

**Quantum-gravity induced Lorentz violation and dynamical mass generation**

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In the eprint by Jean Alexandre [[arXiv:1009.5834](https://arxiv.org/abs/1009.5834)], a minimal extension of  $(3 + 1)$ -dimensional quantum electrodynamics has been proposed, which includes Lorentz violation (LV) in the form of higher-(spatial)-derivative isotropic terms in the gauge sector, suppressed by a mass scale  $M$ . The model can lead to dynamical mass generation for charged fermions. In this article, I elaborate further on this idea and I attempt to connect it to specific quantum-gravity models, inspired from string/brane theory. Specifically, in the first part of the article, I comment briefly on the gauge dependence of the dynamical mass generation in the approximations of J. Alexandre [[arXiv:1009.5834](https://arxiv.org/abs/1009.5834)], and I propose a possible avenue for obtaining the true gauge-parameter-independent value of the mass by means of pinch technique argumentations. In the second part of the work, I embed the LV QED model into multibrane world scenarios with a view to provide a geometrical way of enhancing the dynamical mass to phenomenologically realistic values by means of bulk warp metric factors, in an (inverse) Randall-Sundrum hierarchy. Finally, in the third part of this paper, I demonstrate that such Lorentz-violating QED models may represent parts of a low-energy effective action (of Finsler-Born-Infeld type) of open strings propagating in quantum D0-particle stochastic space-time foam backgrounds, which are viewed as consistent quantum-gravity configurations. To capture correctly the quantum-fluctuating nature of the foam background, I replace the D0-recoil-velocity parts of this action by appropriate gradient operators in three-space, keeping the photon field part intact. This is consistent with the summation over world-sheet genera in the first-quantized string approach. I identify a class of quantum orderings which leads to the LV QED action of J. Alexandre, [arXiv:1009.5834](https://arxiv.org/abs/1009.5834). In this way I argue, following the logic in that work, that the D-foam can lead to dynamically generated masses for charged-matter (fermionic) excitations interacting with it.

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**I. INTRODUCTION**

Lorentz violation (LV) in the standard model is a relatively old subject, originating from works by Kostelecky and collaborators [1]. There is extensive literature over the past 20 years on tests of LV based on a (phenomenological) effective Lagrangian approach, the so-called standard model extension (SME). However, there is no way in this approach of knowing the precise magnitude of the various LV parameters appearing in the SME, and thus we cannot have a feeling on the expected order of magnitude of the violations, so as to guide (or discourage!) experimental searches in a sensible and constructive way.

It is therefore desirable to have an ultraviolet completion of such phenomenological theories, which would guide us in our quest for an understanding of the quantum structure of space time at microscopic scales. In this latter respect, Horava recently revisited the issue of Lorentz violation by suggesting an anisotropic time-space scaling for gravity, which in the ultraviolet regime rendered the quantum theory of this gravitation renormalizable [2] (Horava-Lifshitz theories). This approach triggered an enormous interest among the relevant communities. Although in its

original formulation the model required a cosmological constant with the wrong sign to be compatible with observations, nevertheless, phenomenologically realistic improved versions [3] and appropriate extensions of the standard model [4] have recently been constructed. Moreover, LV quantum field theories with anisotropic scalings (therefore of Lifshitz type, but not related necessarily to gravity) existed long before the Horava-Lifshitz gravity, and in fact their anisotropic scaling has been used as a regulator of the ultraviolet divergences of quantum field theory [5]. Dynamical mass generation in such theories has also been discussed [6].

Whether LV is ultimately related to quantum gravity is not known at present, since all such theories are at the level of effective (low-energy) field theories and their microscopic understanding is lacking. However, quantum gravity may not admit a local effective Lagrangian description, as we have argued some time ago, based on a toy model of space-time foam, originating from string theory [7–10]. In fact, in these works, the space-time foamy structures are provided by stringy defects, interacting with matter string excitations on brane universes. The defects break Poincaré invariance, and their recoil during the interaction with open strings also breaks local Lorentz invariance. The D-particles, being string theory solitons, quantum fluctuate and their fluctuations give the background space time a

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foamy nature (“D-foam”). There is a nontrivial momentum transfer during the interactions of matter strings with D-foam defects, which necessitates the emission of non-local intermediate string states. The latter do not admit a local effective action description. In this kind of models, the so-called ultraviolet completion is provided by string/brane theory itself. We have argued [9,11], however, that, despite the lack of an efficient local effective action description, there is a rich astroparticle phenomenology associated with either the induced vacuum refraction, characterizing the D-foam, or with global aspects of it, associated with modifications of the energy budget of the dark sector of the Universe [8,9,12].

Nevertheless, as we shall discuss below, under some approximations, it is after all possible to write down a LV local effective action (expanded in powers of derivatives), which describes part of the effects of an isotropic, stochastically fluctuating, D-foam on radiation and, indirectly, on charged fermionic matter coupled to it, in particular dynamical mass generation. However, there are some other aspects of the foam, such as the induced time delays [9], during the interaction of a photon with a D-particle, and the associated vacuum refractive index, that such an action cannot fully capture. The latter effects, being associated with stringy time-space uncertainties, require the full string theory machinery to be quantified.

The quantum fluctuations of the D-particle defects in target space lead to a novel *correspondence principle* by means of which a B-antisymmetric tensor background field in phase space, representing the recoil velocity of the defect during its interaction with matter, is mapped [13] into a spatial-derivative operator along the direction of the recoil. In this way, the resulting Finsler-Born-Infeld (FBI) Lagrangian, which describes the (low-energy) dynamics of open strings on a brane world in interaction with the D-particles, may be transformed into an effective Lagrangian with Lorentz-violating higher spatial-derivative terms. The latter are of the form studied in a minimal LV extension of QED considered recently in [14]. That model has been argued to provide a novel way of generating electron masses dynamically. The model is not of Lifshitz type, in the sense that there is no anisotropic scaling between time and space coordinates. The presence of LV is manifested through higher-spatial-derivative terms, respecting rotational invariance in three space, which are suppressed by an effective mass scale. The presence of this scale and the LV terms *catalyze* dynamical mass generation for charged fermions, for arbitrarily weak gauge fields.

In view of our link of such a Lagrangian with the D-foam, and ultimately with more general quantum gravitational structures, this constitutes an explicit realization of the effects of a foam medium on “slowing” down some particles via mass generation. In our case, the suppression mass scale of the LV terms is expressed in terms of the

string mass scale and fundamental parameters of the foam, such as the variance of its quantum fluctuations. There are issues associated with quantum ordering ambiguities in our construction, which I attempt to resolve by appealing to the important role of the dynamical mass generation in curing infrared instabilities in the model. In this way, I select as the *physical* class of *quantum orderings* the one that maps the FBI effective action into *the precise form* of the LV QED action of [14] at low energies. Any further ordering ambiguities within this class of models is absorbed into the quantum fluctuation parameters of the D-foam, which at this stage are viewed as phenomenological.

The structure of the article is as follows: in Sec. II, I review the basic features of the model of [14]. In Sec. III, I comment briefly on the gauge dependence of the dynamically generated mass for fermions in this model, and propose a way to extract the gauge-fixing-parameter independent part of it, which we argue to be provided by its value in the Feynman gauge. The argumentation is based on the so-called pinch technique [15,16], which applies straightforwardly to our LV case, as a result of the very minimal form of the LV terms in the model of [14]. In Sec. IV, I discuss geometric ways of enhancing the dynamically generated mass through an inverse Randall-Sundrum-like hierarchy, which is achieved when the model of [14] is embedded appropriately into a multibrane world scenario, with warped bulk geometries. As a rather intriguing aspect, in Sec. V, I discuss a possible microscopic origin of the LV terms in the model of [14]. I show that such terms may constitute parts of an effective action describing the interaction of photons with quantum-fluctuating (in target-space) D-particles in the stochastic stringy space-time foam model of [8,9]. In this case, the LV scale  $M$  in the action of [14] is identified with a specific function of the foam parameters and the string scale and coupling. Conclusions and Outlook, in particular possible extensions of the model to incorporate flavored neutrinos, are presented in Sec. VI. Details on the application of the pinch technique to our one-loop approximate Schwinger-Dyson (SD) equations, relevant for dynamical mass generation in the analysis of [14], and some remarks on its current status in gauge theories in general, are given in an Appendix to the work.

## II. REVIEW OF A MINIMAL LORENTZ-VIOLATING (LV) QED MODEL

In [14] dynamical mass generation for fermions has been studied in the context of a  $(3 + 1)$ -dimensional QED field theory with higher-order spatial derivatives in the photon sector, that violated four-dimensional Lorentz symmetry but preserved spatial rotations, and a standard form for the fermion sector. This LV model is not of Lifshitz type, in the sense that there is isotropic scaling between time and space coordinates, but there is a mass scale to suppress

the LV spatial-derivative terms. The Lorentz-violating Lagrangian considered in [14] reads

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}\left(1 - \frac{\Delta}{M^2}\right)F_{\mu\nu} - \frac{\xi}{2}\partial_\mu A^\mu\left(1 - \frac{\Delta}{M^2}\right)\partial_\nu A^\nu + i\bar{\psi}\not{D}\psi, \quad (2.1)$$

where  $\xi$  is a covariant gauge-fixing parameter (GFP),  $D_\mu = \partial_\mu + ieA_\mu$ , and  $\Delta = \partial_i\partial^i = \vec{\partial} \cdot \vec{\partial}$ . Our conventions for the metric are  $(-1, 1, 1, 1)$ .

As emphasized in [14], no higher-order space derivatives are introduced for the fermions, to avoid the introduction of extra nonrenormalizable couplings into the theory. Indeed, in order to respect gauge invariance, which is a crucial assumption of the model, such terms would need to be of the form

$$\frac{1}{M^{n-1}}\bar{\psi}(i\vec{D} \cdot \vec{\gamma})^n\psi \quad n \geq 2, \quad (2.2)$$

thereby leading to nonrenormalizable couplings.

The standard  $(3+1)$ -dimensional QED in a covariant gauge is recovered in the limit  $M \rightarrow \infty$ . This scale parametrizes the region of energies in which the Lorentz-violating effects become important. It may or may not be the Planck scale, depending on the microscopic origin of Lorentz violation.

As discussed in [14], the Lorentz-violating terms play a dual role:

- (i) First, to introduce a mass scale,  $M$ , necessary to generate a fermion mass [14],

$$m_{\text{dyn}} = M \exp\left(-\frac{2\pi}{(4 + (\xi - 1))\alpha}\right), \quad (2.3)$$

where  $\alpha = e^2/4\pi$  is the fine structure constant. Here one draws an analogy with the magnetic catalysis phenomenon of standard QED, studied extensively in the past [17,18], according to which the presence of a sufficiently strong magnetic field catalyzes dynamical generation of a fermion mass for arbitrarily weak QED couplings. This case is also an example of a Lorentz-violating situation: the Lorentz symmetry breaking is provided by the direction of the external background field. However, there are two important differences from the model of [14]. The magnetic field breaks three-dimensional rotational symmetry, and moreover induces an effective dimensional reduction to two dimensions, as a result of the  $(1+1)$ -dimensional form of the fermion propagator of the lowest Landau level, which plays a dominant role in the strong magnetic-field case.

- (ii) Second, the higher-derivative Lorentz-violating terms provide an effective regularization of the theory, leading to finite gap equations [14]. An important remark is in order here, to avoid confusion. In this approach, the scale  $M$  is *not* the regulator of the theory (2.1), since it regularizes loops

with an internal photon line only. It is rather a parameter of the model, which physical quantities, like the dynamically generated mass, will depend upon. As we shall discuss later on, in Sec. V, in our stringy quantum-gravity model, which leads to the Lagrangian (2.1) as a low-energy field theory limit, this scale will be expressed in terms of fundamental parameters of the underlying string theory.

### III. ON DYNAMICAL MASS GENERATION IN LV QED AND ITS GAUGE (IN)DEPENDENCE

In this section we present for completeness a concise brief review of the results of [14] on dynamical mass generation (2.3) in the theory (2.1). We shall put emphasis on the gauge dependence of the result and propose a resolution to this problem, which will lead us to the gauge-parameter-independent, physically observable, value of the dynamical mass.

From the Lagrangian (2.1), the bare photon propagator is given by [14]

$$D_{\mu\nu}^{\text{bare}}(\omega, \vec{p}) = -\frac{i}{1 + p^2/M^2} \times \left( \frac{\eta_{\mu\nu}}{-\omega^2 + p^2} + (\xi - 1) \frac{p_\mu p_\nu}{(-\omega^2 + p^2)^2} \right), \quad (3.1)$$

where  $p^0 = \omega$  and  $p^2 = \vec{p} \cdot \vec{p}$ . We thus observe that, because the pole structure is *not* affected by the LV terms, the *photon* remains *massless* in this minimally LV model [14].

The SD equation for the fermion propagator, used in the derivation of (2.3) is (cf.. for instance [19])

$$S^{-1} - S_{\text{bare}}^{-1} = \int D_{\mu\nu}(e\gamma^\mu)S\Gamma^\nu, \quad (3.2)$$

where  $\Gamma^\nu$ ,  $S$ , and  $D_{\mu\nu}$  are, respectively, the dressed vertex, the dressed fermion propagator, and the dressed photon propagator. This equation gives an exact self-consistent relation between *dressed*  $n$ -point functions, and thus is *nonperturbative*. As a consequence, no redefinition of the bare parameters in the theory can be done in order to absorb the would-be divergences, and for this reason one needs this equation to be regularized by the scale  $M$ , which thus acquires physical significance.

In [14] the so-called *ladder* approximation has been assumed in order to solve the SD equation (3.2). According to this approximation, corrections to the vertex function, which otherwise would have led to a system of coupled SD equations and would complicate matters significantly, are ignored. It is well known that this approximation is *not gauge invariant* [19], as is the case actually with all off-shell field-theoretic quantities that are involved at intermediate stages of calculations of physical quantities in field theory. There are some gauges, termed nonlocal

gauges, in which the bare approximation to the vertex is argued to be an exact ansatz [18]. In our discussion below we shall restrict our analysis to one loop but attempt to determine the gauge-parameter-independent part of the dynamical mass generation.

In this spirit, Ref. [14] neglected the loop corrections to the photon propagator, as well as the fermion wave function renormalization, keeping only the corrections to the electron self-energy. The dressed fermion propagator can then be expressed as (in our conventions)

$$S(\omega, \vec{p}) = i \frac{p_\mu \gamma^\mu - m_{\text{dyn}}}{p_\mu p^\mu + m_{\text{dyn}}^2}, \quad (3.3)$$

where  $m_{\text{dyn}}$  is the fermion dynamical mass.

With these approximations, the SD equation (3.2), involving a convergent integral, due to the  $M$  dependent Lorentz-violating terms, becomes

$$m_{\text{dyn}} = \frac{\alpha}{\pi^2} \int \frac{d\omega p^2 dp}{1 + p^2/M^2} \times \frac{m_{\text{dyn}}(3 + \xi)}{(-\omega^2 + p^2)(-\omega^2 + p^2 + m_{\text{dyn}}^2)}, \quad (3.4)$$

where the fine structure constant is  $\alpha = e^2/4\pi$ . This equation has the obvious solution  $m_{\text{dyn}} = 0$ , and potentially a second solution, which must satisfy the following gap equation, obtained after integration over the frequency  $\omega$ ,

$$\frac{\pi}{(3 + \xi)\alpha} = \int_0^\infty \frac{xdx}{1 + \mu^2 x^2} \left(1 - \frac{x}{\sqrt{1 + x^2}}\right), \quad (3.5)$$

where  $\mu = m_{\text{dyn}}/M$  is the dimensionless dynamical mass, expected to be very small.

Both terms in the last equation, if taken separately, lead to diverging integrals. However, in this specific combination the divergences cancel each other. As explained in [14], after some approximations in the limit  $\mu \ll 1$ , the nontrivial fermion dynamical mass arises as a consistent solution of Eq. (3.5), and is given by (2.3). The selection by the physical system of the dynamical mass  $m_{\text{dyn}}$  takes place in order to avoid *infrared (IR) instabilities*, which would otherwise occur.

The expression (2.3) for  $m_{\text{dyn}}$  is not analytic in  $\alpha$ , so a perturbative expansion cannot lead to such a result. This justifies the use of a nonperturbative approach, like the SD in [14].

There is, however, an obvious drawback of this solution, namely its *dependence* on the *gauge-fixing parameter*  $\xi$ . The latter has consequences on the value of  $m_{\text{dyn}}$ , and prompts the question as to what is, if any, the true physical value of the dynamically generated mass. A possible resolution of this problem, which could lead to the true *physical value* of the dynamical mass, may be achieved by the

so-called pinch technique (PT) [15] and amounts to computing the dynamical mass in the *Feynman gauge*  $\xi = 1$ , ignoring in the relevant SD analyses the longitudinal parts of the gauge boson propagator. A brief review of this technique, which is widely used in related problems, not only in particle physics [16] but also in some condensed-matter systems [20] (involving energy gap generation of relativistic quasiparticle excitations, similar to the mass generation in particle physics), is given in the Appendix. We also present there the rather straightforward adaptation of the (one-loop) PT analysis of the Lorentz-invariant QED to the LV model (2.1), of interest to us here. This is feasible due to the special (minimal) form of LV in this model.

The consequences of the application of PT for the dynamical mass generation (2.3) in the theory (2.1) may be summarized in the statement that the physical (*gauge-parameter-independent*) value of the infrared fermion mass is the one corresponding to the Feynman gauge  $\xi = 1$ , in which all longitudinal parts of the photon propagator vanish:

$$m_{\text{dyn}} = M \exp\left(-\frac{\pi}{2\alpha}\right). \quad (3.6)$$

Unfortunately, for standard QED coupling,  $\alpha \simeq 1/137$ , this value is too small to have any phenomenological significance, even for scales  $M$  as high as the Planck scale. One needs enormously trans-Planckian  $M$  in order to get values of  $m_{\text{dyn}}$  that are near 0.5 MeV, and thus identify the above model as physical.

One might think of using improved ladder approximations, so as to replace the bare QED coupling  $\alpha$  by a running one, which could be stronger and thus increase the value of the dynamical mass. But it is doubtful that such improved analyses, which at any rate have to be performed for completeness, will yield phenomenologically acceptable values for the electron mass. The only case where such analysis works, in the sense of enhancing significantly dynamical masses, compared to the ladder approximation, is the magnetic catalysis [18] in the presence of an external magnetic field, which, as mentioned already, is another example of explicit Lorentz violation due to the direction of the field. However, the reason why in such a case the dynamical mass generation is not so suppressed in the improved approximation is the dimensional reduction to effectively two dimensions induced by the magnetic field. There is no such a reduction in our LV case.

Nevertheless, the use of Lorentz-violating theories as seeds for dynamical mass generation is interesting, and may lead to phenomenologically realistic situations if embedded in more complicated theoretical frameworks, like those involving, e.g., hidden sectors in string or other higher-dimensional extensions of the standard model *à la* Randall-Sundrum scenarios [21], as we now proceed to discuss.

#### IV. GEOMETRICAL ENHANCEMENT OF DYNAMICAL MASS

In this section we shall present a geometric mechanism for the enhancement of the dynamical mass (3.6), by embedding the model in higher-dimensional setups involving brane worlds in warped bulk geometries, following the spirit of [21]. In [21] a large mass hierarchy between Planck masses and TeV scales comes from brane world scenarios in which our world is a negative tension brane, lying at a distance  $r_c\pi$  in a five-dimensional bulk from a hidden brane. The five-dimensional metrics, which are solutions of Einstein's equations in a (necessarily) anti-de-Sitter bulk space, have warped factors

$$ds^2 = e^{-\sigma(z)}\eta_{\mu\nu}dx^\mu dx^\nu + dz^2, \quad (4.1)$$

where  $z$  is the bulk (fifth) dimension, and  $x^\mu$  are coordinates in our four-dimensional space time.

Because of the warp factor  $e^{-\sigma(z)}$  in the metric, fields on our world of mass  $m_0$ , with canonically normalized kinetic terms, will have physical masses of the form

$$m_{\text{phys}} = m_0 e^{-\sigma(z_i)}, \quad (4.2)$$

where  $z_i$  denotes the location of our brane world along the bulk dimension. If, as natural,  $m_0$  is of the order of the (reduced) four-dimensional Planck mass  $2 \times 10^{18}$  GeV, then a large hierarchy between the Planck scale and the particle masses  $m_{\text{phys}}$  in our world may be provided, depending on the size of the exponent  $\sigma(z_i)$  of the warp factor.

In the Randall-Sundrum (RS) scenario [21], involving just two branes, with opposite tensions, our world is identified with the negative tension one, located at  $z_i = r_c\pi$  and  $\sigma = -k|z|$ ,  $k > 0$ . In this scenario, the hierarchy is resolved for  $kr_c\pi \approx \mathcal{O}(50)$ . Notice that in RS the exponent  $\sigma(z_i)$  is positive, and thus the hierarchy factor only decreases the masses as compared to  $m_0$ .

In [22], a more complicated scenario, involving many brane worlds, and higher-order curvature terms in the bulk (of Gauss-Bonnet type) has revealed the possibility of exact solutions to the respective bulk Einstein equations, which however allowed for our world to be identified with a positive tension brane, and moreover, and more importantly for our purposes here, for the exponent of the warp factor to take *negative* values,  $\sigma(z_i) < 0$ , thus introducing an inverse RS hierarchy.

The bulk geometry is characterized by a gravitational action which includes Gauss-Bonnet higher-derivative corrections:

$$S = S_5 + S_4, \quad (4.3)$$

where

$$S_5 = \int d^5x \sqrt{-g} \left[ -R - \frac{4}{3}(\nabla_\mu \Phi)^2 + f(\Phi)(R^2 - 4R^2_{\mu\nu} + R^2_{\mu\nu\rho\sigma}) + \xi(z)e^{\zeta\Phi} + c_2 f(\Phi)(\nabla_\mu \Phi)^4 + \dots \right] \quad (4.4)$$

with  $\Phi(z)$  the dilaton field, and the  $\dots$  denoting other types of contraction of the four-derivative dilaton terms which will not be of interest to us here, given that by appropriate field redefinitions, which leave the (perturbative) string amplitudes invariant, one can always cast such terms in one of the above forms.

The above action is compatible with closed string amplitude computations in the five-dimensional space times. Such a compatibility is necessary in view of the assumption of closed string propagation in the bulk. In the stringy case, one has [22]  $f(\Phi) = \lambda e^{\theta\Phi}$ ,  $\lambda = \alpha'/8g_s^2 > 0$ ,  $c_2 = \frac{16}{9} \frac{D-4}{D-2}$ ,  $\zeta = -\theta = \frac{4}{\sqrt{3(D-2)}}$ , where  $\alpha' = 1/M_s^2$  is the Regge slope,  $M_s$  is the string mass scale,  $g_s$  is the string coupling, and  $D(= 5)$  is the number of space-time dimensions.

The four-dimensional part  $S_4$  of the action (4.3) is defined as

$$S_4 = \sum_i \int d^4x \sqrt{-g_{(4)}} e^{\omega\Phi} \mathcal{V}(z_i), \quad (4.5)$$

where

$$g_{(4)}^{\mu\nu} = \begin{cases} g^{\mu\nu}, & \mu, \nu < 5 \\ 0, & \text{otherwise} \end{cases} \quad (4.6)$$

and the sum over  $i$  extends over D-brane walls located at  $z = z_i$  along the fifth dimension. Embedding the model (2.1) in such scenarios, identifies (2.1) with the effective four-dimensional field theory Lagrangian that describes the low-energy dynamics of open strings (representing the photons) with their ends attached to one of the branes that corresponds to the physical world.

On assuming warp five-dimensional geometries, of the form (4.1), the analysis of [22], where we refer the interested reader for details, has demonstrated that for the actions (4.3), the multibrane situation depicted in Fig. 1, which involves bulk singularities restricting dynamically the available bulk space, is an exact solution. In the bulk regions adjacent to the bulk singularities ( $z \sim z_s$ ), one has a logarithmic solution for the warp factor,

$$\sigma(z) = \sigma_2 + \sigma_1 \log|z - z_s|,$$

while in the other segments of the bulk space, induced by the various brane worlds, one has linear solutions,

$$\sigma(z) = \sigma_0 + kz,$$

with the parameter  $k$  alternating sign between the various segments of the bulk space, as indicated in Fig. 1. Matching the various solutions on each brane yields a consistent scenario.

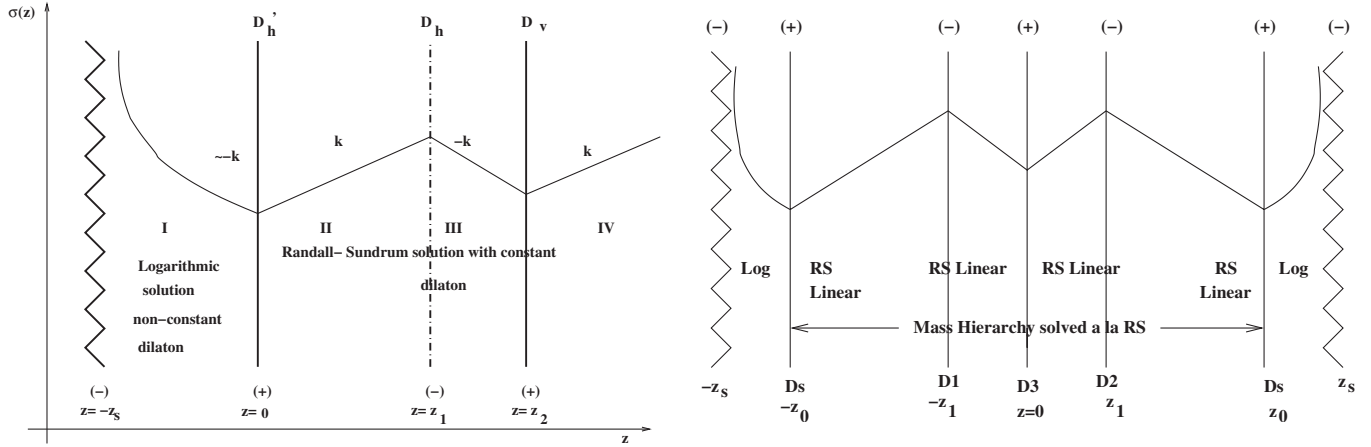


FIG. 1. Left: A multibrane scenario, in which our world is represented as a positive tension brane (at  $z = z_2$ ), having from the left branes with alternating-sign tensions, which shield a bulk naked singularity [which may be thought of as a limiting (singular) case of a negative tension brane]. To the right of the brane world, on the other hand, the bulk dimension extends to infinity. Right: A multibrane scenario, in which our world is represented as a positive tension brane (at  $z = 0$ ), surrounded by branes with alternating-sign tensions that shield two symmetrically positioned bulk naked singularities.

The detailed analysis of [22] has proven the existence of solutions of the low-energy gravitational bulk equations which imply mass hierarchies of the form

$$m_{\text{phys}} = m_0 e^{k(2z_1 - z_2)}, \quad k = \sqrt{\frac{2}{3}} g_s M_s > 0, \quad (4.7)$$

where  $z_2$  is the location of our world ( $z_2 = 0$  in the symmetric scenario of the right picture of Fig. 1). The bulk string scale,  $M_s$ , is an arbitrary scale in the modern version of string theory and thus can be very different from the four-dimensional Planck scale  $M_P$ . In this scenario, though,  $M_P$  is not lying far from  $M_s$ , e.g. it can be of order [22]  $M_P \sim M_s / \sqrt{g_s}$ . We stress again that, in these scenarios, our physical world is a *positive* tension brane, which makes physical sense.

It is clear that in such a situation, it is possible to have *inverse RS hierarchies*, by arranging appropriately the positions of the various branes. For instance, we may have  $z_2 < 2z_1$ . One may then identify  $m_0$  in (4.7) with our dynamically generated gauge-invariant mass (3.6), in which case the physical mass we would observe in our brane world would be

$$m_{\text{dyn}} = M \exp\left(-\frac{\pi}{2\alpha} + \sqrt{\frac{2}{3}} g_s M_s |z_2 - 2z_1|\right). \quad (4.8)$$

In this way, phenomenologically desired electron masses are obtained by arranging appropriately the distance  $|z_2 - 2z_1|$  in the arrangement of Fig. 1.

We finally note that, once we puncture the bulk space in such warped scenarios with D-particles, then the quantum-gravity D-foam scenario, discussed in Sec. V, may be in operation, in which the scale  $M$  is related to the string scale  $M_s$  and the foam fluctuation parameter via (5.23) below.

### V. LV QED AS AN EFFECTIVE THEORY OF A D-PARTICLE SPACE-TIME FOAM MODEL

In this section we consider it as useful, or at least intriguing, to attempt and discuss a possible microscopic origin of the model (2.1) in the context of a stringy space-time foam scenario [7–9]. We should stress that the material presented here may be omitted by those readers who are not interested in such explanations. Nevertheless, the fact that LV structures, like the ones appearing in (2.1), characterize parts of the effective action describing the interactions of a space-time foamy defect with photons, gives the model a rather different perspective: it provides, for the first time in our opinion, a concrete realization of the conjecture that space-time foam as a vacuum medium may be responsible for “slowing down” particles by giving them a mass. Caution, however, is needed here. The so-generated mass refers to the charged electrons here, the photon remaining massless, as it is revealed by the pole structure of the gauge propagator (3.1) in the model. However, this masslessness should be *disentangled* from an induced nontrivial *vacuum refractive index* in the model, which is associated with purely stringy effects [9], namely, time-space uncertainties, not captured by the local effective action.

Below we shall demonstrate that LV Lagrangians of the form (2.1) may arise as (parts of) the low-energy, weak-photon-field limits of an effective Born-Infeld Lagrangian describing the propagation of photons (represented as open strings) on a three spatial brane, punctured by a uniform background of stochastically fluctuating pointlike D0-brane defects. Such models have been termed D-particle foam [7–10] and may find a variety of applications, ranging from providing microscopic string-inspired situations with Lorentz-violating vacuum

refractive indices [9–11], to enhancing dark matter thermal relics, with interesting astro-particle phenomenology [12].

### A. Features of a D-foam model

The basic idea of the D-foam can be described as follows (see Fig. 2). A (possibly compactified from higher-dimensions) three-brane world is moving in a higher-dimensional bulk space, punctured by D-particles. Depending on the string theory considered, the latter can be either pointlike D0-branes (type IIA strings) [7–9] or D3-branes wrapped up around appropriate three cycles (type IIB strings [10]). As the brane world and the D-particles move in the bulk, they cross each other, and thus, for an observer on the D3-brane, the D-particle defects appear as flashing on and off *vacuum structures*. In these scenarios, ordinary matter and radiation are represented by open strings with their ends attached on the

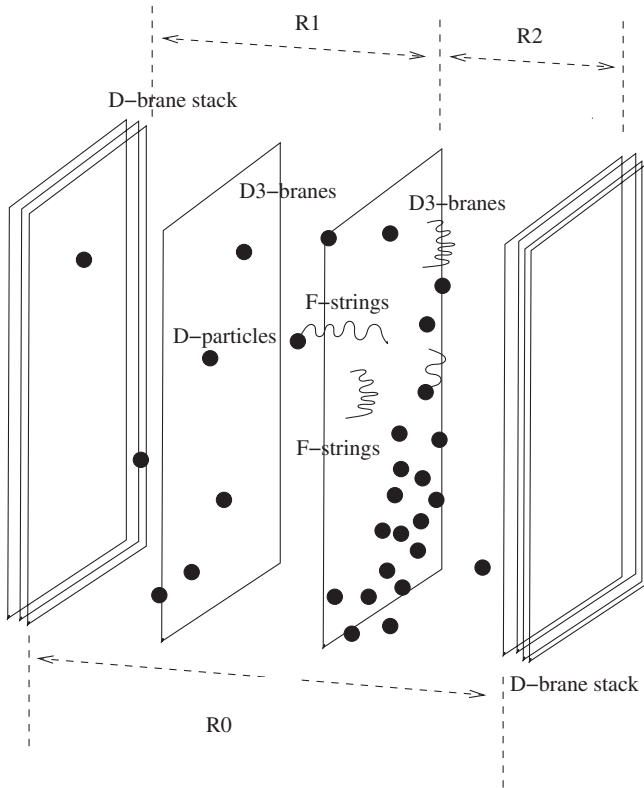


FIG. 2. Schematic representation of a generic D-particle space-time foam model. The model of Ref. [8], which acts as a prototype of a D-foam, involves two stacks of D8-branes, each stack being attached to an orientifold plane. Owing to their special reflective properties, the latter provide a natural compactification of the bulk dimension. The bulk is punctured by D0-branes (D-particles), which are allowed in the type IA string theory of [8]. The presence of a D-brane is essential due to gauge flux conservation, since an isolated D-particle cannot exist. Open strings live on the brane world, representing standard model matter and they can interact in a topologically nontrivial way with the D-particle defects in the foam.

D3-brane. Gravitational degrees of freedom propagate in the bulk.

There are nontrivial interactions of open strings with D-particles, provided there is *no electric charge* flux along the open-string excitations, that is, provided the pertinent excitations are electrically neutral.<sup>1</sup> This is because the electrically neutral D-particles can “cut” an open string, leading to the emission of intermediate strings stretched between the D-particles and the D3 brane [9,11]. If the open-string state carries electric flux, such a cutting procedure is not allowed, due to charge conservation.

As discussed in [11], the first-quantization picture for an open-string-D-particle interaction is provided by a world-sheet  $\sigma$ model with the following deformation:

$$\mathcal{V}_{\text{recoil velocity part}}^{\text{impulse}} = \frac{1}{2\pi\alpha'} \sum_{i=1}^D \int_{\partial D} d\tau u_i X^0 \Theta(X^0) \partial_n X^i, \quad (5.1)$$

where  $M_s$  is the string (mass) scale,  $g_s$  is the string coupling, and  $u_i$  is the recoil velocity of the D-particle, the latter assumed heavy (in fact  $u^i$  is the spatial part of the *four-velocity* of the D-particle, but for nonrelativistic, slow moving, heavy D-particles this is well approximated by the ordinary velocity, to leading order).  $D$  in the sum denotes the appropriate number of spatial target-space dimensions. For a recoiling D-particle confined on a D3 brane, as is our case here,  $D = 3$ . The operator  $\Theta_\varepsilon(X^0) = -i \int_{-\infty}^{\infty} \frac{d\omega}{\omega + i\varepsilon}$ ,  $\varepsilon \rightarrow 0^+$ , is a regularized Heaviside world-sheet operator.

There is a specific type of conformal algebra, termed logarithmic conformal algebra [23], that the recoil operators satisfy [13,24]. This algebra is the limiting case of world-sheet algebras that can still be classified by conformal blocks. The impulse operator  $\Theta(X^0)$  is regularized so that the logarithmic conformal field theory algebra is respected.<sup>2</sup> The conformal algebra is consistent with momentum conservation during recoil [13,24], which allows for the expression of the recoil velocity  $u_i$  in terms of momentum transfer during the scattering,

$$u_i = g_s \frac{p_1 - p_2}{M_s}, \quad (5.2)$$

with  $\frac{M_s}{g_s}$  being the D-particle “mass” and  $\Delta p \equiv p_1 - p_2$  the associated momentum transfer of a string state during its scattering with the D-particle.

<sup>1</sup>For type IIB strings, where the D-particles are not pointlike, one may have such interactions between electrically charged excitations and the D-particles, however the foam effects on such charged particles are suppressed [10] compared to those on neutral particles. In our discussion below, therefore, we shall not differentiate between these two cases.

<sup>2</sup>This can be done by using the world-sheet scale,  $\varepsilon^{-2} \equiv \ln(L/a)^2$ , with  $a$  an ultraviolet scale and  $L$  the world-sheet area, as a regulator [13,24]:  $\Theta_\varepsilon(X^0) = -i \int_{-\infty}^{\infty} \frac{d\omega}{\omega - i\varepsilon} e^{i\omega X^0}$ . The quantity  $\varepsilon \rightarrow 0^+$  at the end of the calculations.

We next note that one can write the boundary recoil/capture operator (5.1) as a total derivative over the bulk of the world sheet, by means of the two-dimensional version of Stokes theorem. Omitting from now on the explicit summation over repeated  $i$  indices, which is understood to be over the spatial indices of the D3-brane world, we write then

$$\begin{aligned} \mathcal{V}_{\text{recoil valocity part}}^{\text{impulse}} &= \frac{1}{2\pi\alpha'} \int_D d^2z \epsilon_{\alpha\beta} \partial^\beta ([u_i X^0] \Theta_\varepsilon(X^0) \partial^\alpha X^i) \\ &= \frac{1}{4\pi\alpha'} \int_D d^2z (2u_i) \epsilon_{\alpha\beta} \partial^\beta X^0 \\ &\quad \times [\Theta_\varepsilon(X^0) + X^0 \delta_\varepsilon(X^0)] \partial^\alpha X^i, \end{aligned} \quad (5.3)$$

where  $\delta_\varepsilon(X^0)$  is an  $\varepsilon$ -regularized  $\delta$  function.

For relatively large times after the after the moment of impulse,  $X^0 = 0$ , at which the open-string state splits into intermediate open-string ones, as a result of the topologically nontrivial interactions with the D-particle, which we assume for our phenomenological purposes in this work, the expression (5.3) is equivalent to a deformation describing an open string propagating in an antisymmetric  $B_{\mu\nu}$  background ( $B$  field) corresponding to a constant external ‘‘electric’’ field in target space:

$$T^{-1}B_{i0} = -T^{-1}B_{0i} = u_i = \frac{g_s \Delta p_i}{M_s}, \quad T = \frac{1}{2\pi\alpha'}, \quad (5.4)$$

where  $T$  denotes the (open) string tension, 0 is a temporal index, and  $i$  is a spatial index. The reader should notice here the phase-space dependence of the background field, which resembles an electric field background but in momentum space [11], and therefore of Finsler type [25].

In the above analysis we have ignored a possible angular momentum operator, which also arises as a result of the nontrivial scattering of photons with the D-particle defects. At a  $\sigma$ -model level, the latter is also described by a logarithmic conformal algebra deformation, which for a three-brane, which we restrict our attention to here, assumes the form

$$\begin{aligned} \mathcal{V}_{\text{ang mom D-part}}^{\text{impulse}} &= T^{-1} \int_{\partial\Sigma} u^i \epsilon_{ijk} X^j \Theta_\varepsilon(X^0) \partial_n X^k \\ &= T^{-1} \int_\Sigma \varepsilon^{\alpha\beta} (u^i \epsilon_{ijk} \Theta_\varepsilon(X^0) \partial_\alpha X^j \partial_\beta X^k \\ &\quad + u^i \epsilon_{ijk} X^j \delta_\varepsilon(X^0) \partial_\alpha X^0 \partial_\beta X^k), \end{aligned} \quad (5.5)$$

where we have again applied the two-dimensional Stokes theorem ( $\alpha, \beta = 1, 2$  are world-sheet indices, and  $\varepsilon^{\alpha\beta}$  is the world-sheet Levi-Civita tensor). On the other hand,  $\epsilon_{ijk}$  is the antisymmetric symbol in the three spatial dimensions of the brane world. The logarithmic conformal properties [24] of the deformation arise from the  $X^J$  parts. For relatively large times  $X^0 > 0$  after the impulse, we consider here, we may ignore the  $\delta$ -function terms, and in this case the effects of the angular momentum deformation in target space are equivalent to the open string propagating in an

antisymmetric tensor ‘‘magnetic-field’’ type background with spatial components given by

$$T^{-1}B_{ij} = -T^{-1}B_{ji} = \epsilon_{ijk} u^k. \quad (5.6)$$

This should be combined with (5.4) in order to provide a complete description of the *average* interactions of the photons with the D-foam, in a first-quantized version.

## B. Finsler-Born-Infeld (FBI) effective actions

The effective target-space action on the D3-brane world, where the D-particle meets the open-string photon state, is described by the following Born-Infeld Lagrangian [26,27]:

$$S_{\text{BI}} = \frac{T^2}{g_s} \int d^4x \sqrt{\det(g + T^{-1}(B + F))}, \quad (5.7)$$

where  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$  is the Maxwell tensor of the photon field  $A_\mu$ .

In fact, the field theory of photons in the presence of the electric field type background  $B_{0i}$  implies a time-space noncommutativity for the model [11], with noncommutativity parameter  $u_i$  which is of Finsler type [25], in the sense that it is proportional to the photon momentum transfer (5.4). On the other hand, the presence of magnetic-field type backgrounds (5.6) also implies Finsler-type spatial noncommutativity among  $X^j$  target-space coordinates.

As a result of this, it is not in general possible to write down a local effective action for the description of the photon-D-particle interactions. The Lagrangian (5.7) depends on both space-time coordinates and the *momentum transfer*, which cannot be expressed as an ordinary local term in an effective action framework. Nevertheless, as we shall argue now, there is an approximation, namely the *stochastic D-foam* background, for which construction of a low-energy *local* effective action becomes possible and, in fact, for weak photon fields, and under some other special circumstances to be described below, assumes the Lorentz-violating form (2.1).

To this end, we first remark that, in the phase space of a D3-brane world, the function  $u_i$ , [cf. (5.4)], involving a momentum transfer,  $\Delta p_i$ , can be *modeled* by a local operator using the following parametrization [28]:

$$u_i = g_s \frac{\Delta p_i}{M_s} = \frac{g_s}{M_s} r_i p_i, \quad \text{no sum over } i = 1, 2, 3, \quad (5.8)$$

where the (dimensionless) variables  $r_i$ ,  $i = 1, 2, 3$  appearing above, are related to the fraction of momentum that is transferred at a collision with a D-particle in each spatial direction  $i$ . In the *stochastic foam approximation* [28], these parameters are taken as Gaussian normal random variables with a range  $-\infty$  to  $+\infty$  and defining moments

$$\langle r_i \rangle = 0, \quad (5.9)$$

$$\langle r_i r_j \rangle = 0, \quad \text{if } i \neq j \quad (5.10)$$



and

$$\sigma_i^2 = \langle r_i^2 \rangle - \langle r_i \rangle^2 = \langle r_i^2 \rangle \neq 0. \quad (5.11)$$

An isotropic foam situation, which we consider here, would require  $r_i = r$ , for all  $i = 1, 2, 3$ . In such a case the variances

$$\langle (r_i)^2 \rangle = \sigma^2, \quad i = 1, 2, 3, \quad (5.12)$$

are equal along all spatial directions.

However, for fully rotational invariance in three space of the Born-Infeld action (5.7), involving interactions between the velocity fields  $u_i$  and the photon Maxwell tensor  $F^{\mu\nu}$ , one more requirement is necessary. When considering the application of the above averages to the recoil velocities, the latter must assume the form

$$\langle u_i \rangle = 0, \quad \langle u_i u_j \rangle = \delta_{ij} \frac{\sigma^2 g_s^2}{M_s^2} p^k p_k. \quad (5.13)$$

The averages  $\langle \dots \rangle$  denote *both* statistical averages, over populations of D-particle defects in the foam (c.f. Fig. 2), as well as target-space quantum fluctuations. The latter can be induced by considering appropriate summation over world-sheet genera [13]. This is quite important for our purposes and we shall come back to this issue and its implications later.

For the moment, we remark that the stochasticity conditions (5.9) imply *restoration* of Lorentz invariance as an average phenomenon, with nontrivial fluctuations expressed in the isotropic and three-space rotationally invariant case by (5.10), (5.12), and (5.13). Hence, although at individual scatterings of photons with D-particles, Lorentz invariance (and isotropy of space) will be lost locally, due to the presence of the recoil velocity of the D-particle,  $u_i$ , the isotropic foam washes out on average such violations and isotropy and rotational invariance are restored.

Let us now go back to the effective action (5.7). On defining the generalized field  $\mathcal{F}_{\mu\nu} \equiv F_{\mu\nu} + B_{\mu\nu}$ , we make use of the fact [29] that in four space-time dimensions (assumed Minkowski flat for concreteness from now on), the determinant  $\det_4(\eta + T^{-1}F)$  has special properties, that allow the following representation of the Born-Infeld action (we work from now on in units where the string tension is  $T = 1$ ):<sup>3</sup>

<sup>3</sup>The reader is reminded at this stage that the indices are raised and lowered with the background metric  $g$ , which in our case, has been assumed to be Minkowski flat, for brevity and definiteness. In a general situation, where the metric  $g$  is not trivial, the pure foam contributions, proportional to the various powers of  $u_i^2$  contribute to (dark) vacuum energy [8,9] and can be constrained by cosmological considerations, for instance. We ignore such terms for our discussion here.

$$\begin{aligned} S_{\text{BI}} &= \frac{1}{g_s} \int d^4x (I_2 + I_4 (1 + \mathcal{O}(\mathcal{F}^2))) + \text{const} \\ I_2 &= \frac{1}{4} \mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu}, \\ I_4 &= -\frac{1}{8} \left[ \mathcal{F}_{\mu\nu} \mathcal{F}^{\nu\rho} \mathcal{F}_{\rho\lambda} \mathcal{F}^{\lambda\mu} - \frac{1}{4} (\mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu})^2 \right]. \end{aligned} \quad (5.14)$$

In the weak-photon-field approximation, of interest to us here, we shall ignore terms of order higher than quadratic in the photon field and the (small) recoil velocity  $u_i$  field. This is a consistent approximation for relatively heavy D-particles, whose recoil is suppressed by their mass. We also take a quantum average over stochastic fluctuations of the  $B$ -field, using (5.13), and keep terms quadratic in photon ( $A$ ) or (averaged) recoil ( $\langle u^2 \rangle$ ) fields, including mixed terms of order  $A^2 \langle u^2 \rangle$ .

### C. Target-space quantization proposal for the FBI action and minimal LV QED

There is an important and novel feature associated with the proper *quantization* in target space of the Finsler background  $B_{0i}$  (5.4), which is provided by summation over world-sheet genera. As discussed in detail in Ref. [13], making proper use of the (logarithmic) conformal field theory properties of the D-particle recoil vertex operator in a  $\sigma$ -model approach [24], the summation over world-sheet topologies results in quantum uncertainty relations involving the fluctuations of the recoil velocities and of the collective coordinates describing the initial position of the D-particle. Such uncertainties have been shown to correspond to those induced by canonically quantized collective D-particle momentum and position operators in target space. In this way, it has been argued in [13] that summation over world-sheet genera converts, via (5.8), the recoil velocity fields  $u_i$  into quantum-fluctuating momentum operators in target space.<sup>4</sup>

More specifically, one has the correspondence

$$\begin{aligned} B_{i0} = u_i &\Rightarrow \hat{B}_{i0} = \hat{u}_i = -ig_s \frac{r_i}{M_s} \hbar \frac{\partial}{\partial X^i} \equiv -ig_s \frac{r_i}{M_s} \hbar \nabla_i, \\ &\text{no sum over } i = 1, 2, 3. \\ B_{ij} = \epsilon_{ijk} u^k &\Rightarrow \hat{B}_{ij} = \epsilon_{ijk} \hat{u}^k. \end{aligned} \quad (5.15)$$

In view of this, the noncommutativity uncertainty relations  $[X^i, t] \sim u^i$  of the classical recoil background [11] become now complicated quantum operator relations. *En passant*, we also note that the correspondence (5.15) can also lead to

<sup>4</sup>Technically speaking, we should mention here that the effects of the summation over genera on the fluctuations of the background fields  $u_i$  can be expressed in closed form only in the bosonic string case. In the world-sheet supersymmetric case [30], such closed expressions were not possible. Nevertheless, this does not invalidate our arguments on the correspondence principle (5.15), which we conjecture to characterize all types of string models.

master equations for the study of the induced decoherence of quantum matter propagating in the stochastic D-foam quantum-fluctuating backgrounds [28].

We now wish to make a technical but important observation regarding the correspondence (5.15), which we shall make use of in our analysis below. As already mentioned, we have inferred this correspondence by studying the corresponding  $\sigma$ -model approach to recoil [24], in which, in order to guarantee the convergence of the world-sheet path integrals, one is forced to use time fields  $X^0$  with *Euclidean* signature. In this sense, the correspondence (5.15), should be augmented to include the following:

$$B^{i0} \Rightarrow \hat{B}^{i0} = g_E^{00} g_E^{ik} \hat{B}_{k0} = +\hat{B}_{i0}, \quad (5.16)$$

where the subscript  $E$  indicate *Euclidean signature*. Only at the very end of the computations, after we replace the background  $B$ -field by appropriate operators, and only then, we revert back to Minkowski signature, by means of *analytic continuation*.

In this picture, where the quantum-fluctuating aspects of the recoil operators are taken care of by means of the correspondence (5.15) and (5.16), the statistical aspects of the foam are implemented by averaging the momentum transfer statistical variable  $r$  ( $\langle \cdot \cdot \cdot \rangle$ ) over populations of quantum-fluctuating D-particles, using the relations (5.13) for the case of isotropic foam. In fact, we now have the correspondence:

$$\begin{aligned} \langle u_i \rangle &= 0, \\ \langle u_i u_j \rangle &= \delta_{ij} \frac{\sigma^2 g_s^2}{M_s^2} p^k p_k \Rightarrow -\hbar^2 \frac{\sigma^2 g_s^2}{M_s^2} \delta_{ij} \Delta, \quad \Delta \equiv \vec{\nabla} \cdot \vec{\nabla}. \end{aligned} \quad (5.17)$$

This is our *prescription*, which, as we shall show below, maps the effective action (5.14) to particularly simple local effective actions containing terms of the form (2.1).

Indeed, on implementing (5.15) and (5.16), the relevant photon-dependent terms of the Finsler-Born-Infeld (FBI) action, which we restrict our attention upon in this work, assume the form

$$\begin{aligned} S_{\text{BI}} \ni & \frac{1}{g_s} \int d^4x: \left[ \frac{1}{4} F_{\mu\nu} \left( 1 + \frac{b}{16} \hat{B}_{\alpha\beta} \hat{B}^{\alpha\beta} \right) F^{\mu\nu} \right. \\ & \left. + \frac{a}{64} F_{\mu\nu} \hat{B}^{\mu\nu} \hat{B}_{\alpha\beta} F^{\alpha\beta} - \frac{1}{8} F_{\mu\nu} \hat{B}^{\nu\rho} \hat{B}_{\rho\beta} F^{\beta\mu} \right]: + \dots \end{aligned} \quad (5.18)$$

The  $\dots$  represent higher-order terms in the fields  $B$  and  $F$  and the symbol  $:\dots:$  denotes *appropriate quantum operator ordering* and the ordering constants  $a, b$  are such that

$$a + b = 2. \quad (5.19)$$

Above we have ignored terms involving odd powers of the operators  $\hat{B}_{\mu\nu}$  since such terms vanish in our stochastic Gaussian background (5.17). Also we dropped terms

involving the operators  $\hat{B}_{0i}$  or  $\hat{B}_{ij}$  lying on the far left-hand side of the integrand in (5.18), as these correspond to total spatial-derivative terms and, hence, do not contribute, upon the assumption that the fields decay away at spatial infinity on the brane world.

Upon recalling that, when making the correspondence (5.17), one uses as an intermediate step target-space times with Euclidean signature, i.e. (5.16), we observe, after some straightforward algebra, that (5.18) reduces to [using (5.19)]

$$\begin{aligned} S_{\text{BI}} \ni & \frac{1}{g_s} \int d^4x \left[ \frac{1}{4} F_{\mu\nu} \left( 1 + \frac{1}{4} \left( 1 - \frac{b}{2} \right) \frac{g_s^2 \sigma^2}{M_s^2} \Delta \right) F^{\mu\nu} \right] \\ & + \dots, \end{aligned} \quad (5.20)$$

where the  $\dots$  denote terms higher order in derivatives and the Maxwell tensor  $F_{\mu\nu}$ , and  $\Delta$  is the 3-space Laplacian,  $\Delta \equiv \nabla_i \nabla^i = \vec{\nabla} \cdot \vec{\nabla}$ .

In the classical limit the FBI action is recovered trivially. The quantum ordering ambiguities are an issue here. Usually, quantum ordering of operators is associated with Hermiticity of the effective Lagrangian, which is not an issue here, due to the foam-background stochasticity (5.17). The physics which selects the correct quantum ordering in this picture is encoded in the full (still elusive) underlying theory of quantum gravity in this context, and at this stage the so-constructed effective action should be considered as somewhat phenomenological. For instance, the choice  $b = 2$  would eliminate any LV terms, however, from the point of view of avoiding infrared (IR) divergences and the associated instabilities, the solution that leads to dynamical mass generation is preferred. Adopting the point of view that the full quantum-gravity theory should act as an IR regulator, one then is forced to select an ordering with  $b \neq 2$ .

Moreover, we observe that any ordering with  $b < 2$  would lead to terms which would affect the three-space pole structure of the photon propagator. Indeed, the photon propagator stemming from (5.20), assumes the form (up to gauge-fixing terms that we do not write explicitly here): the bare photon propagator is given by [14]

$$\begin{aligned} D_{\mu\nu}^{\text{bare}}(\omega, \vec{p}) &= -\frac{i}{1 - p^2/M^2} \left( \frac{\eta_{\mu\nu}}{-\omega^2 + p^2} + \text{gauge fixing} \right), \\ M^2 &\equiv \frac{4M_s^2}{(1 - b/2)g_s^2\sigma^2} > 0. \end{aligned} \quad (5.21)$$

Thus, in Fourier space, this would imply that the effective action would no longer be unitary for momenta above the scale  $M$ , since the overall signature of the photon propagator would change. This would be fine for a low-energy effective action in a classical background, and indeed this is exactly what happens [9,11] when the classical recoil velocity (5.2) exceeds the speed of light *in vacuo*. Such mass scales define the range of validity of the low-energy local effective action.

However, in our prescription we would like to go beyond such classical effective field theories. If we could find a quantum ordering which allows extension of the action beyond this cutoff then, in principle, we could describe some aspects of the foam on radiation for arbitrarily large momenta. This would provide a sort of partial ultraviolet (UV) completion of the low-energy theory (as far as dynamical mass generation is concerned—the reader’s attention is called at this stage to the fact that other aspects of the foam, such as vacuum refraction induced photon delays, cannot be described within the framework of local effective field theories [9,11], see below). Dynamical mass generation is, of course, an IR phenomenon and one would have thought that the detailed structure of the theory at the UV would not affect it. This is, for instance, what arguably may happen with the Landau pole of QED, whose presence may not affect dynamical mass generation (there is support towards this result from lattice calculations but, as far as we are aware of, no rigorous proof exists as yet [18,31]). However, this is not quite the case if an effective theory itself breaks down at a given momentum scale, in the sense, as is the case here, that the unitarity of the effective Lagrangian (sign of photon propagator terms) is lost above such scales. If one were to study dynamical generation in such cases, the momentum integrals in the SD equations had to be cut off from above at the specific scale  $M$ . This would be an unwelcome feature for a consistent SD treatment, such as the one in [14], which requires cancellation of potential UV divergences, and thus implicit extension of the model beyond any UV cutoff.

Fortunately in our model, such UV cutoffs can be avoided by a judicious choice of quantum ordering. A minimal class of such orderings, which respect the pole structure of the photon propagator (5.21) in three space, is the one in which the ordering parameter is in the range

$$b > 2. \tag{5.22}$$

After a formal analytic continuation back to Minkowski space-time, the corresponding actions (5.20) become *precisely* of the form (2.1) of Ref. [14] with the mass scale:

$$M = \frac{M_s}{g_s \sqrt{\tilde{\sigma}^2}}, \quad \tilde{\sigma}^2 \equiv \sigma^2 \frac{1}{4} \left| 1 - \frac{b}{2} \right|. \tag{5.23}$$

Any residual ordering ambiguity is thus absorbed in the fluctuations of the foam, which is thus a (small) phenomenological parameter in our first-quantized approach. It is hoped that when a full, second-quantized quantum-gravity model for D-foam fluctuations becomes available, such ambiguities will be removed.<sup>5</sup> In this sense the so-chosen

<sup>5</sup>We also note that for the unique (in this range of  $b$ ) value  $b = 10$ ,  $\tilde{\sigma}^2 = \sigma^2$  and thus the scale  $M$  of the LV terms in the action (5.20) becomes identical to the one at which the foam-averaged recoil velocity equals the speed of light *in vacuo* [9,11]. This choice of ordering then allows for a unique quantum-gravity scale to enter in the model in various forms.

quantum ordering leads to an effective action that is characterized by *maximal suppression* of the LV effects, and through mass generation, cures IR instabilities.

We next come to the fermion sector. In view of the properties of the D-foam, according to which the latter is *transparent* to *charged fermion fields*, due to charge conservation, the lowest order (in weak field) effective action term in the fermion sector will be given by the *ordinary* QED fermion-photon coupling. There will be no tree-level coupling of fermions to the foam recoil velocity field  $u_i$ :

$$S_\psi = \int d^4x \bar{\psi} \gamma^\mu i D_\mu \psi, \quad D_\mu = \partial_\mu + iA_\mu. \tag{5.24}$$

Thus, the *renormalizability* argument of [14] on the absence of higher-derivative noncovariant fermion-gauge-boson couplings (2.2), is *replaced* here by the *transparency of the D-foam* to charged fermions, as a consequence of charge conservation.<sup>6</sup>

In view of the dynamical mass generation arguments, presented previously, one expects the D-foam models to lead to masses for fermions of the type (2.3) [or (3.6)], with the scale  $M$  being replaced by (5.23). Since, in the context of our string-inspired foam and the Born-Infeld action, the gauge coupling is directly related to the string coupling  $\sqrt{g_s}$ , the dynamical mass (2.3) [or (3.6)] in this case is *nonanalytic* in *both* the (weak) string coupling and the (small) D-foam fluctuation parameter  $\tilde{\sigma}^2$ .

The effective action (5.20) and (5.24) describes part of the effects of foam on radiation and charged matter. However, as already discussed in the relevant literature [9,11], the induced vacuum refraction cannot be captured solely by this local effective action. The *causal* time delays of the reemitted photons after their topologically nontrivial interactions with the D-foam, which scale linearly with the photon energy, and are thus suppressed only by a single power of the string mass scale, are purely stringy effects, associated with time-space uncertainties [9–11,27] as a result of intermediate string (and hence nonlocal) states, stretched between the D-particles and the D3 brane universes. On the other hand, the modifications due to the foam in the local string effective action (5.20) are quadratically suppressed by the string scale, (5.23), as a result of the stochasticity assumption (5.17).

Finally, before closing this section, we mention that by embedding the D-foam model into the multibrane world scenarios of Sec. IV, we may enhance the dynamical mass generation to phenomenologically acceptable values, via the (inverse) Randall-Sundrum hierarchies (4.7) and (4.8).

<sup>6</sup>This argument is strictly speaking valid only for type IIA string theory D-foam, in which the D-particles are pointlike. For type IIB string theory D-foam models, on the other hand, D-particles are compactified D3-branes, and as such there are nontrivial, but much more *suppressed* fermion-foam couplings [10].

## VI. CONCLUSIONS AND OUTLOOK

In this note I have elaborated on a recently proposed model [14] of quantum electrodynamics with minimal Lorentz violation, associated with higher-order spatial-derivative terms in the Lagrangian, suppressed by a scale  $M$ , which lead to dynamical mass generation for the charged fermions. I have proposed a way of extracting the physical part of the fermion mass function, which is independent of the gauge-fixing parameter. My arguments were based on the so-called pinch technique of particle physics. According to this approach, only a certain class of Feynman diagrams can be resummed and play a role in the evaluation of the final physical quantities, such as the observable value of the dynamical fermion mass. The upshot of the analysis was that the physical value of the dynamically generated mass corresponds to the one derived in the Feynman gauge,  $\xi = 1$ . For the problem at hand, this, unfortunately, yields phenomenologically unrealistic small masses for the fermions.

However, if the model is embedded in a multibrane world scenario, with five-dimensional warp bulk geometries, then an enhancement mechanism is possible, through the dependence of the masses on the nontrivial warp factor in the observable (positive tension) brane world. The enhancement presupposes a reversed Randall-Sundrum hierarchy, which is possible in five-dimensional models with higher curvature gravitational corrections, of Gauss-Bonnet type (the latter is consistent with the effective target-space-time action corresponding to string theory scattering amplitudes in the bulk space).

I have also discussed a possible microscopic origin of these LV terms in the context of stringy space-time foam models, in which a brane world, over which photons propagate, is punctured by D-particle defects, with which the photons interact nontrivially. Electric charge conservation arguments necessitate that tree-level couplings between the foam and the fermions vanish, thereby making the effects of the D-particle quantum fluctuations on the fermions felt only through their coupling with the gauge (photon) fields. On the other hand, the interactions of the photons with the D-particles can be partly summarized by means of a Born-Infeld type effective Lagrangian. The latter involves, in addition to the Maxwell field strength of the photon field, also LV B-type fields, representing the recoil velocity fields of the D-particle defect during its scattering with the photon. The B-fields become quantum derivative momentum operators upon summing up world-sheet genera, and thus a true quantum D-foam situation is described by a complicated Finsler-Born-Infeld (FBI) Lagrangian, which is, in general, nonlocal.

However, for weak fields and foam, appropriate truncations and approximations can be made, with the implication that the relevant parts of the effective action, describing the interactions of photons and electrons with the foam, are provided by a (local) effective Lagrangian of

the form (2.1), with minimal LV. This exercise had provided the first concrete realization of the conjecture that the stochastic D-foam is responsible for generating masses for the fermions, by catalyzing, through its Lorentz-violating terms in the gauge sector, weak-gauge-coupling dynamical fermion mass generation. The reader should bear in mind, though, that this local effective action is *not* capable of capturing the vacuum refraction aspects of the D-foam. The latter are associated [9] with purely stringy (and thus nonlocal) time-space uncertainty effects, associated with intermediate string states of finite length, stretched between the D-particle and the brane world, during the quantum scattering of photons off the foamy defects.

Before closing, I would like to make some general remarks about effects of the space-time foam on neutral fermions, such as neutrinos. In our mechanism, dynamical mass is *catalyzed* by the LV terms in the gauge action, even for weak gauge couplings, and proceeds through the coupling of the gauge fields to the *charged* fermions. In this sense, our space-time foam mechanism does not apply to neutrino (assumed electrically neutral), at least to leading order. However, the neutrinos of the standard model eventually couple to photons via effective higher-loop vertices involving the weak-interaction gauge bosons. In this way, one obtains an “effective running neutrino charge” and an associated “radius” [32]. In this sense, the D-foam, through its LV coupling to photons, may also catalyze a (much more suppressed) neutrino mass, and in fact the electric neutrality of the neutrino, would provide a natural reason for the observed hierarchy of the neutrino masses, as compared to the electron mass. This is something one should look at more carefully. Moreover, since the stochastic effects of the D-foam will violate *CPT* [33], the so-generated neutrino masses may experience a medium-induced *CPT* violation [34]. It would be interesting also to see whether a mass hierarchy among different flavors can be generated this way, for example, by means of a judicious combination of LV terms in the gauge sector of the standard model and flavored neutrino interactions.

We hope to come back to such fundamental issues in future works. For the moment, we can only provide the reader with the above-described toy ideas and speculations on how Lorentz-violating effects in the standard model sector, that may have a microscopic origin in some foamy structures of quantum space time, may be responsible for the generation of fermion masses, through the coupling of the foam “medium” with the gauge fields that, in turn, interact with matter fermions. In the toy model we have discussed in this work, the photon gauge field remains massless, as evidenced by the structure of its propagator and one-loop vacuum polarization. In general, this may not be true in more complicated situations of anisotropic foam.

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**APPENDIX: THE PINCH TECHNIQUE (PT)  
AND GAUGE-FIXING-PARAMETER  
INDEPENDENT MASS**

**1. General remarks on PT**

It is well known that off-shell Green’s functions depend in general on the gauge-fixing procedure used to quantize the theory, and, in particular, on the gauge-fixing parameter (GFP) chosen within a given scheme. The fermion self-energy  $\Sigma(p)$ , for example, of interest to us here, is GFP-dependent already at the one-loop level. The dependence on the GFP is in general nontrivial and affects the properties of a given Green’s function. It is understood that, when forming observables, the gauge dependencies of the Green’s functions cancel among each other order by order in perturbation theory, due to powerful field-theoretical properties. Nevertheless, these dependencies pose a major difficulty when one attempts to extract physically meaningful information from individual Green’s functions. This is the case when studying the SD equations; this infinite system of coupled nonlinear integral equations for all Green’s functions of the theory is inherently non-perturbative and can accommodate phenomena such as chiral symmetry breaking and dynamical mass generation.

The main problem in this context is that the SD equations are built out of gauge-dependent Green’s functions; since the cancellation mechanism is very subtle, involving a delicate conspiracy of terms from *all orders*, a casual truncation often gives rise to gauge-dependent approximations for ostensibly gauge-independent quantities [35]. The study of SD equations, and especially of “gap equations,” has been particularly popular in many studies.

To address the problems of the gauge-dependence of off-shell Green’s functions a method known as the *pinch technique* (PT) has been introduced [15]. For a detailed and up-to-date review on this technique and its diverse applications, the reader is referred to [16]. For a comprehensive review, of relevance to our (one-loop approximate) discussion here, and its application to the infrared structure on low-dimensional gauge theories, with potential application to condensed-matter systems, such as high-temperature superconductors, see [20].

The PT is a diagrammatic method which exploits the underlying symmetries encoded in a *physical* amplitude such as an *S*-matrix element, or a Wilson loop, in order to construct effective Green’s functions with special properties. The aforementioned symmetries, even though they are always present, are usually concealed by the gauge-fixing procedure. The PT makes them manifest by means of a fixed algorithm, which does *not* depend on the gauge-fixing scheme one uses in order to quantize the theory, i.e.

regardless of the set of Feynman rules used when writing down the *S*-matrix element. The method exploits the elementary Ward identities triggered by the *longitudinal momenta* appearing inside Feynman diagrams in order to enforce massive cancellations. The realization of these cancellations mixes nontrivially contributions stemming from diagrams of different kinematic nature (propagators, vertices, boxes). Thus, a given physical amplitude is reorganized into subamplitudes, which have the same kinematic properties as conventional *n*-point functions and, in addition, are endowed with desirable physical properties, such as GFP independence.

Finally, the PT amounts to a nontrivial *reorganization of the perturbative expansion*. The role of the PT when dealing with SD equations is to (eventually) trade the conventional SD series for another, written in terms of the new, gauge-independent building blocks [15,36,37]. The upshot of this program would then be to truncate this new series, by keeping only a few terms in a “dressed-loop” expansion, and maintain exact gauge-invariance, while at the same time accommodating nonperturbative effects. We hasten to emphasize that the aforementioned program is still *not* complete; however, a great deal of important insight on the precise GFP-cancellation mechanism has been accumulated, and the field-theoretic properties of gauge-independent Green’s functions have been established in detail. The generalization of the PT to all orders for Lorentz-invariant quantum gauge field theories has been recently accomplished [38].

It would be interesting to discuss extensions of these ideas to LV gauge theories, either of Lifshitz type or in the framework of the standard model extension [1]. In the model (2.1), however, the validity of the PT is straightforward, due to the specific minimal form of LV, as we now proceed to discuss.

***One-loop application of PT to dynamical mass generation in LV QED***

In this part of the Appendix, we explain how the PT gives rise to effective, gauge-independent fermion self-energies at one-loop, in our Lorentz-violating QED model. The discussion parallels the cases of Lorentz-invariant theories of QED [20,38], due to the specific form of the (gauge fixed) photon propagator (3.1), which we may write compactly as

$$\Delta_{\mu\nu}(\ell, \xi) = (\Delta_{LV}(\vec{\ell} \cdot \vec{\ell})) \left( -\frac{i}{\ell^2} \left[ g_{\mu\nu} - (1 - \xi) \frac{\ell_\mu \ell_\nu}{\ell^2} \right] \right),$$

$$\Delta_{LV}(\vec{\ell} \cdot \vec{\ell}) = \frac{1}{1 + \frac{\vec{\ell} \cdot \vec{\ell}}{M^2}}. \tag{A1}$$

The off-shell propagator depends of course on the gauge-fixing parameter (GFP)  $\xi$ . It is important for our discussion below that the Lorentz-violating (LV) terms  $\Delta_{LV}(\vec{\ell} \cdot \vec{\ell})$ , factorize and do not introduce any extra pole singularities

in momentum space. The quantity  $\ell^\mu$  denotes the contravariant four momentum  $\ell^\mu = (\omega, \vec{p})$ .

The form of  $\Delta_{\mu\nu}(\ell, \xi)$  for the special choice  $\xi = 1$  (Feynman gauge) will be of central importance for the PT application in our discussion below. We denote it by  $\Delta_{\mu\nu}^F(\ell)$ , i.e.

$$\Delta_{\mu\nu}(\ell, 1) \equiv \Delta_{\mu\nu}^F(\ell) = \left( -\frac{i}{\ell^2} g_{\mu\nu} \right) (\Delta_{\text{LV}}(\vec{\ell} \cdot \vec{\ell})). \quad (\text{A2})$$

$\Delta_{\mu\nu}(\ell, \xi)$  and  $\Delta_{\mu\nu}^F(\ell)$  will be denoted graphically as in Fig. 3 [the LV factor  $(\Delta_{\text{LV}}(\vec{\ell} \cdot \vec{\ell}))$  is suppressed in the graphs, but its presence is always understood as a factor] [16,20].

For the diagrammatic proofs that will follow, in addition to the propagators  $\Delta_{\mu\nu}(\ell)$  and  $\Delta_{\mu\nu}^F(\ell)$  introduced above, we will need six auxiliary propagatorlike structures, as shown in Fig. 4 [again suppression of the LV factors  $(\Delta_{\text{LV}}(\vec{\ell} \cdot \vec{\ell}))$  is understood].

All of these six structures will arise from algebraic manipulations of the original  $\Delta_{\mu\nu}(\ell)$ . For example, in terms of the above notation we have the simple diagrammatic relation depicted in Fig. 5 (we will set  $\lambda \equiv \xi - 1$ ).

We next turn to the study of the gauge dependence of the fermion self-energy in our LV QED (but of course the discussion parallels that of electron in QED, quarks in QCD [16]). First we observe that, in view of the fact that in the theory (2.1) of [14] the fermion part of the action retains its form in Lorentz-invariant QED, the inverse electron propagator of order  $n$  in the perturbative expansion has the standard form:

$$S_n^{-1}(p, \xi) = \not{p} - m - \Sigma^{(n)}(p, \xi), \quad (\text{A3})$$

where  $\Sigma^{(n)}(p, \xi)$  is the  $n$ th order self-energy. Clearly,  $\Sigma^{(0)} = 0$ , and  $S_0^{-1}(p) = \not{p} - m$ . The quantity  $\Sigma^{(n)}(p, \xi)$  depends explicitly on  $\xi$  already for  $n = 1$ . In particular,

$$\begin{aligned} \Sigma^{(1)}(p, \xi) &= \int [d\ell] \gamma^\mu S_0(p + \ell) \gamma^\nu \Delta_{\mu\nu}(\ell, \xi) \\ &= \Sigma_F^{(1)}(p) + \lambda \Sigma_L^{(1)}(p) \end{aligned} \quad (\text{A4})$$

with  $[d\ell] \equiv g^2 \frac{d^4\ell}{(2\pi)^4}$  and

$$\Sigma_F^{(1)}(p) \equiv \Sigma^{(1)}(p, 1) = \int [d\ell] \gamma^\mu S_0(p + \ell) \gamma^\nu \Delta_{\mu\nu}^F(\ell) \quad (\text{A5})$$

and

$$\text{curly line} \equiv i\Delta_{\mu\nu}(\ell, \xi), \quad \text{wavy line} \equiv i\Delta_{\mu\nu}^F(\ell).$$

FIG. 3. Graphic representation (Feynman rules) for gauge boson (photon) propagator in the Lorentz-violating (LV) QED model. The curly line denotes the full propagator in the covariant gauge  $\xi$ , while the wavy line denotes the photon propagator in the Feynman gauge  $\xi = 1$ .

$$\begin{aligned} \text{curly line} &\equiv \frac{\ell_\mu \ell_\nu}{\ell^4} & \text{curly line with dot} &\equiv \frac{\ell_\mu \ell_\nu}{\ell^2} \\ \text{curly line with dot} &\equiv \frac{\ell_\mu}{\ell^4} & \text{curly line with dot and dot} &\equiv \frac{\ell_\mu}{\ell^2} \\ \text{curly line with dot and dot} &\equiv \frac{1}{\ell^4} & \text{curly line with dot and dot and dot} &\equiv \frac{1}{\ell^2} \end{aligned}$$

FIG. 4. Auxiliary propagatorlike structures, needed for diagrammatic proof of certain identities in the LV QED model.

$$\text{curly line} \equiv \text{wavy line} + \lambda \text{curly line with dot}$$

FIG. 5. Simple diagrammatic relation, which can be proven using the structures of Figs. 3 and 4.

$$\begin{aligned} \Sigma_L^{(1)}(p) &= -S_0^{-1}(p) \int \frac{[d\ell]_{\text{LV}}}{\ell^4} S_0(p + \ell) \gamma^\nu \ell_\nu \\ &= -\int \frac{[d\ell]_{\text{LV}}}{\ell^4} \ell_\mu \gamma^\mu S_0(p + \ell) S_0^{-1}(p) \\ &= S_0^{-1}(p) \int \frac{[d\ell]_{\text{LV}}}{\ell^4} S_0(p + \ell) S_0^{-1}(p) \\ &\quad - S_0^{-1}(p) \int \frac{[d\ell]_{\text{LV}}}{\ell^4}. \end{aligned} \quad (\text{A6})$$

Notice that in the above formulas we have absorbed the LV factors of the propagator (3.1) into the integration measure for brevity,

$$\text{curly line} \equiv \text{curly line with dot} - \text{curly line with dot and dot}$$

FIG. 6. The elementary Ward identity in Lorentz-violating QED.

$$\begin{aligned} \text{curly line} &\equiv \text{curly line with dot} - \lambda \text{curly line with dot and dot} \\ &= \text{curly line with dot} + \lambda \text{curly line with dot and dot} - \lambda \text{curly line with dot and dot and dot} \end{aligned}$$

FIG. 7. Combination of graphs used in the diagrammatic representation of Eqs. (A4)–(A6).

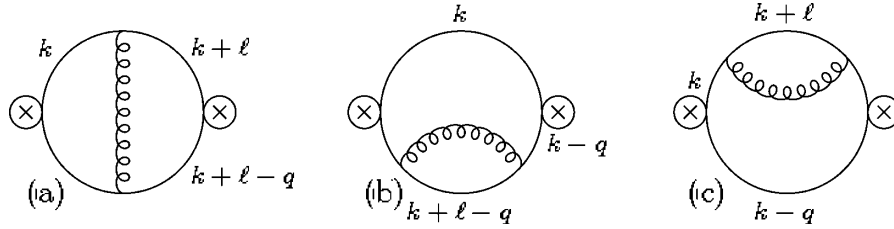


FIG. 8. One-loop diagram contributing to the Lorentz-violating QED fermion self-energy. The diagram is formally the same as in the Lorentz-invariant case due to the specific form of the action (2.1).

$$[d\ell]_L V \equiv g^2 \Delta_{LV}(\vec{\ell} \cdot \vec{\ell}) \frac{d^4 \ell}{(2\pi)^4}, \quad (A7)$$

$$\Delta_{LV}(\vec{\ell} \cdot \vec{\ell}) \equiv \frac{1}{1 + \frac{\vec{\ell} \cdot \vec{\ell}}{M^2}}.$$

The role of these LV terms as ultraviolet regulators (as  $p \rightarrow \infty$ ) in graphs with internal photon lines is clear. The ordinary Lorentz-invariant case is recovered in the limit  $M \rightarrow \infty$ . In the latter limit of course,  $[d\ell]_{M \rightarrow \infty} \rightarrow [d\ell]$  needs regularization (e.g. dimensional [20]). In our discussion  $M$  is fixed and finite and such a regularization is unnecessary for our discussion below. The quantity  $g$  is the gauge coupling ( $g \equiv e$  for ordinary QED). The subscripts “F” and “L” stand for “Feynman” and “Longitudinal,” respectively. Notice that  $\Sigma_L^{(1)}$  is proportional to  $S_0^{-1}(p)$  and thus vanishes “on-shell”. The most direct way to arrive at the results of Eq. (A6) is to employ the fundamental Ward identity (WI)

$$\ell = S_0^{-1}(p + \ell) - S_0^{-1}(p), \quad (A8)$$

which is valid in our case in view of the unaltered form of the fermion sector of the theory (2.1), as compared to the standard QED.

This WI is triggered every time the longitudinal momenta of  $\Delta_{\mu\nu}(\ell, \xi)$  gets contracted with the appropriate  $\gamma$  matrix appearing in the vertices. Diagrammatically, this elementary WI gets translated to the combination shown in Fig. 6 [again appropriate LV factors  $\Delta_{LV}(\vec{\ell} \cdot \vec{\ell})$  are understood].

Then, the diagrammatic representation of Eqs. (A4)–(A6) will be given by the combination of graphs depicted in Fig. 7.

This diagrammatic analysis is all one needs to prove the validity of the PT to our case. Indeed, when considering physical amplitudes, the characteristic structure of the longitudinal parts established above allows for their cancellation against identical contributions originating from diagrams which are kinematically different from fermion self-energies, such as vertex graphs or boxes, *without* the need for integration over the internal virtual momenta. This last property is important because in this way the original kinematical identity is guaranteed to be

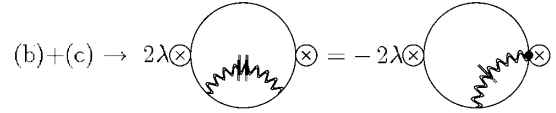


FIG. 9. A typical diagrammatic relation in (LV) QED, used to demonstrate the appearance of unphysical vertices in off-shell processes.

FIG. 10. Feynman rule for the unphysical effective vertex describing an interaction of the form  $\gamma\gamma\bar{\psi}\psi$ , where  $\gamma$  denotes a photon field and  $\mu$  is the index of the external current.

maintained; instead, loop integrations generally mix the various kinematics.<sup>7</sup> Diagrammatically, the action of the WI is very distinct: it always gives rise to *unphysical* effective vertices, i.e. vertices which do not appear in the original Lagrangian; all such vertices *cancel* in the full, gauge-invariant amplitude.

To actually pursue these special cancellations explicitly, one may choose among a variety of gauge-invariant quantities. For our interest, such an example is provided by the fermion-current correlation function  $I_{\mu\nu}$  defined as (in momentum space)

$$I_{\mu\nu}(q) = i \int d^4 x e^{iq \cdot x} \langle 0 | T [J_\mu(x) J_\nu(0)] | 0 \rangle$$

$$= (g_{\mu\nu} q^2 - q_\mu q_\nu) I(q^2) I_{LV}(\vec{q} \cdot \vec{q}), \quad (A9)$$

where the current  $J_\mu(x)$  is given by  $J_\mu(x) = :\bar{\psi}(x)\gamma_\mu\psi(x):$ .  $I_{\mu\nu}(q)$  coincides with the photon vacuum polarization of QED. The quantity  $q$  denotes four momentum,  $\vec{q}$  the spatial part, and the factoring out of the LV term  $I(\vec{q} \cdot \vec{q})$  is a consequence of the photon propagator (2.1), which maintains gauge invariance

<sup>7</sup>It is now clear to the reader that it is also this property that guarantees the straightforward application of the PT of Lorentz-invariant gauge theories to our LV QED (2.1), as a result of the factorization of the LV terms in the photon propagator.

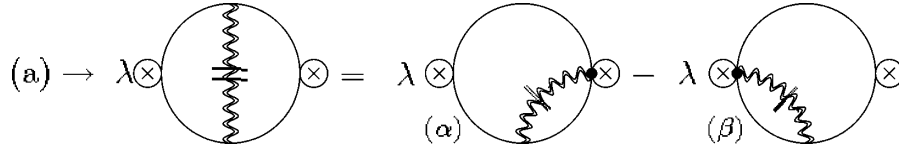


FIG. 11. Diagrammatic representation of the Ward identity used to demonstrate the cancellation of unphysical contributions inside the two-point fermion-current correlation function  $I_{\mu\nu}$ , Eq. (A9).

(and hence masslessness of the photon) in the presence of LV [14].

To see explicitly the mechanism enforcing these cancellations, we first consider the one-loop photonic corrections to the quantity  $I_{\mu\nu}$ . Clearly such corrections are GFP independent, since the current  $J_\mu(x)$  is invariant under both the  $U(1)$  gauge transformations. The relevant diagrams are those shown in Fig. 8.

To see the appearance of the unphysical vertices, we carry out the manipulations presented in Eqs. (A4)–(A6), or, equivalently, in the diagrammatic equation depicted in Fig. 7, this time embedded inside  $I_{\mu\nu}(q)$ . Thus, from diagrams (b) and (c) we arrive at the relation depicted in Fig. 9.

Again, the reader should understand the presence of LV factors  $\Delta_{LV}(\vec{k}, \vec{k})$  in the appropriate parts of the graphs, but they do not affect the general arguments on the GFP dependence cancellation.

We thus see that, since the action of the elementary WI of Eq. (A8) amounts to the cancellation of internal propagators, its diagrammatic consequence is that of introducing an unphysical effective vertex, describing an interaction of the form  $\gamma\gamma\bar{\psi}\psi$ , where  $\gamma$  denotes a photon field. This type of vertex may be depicted by means of a Feynman rule of the form shown in Fig. 10, where  $\mu$  is the index of the external current.

To see how the above unphysical contributions cancel inside  $I_{\mu\nu}$  we turn to diagram (a). The action of the WI

may be translated to the diagrammatic picture depicted in Fig. 11.

It is then elementary to establish that the two diagrams on the right-hand side of the above diagrammatic equation add up.

Summing up the two equations above, it is clear how the gauge-dependent part of the one-loop amplitude cancels completely. Having proved that the GFP-dependent contributions coming from the original graphs containing  $\Sigma^{(1)}(p, \xi)$ , i.e. Figs. 8(b) and 8(c), cancel exactly against equal but opposite *propagatorlike* contributions coming from Fig. 8(a), one is left with the “pure” GFP-independent one-loop fermion self-energy,  $\hat{\Sigma}^{(1)}(p)$ . Clearly, it coincides with the  $\Sigma_F^{(1)}(p)$  of Eq. (A5), i.e.

$$\hat{\Sigma}^{(1)}(p) = \Sigma_F^{(1)}(p). \quad (\text{A10})$$

This implies that in any physically measurable quantities, such as scattering amplitudes or dynamically generated masses, of interest at hand, the only relevant part of the self-energy that will contribute is the one (A10) associated with the Feynman gauge  $\xi = 1$ . With this in mind, the relevant computations in the SD analysis for dynamical mass generation, for instance, are simplified by going to this gauge and dropping the longitudinal parts of the photon propagator [16,20], for reasons explained above.

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