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Charmless hadronic B decays into a tensor meson

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Two-body charmless hadronic B decays involving a tensor meson in the final state are studied within the framework of QCD factorization (QCDF). Because of the G-parity of the tensor meson, both the chiral-even and chiral-odd two-parton light-cone distribution amplitudes of the tensor meson are antisymmetric under the interchange of momentum fractions of the quark and antiquark in the SU(3) limit. Our main results are: (i) In the naïve factorization approach, the decays such as $B^- \to \bar{K}_2^{*0}\pi^-$ and $\bar{B}^0 \to K_2^{*-}\pi^+$ with a tensor meson emitted are prohibited because a tensor meson cannot be created from the local $V - A$ or tensor current. Nevertheless, the decays receive nonfactorizable contributions in QCDF from vertex, penguin and hard spectator corrections. The experimental observation of $B^- \to \bar{K}_2^{*0}\pi^-$ indicates the importance of nonfactorizable effects. (ii) For penguin-dominated $B \to TP$ and TV decays, the predicted rates in naïve factorization are usually too small by 1 to 2 orders of magnitude. In QCDF, they are enhanced by power corrections from penguin annihilation and nonfactorizable contributions. (iii) The dominant penguin contributions to $B \to K_2^*\eta^{(\prime)}$ arise from the processes: (a) $b \to ss\bar{s} \to s\eta_s$ and (b) $b \to sq\bar{q} \to q\bar{K}_2^*$ with $\eta_q = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\eta_s = s\bar{s}$. The interference, constructive for $K_2^*\eta'$ and destructive for $K_2^*\eta$, explains why $\Gamma(B \to K_2^*\eta') \gg \Gamma(B \to K_2^*\eta)$. (iv) We use the measured rates of $B \to K_2^*(\omega, \phi)$ to extract the penguin-annihilation parameters ρ_A^{TV} and ρ_A^{VT} and the observed longitudinal polarization fractions $f_L(K_2^*\omega)$ and $f_L(K_2^*\phi)$ to fix the phases ϕ_A^{VT} and ϕ_A^{TV}. (v) The experimental observation that $f_T/f_L \ll 1$ for $B \to K_2^*(1430)\phi$, whereas $f_T/f_L \sim 1$ for $B \to K_2^*(1430)\omega$ with f_T being the transverse polarization fraction, can be *accommodated* in QCDF, but it cannot be *dynamically explained* at first place. For penguin-dominated $B \to TV$ decays, we find $f_L(K_2^*\rho) \sim f_L(K_2^*\omega) \sim 0.65$, whereas $f_L(K^*f_2) \sim 0.93$. It will be of great interest to measure f_L for these modes to test QCDF. Theoretically, transverse polarization is expected to be small in tree-dominated $\bar{B} \to TV$ decays except for the $a_2^-\rho^0$, $a_2^-\rho^+$, $K_2^{*0}K^{*-}$ and $K_2^{*0}\bar{K}^{*0}$ modes. (vi) For tree-dominated decays, their rates are usually very small except for the $a_2^0(\pi^-, \rho^-)$, $a_2^+(\pi^-, \rho^-)$ and $f_2(\pi^-, \rho^-)$ modes with branching fractions of order 10^{-6} or even larger.

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I. INTRODUCTION

In the past few years, *BABAR* and Belle [1–14] have begun to measure several charmless B decay modes involving a light tensor meson T in the final states, with the results summarized in Table I. From the theoretical point of view, the hadronic decays $B \to TM$ with $M = P, V, A$ are of great interest for two reasons: rate deficit and polarization puzzles. First, these decays have been studied in the naïve factorization approach [15–23]. The predicted rates are in general too small by 1 to 2 orders of magnitude. This implies the importance of $1/m_b$ power corrections. Since the nonfactorizable amplitudes such as vertex and penguin corrections, spectator interactions cannot be tackled in naïve factorization, it is necessary to go beyond the naïve factorization framework. The theoretical frameworks suitable for this purpose include QCD factorization (QCDF) [24], perturbative QCD (pQCD) [25] and soft collinear effective theory (SCET) [26].

Second, it is known that an unexpectedly large fraction of transverse polarization has been observed in the penguin-dominated $B \to VV$ channels, such as $B \to \phi K^*, \rho K^*$, contrary to the naïve expectation of the longitudinal polarization dominance. (For a review, see [27].) However, while the polarization measurement in $B \to \omega K_2^*(1430)$ indicates a large fraction of transverse polarization f_T (see Table I), the measurement in $B \to \phi K_2^*(1430)$ is consistent with the longitudinal polarization dominance. Therefore, it is important to understand why $f_T/f_L \ll 1$ for $B \to \phi K_2^*(1430)$, whereas $f_T/f_L \sim 1$ for $B \to \omega K_2^*(1430)$, even though both are penguin-dominated. The polarization studies for $B \to TV, TA, TT$ will further shed light on the underlying helicity structure of the decay mechanism.

In the present work we shall study charmless $B \to TM$ decays within the framework of QCD factorization. One unique feature of the tensor meson is that it cannot be created from the $V - A$ or tensor current. Hence, the decay

TABLE I. Experimental branching fractions (in units of 10^{-6}) and the longitudinal polarization fractions f_L for B decays to final states containing a tensor meson. Data are taken from [1–14].

Mode	$\mathcal{B}$	f_L	Mode	$\mathcal{B}$	f_L
$\mathcal{B}(B^+ \to K_2^*(1430)^+\omega)$	21.5 ± 4.3	0.56 ± 0.11	$\mathcal{B}(B^0 \to K_2^*(1430)^0\omega)$	10.1 ± 2.3	0.45 ± 0.12
$\mathcal{B}(B^+ \to K_2^*(1430)^+\phi)$	8.4 ± 2.1	0.80 ± 0.10	$\mathcal{B}(B^0 \to K_2^*(1430)^0\phi)$	7.5 ± 1.0	$0.901^{+0.059}_{-0.069}$
$\mathcal{B}(B^+ \to K_2^*(1430)^+\eta)$	9.1 ± 3.0		$\mathcal{B}(B^0 \to K_2^*(1430)^0\eta)$	9.6 ± 2.1	
$\mathcal{B}(B^+ \to K_2^*(1430)^+\eta')$	$28.0^{+5.3}_{-5.0}$		$\mathcal{B}(B^0 \to K_2^*(1430)^0\eta')$	$13.7^{+3.2}_{-3.1}$	
$\mathcal{B}(B^+ \to K_2^*(1430)^0\pi^+)$	$5.6^{+2.2}_{-1.4}$		$\mathcal{B}(B^0 \to K_2^*(1430)^+\pi^-)$	<6.3	
$\mathcal{B}(B^+ \to K_2^*(1430)^0K^+)$	<1.1		$\mathcal{B}(B^0 \to K_2^*(1430)^0\pi^0)$	<4.0	
$\mathcal{B}(B^+ \to f_2(1270)K^+)$	$1.06^{+0.28}_{-0.29}$		$\mathcal{B}(B^0 \to f_2(1270)K^0)$	$2.7^{+1.3}_{-1.2}$	
$\mathcal{B}(B^+ \to f_2(1270)\pi^+)$	$1.57^{+0.69}_{-0.49}$				
$\mathcal{B}(B^+ \to f_2'(1525)K^+)$	<7.7 [a]				
$\mathcal{B}(B^+ \to a_2(1320)^0K^+)$	<45				

[a]From the measurement of $\mathcal{B}(B^+ \to f_2'(1525)^0K^+ \to K^+K^+K^-) < 3.4 \times 10^{-6}$ [6].

with a tensor meson emitted, for example, $B^- \to \bar{K}_2^{*0}\pi^-$, is prohibited in naïve factorization. The experimental observation of this penguin-dominated mode with a sizable rate implies the importance of nonfactorizable effects which will be addressed in QCDF.

The layout of this work is as follows. We study the physical properties of tensor mesons such as decay constants, form factors, light-cone distribution amplitudes and helicity projection operators in Sec. II and specify various input parameters. Then we work out in detail the next-to-leading order corrections to $B \to TP, TV$ decays in Sec. III and present numerical results and discussions in Sec. IV. Conclusions are given in Sec. V. Appendix A is devoted to a recapitulation of the ISGW model. Decay amplitudes and explicit expressions of helicity-dependent annihilation amplitudes are shown in Appendixes B and C, respectively. A minireview of the $\eta - \eta'$ mixing is given in Appendix D.

II. PHYSICAL PROPERTIES OF TENSOR MESONS

A. Tensor mesons

The observed $J^P = 2^+$ tensor mesons $f_2(1270)$, $f_2'(1525)$, $a_2(1320)$ and $K_2^*(1430)$ form an SU(3) 1^3P_2 nonet. The $q\bar{q}$ content for isodoublet and isovector tensor resonances is obvious. Just like the η-η' mixing in the pseudoscalar case, the isoscalar tensor states $f_2(1270)$ and $f_2'(1525)$ also have a mixing, and their wave functions are defined by

$$f_2(1270) = \frac{1}{\sqrt{2}}(f_2^u + f_2^d)\cos\theta_{f_2} + f_2^s\sin\theta_{f_2},$$
$$f_2'(1525) = \frac{1}{\sqrt{2}}(f_2^u + f_2^d)\sin\theta_{f_2} - f_2^s\cos\theta_{f_2}, \tag{1}$$

with $f_2^u \equiv u\bar{u}$ and likewise for $f_2^{d,s}$. Since $\pi\pi$ is the dominant decay mode of $f_2(1270)$ whereas $f_2'(1525)$ decays predominantly into $K\bar{K}$ (see Ref. [28]), it is obvious that this mixing angle should be small. More precisely, it is found that $\theta_{f_2} = 7.8°$ [29] and $(9 \pm 1)°$ [28]. Therefore, $f_2(1270)$ is primarily a $(u\bar{u} + d\bar{d})/\sqrt{2}$ state, while $f_2'(1525)$ is dominantly $s\bar{s}$.

For a tensor meson, the polarization tensors $\epsilon^{\mu\nu}_{(\lambda)}$ with helicity λ can be constructed in terms of the polarization vectors of a massive vector state moving along the z-axis [30]

$$\epsilon(0)^{*\mu} = (P_3, 0, 0, E)/m_T, \quad \epsilon(\pm 1)^{*\mu} = (0, \mp 1, +i, 0)/\sqrt{2}, \tag{2}$$

and are given by

$$\epsilon^{\mu\nu}_{(\pm 2)} \equiv \epsilon(\pm 1)^\mu \epsilon(\pm 1)^\nu, \tag{3}$$

$$\epsilon^{\mu\nu}_{(\pm 1)} \equiv \sqrt{\frac{1}{2}}[\epsilon(\pm 1)^\mu \epsilon(0)^\nu + \epsilon(0)^\mu \epsilon(\pm 1)^\nu], \tag{4}$$

$$\epsilon^{\mu\nu}_{(0)} \equiv \sqrt{\frac{1}{6}}[\epsilon(+1)^\mu \epsilon(-1)^\nu + \epsilon(-1)^\mu \epsilon(+1)^\nu] + \sqrt{\frac{2}{3}}\epsilon(0)^\mu \epsilon(0)^\nu. \tag{5}$$

The polarization $\epsilon^{(\lambda)}_{\mu\nu}$ can be decomposed in the frame formed by the two lightlike vectors, z_μ and $p_\nu \equiv P_\nu - z_\nu m_T^2/(2pz)$, with P_ν and m_T being the momentum and mass of the tensor meson, respectively, and their orthogonal plane [31,32]. The transverse component that we use thus reads

$$\epsilon^{(\lambda)}_{\perp\mu\nu}z^\nu = \epsilon^{(\lambda)}_{\mu\nu}z^\nu - \epsilon^{(\lambda)}_{\parallel\mu\nu}z^\nu = \epsilon^{(\lambda)}_{\mu\nu}z^\nu - \frac{\epsilon^{(\lambda)}_{\alpha\nu}z^\alpha z^\nu}{pz}\left(p_\mu - \frac{m_T^2}{2pz}z_\mu\right). \quad (6)$$

The polarization tensor $\epsilon^{(\lambda)}_{\alpha\beta}$ satisfies the relations

$$\begin{aligned} \epsilon^{(\lambda)}_{\mu\nu} &= \epsilon^{(\lambda)}_{\nu\mu}, \\ \epsilon^{\mu}_{(\lambda)\mu} &= 0, \\ P_\mu\epsilon^{\mu\nu}_{(\lambda)} &= P_\nu\epsilon^{\mu\nu}_{(\lambda)} = 0, \\ \epsilon^{(\lambda)}_{\mu\nu}(\epsilon^{(\lambda')\mu\nu})^* &= \delta_{\lambda\lambda'}. \end{aligned} \quad (7)$$

The completeness relation reads

$$\sum_\lambda \epsilon^{(\lambda)}_{\mu\nu}(\epsilon^{(\lambda)}_{\rho\sigma})^* = \frac{1}{2}M_{\mu\rho}M_{\nu\sigma} + \frac{1}{2}M_{\mu\sigma}M_{\nu\rho} - \frac{1}{3}M_{\mu\nu}M_{\rho\sigma}, \quad (8)$$

where $M_{\mu\nu} = g_{\mu\nu} - P_\mu P_\nu/m_T^2$.

B. Decay constants

Decay constants of the vector meson are defined as

$$\begin{aligned} \langle V(P,\epsilon)|\bar{q}_2\gamma_\mu q_1|0\rangle &= -if_V m_V\epsilon^*_\mu, \\ \langle V(P,\epsilon)|\bar{q}_2\sigma_{\mu\nu} q_1|0\rangle &= -f_V^\perp(\epsilon^*_\mu P_\nu - \epsilon^*_\nu P_\mu). \end{aligned} \quad (9)$$

Contrary to the vector meson case, a $^3P_2$ tensor meson with $J^{PC} = 2^{++}$ cannot be produced through the local $V - A$ and tensor currents. To see this, we notice that

$$\langle T(P,\lambda)|V_\mu|0\rangle = a\epsilon^{*(\lambda)}_{\mu\nu}P^\nu + b\epsilon^{*(\lambda)\nu}_{\nu}P_\mu = 0, \quad (10)$$

$$\langle T(P,\lambda)|A_\mu|0\rangle = \varepsilon_{\mu\nu\rho\sigma}P^\nu\epsilon^{\rho\sigma*}_{(\lambda)} = 0, \quad (11)$$

where use of Eq. (7) has been made. Nevertheless, a tensor meson can be created from these local currents involving covariant derivatives:

$$\begin{aligned} \langle T(P,\lambda)|J_{\mu\nu}(0)|0\rangle &= f_T m_T^2\epsilon^{*(\lambda)}_{\mu\nu}, \\ \langle T(P,\lambda)|J^\perp_{\mu\nu\alpha}(0)|0\rangle &= -if_T^\perp m_T(\epsilon^{(\lambda)*}_{\mu\alpha}P_\nu - \epsilon^{(\lambda)*}_{\nu\alpha}P_\mu), \end{aligned} \quad (12)$$

where

$$\begin{aligned} J_{\mu\nu}(0) &= \frac{1}{2}(\bar{q}_1(0)\gamma_\mu i\overleftrightarrow{D}_\nu q_2(0) + \bar{q}_1(0)\gamma_\nu i\overleftrightarrow{D}_\mu q_2(0)), \\ J^\perp_{\mu\nu\alpha}(0) &= \bar{q}_1(0)\sigma_{\mu\nu}i\overleftrightarrow{D}_\alpha q_2(0), \end{aligned} \quad (13)$$

and $\overleftrightarrow{D}_\mu = \vec{D}_\mu - \overleftarrow{D}_\mu$ with $\vec{D}_\mu = \vec{\partial}_\mu + ig_sA^a_\mu\lambda^a/2$ and $\overleftarrow{D}_\mu = \overleftarrow{\partial}_\mu - ig_sA^a_\mu\lambda^a/2$.

The decay constant f_T of the tensor meson has been estimated using QCD sum rules for the tensor mesons $f_2(1270)$ [33] and $K_2^*(1430)$ [34] and the tensor-meson dominance hypothesis for $f_2(1270)$ [33,35,36]. The previous sum rule predictions are [33,34][1]

$$\begin{aligned} f_{f_2(1270)}(\mu = 1\ \text{GeV}) &\simeq 0.08m_{f_2(1270)} = 102\ \text{MeV}, \\ f_{K_2^*(1430)}(\mu = 1\ \text{GeV}) &\simeq (0.10 \pm 0.01)m_{K_2^*(1430)} \\ &= (143 \pm 14)\ \text{MeV}. \end{aligned} \quad (14)$$

Recently, we have derived a sum rule for $f_T(\mu)f_T^\perp(\mu)$ and revisited the sum rule analysis for $f_T(\mu)$. Our results of f_T and $f_T^\perp$ for various tensor mesons are shown in Table IV below [40]. Our sum rule results are in good agreement with [33] for $f_{f_2(1270)}$, but smaller than the results of [34] for $f_{K_2^*(1430)}$. The decay constants for $f_2(1270)$ and $f_2'(1525)$ can also be extracted based on the hypothesis of tensor-meson dominance together with the data of $\Gamma(f_2 \to \pi\pi)$ and $\Gamma(f_2' \to K\bar{K})$. We found that [40]

$$\begin{aligned} f_{f_2(1270)} &\simeq (0.085 \pm 0.001)m_{f_2(1270)} = (108 \pm 1)\ \text{MeV}, \\ f_{f_2'(1525)} &\simeq (0.089 \pm 0.003)m_{f_2'(1525)} = (136 \pm 5)\ \text{MeV}. \end{aligned} \quad (15)$$

They are in accordance with the sum rule predictions shown in Table IV.

C. Form factors

Form factors for $B \to P, V, T$ transitions are defined by [42–44]

[1]The dimensionless decay constant f_T defined in [33,34] differs from ours by a factor of $2m_T$. The factor of 2 comes from a different definition of $\overleftrightarrow{D}_\mu$ there.

$$
\begin{aligned}
\langle P(P)|V_\mu|B(p_B)\rangle &= \left(P_\mu + (p_B)_\mu - \frac{m_B^2 - m_P^2}{q^2} q_\mu\right) F_1^{BP}(q^2) + \frac{m_B^2 - m_P^2}{q^2} q_\mu F_0^{BP}(q^2),\\
\langle V(P,\lambda)|V_\mu|B(p_B)\rangle &= -i\frac{2}{m_B + m_V}\varepsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}_{(\lambda)} p_B^\alpha P^\beta V^{BV}(q^2),\\
\langle V(P,\lambda)|A_\mu|B(p_B)\rangle &= 2m_V \frac{\epsilon^{(\lambda)*}\cdot p_B}{q^2} q_\mu A_0^{BV}(q^2) + (m_B + m_V)\left[\epsilon_\mu^{(\lambda)*} - \frac{e^{(\lambda)*}\cdot p_B}{q^2} q_\mu\right] A_1^{BV}(q^2)\\
&\quad - \frac{\epsilon^{(\lambda)*}\cdot p_B}{m_B + m_V}\left[P_\mu + (p_B)_\mu - \frac{m_B^2 - m_V^2}{q^2} q_\mu\right] A_2^{BV}(q^2)\\
\langle T(P,\lambda)|V_\mu|B(p_B)\rangle &= -i\frac{2}{m_B + m_T}\varepsilon_{\mu\nu\alpha\beta} e^{*\nu}_{(\lambda)} p_B^\alpha P^\beta V^{BT}(q^2),\\
\langle T(P,\lambda)|A_\mu|B(p_B)\rangle &= 2m_T \frac{e^{(\lambda)*}\cdot p_B}{q^2} q_\mu A_0^{BT}(q^2) + (m_B + m_T)\left[e_\mu^{(\lambda)*} - \frac{e^{(\lambda)*}\cdot p_B}{q^2} q_\mu\right] A_1^{BT}(q^2)\\
&\quad - \frac{e^{(\lambda)*}\cdot p_B}{m_B + m_T}\left[P_\mu + (p_B)_\mu - \frac{m_B^2 - m_T^2}{q^2} q_\mu\right] A_2^{BT}(q^2),
\end{aligned}
\tag{16}
$$

where $q_\mu = (p_B - P)_\mu$ and $e^{*\mu}_{(\lambda)} \equiv \epsilon^{*\mu\nu}_{(\lambda)} p_{B\nu}/m_B$. Throughout the paper we have adopted the convention $\varepsilon^{0123} = -1$.

In the Isgur-Scora-Grinstein-Wise (ISGW) model [45], the general expression for the $B \to T$ transition is parametrized as

$$
\begin{aligned}
&\langle T(P,\lambda)|(V-A)_\mu|B(p_B)\rangle\\
&\quad = ih(q^2)\varepsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu\alpha} p_{B\alpha}(p_B + P)^\rho q^\sigma - k(q^2)\epsilon^*_{\mu\nu} p_B^\nu\\
&\qquad - b_+(q^2)\epsilon^*_{\alpha\beta} p_B^\alpha p_B^\beta (p_B + P)_\mu - b_-(q^2)\epsilon^*_{\alpha\beta} p_B^\alpha p_B^\beta q_\mu,
\end{aligned}
\tag{17}
$$

where the form factor k is dimensionless, and the canonical dimension of h, b_+ and b_- is -2. The relations between these two different sets of form factors are

$$
\begin{aligned}
V^{BT}(q^2) &= m_B(m_B + m_T)h(q^2),\\
A_1^{BT}(q^2) &= \frac{m_B}{m_B + m_T} k(q^2),\\
A_2^{BT}(q^2) &= -m_B(m_B + m_T)b_+(q^2),\\
A_0^{BT}(q^2) &= \frac{m_B}{2m_T}[k^2(q^2) + (m_B^2 - m_T^2)b_+(q^2) + q^2 b_-(q^2)].
\end{aligned}
\tag{18}
$$

The $B \to T$ transition form factors have been evaluated in the ISGW model [45] and its improved version, ISGW2 [46], the covariant light-front quark model (CLFQ) [47], the light-cone sum rule (LCSR) approach [48], the large energy effective theory (LEET) [49–51] and the pQCD approach [44]. In LEET, form factors are evaluated at large recoil and all the form factors in the LEET limit to be specified below can be parametrized in terms of two independent universal form factors $\zeta_\perp$ and $\zeta_\parallel$ [43]:

$$
\begin{aligned}
V^{BT}(q^2) &= \frac{m_T}{|\vec{p}_T|}\left(1 + \frac{m_T}{m_B}\right)\zeta_\perp(q^2),\\
A_0^{BT}(q^2) &= \frac{m_T}{|\vec{p}_T|}\left[\left(1 - \frac{m_T^2}{m_B E_T}\right)\zeta_\parallel(q^2) + \frac{m_T}{m_B}\zeta_\perp(q^2)\right],\\
A_1^{BT}(q^2) &= \frac{m_T}{|\vec{p}_T|}\left(\frac{2E_T}{m_B + m_T}\right)\zeta_\perp(q^2),\\
A_2^{BT}(q^2) &= \frac{m_T}{|\vec{p}_T|}\left(1 + \frac{m_T}{m_B}\right)\left[\zeta_\perp(q^2) - \frac{m_T}{E_T}\zeta_\parallel(q^2)\right],
\end{aligned}
\tag{19}
$$

where E_T is the energy of the tensor meson

$$
E_T = \frac{m_B}{2}\left(1 + \frac{m_T^2 - q^2}{m_B^2}\right). \tag{20}
$$

In the LEET limit,

$$
E_T, m_B \gg m_T, \Lambda_{\rm QCD}. \tag{21}
$$

Using the recent analysis of tensor meson distribution amplitudes [40], one of us (KCY) has calculated the form factors of B decays into tensor mesons using the LCSR

TABLE II. $B \to T$ transition form factors at $q^2 = 0$ evaluated in the ISGW2, CLFQ, LCSR, LEET and pQCD models. The CLFQ results are obtained by first calculating the form factors $h(q^2)$, $b_+(q^2)$ and $b_-(q^2)$ using the covariant light-front approach and $k(q^2)$ from the heavy quark symmetry relation Eq. (22) and then converting them into the form-factor set $V(q^2)$ and $A_{0,1,2}(q^2)$. To compute the form factors in LEET, we have applied $\zeta_\perp(0) = 0.28 \pm 0.04$ and $\zeta_\parallel(0) = 0.22 \pm 0.03$. LCSR and pQCD results are taken from [44,48], respectively.

F	ISGW2	CLFQ	LCSR	LEET	pQCD	F	ISGW2	CLFQ	LCSR	LEET	pQCD
V^{Ba_2}	0.32	0.28	0.18 ± 0.02	0.18 ± 0.03	$0.18^{+0.05}_{-0.04}$	$A_0^{Ba_2}$	0.20	0.24	0.21 ± 0.04	0.14 ± 0.02	$0.18^{+0.06}_{-0.04}$
$A_1^{Ba_2}$	0.16	0.21	0.14 ± 0.02	0.13 ± 0.02	$0.11^{+0.03}_{-0.03}$	$A_2^{Ba_2}$	0.14	0.19	0.09 ± 0.02	0.13 ± 0.02	$0.06^{+0.02}_{-0.01}$
$V^{Bf_{2q}}$	0.32	0.28	0.18 ± 0.02	0.18 ± 0.02	$0.12^{+0.03}_{-0.03}$	$A_0^{Bf_{2q}}$	0.20	0.25	0.20 ± 0.04	0.13 ± 0.02	$0.13^{+0.04}_{-0.03}$
$A_1^{Bf_{2q}}$	0.16	0.21	0.14 ± 0.02	0.12 ± 0.02	$0.08^{+0.02}_{-0.02}$	$A_2^{Bf_{2q}}$	0.14	0.19	0.10 ± 0.02	0.13 ± 0.02	$0.04^{+0.01}_{-0.01}$
$V^{BK_2^*}$	0.38	0.29	0.16 ± 0.02	0.21 ± 0.03	$0.21^{+0.06}_{-0.05}$	$A_0^{BK_2^*}$	0.27	0.23	0.25 ± 0.04	0.15 ± 0.02	$0.18^{+0.05}_{-0.04}$
$A_1^{BK_2^*}$	0.24	0.22	0.14 ± 0.02	0.14 ± 0.02	$0.13^{+0.04}_{-0.03}$	$A_2^{BK_2^*}$	0.22	0.21	0.05 ± 0.02	0.14 ± 0.02	$0.08^{+0.03}_{-0.02}$

approach [48]. The LCSR results are close to LEET and pQCD calculations.

The $B \to a_2(1320)$, $f_{2q} = (u\bar{u} + d\bar{d})/\sqrt{2}$, $K_2^*(1430)$ transition form factors calculated in various models at the maximal recoil $q^2 = 0$ are summarized in Table II. The ISGW model [45] is based on the nonrelativistic constituent quark picture. In general, the form factors evaluated in the ISGW model are reliable only at $q^2 = q_m^2 \equiv (m_B - m_T)^2$, the maximum momentum transfer. The reason is that the form-factor q^2 dependence in the ISGW model is proportional to $\exp[-(q_m^2 - q^2)]$, and hence the form factor decreases exponentially as a function of $(q_m^2 - q^2)$. (See Appendix A for details.) This has been improved in the ISGW2 model [46], in which the form factor has a more realistic behavior at large $(q_m^2 - q^2)$ which is expressed in terms of a certain polynomial term. As noticed in [20], form factors are increased in the ISGW2 model so that the branching fractions of $B \to TM$ decays are enhanced by about an order of magnitude compared to the estimates based on the ISGW model.

The CLFQ model is a relativistic quark model in which a consistent and fully relativistic treatment of quark spins and the center-of-mass motion is carried out. This model is very suitable to study hadronic form factors. Especially as the recoil momentum increases (corresponding to a decreasing q^2), we need to start considering relativistic effects seriously. In particular, at the maximum recoil point $q^2 = 0$ where the final-state meson could be highly relativistic, it is expected that the corrections to the nonrelativistic quark model will be sizable in this case.

The CLFQ and ISGW2 model predictions for $B \to T$ transition form factors differ mainly in two aspects: (i) When q^2 increases, $h(q^2)$, $b_+(q^2)$ and $b_-(q^2)$ increase more rapidly in the former and (ii) The form factor k obtained in both models is quite different. For example, $k^{BK_2^*}(0) = 0.015$ in the former and 0.293 in the latter. Indeed, it has been noticed [47] that among the four $B \to T$ transition form factors, the one $k(q^2)$ is particularly sensitive to β_T, a parameter describing the tensor-meson wave function, and that $k(q^2)$ at zero recoil shows a large deviation from the heavy quark symmetry relation. It is not clear to us if the very complicated analytic expression for $k(q^2)$ in Eq. (3.29) of [47] is complete. To overcome this difficulty, it was pointed out in [47] that one may apply the heavy quark symmetry relation to obtain $k(q^2)$ for $B \to T$ transition

$$k(q^2) = m_B m_T\left(1 + \frac{m_B^2 + m_T^2 - q^2}{2m_B m_T}\right) \times \left[h(q^2) - \frac{1}{2}b_+(q^2) + \frac{1}{2}b_-(q^2)\right]. \qquad (22)$$

In Table II the CLFQ results were obtained by first calculating the form factors $h(q^2)$, $b_+(q^2)$ and $b_-(q^2)$ using the covariant light-front approach [47] and $k(q^2)$ from the heavy quark symmetry relation Eq. (22) and then converting them into the form-factor set $V(q^2)$ and $A_{0,1,2}(q^2)$.

Form factors in the CLFQ model are first calculated in the spacelike region and their momentum dependence is fitted to a 3-parameter form:

$$F(q^2) = \frac{F(0)}{1 - a(q^2/m_B^2) + b(q^2/m_B^2)^2}. \qquad (23)$$

The parameters a, b and $F(0)$ are first determined in the spacelike region. This parametrization is then analytically continued to the timelike region to determine the physical form factors at $q^2 \geq 0$. The results are exhibited in Table III. The momentum dependence of the form factors in the LCSR approach can be found in [48], while a slightly different parametrization,

$$F(q^2) = \frac{F(0)}{(1 - q^2/m_B^2)[1 - aq^2/m_B^2 + bq^4/m_B^4]}, \quad (24)$$

is used in the pQCD approach for the calculations of the form-factor q^2 dependence [44].

For the calculation in LEET, we have followed [52] to use $\zeta_\perp(0) = 0.28 \pm 0.04$ and $\zeta_\parallel(0) = 0.22 \pm 0.03$. For the q^2 dependence, we shall use

$$\zeta^T_{\perp,\parallel}(q^2) = \frac{\zeta^T_{\perp,\parallel}(0)}{(1 - q^2/m_B^2)^2}. \quad (25)$$

For the ISGW2 model, the q^2 dependence of the form factors is governed by Eq. (A1).

TABLE III. $B \to T$ transition form factors obtained in the covariant light-front model and fitted to the 3-parameter form Eq. (23).

F	$F(0)$	a	b	F	$F(0)$	a	b
V^{Ba_2}	0.28	2.19	2.22	$A_0^{Ba_2}$	0.24	1.28	0.84
$A_1^{Ba_2}$	0.21	1.38	0.47	$A_2^{Ba_2}$	0.19	1.93	1.69
$V^{Bf_{2q}}$	0.28	2.19	2.22	$A_0^{Bf_{2q}}$	0.25	1.37	0.95
$A_1^{Bf_{2q}}$	0.21	1.39	0.46	$A_2^{Bf_{2q}}$	0.19	1.93	1.69
$V^{BK_2^*}$	0.29	2.17	2.22	$A_0^{BK_2^*}$	0.23	1.23	0.74
$A_1^{BK_2^*}$	0.22	1.42	0.50	$A_2^{BK_2^*}$	0.21	1.96	1.79

D. Light-cone distribution amplitudes

The light-cone distribution amplitudes (LCDAs) of the tensor meson are defined as [40][2]

$$\langle T(P,\lambda)|\bar q_1(y)\gamma_\mu q_2(x)|0\rangle = -if_T m_T^2 \int_0^1 du e^{i(uPy+\bar uPx)}\bigg\{P_\mu \frac{\epsilon^{(\lambda)*}_{\alpha\beta} z^\alpha z^\beta}{(Pz)^2}\Phi^T_\parallel(u) + \bigg(\frac{\epsilon^{(\lambda)*}_{\mu\alpha} z^\alpha}{Pz} - P_\mu \frac{\epsilon^{(\lambda)*}_{\beta\alpha} z^\beta z^\alpha}{(Pz)^2}\bigg) g_v(u) - \frac{1}{2} z_\mu \frac{\epsilon^{(\lambda)*}_{\alpha\beta} z^\alpha z^\beta}{(Pz)^3} m_T^2 \bar g_3(u) + \mathcal{O}(z^2)\bigg\}, \quad (26)$$

$$\langle T(P,\lambda)|\bar q_1(y)\gamma_\mu\gamma_5 q_2(x)|0\rangle = -if_T m_T^2 \int_0^1 du e^{i(uPy+\bar uPx)} \varepsilon_{\mu\nu\alpha\beta} z^\nu P^\alpha \epsilon^{*\beta\delta}_{(\lambda)} z_\delta \frac{1}{2Pz} g_a(u), \quad (27)$$

$$\langle T(P,\lambda)|\bar q_1(y)\sigma_{\mu\nu} q_2(x)|0\rangle = -f_T^\perp m_T \int_0^1 du e^{i(uPy+\bar uPx)}\bigg\{[\epsilon^{(\lambda)*}_{\mu\alpha} z^\alpha P_\nu - \epsilon^{(\lambda)*}_{\nu\alpha} z^\alpha P_\mu]\frac{1}{Pz}\Phi^T_\perp(u) + (P_\mu z_\nu - P_\nu z_\mu) \times \frac{m_T^2 \epsilon^{(\lambda)*}_{\alpha\beta} z^\alpha z^\beta}{(Pz)^3}\bar h_t(u) + \frac{1}{2}[\epsilon^{(\lambda)*}_{\mu\alpha} z^\alpha z_\nu - \epsilon^{(\lambda)*}_{\nu\alpha} z^\alpha z_\mu]\frac{m_T^2}{(Pz)^2}\bar h_3(u) + \mathcal{O}(z^2)\bigg\}, \quad (28)$$

$$\langle T(P,\lambda)|\bar q_1(y) q_2(x)|0\rangle = -f_T^\perp m_T^3 \int_0^1 du e^{i(uPy+\bar uPx)} \frac{\epsilon^{(\lambda)*}_{\alpha\beta} z^\alpha z^\beta}{2Pz} h_s(u), \quad (29)$$

where $\bar g_3 = g_3 + \Phi^T_\parallel - 2g_v$, $\bar h_t = h_t - \frac{1}{2}(\Phi^T_\perp + h_3)$, $\bar h_3 = h_3 - \Phi^T_\perp$, and $z \equiv y - x$. Here $\Phi^T_\parallel$, $\Phi^T_\perp$ are twist-2 LCDAs,[3] g_v, g_a, h_t, h_s twist-3 ones, and g_3, h_3 twist-4. In the SU(3) limit, due to the G-parity of the tensor meson, $\Phi^T_\parallel$, $\Phi^T_\perp$, g_v, g_a, h_t, h_s, g_3 and h_3 are antisymmetric under the replacement $u \to 1 - u$ [40].

Using the QCD equations of motion [31,32], the two-parton distribution amplitudes g_v, g_a, h_t and h_s can be represented in terms of $\Phi^T_{\parallel,\perp}$ and three-parton distribution amplitudes. Neglecting the three-parton distribution amplitudes containing gluons and terms proportional to light quark masses, twist-3 LCDAs g_a, g_v, h_t and h_s are related to twist-2 ones through the Wandzura-Wilczek relations:

$$g_v^{WW}(u) = \int_0^u dv \frac{\Phi^T_\parallel(v)}{\bar v} + \int_u^1 dv \frac{\Phi^T_\parallel(v)}{v},$$

$$g_a^{WW}(u) = 2\bar u \int_0^u dv \frac{\Phi^T_\parallel(v)}{\bar v} + 2u \int_u^1 dv \frac{\Phi^T_\parallel(v)}{v},$$

$$h_t^{WW}(u) = \frac{3}{2}(2u-1)\bigg(\int_0^u dv \frac{\Phi^T_\perp(v)}{\bar v} - \int_u^1 dv \frac{\Phi^T_\perp(v)}{v}\bigg),$$

$$h_s^{WW}(u) = 3\bigg(\bar u \int_0^u dv \frac{\Phi^T_\perp(v)}{\bar v} + u \int_u^1 dv \frac{\Phi^T_\perp(v)}{v}\bigg). \quad (30)$$

[2] The LCDAs of the tensor meson were first studied in [53].

[3] Since in the transversity basis, one denotes the corresponding parallel and perpendicular states by $A_\parallel$ and $A_\perp$, a better notation for the longitudinal and transverse LCDAs will be Φ_L and Φ_T, respectively, rather than $\Phi_\parallel$ and $\Phi_\perp$. Indeed, the transverse polarization includes both parallel and perpendicular polarizations. In the present work we follow the conventional notation for LCDAs.

These Wandzura-Wilczek relations further give us

$$\begin{aligned}
&\frac{1}{4}g_a'(u)+\frac{1}{2}g_v(u)=\int_u^1\frac{\Phi_\parallel^T(v)}{v}dv\equiv\Phi_+^T(u),\qquad \frac{1}{4}g_a'(u)-\frac{1}{2}g_v(u)=-\int_0^u\frac{\Phi_\parallel^T(v)}{\bar v}dv\equiv-\Phi_-^T(u),\\
&h_s'(u)=-3\bigg[\int_0^u\frac{\Phi_\perp^T(v)}{\bar v}dv-\int_u^1\frac{\Phi_\perp^T(v)}{v}dv\bigg]\equiv-3\Phi_t(u),\\
&\int_0^u dv\bigg(\Phi_\perp^T(v)-\frac{2}{3}h_t(v)\bigg)=u\bar u\bigg[\int_0^u\frac{\Phi_\perp^T(v)}{\bar v}dv-\int_u^1\frac{\Phi_\perp^T(v)}{v}dv\bigg]=u\bar u\Phi_t(u),\\
&\int_0^u dv\bigg(\Phi_\parallel^T(v)-\frac{1}{2}g_v(v)\bigg)=\frac{1}{2}\bigg[\bar u\int_0^u\frac{\Phi_\parallel^T(v)}{\bar v}dv-u\int_u^1\frac{\Phi_\parallel^T(v)}{v}dv\bigg]=\frac{1}{2}(\bar u\Phi_-^T(u)-u\Phi_+^T(u)).
\end{aligned}\tag{31}$$

The LCDAs $\Phi_{\parallel,\perp}^T(u,\mu)$ and $\Phi_t(u,\mu)$ can be expanded as

$$\begin{aligned}
\Phi_{\parallel,\perp}^T(u,\mu)&=6u(1-u)\sum_{l=0}^\infty a_l^{(\parallel,\perp),T}(\mu)C_l^{3/2}(2u-1),\\
\Phi_t(u,\mu)&=3\sum_{l=0}^\infty a_l^{\perp,T}(\mu)P_{l+1}(2u-1),
\end{aligned}\tag{32}$$

where the Gegenbauer moments $a_l^{(\parallel,\perp),T}$ with l being even vanish in the SU(3) limit, μ is the normalization scale and $P_n(x)$ are the Legendre polynomials. In the present study the distribution amplitudes are normalized to be

$$\begin{aligned}
&\int_0^1 du(2u-1)\Phi_\parallel^T(u)=\int_0^1 du(2u-1)\Phi_\perp^T(u)=1,\\
&\int_0^1 du\Phi_t(u)=0.
\end{aligned}\tag{33}$$

Consequently, the first Gegenbauer moments are fixed to be $a_1^{\parallel,T}=a_1^{\perp,T}=\frac{5}{3}$. Moreover, we have

$$\begin{aligned}
&3\int_0^1 du(2u-1)g_a(u)=\int_0^1 du(2u-1)g_v(u)=1,\\
&2\int_0^1 du(2u-1)h_s(u)=\int_0^1 du(2u-1)h_t(u)=1,
\end{aligned}\tag{34}$$

which hold even if the complete leading twist distribution amplitudes and corrections from the three-parton distribution amplitudes containing gluons are included. The asymptotic wave function is therefore

$$\Phi_{\parallel,\perp}^{T,\mathrm{as}}(u)=30u(1-u)(2u-1),\tag{35}$$

and the corresponding expressions for the twist-3 distributions are

$$\begin{aligned}
g_v^{\mathrm{as}}(u)&=5(2u-1)^3,\\
g_a^{\mathrm{as}}(u)&=10u(1-u)(2u-1),\\
h_t^{\mathrm{as}}(u)&=\frac{15}{2}(2u-1)(1-6u+6u^2),\\
h_s^{\mathrm{as}}(u)&=15u(1-u)(2u-1),
\end{aligned}\tag{36}$$

and

$$\Phi_t^{\mathrm{as}}(u)=5(1-6u+6u^2).\tag{37}$$

Note that, contrary to the twist-2 LCDA $\Phi_{\parallel,\perp}^T(u)$, the twist-3 one $\Phi_t(u)$ is even under the replacement $u\to 1-u$ in the SU(3) limit.

For vector mesons, the general expressions of LCDAs are

$$\Phi_V(x,\mu)=6x(1-x)\bigg[1+\sum_{n=1}^\infty a_n^{\parallel,V}(\mu)C_n^{3/2}(2x-1)\bigg],\tag{38}$$

and

$$\Phi_v(x,\mu)=3\bigg[2x-1+\sum_{n=1}^\infty a_n^{\perp,V}(\mu)P_{n+1}(2x-1)\bigg].\tag{39}$$

Likewise, for pseudoscalar mesons,

$$\begin{aligned}
\Phi_P(x,\mu)&=6x(1-x)\bigg[1+\sum_{n=1}^\infty a_n^P(\mu)C_n^{3/2}(2x-1)\bigg],\\
\Phi_p(x,\mu)&=1.
\end{aligned}\tag{40}$$

E. Helicity projection operators

In the QCDF calculation, we need to know the helicity projection operators in the momentum space. To do this, using the identity

$$\begin{aligned}
\bar q_\alpha^1(y)q_\delta^2(x)=\frac{1}{4}\Big\{&\mathbf{1}[\bar q^1(y)q^2(x)]+\gamma_5[\bar q^1(y)\gamma_5 q^2(x)]\\
&+\gamma^\rho[\bar q_1(y)\gamma_\rho q^2(x)]-\gamma^\rho\gamma_5[\bar q^1(y)\gamma_\rho\gamma_5 q^2(x)]\\
&+\frac{1}{2}\sigma^{\rho\lambda}[\bar q^1(y)\sigma_{\rho\lambda}q^2(x)]\Big\}_{\delta\alpha}
\end{aligned}\tag{41}$$

and Eqs. (26)–(29), we obtain

$$\langle T(P,\lambda)|\bar q^1_\alpha(y)q^2_\delta(x)|0\rangle = -\frac{i}{4}\int_0^1 du e^{i(uPy+\bar uPx)}\Big\{f_T m_T^2\Big[\not P\frac{\epsilon^{*(\lambda)}_{\mu\nu}z^\mu z^\nu}{(Pz)^2}\Phi^T_\parallel(u) - \frac{1}{2}\not z\frac{\epsilon^{*(\lambda)}_{\mu\nu}z^\mu z^\nu}{(Pz)^3}m_T^2\bar g_3(u) + \Big(\frac{\epsilon^{*(\lambda)}_{\mu\nu}z^\nu}{Pz} - P_\mu\frac{\epsilon^{*(\lambda)}_{\nu\beta}z^\nu z^\beta}{(Pz)^2}\Big)\gamma^\mu g_v(u) + \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\epsilon^{*\nu\beta}_{(\lambda)}z_\beta P^\rho z^\sigma\gamma_5\frac{1}{Pz}g_a(u)\Big] - \frac{i}{2}f_T^\perp m_T\Big[\sigma^{\mu\nu}(\epsilon^{(\lambda)*}_{\mu\beta}z^\beta P_\nu - \epsilon^{(\lambda)*}_{\nu\beta}z^\beta P_\mu)\frac{1}{Pz}\Phi^T_\perp(u) + \sigma^{\mu\nu}(P_\mu z_\nu - P_\nu z_\mu)\frac{m_T^2\epsilon^{(\lambda)*}_{\rho\beta}z^\rho z^\beta}{(Pz)^3}\bar h_t(u) + \frac{1}{2}\sigma^{\mu\nu}(\epsilon^{(\lambda)*}_{\mu\beta}z^\beta z_\nu - \epsilon^{(\lambda)*}_{\nu\beta}z^\beta z_\mu)\frac{m_T^2}{(Pz)^2}\bar h_3(u) + \epsilon^{*(\lambda)}_{\mu\nu}z^\mu z^\nu\frac{m_T^2}{Pz}h_s(u)\Big] + \mathcal{O}[(x-y)^2]\Big\}_{\delta\alpha}. \tag{42}$$

Since any four momentum can be split into light-cone and transverse components as $k^\mu = k^\mu_- + k^\mu_+ + k^\mu_\perp$, we shall assign the momenta

$$k_1^\mu = uEn^\mu_- + \frac{k_\perp^2}{4uE}n^\mu_+ + k^\mu_\perp,\qquad k_2^\mu = \bar uEn^\mu_- + \frac{k_\perp^2}{4\bar uE}n^\mu_+ - k^\mu_\perp \tag{43}$$

to the quark and antiquark, respectively, in an energetic light final-state meson with the momentum P^μ and mass m, satisfying the relation $P^\mu = En^\mu_- + m^2n^\mu_+/(4E) \simeq En^\mu_-$, where we have defined two lightlike vectors $n^\mu_\pm$ with $n^\mu_- \equiv (1,0,0,-1)$ and $n^\mu_+ \equiv (1,0,0,1)$ and assumed that the meson moves along the n^μ_- direction. To obtain the light-cone projection operator of the meson in the momentum space, we take the Fourier transformation of Eq. (42) and apply the following substitution in the calculation:

$$z^\mu \to -i\frac{\partial}{\partial k_{1\mu}} \simeq -i\Big(\frac{n^\mu_+}{2E}\frac{\partial}{\partial u} + \frac{\partial}{\partial k_{\perp\mu}}\Big), \tag{44}$$

where terms of order $k_\perp^2$ have been omitted. The longitudinal projector reads

$$M^T_\parallel(\lambda=0) = -i\frac{f_T}{4}E\Big\{\Big[\epsilon^{(\lambda)*}_{\alpha\beta}n^\alpha_+n^\beta_+\Big(\frac{m_T}{2E}\Big)^2\Big]\not n_-\Phi^T_\parallel(u) + \frac{f_T^\perp}{f_T}\frac{m_T}{E}\Big[\epsilon^{(\lambda)*}_{\alpha\beta}n^\alpha_+n^\beta_+\Big(\frac{m_T}{2E}\Big)^2\Big]\times\Big[-\frac{i}{2}\sigma_{\mu\nu}n^\mu_-n^\nu_+h_t(u) - iE\int_0^u dv(\Phi^T_\perp(v) - h_t(v))\sigma_{\mu\nu}n^\mu_-\frac{\partial}{\partial k_{\perp\nu}} + \frac{h_s'(u)}{2}\Big] + \frac{f_T^\perp}{f_T}\frac{m_T}{E}iE\sigma_{\mu\nu}n^\nu_-\epsilon^{(\lambda)*\mu\alpha}\delta_{\lambda,0}\frac{\partial}{\partial k^\alpha_\perp}\int_0^u dv\Phi^T_\perp(v) + \mathcal{O}\Big(\frac{m_T^2}{E^2}\Big)\Big\}$$
$$= -i\frac{f_T}{4}E\Big[\epsilon^{(\lambda)*}_{\alpha\beta}n^\alpha_+n^\beta_+\Big(\frac{m_T}{2E}\Big)^2\Big]\Big\{\not n_-\Phi^T_\parallel(u) + \frac{f_T^\perp}{f_T}\frac{m_T}{E}\Big[-\frac{i}{2}\sigma_{\mu\nu}n^\mu_-n^\nu_+h_t(u) - \frac{3i}{2}E\int_0^u dv\Big(\Phi^T_\perp(v) - \frac{2}{3}h_t(v)\Big)\times\sigma_{\mu\nu}n^\mu_-\frac{\partial}{\partial k_{\perp\nu}} + \frac{h_s'(u)}{2}\Big] + \mathcal{O}\Big(\frac{m_T^2}{E^2}\Big)\Big\}, \tag{45}$$

and the transverse projectors have the form

$$M^T_\perp(\lambda=\pm1) = -i\frac{f_T^\perp}{4}E\Big[\epsilon^{*(\lambda)}_{\perp\mu\alpha}n^\alpha_+\Big(\frac{m_T}{2E}\Big)\Big]\Big\{\gamma^\mu\not n_-\Phi^T_\perp(u) + \frac{f_T}{f_T^\perp}\frac{m_T}{E}\Big[\gamma^\mu g_v(u) - E\int_0^u dv(2\Phi^T_\parallel(v) - g_v(v))\not n_-\frac{\partial}{\partial k_{\perp\mu}} - i\varepsilon_{\mu\nu\rho\sigma}\gamma^\nu n^\rho_-\gamma_5\Big(n^\sigma_+\frac{g_a'(u)}{4} - E\frac{g_a(u)}{2}\frac{\partial}{\partial k_{\perp\sigma}}\Big)\Big] + \mathcal{O}\Big(\frac{m_T^2}{E^2}\Big)\Big\}, \tag{46}$$

and

$$M^T_\perp(\lambda=\pm2) = -i\frac{f_T^\perp}{4}E\Big\{\frac{m_T}{E}iE\sigma_{\mu\nu}n^\nu_-\epsilon^{(\lambda)*\mu\alpha}\delta_{\lambda,\pm2}\frac{\partial}{\partial k^\alpha_\perp}\int_0^u dv\Phi^T_\perp(v) + \mathcal{O}\Big(\frac{m_T^2}{E^2}\Big)\Big\}. \tag{47}$$

The exactly longitudinal and transverse polarization tensors of the tensor meson, which are independent of the coordinate variable $z = y - x$, have the expressions

$$\epsilon^{*(0)\mu\nu}n_\nu^+ = \sqrt{\frac{2}{3}}\frac{2E^2}{m_T^2}\left[\left(1-\frac{m_T^2}{4E^2}\right)n_-^\mu - \frac{m_T^2}{4E^2}n_+^\mu\right],$$
$$\epsilon_\perp^{*(\lambda)\mu\nu}n_\nu^+ = \left(\epsilon^{*(\lambda)\mu\nu} - \frac{\epsilon^{*(\lambda)\nu\alpha}n_\alpha^+}{2}n_-^\mu - \frac{\epsilon^{*(\lambda)\nu\alpha}n_\alpha^-}{2}n_+^\mu\right)n_\nu^+\delta_{\lambda,\pm1}, \quad (48)$$

which in turn imply that

$$\epsilon^{*(\lambda)}_{\mu\nu}n_+^\mu n_+^\nu\left(\frac{m_T}{2E}\right)^2 = \sqrt{\frac{2}{3}}\delta_{\lambda,0},$$
$$\epsilon^{*(\lambda)}_{\perp\mu\nu}n_+^\nu\left(\frac{m_T}{2E}\right) = \sqrt{\frac{1}{2}}\epsilon_\mu^*(\pm1)\delta_{\lambda,\pm1}. \quad (49)$$

The projector on the transverse polarization states in the helicity basis reads

$$M_{\mp1}^T(u) = -i\frac{f_T^\perp}{4\sqrt{2}}E\Bigg\{\not\epsilon^*(\mp1)\not n_-\Phi_\perp^T(u) + \frac{1}{2}\frac{f_T}{f_T^\perp}\frac{m_T}{E}\Bigg[\not\epsilon^*(\mp1)(1-\gamma_5)\left(g_v(u)\pm\frac{g_a'(u)}{2}\right) + \not\epsilon^*(\mp1)(1+\gamma_5)$$
$$\times\left(g_v(u)\mp\frac{g_a'(u)}{2}\right) - E\not n_-(1-\gamma_5)\left(\int_0^u dv(2\Phi_\parallel^T(v)-g_v(v))\mp\frac{g_a(u)}{2}\right)\epsilon_\nu^*(\mp1)\frac{\partial}{\partial k_{\perp\nu}} - E\not n_-(1+\gamma_5)$$
$$\times\left(\int_0^u dv(2\Phi_\parallel^T(v)-g_v(v))\pm\frac{g_a(u)}{2}\right)\epsilon_\nu^*(\mp1)\frac{\partial}{\partial k_{\perp\nu}}\Bigg] + \mathcal{O}\left(\frac{m_T^2}{E^2}\right)\Bigg\}. \quad (50)$$

After applying the Wandzura-Wilczek relations Eq. (31), the transverse helicity projector (50) can be simplified to

$$M_{\mp1}^T(u) = -i\frac{f_T^\perp}{4\sqrt{2}}E\Bigg\{\not\epsilon^*(\mp1)\not n_-\Phi_\perp^T(u) + \frac{f_T}{f_T^\perp}\frac{m_T}{E}\Bigg[\epsilon_\nu^*(\mp1)\Phi_+^T(u)\left(\gamma^\nu(1\mp\gamma_5) + uE\not n_-(1\mp\gamma_5)\frac{\partial}{\partial k_{\perp\nu}}\right)$$
$$+\epsilon_\nu^*(\mp1)\Phi_-^T(u)\left(\gamma^\nu(1\pm\gamma_5) - \bar uE\not n_-(1\pm\gamma_5)\frac{\partial}{\partial k_{\perp\nu}}\right)\Bigg] + \mathcal{O}\left(\frac{m_T^2}{E^2}\right)\Bigg\}, \quad (51)$$

to be compared with

$$M_{\mp1}^V(u) = -i\frac{f_V^\perp}{4}E\Bigg\{\not\epsilon^*(\mp1)\not n_-\Phi_\perp^V(u) + \frac{f_V}{f_V^\perp}\frac{m_V}{E}\Bigg[\epsilon_\nu^*(\mp1)\Phi_+^V(u)\left(\gamma^\nu(1\mp\gamma_5) + uE\not n_-(1\mp\gamma_5)\frac{\partial}{\partial k_{\perp\nu}}\right)$$
$$+\epsilon_\nu^*(\mp1)\Phi_-^V(u)\left(\gamma^\nu(1\pm\gamma_5) - \bar uE\not n_-(1\pm\gamma_5)\frac{\partial}{\partial k_{\perp\nu}}\right)\Bigg] + \mathcal{O}\left(\frac{m_V^2}{E^2}\right)\Bigg\} \quad (52)$$

for the vector meson. The longitudinal projector for the tensor meson can be recast as

$$M_\parallel^T(\lambda=0) = -i\frac{f_T}{4}\sqrt{\frac{2}{3}}E\Bigg\{\not n_-\Phi_\parallel^T(u) + \frac{3}{2}\frac{f_T^\perp}{f_T}\frac{m_T}{E}\Phi_t(u)\times\left[-\frac{i}{2}\sigma_{\mu\nu}n_-^\mu n_+^\nu(u-\bar u) - iEu\bar u\sigma_{\mu\nu}n_-^\mu\frac{\partial}{\partial k_{\perp\nu}} - \mathbf{1}\right] + \mathcal{O}\left(\frac{m_T^2}{E^2}\right)\Bigg\}, \quad (53)$$

to be compared with

$$M_\parallel^V(\lambda=0) = -i\frac{f_V}{4}E\Bigg\{\not n_-\Phi_\parallel^V(u) + \frac{f_V^\perp}{f_V}\frac{m_V}{E}\Phi_v(u)\times\left[-\frac{i}{2}\sigma_{\mu\nu}n_-^\mu n_+^\nu(u-\bar u) - iEu\bar u\sigma_{\mu\nu}n_-^\mu\frac{\partial}{\partial k_{\perp\nu}} - \mathbf{1}\right] + \mathcal{O}\left(\frac{m_V^2}{E^2}\right)\Bigg\} \quad (54)$$

for the vector meson.

F. A summary of input parameters

It is useful to summarize all the input parameters we have used in this work. Some of the input quantities are collected in Table IV.

The Wilson coefficients $c_i(\mu)$ at various scales, $\mu =$ 4.4 GeV, 2.1 GeV, 1.45 GeV and 1 GeV, are taken from [54]. For the renormalization scale of the decay amplitude, we choose $\mu = m_b(m_b)$. However, as will be discussed below, the hard spectator and annihilation contributions will be evaluated at the hard-collinear scale $\mu_h = \sqrt{\mu\Lambda_h}$ with $\Lambda_h \approx 500$ MeV.

III. $B \to TP$, TV DECAYS

Within the framework of QCD factorization [24], the effective Hamiltonian matrix elements are written in the form

$$\langle M_1M_2|\mathcal{H}_{\rm eff}|\bar B\rangle = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(q)}\langle M_1M_2|\mathcal{T}_{\mathcal{A}}^{p,h}+\mathcal{T}_{\mathcal{B}}^{p,h}|\bar B\rangle, \quad (55)$$

TABLE IV. Input parameters. The values of the scale-dependent quantities $f_V^\perp(\mu)$ and $a_{1,2}^{\perp,V}(\mu)$ are given for $\mu = 1$ GeV. The values of Gegenbauer moments are taken from [37] and Wolfenstein parameters from [38].

Light vector mesons [37,39]

V	f_V (MeV)	$f_V^\perp$ (MeV)	$a_1^{\parallel,V}$	$a_2^{\parallel,V}$	$a_1^{\perp,V}$	$a_2^{\perp,V}$
ρ	216 ± 3	165 ± 9	0	0.15 ± 0.07	0	0.14 ± 0.06
ω	187 ± 5	151 ± 9	0	0.15 ± 0.07	0	0.14 ± 0.06
ϕ	215 ± 5	186 ± 9	0	0.18 ± 0.08	0	0.14 ± 0.07
K^*	220 ± 5	185 ± 10	0.03 ± 0.02	0.11 ± 0.09	0.04 ± 0.03	0.10 ± 0.08

Light tensor mesons [40]

T	f_T (MeV)	$f_T^\perp$ (MeV)	$a_1^{\parallel,T}$, $a_1^{\perp,T}$
$f_2(1270)$	102 ± 6	117 ± 25	$\frac{5}{3}$
$f_2'(1525)$	126 ± 4	65 ± 12	$\frac{5}{3}$
$a_2(1320)$	107 ± 6	105 ± 21	$\frac{5}{3}$
$K_2^*(1430)$	118 ± 5	77 ± 14	$\frac{5}{3}$

B mesons

B	m_B (GeV)	τ_B (ps)	f_B (MeV)	λ_B (MeV)
B_u	5.279	1.638	210 ± 20	300 ± 100
B_d	5.279	1.525	210 ± 20	300 ± 100

Form factors at $q^2 = 0$ [37,41]

$F_{0,1}^{BK}(0)$	$A_0^{BK^*}(0)$	$A_1^{BK^*}(0)$	$A_2^{BK^*}(0)$	$V_0^{BK^*}(0)$
0.35 ± 0.04	0.374 ± 0.033	0.292 ± 0.028	0.259 ± 0.027	0.411 ± 0.033
$F_{0,1}^{B\pi}(0)$	$A_0^{B\rho}(0)$	$A_1^{B\rho}(0)$	$A_2^{B\rho}(0)$	$V_0^{B\rho}(0)$
0.25 ± 0.03	0.303 ± 0.029	0.242 ± 0.023	0.221 ± 0.023	0.323 ± 0.030
$F_{0,1}^{B\eta_q}(0)$	$A_0^{B\omega}(0)$	$A_1^{B\omega}(0)$	$A_2^{B\omega}(0)$	$V_0^{B\omega}(0)$
0.296 ± 0.028	0.281 ± 0.030	0.219 ± 0.024	0.198 ± 0.023	0.293 ± 0.029

Quark masses

$m_b(m_b)$/GeV	$m_c(m_b)$/GeV	$m_c^{\rm pole}/m_b^{\rm pole}$	$m_s(2.1\ {\rm GeV})$/GeV
4.2	0.91	0.3	0.095 ± 0.020

Wolfenstein parameters [38]

A	λ	$\bar\rho$	$\bar\eta$	γ
0.812	0.22543	0.144	0.342	$(67.2 \pm 3.9)°$

where $\lambda_p^{(q)} \equiv V_{pb}V_{pq}^*$ with $q = s$, d, and the superscript h denotes the helicity of the final-state meson. For decays involving a pseudoscalar in the final state, h is equivalent to zero. $\mathcal{T}_{\mathcal{A}}{}^{p,h}$ describes contributions from naïve factorization, vertex corrections, penguin contractions and spectator scattering expressed in terms of the flavor operators $a_i^{p,h}$, while $\mathcal{T}_{\mathcal{B}}{}^{p,h}$ contains annihilation topology amplitudes characterized by the annihilation operators $b_i^{p,h}$. In general, $\mathcal{T}_{\mathcal{A}}{}^{p,h}$ can be expressed in terms of $c\alpha_i^{p,h}(M_1M_2)\times X^{(\bar BM_1,M_2)}$ for $M_1 = T$ or $c\alpha_i^{p,h}(M_1M_2)\bar X^{(\bar BM_1,M_2)}$ for $M_2 = T$, where c contains factors arising from flavor structures of final-state mesons, α_i are functions of the Wilson coefficients (see Eqs. (B1) and (B2)), and we have defined the notations

$$X^{(\bar BT,P)} \equiv \langle P|J^\mu|0\rangle\langle T|J'_\mu|\bar B\rangle = -i2f_PA_0^{BT}(m_P^2)\frac{m_T}{m_B}\epsilon^{*\mu\nu}(0)p_{B\mu}p_{B\nu}, \quad (56)$$

$$\bar X^{(\bar BP,T)} \equiv -2if_Tm_Bp_cF_1^{BP}(m_T^2), \quad (57)$$

for the decays $\bar B \to TP$, and

$$X_h^{(\bar{B}T,V)} \equiv \langle V|J^\mu|0\rangle\langle T|J'_\mu|\bar{B}\rangle = -if_V m_V\bigg[(e_T^*\cdot\epsilon_V^*)(m_B+m_T)A_1^{BT}(m_V^2) - (e_T^*\cdot p_B)(\epsilon_V^*\cdot p_B)\frac{2A_2^{BT}(m_V^2)}{m_B+m_T} + i\varepsilon_{\mu\nu\alpha\beta}\epsilon_V^{*\mu}e_T^{*\nu}p_B^\alpha p_T^\beta\frac{2V^{BT}(m_V^2)}{m_B+m_T}\bigg], \tag{58}$$

$$\bar{X}_h^{(\bar{B}V,T)} \equiv \begin{cases} \frac{if_T}{2m_V}\Big[(m_B^2-m_V^2-m_T^2)(m_B+m_V)A_1^{BV}(m_T^2) - \frac{4m_B^2p_c^2}{m_B+m_V}A_2^{BV}(m_T^2)\Big] & \text{for } h=0, \\ -if_T m_B m_T\Big[\Big(1+\frac{m_V}{m_B}\Big)A_1^{BV}(m_T^2) \mp \frac{2p_c}{m_B+m_V}V^{BV}(m_T^2)\Big] & \text{for } h=\pm 1, \end{cases} \tag{59}$$

for the decays $\bar{B}\to TV$, where $\bar{X}^{(\bar{B}P,T)}$ and $\bar{X}_h^{(\bar{B}V,T)}$ are expressed in the B rest frame. Note that in the factorization limit, the factorizable amplitude $X^{(\bar{B}M_1,T)} \equiv \langle T|J^\mu|0\rangle \times \langle M|J'_\mu|\bar{B}\rangle$ vanishes as the tensor meson cannot be produced through the $V-A$ or tensor current. Nevertheless, beyond the factorization approximation, contributions proportional to the decay constant f_T of the tensor meson defined in Eq. (12) can be produced from vertex, penguin and spectator-scattering corrections.

To evaluate the helicity amplitudes of $B\to TV$, we work in the rest frame of the B meson and assume that the tensor (vector) meson moves along the $-z$ (z) axis. The momenta are thus given by

$$p_B^\mu = (m_B,0,0,0),\quad p_T^\mu = (E_T,0,0,-p_c),\quad p_V^\mu = (E_V,0,0,p_c). \tag{60}$$

The polarization tensor $\epsilon_{(\lambda)}^{\mu\nu}$ of the massive tensor meson with helicity λ can be constructed in terms of the polarization vectors of a massive vector state

$$\epsilon_T^{*\mu}(0) = (p_c,0,0,-E_T)/m_T,\quad \epsilon_T^{*\mu}(\pm) = (0,\mp 1,-i,0)/\sqrt{2}. \tag{61}$$

For the vector meson moving along the z direction, its polarization vectors are

$$\epsilon_V^{*\mu}(0) = (p_c,0,0,E_V)/m_V,\quad \epsilon_V^{*\mu}(\pm) = (0,\mp 1,i,0)/\sqrt{2}, \tag{62}$$

where we have followed the Jackson convention, namely, in the $\bar{B}$ rest frame, one of the vector or tensor mesons is moving along the z axis of the coordinate system and the other along the $-z$ axis, while the x axes of both daughter particles are parallel [55]. The longitudinal ($h=0$) and transverse ($h=\pm 1$) components of factorization amplitudes $X_h^{(\bar{B}T,V)}$ then have the expressions

$$X_0^{(\bar{B}T,V)} = \frac{if_V}{2m_T^2}p_c\sqrt{\frac{2}{3}}\bigg[(m_B^2-m_V^2-m_T^2)(m_B+m_T) \times A_1^{BT}(m_V^2) - \frac{4m_B^2p_c^2}{m_B+m_T}A_2^{BT}(m_V^2)\bigg],$$
$$X_\pm^{(\bar{B}T,V)} = -if_V m_B m_V\frac{p_c}{\sqrt{2}m_T}\bigg[\bigg(1+\frac{m_T}{m_B}\bigg)A_1^{BT}(m_V^2) \mp \frac{2p_c}{m_B+m_T}V^{BT}(m_V^2)\bigg]. \tag{63}$$

Likewise, the factorizable $B\to TP$ amplitude can be simplified to

$$X^{(\bar{B}T,P)} = -i2\sqrt{\frac{2}{3}}f_P\frac{m_B}{m_T}p_c^2A_0^{BT}(m_P^2). \tag{64}$$

The flavor operators $a_i^{p,h}$ are basically the Wilson coefficients in conjunction with short-distance nonfactorizable corrections such as vertex corrections and hard spectator interactions. In general, they have the expressions [24,56]

$$a_i^{p,h}(M_1M_2) = \bigg(c_i+\frac{c_{i\pm1}}{N_c}\bigg)N_i^h(M_2) + \frac{c_{i\pm1}}{N_c}\frac{C_F\alpha_s}{4\pi}\bigg[V_i^h(M_2) + \frac{4\pi^2}{N_c}H_i^h(M_1M_2)\bigg] + P_i^{p,h}(M_2), \tag{65}$$

where $i=1,\cdots,10$, the upper (lower) signs apply when i is odd (even), c_i are the Wilson coefficients, $C_F=(N_c^2-1)/(2N_c)$ with $N_c=3$, M_2 is the emitted meson and M_1 shares the same spectator quark with the B meson. The quantities $V_i^h(M_2)$ account for vertex corrections, $H_i^h(M_1M_2)$ for hard spectator interactions with a hard gluon exchange between the emitted meson and the spectator quark of the B meson and $P_i(M_2)$ for penguin contractions. The expression of the quantities $N_i^h(M_2)$, which are relevant to the factorization amplitudes, reads

$$N_i^h(V) = \begin{cases} 0 & i=6,8, \\ 1 & \text{else}, \end{cases}\qquad N_i^h(T)=0,\qquad N_i(P)=1. \tag{66}$$

A. Vertex corrections

The vertex corrections are given by

$$V_i^0(M_2) = \begin{cases} C(M_2)\int_0^1 dx \Phi_\parallel^{M_2}(x)\Big[12\ln\frac{m_b}{\mu} - 18 + g(x)\Big] & (i = 1-4, 9, 10), \\ C(M_2)\int_0^1 dx \Phi_\parallel^{M_2}(x)\Big[-12\ln\frac{m_b}{\mu} + 6 - g(1-x)\Big] & (i = 5, 7), \\ \frac{1}{C(M_2)}\int_0^1 dx \Phi_{m_2}(x)[-6 + h(x)] & (i = 6, 8), \end{cases} \tag{67}$$

$$V_i^\pm(M_2) = \begin{cases} D(M_2)\int_0^1 dx \Phi_\pm^{M_2}(x)\Big[12\ln\frac{m_b}{\mu} - 18 + g_T(x)\Big] & (i = 1-4, 9, 10), \\ D(M_2)\int_0^1 dx \Phi_\mp^{M_2}(x)\Big[-12\ln\frac{m_b}{\mu} + 6 - g_T(1-x)\Big] & (i = 5, 7), \\ 0 & (i = 6, 8), \end{cases} \tag{68}$$

with

$$g(x) = 3\left(\frac{1-2x}{1-x}\ln x - i\pi\right) + \left[2\mathrm{Li}_2(x) - \ln^2 x + \frac{2\ln x}{1-x} - (3+2i\pi)\ln x - (x \leftrightarrow 1-x)\right],$$
$$h(x) = 2\mathrm{Li}_2(x) - \ln^2 x - (1+2i\pi)\ln x - (x \leftrightarrow 1-x), \qquad g_T(x) = g(x) + \frac{\ln x}{\bar{x}}, \tag{69}$$

and

$$C(P) = C(V) = D(V) = 1, \qquad C(T) = \sqrt{\frac{2}{3}},$$
$$D(T) = \frac{1}{\sqrt{2}}, \qquad D(P) = 0, \tag{70}$$

where $\bar{x} = 1 - x$, $\Phi_\parallel^M$ is a twist-2 light-cone distribution amplitude of the meson M, Φ_m (for the longitudinal component), and $\Phi_\pm$ (for transverse components) are twist-3 ones. Specifically, $\Phi_m = \Phi_t, \Phi_v, \Phi_p$ for $M = T, V, P$, respectively. The expressions of $C(T)$ and $D(T)$ are obtained by comparing Eqs. (51)–(54).

B. Hard spectator terms

$H_i^h(M_1M_2)$ arise from hard spectator interactions with a hard gluon exchange between the emitted meson and the spectator quark of the $\bar{B}$ meson. $H_i^0(M_1M_2)$ have the expressions:

$$H_i^0(M_1M_2) = \pm\frac{if_Bf_{M_1}f_{M_2}}{X_0^{(\bar{B}M_1,M_2)}}\frac{m_B}{\lambda_B}C(M_2) \times \int_0^1 dudv\left[C(M_1)\frac{\Phi_\parallel^{M_1}(u)\Phi_\parallel^{M_2}(v)}{\bar{u}\,\bar{v}} + r_\chi^{M_1}\frac{\Phi_{m_1}(u)\Phi_\parallel^{M_2}(v)}{C(M_1)\bar{u}v}\right], \tag{71}$$

for $i = 1$–4, 9, 10,

$$H_i^0(M_1M_2) = \mp\frac{if_Bf_{M_1}f_{M_2}}{X_0^{(\bar{B}M_1,M_2)}}\frac{m_B}{\lambda_B}C(M_2) \times \int_0^1 dudv\left[C(M_1)\frac{\Phi_\parallel^{M_1}(u)\Phi_\parallel^{M_2}(v)}{\bar{u}v} + r_\chi^{M_1}\frac{\Phi_{m_1}(u)\Phi_\parallel^{M_2}(v)}{C(M_1)\bar{u}\,\bar{v}}\right], \tag{72}$$

for $i = 5$, 7, and $H_i^0(M_1M_2) = 0$ for $i = 6$, 8, where the upper signs are for TV modes and the lower ones for TP modes. The transverse hard spectator terms $H_i^\pm(M_1M_2)$ read

$$H_i^-(M_1M_2) = \frac{\sqrt{2}if_Bf_{M_1}^\perp f_{M_2}m_{M_2}}{m_BX_-^{(\bar{B}M_1,M_2)}}\frac{m_B}{\lambda_B} \times \int_0^1 dudv\frac{\Phi_\perp^{M_1}(u)\Phi_-^{M_2}(v)}{\bar{u}^2v}, \tag{73}$$

$$H_i^+(M_1M_2) = -\frac{\sqrt{2}if_Bf_{M_1}f_{M_2}m_{M_1}m_{M_2}}{m_B^2X_+^{(\bar{B}M_1,M_2)}}\frac{m_B}{\lambda_B} \times \int_0^1 dudv\frac{(\bar{u}-v)\Phi_+^{M_1}(u)\Phi_+^{M_2}(v)}{\bar{u}^2\bar{v}^2}, \tag{74}$$

for $i = 1$–4, 9, 10, and

$$H_i^-(M_1M_2) = -\frac{\sqrt{2}if_Bf_{M_1}^\perp f_{M_2}m_{M_2}}{m_BX_-^{(\bar{B}M_1,M_2)}}\frac{m_B}{\lambda_B} \times \int_0^1 dudv\frac{\Phi_\perp^{M_1}(u)\Phi_+^{M_2}(v)}{\bar{u}^2\bar{v}}, \tag{75}$$

$$H_i^+(M_1M_2) = -\frac{\sqrt{2}if_Bf_{M_1}f_{M_2}m_{M_1}m_{M_2}}{m_B^2X_+^{(\bar{B}M_1,M_2)}}\frac{m_B}{\lambda_B} \times \int_0^1 dudv\frac{(u-v)\Phi_+^{M_1}(u)\Phi_-^{M_2}(v)}{\bar{u}^2v^2}, \quad (76)$$

for $i = 5, 7$, and

$$H_i^-(M_1M_2) = -\frac{if_Bf_{M_1}f_{M_2}m_{M_2}}{\sqrt{2}m_BX_-^{(\bar{B}M_1,M_2)}}\frac{m_Bm_{M_1}}{m_{M_2}^2}\frac{m_B}{\lambda_B} \times \int_0^1 dudv\frac{\Phi_+^{M_1}(u)\Phi_\perp^{M_2}(v)}{v\bar{u}\,\bar{v}}, \quad (77)$$

$$H_i^+(M_1M_2) = 0, \quad (78)$$

for $i = 6, 8$. Since we consider only TP and TV modes in the present work, it is obvious that $M_1M_2 = TV$ or VT for the transverse components.

C. Penguin terms

At order α_s, corrections from penguin contractions are present only for $i = 4, 6$. For $i = 4$ we obtain

$$P_4^{p,h}(M_2) = \frac{C_F\alpha_s}{4\pi N_c}\Big\{c_1[G_{M_2}^h(s_p) + g_{M_2}] + c_3[G_{M_2}^h(s_s) + G_{M_2}^h(1) + 2g_{M_2}^h] + (c_4 + c_6)\sum_{i=u}^{b}[G_{M_2}^h(s_i) + g'^h_{M_2}] - 2c_{8g}^{\rm eff}G_g^h\Big\}, \quad (79)$$

where $s_i = m_i^2/m_b^2$ and the function $G_{M_2}^h(s)$ is given by

$$G_{M_2}^h(s) = 4\int_0^1 du\Phi^{M_2,h}(u)\int_0^1 dxx\bar{x}\ln[s - \bar{u}x\bar{x} - i\epsilon],$$
$$g_{M_2}^h = \left(\frac{4}{3}\ln\frac{m_b}{\mu} + \frac{2}{3}\right)\int_0^1 \Phi^{M_2,h}(x)dx, \quad (80)$$
$$g'^h_{M_2} = \frac{4}{3}\ln\frac{m_b}{\mu}\int_0^1 \Phi^{M_2,h}(x)dx,$$

with $\Phi^{M_2,0} = C(M_2)\Phi_\parallel^{M_2}$, $\Phi^{M_2,\pm} = D(M_2)\Phi_\pm^{M_2}$. For $i = 6$, the result for the penguin contribution is

$$P_6^{p,h}(M_2) = \frac{C_F\alpha_s}{4\pi N_c}\Big\{c_1\hat{G}_{M_2}^h(s_p) + c_3[\hat{G}_{M_2}^h(s_s) + \hat{G}_{M_2}^h(1)] + (c_4 + c_6)\sum_{i=u}^{b}\hat{G}_{M_2}^h(s_i)\Big\}. \quad (81)$$

In analogy with (80), the function $\hat{G}_{M_2}(s)$ is defined as

$$\hat{G}_{M_2}^0(s) = \frac{4}{C(M_2)}\int_0^1 du\Phi_{m_2}(u)\int_0^1 dxx\bar{x}\ln[s - \bar{u}x\bar{x} - i\epsilon],$$
$$\hat{G}_{M_2}^\pm(s) = 0. \quad (82)$$

Therefore, the transverse penguin contractions vanish for $i = 6, 8$: $P_{6,8}^{\pm,p} = 0$. Note that we have factored out the $r_\chi^{M_2}$ term in Eq. (81) so that when the vertex correction $V_{6,8}$ is neglected, a_6^0 will contribute to the decay amplitude in the product $r_\chi^{M_2}a_6^0 \approx r_\chi^{M_2}P_6^0$.

For $i = 8, 10$ we find

$$P_8^{p,h}(M_2) = \frac{\alpha_{\rm em}}{9\pi N_c}(c_1 + N_cc_2)\hat{G}_{M_2}^h(s_p), \quad (83)$$

$$P_{10}^{p,h}(M_2) = \frac{\alpha_{\rm em}}{9\pi N_c}\{(c_1 + N_cc_2)[G_{M_2}^h(s_p) + 2g_{M_2}] - 3c_\gamma^{\rm eff}G_g^h\}. \quad (84)$$

For $i = 7, 9$,

$$P_{7,9}^{-,p}(M_2) = -\frac{\alpha_{\rm em}}{3\pi}C_{7\gamma}^{\rm eff}\frac{m_Bm_b}{m_{M_2}^2} + \frac{2\alpha_{\rm em}}{27\pi}(c_1 + N_cc_2) \times\left[\delta_{pc}\ln\frac{m_c^2}{\mu^2} + \delta_{pu}\ln\frac{\nu^2}{\mu^2} + 1\right], \quad (85)$$

for $M_2 = \rho^0$, ω, ϕ, a_2^0, $f_2(1270)$, $f_2'(1525)$, and vanish otherwise. Here the first term is an electromagnetic penguin contribution to the transverse helicity amplitude enhanced by a factor of $m_Bm_b/m_{M_2}^2$, as first pointed out in [57]. Note that the quark loop contains an ultraviolet divergence for both transverse and longitudinal components which must be subtracted in accordance with the scheme used to define the Wilson coefficients. The scale and scheme dependence after subtraction is required to cancel the scale and scheme dependence of the electroweak penguin coefficients. Therefore, the scale μ in the above equation is the same as the one appearing in the expressions for the penguin corrections, e.g., Eq. (80). On the other hand, the scale ν is referred to the scale of the decay constant $f_{M_2}(\nu)$ as the operator $\bar{q}\gamma^\mu q$ has a nonvanishing anomalous dimension in the presence of electromagnetic interactions [56]. The ν dependence of Eq. (85) is compensated by that of $f_{M_2}(\nu)$.

The relevant integrals for the dipole operators $O_{g,\gamma}$ are

$$G_g^0 = C(M_2)\int_0^1 du\frac{\Phi_\parallel^{M_2}(u)}{\bar{u}},$$
$$G_g^\pm = D(M_2)\int_0^1\frac{du}{\bar{u}}\left[\frac{1}{2}(\bar{u}\Phi_-^{M_2}(u) - u\Phi_+^{M_2}(u)) - \bar{u}\Phi_\pm^{M_2}(u) + \frac{1}{2}(\bar{u}\Phi_-^{M_2}(u) + u\Phi_+^{M_2}(u))\right]. \quad (86)$$

Using Eq. (31), $G_g^\pm$ can be further reduced to

$$G_g^+ = D(M_2)\int_0^1 du[\Phi_-^{M_2}(u) - \Phi_+^{M_2}(u)] = 0,$$
$$G_g^- = 0. \quad (87)$$

Hence, $G_g^\pm$ in Eq. (87) are actually equal to zero. It was first pointed out by Kagan [58] that the dipole operators Q_{8g}

and $Q_{7\gamma}$ do not contribute to the transverse penguin amplitudes at $\mathcal{O}(\alpha_s)$ due to angular momentum conservation.

D. Annihilation topologies

The weak annihilation contributions to the decay $\bar{B} \to M_1 M_2$ (with $M_1 M_2 \equiv VT$ or TV) can be described in terms of the building blocks $b_i^{p,h}$ and $b_{i,\mathrm{EW}}^{p,h}$:

$$\frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \langle M_1 M_2 | \mathcal{T}_{\mathcal{B}}{}^{p,h} | \bar{B}^0 \rangle = i \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p f_B f_{M_1} f_{M_2} \sum_i (d_i b_i^{p,h} + d_i' b_{i,\mathrm{EW}}^{p,h}). \quad (88)$$

The building blocks have the expressions

$$\begin{aligned} b_1 &= \frac{C_F}{N_c^2} c_1 A_1^i, \\ b_3 &= \frac{C_F}{N_c^2} [c_3 A_1^i + c_5 (A_3^i + A_3^f) + N_c c_6 A_3^f], \\ b_2 &= \frac{C_F}{N_c^2} c_2 A_1^i, \\ b_4 &= \frac{C_F}{N_c^2} [c_4 A_1^i + c_6 A_2^f], \\ b_{3,\mathrm{EW}} &= \frac{C_F}{N_c^2} [c_9 A_1^i + c_7 (A_3^i + A_3^f) + N_c c_8 A_3^i], \\ b_{4,\mathrm{EW}} &= \frac{C_F}{N_c^2} [c_{10} A_1^i + c_8 A_2^i], \end{aligned} \quad (89)$$

where for simplicity we have omitted the superscripts p and h in the above expressions. The subscripts 1, 2, 3 of $A_n^{i,f}$ denote the annihilation amplitudes induced from $(V-A)(V-A)$, $(V-A)(V+A)$ and $(S-P)(S+P)$ operators, respectively, and the superscripts i and f refer to gluon emission from the initial and final-state quarks, respectively. Following [56] we choose the convention that M_2 contains an antiquark from the weak vertex with longitudinal fraction $\bar{v}$, while M_1 contains a quark from the weak vertex with momentum fraction u. The explicit expressions of weak annihilation amplitudes are:

$$A_1^{i,0}(M_1 M_2) = \sqrt{\frac{2}{3}} \pi \alpha_s \int_0^1 du dv \left\{ \Phi_\parallel^{M_1}(u) \Phi_\parallel^{M_2}(v) \times \left[\frac{1}{u(1-\bar{u}v)} + \frac{1}{u\bar{v}^2} \right] \mp \frac{3}{2} r_\chi^{M_1} r_\chi^{M_2} \Phi_{m_1}(u) \Phi_{m_2}(v) \frac{2}{u\bar{v}} \right\}, \quad (90)$$

$$A_1^{i,-}(M_1 M_2) = -\pi \alpha_s \frac{\sqrt{2} m_{M_1} m_{M_2}}{m_B^2} \times \int_0^1 du dv \left\{ \Phi_-^{M_1}(u) \Phi_-^{M_2}(v) \times \left[\frac{\bar{u}+\bar{v}}{u^2 \bar{v}^2} + \frac{1}{(1-\bar{u}v)^2} \right] \right\}, \quad (91)$$

$$A_1^{i,+}(M_1 M_2) = -\pi \alpha_s \frac{\sqrt{2} m_{M_1} m_{M_2}}{m_B^2} \times \int_0^1 du dv \left\{ \Phi_+^{M_1}(u) \Phi_+^{M_2}(v) \times \left[\frac{2}{u\bar{v}^3} - \frac{v}{(1-\bar{u}v)^2} - \frac{v}{\bar{v}^2(1-\bar{u}v)} \right] \right\}, \quad (92)$$

$$A_2^{i,0}(M_1 M_2) = \sqrt{\frac{2}{3}} \pi \alpha_s \int_0^1 du dv \left\{ \Phi_\parallel^{M_1}(u) \Phi_\parallel^{M_2}(v) \times \left[\frac{1}{\bar{v}(1-\bar{u}v)} + \frac{1}{u^2 \bar{v}} \right] \mp \frac{3}{2} r_\chi^{M_1} r_\chi^{M_2} \Phi_{m_1}(u) \Phi_{m_2}(v) \frac{2}{u\bar{v}} \right\}, \quad (93)$$

$$A_2^{i,-}(M_1 M_2) = -\pi \alpha_s \frac{\sqrt{2} m_{M_1} m_{M_2}}{m_B^2} \times \int_0^1 du dv \left\{ \Phi_+^{M_1}(u) \Phi_+^{M_2}(v) \times \left[\frac{u+v}{u^2 \bar{v}^2} + \frac{1}{(1-\bar{u}v)^2} \right] \right\}, \quad (94)$$

$$A_2^{i,+}(M_1 M_2) = -\pi \alpha_s \frac{\sqrt{2} m_{M_1} m_{M_2}}{m_B^2} \times \int_0^1 du dv \left\{ \Phi_-^{M_1}(u) \Phi_-^{M_2}(v) \times \left[\frac{2}{u^3 \bar{v}} - \frac{\bar{u}}{(1-\bar{u}v)^2} - \frac{\bar{u}}{u^2(1-\bar{u}v)} \right] \right\}, \quad (95)$$

$$A_3^{i,0}(M_1 M_2) = \pi \alpha_s \int_0^1 du dv \left\{ \frac{C(M_2)}{C(M_1)} r_\chi^{M_1} \Phi_{m_1}(u) \Phi_\parallel^{M_2}(v) \times \frac{2\bar{u}}{u\bar{v}(1-\bar{u}v)} + \frac{C(M_1)}{C(M_2)} r_\chi^{M_2} \Phi_\parallel^{M_1}(u) \Phi_{m_2}(v) \times \frac{2v}{u\bar{v}(1-\bar{u}v)} \right\}, \quad (96)$$

$$A_3^{i,-}(M_1M_2) = -\frac{\pi\alpha_s}{\sqrt{2}}\int_0^1 dudv\bigg\{-\frac{m_{M_2}}{m_{M_1}}r_\chi^{M_1}\Phi_\perp^{M_1}(u)\Phi_-^{M_2}(v) \times\frac{2}{u\bar{v}(1-\bar{u}v)}+\frac{m_{M_1}}{m_{M_2}}r_\chi^{M_2}\Phi_+^{M_1}(u)\Phi_\perp^{M_2}(v) \times\frac{2}{u\bar{v}(1-\bar{u}v)}\bigg\}, \quad (97)$$

$$A_3^{f,0}(M_1M_2) = \pi\alpha_s\int_0^1 dudv\bigg\{\frac{C(M_2)}{C(M_1)}r_\chi^{M_1}\Phi_{m_1}(u)\Phi_\parallel^{M_2}(v) \times\frac{2(1+\bar{v})}{u\bar{v}^2}-\frac{C(M_1)}{C(M_2)}r_\chi^{M_2}\Phi_\parallel^{M_1}(u)\Phi_{m_2}(v) \times\frac{2(1+u)}{u^2\bar{v}}\bigg\}, \quad (98)$$

$$A_3^{f,-}(M_1M_2) = -\frac{\pi\alpha_s}{\sqrt{2}}\int_0^1 dudv\bigg\{\frac{m_{M_2}}{m_{M_1}}r_\chi^{M_1}\Phi_\perp^{M_1}(u)\Phi_-^{M_2}(v) \times\frac{2}{u^2\bar{v}}+\frac{m_{M_1}}{m_{M_2}}r_\chi^{M_2}\Phi_+^{M_1}(u)\Phi_\perp^{M_2}(v)\frac{2}{u\bar{v}^2}\bigg\}, \quad (99)$$

and $A_1^{f,h} = A_2^{f,h} = A_3^{i,+} = A_3^{f,+} = 0$. Here in the helicity amplitudes with $h = 0$, the upper signs correspond to $(M_1, M_2) = (T, V)$, (V, T), and (V, P) and the lower ones to $(M_1, M_2) = (P, V)$. When $(M_1, M_2) = (V, P)$, one has to add an overall minus sign to $A_2^{i,0}$. For $(M_1, M_2) = (P, V)$, one has to change the sign of the second term of $A_2^{i,0}$. Note that in this paper, we adopt the notations $A_j^{(i,f),0} \equiv A_j^{(i,f)}$ for the TP modes.

Since the annihilation contributions $A_{1,2}^{i,\pm}$ are suppressed by a factor of m_1m_2/m_B^2 relative to other terms, in the numerical analysis we will consider only the annihilation contributions due to $A_3^{f,0}$, $A_3^{f,-}$, $A_{1,2,3}^{i,0}$ and $A_3^{i,-}$.

Finally, two remarks are in order: (i) Although the parameters $a_i (i \neq 6, 8)$ and $a_{6,8}r_\chi$ are formally renormalization scale and γ_5-scheme independent, in practice there exists some residual scale dependence in $a_i(\mu)$ to finite order. To be specific, we shall evaluate the vertex corrections to the decay amplitude at the scale $\mu = m_b$. In contrast, as stressed in [24], the hard spectator and annihilation contributions should be evaluated at the hard-collinear scale $\mu_h = \sqrt{\mu\Lambda_h}$ with $\Lambda_h \approx 500$ MeV. (ii) Power corrections in QCDF always involve troublesome endpoint divergences. For example, the annihilation amplitude has endpoint divergences even at twist-2 level and the hard spectator scattering diagram at twist-3 order is power suppressed and possesses soft and collinear divergences arising from the soft spectator quark. Since the treatment of endpoint divergences is model dependent, subleading power corrections generally can be studied only in a phenomenological way. We shall follow [24] to model the endpoint divergence $X \equiv \int_0^1 dx/\bar{x}$ in the annihilation and hard spectator scattering diagrams as

$$X_A = \ln\left(\frac{m_B}{\Lambda_h}\right)(1+\rho_A e^{i\phi_A}), \qquad X_H = \ln\left(\frac{m_B}{\Lambda_h}\right)(1+\rho_H e^{i\phi_H}), \quad (100)$$

with the unknown real parameters $\rho_{A,H}$ and $\phi_{A,H}$. For simplicity, we shall assume that X_A^h and X_H^h are helicity independent; that is, $X_A^- = X_A^+ = X_A^0$ and $X_H^- = X_H^+ = X_H^0$.

IV. NUMERICAL RESULTS

Let the general amplitude of $\bar{B} \to TP$ be

$$\mathcal{A}_{\bar{B}\to TP} = \frac{G_F}{\sqrt{2}}(aX^{(\bar{B}T,P)} + \bar{a}\bar{X}^{(\bar{B}P,T)}). \quad (101)$$

Its decay rate is given by

$$\Gamma_{\bar{B}\to TP} = \frac{p_c}{8\pi m_B^2}|\mathcal{A}_{\bar{B}\to TP}|^2. \quad (102)$$

It follows from Eqs. (57) and (64) that

$$\Gamma_{\bar{B}\to TP} = \frac{G_F^2}{6\pi}\frac{f_P^2p_c^5}{m_T^2}\left|aA_0^{BT}(m_P^2)+\sqrt{\frac{3}{2}}\bar{a}\frac{f_T}{f_P}\frac{m_T}{p_c}\frac{F_1^{BP}(m_T^2)}{A_0^{BT}(m_P^2)}\right|^2, \quad (103)$$

where p_c is the center-of-mass momentum of the final-state particle T or P. Note that the coefficient $\bar{a}$ vanishes in naïve factorization.

The decay amplitude of $\bar{B} \to TV$ can be decomposed into three components, one for each helicity of the final state: $\mathcal{A}_0$, $\mathcal{A}_+$, $\mathcal{A}_-$. The transverse amplitudes defined in the transversity basis are related to the helicity ones via

$$\mathcal{A}_\parallel = \frac{\mathcal{A}_+ + \mathcal{A}_-}{\sqrt{2}}, \qquad \mathcal{A}_\perp = \frac{\mathcal{A}_+ - \mathcal{A}_-}{\sqrt{2}}. \quad (104)$$

The decay rate can be expressed in terms of these amplitudes as

$$\Gamma_{\bar{B}\to TV} = \frac{p_c}{8\pi m_B^2}(|\mathcal{A}_0|^2+|\mathcal{A}_+|^2+|\mathcal{A}_-|^2) = \frac{p_c}{8\pi m_B^2}(|\mathcal{A}_L|^2+|\mathcal{A}_\parallel|^2+|\mathcal{A}_\perp|^2). \quad (105)$$

Writing the general helicity amplitudes as

$$\mathcal{A}_0(\bar{B}\to TV) = \frac{G_F}{\sqrt{2}}(b^0X_0^{(\bar{B}T,V)} + \bar{b}^0\bar{X}_0^{(\bar{B}V,T)}), \quad (106)$$

$$\mathcal{A}_\pm(\bar{B}\to TV) = \frac{G_F}{\sqrt{2}}(b^\pm X_\pm^{(\bar{B}T,V)} + \bar{b}^\pm\bar{X}_\pm^{(\bar{B}V,T)}), \quad (107)$$

where $X_{0,\pm}^{(\bar{B}T,V)}$ and $\bar{X}_{0,\pm}^{(\bar{B}V,T)}$ are given in Eqs. (63) and (59), respectively, and it is understood that the relevant CKM factors should be put back by the end of calculations, the decay rate has the following explicit expression

$$\Gamma_{\bar{B}\to TV} = \frac{G_F^2}{48\pi}\frac{f_V^2}{m_T^4}(\alpha p_c^7 + \beta p_c^5 + \gamma p_c^4 + \lambda p_c^3), \quad (108)$$

with

$$\begin{aligned}\alpha &= 8\frac{m_B^2}{(m_B+m_T)^2}|b^0\mathbf{A}_2^{BT}|^2,\\ \beta &= 6\frac{m_V^2 m_T^2}{(m_B+m_T)^2}(|b^+|^2(\mathbf{V}_+^{BT})^2 + |b^-|^2(\mathbf{V}_-^{BT})^2)\\ &\quad - 4(m_B^2 - m_V^2 - m_T^2)|b^0|^2\mathbf{A}_1^{BT}\mathbf{A}_2^{BT},\\ \gamma &= 6\frac{m_V^2 m_T^2}{m_B}(|b^-|^2\mathbf{A}_{1,-}^{BT}\mathbf{V}_-^{BT} - |b^+|^2\mathbf{A}_{1,+}^{BT}\mathbf{V}_+^{BT}),\\ \lambda &= \frac{(m_B+m_T)^2}{2m_B^2}\Big[3(|b^+|^2(\mathbf{A}_{1,+}^{BT})^2 + |b^-|^2(\mathbf{A}_{1,-}^{BT})^2)m_V^2 m_T^2\\ &\quad + |b^0|^2(m_B^2 - m_V^2 - m_T^2)^2(\mathbf{A}_1^{BT})^2\Big], \end{aligned} \quad (109)$$

where we have adopted the shorthand notations,

$$\mathbf{A}_1^{BT} \equiv A_1^{BT}(m_V^2) + \frac{\bar{b}^0}{b^0}\sqrt{\frac{3}{2}}\frac{f_T}{f_V}\frac{m_T}{m_V}\frac{m_T}{p_c}\frac{m_B+m_V}{m_B+m_T}A_1^{BV}(m_T^2), \quad (110)$$

$$\mathbf{A}_2^{BT} \equiv A_2^{BT}(m_V^2) + \frac{\bar{b}^0}{b^0}\sqrt{\frac{3}{2}}\frac{f_T}{f_V}\frac{m_T}{m_V}\frac{m_T}{p_c}\frac{m_B+m_T}{m_B+m_V}A_2^{BV}(m_T^2), \quad (111)$$

$$\mathbf{A}_{1,\pm}^{BT} \equiv A_1^{BT}(m_V^2) + \frac{\bar{b}^\pm}{b^\pm}\sqrt{2}\frac{f_T}{f_V}\frac{m_T}{m_V}\frac{m_T}{p_c} \times \frac{m_B+m_V}{m_B+m_T}A_1^{BV}(m_T^2), \quad (112)$$

$$\mathbf{V}_\pm^{BT} \equiv V^{BT}(m_V^2) + \frac{\bar{b}^\pm}{b^\pm}\sqrt{2}\frac{f_T}{f_V}\frac{m_T}{m_V}\frac{m_T}{p_c}\frac{m_B+m_T}{m_B+m_V}V^{BV}(m_T^2). \quad (113)$$

Note that Eqs. (103) and (108) are in agreement with [16] for the special case that $a = b^0 = b^\pm = 1$ and $\bar{a} = \bar{b}^0 = \bar{b}^\pm = 0$. As stressed in [16], the p_c^5 dependence in Eq. (103) indicates that only the $L = 2$ wave is allowed

TABLE V. CP-averaged branching fractions (in units of 10^{-6}) and direct CP asymmetries (%) for $B \to PT$ decays with $\Delta S = 1$. The parameters ρ_A and ϕ_A are taken from Eq. (114). The theoretical errors correspond to the uncertainties due to the variation of Gegenbauer moments, decay constants, quark masses, form factors, the λ_B parameter for the B meson wave function and the power-correction parameters $\rho_{A,H}$, $\phi_{A,H}$. Then they are added in quadrature. The experimental data are taken from [60], and the model predictions of [20] are for $1/N_c^{\rm eff} = 0.3$.

	$\mathcal{B}$				
Decay	QCDF	Kim-Lim-Oh [20]	Munoz-Quintero [21]	Experiment	A_{CP}
$B^- \to \bar{K}_2^*(1430)^0\pi^-$	$3.1^{+8.3}_{-3.1}$			$5.6^{+2.2}_{-1.4}$	$1.6^{+2.2}_{-1.8}$
$B^- \to K_2^*(1430)^-\pi^0$	$2.2^{+4.7}_{-1.9}$	0.090	0.15		$0.2^{+17.8}_{-14.8}$
$\bar{B}^0 \to K_2^*(1430)^-\pi^+$	$3.3^{+8.5}_{-3.2}$			<6.3	$1.7^{+4.2}_{-5.2}$
$\bar{B}^0 \to \bar{K}_2^*(1430)^0\pi^0$	$1.2^{+4.3}_{-1.3}$	0.084	0.13	<4.0	$7.1^{+23.5}_{-24.1}$
$B^- \to a_2(1320)^0K^-$	$4.9^{+8.4}_{-4.2}$	0.311	0.39	<45	$-27.1^{+33.3}_{-41.1}$
$B^- \to a_2(1320)^-\bar{K}^0$	$8.4^{+16.1}_{-7.2}$	0.011	0.015		$-0.6^{+0.4}_{-0.8}$
$\bar{B}^0 \to a_2(1320)^+K^-$	$9.7^{+17.2}_{-8.1}$	0.584	0.73		$-21.5^{+28.9}_{-35.0}$
$\bar{B}^0 \to a_2(1320)^0\bar{K}^0$	$4.2^{+8.3}_{-3.5}$	0.005	0.014		$6.7^{+6.5}_{-6.9}$
$B^- \to f_2(1270)K^-$	$3.8^{+7.8}_{-3.0}$	0.344		$1.06^{+0.28}_{-0.29}$	$-39.5^{+49.4}_{-25.5}$
$\bar{B}^0 \to f_2(1270)\bar{K}^0$	$3.4^{+8.5}_{-3.1}$	0.005		$2.7^{+1.3}_{-1.2}$	$-7.3^{+8.4}_{-7.9}$
$B^- \to f_2'(1525)K^-$	$4.0^{+7.4}_{-3.6}$	0.004		<7.7	$-0.6^{+4.3}_{-6.0}$
$\bar{B}^0 \to f_2'(1525)\bar{K}^0$	$3.8^{+7.3}_{-3.5}$	7×10^{-5}			$0.8^{+1.2}_{-0.7}$
$B^- \to K_2^*(1430)^-\eta$	$6.8^{+13.5}_{-8.7}$	0.031	1.19	9.1 ± 3.0	$1.5^{+7.4}_{-5.6}$
$B^- \to K_2^*(1430)^-\eta'$	$12.1^{+20.7}_{-12.1}$	1.405	2.70	$28.0^{+5.3}_{-5.0}$	$-1.7^{+3.2}_{-3.9}$
$\bar{B}^0 \to \bar{K}_2^*(1430)^0\eta$	$6.6^{+13.5}_{-8.7}$	0.029	1.09	9.6 ± 2.1	$3.2^{+16.5}_{-4.8}$
$\bar{B}^0 \to \bar{K}_2^*(1430)^0\eta'$	$12.4^{+21.3}_{-12.4}$	1.304	2.46	$13.7^{+3.2}_{-3.1}$	$-2.2^{+3.3}_{-4.0}$

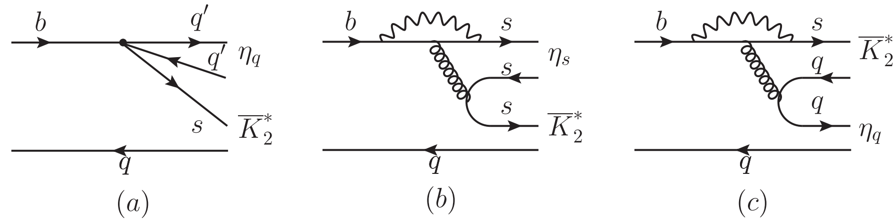

FIG. 1. Three different penguin contributions to $\bar{B} \to \bar{K}_2^* \eta^{(\prime)}$. Figure 1(a) is induced by the penguin operators $O_{3,5,7,9}$.

for the TP system, while in the TV modes the $L = 1, 2$ and 3 waves are simultaneously allowed, as expected.

A. $\bar{B} \to PT$ decays

As noticed in [59], since the penguin-annihilation effects are different for $B \to VP$ and $B \to PV$ decays, the penguin-annihilation parameters X_A^{VP} and X_A^{PV} are not necessarily the same. Indeed, a fit to the $B \to VP$, PV decays yields $\rho_A^{VP} \approx 1.07$, $\phi_A^{VP} \approx -70°$ and $\rho_A^{PV} \approx 0.87$, $\phi_A^{PV} \approx -30°$ [59]. Likewise, for $B_{u,d} \to TP$ decays we find that the data of $B_{u,d} \to TP$ can be described by the penguin-annihilation parameters

$$\rho_A^{TP} = 0.83, \qquad \phi_A^{TP} = -70°,$$
$$\rho_A^{PT} = 0.75, \qquad \phi_A^{PT} = -30°. \tag{114}$$

For $B \to T$ transition form factors, the LEET or pQCD predictions are favored by the experimental data of $B \to f_2(1270)K$ and $B \to f_2(1270)\pi$, while the CLFQ or ISGW2 model results are preferred by the measurements of $B \to K_2^*(1430)\eta^{(\prime)}$, $K_2^*(1430)\omega$, $K_2^*(1430)\phi$. For example, the branching fractions (in units of 10^{-6}; only the central values are quoted here) for the $f_2\pi^-$, $f_2\bar{K}^0$ and f_2K^- modes are found to be 8.1 (2.7), 5.0 (3.4) and 6.4 (3.8), respectively, using the CLFQ (LEET) model for $B \to f_2$ transition form factors. The corresponding experimental values are $1.57^{+0.69}_{-0.49}$, $2.7^{+1.3}_{-1.2}$, and $1.06^{+0.28}_{-0.29}$. Therefore, it is evident that the data favor LEET over the CLFQ model for decays involving $B \to f_2$ transitions. Likewise, the branching fractions for the modes $K_2^{*-}\phi$, $K_2^{*-}\eta$ and $\bar{K}^{*0}\eta'$ are calculated to be 7.4 (4.7), 6.8 (4.7) and 12.4 (8.4), respectively, using the CLFQ (LEET) model for $B \to K_2^*$ transition form factors. The corresponding experimental values are 8.4 ± 2.1, 9.1 ± 3.0, and $13.7^{+3.2}_{-3.1}$. It is clear that the CLFQ model works better for decays involving $B \to K_2^*$ transitions. In this work we shall use the $B \to K_2^*(1430)$ form factors obtained in the CLFQ model and $B \to a_2(1270)$ and $B \to f_2$ ones from LEET (see Table II).

Branching fractions and CP asymmetries for $B \to TP$ decays are shown in Tables V and VI. The theoretical errors correspond to the uncertainties due to the variation of (i) the Gegenbauer moments, the decay constants, (ii) the heavy-to-light form factors and the strange quark mass, (iii) the wave function of the B meson characterized by the parameter λ_B, and (iv) the power corrections due to weak annihilation and hard spectator interactions described by the parameters $\rho_{A,H}$, $\phi_{A,H}$. We allow the variation of ρ_A and ϕ_A to be ± 0.4 and $\pm 50°$, respectively, and put ρ_H and ϕ_H in the respective ranges $0 \le \rho_H \le 1$ and $0 \le \phi_H \le 2\pi$. To obtain the errors shown in these tables, we first scan randomly the points in the allowed ranges of the above-mentioned parameters and then add errors in quadrature. Power corrections beyond the heavy quark limit generally give the major theoretical uncertainties.

For $\bar{B} \to \bar{K}_2^*\eta^{(\prime)}$ decays, there exist three different penguin contributions as depicted in Fig. 1: (i) $b \to sq\bar{q} \to s\eta_q$, (ii) $b \to ss\bar{s} \to s\eta_s$, and (iii) $b \to sq\bar{q} \to q\bar{K}_2^*$, corresponding to Figs. 1(a)–1(c), respectively. The dominant contributions come from Figs. 1(b) and 1(c). Since the relative sign of the η_s state with respect to the η_q is negative for the η and positive for the η' [see Eq. (D1)], it is evident that the interference between Figs. 1(b) and 1(c) is destructive for $K_2^*\eta$ and constructive for $K_2^*\eta'$. This explains why $K_2^*\eta'$ has a rate larger than $K_2^*\eta$. It was known that the predicted rates in naïve factorization are too small by 1 order of magnitude, of order 1.0×10^{-6} for $\mathcal{B}(\bar{B}^0 \to \bar{K}_2^{*0}\eta)$ and 2.5×10^{-6} for $\mathcal{B}(\bar{B}^0 \to \bar{K}_2^{*0}\eta')$ [21,61].[4] One reason is that the factorizable contribution to Fig. 1(c) vanishes in the naïve factorization approach. The rates of $K_2^*\eta^{(\prime)}$ are greatly enhanced in QCDF owing to the large power corrections from penguin annihilation and the sizable nonfactorizable contributions to Fig. 1(c).

From Tables V and VI we see that the predicted branching fractions for penguin-dominated $B \to TP$ decays in QCDF are larger than those of [20,21] by 1 to 2 orders of magnitude through the aforementioned two mechanisms for enhancement, while the predicted rates in QCDF are consistent with [20] for the leading tree-dominated modes such as $a_2^0\pi^-$, $a_2^+\pi^-$, $f_2\pi^-$. Note that the branching fractions of $B^- \to \bar{K}_2^{*0}\pi^-$ and $\bar{B}^0 \to K_2^{*-}\pi^+$ vanish in naïve factorization, while experimentally it is $(5.6^{+2.2}_{-1.4}) \times 10^{-6}$ for the former. The QCDF calculation indicates that the nonfactorizable contributions arising from vertex, penguin and spectator corrections are sizable to account for the data.

[4]The rate of $\bar{B}^0 \to \bar{K}_2^{*0}\eta$ was predicted to be very suppressed in [20] (see Table V) due to the use of a wrong matrix element for $\langle \eta^{(\prime)}|\bar{s}\gamma_5 s|0\rangle$ [61].

TABLE VI. Same as Table V except for $B \to PT$ decays with $\Delta S = 0$.

	$\mathcal{B}$				
Decay	QCDF	Kim-Lim-Oh [20]	Munoz-Quintero [21]	Experiment	A_{CP}
$B^- \to a_2(1320)^0\pi^-$	$3.0^{+1.4}_{-1.2}$	2.602	4.38		$9.6^{+47.9}_{-46.6}$
$B^- \to a_2(1320)^-\pi^0$	$0.24^{+0.79}_{-0.31}$	0.001	0.015		$-24.3^{+124.3}_{-75.7}$
$\bar{B}^0 \to a_2(1320)^+\pi^-$	$5.2^{+1.8}_{-1.8}$	4.882	8.19		$37.3^{+23.9}_{-40.4}$
$\bar{B}^0 \to a_2(1320)^-\pi^+$	$0.21^{+0.43}_{-0.17}$				$-26.6^{+111.6}_{-82.9}$
$\bar{B}^0 \to a_2(1320)^0\pi^0$	$0.24^{+0.42}_{-0.19}$	0.0003	0.007		$-86.2^{+128.9}_{-26.4}$
$B^- \to a_2(1320)^-\eta$	$0.11^{+0.28}_{-0.11}$	0.294	45.8		$27.6^{+73.4}_{-127.6}$
$B^- \to a_2(1320)^-\eta'$	$0.11^{+0.47}_{-0.12}$	1.310	71.3		$31.3^{+61.6}_{-131.3}$
$\bar{B}^0 \to a_2(1320)^0\eta$	$0.06^{+0.16}_{-0.05}$	0.138	25.2		$-76.7^{+100}_{-19.2}$
$\bar{B}^0 \to a_2(1320)^0\eta'$	$0.05^{+0.22}_{-0.04}$	0.615	43.3		$-66.0^{+154.0}_{-41.1}$
$B^- \to f_2(1270)\pi^-$	$2.7^{+1.4}_{-1.2}$	2.874		$1.57^{+0.69}_{-0.49}$	$60.2^{+27.1}_{-72.3}$
$\bar{B}^0 \to f_2(1270)\pi^0$	$0.15^{+0.42}_{-0.14}$	0.0003			$-37.2^{+103.8}_{-85.5}$
$\bar{B}^0 \to f_2(1270)\eta$	$0.17^{+0.23}_{-0.12}$	0.152			$69.7^{+25.7}_{-102.7}$
$\bar{B}^0 \to f_2(1270)\eta'$	$0.13^{+0.22}_{-0.13}$	0.680			$82.3^{+22.9}_{-94.8}$
$B^- \to f_2'(1525)\pi^-$	$0.009^{+0.024}_{-0.009}$	0.037			0
$\bar{B}^0 \to f_2'(1525)\pi^0$	$0.005^{+0.012}_{-0.005}$	4×10^{-6}			0
$\bar{B}^0 \to f_2'(1525)\eta$	$0.002^{+0.006}_{-0.003}$	0.002			0
$\bar{B}^0 \to f_2'(1525)\eta'$	$0.008^{+0.008}_{-0.005}$	0.009			0
$B^- \to K_2^*(1430)^-K^0$	$0.44^{+0.74}_{-0.41}$	4×10^{-5}	7.8×10^{-4}		$30.3^{+51.2}_{-33.7}$
$B^- \to K_2^*(1430)^0K^-$	$0.12^{+0.52}_{-0.12}$				$-0.26^{+0.23}_{-0.27}$
$\bar{B}^0 \to K_2^*(1430)^-K^+$	$0.03^{+0.07}_{-0.02}$				$-15.0^{+22.7}_{-25.1}$
$\bar{B}^0 \to K_2^*(1430)^+K^-$	$0.13^{+0.16}_{-0.10}$				$18.6^{+27.2}_{-27.4}$
$\bar{B}^0 \to \bar{K}_2^*(1430)^0K^0$	$0.54^{+0.88}_{-0.49}$	3×10^{-5}	7.2×10^{-4}		$-2.1^{+4.1}_{-2.0}$
$\bar{B}^0 \to K_2^*(1430)^0\bar{K}^0$	$0.22^{+0.54}_{-0.22}$				$-14.0^{+13.7}_{-60.1}$

Just as $\bar{B} \to a_0\pi$ with $a_0 = a_0(980)$ or $a_0(1450)$ and $\bar{B} \to b_1(1235)\pi$ decays, we see from Table VI that for $\bar{B} \to a_2(1320)\pi$ decays, the $a_2^-\pi^+$ and $a_2^-\pi^0$ modes are highly suppressed relative to $a_2^+\pi^-$ and $a_2^0\pi^-$, respectively. Since (see Appendix B)

$$\mathcal{A}_{\bar{B}^0 \to a_2^-\pi^+} \propto \sum_{p=u,c} V_{pb}V_{pd}^*[\delta_{pu}\alpha_1 + \alpha_4^p]\bar{X}^{(\bar{B}\pi,a_2)},$$
$$\mathcal{A}_{\bar{B}^0 \to a_2^+\pi^-} \propto \sum_{p=u,c} V_{pb}V_{pd}^*[\delta_{pu}\alpha_1 + \alpha_4^p]X^{(\bar{B}a_2,\pi)}, \quad (115)$$

it is tempting to argue that $\Gamma(\bar{B}^0 \to a_2^+\pi^-) \gg \Gamma(\bar{B}^0 \to a_2^-\pi^+)$ is a natural consequence of naïve factorization as the tensor meson cannot be created from the $V - A$ current. However, the suppression of $a_2^-\pi^+$ relative to $a_2^+\pi^-$ in QCDF stems from a different reasoning. The amplitude $\bar{X}^{(\bar{B}\pi,a_2)}$ does not vanish in QCDF owing to the nonfactorizable corrections. Indeed, $\bar{X}^{(\bar{B}\pi,a_2)} = 0.80$ and $X^{(\bar{B}a_2,\pi)} = 0.69$ are numerically comparable. Therefore, one may wonder how to view the aforementioned suppression? The key is the quantity $N_i(M_2)$ appearing in the expression for the effective parameter a_i [see Eq. (65)]. This quantity vanishes for the tensor meson [cf. Eq. (66)]. As a result, the parameter $a_1(\pi a_2)$ is not of order unity as it receives contributions only from vertex corrections and hard spectator interactions, both suppressed by factors of $\alpha_s/(4\pi)$. Numerically, we have $a_1(\pi a_2) = -0.035 + i0.014$. By contrast, $a_1(a_2\pi)$ is of order unity. This explains why $\bar{B}^0 \to a_2^+\pi^-$ has a rate greater than $a_2^-\pi^+$ and why $B^- \to a_2^-\pi^0$ is suppressed relative to $a_2^0\pi^-$.[5] The same pattern also occurs in $\bar{B} \to a_2\rho$ decays; see Table VIII.

The branching fractions of $B \to a_2\eta^{(\prime)}$ of order 10^{-7} in QCDF are in sharp contrast to the predictions of [21], ranging from 25×10^{-6} to 70×10^{-6} (see Table VI). It

[5]The same argument also explains the suppression of $\bar{B}^0 \to b_1^-\pi^+$ relative to $b_1^+\pi^-$ in QCDF [62].

TABLE VII. CP-averaged branching fractions (in units of 10^{-6}), direct CP asymmetries (%) and the longitudinal polarization fractions f_L for $B \to VT$ decays with $\Delta S = 1$. The parameters ρ_A and ϕ_A are taken from Eq. (124).

Decay	$\mathcal{B}$ QCDF	$\mathcal{B}$ Kim-Lim-Oh [20]	$\mathcal{B}$ Munoz-Quintero [21]	$\mathcal{B}$ Experiment	f_L QCDF	f_L Experiment	A_{CP}
$B^- \to \bar{K}_2^*(1430)^0\rho^-$	$18.6^{+50.1}_{-17.2}$				$0.63^{+0.10}_{-0.09}$		$-1.0^{+0.8}_{-1.0}$
$B^- \to K_2^*(1430)^-\rho^0$	$10.4^{+18.8}_{-9.2}$	0.253	0.74		$0.66^{+0.06}_{-0.07}$		$2.1^{+11.1}_{-9.9}$
$\bar{B}^0 \to K_2^*(1430)^-\rho^+$	$19.8^{+52.0}_{-18.2}$				$0.64^{+0.07}_{-0.03}$		$-1.5^{+2.6}_{-2.0}$
$\bar{B}^0 \to \bar{K}_2^*(1430)^0\rho^0$	$9.5^{+33.4}_{-9.5}$	0.235	0.68		$0.64^{+0.15}_{-0.37}$		$-4.0^{+14.1}_{-10.8}$
$B^- \to K_2^*(1430)^-\omega$	$7.5^{+19.7}_{-7.0}$	0.112	0.06	21.5 ± 4.3	$0.64^{+0.08}_{-0.07}$	0.56 ± 0.11	$2.0^{+12.2}_{-10.5}$
$\bar{B}^0 \to \bar{K}_2^*(1430)^0\omega$	$8.1^{+21.7}_{-7.6}$	0.104	0.053	10.1 ± 2.3	$0.66^{+0.11}_{-0.15}$	0.45 ± 0.12	$4.4^{+10.9}_{-10.0}$
$B^- \to K_2^*(1430)^-\phi$	$7.4^{+25.8}_{-5.2}$	2.180	9.24	8.4 ± 2.1	$0.85^{+0.16}_{-0.77}$	0.80 ± 0.10	$0.1^{+1.2}_{-0.5}$
$\bar{B}^0 \to \bar{K}_2^*(1430)^0\phi$	$7.7^{+26.9}_{-5.5}$	2.024	8.51	7.5 ± 1.0	$0.86^{+0.16}_{-0.77}$	$0.901^{+0.059}_{-0.069}$	$0.09^{+0.82}_{-0.21}$
$B^- \to a_2(1320)^0K^{*-}$	$2.9^{+11.7}_{-2.5}$	1.852	2.80		$0.73^{+0.22}_{-0.33}$		$-15.0^{+56.0}_{-15.0}$
$B^- \to a_2(1320)^-\bar{K}^{*0}$	$6.1^{+23.8}_{-5.4}$	4.495	8.62		$0.79^{+0.20}_{-0.64}$		$-0.1^{+1.3}_{-0.3}$
$\bar{B}^0 \to a_2(1320)^+K^{*-}$	$6.1^{+24.3}_{-5.3}$	3.477	7.25		$0.77^{+0.19}_{-0.46}$		$-13.3^{+38.2}_{-7.0}$
$\bar{B}^0 \to a_2(1320)^0\bar{K}^{*0}$	$3.4^{+12.4}_{-2.8}$	2.109	4.03		$0.82^{+0.14}_{-0.67}$		$1.2^{+7.0}_{-13.3}$
$B^- \to f_2(1270)K^{*-}$	$8.3^{+17.3}_{-6.7}$	2.032			$0.93^{+0.07}_{-0.63}$		$-8.1^{+13.7}_{-7.1}$
$\bar{B}^0 \to f_2(1270)\bar{K}^{*0}$	$9.1^{+18.8}_{-7.3}$	2.314			$0.94^{+0.06}_{-0.69}$		$-0.08^{+4.3}_{-3.1}$
$B^- \to f_2'(1525)K^{*-}$	$12.6^{+24.0}_{-11.1}$	0.025			$0.65^{+0.28}_{-0.38}$		$0.6^{+2.5}_{-2.9}$
$\bar{B}^0 \to f_2'(1525)\bar{K}^{*0}$	$13.5^{+25.4}_{-11.9}$	0.029			$0.66^{+0.27}_{-0.38}$		$0.2^{+0.3}_{-0.4}$

seems to us that it is extremely unlikely that the rate of $a_2^-\eta^{(\prime)}$ can be greater than $a_2^-\pi^0$ by 4 orders of magnitude as claimed in [21]. It appears that the former should be slightly smaller than the latter in rates. This can be tested in the future. It is also interesting to notice that, while $\bar{B} \to \bar{K}_2^* K$ decays are quite suppressed in naïve factorization, their branching fractions are a few $\times 10^{-7}$ in QCDF. Finally, it is worth remarking that $\bar{B}^0 \to K_2^{*+}K^-$ and $\bar{B}^0 \to K_2^{*-}K^+$ can only proceed through weak annihilation.

B. $B \to TV$ decays

Branching fractions, direct CP asymmetries and the longitudinal polarization fractions for $B \to TV$ decays are shown in Tables VII and VIII. Thus far only four of the $B \to TV$ decays have been measured: $B^- \to K_2^{*-}(\phi, \omega)$ and $\bar{B}^0 \to \bar{K}_2^{*0}(\phi, \omega)$. They can be used to fix the penguin-annihilation parameters. From Eqs. (B38) and (B40) we have

$$\begin{aligned} \sqrt{2}\mathcal{A}^h_{B^- \to K_2^{*-}\omega} &\approx \sqrt{2}\mathcal{A}^h_{\bar{B}^0 \to \bar{K}_2^{*0}\omega} \\ &\approx \{[\alpha_4^{p,h} + \beta_3^{p,h}]\bar{X}_h^{(\bar{B}\omega, \bar{K}_2^*)} + [2\alpha_3^{p,h}]X_h^{(\bar{B}\bar{K}_2^*, \omega)}\}, \\ \mathcal{A}^h_{B^- \to K_2^{*-}\phi} &\approx \mathcal{A}^h_{\bar{B}^0 \to \bar{K}_2^{*0}\phi} \\ &\approx [\alpha_3^{p,h} + \alpha_4^{p,h} + \beta_3^{p,h} + \beta_{3,\mathrm{EW}}^{p,h}]X_h^{(\bar{B}\bar{K}_2^*, \phi)}. \end{aligned} \tag{116}$$

Since $\bar{X}_0^{(\bar{B}\omega, \bar{K}_2^*)}/X_0^{(\bar{B}\bar{K}_2^*, \phi)} = 0.56$, it is expected that $\mathcal{B}(B^- \to K_2^{*-}\omega)/\mathcal{B}(B^- \to K_2^{*-}\phi) \approx |\bar{X}_0^{(\bar{B}\omega, \bar{K}_2^*)}/X_0^{(\bar{B}\bar{K}_2^*, \phi)}|^2/2 \approx 0.15$, provided that the penguin-annihilation parameters are the same for $K_2^*\omega$ and $K_2^*\phi$, i.e., $\rho_A^{K_2^*\omega} = \rho_A^{\phi K_2^*}$ and likewise for ϕ_A. However, it is the other way around experimentally: the rate of $K_2^*\omega$ is larger than that of $K_2^*\phi$. Since, in the $B \to VV$ sector, $\mathcal{B}(B^- \to K^{*-}\omega)/\mathcal{B}(B^- \to K^{*-}\phi) \approx 0.3$ [60], it is thus puzzling as to why $K_2^*\omega$ behaves so differently from $K^*\omega$ in terms of branching fractions. It is clear from Eq. (116) that the $B \to K_2^*\phi$ decay receives penguin annihilation via ρ_A^{TV} and ϕ_A^{TV}, while $B \to K_2^*\omega$ is governed by ρ_A^{VT} and ϕ_A^{VT}. Therefore, we should have $\rho_A^{VT} \gg \rho_A^{TV}$ in order to account for their rates (see Eq. (124) below).

The branching fractions of the tree-dominated modes $a_2\phi$, $f_2\phi$, $f_2'\phi$ are very small, of order 10^{-9} (see Table VIII), as they proceed only through QCD and electroweak penguins.

For charmless $\bar{B} \to TV$ decays, it is naïvely expected that the helicity amplitudes $\mathcal{A}_h$ (helicities $h = 0, -, +$) for both tree- and penguin-dominated $\bar{B} \to TV$ decays respect the hierarchy pattern

$$\mathcal{A}_0 : \mathcal{A}_- : \mathcal{A}_+ = 1 : \left(\frac{\Lambda_{\mathrm{QCD}}}{m_b}\right) : \left(\frac{\Lambda_{\mathrm{QCD}}}{m_b}\right)^2. \tag{117}$$

TABLE VIII. Predicted branching fractions (in units of 10^{-6}), direct CP asymmetries (%) and the longitudinal polarization fractions f_L for $B \to VT$ decays with $\Delta S = 0$.

	$\mathcal{B}$				
Decay	QCDF	Kim-Lim-Oh [20]	Munoz-Quintero [21]	f_L	A_{CP}
$B^- \to a_2(1320)^0\rho^-$	$8.4^{+4.7}_{-2.9}$	7.342	19.34	$0.88^{+0.05}_{-0.14}$	$31.0^{+16.0}_{-45.5}$
$B^- \to a_2(1320)^-\rho^0$	$0.82^{+2.30}_{-0.95}$	0.007	0.071	$0.56^{+0.20}_{-0.31}$	$-13.7^{+74.8}_{-83.2}$
$\bar{B}^0 \to a_2(1320)^+\rho^-$	$11.3^{+5.3}_{-4.6}$	14.686	36.18	$0.91^{+0.03}_{-0.10}$	$7.6^{+10.2}_{-23.7}$
$\bar{B}^0 \to a_2(1320)^-\rho^+$	$1.2^{+2.6}_{-1.0}$			$0.64^{+0.16}_{-0.05}$	$49.0^{+24.0}_{-66.8}$
$\bar{B}^0 \to a_2(1320)^0\rho^0$	$0.39^{+1.35}_{-0.20}$	0.003	0.03	$0.91^{+0.07}_{-0.60}$	$55.2^{+31.9}_{-144.7}$
$B^- \to a_2(1320)^-\omega$	$0.38^{+1.84}_{-0.36}$	0.010	0.14	$0.73^{+0.09}_{-0.46}$	$-36.2^{+127.8}_{-57.2}$
$B^- \to a_2(1320)^-\phi$	$0.003^{+0.013}_{-0.001}$	0.004	0.019	$0.93^{+0.34}_{-0.00}$	$0.06^{+0.07}_{-0.07}$
$\bar{B}^0 \to a_2(1320)^0\omega$	$0.25^{+1.14}_{-0.19}$	0.005	0.07	$0.78^{+0.05}_{-0.42}$	$60.5^{+25.7}_{-141.2}$
$\bar{B}^0 \to a_2(1320)^0\phi$	$0.001^{+0.006}_{-0.001}$	0.002	0.009	$0.93^{+0.03}_{-0.56}$	$0.06^{+0.07}_{-0.07}$
$B^- \to f_2(1270)\rho^-$	$7.7^{+4.8}_{-2.9}$	8.061		$0.90^{+0.04}_{-0.18}$	$-18.2^{+41.1}_{-20.0}$
$\bar{B}^0 \to f_2(1270)\rho^0$	$0.42^{+1.90}_{-0.44}$	0.004		$0.82^{+0.11}_{-0.86}$	$38.1^{+49.4}_{-113.3}$
$\bar{B}^0 \to f_2(1270)\omega$	$0.69^{+0.97}_{-0.36}$	0.005		$0.91^{+0.07}_{-0.40}$	$-73.3^{+105.1}_{-11.0}$
$\bar{B}^0 \to f_2(1270)\phi$	$0.001^{+0.007}_{-0.000}$	0.002		$0.92^{+0.04}_{-0.59}$	$0.07^{+0.76}_{-0.78}$
$B^- \to f_2'(1525)\rho^-$	$0.07^{+0.11}_{-0.04}$	0.103		$0.96^{+0.03}_{-0.48}$	$-0.02^{+0.08}_{-0.07}$
$\bar{B}^0 \to f_2'(1525)\rho^0$	$0.03^{+0.06}_{-0.02}$	5×10^{-5}		$0.96^{+0.03}_{-0.48}$	$-0.02^{+0.08}_{-0.07}$
$\bar{B}^0 \to f_2'(1525)\omega$	$0.03^{+0.04}_{-0.01}$	6×10^{-5}		$0.95^{+0.04}_{-0.51}$	$-0.03^{+0.09}_{-0.08}$
$\bar{B}^0 \to f_2'(1525)\phi$	$0.006^{+0.034}_{-0.005}$	2×10^{-5}		1	0
$B^- \to K_2^*(1430)^-K^{*0}$	$0.56^{+1.09}_{-0.38}$	0.014	0.59	$0.85^{+0.09}_{-0.57}$	$-14.6^{+14.5}_{-10.7}$
$B^- \to K_2^*(1430)^0K^{*-}$	$2.1^{+4.2}_{-1.8}$			$0.54^{+0.06}_{-0.05}$	$10.1^{+16.0}_{-8.2}$
$\bar{B}^0 \to K_2^*(1430)^-K^{*+}$	$0.06^{+0.09}_{-0.03}$			1	$-58.3^{+135.1}_{-43.9}$
$\bar{B}^0 \to K_2^*(1430)^+K^{*-}$	$0.43^{+0.54}_{-0.31}$			1	$15.0^{+40.9}_{-39.2}$
$\bar{B}^0 \to \bar{K}_2^*(1430)^0K^{*0}$	$0.44^{+0.88}_{-0.30}$	0.026	0.55	$0.90^{+0.08}_{-0.73}$	$-2.1^{+4.0}_{-10.4}$
$\bar{B}^0 \to K_2^*(1430)^0\bar{K}^{*0}$	$1.1^{+2.9}_{-1.0}$			$0.60^{+0.14}_{-0.23}$	$-2.3^{+0.1}_{-0.1}$

Hence, they are dominated by the longitudinal polarization states and satisfy the scaling law, namely [58],

$$f_T \equiv 1 - f_L = \mathcal{O}\left(\frac{m_{V,T}^2}{m_B^2}\right), \qquad \frac{f_\perp}{f_\parallel} = 1 + \mathcal{O}\left(\frac{m_{V,T}}{m_B}\right), \tag{118}$$

with $f_L, f_\perp, f_\parallel$ and f_T being the longitudinal, perpendicular, parallel and transverse polarization fractions, respectively, defined as

$$f_\alpha \equiv \frac{\Gamma_\alpha}{\Gamma} = \frac{|\mathcal{A}_\alpha|^2}{|\mathcal{A}_0|^2 + |\mathcal{A}_\parallel|^2 + |\mathcal{A}_\perp|^2}, \tag{119}$$

with $\alpha = L, \parallel, \perp$.

The so-called polarization puzzle in $B \to VV$ decays is the enigma of why the transverse polarization fraction f_T in the penguin-dominated channels such as $B \to \phi K^*$, ρK^* is comparable to f_L, namely, $f_T/f_L \sim 1$. This poses an interesting challenge for any theoretical interpretation. For $B \to TV$ decays, the experimental measurement indicates that $f_T/f_L \ll 1$ for $B \to \phi K_2^*(1430)$, whereas $f_T/f_L \sim 1$ for $B \to \omega K_2^*(1430)$, even though both are penguin-dominated.

Consider the ratio of negative- and longitudinal-helicity amplitudes:

$$\begin{aligned} \frac{\mathcal{A}_-}{\mathcal{A}_0}\bigg|_{B^-\to K_2^{*-}\omega} &\approx \left(\frac{\alpha_4^{c,-} + \beta_3^-}{\alpha_4^{c,0} + \beta_3^0}\right)_{\omega K_2^*}\left(\frac{\bar{X}^-_{\omega K_2^*}}{\bar{X}^0_{\omega K_2^*}}\right), \\ \frac{\mathcal{A}_-}{\mathcal{A}_0}\bigg|_{B^-\to K_2^{*-}\phi} &\approx \left(\frac{\alpha_4^{c,-} + \beta_3^-}{\alpha_4^{c,0} + \beta_3^0}\right)_{K_2^*\phi}\left(\frac{X^-_{K_2^*\phi}}{X^0_{K_2^*\phi}}\right). \end{aligned} \tag{120}$$

The longitudinal polarization fraction can be approximated as

$$f_L(\omega K_2^*) \simeq 1 - \frac{|\alpha_4^{c,-} + \beta_3^-|^2 |\bar{X}^-_{\omega K_2^*}|^2}{\sum_{h=0,-} |\alpha_4^{c,h} + \beta_3^h|^2 |\bar{X}^h_{\omega K_2^*}|^2},$$
$$f_L(K_2^*\phi) \simeq 1 - \frac{|\alpha_4^{c,-} + \beta_3^-|^2 |X^-_{K_2^*\phi}|^2}{\sum_{h=0,-} |\alpha_4^{c,h} + \beta_3^h|^2 |X^h_{K_2^*\phi}|^2}. \tag{121}$$

We have

$$|\bar{X}^0_{\omega K_2^*}|:|\bar{X}^-_{\omega K_2^*}|:|\bar{X}^+_{\omega K_2^*}| = 1{:}0.51{:}0.04,$$
$$|X^0_{K_2^*\phi}|:|X^-_{K_2^*\phi}|:|X^+_{K_2^*\phi}| = 1{:}0.38{:}0.06. \tag{122}$$

In the absence of penguin annihilation, we find $f_L(\omega K_2^*) \simeq f_L(K_2^*\phi) \approx 0.72$. As we have stressed in [63], in the presence of next-to-leading-order nonfactorizable corrections, e.g., vertex, penguin and hard spectator scattering contributions, the parameters a_i^h are helicity dependent. Although the factorizable helicity amplitudes X^0, X^- and X^+ or $\bar{X}^0$, $\bar{X}^-$, $\bar{X}^+$ respect the scaling law (117) with $\Lambda_{\rm QCD}/m_b$ replaced by $2m_{V,T}/m_B$ for the tensor and vector meson productions, one needs to consider the effects of helicity-dependent Wilson coefficients: $\mathcal{A}_-/\mathcal{A}_0 = f(a_i^-)X^-/[f(a_i^0)X^0]$. The constructive (destructive) interference in the negative-helicity (longitudinal-helicity) amplitude of the penguin-dominated $\bar{B} \to TV$ decay will render $f(a_i^-) \gg f(a_i^0)$ so that $\mathcal{A}_-$ is comparable to $\mathcal{A}_0$ and the transverse polarization is enhanced. Indeed, we find $f_T(\omega K_2^*) \simeq f_T(K_2^*\phi) \approx 0.28$. Therefore, when next-to-leading-order effects are turned on, their corrections on a_i^- will render the negative-helicity amplitude $\mathcal{A}_-(\bar{B} \to \bar{K}_2^*\phi)$ comparable to the longitudinal one $\mathcal{A}_0(\bar{B} \to \bar{K}_2^*\phi)$ so that even at the short-distance level, f_L for $\bar{B}^0 \to \bar{K}_2^*\phi$ is reduced to the level of 70% and likewise for $\bar{B}^0 \to \bar{K}_2^*\omega$.

As noticed in passing, penguin annihilation is needed in order to account for the observed rates. This is because, in the absence of power corrections, QCDF predicts too small rates for penguin-dominated $\bar{B} \to TV$ and VV, VA decays. For example, the calculated $\bar{B} \to \bar{K}_2^*\phi$ rate is too small by a factor of 2.5 and $\bar{B} \to \bar{K}_2^*\omega$ by 2 orders of magnitude. We shall rely on power corrections from penguin annihilation to enhance the rates. A nice feature of the $(S-P)(S+P)$ penguin annihilation is that it contributes to $\mathcal{A}_0$ and $\mathcal{A}_-$ with the same order of magnitude [64]:

$$\mathcal{A}_0^{\rm PA}{:}\mathcal{A}_-^{\rm PA}{:}\mathcal{A}_+^{\rm PA} = \left(\frac{\Lambda_{\rm QCD}}{m_b}\ln\frac{m_b}{\Lambda_h}\right)^2{:}\left(\frac{\Lambda_{\rm QCD}}{m_b}\ln\frac{m_b}{\Lambda_h}\right)^2{:}\left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^4. \tag{123}$$

The logarithmic divergences are associated with the limit in which both the s and $\bar{s}$ quarks originating from the gluon are soft [64]. As for the power counting, the annihilation topology is of order $1/m_b$ and each remaining factor of $1/m_b$ is associated with a quark helicity flip. The fact that $\mathcal{A}_-^{\rm PA}$ and $\mathcal{A}_0^{\rm PA}$ have the same power counting explains why penguin annihilation is helpful to resolve the polarization puzzle. The relative size of $\mathcal{A}_-^{\rm PA}$ and $\mathcal{A}_0^{\rm PA}$ depends mainly on the phase ϕ_A. It turns out that the longitudinal polarization fraction for $B \to K_2^*\phi$ is quite sensitive to the phase ϕ_A^{TV}, while $f_L(\omega K_2^*)$ is not so sensitive to ϕ_A^{VT}. For example, $f_L(K_2^*\phi) = 0.88$, 0.72, 0.48, respectively, for $\phi_A^{TV} = -30°$, $-45°$, $-60°$ and $f_L(\omega K_2^*) = 0.68$, 0.66, 0.64, respectively, for $\phi_A^{VT} = -30°$, $-45°$, $-60°$. Hence, we can use the experimental measurements of f_L to fix the phases ϕ_A^{VT} and ϕ_A^{TV} and branching fractions to pin down the parameters $\rho_A(VT)$ and $\rho_A(TV)$:

$$\rho_A^{TV} = 0.65, \qquad \phi_A^{TV} = -33°,$$
$$\rho_A^{VT} = 1.20, \qquad \phi_A^{VT} = -60°. \tag{124}$$

It should be stressed that, although the experimental observation of the longitudinal polarization in $B \to K_2^*\phi$ and $B \to K_2^*\omega$ decays can be accommodated in the QCDF approach, no dynamical explanation is offered for the smallness of $f_T(K_2^*\phi)$ and the sizable $f_T(\omega K_2^*)$.

For penguin-dominated $B \to TV$ decays, we find $f_L(K_2^*\rho) \sim f_L(K_2^*\omega) \sim 0.65$, whereas $f_L(f_2K^*) \sim 0.93$ (cf. Table VII). It will be very interesting to measure f_L for these modes to test the approach of QCDF. Theoretically, transverse polarization is expected to be small in tree-dominated $\bar{B} \to TV$ decays except for the $a_2^-\rho^0$, $a_2^-\rho^+$, $K_2^{*0}K^{*-}$ and $K_2^{*0}\bar{K}^{*0}$ modes.

V. CONCLUSIONS

We have studied in this work the charmless hadronic B decays with a tensor meson in the final state within the framework of QCD factorization. Because of the G-parity of the tensor meson, both the chiral-even and chiral-odd two-parton LCDAs of the tensor meson are antisymmetric under the interchange of momentum fractions of the *quark* and *antiquark* in the SU(3) limit. The main results of this work are as follows:

(1) We have worked out the longitudinal and transverse helicity projection operators for the tensor meson. They are very similar to the projectors for the vector meson. Consequently, the nonfactorizable contributions such as vertex, penguin and hard spectator corrections to $B \to T(P, V)$ decays can be directly obtained from $B \to VP$, VV ones by making some suitable replacement.

(2) The factorizable amplitude with a tensor meson emitted vanishes under the factorization hypothesis owing to the fact that a tensor meson cannot be created from the local $V-A$ and tensor currents. As a result, $B^- \to \bar{K}_2^{*0}\pi^-$ and $\bar{B}^0 \to K_2^{*-}\pi^+$ vanish

in naïve factorization. The experimental observation of the former implies the importance of nonfactorizable effects.

(3) Five different models for $B \to T$ transition form factors were considered. While the predictions of $B \to f_2(1270)$ form factors based on large energy effective theory or pQCD are favored by experiment, the covariant light-front quark model or the ISGW2 model for $B \to K_2^*(1430)$ ones is preferred by the data.

(4) For penguin-dominated $B \to TP$ and TV decays, the predicted rates in naïve factorization are normally too small by 1 to 2 orders of magnitude. In QCDF, they are enhanced by the power corrections from penguin annihilation and nonfactorizable contributions.

(5) Three distinct types of penguin contributions to $B \to K_2^* \eta^{(\prime)}$ exist: (i) $b \to sq\bar{q} \to s\eta_q$, (ii) $b \to ss\bar{s} \to s\eta_s$, and (iii) $b \to sq\bar{q} \to q\bar{K}_2^*$ with $\eta_q = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\eta_s = s\bar{s}$. The dominant effects arise from the last two penguin contributions. The interference, constructive for $K_2^*\eta'$ and destructive for $K_2^*\eta$ between type (ii) and type (iii) diagrams, explains why $\Gamma(B \to K_2^*\eta') \gg \Gamma(B \to K_2^*\eta)$.

(6) We use the measured rates of $K_2^*\omega$ and $K_2^*\phi$ modes to extract the penguin-annihilation parameters ρ_A^{TV} and ρ_A^{VT} and the observed longitudinal polarization fractions $f_L(K_2^*\omega)$ and $f_L(K_2^*\phi)$ to fix the phases ϕ_A^{VT} and ϕ_A^{TV}. The unexpectedly large rate of $B \to K_2^*\omega$ relative to $B \to K_2^*\phi$ implies that $\rho_A^{VT} \gg \rho_A^{VT}$. However, it may be hard to offer more intuitive understanding for the large disparity in magnitude between ρ_A^{TV} and ρ_A^{VT}.

(7) The experimental observation that $f_T/f_L \ll 1$ for $B \to \phi K_2^*(1430)$, whereas $f_T/f_L \sim 1$ for $B \to \omega K_2^*(1430)$, can be *accommodated* in QCDF, but cannot be *dynamically explained* at first place. For penguin-dominated $B \to TV$ decays, we find $f_L(K_2^*\rho) \sim f_L(K_2^*\omega) \sim 0.65$ and $f_L(K^* f_2) \sim 0.93$. It will be of great interest to measure f_L for these modes to test QCDF. Theoretically, transverse polarization is expected to be small in tree-dominated $\bar{B} \to TV$ decays except for the $a_2^- \rho^0$, $a_2^- \rho^+$, $K_2^{*0} K^{*-}$ and $K_2^{*0} \bar{K}^{*0}$ modes.

(8) For tree-dominated decays, their rates are usually very small except for the $a_2^0(\pi^-, \rho^-)$, $a_2^+(\pi^-, \rho^-)$ and $f_2(\pi^-, \rho^-)$ modes with branching fractions of order 10^{-6} or even bigger.

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APPENDIX A: FORM FACTORS IN THE ISGW2 QUARK MODEL

Consider the transition $B \to T$ in the ISGW2 quark model [46], where the tensor meson T has the quark content $q_1\bar{q}_2$ with $\bar{q}_2$ being the spectator quark. We begin with the definition [46]

$$F_n = \left(\frac{\tilde{m}_T}{\tilde{m}_B}\right)^{1/2} \left(\frac{\beta_B \beta_T}{\beta_{BT}^2}\right)^{n/2} \left[1 + \frac{1}{18} r^2 (t_m - t)\right]^{-3}, \quad \text{(A1)}$$

where

$$r^2 = \frac{3}{4m_b m_1} + \frac{3m_2^2}{2\bar{m}_B \bar{m}_T \beta_{BT}^2} + \frac{1}{\bar{m}_B \bar{m}_T}\left(\frac{16}{33 - 2n_f}\right) \times \ln\left[\frac{\alpha_s(\mu_{\rm QM})}{\alpha_s(m_1)}\right], \quad \text{(A2)}$$

$\tilde{m}$ is the sum of the meson's constituent quarks' masses, $\bar{m}$ is the hyperfine-averaged mass (for example, $\bar{m}_B = \frac{3}{4} m_{B^*} + \frac{1}{4} m_B$), $t_m = (m_B - m_T)^2$ is the maximum momentum transfer, and

$$\mu_\pm = \left(\frac{1}{m_1} \pm \frac{1}{m_b}\right)^{-1}, \quad \text{(A3)}$$

with m_1 and m_2 being the masses of the quarks q_1 and $\bar{q}_2$, respectively. In Eq. (A1), the values of the parameters β_B and β_T are available in [46] and $\beta_{BT}^2 = \frac{1}{2}(\beta_B^2 + \beta_T^2)$.

The form factors defined by Eq. (17) have the following expressions in the ISGW2 model:

$$\begin{aligned} h &= \frac{m_2}{2\sqrt{2}\tilde{m}_B \beta_B}\left[\frac{1}{m_1} - \frac{m_2 \beta_B^2}{2\mu_- \tilde{m}_T \beta_{BT}^2}\right] F_5^{(h)}, \\ k &= \frac{m_2}{\sqrt{2}\beta_B}(1 + \tilde{\omega}) F_5^{(k)}, \\ b_+ + b_- &= \frac{m_2^2}{4\sqrt{2} m_q m_b \tilde{m}_B \beta_B} \frac{\beta_T^2}{\beta_{BT}^2}\left(1 - \frac{m_2}{2\tilde{m}_B}\frac{\beta_T^2}{\beta_{BT}^2}\right) F_5^{(b_+ + b_-)}, \\ b_+ - b_- &= -\frac{m_2}{\sqrt{2} m_b \tilde{m}_T \beta_B}\left[1 - \frac{m_2 m_b}{2\mu_+ \tilde{m}_B}\frac{\beta_T^2}{\beta_{BT}^2} + \frac{m_2}{4m_q}\frac{\beta_T^2}{\beta_{BT}^2}\left(1 - \frac{m_2}{2\tilde{m}_B}\frac{\beta_T^2}{\beta_{BT}^2}\right)\right] F_5^{(b_+ - b_-)}, \end{aligned} \quad \text{(A4)}$$

where

$$\begin{aligned} F_5^{(h)} &= F_5\left(\frac{\bar{m}_B}{\tilde{m}_B}\right)^{-3/2}\left(\frac{\bar{m}_T}{\tilde{m}_T}\right)^{-1/2}, \\ F_5^{(k)} &= F_5\left(\frac{\bar{m}_B}{\tilde{m}_B}\right)^{-1/2}\left(\frac{\bar{m}_T}{\tilde{m}_T}\right)^{1/2}, \\ F_5^{(b_+ + b_-)} &= F_5\left(\frac{\bar{m}_B}{\tilde{m}_B}\right)^{-5/2}\left(\frac{\bar{m}_T}{\tilde{m}_T}\right)^{1/2}, \\ F_5^{(b_+ - b_-)} &= F_5\left(\frac{\bar{m}_B}{\tilde{m}_B}\right)^{-3/2}\left(\frac{\bar{m}_T}{\tilde{m}_T}\right)^{-1/2}, \end{aligned} \tag{A5}$$

and

$$\tilde{\omega} - 1 = \frac{t_m - t}{2\bar{m}_B\bar{m}_T}. \tag{A6}$$

In the original version of the ISGW model [45], the function F_n has a different expression in its $(t_m - t)$ dependence:

$$F_n = \left(\frac{\tilde{m}_T}{\tilde{m}_B}\right)^{1/2}\left(\frac{\beta_B\beta_T}{\beta_{BT}^2}\right)^{n/2}\exp\left\{-\frac{m_2}{4\tilde{m}_B\tilde{m}_T}\frac{t_m - t}{\kappa^2\beta_{BT}^2}\right\}, \tag{A7}$$

where $\kappa = 0.7$ is the relativistic correction factor. The form factors are then given by

$$\begin{aligned} h &= \frac{m_2}{2\sqrt{2}\tilde{m}_B\beta_B}\left[\frac{1}{m_1} - \frac{m_2}{2\tilde{m}_T\mu_-}\frac{\beta_B^2}{\beta_{BT}^2}\right]F_5, \\ k &= \frac{m_2}{\sqrt{2}\beta_B}F_5, \\ b_+ &= -\frac{m_2}{2\sqrt{2}m_b\tilde{m}_T\beta_B}\left[1 - \frac{m_2 m_b}{2\mu_+\tilde{m}_B}\frac{\beta_T^2}{\beta_{BT}^2}\right. \\ &\quad \left. + \frac{m_2 m_b}{4\tilde{m}_B\mu_-}\frac{\beta_T^2}{\beta_{BT}^2}\left(1 - \frac{m_2}{2\tilde{m}_B}\frac{\beta_T^2}{\beta_{BT}^2}\right)\right]F_5. \end{aligned} \tag{A8}$$

Note that the expressions in Eq. (A4) in the ISGW2 model allow one to determine the form factor b_-, which vanishes in the ISGW model.

APPENDIX B: DECAY AMPLITUDES

The coefficients of the flavor operators $\alpha_i^{p,(h)}$ can be expressed in terms of $a_i^{p,(h)}$ in the following:

$$\begin{aligned} \alpha_1(M_1M_2) &= a_1(M_1M_2), \\ \alpha_2(M_1M_2) &= a_2(M_1M_2), \\ \alpha_3^p(M_1M_2) &= \begin{cases} a_3^p(M_1M_2) - a_5^p(M_1M_2) & \text{for } M_1M_2 = TP, \\ a_3^p(M_1M_2) + a_5^p(M_1M_2) & \text{for } M_1M_2 = PT, \end{cases} \\ \alpha_4^p(M_1M_2) &= \begin{cases} a_4^p(M_1M_2) - r_\chi^{M_2}a_6^p(M_1M_2) & \text{for } M_1M_2 = TP \\ a_4^p(M_1M_2) + r_\chi^{M_2}a_6^p(M_1M_2) & \text{for } M_1M_2 = PT, \end{cases} \\ \alpha_{3,\mathrm{EW}}^p(M_1M_2) &= \begin{cases} a_9^p(M_1M_2) - a_7^p(M_1M_2) & \text{for } M_1M_2 = TP, \\ a_9^p(M_1M_2) + a_7^p(M_1M_2) & \text{for } M_1M_2 = PT, \end{cases} \\ \alpha_{4,\mathrm{EW}}^p(M_1M_2) &= \begin{cases} a_{10}^p(M_1M_2) - r_\chi^{M_2}a_8^p(M_1M_2) & \text{for } M_1M_2 = TP, \\ a_{10}^p(M_1M_2) + r_\chi^{M_2}a_8^p(M_1M_2) & \text{for } M_1M_2 = PT, \end{cases} \end{aligned} \tag{B1}$$

for $\bar{B} \to TP$ decays, and

$$\begin{aligned} \alpha_1^h(M_1M_2) &= a_1^h(M_1M_2), \\ \alpha_2^h(M_1M_2) &= a_2^h(M_1M_2), \\ \alpha_3^{p,h}(M_1M_2) &= a_3^{p,h}(M_1M_2) + a_5^{p,h}(M_1M_2), \\ \alpha_4^{p,h}(M_1M_2) &= a_4^{p,h}(M_1M_2) - r_\chi^{M_2}a_6^{p,h}(M_1M_2), \\ \alpha_{3,\mathrm{EW}}^{p,h}(M_1M_2) &= a_9^{p,h}(M_1M_2) + a_7^{p,h}(M_1M_2), \\ \alpha_{4,\mathrm{EW}}^{p,h}(M_1M_2) &= a_{10}^{p,h}(M_1M_2) - r_\chi^{M_2}a_8^{p,h}(M_1M_2), \end{aligned} \tag{B2}$$

for $\bar{B} \to TV$ decays with $(M_1M_2) \equiv (TV)$ or (VT). It should be noted that the order of the arguments of $\alpha_i^p(M_1M_2)$ and $a_i^p(M_1M_2)$ is relevant. The chiral factor r_χ^M is given by

$$\begin{aligned} r_\chi^{\pi,K}(\mu) &= \frac{2m_{\pi,K}^2}{m_b(\mu)(m_{q_1} + m_{q_2})(\mu)}, \\ r_\chi^{\eta_{q,s}^{(\prime)}}(\mu) &= \frac{h_{q,s}}{f_{q,s}^{(\prime)}m_b(\mu)m_{q,s}(\mu)} \end{aligned} \tag{B3}$$

for the pseudoscalar mesons,

$$r_\chi^V(\mu) = \frac{2m_V}{m_b(\mu)}\frac{f_V^\perp(\mu)}{f_V} \tag{B4}$$

for the vector meson, and

$$r_\chi^T(\mu) = \frac{2m_T}{m_b(\mu)}\frac{f_T^\perp(\mu)}{f_T} \tag{B5}$$

for the tensor meson. See Appendix D for further discussions on the parameters $h_{q,s}$ and $f_{q,s}^{(\prime)}$.

In the following decay amplitudes, the order of the arguments of $\alpha_i^{p(,h)}(M_1M_2)$ and $\beta_i^{p(,h)}(M_1M_2)$ is consistent with the order of the arguments of $X_{(h)}^{(\bar{B}M_1,M_2)}$ or $\bar{X}_{(h)}^{(\bar{B}M_1,M_2)}$, where

$$\beta_i^p(TP) = \frac{-if_Bf_Tf_P}{X^{(\bar{B}T,P)}} b_i^p, \qquad \beta_i^p(PT) = \frac{-if_Bf_Tf_P}{\bar{X}^{(\bar{B}P,T)}} b_i^p, \quad \text{for } TP \text{ modes}, \tag{B6}$$

$$\beta_i^{p,h}(TV) = \frac{if_Bf_Tf_V}{X_h^{(\bar{B}T,V)}} b_i^{p,h}, \qquad \beta_i^{p,h}(VT) = \frac{if_Bf_Tf_V}{\bar{X}_h^{(\bar{B}V,T)}} b_i^{p,h}, \quad \text{for } TV \text{ modes.} \tag{B7}$$

The decay amplitudes for $\bar{B} \to TP, TV$ are summarized as follows:

1. $\bar{B} \to TP$ decays

A. Decay amplitudes with $\Delta S = 0$

$$\sqrt{2}\mathcal{A}_{B^-\to f_2\pi^-} = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(d)}\bigg\{\bigg[\delta_{pu}(\alpha_2+\beta_2)+2\alpha_3^p+\alpha_4^p+\frac{1}{2}\alpha_{3,\rm EW}^p-\frac{1}{2}\alpha_{4,\rm EW}^p+\beta_3^p+\beta_{3,\rm EW}^p\bigg]\bar{X}^{(\bar{B}\pi,f_2^q)} + \sqrt{2}\bigg[\alpha_3^p-\frac{1}{2}\alpha_{3,\rm EW}^p\bigg]\bar{X}^{(\bar{B}\pi,f_2^s)}+[\delta_{pu}(\alpha_1+\beta_2)+\alpha_4^p+\alpha_{4,\rm EW}^p+\beta_3^p+\beta_{3,\rm EW}^p]X^{(\bar{B}f_2^q,\pi)}\bigg\}, \tag{B8}$$

$$-2\mathcal{A}_{\bar{B}^0\to f_2\pi^0} = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(d)}\bigg\{\bigg[\delta_{pu}(\alpha_2-\beta_1)+2\alpha_3^p+\alpha_4^p+\frac{1}{2}\alpha_{3,\rm EW}^p-\frac{1}{2}\alpha_{4,\rm EW}^p+\beta_3^p-\frac{1}{2}\beta_{3,\rm EW}^p-\frac{3}{2}\beta_{4,\rm EW}^p\bigg]\bar{X}^{(\bar{B}\pi,f_2^q)} + \sqrt{2}\bigg[\alpha_3^p-\frac{1}{2}\alpha_{3,\rm EW}^p\bigg]\bar{X}^{(\bar{B}\pi,f_2^s)}+\bigg[\delta_{pu}(-\alpha_2-\beta_1)+\alpha_4^p-\frac{3}{2}\alpha_{3,\rm EW}^p-\frac{1}{2}\alpha_{4,\rm EW}^p+\beta_3^p-\frac{1}{2}\beta_{3,\rm EW}^p - \frac{3}{2}\beta_{4,\rm EW}^p\bigg]X^{(\bar{B}f_2^q,\pi)}\bigg\}, \tag{B9}$$

$$2\mathcal{A}_{\bar{B}^0\to f_2\eta} = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(d)}\bigg\{\bigg[\delta_{pu}(\alpha_2+\beta_1)+2\alpha_3^p+\alpha_4^p+\frac{1}{2}\alpha_{3,\rm EW}^p-\frac{1}{2}\alpha_{4,\rm EW}^p+\beta_3^p+2\beta_4^p-\frac{1}{2}\beta_{3,\rm EW}^p+\frac{1}{2}\beta_{4,\rm EW}^p\bigg]X^{(\bar{B}f_2^q,\eta_q)} + \sqrt{2}\bigg[\alpha_3^p-\frac{1}{2}\alpha_{3,\rm EW}^p\bigg]X^{(\bar{B}f_2^q,\eta_s)}+\sqrt{2}[\delta_{pc}\alpha_2+\alpha_3^p]X^{(\bar{B}f_2^q,\eta_c)}-2if_Bf_2^sf_\eta^s\bigg[b_4^p-\frac{1}{2}b_{4,\rm EW}^p\bigg]_{f_2^s\eta_s} + \bigg[\delta_{pu}(\alpha_2+\beta_1)+2\alpha_3^p+\alpha_4^p+\frac{1}{2}\alpha_{3,\rm EW}^p-\frac{1}{2}\alpha_{4,\rm EW}^p+\beta_3^p+2\beta_4^p-\frac{1}{2}\beta_{3,\rm EW}^p+\frac{1}{2}\beta_{4,\rm EW}^p\bigg]\bar{X}^{(\bar{B}\eta_q,f_2^q)} + \sqrt{2}\bigg[\alpha_3^p-\frac{1}{2}\alpha_{3,\rm EW}^p\bigg]\bar{X}^{(\bar{B}\eta_q,f_2^s)}-2if_Bf_2^sf_\eta^s\bigg[b_4^p-\frac{1}{2}b_{4,\rm EW}^p\bigg]_{\eta_sf_2^s}\bigg\}, \tag{B10}$$

and the amplitudes for $\bar{B} \to f_2\eta'$ can be obtained from $\bar{B} \to f_2\eta$ with the replacement $(f_2, \eta) \to (f_2, \eta')$:

$$\sqrt{2}\mathcal{A}_{B^-\to a_2^0\pi^-} = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(d)}\bigg\{\bigg[\delta_{pu}(\alpha_2-\beta_2)-\alpha_4^p+\frac{3}{2}\alpha_{3,\rm EW}^p+\frac{1}{2}\alpha_{4,\rm EW}^p-\beta_3^p-\beta_{3,\rm EW}^p\bigg]\bar{X}^{(\bar{B}\pi,a_2)} + [\delta_{pu}(\alpha_1+\beta_2)+\alpha_4^p+\alpha_{4,\rm EW}^p+\beta_3^p+\beta_{3,\rm EW}^p]X^{(\bar{B}a_2,\pi)}\bigg\}, \tag{B11}$$

$$\sqrt{2}\mathcal{A}_{B^-\to a_2^-\pi^0} = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(d)}\bigg\{\bigg[\delta_{pu}(\alpha_2-\beta_2)-\alpha_4^p+\frac{3}{2}\alpha_{3,\rm EW}^p+\frac{1}{2}\alpha_{4,\rm EW}^p-\beta_3^p-\beta_{3,\rm EW}^p\bigg]X^{(\bar{B}a_2,\pi)} + [\delta_{pu}(\alpha_1+\beta_2)+\alpha_4^p+\alpha_{4,\rm EW}^p+\beta_3^p+\beta_{3,\rm EW}^p]\bar{X}^{(\bar{B}\pi,a_2)}\bigg\}, \tag{B12}$$

$$\mathcal{A}_{\bar{B}^0\to a_2^-\pi^+} = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(d)}\bigg\{\bigg[\delta_{pu}\alpha_1 + \alpha_4^p + \alpha_{4,\mathrm{EW}}^p + \beta_3^p + \beta_4^p - \frac{1}{2}\beta_{3,\mathrm{EW}}^p - \frac{1}{2}\beta_{4,\mathrm{EW}}^p\bigg]\bar{X}^{(\bar{B}\pi,a_2)} + [\delta_{pu}\beta_1 + \beta_4^p + \beta_{4,\mathrm{EW}}^p]X^{(\bar{B}a_2,\pi)}\bigg\}, \tag{B13}$$

$$\mathcal{A}_{\bar{B}^0\to a_2^+\pi^-} = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(d)}\bigg\{\bigg[\delta_{pu}\alpha_1 + \alpha_4^p + \alpha_{4,\mathrm{EW}}^p + \beta_3^p + \beta_4^p - \frac{1}{2}\beta_{3,\mathrm{EW}}^p - \frac{1}{2}\beta_{4,\mathrm{EW}}^p\bigg]X^{(\bar{B}a_2,\pi)} + [\delta_{pu}\beta_1 + \beta_4^p + \beta_{4,\mathrm{EW}}^p]\bar{X}^{(\bar{B}\pi,a_2)}\bigg\}, \tag{B14}$$

$$-2\mathcal{A}_{\bar{B}^0\to a_2^0\pi^0} = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(d)}\bigg\{\bigg[\delta_{pu}(\alpha_2 - \beta_1) - \alpha_4^p + \frac{3}{2}\alpha_{3,\mathrm{EW}}^p + \frac{1}{2}\alpha_{4,\mathrm{EW}}^p - \beta_3^p - 2\beta_4^p + \frac{1}{2}\beta_{3,\mathrm{EW}}^p - \frac{1}{2}\beta_{4,\mathrm{EW}}^p\bigg]\bar{X}^{(\bar{B}\pi,a_2)} + \bigg[\delta_{pu}(\alpha_2 - \beta_1) - \alpha_4^p + \frac{3}{2}\alpha_{3,\mathrm{EW}}^p + \frac{1}{2}\alpha_{4,\mathrm{EW}}^p - \beta_3^p - 2\beta_4^p + \frac{1}{2}\beta_{3,\mathrm{EW}}^p - \frac{1}{2}\beta_{4,\mathrm{EW}}^p\bigg]X^{(\bar{B}a_2,\pi)}\bigg\}, \tag{B15}$$

$$\sqrt{2}\mathcal{A}_{B^-\to a_2^-\eta^{(\prime)}} = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(d)}\bigg\{\bigg[\delta_{pu}(\alpha_2 + \beta_2) + 2\alpha_3^p + \alpha_4^p + \frac{1}{2}\alpha_{3,\mathrm{EW}}^p - \frac{1}{2}\alpha_{4,\mathrm{EW}}^p + \beta_3^p + \beta_{3,\mathrm{EW}}^p\bigg]X^{(\bar{B}a_2,\eta_q^{(\prime)})} + [\delta_{pu}(\alpha_1 + \beta_2) + \alpha_4^p + \alpha_{4,\mathrm{EW}}^p + \beta_3^p + \beta_{3,\mathrm{EW}}^p]\bar{X}^{(\bar{B}\eta_q^{(\prime)},a_2)} + \sqrt{2}\bigg[\alpha_3^p - \frac{1}{2}\alpha_{3,\mathrm{EW}}^p\bigg]X^{(\bar{B}a_2,\eta_s^{(\prime)})} + \sqrt{2}[\delta_{pc}\alpha_2 + \alpha_3^p]X^{(\bar{B}a_2,\eta_c^{(\prime)})}\bigg\}, \tag{B16}$$

$$-2\mathcal{A}_{\bar{B}^0\to a_2^0\eta^{(\prime)}} = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(d)}\bigg\{\bigg[\delta_{pu}(\alpha_2 - \beta_1) + 2\alpha_3^p + \alpha_4^p + \frac{1}{2}\alpha_{3,\mathrm{EW}}^p - \frac{1}{2}\alpha_{4,\mathrm{EW}}^p + \beta_3^p - \frac{1}{2}\beta_{3,\mathrm{EW}}^p - \frac{3}{2}\beta_{4,\mathrm{EW}}^p\bigg]X^{(\bar{B}a_2,\eta_q^{(\prime)})} + \bigg[\delta_{pu}(-\alpha_2 - \beta_1) + \alpha_4^p - \frac{3}{2}\alpha_{3,\mathrm{EW}}^p - \frac{1}{2}\alpha_{4,\mathrm{EW}}^p + \beta_3^p - \frac{1}{2}\beta_{3,\mathrm{EW}}^p - \frac{3}{2}\beta_{4,\mathrm{EW}}^p\bigg]\bar{X}^{(\bar{B}\eta_q^{(\prime)},a_2)} + \sqrt{2}\bigg[\alpha_3^p - \frac{1}{2}\alpha_{3,\mathrm{EW}}^p\bigg]X^{(\bar{B}a_2,\eta_s^{(\prime)})} + \sqrt{2}[\delta_{pc}\alpha_2 + \alpha_3^p]X^{(\bar{B}a_2,\eta_c^{(\prime)})}\bigg\}, \tag{B17}$$

$$\mathcal{A}_{B^-\to K_2^{*-}K^0} = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(d)}\bigg[\delta_{pu}\beta_2 + \alpha_4^p - \frac{1}{2}\alpha_{4,\mathrm{EW}}^p + \beta_3^p + \beta_{3,\mathrm{EW}}^p\bigg]X^{(\bar{B}\bar{K}_2^*,K)}, \tag{B18}$$

$$\mathcal{A}_{B^-\to K_2^{*0}K^-} = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(d)}\bigg[\delta_{pu}\beta_2 + \alpha_4^p - \frac{1}{2}\alpha_{4,\mathrm{EW}}^p + \beta_3^p + \beta_{3,\mathrm{EW}}^p\bigg]\bar{X}^{(\bar{B}\bar{K},K_2^*)}, \tag{B19}$$

$$\mathcal{A}_{\bar{B}^0\to K_2^{*-}K^+} = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(d)}\bigg\{[\delta_{pu}\beta_1 + \beta_4^p + \beta_{4,\mathrm{EW}}^p]X^{(\bar{B}\bar{K}_2^*,K)} - icf_Bf_{K_2^*}f_K\bigg[b_4^p - \frac{1}{2}b_{4,\mathrm{EW}}^p\bigg]_{KK_2^*}\bigg\}, \tag{B20}$$

$$\mathcal{A}_{\bar{B}^0\to K_2^{*+}K^-} = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(d)}\bigg\{[\delta_{pu}\beta_1 + \beta_4^p + \beta_{4,\mathrm{EW}}^p]\bar{X}^{(\bar{B}\bar{K},K_2^*)} - icf_Bf_{K_2^*}f_K\bigg[b_4^p - \frac{1}{2}b_{4,\mathrm{EW}}^p\bigg]_{K_2^*K}\bigg\}, \tag{B21}$$

$$\mathcal{A}_{\bar{B}^0\to \bar{K}_2^{*0}K^0} = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(d)}\bigg\{\bigg[\alpha_4^p - \frac{1}{2}\alpha_{4,\mathrm{EW}}^p + \beta_3^p + \beta_4^p - \frac{1}{2}\beta_{3,\mathrm{EW}}^p - \frac{1}{2}\beta_{4,\mathrm{EW}}^p\bigg]X^{(\bar{B}\bar{K}_2^*,K)}, -icf_Bf_{K_2^*}f_K\bigg[b_4^p - \frac{1}{2}b_{4,\mathrm{EW}}^p\bigg]_{K\bar{K}_2^*}\bigg\}, \tag{B22}$$

$$\mathcal{A}_{\bar{B}^0\to K_2^{*0}\bar{K}^0} = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(d)}\left\{\left[\alpha_4^p - \frac{1}{2}\alpha_{4,\mathrm{EW}}^p + \beta_3^p + \beta_4^p - \frac{1}{2}\beta_{3,\mathrm{EW}}^p - \frac{1}{2}\beta_{4,\mathrm{EW}}^p\right]\bar{X}^{(\bar{B}\bar{K},K_2^*)}, -icf_Bf_{K_2^*}f_K\left[b_4^p - \frac{1}{2}b_{4,\mathrm{EW}}^p\right]_{K_2^*\bar{K}}\right\},\tag{B23}$$

with $c = 1$.

B. Decay amplitudes with $\Delta S = 1$

$$\begin{aligned}\sqrt{2}\mathcal{A}_{B^-\to f_2K^-} = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(s)}\bigg\{&\left[\delta_{pu}\alpha_2 + 2\alpha_3^p + \frac{1}{2}\alpha_{3,\mathrm{EW}}^p\right]\bar{X}^{(\bar{B}\bar{K},f_2^q)}\\ &+ \sqrt{2}\left[\delta_{pu}\beta_2 + \alpha_3^p + \alpha_4^p - \frac{1}{2}\alpha_{3,\mathrm{EW}}^p - \frac{1}{2}\alpha_{4,\mathrm{EW}}^p + \beta_3^p + \beta_{3,\mathrm{EW}}^p\right]\bar{X}^{(\bar{B}\bar{K},f_2^s)}\\ &+ [\delta_{pu}(\alpha_1 + \beta_2) + \alpha_4^p + \alpha_{4,\mathrm{EW}}^p + \beta_3^p + \beta_{3,\mathrm{EW}}^p]X^{(\bar{B}f_2^q,\bar{K})},\end{aligned}\tag{B24}$$

$$\begin{aligned}\sqrt{2}\mathcal{A}_{\bar{B}^0\to f_2K^0} = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(s)}\bigg\{&\left[\delta_{pu}\alpha_2 + 2\alpha_3^p + \frac{1}{2}\alpha_{3,\mathrm{EW}}^p\right]\bar{X}^{(\bar{B}\bar{K},f_2^q)}\\ &+ \sqrt{2}\left[\alpha_3^p + \alpha_4^p - \frac{1}{2}\alpha_{3,\mathrm{EW}}^p - \frac{1}{2}\alpha_{4,\mathrm{EW}}^p + \beta_3^p - \frac{1}{2}\beta_{3,\mathrm{EW}}^p\right]\bar{X}^{(\bar{B}\bar{K},f_2^s)}\\ &+ \left[\alpha_4^p - \frac{1}{2}\alpha_{4,\mathrm{EW}}^p + \beta_3^p - \frac{1}{2}\beta_{3,\mathrm{EW}}^p\right]X^{(\bar{B}f_2^q,\bar{K})},\end{aligned}\tag{B25}$$

$$\sqrt{2}\mathcal{A}_{B^-\to a_2^0K^-} = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(s)}\left\{[\delta_{pu}(\alpha_1 + \beta_2) + \alpha_4^p + \alpha_{4,\mathrm{EW}}^p + \beta_3^p + \beta_{3,\mathrm{EW}}^p]X^{(\bar{B}a_2,\bar{K})} + \left[\delta_{pu}\alpha_2 + \frac{3}{2}\alpha_{3,\mathrm{EW}}^p\right]\bar{X}^{(\bar{B}\bar{K},a_2)}\right\},\tag{B26}$$

$$\mathcal{A}_{B^-\to a_2^-K^0} = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(s)}\left[\delta_{pu}\beta_2 + \alpha_4^p - \frac{1}{2}\alpha_{4,\mathrm{EW}}^p + \beta_3^p + \beta_{3,\mathrm{EW}}^p\right]X^{(\bar{B}a_2,\bar{K})},\tag{B27}$$

$$\mathcal{A}_{\bar{B}^0\to a_2^+K^-} = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(s)}\left[\delta_{pu}\alpha_1^h + \alpha_4^p + \alpha_{4,\mathrm{EW}}^p + \beta_3^p - \frac{1}{2}\beta_{3,\mathrm{EW}}^p\right]X^{(\bar{B}a_2,\bar{K})},\tag{B28}$$

$$\sqrt{2}\mathcal{A}_{\bar{B}^0\to a_2^0K^0} = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(s)}\left\{\left[-\alpha_4^p + \frac{1}{2}\alpha_{4,\mathrm{EW}}^p - \beta_3^p + \frac{1}{2}\beta_{3,\mathrm{EW}}^p\right]X^{(\bar{B}a_2,\bar{K})} + \left[\delta_{pu}\alpha_2 + \frac{3}{2}\alpha_{3,\mathrm{EW}}^p\right]\bar{X}^{(\bar{B}\bar{K},a_2)}\right\},\tag{B29}$$

$$\begin{aligned}\sqrt{2}\mathcal{A}_{B^-\to K_2^{*-}\eta^{(\prime)}} = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(s)}\bigg\{&\left[\delta_{pu}\alpha_2 + 2\alpha_3^p + \frac{1}{2}\alpha_{3,\mathrm{EW}}^p\right]X^{(\bar{B}\bar{K}_2^*,\eta_q^{(\prime)})}\\ &+ \sqrt{2}\left[\delta_{pu}\beta_2 + \alpha_3^p + \alpha_4^p - \frac{1}{2}\alpha_{3,\mathrm{EW}}^p - \frac{1}{2}\alpha_{4,\mathrm{EW}}^p + \beta_3^p + \beta_{3,\mathrm{EW}}^p\right]X^{(\bar{B}\bar{K}_2^*,\eta_s^{(\prime)})}\\ &+ \sqrt{2}[\delta_{pc}\alpha_2 + \alpha_3^p]X^{(\bar{B}\bar{K}_2^*,\eta_c^{(\prime)})} + [\delta_{pu}(\alpha_1 + \beta_2) + \alpha_4^p + \alpha_{4,\mathrm{EW}}^p + \beta_3^p + \beta_{3,\mathrm{EW}}^p]X^{(\bar{B}\eta_q^{(\prime)},\bar{K}_2^*)}\bigg\},\end{aligned}\tag{B30}$$

$$\begin{aligned}\sqrt{2}\mathcal{A}_{\bar{B}^0\to \bar{K}_2^{*0}\eta^{(\prime)}} = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(s)}\bigg\{&\left[\delta_{pu}\alpha_2 + 2\alpha_3^p + \frac{1}{2}\alpha_{3,\mathrm{EW}}^p\right]X^{(\bar{B}\bar{K}_2^*,\eta_q^{(\prime)})}\\ &+ \sqrt{2}\left[\alpha_3^p + \alpha_4^p - \frac{1}{2}\alpha_{3,\mathrm{EW}}^p - \frac{1}{2}\alpha_{4,\mathrm{EW}}^p + \beta_3^p - \frac{1}{2}\beta_{3,\mathrm{EW}}^p\right]X^{(\bar{B}\bar{K}_2^*,\eta_s^{(\prime)})}\\ &+ \sqrt{2}[\delta_{pc}\alpha_2 + \alpha_3^p]X^{(\bar{B}\bar{K}_2^*\bar{B}\bar{K}_2^*,\eta_c^{(\prime)})} + \left[\alpha_4^p - \frac{1}{2}\alpha_{4,\mathrm{EW}}^p + \beta_3^p - \frac{1}{2}\beta_{3,\mathrm{EW}}^p\right]\bar{X}^{(\bar{B}\eta_q^{(\prime)},\bar{K}_2^*)}\bigg\},\end{aligned}\tag{B31}$$

and the amplitudes for $\bar{B}\to\bar{K}_2^*\pi$ can be obtained from $\bar{B}\to\bar{K}a_2$ with the replacement $(\bar{K}, a_2)\to(\bar{K}_2^*, \pi)$.

2. $\bar{B} \to TV$ decays

A. Decay amplitudes with $\Delta S = 0$

The amplitudes for $\bar{B} \to f_2\rho$ can be obtained from $\bar{B} \to f_2\pi$ with the replacement $(f_2, \pi) \to (f_2, \rho)$:

$$2\mathcal{A}_{\bar{B}^0\to f_2\omega} = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(d)}\bigg\{\bigg[\delta_{pu}(\alpha_2^h+\beta_1^h)+2\alpha_3^{p,h}+\alpha_4^{p,h}+\frac{1}{2}\alpha_{3,\rm EW}^{p,h}-\frac{1}{2}\alpha_{4,\rm EW}^{p,h}+\beta_3^{p,h}+2\beta_4^{p,h}-\frac{1}{2}\beta_{3,\rm EW}^{p,h}+\frac{1}{2}\beta_{4,\rm EW}^{p,h}\bigg]X^{(\bar{B}f_2^q,\omega)}$$
$$+\bigg[\delta_{pu}(\alpha_2^h+\beta_1^h)+2\alpha_3^{p,h}+\alpha_4^{p,h}+\frac{1}{2}\alpha_{3,\rm EW}^{p,h}-\frac{1}{2}\alpha_{4,\rm EW}^{p,h}+\beta_3^{p,h}+2\beta_4^{p,h}-\frac{1}{2}\beta_{3,\rm EW}^{p,h}+\frac{1}{2}\beta_{4,\rm EW}^{p,h}\bigg]\bar{X}^{(\bar{B}\omega,f_2^q)}$$
$$+\sqrt{2}\bigg[\alpha_3^{p,h}-\frac{1}{2}\alpha_{3,\rm EW}^{p,h}\bigg]\bar{X}^{(\bar{B}\omega,f_2^s)}\bigg\}, \tag{B32}$$

$$2\mathcal{A}_{\bar{B}^0\to f_2\phi} = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(d)}\bigg\{\sqrt{2}\bigg[\alpha_3^{p,h}-\frac{1}{2}\alpha_{3,\rm EW}^{p,h}\bigg]X^{(\bar{B}f_2^q,\phi)}+2if_Bf_2^sf_\phi\bigg[b_4^{p,h}-\frac{1}{2}b_{4,\rm EW}^{p,h}\bigg]_{f_2^s\phi}$$
$$+2if_Bf_2^sf_\phi\bigg[b_4^{p,h}-\frac{1}{2}b_{4,\rm EW}^{p,h}\bigg]_{\phi f_2^s}\bigg\}, \tag{B33}$$

and the amplitudes for $\bar{B} \to a_2\rho$ can be obtained from $\bar{B} \to a_2\pi$ with the replacement $(a_2, \pi) \to (a_2, \rho)$:

$$\sqrt{2}\mathcal{A}^h_{B^-\to a_2^-\omega} = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(d)}\bigg\{\bigg[\delta_{pu}(\alpha_2^h+\beta_2^h)+2\alpha_3^{p,h}+\alpha_4^{p,h}+\frac{1}{2}\alpha_{3,\rm EW}^{p,h}-\frac{1}{2}\alpha_{4,\rm EW}^{p,h}+\beta_3^{p,h}+\beta_{3,\rm EW}^{p,h}\bigg]X_h^{(\bar{B}a_2,\omega)}$$
$$+[\delta_{pu}(\alpha_1^h+\beta_2^h)+\alpha_4^{p,h}+\alpha_{4,\rm EW}^{p,h}+\beta_3^{p,h}+\beta_{3,\rm EW}^{p,h}]\bar{X}_h^{(\bar{B}\omega,a_2)}\bigg\}, \tag{B34}$$

$$-2\mathcal{A}^h_{\bar{B}^0\to a_2^0\omega} = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(d)}\bigg\{\bigg[\delta_{pu}(\alpha_2^h-\beta_1^h)+2\alpha_3^{p,h}+\alpha_4^{p,h}+\frac{1}{2}\alpha_{3,\rm EW}^{p,h}-\frac{1}{2}\alpha_{4,\rm EW}^{p,h}+\beta_3^{p,h}-\frac{1}{2}\beta_{3,\rm EW}^{p,h}-\frac{3}{2}\beta_{4,\rm EW}^{p,h}\bigg]X_h^{(\bar{B}a_2,\omega)}$$
$$+\bigg[\delta_{pu}(-\alpha_2^h-\beta_1^h)+\alpha_4^{p,h}-\frac{3}{2}\alpha_{3,\rm EW}^{p,h}-\frac{1}{2}\alpha_{4,\rm EW}^{p,h}+\beta_3^{p,h}-\frac{1}{2}\beta_{3,\rm EW}^{p,h}-\frac{3}{2}\beta_{4,\rm EW}^{p,h}\bigg]\bar{X}_h^{(\bar{B}\omega,a_2)}\bigg\}, \tag{B35}$$

$$\mathcal{A}^h_{B^-\to a_2^-\phi} = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(d)}\bigg\{\bigg[\alpha_3^{p,h}-\frac{1}{2}\alpha_{3,\rm EW}^{p,h}\bigg]X_h^{(\bar{B}a_2,\phi)}\bigg\}, \tag{B36}$$

$$-\sqrt{2}\mathcal{A}^h_{\bar{B}^0\to a_2^0\phi} = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(d)}\bigg\{\bigg[\alpha_3^{p,h}-\frac{1}{2}\alpha_{3,\rm EW}^{p,h}\bigg]X_h^{(\bar{B}a_2,\phi)}\bigg\}, \tag{B37}$$

and the amplitudes for $\bar{B} \to \bar{K}_2^* K^*$ can be obtained from $\bar{B} \to \bar{K}_2^* K$ with the replacement $(\bar{K}_2^*, K) \to (\bar{K}_2^*, K^*)$ and $c = -1$.

B. Decay amplitudes with $\Delta S = 1$

The amplitudes for $\bar{B} \to f_2\bar{K}^*$ can be obtained from $\bar{B} \to f_2\bar{K}$ with the replacement $(f_2, \bar{K}) \to (f_2, \bar{K}^*)$, and the ones for $\bar{B} \to \bar{K}_2^*\rho$ and $\bar{B} \to \bar{K}^* a_2$ can be obtained from $\bar{B} \to \bar{K}a_2$ with the replacement $(\bar{K}, a_2) \to (\bar{K}_2^*, \rho)$ and $(\bar{K}, a_2) \to (\bar{K}^*, a_2)$, respectively:

$$\sqrt{2}\mathcal{A}^h_{B^-\to K_2^{*-}\omega} = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(s)}\bigg\{[\delta_{pu}(\alpha_1^h+\beta_2^h)+\alpha_4^{p,h}+\alpha_{4,\rm EW}^{p,h}+\beta_3^{p,h}+\beta_{3,\rm EW}^{p,h}]\bar{X}_h^{(\bar{B}\omega,\bar{K}_2^*)}$$
$$+\bigg[\delta_{pu}\alpha_2^h+2\alpha_3^{p,h}+\frac{1}{2}\alpha_{3,\rm EW}^{p,h}\bigg]X_h^{(\bar{B}\bar{K}_2^*,\omega)}\bigg\}, \tag{B38}$$

$$\sqrt{2}\mathcal{A}^h_{\bar{B}^0\to \bar{K}_2^{*0}\omega} = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(s)}\bigg\{\bigg[\alpha_4^{p,h}-\frac{1}{2}\alpha_{4,\rm EW}^{p,h}+\beta_3^{p,h}-\frac{1}{2}\beta_{3,\rm EW}^{p,h}\bigg]\bar{X}_h^{(\bar{B}\omega,\bar{K}_2^*)}+\bigg[\delta_{pu}\alpha_2^h+2\alpha_3^{p,h}+\frac{1}{2}\alpha_{3,\rm EW}^{p,h}\bigg]X_h^{(\bar{B}\bar{K}_2^*,\omega)}\bigg\}, \tag{B39}$$

$$\mathcal{A}^{h}_{B^-\to K_2^{*-}\phi} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(s)} \left[\delta_{pu}\beta_2^h + \alpha_3^{p,h} + \alpha_4^{p,h} - \frac{1}{2}\alpha_{3,\mathrm{EW}}^{p,h} - \frac{1}{2}\alpha_{4,\mathrm{EW}}^{p,h} + \beta_3^{p,h} + \beta_{3,\mathrm{EW}}^{p,h} \right] X_h^{(\bar{B}\bar{K}_2^*,\phi)}, \tag{B40}$$

$$\mathcal{A}^{h}_{\bar{B}^0\to \bar{K}_2^{*0}\phi} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(s)} \left[\alpha_3^{p,h} + \alpha_4^{p,h} - \frac{1}{2}\alpha_{3,\mathrm{EW}}^{p,h} - \frac{1}{2}\alpha_{4,\mathrm{EW}}^{p,h} + \beta_3^{p,h} - \frac{1}{2}\beta_{3,\mathrm{EW}}^{p,h} \right] X_h^{(\bar{B}\bar{K}_2^*,\phi)}. \tag{B41}$$

APPENDIX C: EXPLICIT EXPRESSIONS OF ANNIHILATION AMPLITUDES

The general expressions of the helicity-dependent annihilation amplitudes are given in Eqs. (90)–(99). They can be further simplified by considering the asymptotic distribution amplitudes for Φ_V, Φ_v, Φ_T and Φ_t:

$$\begin{aligned} \Phi^V_{\parallel,\perp}(u) &= 6u\bar{u}, \\ \Phi_v(u) &= 3(2u-1), \\ \Phi^T_{\parallel,\perp}(u) &= 30u\bar{u}(2u-1), \\ \Phi_t(u) &= 5(1-6u+6u^2), \\ \Phi^M_+(u) &= \int_u^1 dv \frac{\Phi^M_\parallel(v)}{v}, \\ \Phi^M_-(u) &= \int_0^u dv \frac{\Phi^M_\parallel(v)}{\bar{v}}. \end{aligned} \tag{C1}$$

We find

$$A_3^{f,0}(VT) \approx -30\sqrt{\frac{2}{3}}\pi\alpha_s\left[(6X_A^{02} - 23X_A^0 + 22)r_\chi^V + \frac{3}{2}(2X_A^0 - 1)(X_A^0 - 3)r_\chi^T\right], \tag{C2}$$

$$A_3^{f,-}(VT) \approx -\frac{10}{\sqrt{2}}\pi\alpha_s\left[(6X_A^{-2} - 23X_A^- + 17)\frac{m_T}{m_V}r_\chi^V + 9(2X_A^- - 3)(X_A^- - 2)\frac{m_V}{m_T}r_\chi^T\right], \tag{C3}$$

$$A_3^{f,0}(TV) = -A_3^{f,0}(VT), \qquad A_3^{f,-}(TV) = -A_3^{f,-}(VT), \tag{C4}$$

$$A_3^{i,0}(VT) \approx 30\sqrt{\frac{2}{3}}\pi\alpha_s\left[-3(X_A^{02} - 4X_A^0 - 4 + \pi^2)r_\chi^V + \frac{3}{2}\left(X_A^{02} - 2X_A^0 - 6 + \frac{\pi^2}{3}\right)r_\chi^T\right], \tag{C5}$$

$$A_3^{i,-}(VT) \approx -\frac{30}{\sqrt{2}}\pi\alpha_s\left[-(X_A^{-2} - 2X_A^- - 2)\frac{m_T}{m_V}r_\chi^V + (3X_A^{-2} - 12X_A^- + 2\pi^2)\frac{m_V}{m_T}r_\chi^T\right], \tag{C6}$$

$$A_3^{i,0}(TV) = A_3^{i,0}(VT), \qquad A_3^{i,-}(TV) = A_3^{i,-}(VT), \tag{C7}$$

$$A_1^{i,0}(VT) \approx 30\sqrt{\frac{2}{3}}\pi\alpha_s\left[3X_A^0 + 4 - \pi^2 + \frac{3}{2}(X_A^0 - 3)(X_A^0 - 2)r_\chi^V r_\chi^T\right], \tag{C8}$$

$$A_1^{i,0}(TV) \approx -30\sqrt{\frac{2}{3}}\pi\alpha_s\left[X_A^0 + 29 - 3\pi^2 + \frac{3}{2}(X_A^0 - 3)(X_A^0 - 2)r_\chi^V r_\chi^T\right], \tag{C9}$$

$$A_2^{i,0}(VT) = -A_1^{i,0}(TV), \qquad A_2^{i,0}(TV) = -A_1^{i,0}(VT), \tag{C10}$$

for TV modes, and

$$A_3^f(PT) \approx \sqrt{\frac{2}{3}}\pi\alpha_s[10X_A(6X_A - 11)r_\chi^P + 45(2X_A - 1)(X_A - 3)r_\chi^T], \quad \text{(C11)}$$

$$A_3^i(PT) \approx 30\sqrt{\frac{2}{3}}\pi\alpha_s\left[\left(X_A^2 - 4X_A + 4 + \frac{\pi^2}{3}\right)r_\chi^P + \frac{3}{2}\left(X_A^2 - 2X_A - 6 + \frac{\pi^2}{3}\right)r_\chi^T\right], \quad \text{(C12)}$$

$$A_1^i(PT) \approx 10\sqrt{\frac{2}{3}}\pi\alpha_s\left[3(3X_A + 4 - \pi^2) + \frac{3}{2}X_A(X_A - 3)r_\chi^P r_\chi^T\right], \quad \text{(C13)}$$

$$A_1^i(TP) \approx -10\sqrt{\frac{2}{3}}\pi\alpha_s\left[3(X_A + 29 - 3\pi^2) + \frac{3}{2}X_A(X_A - 3)r_\chi^P r_\chi^T\right], \quad \text{(C14)}$$

$$A_3^f(PT) = A_3^f(TP), \qquad A_3^i(TP) = A_3^i(PT), \quad \text{(C15)}$$

$$A_2^i(TP) = A_1^i(PT), \qquad A_2^i(PT) = A_1^i(TP), \quad \text{(C16)}$$

for TP modes. As pointed out in [63], since the annihilation contributions $A_{1,2}^{i,\pm}$ are suppressed by a factor of $m_1 m_2/m_B^2$ relative to other terms, in numerical analysis we will consider only the annihilation contributions due to $A_3^{f,0}$, $A_3^{f,-}$, $A_{1,2,3}^{i,0}$ and $A_3^{i,-}$.

The logarithmic divergences that occurred in weak annihilation in the above equations are described by the variable X_A:

$$\int_0^1 \frac{du}{u} \to X_A, \qquad \int_0^1 du\frac{\ln u}{u} \to -\frac{1}{2}(X_A)^2. \quad \text{(C17)}$$

Following [24], these variables are parameterized in Eq. (100) in terms of the unknown real parameters ρ_A and ϕ_A. For simplicity, we shall assume in practical calculations that X_A^h are helicity independent, $X_A^- = X_A^+ = X_A^0$.

APPENDIX D: THE $\eta - \eta'$ SYSTEM

For the η and η' particles, it is more convenient to work with the flavor states $q\bar{q} \equiv (u\bar{u} + d\bar{d})/\sqrt{2}$, $s\bar{s}$ and $c\bar{c}$ labeled by the η_q, η_s and η_c^0, respectively. Neglecting the small mixing with η_c^0, we write

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}\begin{pmatrix} |\eta_q\rangle \\ |\eta_s\rangle \end{pmatrix}, \quad \text{(D1)}$$

where $\phi = (39.3 \pm 1.0)^\circ$ [65] is the $\eta - \eta'$ mixing angle in the η_q and η_s flavor basis.[6] Decay constants $f_{\eta^{(\prime)}}^q$, $f_{\eta^{(\prime)}}^s$ and $f_{\eta^{(\prime)}}^c$ are defined by

$$\begin{aligned} \langle 0|\bar{q}\gamma_\mu\gamma_5 q|\eta^{(\prime)}\rangle &= i\frac{1}{\sqrt{2}}f_{\eta^{(\prime)}}^q p_\mu, \\ \langle 0|\bar{s}\gamma_\mu\gamma_5 s|\eta^{(\prime)}\rangle &= if_{\eta^{(\prime)}}^s p_\mu, \\ \langle 0|\bar{c}\gamma_\mu\gamma_5 c|\eta^{(\prime)}\rangle &= if_{\eta^{(\prime)}}^c p_\mu, \end{aligned} \quad \text{(D2)}$$

while the widely studied decay constants f_q and f_s are defined as [65]

$$\begin{aligned} \langle 0|\bar{q}\gamma^\mu\gamma_5 q|\eta_q\rangle &= \frac{i}{\sqrt{2}}f_q p^\mu, \\ \langle 0|\bar{s}\gamma^\mu\gamma_5 s|\eta_s\rangle &= if_s p^\mu. \end{aligned} \quad \text{(D3)}$$

The ansatz made by Feldmann, Kroll and Stech [65] is that the decay constants in the quark flavor basis follow the same pattern of $\eta - \eta'$ mixing given in Eq. (D1):

$$\begin{pmatrix} f_\eta^q & f_\eta^s \\ f_{\eta'}^q & f_{\eta'}^s \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}\begin{pmatrix} f_q & 0 \\ 0 & f_s \end{pmatrix}. \quad \text{(D4)}$$

Empirically, this ansatz works very well [65]. Theoretically, it has been shown recently that this assumption can be justified in the large-N_c approach [67].

It is useful to consider the matrix elements of pseudoscalar densities [68]

$$\begin{aligned} 2m_q\langle 0|\bar{q}\gamma_5 q|\eta^{(\prime)}\rangle &= \frac{i}{\sqrt{2}}h_{\eta^{(\prime)}}^q, \\ 2m_s\langle 0|\bar{s}\gamma_5 s|\eta^{(\prime)}\rangle &= ih_{\eta^{(\prime)}}^s, \end{aligned} \quad \text{(D5)}$$

and define the parameters h_q and h_s in analogue to f_q and f_s

$$\begin{aligned} 2m_q\langle 0|\bar{q}\gamma_5 q|\eta_q\rangle &= \frac{i}{\sqrt{2}}h_q, \\ 2m_s\langle 0|\bar{s}\gamma_5 s|\eta_s\rangle &= ih_s, \end{aligned} \quad \text{(D6)}$$

[6] A different mixing angle $\phi = (35.9 \pm 3.4)^\circ$ was obtained recently in the analysis of [66] based on vector meson radiative decays.

and relate them to $h^{q,s}_{\eta,\eta'}$ by the similar Feldmann, Kroll and Stech ansatz as in Eq. (D4).

In this work, we shall follow [56] to use

$$\begin{aligned} h^q_\eta &= 0.0013\ \mathrm{GeV}^3, & h^s_\eta &= -0.0555\ \mathrm{GeV}^3,\\ h^q_{\eta'} &= 0.0011\ \mathrm{GeV}^3, & h^s_{\eta'} &= 0.068\ \mathrm{GeV}^3,\\ f^q_\eta &= 109\ \mathrm{MeV}, & f^s_\eta &= -111\ \mathrm{MeV},\\ f^q_{\eta'} &= 89\ \mathrm{MeV}, & f^s_{\eta'} &= 136\ \mathrm{MeV},\\ f^c_\eta &= -2.3\ \mathrm{MeV}, & f^c_{\eta'} &= -5.8\ \mathrm{MeV},\\ m_{\eta_q} &= 741\ \mathrm{MeV}, & m_{\eta_s} &= 802\ \mathrm{MeV}, \end{aligned} \tag{D7}$$

where we have used the perturbative result [69]

$$f^c_{\eta^{(\prime)}} = -\frac{m^2_{\eta^{(\prime)}}}{12m_c^2}\frac{f^q_{\eta^{(\prime)}}}{\sqrt{2}}. \tag{D8}$$

The form factors for $B \to \eta^{(\prime)}$ transitions obtained in QCD sum rules are [70]

$$\begin{aligned} F^{B\eta}_{0,1}(0) &= 0.229 \pm 0.024 \pm 0.011,\\ F^{B\eta'}_{0,1}(0) &= 0.188 \pm 0.002 B^g_2 \pm 0.019 \pm 0.009, \end{aligned} \tag{D9}$$

where the flavor-singlet contribution to the $B \to \eta^{(\prime)}$ form factors is characterized by the parameter B^g_2, a gluonic Gegenbauer moment. It appears that the singlet contribution to the form factor is small unless B^g_2 assumes extreme values ~ 40 [70]. Using the relation

$$F^{B\eta}_{0,1} = F^{B\eta_q}_{0,1}\cos\phi, \qquad F^{B\eta'}_{0,1} = F^{B\eta_q}_{0,1}\sin\phi, \tag{D10}$$

we obtain $F^{B\eta_q}_{0,1}(0) = 0.296 \pm 0.028$ as shown in Table IV. The momentum dependence of the form factor can be found in [70].

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