sigma\pi\pi} &= \tilde{c}_d \text{Tr}(u \cdot u) \sigma + \tilde{c}_m \text{Tr}(\chi_+) \sigma, \\ \mathcal{L}_{\sigma\Delta\Delta} &= g_{\sigma} \bar{T}_{\mu} T^{\mu} \sigma, \end{aligned} \quad (35)$$

we get

$$\mathcal{L}_{\text{eff}}^{\sigma} \sim \left(\frac{2g_{\sigma}\tilde{c}_d}{m_{600}^2} - \frac{8g_{\sigma}\tilde{c}_m}{m_{600}^2} \right) \text{Tr}(\mathbf{v} \cdot u \mathbf{v} \cdot u) \bar{T}_{\mu} T^{\mu}. \quad (36)$$

Similarly, the scalar octet containing $\kappa(800)$, $a_0(980)$, and $f_0(980)$ may also be considered. We denote it as κ . The corresponding Lagrangians are

$$\begin{aligned} \mathcal{L}_{\kappa\pi\pi} &= c_d \text{Tr}(u \cdot u \kappa) + c_m \text{Tr}(\chi_+ \kappa), \\ \mathcal{L}_{\kappa\Delta\Delta} &= g_{\kappa} \bar{T}_{\mu} \kappa T^{\mu}, \end{aligned} \quad (37)$$

from which one obtains

$$\begin{aligned} \mathcal{L}_{\text{eff}}^{\kappa} &\sim \frac{2g_{\kappa}c_d}{m_{800}^2} \bar{T}_{\mu}^{ijk} \mathbf{v} \cdot u_i^s \mathbf{v} \cdot u_s^l T_{ljk}^{\mu} - \frac{2g_{\kappa}c_m}{3m_{800}^2} \\ &\times \text{Tr}(\mathbf{v} \cdot u \mathbf{v} \cdot u) \bar{T}_{\mu} T^{\mu} + \frac{2g_{\kappa}c_m}{m_{800}^2} \bar{T}_{\mu} \tilde{\chi}_+ T^{\mu}. \end{aligned} \quad (38)$$

$$G_{N(1520)}^2 = \frac{36\pi\Gamma(N(1520) \rightarrow \Delta\pi) m_{1520} m_{\Delta}^2 f_{\pi}^2}{|\vec{p}_d|(m_{1520} - m_{\Delta})^2 (E_{\Delta} + m_{\Delta})(2E_{\Delta}^2 + 2E_{\Delta}m_{\Delta} + 5m_{\Delta}^2)}. \quad (41)$$

Here \vec{p}_d is the decay momentum, $E_{\pi} = \sqrt{\vec{p}_d^2 + m_{\pi}^2}$, and $E_{\Delta} = \sqrt{\vec{p}_d^2 + m_{\Delta}^2}$. From the mass $m_{1520} = 1520$ MeV, decay width $\Gamma_{1520} = 115$ MeV and $\Gamma(N(1520) \rightarrow \Delta\pi) = 23$ MeV [30], one derives

$$\frac{G_{N(1520)}^2}{m_{1520} - m_{\Delta}} = 0.27 \text{ GeV}^{-1}. \quad (42)$$

With the same procedure and $m_{1600} = 1600$ MeV, $\Gamma_{1600} = 350$ MeV and $\Gamma(\Delta(1600) \rightarrow \Delta\pi) = 192$ MeV [30], we derive the coupling constant $G_{\Delta(1600)}$,

In the pion-nucleon case, the vector meson ρ does not contribute because the contraction of the ρ -meson propagator with the corresponding $\rho\pi\pi$ matrix element vanishes in the forward direction [31]. Here the vector mesons do not contribute for the same reason.

Adding the above effective Lagrangians (32), (34), (36), and (38) together, we have

$$\begin{aligned} \mathcal{L}_{\text{eff}} &= \left(\frac{G_{N(1520)}^2}{m_{1520} - m_{\Delta}} + \frac{2G_{\Delta(1600)}^2}{3(m_{1600} + m_{\Delta})} \right) \bar{T}_{\mu}^{ijk} \mathbf{v} \cdot u_i^s \mathbf{v} \cdot u_j^l T_{slk}^{\mu} \\ &+ \left(\frac{G_{\Delta(1600)}^2}{3(m_{1600} + m_{\Delta})} - \frac{G_{N(1520)}^2}{m_{1520} - m_{\Delta}} + \frac{2g_{\kappa}c_d}{m_{800}^2} \right) \bar{T}_{\mu}^{ijk} \mathbf{v} \cdot u_i^s \\ &\times \mathbf{v} \cdot u_s^l T_{ljk}^{\mu} + \left(\frac{2g_{\sigma}\tilde{c}_d}{m_{600}^2} - \frac{8g_{\sigma}\tilde{c}_m}{m_{600}^2} - \frac{2g_{\kappa}c_d}{3m_{800}^2} \right) \\ &\times \text{Tr}(\mathbf{v} \cdot u \mathbf{v} \cdot u) \bar{T}_{\mu} T^{\mu} + \frac{2g_{\kappa}c_m}{m_{800}^2} \bar{T}_{\mu} \tilde{\chi}_+ T^{\mu}. \end{aligned} \quad (39)$$

Now one estimates the LECs by comparing the Lagrangian (39) with the Lagrangian (9),

$$\begin{aligned} d_1 &= \frac{G_{N(1520)}^2}{m_{1520} - m_{\Delta}} + \frac{2G_{\Delta(1600)}^2}{3(m_{1600} + m_{\Delta})}, \\ d_2 &= \frac{G_{\Delta(1600)}^2}{3(m_{1600} + m_{\Delta})} - \frac{G_{N(1520)}^2}{m_{1520} - m_{\Delta}} + \frac{2g_{\kappa}c_d}{m_{800}^2}, \\ d_3 &= \frac{2g_{\sigma}\tilde{c}_d}{m_{600}^2} - \frac{8g_{\sigma}\tilde{c}_m}{m_{600}^2} - \frac{2g_{\kappa}c_d}{3m_{800}^2}, \\ d_4 &= \frac{2g_{\kappa}c_m}{m_{800}^2}. \end{aligned} \quad (40)$$

It is not difficult to determine the coupling constant $G_{N(1520)}$ according to the decay width. From the Lagrangian (31) and the decay to $\pi\Delta$, one deduces

$$\frac{G_{\Delta(1600)}^2}{m_{1600} + m_{\Delta}} = 0.46 \text{ GeV}^{-1}. \quad (43)$$

For the coupling constants c_d and c_m , we use [27]

$$\begin{aligned} |c_d| &= 3.2 \times 10^{-2} \text{ GeV}, \\ |c_m| &= 4.2 \times 10^{-2} \text{ GeV}, \quad c_d c_m > 0. \end{aligned} \quad (44)$$

Although there is no empirical value of g_{κ} , one may estimate it by comparing d_4 obtained with the resonance saturation method in (40) with that from the mass splitting in (30),

$$|g_{\kappa}| = 3.9, \quad g_{\kappa}c_m > 0. \quad (45)$$

In addition, the coupling constants should obey the nonet relations in the large N_c limit,

$$\begin{aligned} \tilde{c}_d &= \frac{\zeta}{\sqrt{3}}c_d, & \tilde{c}_m &= \frac{\zeta}{\sqrt{3}}c_m, \\ g_{\sigma} &= \frac{\zeta}{\sqrt{3}}g_{\kappa}, & \zeta &= \pm 1. \end{aligned} \quad (46)$$

By combining the above relations, the estimated LECs at $O(\epsilon^2)$ are

$$\begin{aligned} d_1 &= 0.58 \text{ GeV}^{-1}, & d_2 &= 0.28 \text{ GeV}^{-1}, \\ d_3 &= -1.11 \text{ GeV}^{-1}, & d_4 &= 0.51 \text{ GeV}^{-1}. \end{aligned} \quad (47)$$

In a similar way, one may consider the high-order corrections and estimate the $O(\epsilon^3)$ LECs $h_1 - h_5$. Because of the uncertainty of this method, we tend to neglect these counter terms in the following calculation. This assumption was adopted in the study of the meson-baryon scattering lengths in Refs. [8,33]. The counterterms of the πN scattering lengths are found to be much smaller than the loop contributions at this order in Ref. [31]. However, to explore the orders of the counterterms, we have made a very crude estimate about these LECs h_1-h_5 , which are collected in the Appendix .

V. NUMERICAL RESULTS AND DISCUSSIONS

We set λ at $4\pi f_{\pi}$, $4\pi f_K$, and $4\pi f_{\eta}$ respectively for the pion, kaon and η -scattering in our numerical analysis. The results for the T -matrices and the scattering lengths at different orders are shown in Tables I, II, and III. The positive sign of $a_{\pi\Delta}^{(3/2)}$, $a_{\pi\Delta}^{(1/2)}$, $a_{\pi\Sigma^*}^{(1)}$, $a_{\pi\Sigma^*}^{(0)}$, $a_{\pi\Xi^*}^{(1/2)}$, $a_{K\Delta}^{(1)}$, $a_{K\Sigma^*}^{(1/2)}$, $a_{K\Xi^*}^{(1)}$, $a_{K\Xi^*}^{(0)}$, $a_{K\Omega}^{(1/2)}$, $a_{\bar{K}\Delta}^{(1)}$, $a_{\bar{K}\Sigma^*}^{(1/2)}$, $a_{\eta\Sigma^*}^{(1)}$, and $a_{\eta\Xi^*}^{(1/2)}$ indicates that the strong interactions for these channels are attractive.

In Ref. [34], the authors used the extended Nambu-Jona-Lasinio model and got the following $\pi\Delta$ scattering lengths:

$$\begin{aligned} a_{\pi\Delta}^{5/2} &= -0.258m_{\pi}^{-1} = -0.364 \text{ fm}, \\ a_{\pi\Delta}^{3/2} &= 0.172m_{\pi}^{-1} = 0.242 \text{ fm}, \\ a_{\pi\Delta}^{1/2} &= 0.429m_{\pi}^{-1} = 0.604 \text{ fm}. \end{aligned} \quad (48)$$

Somehow their results are consistent with ours.

From the data, one easily sees that the third-order contributions are large and the T -matrices do not converge. The bad convergence may be mainly due to the large mass of kaon and eta [6,8,19,20]. Inaccurate determination of LECs might worsen the convergence. Another reason probably comes from the heavy hadron framework itself. In the meson-baryon case [35] and the meson-heavy meson case [36,37], it was found that the recoil corrections are sizable. The future investigation of the higher order recoil corrections and LECs, etc., may answer whether similar features occur in the meson-decuplet cases.

We also show different contributions of the T -matrices at $O(\epsilon^3)$ in Tables IV, V, and VI. From the values one observes that the loop corrections from the intermediate decuplet states dominate. In the pion scattering channels, only loop diagrams containing the octet baryons could generate the imaginary part. The contributions from the intermediate octet and decuplet baryons have opposite signs in many channels, which is beneficial for the convergence. The consideration of the recoil corrections increases the convergence in most channels.

We compare $T_{\pi\Sigma^*}^{(2,1,0)}$, $T_{\pi\Xi^*}^{(3/2,1/2)}$, $T_{K\Sigma^*}^{(3/2,1/2)}$, $T_{K\Xi^*}^{(1,0)}$, $T_{\bar{K}\Sigma^*}^{(3/2,1/2)}$, and $T_{\bar{K}\Xi^*}^{(1,0)}$ with the corresponding $T_{\pi\Sigma}^{(2,1,0)}$, $T_{\pi\Xi}^{(3/2,1/2)}$, $T_{K\Sigma}^{(3/2,1/2)}$, $T_{K\Xi}^{(1,0)}$, $T_{\bar{K}\Sigma}^{(3/2,1/2)}$, and $T_{\bar{K}\Xi}^{(1,0)}$ obtained in Refs. [21,33], and find that the results of each pair are equal at the leading order, but different at higher order. We also check the following quantities at the leading order:

$$\begin{aligned} T_{\pi^+\Delta^+} &= T_{\pi^+p}, & T_{\pi^+\Delta^0} &= T_{\pi^+n}, \\ T_{K^+\Delta^+} &= T_{K^+p}, & T_{K^+\Delta^0} &= T_{K^+n}, \\ T_{\bar{K}^0\Delta^+} &= T_{\bar{K}^0p}, & T_{\bar{K}^0\Delta^0} &= T_{\bar{K}^0n}. \end{aligned} \quad (49)$$

TABLE I. Pion-decuplet baryon threshold T -matrices order by order in units of fm.

	$O(\epsilon^1)$	$O(\epsilon^2)$	$O(\epsilon^3)$	Total	Scattering length
$T_{\pi\Delta}^{(5/2)}$	-4.84	0.445	-1.37	-5.76	-0.412
$T_{\pi\Delta}^{(3/2)}$	3.23	-0.253	-0.901 - 1.02i	2.07 - 1.02i	0.148 - 0.0728i
$T_{\pi\Delta}^{(1/2)}$	8.06	1.	1.44 + 0.814i	10.5 + 0.814i	0.751 + 0.0582i
$T_{\pi\Sigma^*}^{(2)}$	-3.23	0.361	-2.52 - 0.102i	-5.38 - 0.102i	-0.389 - 0.00736i
$T_{\pi\Sigma^*}^{(1)}$	3.23	0.0819	-1.8 - 0.509i	1.51 - 0.509i	0.109 - 0.0368i
$T_{\pi\Sigma^*}^{(0)}$	6.45	0.781	-0.412 + 0.509i	6.82 + 0.509i	0.493 + 0.0368i
$T_{\pi\Xi^*}^{(3/2)}$	-1.61	0.277	-2.69 - 0.102i	-4.02 - 0.102i	-0.293 - 0.00742i
$T_{\pi\Xi^*}^{(1/2)}$	3.23	0.277	-1.89 - 0.102i	1.61 - 0.102i	0.118 - 0.00742i
$T_{\pi\Omega}^{(1)}$	0	0.194	-1.87	-1.68	-0.123

TABLE II. Kaon-decuplet baryon threshold T -matrices order by order in units of fm.

	$O(\epsilon^1)$	$O(\epsilon^2)$	$O(\epsilon^3)$	Total	Scattering length
$T_{K\Delta}^{(2)}$	-11.4	3.73	-9.27	-17.	-0.965
$T_{K\Delta}^{(1)}$	3.81	0.917	3.67	8.4	0.477
$T_{K\Sigma^*}^{(3/2)}$	-3.81	1.39	$-9.44 + 6.77i$	$-11.9 + 6.77i$	$-0.696 + 0.397i$
$T_{K\Sigma^*}^{(1/2)}$	7.63	2.79	$10.1 + 11.5i$	$20.5 + 11.5i$	$1.2 + 0.673i$
$T_{K\Xi^*}^{(1)}$	3.81	1.39	$-3.22 + 10.9i$	$1.98 + 10.9i$	$0.119 + 0.658i$
$T_{K\Xi^*}^{(0)}$	11.4	4.66	$16.4 + 17.2i$	$32.5 + 17.2i$	$1.95 + 1.04i$
$T_{K\Omega}^{(1/2)}$	11.4	3.73	$9.4 + 12.5i$	$24.6 + 12.5i$	$1.51 + 0.768i$
$T_{\bar{K}\Delta}^{(2)}$	0	1.62	$-2.85 + 8.34i$	$-1.23 + 8.34i$	$-0.0697 + 0.474i$
$T_{\bar{K}\Delta}^{(1)}$	15.3	4.43	$14.6 + 2.78i$	$34.3 + 2.78i$	$1.95 + 0.158i$
$T_{\bar{K}\Sigma^*}^{(3/2)}$	-3.81	2.32	$-4.01 + 11.3i$	$-5.5 + 11.3i$	$-0.323 + 0.662i$
$T_{\bar{K}\Sigma^*}^{(1/2)}$	7.63	0.92	$-0.775 + 2.42i$	$7.77 + 2.42i$	$0.456 + 0.142i$
$T_{\bar{K}\Xi^*}^{(1)}$	-7.63	3.02	$-5.33 + 8.51i$	$-9.93 + 8.51i$	$-0.598 + 0.512i$
$T_{\bar{K}\Xi^*}^{(0)}$	0	-0.249	$-9.72 - 0.533i$	$-9.97 - 0.533i$	$-0.6 - 0.0321i$
$T_{\bar{K}\Omega}^{(1/2)}$	-11.4	3.73	-6.8	-14.5	-0.892

TABLE III. Eta-decuplet baryon threshold T -matrices order by order in units of fm.

	$O(\epsilon^1)$	$O(\epsilon^2)$	$O(\epsilon^3)$	Total	Scattering length
$T_{\eta\Delta}^{(3/2)}$	0	-0.173	$0.338 + 3.76i$	$0.165 + 3.76i$	$0.009\ 09 + 0.207i$
$T_{\eta\Sigma^*}^{(1)}$	0	3.58	$1.79 + 9.96i$	$5.37 + 9.96i$	$0.306 + 0.568i$
$T_{\eta\Xi^*}^{(1/2)}$	0	4.34	$1.23 + 11.2i$	$5.57 + 11.2i$	$0.326 + 0.655i$
$T_{\eta\Omega}^{(0)}$	0	2.1	$-1.35 + 7.44i$	$0.756 + 7.44i$	$0.0453 + 0.446i$

These relations are easy to understand with the Weinberg-Tomazawa formula, $T \sim I_{\text{tot}}(I_{\text{tot}} + 1) - I_B(I_B + 1) - I_M(I_M + 1)$, where I_{tot} , I_B , and I_M are the total, baryon, and meson isospin, respectively.

In summary, we have calculated the S-wave scattering lengths of the pseudoscalar mesons and light decuplet baryons to the third order in the framework of heavy

baryon chiral perturbation theory, and found that some of the T -matrices do not converge very well. We estimate LECs with the resonance saturation method. Our results may be helpful to the model construction of the meson-decuplet baryon interactions. We hope that future lattice simulation or experimental measurements may test the predictions.

TABLE IV. Pion-decuplet baryon threshold T -matrices at $O(\epsilon^3)$ in units of fm.

	Loop: octet contribution	Loop: decuplet contribution	Loop: total	Tree only recoil correction	Total
$T_{\pi\Delta}^{(5/2)}$	0	-1.41	-1.41	0.0345	-1.37
$T_{\pi\Delta}^{(3/2)}$	$-0.518 - 1.02i$	-0.586	$-1.1 - 1.02i$	0.203	$-0.901 - 1.02i$
$T_{\pi\Delta}^{(1/2)}$	$0.414 + 0.814i$	1.08	$1.49 + 0.814i$	-0.0575	$1.44 + 0.814i$
$T_{\pi\Sigma^*}^{(2)}$	$-0.129 - 0.102i$	-2.41	$-2.54 - 0.102i$	0.023	$-2.52 - 0.102i$
$T_{\pi\Sigma^*}^{(1)}$	$-0.337 - 0.509i$	-1.56	$-1.9 - 0.509i$	0.0977	$-1.8 - 0.509i$
$T_{\pi\Sigma^*}^{(0)}$	$0.181 + 0.509i$	-0.548	$-0.366 + 0.509i$	-0.046	$-0.412 + 0.509i$
$T_{\pi\Xi^*}^{(3/2)}$	$-0.129 - 0.102i$	-2.57	$-2.7 - 0.102i$	0.0115	$-2.69 - 0.102i$
$T_{\pi\Xi^*}^{(1/2)}$	$-0.129 - 0.102i$	-1.78	$-1.91 - 0.102i$	0.0222	$-1.89 - 0.102i$
$T_{\pi\Omega}^{(1)}$	0	-1.87	-1.87	0	-1.87

TABLE V. Kaon-decuplet baryon threshold T -matrices at $O(\epsilon^3)$ in units of fm.

	Loop: octet contribution	Loop: decuplet contribution	Loop: total	Tree only recoil correction	Total
$T_{K\Delta}^{(2)}$	0	-9.33	-9.33	0.0607	-9.27
$T_{K\Delta}^{(1)}$	0	3.69	3.69	-0.0202	3.67
$T_{K\Sigma^*}^{(3/2)}$	$0.778 - 0.178i$	$-11. + 6.95i$	$-10.2 + 6.77i$	0.777	$-9.44 + 6.77i$
$T_{K\Sigma^*}^{(1/2)}$	$0.666 + 0.355i$	$9.47 + 11.1i$	$10.1 + 11.5i$	-0.0405	$10.1 + 11.5i$
$T_{K\Xi^*}^{(1)}$	$0.778 - 0.178i$	$-4.99 + 11.1i$	$-4.21 + 10.9i$	0.989	$-3.22 + 10.9i$
$T_{K\Xi^*}^{(0)}$	$0.628 + 0.533i$	$15.8 + 16.7i$	$16.4 + 17.2i$	-0.0607	$16.4 + 17.2i$
$T_{K\Omega}^{(1/2)}$	0	$8.7 + 12.5i$	$8.7 + 12.5i$	0.696	$9.4 + 12.5i$
$T_{\bar{K}\Delta}^{(2)}$	0	$-2.85 + 8.34i$	$-2.85 + 8.34i$	0	$-2.85 + 8.34i$
$T_{\bar{K}\Delta}^{(1)}$	0	$13.6 + 2.78i$	$13.6 + 2.78i$	0.929	$14.6 + 2.78i$
$T_{\bar{K}\Sigma^*}^{(3/2)}$	$0.703 + 0.178i$	$-4.73 + 11.1i$	$-4.03 + 11.3i$	0.0202	$-4.01 + 11.3i$
$T_{\bar{K}\Sigma^*}^{(1/2)}$	$0.816 - 0.355i$	$-3.06 + 2.78i$	$-2.25 + 2.42i$	1.47	$-0.775 + 2.42i$
$T_{\bar{K}\Xi^*}^{(1)}$	$0.703 + 0.178i$	$-6.07 + 8.34i$	$-5.37 + 8.51i$	0.0405	$-5.33 + 8.51i$
$T_{\bar{K}\Xi^*}^{(0)}$	$0.854 - 0.533i$	-12.1	$-11.2 - 0.533i$	1.51	$-9.72 - 0.533i$
$T_{\bar{K}\Omega}^{(1/2)}$	0	-6.86	-6.86	0.0607	-6.8

TABLE VI. Eta-decuplet baryon threshold T -matrices at $O(\epsilon^3)$ in units of fm.

	Loop: octet contribution	Loop: decuplet contribution	Loop: total	Tree only recoil correction	Total
$T_{\eta\Delta}^{(3/2)}$	$0.493 + 0.0439i$	$-0.479 + 3.72i$	$0.0145 + 3.76i$	0.324	$0.338 + 3.76i$
$T_{\eta\Sigma^*}^{(1)}$	$-0.548 + 0.0366i$	$2.33 + 9.92i$	$1.79 + 9.96i$	0	$1.79 + 9.96i$
$T_{\eta\Xi^*}^{(1/2)}$	$-0.403 + 0.0219i$	$1.3 + 11.2i$	$0.902 + 11.2i$	0.324	$1.23 + 11.2i$
$T_{\eta\Omega}^{(0)}$	0.93	$-3.57 + 7.44i$	$-2.64 + 7.44i$	1.29	$-1.35 + 7.44i$

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APPENDIX: ROUGH ESTIMATION OF THE $O(\epsilon^3)$ COUNTERTERMS

At this order, the excited baryons contribute to LECs through both tree diagrams and loop diagrams. Here we assume the former contribution is much smaller than the latter one. This assumption for the estimation is similar to the one used in studying the meson-baryon scattering lengths in Refs. [8,33] and the above numerical calculation. For a very rough estimate at $O(\epsilon^3)$, we only consider

the octet resonance containing $N(1440)$, denoted by R here.

The diagrams one needs to calculate are similar to those in the last column of Fig. 1. A new Lagrangian should be introduced,

$$\begin{aligned} \mathcal{L}_{\phi TR} = & \text{Tr}(\bar{R}(i\nu \cdot \partial - \delta_R)R + \bar{R}[\nu \cdot \Gamma, R]) \\ & + G_{N(1440)}(\bar{T}^\mu u_\mu R + \bar{R}u_\mu T^\mu) + \dots, \quad (\text{A1}) \end{aligned}$$

where $\delta_R \sim 500$ MeV is the mass difference between $N(1440)$ and the nucleon. In our calculation, we need $\tilde{\delta}_R = 208$ MeV, which is the mass difference between $N(1440)$ and Δ .

The T -matrices can be obtained by replacing J and $W(m)$ in Eqs. (13) and (14) with

$$J_{\text{new}} = J + J_R, \quad W_{\text{new}}(m) = W(m) + W_R(m), \quad (\text{A2})$$

where

TABLE VII. Pion-decuplet baryon threshold T -matrices considering $N(1440)$ octet baryon contribution in units of fm.

	$O(\epsilon^1)$	$O(\epsilon^2)$	$O(\epsilon^3)$: old	$O(\epsilon^3)$: $N(1440)$ contribution	$O(\epsilon^3)$: Total, new
$T_{\pi\Delta}^{(5/2)}$	-4.84	0.445	-1.37	0	-1.37
$T_{\pi\Delta}^{(3/2)}$	3.23	-0.253	-0.901 - 1.02 <i>i</i>	0.374	-0.527 - 1.02 <i>i</i>
$T_{\pi\Delta}^{(1/2)}$	8.06	1.	1.44 + 0.814 <i>i</i>	-0.299	1.14 + 0.814 <i>i</i>
$T_{\pi\Sigma^*}^{(2)}$	-3.23	0.361	-2.52 - 0.102 <i>i</i>	-0.0434	-2.56 - 0.102 <i>i</i>
$T_{\pi\Sigma^*}^{(1)}$	3.23	0.0819	-1.8 - 0.509 <i>i</i>	0.106	-1.69 - 0.509 <i>i</i>
$T_{\pi\Sigma^*}^{(0)}$	6.45	0.781	-0.412 + 0.509 <i>i</i>	-0.268	-0.68 + 0.509 <i>i</i>
$T_{\pi\Xi^*}^{(3/2)}$	-1.61	0.277	-2.69 - 0.102 <i>i</i>	-0.0434	-2.73 - 0.102 <i>i</i>
$T_{\pi\Xi^*}^{(1/2)}$	3.23	0.277	-1.89 - 0.102 <i>i</i>	-0.0434	-1.93 - 0.102 <i>i</i>
$T_{\pi\Omega}^{(1)}$	0	0.194	-1.87	0	-1.87

TABLE VIII. Kaon-decuplet baryon threshold T -matrices considering $N(1440)$ octet baryon contribution in units of fm.

	$O(\epsilon^1)$	$O(\epsilon^2)$	$O(\epsilon^3)$: old	$O(\epsilon^3)$: $N(1440)$ contribution	$O(\epsilon^3)$: Total, new
$T_{K\Delta}^{(2)}$	-11.4	3.73	-9.27	0	-9.27
$T_{K\Delta}^{(1)}$	3.81	0.917	3.67	0	3.67
$T_{K\Sigma^*}^{(3/2)}$	-3.81	1.39	-9.44 + 6.77 <i>i</i>	1.62	-7.82 + 6.77 <i>i</i>
$T_{K\Sigma^*}^{(1/2)}$	7.63	2.79	10.1 + 11.5 <i>i</i>	-0.558	9.53 + 11.5 <i>i</i>
$T_{K\Xi^*}^{(1)}$	3.81	1.39	-3.22 + 10.9 <i>i</i>	1.62	-1.6 + 10.9 <i>i</i>
$T_{K\Xi^*}^{(0)}$	11.4	4.66	16.4 + 17.2 <i>i</i>	-1.28	15.1 + 17.2 <i>i</i>
$T_{K\Omega}^{(1/2)}$	11.4	3.73	9.4 + 12.5 <i>i</i>	0	9.4 + 12.5 <i>i</i>
$T_{\bar{K}\Delta}^{(2)}$	0	1.62	-2.85 + 8.34 <i>i</i>	0	-2.85 + 8.34 <i>i</i>
$T_{\bar{K}\Delta}^{(1)}$	15.3	4.43	14.6 + 2.78 <i>i</i>	0	14.6 + 2.78 <i>i</i>
$T_{\bar{K}\Sigma^*}^{(3/2)}$	-3.81	2.32	-4.01 + 11.3 <i>i</i>	0.168	-3.84 + 11.3 <i>i</i>
$T_{\bar{K}\Sigma^*}^{(1/2)}$	7.63	0.92	-0.775 + 2.42 <i>i</i>	2.35	1.57 + 2.42 <i>i</i>
$T_{\bar{K}\Xi^*}^{(1)}$	-7.63	3.02	-5.33 + 8.51 <i>i</i>	0.168	-5.16 + 8.51 <i>i</i>
$T_{\bar{K}\Xi^*}^{(0)}$	0	-0.249	-9.72 - 0.533 <i>i</i>	3.07	-6.65 - 0.533 <i>i</i>
$T_{\bar{K}\Omega}^{(1/2)}$	-11.4	3.73	-6.8	0	-6.8

TABLE IX. Eta-decuplet baryon threshold T -matrices considering $N(1440)$ octet baryon contribution in units of fm.

	$O(\epsilon^1)$	$O(\epsilon^2)$	$O(\epsilon^3)$: old	$O(\epsilon^3)$: $N(1440)$ contribution	$O(\epsilon^3)$: Total, new
$T_{\eta\Delta}^{(3/2)}$	0	-0.173	0.338 + 3.76 <i>i</i>	0.814	1.15 + 3.76 <i>i</i>
$T_{\eta\Sigma^*}^{(1)}$	0	3.58	1.79 + 9.96 <i>i</i>	-0.686	1.1 + 9.96 <i>i</i>
$T_{\eta\Xi^*}^{(1/2)}$	0	4.34	1.23 + 11.2 <i>i</i>	-0.402	0.823 + 11.2 <i>i</i>
$T_{\eta\Omega}^{(0)}$	0	2.1	-1.35 + 7.44 <i>i</i>	1.67	0.319 + 7.44 <i>i</i>

$$J_R = \frac{G_{N(1440)}^2 m_K^2}{12\pi^2 f^4 (m_\eta^2 - m_\pi^2)} \left[\left(\tilde{\delta}_R^3 - \frac{3}{2} m_\eta^2 \tilde{\delta}_R \right) \log \frac{m_\eta}{\lambda} + \left(\frac{3}{2} m_\pi^2 \tilde{\delta}_R - \tilde{\delta}_R^3 \right) \log \frac{m_\pi}{\lambda} \right. \\ \left. + (\tilde{\delta}_R^2 - m_\pi^2)^{3/2} \log \frac{\sqrt{\tilde{\delta}_R^2 - m_\pi^2} - \tilde{\delta}_R}{m_\pi} - (m_\eta^2 - \tilde{\delta}_R^2)^{3/2} \cos^{-1} \left(-\frac{\tilde{\delta}_R}{m_\eta} \right) + (m_\eta^2 - m_\pi^2) \tilde{\delta}_R \right], \quad (\text{A3})$$

$$W_R(m) = \frac{G_{N(1440)}^2}{16\pi^2 f^4} \begin{cases} -2\sqrt{m^2 - \tilde{\delta}_R^2} \cos^{-1} \left(-\frac{\tilde{\delta}_R}{m} \right) - 2\tilde{\delta}_R \log \frac{m}{\lambda} + \tilde{\delta}_R & m > |\tilde{\delta}_R| \\ -2\tilde{\delta}_R \log \frac{m}{\lambda} + 2\sqrt{\tilde{\delta}_R^2 - m^2} \log \frac{\sqrt{\tilde{\delta}_R^2 - m^2} - \tilde{\delta}_R}{m} + \tilde{\delta}_R & m \leq |\tilde{\delta}_R| \end{cases}. \quad (\text{A4})$$

We show the numerical results at order $O(\epsilon^3)$ in Tables VII, VIII, and IX. The roughly estimated LECs are $h_1^r = -1.7 \text{ GeV}^{-2}$, $h_2^r = -4.4 \text{ GeV}^{-2}$, $h_3^r = 3.3 \text{ GeV}^{-2}$, $h_4^r = -0.8 \text{ GeV}^{-2}$, and $h_5^r = 0.0$ at around the scale $4\pi f_\pi$. From the tables, we see that the contribution due to the intermediate $N(1440)$ octet increases the convergence of the chiral expansion in most channels slightly.

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