Cosmological rotating black holes in five-dimensional fake supergravity

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In recent series of papers, we found an arbitrary dimensional, time-evolving, and spatially inhomogeneous solution in Einstein-Maxwell-dilaton gravity with particular couplings. Similar to the supersymmetric case, the solution can be arbitrarily superposed in spite of nontrivial time-dependence, since the metric is specified by a set of harmonic functions. When each harmonic has a single point source at the center, the solution describes a spherically symmetric black hole with regular Killing horizons and the spacetime approaches asymptotically to the Friedmann-Lemaître-Robertson-Walker (FLRW) cosmology. We discuss in this paper that in 5 dimensions, this equilibrium condition traces back to the first-order ''Killing spinor'' equation in ''fake supergravity'' coupled to arbitrary $U(1)$ gauge fields and scalars. We present a five-dimensional, asymptotically FLRW, rotating black-hole solution admitting a nontrivial ''Killing spinor,'' which is a spinning generalization of our previous solution. We argue that the solution admits nondegenerate and rotating Killing horizons in contrast with the supersymmetric solutions. It is shown that the present pseudo-supersymmetric solution admits closed timelike curves around the central singularities. When only one harmonic is time-dependent, the solution oxidizes to 11 dimensions and realizes the dynamically intersecting $M2/M2/M2$-branes in a rotating Kasner universe. The Kaluza-Klein–type black holes are also discussed.

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I. INTRODUCTION

Supersymmetric solutions in supergravity have played an important role in the development of string theory and the anti–de Sitter (AdS)/conformal field theory (CFT) correspondence. A pioneer work in this direction was the great success of microscopic deviation of black-hole entropy from the viewpoint of intersecting D-branes. By virtue of the saturation of the Bogomol'nyi-Prasad-Sommerfield (BPS) bound, the supersymmetric solutions can provide an arena for exploring the nonperturbative limits of string theory. The BPS equality constrains the supersymmetry variation spinor to satisfy the first-order differential equation. Such a covariantly constant spinor is called a Killing spinor, which ensures that the energy is positively bounded by central charges, guaranteeing the stability of the theory. The relationship between vacuum stability and BPS states was suggested by Witten's positive energy theorem [1], and later validated firmly by [2,3].

From the standpoint of a pure gravitating object, black-hole solutions admitting a Killing spinor are sharply distinguished from non-BPS black-hole solutions. These BPS configurations are dynamically very simple. First of all, BPS black-hole solutions necessarily have zero Hawking temperature (the converse is not true), implying that the horizon is degenerate. Accordingly, they are free from thermal excitation. Such a nonbifurcating horizon universally admits a throat infinity and enhanced isometries of $SO(2,1)$ [4]. Secondly, most BPS solutions satisfy the ''no-force'' condition. For example, we are able to superpose the extreme Reissner-Nordström solutions at our disposal due to the delicate compensation between the gravitational attractive force and the electromagnetic repulsive force. The resulting multicenter metric, originally found by Majumdar and Papapetrou, maintains static equilibrium and describes collection of charged black holes [5]. This property can be ascribed to the complete linearization of field equations. Besides these, all the BPS black holes are known to be strictly stationary, viz., the ergoregion does not exist even if the black hole has nonvanishing angular momentum. Dynamically evolving states are not compatible with supersymmetry.

To what extent, then, do these known intuitive properties continue to hold? Motivated by this inquiry, it is important to explore general properties and classify BPS solutions. A first progress was made by Tod, who cataloged all the BPS solutions admitting nontrivial Killing spinors of four-dimensional $\mathcal{N} = 2$ supergravity [6], inspired by the early study of Gibbons and Hull [2]. Recently, Gauntlett *et al.* [7] were able to obtain general supersymmetric solutions in five-dimensional minimal supergravity, exploiting bilinears constructed from a Killing spinor. Since their technique has no restriction upon the spacetime dimensionality, [7] has sparked a considerable development in the classifications of supersymmetric solutions in various supergravities [8–11]. This formalism is useful for finding supersymmetric black holes [12,13] and black rings [14–16], and for proving the uniqueness theorem of certain black holes [17,18]. It turns out that all the BPS black holes

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fulfill the above-mentioned properties except for the equilibrium condition which is valid only in the ungauged case.

On the other hand, non-BPS black-hole solutions—especially the time-dependent black-hole solutions—have been much less understood. In this paper, we address some properties of cosmological black-hole solutions which have an interpretation as arising from the gauged supergravity with noncompact R-symmetry gauged. The simplest theory is the four-dimensional minimal de Sitter supergravity consisting of the graviton, the Maxwell fields, and a *positive* cosmological constant [19]. A time-dependent solution in this theory was found by Kastor and Traschen [20,21], which is the generalization of the Majumdar-Papapetrou solution in the de Sitter background. The Kastor-Traschen solution describes coalescing black holes in the contracting de Sitter universe (or splitting white holes in the expanding de Sitter universe) and inherits some salient characteristics from the Majumdar-Papapetrou solution. The reason why multicenter metric is in mechanical equilibrium irrespective of the time-dependence is attributed to the first-order "BPS equation" that extremizes the action, allowing the complete linearization of field equations. Since these "BPS" states are not truly supersymmetric in the usual sense, they are referred to as pseudo-supersymmetric and the corresponding theory is called a "fake" supergravity. Recently, all pseudo-supersymmetric solutions in four- and five-dimensional fake de Sitter fake supergravity were classified using the spinorial geometry method [22,23] (see [24] for a non-Abelian generalization).

In this paper, we discuss properties of pseudo-supersymmetric solutions of five-dimensional fake supergravity with an arbitrary number of $U(1)$ gauge fields and scalar fields. Some time-dependent black-hole solutions in this theory have been available so far [25,26], but their properties and causal structures are yet to be explored. Even for the simplest case in which the harmonic function is sourced by a single point mass, the spacetime is highly dynamical except in the de Sitter supergravity. In the present case, the background spacetime is the Friedmann-Lemaître-Robertson-Walker (FLRW) cosmology. (In the context of fake supergravity, it is argued that the FLRW cosmologies are duals of supersymmetric domain walls. See [27] for details.) A series of recent papers of present authors [28,29] revealed that the solution of a single point source found in [30,31] actually describes a charged black hole in the FLRW cosmology. Though the metric in [30,31] were shown to be the exact solutions of the Einstein-Maxwell-dilaton system, we show in this paper that the five-dimensional solutions of [29,30] in fact satisfy the first-order BPS equation in fake supergravity. The pseudo-supersymmetry is indeed consistent with an expanding universe. This work will establish new insights for black holes in time-dependent and nonsupersymmetric backgrounds.

The main concern in this paper is to see the effects of black-hole rotation in 5 dimensions by restricting to the single point mass case. As it turns out, rotation makes the properties of spacetime much richer. Our work is organized as follows. In Sec. II, we describe a fake supergravity model and derive (in a gauge different from [32]) a rotating, time-dependent solution preserving the pseudo-supersymmetry. Section III is devoted to exploring physical and geometrical properties of the spacetime. We establish that the black-hole horizon is generated by a rotating Killing horizon, in sharp contrast with the supersymmetric black-hole horizon which admits a nonrotating degenerate Killing horizon without an ergoregion. It is also demonstrated that the solution generally admits closed timelike curves in the vicinity of timelike singularities (with a trivial fundamental group). Combining the analysis of the near-horizon geometries, we shall elucidate the causal structures by illustrating Carter-Penrose diagrams. In Sec. IV, the lift-up and reduction scheme of the five-dimensional solution is accounted for. It is shown that the five-dimensional solutions derived in [29,30] and in Sec. III are elevated to describe the non-BPS dynamically intersecting $M2/M2/M2$-branes in eleven-dimensional supergravity. Upon dimensional reduction, the four-dimensional black hole [30,33] is obtainable. We shall also present some Kaluza-Klein black holes in the FLRW universe. Section V gives final remarks.

We will work in mostly plus metric signature and the standard curvature conventions $2\nabla_{[\rho}\nabla_{\sigma]}V^\mu = R^\mu{}_{\nu\rho\sigma}V^\nu$. Gamma matrix conventions are such that $\gamma_{\mu\nu\rho\sigma\tau} = i\epsilon_{\mu\nu\rho\sigma\tau}$ with $\epsilon_{01234} = 1$ and $\bar{\psi} := i\psi^\dagger\gamma^0$.

II. FIVE-DIMENSIONAL SOLUTIONS IN MINIMAL SUPERGRAVITY

The metrics obtained in [28–31] are the exact solutions of Einstein's equations sourced by two $U(1)$ fields and a scalar field coupled to the gauge fields. Since the solution involves two kinds of harmonic functions, it manifests mechanical equilibrium regardless of time-evolving spacetime. When each harmonic has a point source at the center, the solution in [28–31] describes a spherically symmetric black hole embedded in the FLRW cosmology. In this section, we consider a five-dimensional supergravity-type Lagrangian and present more general (pseudo-) BPS solutions, which encompass the five-dimensional solution in [29,30] as a special limiting case.

Let us start from the minimal five-dimensional gauged supergravity coupled to N abelian vector multiplets. The bosonic action involves graviton $U(1)$ gauge fields $A^{(I)}$ ($I = 1, \ldots, N$) with real scalars ϕ^A ($A = 1, \ldots, N-1$) [34],

$$S = \frac{1}{2\kappa_5^2}\int\Big[({}^5R + 2\mathrm{g}^2 V)\star_5 1 - \mathcal{G}_{AB}d\phi^A\wedge\star_5 d\phi^B - G_{IJ}F^{(I)}\wedge\star_5 F^{(J)} - \frac{1}{6}C_{IJK}A^{(I)}\wedge F^{(J)}\wedge F^{(K)}\Big], \tag{2.1}$$

where $F^{(I)} = dA^{(I)}$ are the field strengths of gauge fields and $\mathfrak{g}$ is the coupling constant corresponding to the reciprocal of the AdS curvature radius. C_{IJK} are constants symmetric in (IJK) and obey the "adjoint identity"

$$C_{IJK}C_{J'(LM}C_{PQ)K'}\delta^{JJ'}\delta^{KK'} = \frac{4}{3}\delta_{I(L}C_{MPQ)}, \qquad (2.2)$$

where the round brackets denote symmetrization of the suffixes. The potential V can be expressed in terms of a superpotential W as

$$V = 6W^2 - \frac{9}{2}\mathcal{G}^{AB}(\partial_A W)(\partial_B W), \qquad (2.3)$$

where $\partial_A X^I := dX^I(\phi)/d\phi^A$. The superpotential takes the form

$$W = V_I X^I, \qquad (2.4)$$

where V_I are constants arising from an Abelian gauging of the $SU(2)$-R-symmetry with the gauge field $A = V_I A^I$ [34]. The N-scalars X^I are constrained by

$$\mathcal{V} := \frac{1}{6}C_{IJK}X^I X^J X^K = 1. \qquad (2.5)$$

It is convenient to define

$$X_I = \frac{1}{6}C_{IJK}X^J X^K, \qquad (2.6)$$

in terms of which Eq. (2.5) is simply $X_I X^I = 1$. The coupling matrix G_{IJ} is the metric of the "very special geometry" [35] defined by

$$G_{IJ} := -\frac{1}{2}\frac{\partial^2}{\partial X^I \partial X^J}\ln \mathcal{V}|_{\mathcal{V}=1} = \frac{9}{2}X_I X_J - \frac{1}{2}C_{IJK}X^K, \qquad (2.7)$$

with its inverse

$$G^{IJ} = 2X^I X^J - 6C^{IJK}X_K, \qquad (2.8)$$

where $C^{IJK} = \delta^{IL}\delta^{JP}\delta^{KQ}C_{LPQ}$. The other coupling matrix $\mathcal{G}_{AB}$ is given by

$$\mathcal{G}_{AB} = G_{IJ}\partial_A X^I \partial_B X^J. \qquad (2.9)$$

It follows that

$$X^I = \frac{9}{2}C^{IJK}X_J X_K, \qquad (2.10)$$

and

$$X_I = \frac{2}{3}G_{IJ}X^J, \qquad X^I = \frac{3}{2}G^{IJ}X_J. \qquad (2.11)$$

From these relations, we obtain useful expressions

$$\begin{aligned} dX_I &= -\frac{2}{3}G_{IJ}dX^J, \\ dX^I &= -\frac{3}{2}G^{IJ}dX_J, \\ X^I dX_I &= X_I dX^I = 0, \\ \mathcal{G}^{AB}\partial_A X^I \partial_B X^J &= G^{IJ} - \frac{2}{3}X^I X^J. \end{aligned} \qquad (2.12)$$

Using these formulae, the potential reads

$$V = 27C^{IJK}V_I V_J X_K. \qquad (2.13)$$

If this theory is derived via gauging the supergravity derived from the Calabi-Yau compactification of M-theory, $\mathcal{V}$ is the intersection form, and X^I and X_I correspond, respectively, to the size of the two- and four-cycles. The constants C_{IJK} are the intersection numbers of the Calabi-Yau threefold and N denotes the Hodge number $h_{1,1}$ [36].

The governing equations are the Einstein equations (varying $g^{\mu\nu}$),

$$\begin{aligned} &{}^5R_{\mu\nu} - \frac{1}{2}({}^5R + 2\mathfrak{g}^2 V)g_{\mu\nu} \\ &= G_{IJ}\Big[(\nabla_\mu X^I)(\nabla_\nu X^J) - \frac{1}{2}(\nabla^\rho X^I)(\nabla_\rho X^J)g_{\mu\nu} \\ &\quad + F_\mu^{(I)\rho}F_{\nu\rho}^{(J)} - \frac{1}{4}F_{\rho\sigma}^{(I)}F^{(J)\rho\sigma}g_{\mu\nu}\Big], \end{aligned} \qquad (2.14)$$

the electromagnetic field equations (varying $A^{(I)}$),

$$\nabla_\nu(G_{IJ}F^{(J)\mu\nu}) - \frac{1}{16}C_{IJK}\epsilon^{\mu\nu\rho\sigma\tau}F_{\nu\rho}^{(J)}F_{\sigma\tau}^{(K)} = 0, \qquad (2.15)$$

where $\epsilon_{\mu\nu\rho\sigma\tau}$ is the metric-compatible volume element, and the scalar-field equations (varying ϕ^A),

$$\begin{aligned} &\Big[\nabla^\mu\nabla_\mu X_I + 6\mathfrak{g}^2 V_L V_M C_{IJK}C^{KLM}X^J \\ &\quad + \Big(C_{IJL}X_K X^L - \frac{1}{6}C_{IJK}\Big)\Big((\nabla_\mu X^J)(\nabla^\mu X^K) \\ &\quad + \frac{1}{2}F_{\mu\nu}^{(J)}F^{(K)\mu\nu}\Big)\Big]\partial_A X^I = 0. \end{aligned} \qquad (2.16)$$

From the condition $X_I dX^I = 0$, the terms in square brackets in the above equation must be proportional to X_I. Denoting it by LX_I, one obtains the expression of L using the relation $X_I X^I = 1$. The scalar equations are then rewritten as

$$\begin{aligned} &\nabla^\mu\nabla_\mu X_I + \Big(\frac{1}{2}C_{JKL}X_I X^L - \frac{1}{6}C_{IJK}\Big)(\nabla^\mu X^J)(\nabla_\mu X^K) \\ &\quad + 6\mathfrak{g}^2 C^{JLM}V_L V_M(6X_I X_J - C_{IJK}X^K) \\ &\quad + \frac{1}{2}\Big(C_{IJL}X_K X^L - \frac{1}{6}C_{IJK} - 6X_I X_J X_K \\ &\quad + \frac{1}{6}C_{JKL}X_I X^L\Big)F_{\mu\nu}^{(J)}F^{(K)\mu\nu} = 0. \end{aligned} \qquad (2.17)$$

The supersymmetric transformations for the gravitino ψ_μ and gauginos λ_A are given by

$$\delta\psi_\mu = \Big[\mathcal{D}_\mu - \frac{3i}{2}\mathfrak{g}V_I A^{(I)}_\mu + \frac{i}{8}X_I(\gamma_\mu{}^{\nu\rho} - 4\delta_\mu{}^\nu\gamma^\rho)F^{(I)}_{\nu\rho} + \frac{1}{2}\mathfrak{g}\gamma_\mu X^I V_I\Big]\epsilon, \tag{2.18}$$

$$\delta\lambda_A = \Big[\frac{3}{8}\gamma^{\mu\nu}F^{(I)}_{\mu\nu}\partial_A X_I - \frac{i}{2}\mathcal{G}_{AB}\gamma^\mu\partial_\mu\phi^B + \frac{3i}{2}\mathfrak{g}V_I\partial_A X^I\Big]\epsilon, \tag{2.19}$$

where ϵ is a spinor generating an infinitesimal supersymmetry transformation. Here and throughout the paper, $\mathcal{D}_\mu$ will be used for a gravitationally covariant derivative defined by

$$\mathcal{D}_\mu\epsilon = \Big(\partial_\mu + \frac{1}{4}\omega_\mu{}^{ab}\gamma_{ab}\Big)\epsilon, \tag{2.20}$$

where $\omega_\mu{}^{ab}$ is a spin connection without torsion. We have used the Dirac spinor instead of the symplectic Majorana spinor. The supersymmetric solutions in this theory have been analyzed [11]. One recovers ungauged supergravity by $\mathfrak{g} \to 0$.

A. Pseudo-supersymmetric solutions in fake supergravity

If we consider a noncompact gauging of R-symmetry, an imaginary coupling arises, $\mathfrak{g} \to ik (k \in \mathbb{R})$. Since only the R-symmetry is gauged, the imaginary coupling reflects the noncompactness of R-symmetry. The Lagrangian (2.1) is neutral under the R-symmetry, so that the theory is free from the ghostlike contribution. This theory is called a fake supergravity. The fake "Killing spinor" equations reduce to

$$\Big[\mathcal{D}_\mu + \frac{3k}{2}V_I A^{(I)}_\mu + \frac{i}{8}X_I(\gamma_\mu{}^{\nu\rho} - 4\delta_\mu{}^\nu\gamma^\rho)F^{(I)}_{\nu\rho} + \frac{i}{2}k\gamma_\mu X^I V_I\Big]\epsilon = 0, \tag{2.21}$$

$$\Big[\frac{3}{8}\gamma^{\mu\nu}F^{(I)}_{\mu\nu}\partial_A X_I - \frac{i}{2}\mathcal{G}_{AB}\gamma^\mu\partial_\mu\phi^B - \frac{3}{2}kV_I\partial_A X^I\Big]\epsilon = 0. \tag{2.22}$$

Here, the supercovariant derivative operator is no longer hermitian for $k \in \mathbb{R}$. This implies that we are unable to use ϵ to prove the positive energy theorem in the usual manner. Still, we presume that the above Eqs. (2.21) and (2.22) continue to be valid for $k \in \mathbb{R}$.

Inferring from the supersymmetric solutions in [11], we assume the standard metric ansatz,

$$ds_5^2 = -f^2(dt+\omega)^2 + f^{-1}h_{mn}dx^m dx^n, \tag{2.23}$$

where the four-metric h_{mn} is orthogonal to $V^\mu = (\partial/\partial t)^\mu$ ($i_V h_{\mu\nu} = 0$) and is supposed to be independent of t ($\mathcal{L}_V h_{\mu\nu} = 0$). The one-form ω corresponds to the $U(1)$ fibration of the transverse base space $(\mathcal{B}, h_{mn})$. In what follows, indices $m, n, \ldots$ are raised and lowered by h_{mn} and its inverse h^{mn}. The connection ω is orthogonal to the timelike vector field V^μ and assumed to be independent of t ($\mathcal{L}_V\omega = 0$). We further suppose that the lapse function is given by

$$f^{-3} = \frac{1}{6}C^{IJK}H_I H_J H_K, \tag{2.24}$$

where H_I's are some functions. We also assume the profiles of the electromagnetic and the scalar fields as

$$A^{(I)} = fX^I(dt+\omega), \qquad X_I = \frac{1}{3}fH_I. \tag{2.25}$$

In the ungauged supersymmetric case (when $\mathfrak{g} = 0$), the condition (2.24) is obtained as a special case of the general supersymmetric solutions, as referred to hereinafter in Sec. IV. In this section, however, we just assume (2.24).

Taking the orthonormal frame

$$e^0 = f(dt+\omega), \qquad e^i = f^{-1/2}\hat{e}^i, \tag{2.26}$$

where $\hat{e}^i$ is the orthonormal frame for h_{mn}, one can calculate the time and spatial components of "Killing spinor" Eq. (2.21), which are given by

$$\Big[\partial_t + kfV_I X^I + \Big\{\frac{1}{2}fkV_I X^I + \frac{1}{4}f^3\partial_{[m}\omega_{n]}\hat{\gamma}^{mn} + \frac{i}{2}f^{1/2}(\partial_m f - \omega_m\partial_t f)\hat{\gamma}^m\Big\}(1 - i\gamma^0)\Big]\epsilon = 0, \tag{2.27}$$

$$\Big[{}^h\mathcal{D}_m - \omega_m\partial_t - \frac{1}{2f}(\partial_m f - \omega_m\partial_t f)i\gamma^0 + \frac{f^{3/2}}{2}\Big(\frac{1}{2}{}^h\epsilon_{mn}{}^{pq}\partial_{[p}\omega_{q]} + \partial_{[m}\omega_{n]}\Big)\hat{\gamma}^n\gamma^0 + \frac{i}{2f^{1/2}}\hat{\gamma}_m\Big(kV_I X^I + \frac{i\gamma^0\partial_t f}{2f^2}\Big) + \Big\{-\frac{1}{4f}(\partial_n f - \omega_n\partial_t f)\hat{\gamma}_m{}^n - if^{3/2}\partial_{[m}\omega_{n]}\Big\} \times (1 - i\gamma^0)\Big]\epsilon = 0, \tag{2.28}$$

where $\hat{\gamma}^m = \hat{e}_i{}^m\gamma^i$. ${}^h\mathcal{D}$ and ${}^h\epsilon$ are, respectively, the Lorentz covariant derivative and the volume element with respect to h_{mn}. From Eqs. (2.24) and (2.25), we have a useful relation

$$kV_I X^I + \frac{1}{2}f^{-2}\partial_t f = \frac{1}{2}f^2 C^{IJK}H_I H_J\Big(kV_I - \frac{1}{6}\partial_t H_I\Big). \tag{2.29}$$

Thus, if $d\omega$ satisfies the anti-self-duality condition,

$$d\omega + \star_h d\omega = 0, \tag{2.30}$$

where $\star_h$ denotes the Hodge dual operator with respect to the base space metric h_{mn} and if H_I's satisfy the differential equations $\partial_t H_I = 6kV_I$, the Killing spinor equations are solved by

$$i\gamma^0\epsilon = \epsilon, \tag{2.31}$$

$$\epsilon = f^{1/2}\zeta. \tag{2.32}$$

Here, ζ is a covariantly constant Killing spinor with respect to the four-dimensional metric h_{mn},

$$^h\mathcal{D}_m\zeta = 0, \tag{2.33}$$

satisfying

$$\hat{\gamma}^{1234}\zeta = \zeta. \tag{2.34}$$

It follows that H_I's take the form

$$H_I(t, x^m) = 6kV_I t + \bar{H}_I(x^m), \tag{2.35}$$

where H_I's are functions on the base space.

The integrability condition of Eq. (2.33) is $^hR_{mnpq}\hat{\gamma}^{pq}\epsilon = 0$. From the chirality condition (2.34), one can find that $\hat{\gamma}_{mn}\epsilon$ is anti-self-dual on the base space. This implies that the Riemann tensor of h_{mn} is self-dual $\star_h(^hR_{mnpq}) = {}^hR_{mnpq}$. Hence, the base space $(\mathcal{B}, h_{mn})$ turns out to be the hyper-Kähler manifold, whose complex structures $\mathfrak{J}^{(i)}$ are anti-self-dual $\star_h\mathfrak{J}^{(i)} = -\mathfrak{J}^{(i)}$. The chirality condition (2.34) is a direct consequence of $i\gamma^0\epsilon = \epsilon$, which is the only projection imposed on the Killing spinor. It follows that the solution preserves at least half of pseudo-supersymmetries. If Eqs. (2.35) and (2.31) are satisfied, one verifies that the dilatino Eq. (2.22) is satisfied automatically.

Let us next turn to the Maxwell Eq. (2.15). Only the 0th component is nontrivial, giving

$$^h\Delta\bar{H}_I = 0, \tag{2.36}$$

where $^h\Delta$ is the Laplacian operator with respect to h_{mn}. This equation manifests the complete linearization.

All the metric components are obtained by use of the Killing spinor and Maxwell equations under our ansatz. We have nowhere solved the scalar and Einstein's equations so far. Nevertheless, these equations are automatically satisfied if the Bianchi identities $dF^{(I)} = 0$ and Maxwell Eqs. (2.15) are satisfied, on account of the integrability conditions for the pseudo-Killing spinor equations.

The procedure for generating time-dependent backgrounds presented here was previously given in [25]. It is, however, observed that the above metric form is not fully general. According to the analysis for the de Sitter supergravity [23], the base space is allowed to have a torsion. We expect that the general classification in this theory is also possible following the same fashion as [23].

B. Rotating black hole in STU theory

To be concrete, let us consider the "STU theory," which is defined by the conditions such that $C_{123} = C_{(123)} = 1$ and the other C_{IJK}'s vanish. In this theory, one has three Abelian gauge fields and two unconstrained scalars. For simplicity, let us choose the flat space as a base space $(\mathcal{B}, h_{mn})$,

$$ds^2_{\mathcal{B}} = dr^2 + r^2(d\vartheta^2 + \sin^2\vartheta d\phi_1^2 + \cos^2\vartheta d\phi_2^2). \tag{2.37}$$

Then, the equation for ω (2.30) is easily solved to give

$$\omega = \frac{J}{r^2}(\sin^2\vartheta d\phi_1 + \cos^2\vartheta d\phi_2), \tag{2.38}$$

where the volume form of $(\mathcal{B}, h_{mn})$ is taken as $dr \wedge (rd\vartheta) \wedge (r\sin\vartheta d\phi_1) \wedge (r\cos\vartheta d\phi_2)$ and J is a constant representing the rotation of the spacetime.

In what follows, we shall specialize to the case where each harmonic function has a point source at the origin $\propto Q_I/r^2$. Denoting

$$t_I = (6kV_I)^{-1}, \tag{2.39}$$

we classify the solutions into the following four cases depending on how many V_I's vanish [37].

(i) $V_1 = V_2 = V_3 = 0$, for which

$$H_1 = 1 + \frac{Q_1}{r^2}, \qquad H_2 = 1 + \frac{Q_2}{r^2}, \qquad H_3 = 1 + \frac{Q_3}{r^2}. \tag{2.40}$$

This is nothing but the solution in the ungauged true supergravity in which the scalar-field potential vanishes. The supersymmetric solutions have been completely classified in [11,15]. This theory can be uplifted to eleven-dimensional supergravity, as described later. The eleven-dimensional solution describes the rotating $M2/M2/M2$-branes preserving $1/8$ supersymmetry. In the following, we do not elaborate this case unless otherwise stated, since its physical properties have been widely discussed in the existing literature [38–40].

(ii) $V_1 \neq 0$, $V_2 = V_3 = 0$, for which

$$H_1 = \frac{t}{t_1} + \frac{Q_1}{r^2}, \qquad H_2 = 1 + \frac{Q_2}{r^2}, \qquad H_3 = 1 + \frac{Q_3}{r^2}. \tag{2.41}$$

This case corresponds also to the zero-potential $V = 27C^{IJK}V_IV_JX_K = 0$ due to $C_{11K} = 0$. It is notable that the potential height V_1 makes a contribution to the pseudo-Killing spinor Eqs. (2.21) and (2.22). This pseudo-supersymmetric solution can be oxidized to 11 dimensions, but the resultant spacetime is not pseudo-supersymmetric since eleven-dimensional supergravity has no potential term. The oxidized solution is interpreted as the intersecting $M2/M2/M2$-branes in the background rotating Kasner universe. The detail is described in Sec. IV A 2.

(iii) $V_1, V_2 \neq 0, V_3 = 0$, for which

$$H_1 = \frac{t}{t_1} + \frac{Q_1}{r^2}, \qquad H_2 = \frac{t}{t_2} + \frac{Q_2}{r^2}, \qquad H_3 = 1 + \frac{Q_3}{r^2}. \tag{2.42}$$

These two cases (ii) and (iii) have not been discussed in [25], although the authors arrived at the same equation as (2.35).

(iv) $V_1, V_2, V_3 \neq 0$, for which

$$H_1 = \frac{t}{t_1} + \frac{Q_1}{r^2}, \qquad H_2 = \frac{t}{t_2} + \frac{Q_2}{r^2}, \qquad H_3 = \frac{t}{t_3} + \frac{Q_3}{r^2}. \tag{2.43}$$

When $t_1 = t_2 = t_3$ and $Q_1 = Q_2 = Q_3$, all scalar fields are trivial. This case corresponds to the fake de Sitter supergravity for which the potential is constant $\mathfrak{g}^2 V = -3/(2t_1^2)$. The complete classification of timelike class for the de Sitter supergravity was done in [22,23].

Even if t_I's and Q_I's are not all identical, this solution inherits many properties of that in de Sitter supergravity, irrespective of nontrivial scalar fields X^I. In fact, by a coordinate transformation

$$r' = r\left(\frac{t}{t_0}\right)^{1/2}, \qquad \ln\left(\frac{t}{t_0}\right) = \frac{t'}{t_0} + \int^{r'} \frac{h_2(r')}{h_1(r')} dr', \qquad \phi_{1,2} = \phi'_{1,2} + \int^{r'} h_2(r') dr', \tag{2.44}$$

where $t_0 \equiv (t_1 t_2 t_3)^{1/3}$ and

$$h_1(r') := \frac{J r'^2 t_0}{H^3 r'^6 - J^2}, \qquad h_2(r') := \frac{2 J r' t_0}{J^2 - H^3 r'^6 + 4 r'^4 t_0^2}, \qquad H^3 := \left(\frac{t_0}{t_1} + \frac{Q_1}{r'^2}\right)\left(\frac{t_0}{t_2} + \frac{Q_2}{r'^2}\right)\left(\frac{t_0}{t_3} + \frac{Q_3}{r'^2}\right), \tag{2.45}$$

the metric (2.43) can be brought to the stationary form,

$$ds^2 = \frac{r'^2 H}{4t_0^2} dt'^2 - H^{-2}\left[dt' + \frac{J}{r'^2}(\sin^2\vartheta d\phi'_1 + \cos^2\vartheta d\phi'_2)\right]^2 + H\left[\frac{dr'^2}{1 - H^3 r'^2/(4t_0^2) + J^2/(4t_0^2 r'^4)} + r'^2(d\vartheta^2 + \sin^2\vartheta d\phi_1'^2 + \cos^2\vartheta d\phi_2'^2)\right]. \tag{2.46}$$

This is asymptotically de Sitter with curvature radius $\ell = 2t_0$ [32].

When the rotation vanishes ($\omega = 0$), these solutions reduce to the ones considered in our previous papers [28,29], describing a spherically symmetric black hole in a five-dimensional FLRW universe. It is then expected that the present solution describes a rotating black hole in the expanding universe. To see this more concretely, let us consider the asymptotic limit $r \to \infty$ of the solutions. Let n denote the number of time-dependent harmonics, i.e., $n = 1, 2$, and 3 are the cases (ii), (iii), and (iv), respectively. Changing to the new time slice

$$\frac{\bar{t}}{\bar{t}_0} = \left(\frac{t}{t_0}\right)^{1-n/3}, \qquad \bar{t}_0 = \frac{3t_0}{3-n}, \tag{2.47}$$

for $n = 1, 2$ and $\bar{t} = t_0 \ln(t/t_0)$ for $n = 3$, one easily finds that each solution (2.40), (2.41), (2.42), and (2.43) approaches the five-dimensional flat FLRW universe,

$$ds_5^2 = -d\bar{t}^2 + a^2 \delta_{mn} dx^m dx^n. \tag{2.48}$$

Here, $\bar{t}$ measures the cosmic time at infinity and the scale factor obeys

$$a = (\bar{t}/\bar{t}_0)^{n/[2(3-n)]}, \tag{2.49}$$

for $n = 1, 2$ and

$$a = e^{\bar{t}/2t_0}, \tag{2.50}$$

for $n = 3$, which are, respectively, the same expansion law as the stiff-matter dominant universe ($n = 1$), the universe filled by fluid with equation of state $P = -\rho/2$ ($n = 2$), and the de Sitter universe with curvature radius $2t_0$ ($n = 3$). In either case, the solution tends to be spatially homogeneous and isotropic in the asymptotic region $r \to \infty$.

On the other hand, when one takes the limit in which r goes to zero *with t kept finite*, the solution (3.1) approaches to a deformed $\mathrm{AdS}_2 \times S^3$:

$$ds^2_{r\to 0} = -\left(\frac{r^2}{\bar{Q}}\right)^2\left[dt + \frac{J}{r^2}(\sin^2\vartheta d\phi_1 + \cos^2\vartheta d\phi_2)\right]^2 + \left(\frac{\bar{Q}}{r^2}\right)^2 dr^2 + \bar{Q} d\Omega_3^2, \tag{2.51}$$

where $\bar{Q} \equiv (Q_1 Q_2 Q_3)^{1/3}$ and $d\Omega_3^2$ denotes the unit line element of S^3. This is the same as the near-horizon geometry of a Breckenridge-Myers-Peet-Vafa (BMPV) black hole [17,40], implying that $r = 0$ is a point at the tip of an infinite throat. Note that when all harmonics are time-independent, the solution reduces to the BMPV black hole with a degenerate horizon at $r = 0$.

It is noteworthy, however, that this metric (2.51) does *not* describe the geometry of a neighborhood of "would-be horizon," since we have fixed the time coordinate when taking the $r \to 0$ limit. As pointed out in [28,29], the null surfaces piercing the throat correspond to the infinite redshift ($t \to +\infty$) and blueshift ($t \to -\infty$) surfaces. The structures of these null surfaces can be analyzed by taking the appropriate "near-horizon" limit, as we will discuss later. As it turns out, the horizon, if it exists, is not extremal in general, contrary to the naïve expectations from (2.51).

The reason why we consider rotating black holes in 5 dimensions is that rotation is compatible with supersymmetry in 5 dimensions. In D dimensions, the gravitational attractive force and centrifugal force behave, respectively, as $-M/r^{D-3}$ and J^2/M^2r^2, so that the balance is maintained only in $D = 5$. The spinning cosmological solution in the Einstein-Maxwell-axion gravity is obtained via dimensional reduction of a chiral null model in 5 dimensions [41].

Incidentally, let us mention the issue of the fact that the action involved several gauge fields. This is a necessary price in order to obtain the finite-sized horizon area. With just a single gauge field, the spacetime becomes nakedly singular unless the scalar-field potential is a pure cosmological constant. A specific example is given in Appendix A within the framework of the Einstein-Maxwell-dilaton gravity.

III. PHYSICAL PROPERTIES OF FIVE-DIMENSIONAL ROTATING BLACK HOLES

Let us explore the physical properties of the solutions (2.40), (2.41), (2.42), and (2.43). For further simplicity of our argument, we shall confine ourselves to the case in which all charges are identical ($Q_1 = Q_2 = Q_3 \equiv Q > 0$) and all the potential heights are the same ($t_1 = t_2 = t_3 \equiv t_0 > 0$). Then, the metric (2.23) is described in a unified way as

$$f = H_T^{-n/3} H_S^{-1+n/3}, \tag{3.1}$$

with

$$H_T := \frac{t}{t_0} + \frac{Q}{r^2}, \qquad H_S := 1 + \frac{Q}{r^2}, \tag{3.2}$$

where $n(= 0, 1, 2,$ or $3)$ counts the number of time-dependent harmonics. This section is devoted to exploring physical properties of the solution (3.1) with (3.2). Here and hereafter, the subscript "T" and "S" will be used consistently for the time-dependent and time-independent quantities. The time-dependent and static scalar fields X_I are given by

$$X_T = \frac{1}{3}\left(\frac{H_T}{H_S}\right)^{1-n/3}, \qquad X_S = \frac{1}{3}\left(\frac{H_T}{H_S}\right)^{-n/3}. \tag{3.3}$$

Similarly, the gauge fields $A^{(I)}$ are

$$A^{(T)} = H_T^{-1}\left(dt + \frac{J}{2r^2}\sigma_3^R\right), \qquad A^{(S)} = H_S^{-1}\left(dt + \frac{J}{2r^2}\sigma_3^R\right). \tag{3.4}$$

The solution reduces to the BMPV solution, describing an asymptotically flat rotating black hole for $n = 0$ [38,39], the Klemm-Sabra solution describing a rotating black hole in the de Sitter universe for $n = 3$ [32].

Our previous solution describing a spherically symmetric black hole in the FLRW universe is recovered when the rotation vanishes $\omega = 0$ [29]. To make contact with the notation of the reference [29], let us define a canonical scalar field

$$\Phi = \sqrt{\frac{n(3-n)}{6}}\ln\left(\frac{H_T}{H_S}\right), \tag{3.5}$$

and make the replacements the electromagnetic fields as

$$(A^{(T)}, A^{(S)}) \to \frac{1}{\sqrt{2\pi}}(A^{(T)}, A^{(S)}). \tag{3.6}$$

Then, the solution (3.1) with (3.2), (3.5), and (3.6) solves the field equations derived from the action,

$$S_5 = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g_5}\bigg[{}^5R - (\nabla\Phi)^2 - \frac{n(n-1)}{2t_0^2}e^{-\lambda_T\Phi} - \sum_{A=T,S} n_A e^{\lambda_A\Phi}F^{(A)}_{\mu\nu}F^{(A)\mu\nu} + 2\epsilon^{\mu\nu\rho\sigma\tau}A^{(T)}_\mu F^{(S)}_{\nu\rho}F^{(S)}_{\sigma\tau}\bigg], \tag{3.7}$$

where $n_T = 3 - n_S = n$ and

$$\lambda_T = 2\sqrt{\frac{2n_S}{3n_T}}, \qquad \lambda_S = -2\sqrt{\frac{2n_T}{3n_S}}, \tag{3.8}$$

which is the $D = 5$ action considered in [29] when the Chern-Simons term does not contribute, i.e., there is no rotation.

When the theory is motivated by supergravity, the parameter n takes an integer value. We should stress that even if n is not an integer, the aforementioned metric (2.23) with (3.1) and (3.2) is still an exact solution of the Einstein-Maxwell scalar system, in which we have two $U(1)$ fields coupled to the scalar field with a Liouville-type exponential potential (3.7). The solution with nonintegral values of $0 < n < 2$ is qualitatively similar to the one with $n = 1$. (The case $2 < n < 3$ has no representative in this paper.) The geometrical properties with $n = 1$ discussed in what follows are also applied to the solution with $0 < n < 2$.

A. Symmetries

At first sight, one might expect that the metric admits $U(1) \times U(1)$ spatial symmetries generated by $\partial/\partial\phi_1$ and $\partial/\partial\phi_2$. In order to see that the solution indeed admits much larger symmetry, let us introduce the Euler angles (θ, ϕ, ψ) by

$$\theta = 2\vartheta, \qquad \phi = \phi_2 - \phi_1, \qquad \psi = \phi_2 + \phi_1, \tag{3.9}$$

which take ranges in $0 \le \theta \le \pi$, $0 \le \phi \le 2\pi$, and $0 \le \psi \le 4\pi$. In terms of the above coordinates, the left-invariant one-forms σ_i^R $(i = 1, 2, 3)$ on $SU(2) \simeq S^3$ are given by

$$\sigma_1^R = -\sin\psi d\theta + \cos\psi \sin\theta d\phi, \tag{3.10}$$

$$\sigma_2^R = \cos\psi d\theta + \sin\psi \sin\theta d\phi, \tag{3.11}$$

$$\sigma_3^R = d\psi + \cos\theta d\phi. \tag{3.12}$$

These one-forms satisfy

$$d\Omega_3^2 = \frac{1}{4}\sum_i (\sigma_i^R)^2, \qquad d\sigma_i^R = \frac{1}{2}\sum_k \epsilon_{ijk}\sigma_j^R \wedge \sigma_k^R. \tag{3.13}$$

The right-invariant vector fields ξ_i^L are the spacetime Killing fields. They are given by

$$\xi_1^L = -\frac{\cos\phi}{\sin\theta}\partial_\psi + \sin\phi\partial_\theta + \cot\theta\cos\phi\partial_\phi, \tag{3.14}$$

$$\xi_2^L = \frac{\sin\phi}{\sin\theta}\partial_\psi + \cos\phi\partial_\theta - \cot\theta\sin\phi\partial_\phi, \tag{3.15}$$

$$\xi_3^L = \partial_\phi, \tag{3.16}$$

which are the generators of the left transformations of $SU(2)$. These Killing vectors satisfy

$$\mathcal{L}_{\xi_i^L}\sigma_j^R = 0, \qquad [\xi_i^L, \xi_j^L] = \sum_k \epsilon_{ijk}\xi_k^L, \qquad \left(\frac{\partial}{\partial\Omega_3}\right)^2 = 4\sum_i \xi_i^L\xi_i^L. \tag{3.17}$$

In addition to these, there exists an additional $U(1)$ Killing field

$$\xi_3^R = \partial_\psi. \tag{3.18}$$

The orbits of ξ_3^R are the fibers of Hopf fibration of S^3. It follows that the metric is invariant under the action of $U(2) \simeq SU(2) \times U(1)$, acting on the three-dimensional orbits which are spacelike at infinity. Thus, the metric is expressed as

$$ds^2 = -f^2\left(dt + \frac{J}{2r^2}\sigma_3^R\right)^2 + f^{-1}(dr^2 + r^2 d\Omega_3^2). \tag{3.19}$$

As discussed in [42], the metric with $U(2)$ symmetry admits a *reducible* Killing tensor

$$\nabla_{(\mu}K_{\nu\rho)} = 0, \qquad K^{\mu\nu} = \sum_i (\xi_i^L)^\mu(\xi_i^L)^\nu, \tag{3.20}$$

which enables us to separate angular variables for the geodesic motion and scalar-field equation. It should be remarked, however, that the solution does not admit a timelike Killing field, so that the geodesic motion is not immediately solved.

B. Singularities

One can immediately find that the scalar fields X_I (3.3) blow up at

$$t = t_s(r) := -\frac{t_0 Q}{r^2} \quad \text{and} \quad r^2 = -Q. \tag{3.21}$$

Straightforward calculations reveal that all the curvature invariants are divergent at these spacetime points, i.e., they are spacetime curvature singularities. For example, the Ricci scalar curvature is given by

$$\begin{aligned} {}^5R = \frac{f^4}{6r^8H_T^2}\Bigg[&\frac{2n(3n-4)r^8}{t_0^2f^6} + J^2\left(24H_T^2 + \frac{n(2-n)r^2}{t_0^2f^3}\right) \\ &- 4Q^2r^2H_T^nH_S^{1-n}\{2(nH_S^2 + (3-n)H_T^2) \\ &- (nH_S + (3-n)H_T)^2\}\Bigg], \end{aligned} \tag{3.22}$$

which diverges at the above spacetime points, as expected.

Note that the $t = 0$ surface and the surface $r = 0$ with t kept finite are not the curvature singularities, where the curvature invariants are bounded. Hence, the big-bang singularity at $t = 0$ is completely smoothed out due to electromagnetic charges. As in the case (i), the surface $r = 0$ is a plausible candidate for event horizon.

C. Closed timelike curves

Since the vector field $\xi_3^R = \partial_\psi$ generates closed orbits of the period 4π, there appear to be closed timelike curves if an orbit of ξ_3^R becomes timelike. Rewrite the metric (3.19) as

$$\begin{aligned} ds^2 = -\frac{f^2}{\Delta_L}dt^2 + \frac{dr^2}{f} + \frac{r^2}{4f}\Bigg[&(\sigma_1^R)^2 + (\sigma_2^R)^2 \\ &+ \Delta_L\left(\sigma_3^R - \frac{2Jf^3}{r^4\Delta_L}dt\right)^2\Bigg], \end{aligned} \tag{3.23}$$

where

$$\Delta_L := 1 - \frac{J^2f^3}{r^6}. \tag{3.24}$$

Inspecting

$$g(\xi_3^R, \xi_3^R) = \frac{f^2}{4}\left(H_T^nH_S^{3-n} - \frac{J^2}{r^6}\right), \tag{3.25}$$

we can see that the first term on the right-hand side vanishes at the singularities. It follows that the Hopf fibres become timelike, i.e., closed timelike curves inevitably emerge in the vicinity of singularities for all values of $J(\neq 0)$. $\Delta_L = 0$ defines the velocity of light surface (VLS), where closed causal curves appear for $\Delta_L < 0$. For $n = 0$ (the BMPV spacetime without time-dependence), the VLS is located at $r^2 = J^{2/3} - Q$, which is inside the horizon for the small rotation $J^{2/3} < Q$; otherwise it is outside the horizon.

For $n \neq 0$, the VLS has the time-dependent profile

$$t_{\rm VLS}(r) := \frac{t_0}{r^2}\left[\left\{\frac{J^2}{(r^2+Q)^{3-n}}\right\}^{1/n} - Q\right]. \tag{3.26}$$

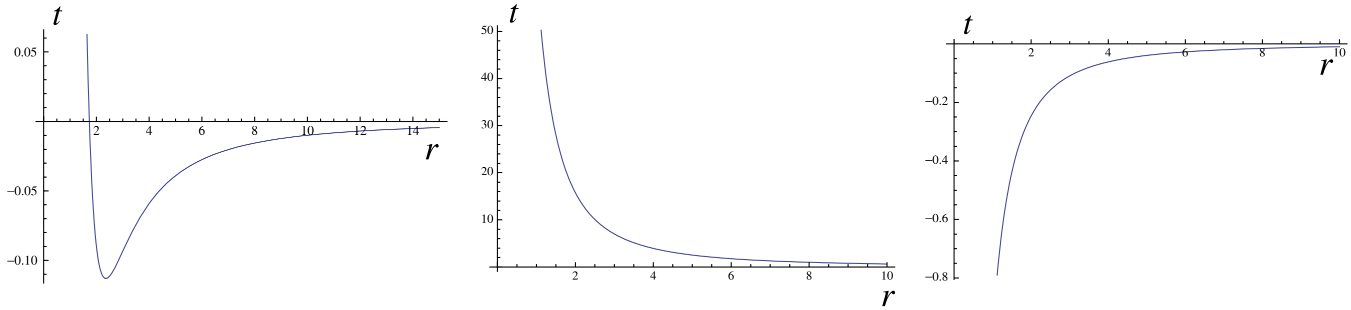

FIG. 1 (color online). Plots of velocity of light surface $t_{\rm VLS}$ against r for $n = 1, 2$ with $J^{2/3} > Q$ (left), for $n = 3$ with $J^{2/3} > Q$ (middle), and for $J^{2/3} < Q$ (right).

Since S^3 is $U(1)$ fibration over S^2, one can introduce the radius of S^2 by

$$R = |r| f^{-1/2}. \tag{3.27}$$

In terms of R, the VLS is positioned at the constant radius,

$$R_{\rm L} = J^{1/3}. \tag{3.28}$$

We shall declare the region $R < R_L$ ($R > R_L$) inside (outside) the VLS. Inside the VLS, ξ_3^R is pointing into the future direction for $J > 0$ and into the past one for $J < 0$. It is obvious that the singularity $t_s(r)$ exists for $r > 0$. As we will see in the next subsection, a horizon is positioned at $r = 0$ (with $t = \infty$), so that these closed timelike curves yield naked time machines—the causally anomalous region that is not hidden behind the event horizon—for every choice of parameters. Since the spacetime is simply connected, these causal pathologies cannot be circumvented by extending to a universal covering space. Hence, the fake supersymmetry fails to get rid of causal pathologies, as occurred for the present BPS rotating solutions.

Figure 1 plots the typical behaviors of VLS. When the angular momentum J is smaller than the critical value $Q^{2/3}$, $t_{\rm VLS}(r) < 0$ is satisfied and $t_{\rm VLS}(r) \to -\infty$ as $r \to 0$. On the other hand, when the angular momentum is larger than $Q^{3/2}$, $t_{\rm VLS}(r) \to +\infty$ as $r \to 0$ and for $n = 3$ $t_{\rm VLS}(r)$ is always positive, whereas it is negative at large value of r for $n = 1, 2$.

Using the radius R, one finds that the singularities (3.21) are both central $R = 0$. Thus, the VLS completely encloses the spacetime singularities.

In the neighborhood of the VLS, $(t - t_{\rm VLS}(r))/t_0 \ll 1$, one finds $f^{-3} \simeq J^2/r^6$. Hence, the neighborhood of the VLS in the present spacetime may be approximated by that in the near-horizon geometry of the BMPV black hole (2.51) with $J^2 = Q^3$. In this case, $\xi_3^R = \partial/\partial\psi$ becomes a hypersurface-orthogonal null Killing vector. Moreover, ψ corresponds to the affine parameter of the null geodesics $(\xi_3^R)^\nu \nabla_\nu (\xi_3^R)^\mu = 0$, so that the spacetime describes the plane-fronted wave [not the plane-fronted wave with parallel rays (pp-wave), since ξ_3^R is not covariantly constant $\nabla_\mu (\xi_3^R)^\nu \neq 0$] [43]. We can expect that properties of the VLS in the present spacetime iare captured by that in the near-horizon geometry of the BMPV black hole with $J = Q^{3/2}$.

D. Scaling limit

Since the event horizon of a black hole is a global concept, it is a difficult task to identify its locus, especially in a time-dependent spacetime. Following the previous papers, we shall argue the "near-horizon geometry" of the present metric and demonstrate that the null surface of the event-horizon candidate is described by a Killing horizon. By solving null geodesics numerically, we can verify that when $J < Q^{2/3}$, these Killing horizons are indeed event horizons in the original spacetimes. (The reason of the restriction $J < Q^{2/3}$ will be discussed later.)

For convenience, we define dimensionless parameters

$$\tau := \frac{t_0}{Q^{1/2}}, \qquad j := \frac{J}{Q^{3/2}}, \tag{3.29}$$

and denote dimensionless variables (normalized by Q) with tilde, e.g., $\tilde{x}^\mu := Q^{-1/2} x^\mu$. Then we can work with the dimensionless metric,

$$d\tilde{s}^2 = -f^2\left(\tau d\tilde{t} + \frac{j}{2\tilde{r}^2}\sigma_3^R\right)^2 + f^{-1}(d\tilde{r}^2 + \tilde{r}^2 d\Omega_3^2),$$
$$f = \tilde{r}^2(\tilde{t}\tilde{r}^2 + 1)^{-n/3}(\tilde{r}^2 + 1)^{(n-3)/3}. \tag{3.30}$$

The parameter j is the reduced angular momentum, and τ denotes the ratio of energy densities of the scalar fields and the Maxwell fields evaluated on the horizon, respectively [28,29]. To simplify the notation, we shall omit the tilde in the following.

We have seen in Eq. (2.51) that the surface $r \to 0$ with t being finite corresponds to the throat infinity. Hence, the null surfaces "intersecting" at the throat should be a candidate of future and past horizons. These surfaces are described by the infinite redshift and blueshift surfaces, respectively. We shall focus on the geometry of the very neighborhood of these horizon candidates. The only well-defined "near-horizon" limit is given by

$$t \to \frac{t}{\epsilon^2}, \qquad r \to \epsilon r, \qquad \epsilon \to 0, \tag{3.31}$$

under which the metric is free from the scaling parameter ϵ. The above scaling limit gives rise to the near-horizon geometry, if a horizon exists, the metric of which is given by

$$ds^2_{\rm NH} = -r^4(tr^2+1)^{-2n/3}\left(\tau dt + \frac{j}{2r^2}\sigma_3^R\right)^2 + r^{-2}(tr^2+1)^{n/3}(dr^2 + r^2 d\Omega_3^2). \tag{3.32}$$

The scalar and gauge fields are also well-defined and given by

$$X_T = \frac{1}{3}(tr^2+1)^{1-n/3}, \quad X_S = \frac{1}{3}(tr^2+1)^{-n/3}, \tag{3.33}$$

and

$$A^{(T)} = r^2(tr^2+1)^{-1}\left(\tau dt + \frac{j}{2r^2}\sigma_3^R\right), \quad A^{(S)} = r^2\left(\tau dt + \frac{j}{2r^2}\sigma_3^R\right). \tag{3.34}$$

This spacetime is pseudo-supersymmetric in its own right, since it admits a nonvanishing Killing spinor of the form (2.31) and (2.32) with $f = r^2(tr^2+1)^{-n/3}$.

The above near-horizon metric (3.32) is still time-evolving and spatially inhomogeneous. Nevertheless, as a consequence of the scaling limit (3.31), the near-horizon metric (3.32) admits a Killing vector

$$\xi^\mu = t\left(\frac{\partial}{\partial t}\right)^\mu - \frac{r}{2}\left(\frac{\partial}{\partial r}\right)^\mu. \tag{3.35}$$

It is then convenient to take ξ^μ to be a coordinate vector so that the metric is independent of that coordinate. A possible coordinate choice (T, R, ψ') is given by

$$T = \ln|t| + \int^R \frac{6R^{6/n-1}(R^6-j^2)dR}{n(R^{6/n}-1)\Delta}, \quad R = (tr^2+1)^{n/6}, \quad \psi' = \psi + \int^R \frac{12j\tau R^{n/6-1}}{n\Delta}dR, \tag{3.36}$$

where

$$\Delta := 4R^4F(R) + j^2, \quad F(R) := \tau^2R^{-4}(R^{6/n}-1)^2 - \frac{1}{4}R^2. \tag{3.37}$$

In this new coordinate system, the Killing field is simply given by $\xi^\mu = (\partial/\partial T)^\mu$, as we desired. After some algebra the near-horizon metric (3.32) is cast into an apparently stationary form,

$$ds^2_{\rm NH} = -F(R)\left[dT + \frac{j\tau(R^{6/n}-1)}{2R^4F(R)}\sigma_3^{\prime R}\right]^2 + \frac{j^2R^2(\sigma_3^{\prime R})^2}{16F(R)} + \frac{36\tau^2R^{12/n}dR^2}{n^2\Delta} + \frac{R^2}{4}[(\sigma_1^R)^2 + (\sigma_2^R)^2 + (\sigma_3^{\prime R})^2]. \tag{3.38}$$

Here, $\sigma_3^{\prime R} = d\psi' + \cos\theta d\phi$. Although its asymptotic structure is highly nontrivial, it is easy to recognize that this spacetime has Killing horizons (if any) at $\Delta = 0$. The Killing horizon is generated by a linear combination of stationary and angular Killing vectors,

$$\zeta = \frac{\partial}{\partial T} + 2\Omega_h\frac{\partial}{\partial\psi'}, \tag{3.39}$$

where

$$\Omega_h = \frac{j}{2\sqrt{R^6-j^2}}\Bigg|_{\rm horizon}. \tag{3.40}$$

Here, Ω_h is the angular velocity of the horizon (associated with $2\partial/\partial\psi' = \partial/\partial\phi_2 + \partial/\partial\phi_1$). The horizon angular velocity Ω_h is constant anywhere on the horizon, which is a generic feature of a Killing horizon [44]. Contrary to (truly) supersymmetric black holes, the angular velocity of the horizon is nonvanishing, i.e., the horizon is *rotating*. In other words, the generator of the event horizon of a supersymmetric black hole is tangent to the stationary Killing field at infinity. Equation (3.39) shows that $\partial/\partial t$ is not the generator of the event horizon. This is a distinguished property not shared by the BPS black holes.

Since Δ fails to have a double root in general, it follows that the horizon is not extremal unless parameters (τ, j) are fine-tuned. The reason of the appearance of the "throat" geometry at $r \to 0$ lies in the fact that (t, r) coordinates cover the "white-hole region" as well as the outside region of a black hole (see Fig. 5 in [29]).

Equations (3.33) and (3.36) imply that the values of scalar fields X_I on the horizon are determined by the horizon radius, which is expressed in terms of the charge Q, (inverse of) potential height t_0, and the angular momentum j. This situation is closely analogous to the attractor mechanism [45], according to which the values of scalar fields on the horizon are expressed by charges and are independent of the asymptotic values of the scalar fields at infinity. As it stands, however, it appears hard to say whether such a mechanism always works in the time-dependent case.

In the following subsections, we shall clarify various physical features of the near-horizon metric (3.38).

1. Horizons

The loci of Killing horizons $\Delta = 0$ can be classified according to the values of τ and j^2. We shall say "under-rotating" when the spacetime (3.32) admits horizons.

Otherwise, it is said to be "over-rotating." The quantity R_- will be consistently used when $\Delta > 0$ for $R < R_-$.

(i) $n = 1$. When the angular momentum parameter $|j|$ is less than the critical value $j_{(1)}$, i.e.,

$$j^2 < j_{(1)}^2 := \frac{1 + 16\tau^2}{16\tau^2}, \tag{3.41}$$

the near-horizon spacetime admits two horizons,

$$R_\pm^6 = \frac{1 + 8\tau^2 \pm \sqrt{1 + 16\tau^2(1 - j^2)}}{8\tau^2}. \tag{3.42}$$

For the over-rotating case $j^2 > j_{(1)}^2$, there exist no horizons. We find the similar results for the case of noninteger values of $n < 2$.

(ii) $n = 2$. This case is further categorized into the following three cases.

(1) $0 < \tau < 1/2$. For any values of j, a single horizon occurs at

$$R_-^3 = \frac{\sqrt{4\tau^2 + (1 - 4\tau^2)j^2} - 4\tau^2}{1 - 4\tau^2}. \tag{3.43}$$

(2) $\tau = 1/2$. For any values of j, a single horizon occurs at

$$R_-^3 = \frac{1 + j^2}{2}. \tag{3.44}$$

(3) $\tau > 1/2$. When the angular momentum parameter $|j|$ is less than the critical value $j_{(2)}$, i.e.,

$$j^2 < j_{(2)}^2 := \frac{4\tau^2}{4\tau^2 - 1}, \tag{3.45}$$

two horizons exist at

$$R_\pm^3 = \frac{4\tau^2 \pm \sqrt{4\tau^2 - (4\tau^2 - 1)j^2}}{4\tau^2 - 1}. \tag{3.46}$$

For the over-rotating case $j^2 > j_{(2)}^2$, no horizons develop.

(iii) $n = 3$. In this case, the metric (3.38) is not the "near-horizon" geometry, but is the original metric itself written in the stationary coordinates. This metric describes a charged rotating black hole in de Sitter space derived by Klemm and Sabra [32]. Let us discuss its horizon structure in detail.

There exists at least one horizon corresponding to the cosmological horizon. For $\tau \le \sqrt{3/2}$, there appears only a cosmological horizon R_c. For $\tau > \sqrt{3/2}$, the number of horizons depend on the value of j^2. Three distinct horizons ($R_- < R_+ < R_c$) exist for $j_{(3)-}^2 < j^2 < j_{(3)+}^2$, where

$$j_{(3)\pm}^2 := \frac{4\tau^2}{27}[\pm 8\sqrt{2}\tau(2\tau^2 - 3)^{3/2} - 32\tau^4 + 9(8\tau^2 - 3)]. \tag{3.47}$$

For $j^2 = j_{(3)+}^2$, inner and outer black-hole horizons are degenerate, while for $j^2 = j_{(3)-}^2$, the outer black-hole horizon and the cosmological horizon are degenerate. $j_{(3)-}^2$ takes real positive values for $\sqrt{3/2} < \tau < 3\sqrt{3}/4$, otherwise the inner horizon does not exist.

A simple calculation reveals that the spacetime (3.32) is regular on and outside the Killing horizon (if anywhere). Only the existing curvature singularity is at $R = 0$. It is almost clear to construct the local coordinate systems that pass through the Killing horizon $\Delta = 0$.

In hindsight, we can understand why the horizon in the present spacetime is not extremal as follows. In the case of the time-independent (truly) BPS solutions such as a BMPV black hole, the Killing horizon lies at $f = 0$ since $V^\mu = (\partial/\partial t)^\mu$ is an everywhere causal Killing field constructed by a Killing spinor ϵ as $V^\mu = i\bar{\epsilon}\gamma^\mu\epsilon$ (see [7]). For the present time-dependent pseudo-supersymmetric black hole, on the other hand, the vector field $V^\mu = (\partial/\partial t)^\mu$ is not the Killing horizon generator: the horizon is generated by $\xi^\mu = t(\partial/\partial t)^\mu - (r/2)(\partial/\partial r)^\mu + \Omega_h(\partial/\partial\psi)^\mu$ given in Eq. (3.39). The vector field V^μ does not give rise to any (asymptotic) symmetry.

Physically speaking, the degeneracy of the horizon is broken by introducing of the time-dependent scalar fields (which do not contribute to the total mass when the spacetime is stationary) or the positive cosmological constant. These ingredients destroy the fine balance between the mass energy and the charges. When the rotation is also added, the centrifugal force gives a negative contribution to the mass energy $M \to M - J^2$—which takes place only in $D = 5$ as discussed before—thus, it exceeds the extremal threshold value if the rotation becomes too large.

2. Ergoregion

An obvious major difference from our previous nonrotating solutions [28,29] is that the near-horizon metric possesses the ergosurface at $F(R) = 0$. Since $\Delta > 4R^4F(R)$, the ergosurface lies strictly outside the horizon, contrary to the four-dimensional Kerr black hole for which the ergosurface touches the horizon at the rotation axis.

When the rotating vanishes ($j = 0$), the roots of $F = 0$ correspond to the loci of horizons [28,29]. Since $\Delta = 0$ reduces to $F(R) = 0$ when $j = 0$, the explicit expression of the ergosphere is given by setting $j = 0$ of the horizon radius. For $n = 1, 2$, they are given by Eqs. (3.42), (3.43), (3.44), and (3.46) with $j = 0$. Note, however, that since the asymptotic structures are quite peculiar when $n = 1, 2$, there may arise an ambiguity concerning the definition of the energy [46]. It may therefore be uncertain, then, whether $R_{\rm erg}$ has a definitive meaning in the $n = 1, 2$ cases.

When $n = 3$ the asymptotic region is described by de Sitter space, so that we can use the standard time translation with respect to the observer at the cosmological horizon to define the energy. Hence, the notion of ergoregion is meaningful in this sense. There exist three distinct roots, $R_{\mathrm{erg},-} < R_{\mathrm{erg},+} < R_{\mathrm{erg},c}$, for $\tau > \tau_{\mathrm{cr}} := 3\sqrt{3}/4$, two roots for $\tau = \tau_{\mathrm{cr}}$ and a single root $R_{\mathrm{erg},-}$ for $\tau < \tau_{\mathrm{cr}}$.

The ergoregion does not arise for the supersymmetric black hole, which inevitably forbids the ergoregion inside which the stationary Killing field becomes spacelike. The ergoregion is intrinsic to a rotating black hole and allows particles to have a negative energy. This means that the rotation energy of a black hole can be subtracted via the Penrose process and the superradiant scattering process. We shall demonstrate in Appendix B that this is indeed the case for the $n = 3$ Klemm-Sabra solution.

3. Closed timelike curves

Write the near-horizon metric (3.38) as

$$ds^2_{\mathrm{NH}} = -\frac{\Delta}{4R^4\Delta_L}dT^2 + \frac{36\tau^2R^{12/n}dR^2}{n^2\Delta} + \frac{R^2}{4}\Big[(\sigma_1^R)^2 + (\sigma_2^R)^2 + \Delta_L\Big(\sigma_3'^R - \frac{2j\tau(R^{6/n}-1)}{R^6\Delta_L}dT\Big)^2\Big], \tag{3.48}$$

where we have also used Δ_L as the near-horizon limit of (3.24):

$$\Delta_L = 1 - \frac{j^2}{R^6}. \tag{3.49}$$

Consequently, the Hopf fibres become timelike inside the VLS ($\Delta_L < 0$), viz. the near-horizon metric (3.38) is also causally unsound. In terms of Δ_L, Δ is

$$\Delta = 4\tau^2(R^{6/n}-1)^2 - R^6\Delta_L. \tag{3.50}$$

It follows that the event horizon ($\Delta = 0$) is outside the VLS [47]. Hence, *the causality-violating region is always hidden behind the horizon* ($R_L < R_+$) in the near-horizon geometry (3.38) [48]. On the other hand, it is naked in the over-rotating case where the horizon does not exist. This should be contrasted with the BMPV or asymptotically AdS ($\mathfrak{g} \in \mathbb{R}$) Klemm-Sabra black hole. In the former case, the VLS is outside the event horizon if the angular momentum is large $J > Q^{2/3}$. In the latter case, a naked time machine inevitably appears outside the event horizon. However, it allows no geodesics to penetrate, so that the horizon exterior is geodesically complete. In the present case, the area of the horizon is given by

$$\mathrm{Area} = 2\pi^2R^3\sqrt{\Delta_L}|_{\mathrm{horizon}} = 4\pi^2\tau(R_+^{6/n}-1), \tag{3.51}$$

which always makes sense contrary to the BMPV or the asymptotically AdS Klemm-Sabra black hole: the latter two spacetimes have an ‘‘imaginary horizon area’’ in the over-rotating case. These formal horizons in the over-rotating case are ‘‘repulsons’’ into which no freely falling orbits penetrate (see, e.g., [42,49–51]).

4. Geodesic motions

It is illustrative to consider geodesics in the near-horizon metric (3.38). For $n = 3$, the following analysis yields the geodesic motion in the exact Klemm-Sabra geometry, not restricted in the neighborhood of its horizons. The particle motion in asymptotically AdS Klemm-Sabra solution ($\mathfrak{g} \in \mathbb{R}$) was previously examined in [49]. Although the behavior of the particle motion in the asymptotically de Sitter case is of course considerably different from that case, the technical method is similar. The analysis in this subsection reveals that the horizon can be reached within a finite affine time from outside.

The Hamilton-Jacobi equation in the near-horizon geometry (3.38) reads

$$-\frac{\partial S}{\partial\lambda} = \frac{1}{2}g^{\mu\nu}_{\mathrm{NH}}\Big(\frac{\partial S}{\partial x^\mu}\Big)\Big(\frac{\partial S}{\partial x^\nu}\Big), \tag{3.52}$$

where the right-hand side of this equation defines a geodesic Hamiltonian and λ is an affine parameter. Assume the separable form of Hamilton's principal function,

$$S = \frac{1}{2}m^2\lambda - ET + L_L\phi + L_R\psi' + S_R(R) + S_\theta(\theta), \tag{3.53}$$

where E, L_R, L_L, and m are constants of motion corresponding to energy, right-rotation, left-rotation, and rest mass of a particle. Since the near-horizon metric keeps the $U(2)$ symmetry, there exists a reducible Killing tensor of the form (3.20), which reads in the coordinates (3.38) as

$$K_{\mu\nu}dx^\mu dx^\nu = \Big[\frac{j\tau}{2R^4}(R^{6/n}-1)dT - \frac{R^2\Delta_L}{4}(\sigma_3'^R)\Big]^2 + \frac{R^4}{16}[(\sigma_1^R)^2 + (\sigma_2^R)^2]. \tag{3.54}$$

Accordingly, in addition to obvious constants of motion (E, L_R, L_L, m) generated by Killing vectors, we have an additional integration constant L^2 with dimensions of angular momentum squared such that

$$L^2 := \sum_i(\xi_i^R S)^2. \tag{3.55}$$

This constant of motion enables us to separate the variables as

$$\Big(\frac{d}{d\theta}S_\theta\Big)^2 + \frac{1}{\sin^2\theta}(L_L^2 + L_R^2 - 2\cos\theta L_RL_L) = L^2. \tag{3.56}$$

The constant L^2 represents the left and right Casimir invariant of the $SU(2)$ subgroup of the $SO(4)$ rotation group. These two Casimirs turn out to be the same for the scalar representation. It follows that the particle motion and the scalar-field equation are Liouville-integrable. The governing equations are obtainable by differentiating the principal function (3.53) by corresponding constants of motion. Using the relation for angular variable (3.56), we obtain a set of useful first-order equations:

$$\frac{dR}{d\lambda} = \pm \frac{n\sqrt{\Delta}}{6\tau R^{6/n}}\left[\frac{E}{F} - m^2 - \frac{4(L^2 - L_R^2)}{R^2} - \frac{16R^2F}{\Delta} \times \left(L_R + \frac{j\tau(R^{6/n}-1)E}{2R^4F}\right)^2\right]^{1/2}, \quad (3.57)$$

$$\frac{d\theta}{d\lambda} = \pm\frac{4}{R^2}\left[L^2 - \frac{1}{\sin^2\theta}(L_L^2 + L_R^2 - 2\cos\theta L_R L_L)\right]^{1/2}, \quad (3.58)$$

$$\frac{dT}{d\lambda} = \frac{4R^4\Delta_L}{\Delta}E - \frac{8j\tau(R^{6/n}-1)L_R}{\Delta R^2}, \quad (3.59)$$

$$\frac{d\phi}{d\lambda} = \frac{4}{R^2\sin^2\theta}(L_L - L_R\cos\theta), \quad (3.60)$$

$$\frac{d\psi'}{d\lambda} = \frac{4(L_R - L_L\cos\theta)}{R^2\sin^2\theta} - \frac{4}{R^2\Delta}[j^2L_R - 2j\tau(R^{6/n}-1)E]. \quad (3.61)$$

If there are no angular momenta of a particle ($L = L_R = L_L = 0$), one sees that there is no motion in directions θ and ϕ, but there is a nonvanishing motion along ψ', encoding the frame-dragging due to the black-hole rotation.

By virtue of the high degree of symmetries, the problem reduces to the one-dimensional radial Eq. (3.57), which is arranged to give

$$\left(\frac{dR}{d\lambda}\right)^2 = \frac{n^2}{9\tau^2R^{4(3/n-1)}}\left[\Delta_L(E - 2\Omega L_R)^2 - \frac{j^2L_R^2\Delta}{R^{12}\Delta_L} - \Delta\left(\frac{L^2}{R^6} + \frac{m^2}{4R^4}\right)\right], \quad (3.62)$$

$$= \frac{n^2\Delta_L}{9\tau^2R^{4(3/n-1)}}(E - V^+)(E - V^-), \quad (3.63)$$

where Ω and $V^\pm$ are the angular velocity of a locally nonrotating observer and the effective potentials, which are defined by

$$\Omega := \frac{j\tau(R^{6/n}-1)}{R^6\Delta_L}, \quad (3.64)$$

$$V^\pm := 2\Omega L_R \pm \frac{\sqrt{\Delta[j^2L_R^2 + \Delta_L R^6(L^2 + m^2R^2/4)]}}{\Delta_L R^6}. \quad (3.65)$$

The allowed region is $E > V^+$ or $E < V^-$ for $\Delta_L > 0$, whereas it is $\min[V^\pm] < E < \max[V^\pm]$ for $\Delta_L < 0$.

Equation (3.59) becomes

$$\frac{dT}{d\lambda} = \frac{4R^4\Delta_L}{\Delta}(E - 2\Omega L_R). \quad (3.66)$$

When $\Delta > 0$ and $\Delta_L > 0$, $E > V^0 := 2\Omega L_R$ follows. Thus, E must be positive for a particle with $\Omega L_R > 0$ moving forward with respect to the time coordinate T. Inside the VLS ($\Delta_L < 0$) where $\Delta > 0$, a particle with $E > V^0$ moves backward with respect to the coordinate T. One also verifies that the horizon $\Delta = 0$ is an infinite redshift surface for the time coordinate T, which is of course a coordinate artifact.

From (3.55) one finds $L^2 \geq L_R^2$. When the equality holds, $L_L = 0$ is satisfied. Thus, Eq. (3.56) implies that the particle motion is confined on the equatorial plane $\theta = \pi/2$ and Eq. (3.60) implies $\phi =$ constant. The same remark applies to the original metric (3.1), since this assertion only comes from the $U(2)$-symmetries of the solution.

It is clear from Eq. (3.62) that massless particles with $L = L_R = 0$ cannot cross the VLS. In the over-rotating case, the geodesics with $L_R = 0$ cannot cross the VLS either, since the right-hand side of (3.62) becomes negative before the VLS is reached.

In the case of $L_R \neq 0$, it is dependent on the parameters whether the geodesic particle moving forward can cross the VLS or not. When $j < 1$, ΩL_R diverges positively (negatively) as $R \to R_L + 0$ for the particle having the opposite (same) spin as the black hole. Hence, the particle with opposite angular momentum ($jL_R < 0$) cannot penetrate the VLS for $j < 1$. Similarly, when $j > 1$, ΩL_R diverges positively (negatively) as $R \to R_L + 0$ for the particle having the same (opposite) spin as the black hole. Thus, the particle with $j > 1$ never penetrates the VLS when it has the same spin as the hole $jL_R > 0$.

Though causal geodesics may cross the VLS, it is shown that they never encounter the singularity at $R = 0$ at least for $n = 2, 3$. For $L > |L_R|$, the function inside the square root of $V^\pm$ (3.65) becomes negative around $R = 0$, so that $V^\pm$ does not exist around $R = 0$ and has a confluent point inside the VLS, which prohibits geodesics to enter inside. For $L = |L_R|$, it can be easily shown that $V^+ < V^0 < V^-$ holds around $R = 0$ and they take the value $2\tau L_R/j$ at $R = 0$. It follows that geodesics with $E = 2\tau L_R/j$ may reach $R = 0$. For $n = 1$, this is indeed the case. By contrast, for $n = 2, 3$, $dV^0/dR < (>)0$ holds around $R = 0$ for $jL_R > (<)0$, which forbids the geodesics to hit the singularity since $E < V^0$ and $V^+ < E < V^-$ are the allowed region for the future-pointing particles. Accordingly, the singularity $R = 0$ has a repulsive nature. We can expect that geodesics also rarely reach the singularity in the dynamical settings.

E. Global structure

We are now ready to discuss the global structures of the time-dependent and rotating spacetime (3.1). The most useful visualization of the causal structure of a spacetime is the conformal diagram. To this end, it is necessary to find a two-dimensional (totally geodesic) integrable submanifold. Now the spacetime is regarded as an $\mathbb{R}^2$ bundle over S^3. Unfortunately, the distribution spanned by $\partial/\partial t$ and $\partial/\partial r$ is not integrable, forbidding us to have a foliation by a two-dimensional conformal diagram. The frame-dragging effect inevitably drives the ψ-motion.

Nevertheless, the two-dimensional metric

$$ds_2^2 = -\frac{\tau^2 f^2}{\Delta_L}dt^2 + \frac{dr^2}{f} \tag{3.67}$$

still contains some information about the spacetime structure and gives us useful visualization [52]. The above metric (3.67) is associated with the null geodesics with $\theta = \pi/2$ and $\phi =$ constant corresponding to $L = L_R = L_L = 0$: hence, they cannot penetrate the VLS, which is found to be a timelike or null surface. As in the BMPV case, the two-dimensional metric (3.67) is not Lorentzian inside the VLS.

From the analysis of the previous subsection, we found that the original time-independent metric (3.38) admits Killing horizons at $\Delta = 0$. In the nonrotating case ($j = 0$), the null surfaces $r = 0$ with $t = \pm\infty$ are also Killing horizons for the original spacetime [29], since the Killing vector is parallel to the generators of horizons.

When a rotation is present, we must be careful. Now, there exists a VLS (3.26), which is bounded below when $j > 1$ (see left and middle plots in Fig. 1), so that the past horizon $t \to -\infty$ may not exist (since we are focusing on the two-dimensional metric (3.67), no causal geodesics penetrate the VLS: inside the VLS is not the physical region of spacetime). Even if the near-horizon metric [29] admits some Killing horizons, we cannot immediately conclude that they are also Killing horizons in the original metric.

The analysis of singularities, asymptotic infinity, behaviors of VLS (Fig. 1), and the near-horizon geometries have provided us sufficient information to deduce Carter-Penrose diagrams. As a striking confirmation, we have solved the geodesic equations numerically and obtained the conformal diagrams displayed in Fig. 2, which may be summarized as follows (we have excluded the special case of the degenerate horizons).

(i) $n = 1$. The asymptotic region is approximated by an FLRW universe, obeying a decelerating expansion $a = (\bar{t}/\bar{t}_0)^{1/4}$ caused by a massless scalar field. Then, the null infinity $\mathcal{I}^-$ possesses an ingoing null structure. When $j < j_{(1)}$, two Killing horizons $R_\pm$ arise (3.42). Since the VLS diverges negatively as $r \to 0$ when $j < 1$ (right plots in Fig. 1), the conformal diagram is (I). Even if two horizons exist in the near-horizon geometry for $1 < j < j_{(1)}$, the VLS conceals the past horizon R_- (corresponding to $t \to -\infty$), since the VLS diverges positively as $r \to$ (left plots in Fig. 2). Then, diagram (II) is obtained. Note that the $R_- =$ constant surface asymptotically approaches null as $t \to \infty$, and R_L is timelike almost everywhere (it happens to be null precisely at one point). For the over-rotation $j > j_{(1)}$, no Killing horizons arise. Hence, the conformal diagram is (V).

(ii) $n = 2$. The spacetime approaches to the marginally accelerating universe, expanding linearly with cosmic time $a = \bar{t}/\bar{t}_0$. This is caused by the fluid with equations of state $P = -\rho/2$. For $\tau > 1/2$, there exist two Killing horizons (3.42), so that conformal diagram is the same as case (i); it is (I) for $0 < j < 1$, (II) for $1 < j < j_{(2)}$ and (V) for $j > j_{(2)}$. An essential difference from the $n = 1$ case arises when $\tau \leq 1/2$, in which case there exists an internal null infinity $\mathcal{I}^+_{\rm in}$ where $R \to \infty$ with $r \to 0$ and $t \to \infty$. Only ingoing null particles can get to $\mathcal{I}^+_{\rm in}$. The existence of internal null infinity can be shown by solving the geodesics asymptotically as in 4 dimensions [29]. It follows that conformal diagrams for $\tau \leq 1/2$ are (III) when $j < 1$ and (IV) when $j > 1$.

(iii) $n = 3$. The conformal diagrams are similar to the Kerr–de Sitter spacetime. Infinity $\mathcal{I}^+$ consists of a spacelike slice due to the acceleration of the universe. First, consider the case in which the near-horizon metric (3.38) admits three distinct horizons, $R_\pm$ and R_c. This occurs when $\sqrt{3/2} < \tau < 3\sqrt{3}/4$ with $j_{(3)-} < j < j_{(3)+}$ and $\tau > 3\sqrt{3}/4$ with $(0 \leq) j < j_{(3)+}$. We must take into account the fact that for $j < 1$ the VLS $t_{\rm VLS}(r)$ diverges negatively as $r \to 0$, which removes past horizons ($t \to -\infty$ and $r \to 0$ with tr^2 finite) in the near-horizon geometry (3.32). Therefore, when $\tau > 3\sqrt{3}/4$ with $(0 \leq) j < 1$ the conformal diagram is (VI), whereas it is (VI′) when $1 < j < j_{(3)+}$ with $\tau > 3\sqrt{3}/4$, or $(1<) j_{(3)-} < j < j_{(3)+}$ with $\sqrt{3/2} < \tau < 3\sqrt{3}/4$. These two are essentially the same: they constitute the different coordinate patches depending on the value of j. In (V′) the slice $t = 0$ and $r \to \infty$ with tr^2 finite comprises a null boundary. When there appears only a cosmological horizon R_c (i.e., $\tau < \sqrt{3/2}$, $\sqrt{3/2} < \tau < 3\sqrt{3}/4$ with $j < j_{(3)-}$, or $j > j_{(3)+}$ and $\tau > 3\sqrt{3}/4$ with $j > j_{(3)+}$), the spacetime diagram is (VII) for $j < 1$ and (VII′) otherwise. Again, (VII) and (VII′) are essentially identical. In (VII′), the slice $t = 0$ and $r \to \infty$ with tr^2 finite is also a null surface.

To summarize, the cases (I), (II), (VI), and (VI’) correspond to the rotating black-hole geometry.

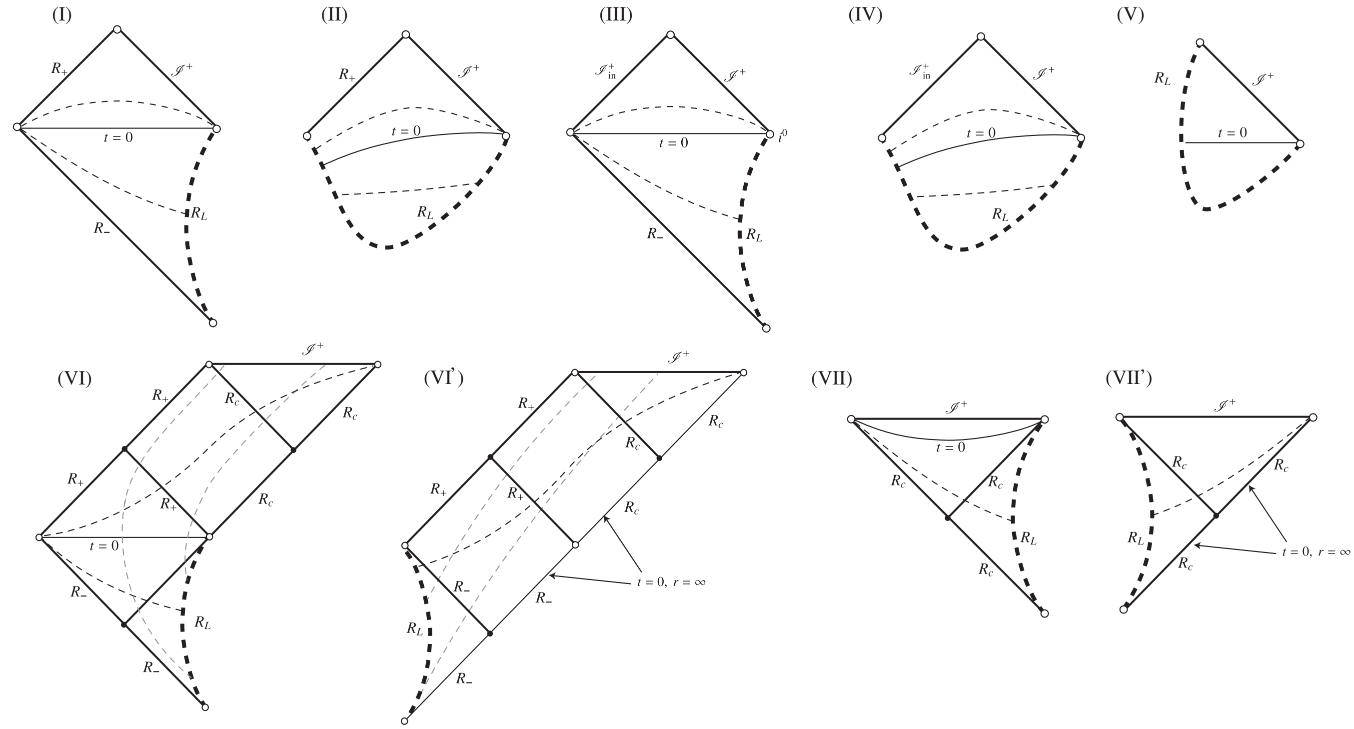

FIG. 2. Conformal diagrams of the two-dimensional spacetime (3.67), by which null geodesics with zero angular momentum is described. R_+, R_-, and R_c are all Killing horizons corresponding, respectively, to the black-hole event horizon, the white-hole horizon, and the cosmological horizon. The thick dotted curves represent the VLS. Thin black and gray dotted curves are $t =$ constant and $r =$ constant surfaces, respectively. White and black circles are infinities (including throat) and bifurcation surfaces. Since the two-dimensional metric (3.67) becomes Riemannian inside the VLS, the diagrams come to an end at R_L. We remark that we are formally writing the two-dimensional figures, but there still remains the angular motion because of the frame-dragging: these figures do not display all the causal information. Though these diagrams are restricted to the $r^2 > 0$ region, the spacetime can be extended across the null surfaces $R_\pm$ and R_c, which are nothing but the ordinary chart boundaries. The conformal diagrams are (I) for $n = 1$ with $j < 1$, and for $n = 2$ with $j < 1$ and $\tau > 1/2$, (II) for $n = 1$ with $1 < j < j_{(1)}$, and for $n = 2$ with $1 < j < j_{(2)}$ and $\tau > 1/2$, (III) for $n = 2$ with $\tau \leq 1/2$ and $j < 1$, (IV) for $n = 2$ with $\tau \leq 1/2$ and $j > 1$, (V) for $n = 1$ with $j > j_{(1)}$ and for $n = 2$ with $\tau > 1/2$ and $j > j_{(2)}$, whereas diagrams (VI–VII') correspond to $n = 3$: (VI) for $\tau > 3\sqrt{3}/4$ with $j < 1$, (VI') for $\tau > 3\sqrt{3}/4$ with $1 < j < j_{(3)+}$, and for $\sqrt{3/2} < \tau < 3\sqrt{3}/4$ with $j_{(3)-} < j < j_{(3)+}$, (VII) for $\tau < \sqrt{3/2}$ with $j < 1$ and for $\sqrt{3/2} < \tau < 3\sqrt{3}/4$ with $j < 1$, and (VII'') for $\tau < \sqrt{3/2}$ with $j > 1$, for $\sqrt{3/2} < \tau < 3\sqrt{3}/4$ with $1 < j < j_{(3)-}$ or $j > j_{(3)+}$ and for $\tau > 3\sqrt{3}/4$ with $j > j_{(3)+}$.

IV. DIMENSIONAL OXIDIZATION AND REDUCTION

In the previous sections, some black-hole solutions in the STU theory have been elaborated in the framework of the five-dimensional theory. We shall discuss in this section the lift-up and compactification procedure to other numbers of dimensions.

A. Lift-up to *M*-theory

The time-evolving and spatially inhomogeneous solutions in 4 and 5 dimensions were originally derived from the dimensional reduction of intersecting *M*-branes in eleven-dimensional supergravity. Now, we argue that the solutions of case (ii)—where two V_I's vanish—can be embedded in eleven-dimensional supergravity.

The eleven-dimensional supergravity action is given by

$$S_{11} = \frac{1}{2\kappa_{11}^2}\int\Big({}^{11}R\star_{11}1 - \frac{1}{2}\mathcal{F}\wedge\star_{11}\mathcal{F} - \frac{1}{6}\mathcal{A}\wedge\mathcal{F}\wedge\mathcal{F}\Big), \tag{4.1}$$

where $\mathcal{F} = d\mathcal{A}$ is the four-form field strength. The equations of motion are Einstein's equations,

$${}^{11}R_{AB} - \frac{1}{2}{}^{11}Rg_{AB} = \frac{1}{2\cdot 3!}\Big(\mathcal{F}_{ACDE}\mathcal{F}_B{}^{CDE} - \frac{1}{8}g_{AB}\mathcal{F}_{CDEF}\mathcal{F}^{CDEF}\Big), \tag{4.2}$$

and the equations

$$d\star_{11}\mathcal{F} + \frac{1}{2}\mathcal{F}\wedge\mathcal{F} = 0. \tag{4.3}$$

In this section, $A, B, \ldots$ denote the eleven-dimensional indices.

Let us consider the "intersecting $M2/M2/M2$ metric" of the following form [15],

$$ds_{11}^2 = ds_5^2 + X^1(dy_1^2 + dy_2^2) + X^2(dy_3^2 + dy_4^2) + X^3(dy_5^2 + dy_6^2), \tag{4.4}$$

$$\mathcal{A} = A^{(1)} \wedge dy_1 \wedge dy_2 + A^{(2)} \wedge dy_3 \wedge dy_4 + A^{(3)} \wedge dy_5 \wedge dy_6, \tag{4.5}$$

where the metric is independent of the brane coordinates $y_1, \ldots, y_6$. This solution is specified by the five-dimensional metric,

$$ds_5^2 = -(H_1H_2H_3)^{-2/3}(dt + \omega)^2 + (H_1H_2H_3)^{1/3}h_{mn}dx^m dx^n, \tag{4.6}$$

as well as three scalars $X^I (I = 1, 2, 3)$ and three one-forms $A^{(I)}$, which are given by

$$A^{(I)} = H_I^{-1}(dt + \omega), \qquad X^I = H_I^{-1}(H_1H_2H_3)^{1/3}. \tag{4.7}$$

Here, h_{mn} is the metric on the four-dimensional base space. $\omega = \omega_m dx^m$ is viewed as a one-form on the base space, i.e., $\omega_\mu V^\mu = 0$ where $V^\mu = (\partial/\partial t)^\mu$.

Since the metric ansatz (4.6) is independent of the coordinates $y_1, \ldots, y_6$, the solution can be dimensionally reduced to 5 dimensions. Noting that the six-torus T^6 has a constant volume $X^1X^2X^3 = 1$, it turns out that the five-dimensional metric ds_5^2 is the five-dimensional Einstein-frame metric. Thus, the metric ansatz (4.4) gives the five-dimensional action of gravity sector as

$$S_g = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g_5}\Big[{}^5R - \frac{1}{2}\sum_I (\nabla^\mu \ln X^I)(\nabla_\mu \ln X^I)\Big], \tag{4.8}$$

where we have used $X^1X^2X^3 = 1$. We can proceed with the form field sector analogously. Letting $F^{(I)} := dA^{(I)}$ denote the two-form field strengths, we find

$$\mathcal{F}_{ABCD}\mathcal{F}^{ABCD} = 12\sum_I (X^I)^{-2}F^{(I)}_{\mu\nu}F^{(I)\mu\nu}, \tag{4.9}$$

$$\mathcal{A}\wedge\mathcal{F}\wedge\mathcal{F} = 2(A^{(1)}\wedge F^{(2)}\wedge F^{(3)} + A^{(2)}\wedge F^{(3)}\wedge F^{(1)} + A^{(3)}\wedge F^{(1)}\wedge F^{(3)})\wedge \mathrm{Vol}(T^6), \tag{4.10}$$

then the Lagrangian for the gauge fields reads

$$S_F = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g_5}\Big[-\frac{1}{4}\sum_I (X^I)^{-2}F^{(I)}_{\mu\nu}F^{(I)\mu\nu} + \frac{1}{12}\epsilon^{\mu\nu\rho\sigma\tau}(A^{(1)}_\mu F^{(2)}_{\nu\rho}F^{(3)}_{\sigma\tau} + A^{(2)}_\mu F^{(3)}_{\nu\rho}F^{(1)}_{\sigma\tau} + A^{(3)}_\mu F^{(1)}_{\nu\rho}F^{(2)}_{\sigma\tau})\Big], \tag{4.11}$$

where $\epsilon_{\mu\nu\rho\sigma\tau}$ is the volume element compatible with the five-dimensional metric ds_5^2 and $\kappa_5^2 := \kappa_{11}^2/\mathrm{Vol}(T^6)$. It follows that the reduced action $S_5 = S_g + S_F$ exactly coincides with that of the STU theory; the five-dimensional minimal ungauged ($\mathfrak{g} = 0$) $U(1)^3$ supergravity (2.1) with the metric of the potential space given by

$$G_{IJ} = \frac{1}{2}\mathrm{diag}[(X^1)^{-2}, (X^2)^{-2}, (X^3)^{-2}], \tag{4.12}$$

and the constants C_{IJK} are totally symmetric in (IJK), with $C_{123} = 1$ and 0 otherwise.

If we consider three equal harmonics $H_1 = H_2 = H_3 := H$ (i.e., $X^I = 1$, $A^1 = A^2 = A^3 =: (2/\sqrt{3})A$ and $F = dA$), all scalar fields are trivial. Then, the action $S_5 = S_g + S_F$ reduces to that of the minimal supergravity in 5 dimensions [7], the action of which is given by

$$S_5 = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g_5}\Big({}^5R - F_{\mu\nu}F^{\mu\nu} + \frac{2}{3\sqrt{3}}\epsilon^{\mu\nu\rho\sigma\tau}A_\mu F_{\nu\rho}F_{\sigma\tau}\Big). \tag{4.13}$$

1. Supersymmetric solution in ungauged theory

Let us first consider the case where the five-dimensional spacetime is supersymmetric, i.e., there exists a nontrivial Killing spinor satisfying (2.18) and (2.19) with $\mathfrak{g} = 0$ [11,15]. For the timelike family of solutions for which $V = \partial/\partial t$ is a timelike Killing vector, the supersymmetry requires that the base space is hyper-Kähler and the Maxwell fields are expressed as

$$F^{(I)} = d[fX^I(dt + \omega)] + \Theta^I, \tag{4.14}$$

where Θ^I are self-dual two-forms on the base space satisfying $X_I\Theta^I = -f(d\omega + \star_h d\omega)/3$. The Bianchi identity for $F^{(I)}$ requires $d\Theta^I = 0$, and the Maxwell equation leads to

$${}^h\Delta(f^{-1}X_I) = \frac{1}{12}C_{IJK}\Theta^{(J)mn}\Theta^{(K)}_{mn}. \tag{4.15}$$

For $\Theta^I = 0$, the solution reduces precisely to the one assumed for the pseudo-supersymmetric solutions (2.24).

If we set $h_{mn} = \delta_{mn}$, $\omega = 0$, and $H_I = 1 + Q_I/r^2$, the metric describes the standard static intersecting $M2/M2/M2$-branes with corresponding charges Q_I. In this case, the eleven-dimensional solution admits a Killing spinor $\varepsilon = (H_1H_2H_3)^{-1/6}\varepsilon_\infty$ with

$$i\Gamma^{0\hat{y}_1\hat{y}_2}\varepsilon_\infty = \varepsilon_\infty, \qquad i\Gamma^{0\hat{y}_3\hat{y}_4}\varepsilon_\infty = \varepsilon_\infty, \qquad i\Gamma^{0\hat{y}_5\hat{y}_6}\varepsilon_\infty = \varepsilon_\infty, \tag{4.16}$$

satisfying

$$\left[{}^{11}\mathcal{D}_A + \frac{i}{288}(\Gamma_A{}^{BCDE} - 8\delta_A{}^B\Gamma^{CDE})\mathcal{F}_{BCDE}\right]\varepsilon = 0, \tag{4.17}$$

where Γ is the eleven-dimensional gamma matrix. It deserves to be mentioned that the fact that ds_5^2 in (4.4) is the five-dimensional Einstein-frame metric means that the causal pathologies are not cured by lifting up to M-theory.

2. Dynamically intersecting M2/M2/M2-branes

Let us next consider the nonsupersymmetric case where the metric is time-dependent. The importance of dynamically intersecting branes in supergravity theory lies in their applications to cosmology and dynamical black holes. The dynamically intersecting branes without rotation are analyzed in detail in [30]. We are going to discuss its rotating version.

The potential $V = 27C^{IJK}V_IV_JX_K$ vanishes identically for the STU theory with the case (ii) $V_1 \neq 0$, $V_2 = V_3 = 0$. This lies at the heart of why the pseudo-supersymmetric solution of the case (ii) derived in Sec. II [see Eq. (2.41)] can be embedded into eleven-dimensional supergravity. Note, however, that the eleven-dimensional configuration is no longer (true nor fake) supersymmetric. Nevertheless, the five-dimensional pseudo-supersymmetry justifies the mechanical equilibrium of dynamically intersecting branes. If $\bar{H}_1$, H_2, and H_3 represent harmonics with a single point source on the Euclidean 4-space, the solution describes the dynamically intersecting rotating $M2/M2/M2$-branes obeying the harmonic superposition rule. For the vanishing charges $\bar{H}_1 = 0$ and $H_2 = H_3 = 1$, the background metric is obtained, which is the eleven-dimensional ''rotating'' Kasner universe,

$$\begin{aligned} ds_{11}^2 = &-\left[d\bar{t} + \frac{J}{2r^2(\bar{t}/\bar{t}_0)^{1/2}}(\sin^2\vartheta d\phi_1 + \cos^2\vartheta d\phi_2)\right]^2 \\ &+ (\bar{t}/\bar{t}_0)^{1/2}[dr^2 + r^2(d\vartheta^2 + \sin^2\vartheta d\phi_1^2 \\ &+ \cos^2\vartheta d\phi_2^2)] + (\bar{t}/\bar{t}_0)^{-1}(dy_1^2 + dy_2^2) \\ &+ (\bar{t}/\bar{t}_0)^{1/2}(dy_3^2 + \cdots + dy_6^2). \end{aligned} \tag{4.18}$$

Here, $\bar{t} \propto t^{2/3}$ measures the cosmic time. The eleven-dimensional universe collapses into the y_1-y_2 directions and expands in other directions [53]. It follows that the three kinds of branes are intersecting in the background of the Kasner universe. The case of $J = 0$ recovers the conventional vacuum Kasner solution.

3. The cases (iii) and (iv)

For the cases (iii) and (iv), there exists a nonzero potential in the fake supergravity theory. It might be reasonable to expect that the FLRW universe may be realized from the viewpoint of intersecting branes, which are the fundamental constituents of supergravity. Assuming the brane intersection rule [30] and making the nonzero vacuum expectation values of the four-form $\mathcal{F}$, we have tried to uplift the solutions (3.1) with (3.2) into 11 dimensions, but failed. Whether the present solutions are obtainable from the brane picture is an outstanding issue at present. We leave this possibility to future work.

B. Compactification to 4 dimensions

When discussing the FLRW spacetime, it is much more reasonable to argue within the four-dimensional effective theory. In this section we shall demonstrate how to achieve this.

1. Dimensional reduction via Gibbons-Hawking space and Kaluza-Klein black hole

One can obtain the four-dimensional solutions in [29,31] via dimensional reduction of five-dimensional solutions (2.23) as follows. We employ the Gibbons-Hawking space [54] as a four-dimensional base space,

$$ds_{\mathcal{B}}^2 = h^{-1}(dx^5 + \chi_i dx^i)^2 + h\delta_{ij}dx^i dx^j, \tag{4.19}$$

where $i, j, \ldots$ denote three-dimensional indices (hence, no distinction is made for upper and lower indices) and

$$\vec{\nabla} \times \vec{\chi} = \vec{\nabla} h. \tag{4.20}$$

$\vec{\nabla}$ is the derivative operator on the flat Euclidean 3-space and the usual vector convention will henceforth be used for the quantities on the Euclidean space. The integrability condition of (4.20) implies that h is a harmonic function on the Euclidean space $\vec{\nabla}^2 h = 0$. In the Gibbons-Hawking base space, $\partial/\partial x^5$ is a Killing vector preserving the three complex structures, which are given by [55]

$$\mathfrak{J}^{(i)} = (dx^5 + \chi) \wedge dx^i - \frac{1}{2}h\epsilon_{ijk}dx^j \wedge dx^k. \tag{4.21}$$

The orientation is chosen in such a way that the complex structures are anti-self-dual, viz., the volume form is given by $hdx^5 \wedge dx^1 \wedge dx^2 \wedge dx^3$. Under the change of Killing coordinates $x^5 \to x^5 + g(x^i)$, where g is an arbitrary function of x^i, χ_i transforms as $\chi_i \to \chi_i - \partial_i g$ and h is unchanged in order to preserve the metric form. Prime examples of Gibbons-Hawking space are the flat space ($h = 1$ or $M/|\vec{x}|$), the Taub-NUT space ($h = 1 + M/|\vec{x}|$), and the Eguchi-Hanson space ($h = M/|\vec{x} - \vec{x}_1| + M/|\vec{x} - \vec{x}_2|$).

Assuming that the vector field $\partial/\partial x^5$ is also a Killing vector for the whole five-dimensional spacetime, it turns out that functions H_I are also harmonics on the Euclidean 3-space $\vec{\nabla}^2 H_I = 0$ (and the linear time-dependence remains intact). Let ω decompose as

$$\omega = \omega_5(dx^5 + \chi_i dx^i) + \omega_i dx^i, \tag{4.22}$$

and let us write the metric as

$$ds_5^2 = \Lambda[dx^5 + \chi_i dx^i - f^2\omega_5\Lambda^{-1}(dt + \omega_i dx^i)]^2 - fh^{-1}\Lambda^{-1}(dt + \omega_i dx^i)^2 + f^{-1}h\delta_{ij}dx^i dx^j, \tag{4.23}$$

$$=: e^{-4\sigma/\sqrt{3}}(dx^5 + B_\alpha dx^\alpha)^2 + e^{2\sigma/\sqrt{3}}g_{\alpha\beta}dx^\alpha dx^\beta, \tag{4.24}$$

where $\Lambda = f^{-1}h^{-1} - f^2\omega_5^2$, $g_{\alpha\beta}$ is the four-dimensional Einstein-frame metric, $B_\alpha dx^\alpha dx^\alpha = \chi_i dx^i - f^2\omega_5\Lambda^{-1}(dt + \omega_i dx^i)$ is the Kaluza-Klein gauge field, and $\sigma = -\frac{\sqrt{3}}{4}\ln\Lambda$ is a dilaton field. The anti-self-duality of Sagnac curvature $d\omega + \star_h d\omega = 0$ [42] reduces to

$$\vec{\nabla}\times\vec{\omega} = h^2\vec{\nabla}(h^{-1}\omega_5). \tag{4.25}$$

The integrability condition of this equation is $\vec{\nabla}^2\omega_5 = 0$, i.e., ω_5 is another harmonic function. The Einstein-frame metric $g_{\alpha\beta}$ is given by

$$ds_4^2 = -\Xi(dt + \omega_i dx^i)^2 + \Xi^{-1}\delta_{ij}dx^i dx^j, \tag{4.26}$$

with

$$\Xi := fh^{-1}\Lambda^{-1/2}, \tag{4.27}$$

where $\vec{\omega}$ is determined by (4.25) up to a gradient.

When ω_5 is proportional to h, Eq. (4.25) implies that $\vec{\omega}$ is written as a gradient of some scalar function, which can be made to vanish by redefinition of t and harmonic functions if we work in a "Coulomb gauge" $\vec{\nabla}\cdot\vec{\omega} = 0$. Thus, the four-dimensional rotation vanishes ($\vec{\omega} = 0$) in this case. If two harmonics are equal ($H_2 = H_3$) in the STU theory and $\omega_5 = 0$, the four-dimensional solutions given in [28,29] except the $n_T = 4$ case are recovered. Since the dimensional reduction does not spoil the fraction of supersymmetries, it turns out that the four-dimensional solutions in [28,29] are also pseudo-supersymmetric in the context of fake supergravity.

The resulting four-dimensional theory involves many scalar and vector multiplets. To see this, we consider the general Kaluza-Klein ansatz (4.24). Defining

$$H_{\alpha\beta} = 2\partial_{[\alpha}B_{\beta]}, \qquad A^{(I)} = A'^{(I)}_\alpha dx^\alpha + \theta^{(I)}dx^5,$$
$$F'^{(I)}_{\alpha\beta} = 2\partial_{[\alpha}A'^{(I)}_{\beta]}, \qquad {}^4F^{(I)}_{\alpha\beta} = F'^{(I)}_{\alpha\beta} - 2\partial_{[\alpha}\theta^{(I)}B_{\beta]}, \tag{4.28}$$

one finds that the five-dimensional theory (2.1) leads to the following four-dimensional effective Lagrangian,

$$\begin{aligned} L_4 = {}&{}^4R - 2k^2Ve^{2\sigma/\sqrt{3}} - 2g^{\alpha\beta}\partial_\alpha\sigma\partial_\beta\sigma \\ &- \frac{1}{4}e^{-2\sqrt{3}\sigma}H_{\alpha\beta}H^{\alpha\beta} - \mathcal{G}_{AB}g^{\alpha\beta}\partial_\alpha\phi^A\partial_\beta\phi^B \\ &- \frac{1}{2}e^{-2\sigma/\sqrt{3}}G_{IJ}\,{}^4F^{(I)}_{\alpha\beta}\,{}^4F^{(J)\alpha\beta} \\ &- e^{4\sigma/\sqrt{3}}G_{IJ}g^{\alpha\beta}\partial_\alpha\theta^{(I)}\partial_\beta\theta^{(J)} \\ &- \frac{1}{8}\epsilon^{\alpha\beta\gamma\delta}C_{IJK}\theta^{(I)}\Big({}^4F^{(J)}_{\alpha\beta}\,{}^4F^{(K)}_{\gamma\delta} - \theta^{(J)}\cdot{}^4F^{(K)}_{\alpha\beta}H_{\gamma\delta} \\ &+ \frac{1}{3}\theta^{(J)}\theta^{(K)}H_{\alpha\beta}H_{\gamma\delta}\Big). \end{aligned} \tag{4.29}$$

Thus, the four-dimensional effective theory derived from the Lagrangian (2.1) comprises $2N$ scalars (σ, ϕ^A, $\theta^{(I)}$) and $N+1$ gauge fields ($A'^{(I)}_\mu$, B_μ) in general. Meanwhile, its supersymmetric solution is specified by $N+2$ harmonics (H_I, h, ω_5).

As an obvious application, let us consider the case where the four-dimensional base space ($\mathcal{B}$, h_{mn}) is the Taub-NUT space. The Taub-NUT metric can be written as a Gibbons-Hawking form (4.19) as

$$ds_{\rm TN}^2 = \left(\varepsilon + \frac{M}{\rho}\right)^{-1}M^2(\sigma_R^3)^2 + \left(\varepsilon + \frac{M}{\rho}\right)[d\rho^2 + \rho^2\{(\sigma_R^1)^2 + (\sigma_R^2)^2\}], \tag{4.30}$$

where $\rho := |\vec{x}|$ and $M(>0)$ corresponds to the NUT parameter. For later convenience, we have introduced a parameter ε, which is unity for the Taub-NUT space.

A natural five-dimensional background ($|\vec{x}| \to \infty$) in this case is

$$ds_{\rm GPS}^2 = -d\bar{t}^2 + a(\bar{t})^2 ds_{\rm TN}^2. \tag{4.31}$$

where the scale factor $a(\bar{t})$ is given by (2.49) and (2.50). This is the Gross-Perry-Sorkin-type monopole [56] immersed in the FLRW universe. At large distance $|\vec{x}| \to \infty$ it may be rewritten as a $U(1)$ fibration over the FLRW universe $M_5 \simeq M_4 \times S^1$,

$$ds_{\rm GPS}^2 = ds_{\rm FLRW}^2 + Ma(\bar{t})^2\rho(\sigma_R^3)^2. \tag{4.32}$$

Thus, the spacetime is effectively four-dimensional at infinity. Since the metric (4.31) admits a homothetic Killing field, one can analyze its causal structures analytically. The conformal diagrams are the same as the five-dimensional FLRW universe.

Recalling the fact that the flat Euclidean space is recovered when $\varepsilon = 0$ in the metric (4.30) (note that in this case M is not the NUT charge), the spacetime structure as $\rho \to 0$ (with or without $t \to \pm\infty$) is identical to that for the solution (3.1). Then, the vicinity of horizons is indeed five-dimensional. Therefore, this geometry describes a Kaluza-Klein-type black hole [57].

2. A caged black hole

As discussed in [58,59] for the supersymmetric case, a caged black-hole geometry is obtained by superimposing an infinite number of black holes aligned in one direction with an equal separation. Since the present time-dependent solution found in Sec. II is linearized in space, we can construct similar configurations easily. Decomposing the Euclidean 4-space coordinates as $x^m = (x, y, z, w)$ with the orientation $dx \wedge dy \wedge dz \wedge dw$ and putting the same point sources along the w axis with an equal spacing of $2\pi R_5$, we obtain

$$H_S = 1 + Q_S \sum_{k=-\infty}^{\infty} \frac{1}{\rho^2 + (w + 2\pi k R_5)^2} = 1 + \frac{Q_S}{2R_5^2} \frac{\sinh\bar{\rho}}{\bar{\rho}(\cosh\bar{\rho} - \cos\bar{w})}, \tag{4.33}$$

$$H_T = \frac{t}{t_0} + Q_T \sum_{k=-\infty}^{\infty} \frac{1}{\rho^2 + (w + 2\pi k R_5)^2} = \frac{t}{t_0} + \frac{Q_T}{2R_5^2} \frac{\sinh\bar{\rho}}{\bar{\rho}(\cosh\bar{\rho} - \cos\bar{w})}, \tag{4.34}$$

$$\omega_{\phi_1} = J \sum_{k=-\infty}^{\infty} \frac{x^2 + y^2}{[\rho^2 + (w + 2\pi k R_5)^2]^2} = \frac{J}{4R_5^2} \frac{(\bar{x}^2 + \bar{y}^2)}{\bar{\rho}^2} \left[\frac{(\cosh\bar{\rho}\cos\bar{w} - 1)}{(\cosh\bar{\rho} - \cos\bar{w})^2} + \frac{\sinh\bar{\rho}}{\bar{\rho}(\cosh\bar{\rho} - \cos\bar{w})} \right], \tag{4.35}$$

$$\omega_{\phi_2} = J \sum_{k=-\infty}^{\infty} \frac{z^2 + (w + 2\pi k R_5)^2}{[\rho^2 + (w + 2\pi k R_5)^2]^2} = \frac{J}{4R_5^2} \left[-\frac{(\bar{x}^2 + \bar{y}^2)}{\bar{\rho}^2} \frac{(\cosh\bar{\rho}\cos\bar{w} - 1)}{(\cosh\bar{\rho} - \cos\bar{w})^2} + \frac{(\bar{\rho}^2 + \bar{z}^2)}{\bar{\rho}^2} \frac{\sinh\bar{\rho}}{\bar{\rho}(\cosh\bar{\rho} - \cos\bar{w})} \right], \tag{4.36}$$

where $\rho^2 \equiv x^2 + y^2 + z^2$, and we have introduced dimension-free coordinates $\bar{x}^m = x^m/R_5$ and $\bar{\rho} = \rho/R_5$. To derive these expressions we have used a series expansion:

$$\sum_{k=-\infty}^{\infty} \frac{1}{\xi^2 + (\eta + 2\pi k)^2} = \frac{\sinh\xi}{2\xi(\cosh\xi - \cos\eta)}. \tag{4.37}$$

Since this solution is periodic in the w direction by identifying $w = 0$ and $2\pi R_5$, it can be regarded as a deformed BMPV "black hole" in a compactified five-dimensional spacetime ($0 \le w \le 2\pi R_5$) with pseudo-supersymmetry.

Introducing the three-dimensional spherical coordinates (ρ, Θ, Φ), which are defined by

$$x = \rho \sin\Theta \cos\Phi, \quad y = \rho \sin\Theta \sin\Phi, \quad z = \rho \cos\Theta, \tag{4.38}$$

the four-dimensional Einstein-frame metric in the asymptotic region ($\rho \gg \pi R_5$) reads

$$d\bar{s}_4^2 = -f^{3/2}(d\bar{t} + \bar{\omega}_\Phi d\Phi)^2 + f^{-3/2}[d\bar{\rho}^2 + \bar{\rho}^2(d\Theta^2 + \sin^2\Theta d\Phi^2)], \tag{4.39}$$

where

$$f = \left(1 + \frac{1}{2R_5^2}\frac{Q_S}{\bar{\rho}}\right)^{-n/3}\left(\frac{t}{t_0} + \frac{1}{2R_5^2}\frac{Q_T}{\bar{\rho}}\right)^{-1+n/3}, \quad \bar{\omega}_\Phi = \frac{1}{4R_5^3}\frac{J}{\bar{\rho}}\sin^2\Theta. \tag{4.40}$$

In the asymptotic limit $\rho \to \infty$, the metric (4.39) describes an FLRW universe with the power exponent of the scale factor being $p = 1/(4 - n)$. One might therefore expect that this solution describes a caged black hole in the effective four-dimensional FLRW universe. However, we have to be careful to judge whether it is a black hole or not. A two-black-hole system in the Kastor-Traschen spacetime [the case (iv) without rotation] will collide and merge to form a single black hole in the contracting universe ($t_0 < 0$). In the expanding universe, the solution describes the time-reversal one. Namely, it corresponds to the two-white-hole system, since one object disrupts into two objects, which is possible for a white hole but not for a black hole. In the present case we have infinite numbers of point sources before identification, so that we can expect a similar result. It therefore appears that the object in the expanding universe corresponds to a splitting "white string" into an array of white holes. In order to clarify this rigorously, we have to analyze (numerically) the horizons of a multi-object system in the expanding universe. One especially important question to be answered is whether black holes will collide in a contracting universe for any value of n.

V. CONCLUDING REMARKS

We have presented pseudo-supersymmetric solutions to five-dimensional fake supergravity coupled to arbitrary $U(1)$ gauge fields and scalar fields. The noncompact gaugings of R-symmetry correspond to the Wick rotation of the gauge coupling constant ($\mathfrak{g} \to ik$). Since the bosonic action is not charged with respect to R-symmetry, no ghosts appear in this sector, i.e., all kinetic terms possess the correct sign. The net effect of imaginary coupling produces a positive potential for the scalar fields. Hence, the background spacetime is generally dynamical, contrary to the supersymmetric case.

The metric solves the first-order Killing spinor equation, which automatically guarantees that the Einstein equations and the scalar-field equations are satisfied if the Maxwell equations are solved. The solution is specified by time-dependent and time-independent harmonics H_I on a hyper-Kähler base space. This encodes the balances of forces of the solution: the gravitational attraction is adjusted to cancel the electromagnetic repulsive force (the scalar fields can contribute both sides depending on the potential). We specialized to the case in which a single point source on the Euclidean 4-space and explored its physical properties. The solutions we found are the rotating generalizations of our previous solutions [29,31] describing a black hole in the FLRW universe. The present metric has four parameters: the Maxwell charge Q, the angular momentum J, the number of time-dependent harmonics n, and the ratio of energy densities of the Maxwell field and the scalar field at the horizon τ. The spacetime approaches to the rotating $\mathrm{AdS}_2 \times S^3$ for small radii, while it asymptotes to the FLRW cosmology for large radii. Thus, the solution is a BMPV black hole immersed in the time-dependent background cosmology. Except the asymptotic de Sitter case, one cannot introduce a stationary coordinate patch even in the single-centered case. Though we have made some simplifications, it turns out that the solution enjoys much richer physical properties than stationary ones.

The analysis of near-horizon geometry uncovers that the horizon is described by a Killing horizon. Hence, the ambient materials fail to accrete onto the black hole irrespective of the dynamical background. This property may be attributed to the pseudo-supersymmetry. The BPS solution maintains equilibrium, forbidding the horizon to grow.

An important issue to be noted is that the event horizon is not extremal in general. This is due to the fact that the event horizon is not generated by the coordinate vector field in the metric (2.23). Furthermore, the event horizon is rotating, i.e., the event horizon is generated by a linear combination of time and angular Killing vectors (3.39). This is in sharp contrast to the supersymmetric BMPV black hole with vanishing angular velocity. The nonvanishing angular velocity of the horizon indicates that there exists an ergoregion lying strictly outside the horizon. The presence of an ergoregion implies the possibility of a rotating energy removal process via the Penrose process and the superradiant scattering [60]. We can find that this is indeed the case for $n = 3$, as shown in Appendix B. For other values of n, the energy of a particle and a wave is not conserved, so it is not a straightforward issue to conclude whether such an energy extraction process is actually realizable under a dynamical setting. This is an interesting future work to be argued.

We have also revealed that rotating solutions generically suffer from causal violation in the neighborhood of singularities. The pseudo-supersymmetry cannot elude naked time machines. The reason is obvious: the (pseudo-) supersymmetry variations (2.21) and (2.22) are local, so that they make no direct mention of a global structure of spacetime such as closed timelike curves. In particular, the timelike singularity $t = t_s(r)$ in the $r^2 > 0$ domain is repulsive.

The original time-dependent equilibrium solution was derived via compactification of $M2/M2/M5/M5$-branes in eleven-dimensional supergravity [30]. We discussed in section IVA 2 that the present metric with a single time-dependent harmonic function can be embedded into 11 dimensions, describing a dynamically intersecting $M2/M2/M2$-branes in a rotating Kasner universe. It is shown that the four-dimensional solution [30] was also derived from compactification of a five-dimensional solution on the Gibbons-Hawking space. Unfortunately, such a lift-up procedure fails to act as a chronology protector. It is of particular interest to see whether it oxidizes to a causally well-behaved solution in ten-dimensional supergravity, as in [50]. It appears appealing to examine if the occurrence of closed timelike curves corresponds to the loss of unitarity in the context of de Sitter/CFT correspondence.

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Note added.—During the completion of this work, we noticed the work of [64], which classifies all the pseudo-supersymmetric solutions of the theory (2.1). It is intriguing to examine if more general classes of solutions admit black-hole horizons in the expanding universe.

APPENDIX A: DILATONIC "BLACK HOLE" IN THE FLRW UNIVERSE

In the body of text, we considered several gauge fields in order to make the horizon area nonvanishing. To see this more concretely, let us consider the four-dimensional Einstein-Maxwell-dilaton gravity in which a single gauge field exists,

$$S = \frac{1}{2\kappa_4^2} \int (R \star_4 1 - 2d\sigma \wedge \star_4 d\sigma - 2e^{-2\alpha\sigma} F \wedge \star_4 F), \tag{A1}$$

where α is a coupling constant. The BPS equations are [61]

$$\left(\mathcal{D}_\alpha + \frac{i}{4\sqrt{1+\alpha^2}} e^{-\alpha\sigma} \gamma^{ab} \gamma_\alpha F_{ab}\right) \epsilon = 0, \tag{A2}$$

$$\left(\gamma^\alpha \partial_\alpha \sigma - \frac{i\alpha}{2\sqrt{1+\alpha^2}} e^{-\alpha\sigma} \gamma^{ab} F_{ab}\right) \epsilon = 0. \tag{A3}$$

[We remark that the second term in the dilatino Eq. (A3) has a factor 2 discrepancy with the result in [61], which seems to be a typo.] This theory admits a static and spherically symmetric black-hole solution [33], whose BPS limit is given by

$$ds^2 = -U^{-2/(1+\alpha^2)}dt^2 + U^{2/(1+\alpha^2)}\delta_{ij}dx^i dx^j,$$
$$A = \frac{1}{\sqrt{1+\alpha^2 U}}dt, \tag{A4}$$
$$\sigma = -\frac{\alpha}{1+\alpha^2}\ln U,$$

where $U = 1 + Q/|\vec{x}|$. This metric admits a Killing spinor $\epsilon = U^{-1/[2(1+\alpha^2)]}\epsilon_\infty$, where ϵ_∞ denotes the constant spinor (corresponding to the asymptotic value of ϵ) satisfying $i\gamma^0\epsilon_\infty = \epsilon_\infty$. We find that any harmonic function U on the flat 3-space solves the Maxwell equations, hence this metric, which is a cousin of a Majumdar-Papapetrou solution, can describe multiple configurations.

This solution can be immersed in an FLRW background by setting $U = t/t_0 + \bar{U}(x)$ where $\bar{U}$ is any harmonic function, and by introducing a Liouville-type exponential potential

$$V = V_0 e^{2\alpha\sigma}, \qquad V_0 = \frac{2(3-\alpha^2)}{(1+\alpha^2)^2 t_0^2}. \tag{A5}$$

This is a generalization of the solution given in [62] to any values of α. This spacetime is dynamical and approaches the flat FLRW universe filling with the fluid of equation of state $P = [(2\alpha^2 - 3)/3]\rho$. Unfortunately, the metric fails to have a regular horizon in either case. These solutions exhibit timelike singularities at $U = 0$: the $r \to 0$ limit fails to give a throat geometry and the well-defined scaling limit does not exist either. This illustrates that only a single gauge field cannot sustain a black hole.

Finally, we briefly comment on the $\alpha = \sqrt{3}$ case, in which the theory can be oxidized to the five-dimensional vacuum Einstein gravity ${}^5R_{\mu\nu} = 0$ via the Kaluza-Klein lift (4.24). When $U = 1 + Q/|\vec{x}|$, the five-dimensional metric admits a covariantly constant *null* Killing vector $V^\mu = (\partial/\partial t)^\mu$, hence the spacetime describes a pp-wave. This means that the BPS solution (A4) belongs to the null family of solutions (see Eq. (4.42) of [7]), so its time-dependent generalization $U = t/t_0 + Q/|\vec{x}|$ does not give a black hole.

APPENDIX B: SUPERRADIANCE FROM THE KLEMM-SABRA SOLUTION

We have found that the black holes preserving pseudo-supersymmetry (2.40), (2.41), (2.42), and (2.43) are rotating and possess an ergoregion. Hence, we expect superradiance. For a spacetime which is an asymptotically FLRW universe, however, it is difficult to argue the wave propagation since the background is dynamical: the particle energy with an asymptotic observer is not conserved. In order to discuss the superradiant phenomena without such an ambiguity, we shall address the wave propagation in the background of the Klemm-Sabra solution (2.43) [case (iv)], in which case the particle energy with respect to an observer resting at the cosmological horizon is conserved since we are able to introduce a stationary coordinate patch (2.46) [we shall restrict our example to the under-rotating case and drop the primes in the coordinates (2.46)].

Since the stationary Killing field for an observer rest at the cosmological horizon becomes spacelike inside the ergoregion, the energy measured by that observer can be negative. Hence, if a wave is scattered off by the black hole, these negative energy modes are excited and fall into the black hole, allowing the outside observer to have an amplified wave coming out of the horizon.

For the purpose of simplicity, let us consider a massless scalar field Ψ, which evolves according to

$$\nabla^\mu\nabla_\mu\Psi = 0. \tag{B1}$$

Assuming

$$\Psi = e^{-i\omega t + im_1\phi_1 + im_2\phi_2} r^{-1} R(r)\Theta(\vartheta), \tag{B2}$$

the massless scalar-field Eq. (B1) is separable. The angular equation is the spin-weighted spherical harmonic with spin weight $s = (m_1 - m_2)/2$. The angular function Θ satisfies

$$\frac{1}{\sin\vartheta\cos\vartheta}\frac{d}{d\vartheta}\left(\sin\vartheta\cos\vartheta\frac{d}{d\vartheta}\Theta\right) + \left[\ell(\ell+2) - \frac{m_1^2}{\sin^2\vartheta} - \frac{m_2^2}{\cos^2\vartheta}\right]\Theta = 0, \tag{B3}$$

where $\ell = 0, 1, 2\ldots.$ Incidentally, $\Theta e^{im_1\phi_1 + im_2\phi_2}$ is proportional to the Wigner D-function, an irreducible representation of $SU(2)$ [63]. Note that the above angular equation does not involve ω, contrary to the Kerr case.

Define the tortoise coordinate r_* by

$$dr_* = \frac{2t_0}{r\Delta_{KS}}, \qquad \Delta_{KS} := 1 - \frac{Hr^2}{4t_0^2} + \frac{J^2}{4t_0^2 r^4}, \tag{B4}$$

so that $r_* \to \infty$ as $r \to r_c$ and $r_* \to -\infty$ as $r \to r_+$, where r_+ and $r_c(>r_+)$ denote, respectively, the loci of event and cosmological horizons with $\Delta_{KS}(r_+) = \Delta_{KS}(r_c) = 0$. It follows that the radial equation obeys the Schrödinger-type equation

$$\frac{d^2}{dr_*^2}R + \left[\left(\omega - \frac{(m_1+m_2)J}{4t_0^2 r^2}\right)^2 - \frac{\Delta_{KS}}{4t_0^2}\left\{\ell(\ell+2) + 4t_0^2\omega^2 + \frac{d}{dr}(r\Delta_{KS})\right\}\right]R = 0. \tag{B5}$$

It turns out that the reflected wave is more amplified than the incident wave if the frequency lies in the superradiant regime

$$\frac{(m_1+m_2)J}{4t_0^2 r_c^2} < \omega < \frac{(m_1+m_2)J}{4t_0^2 r_+^2}. \tag{B6}$$

Such a superradiant amplification is characteristic of a rotating black hole with an ergoregion. This phenomenon does not occur for the supersymmetric black hole, for which the stationary Killing field is always timelike outside the horizon. This means that in the pseudo-supersymmetric case, the energy measured by a local observer is not necessarily positive, which makes super-radiance possible. For the black holes in cases (ii) and (iii), or even for those with an arbitrary value of n, we expect that a similar superradiant phenomena will occur, although there exists a technical difficulty to define a particle state (or the positive frequency states) in a time-dependent spacetime.

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