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Charge separation instability in an unmagnetized disk plasma around a Kerr black hole

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In almost all of plasma theories for astrophysical objects, we have assumed the charge quasineutrality of unmagnetized plasmas in global scales. This assumption has been justified because if there is a charged plasma, it induces the electric field which attracts the opposite charge, and this opposite charge reduces the charge separation. Here, we report a newly discovered instability which causes a charge separation in a rotating plasma inside of an innermost stable circular orbit (ISCO) around a black hole. The growth rate of the instability is smaller than that of the disk instability even in the unstable disk region and is forbidden in the stable disk region outside of the ISCO. However, this growth rate becomes comparable to that of the disk instability when the plasma density is much lower than a critical density inside of the ISCO. In such case, the charge separation instability would become apparent and cause the charged accretion into the black hole, thus charge the hole.

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I. INTRODUCTION

In a scale larger than several Debye lengths, unmagnetized plasmas in the Universe have been assumed to be quasineutral in charge. This is because even a small charge imbalance would result in very large electric fields which would cause the very small time scale electrostatic oscillation called plasma oscillation, and the damping or averaging of the oscillation over the mesoscopic time scale restores neutrality [e.g., [1,2]]. On the other hand, if we assume the ideal MHD condition on a magnetized, relativistically moving plasma, the charge density becomes significant, and this charge density is called the Goldreich-Julian density, $\rho_{\rm GJ}$ [3]. It is noted the Goldreich-Julian density is required so that the electric field induced by the charge density vanishes in the comoving frame of the plasma. In such case, it seems that we cannot assume the charge quasineutrality. However, we point out here that the difference between the net charge density of the magnetized plasma $\rho_{\rm e}$ and the Goldreich-Julian density, $\Delta\rho_{\rm e} = \rho_{\rm e} - \rho_{\rm GJ}$, plays the same role as the charge in the unmagnetized plasma. In this paper, we call the difference $\Delta\rho_{\rm e}$ the "free charge density", which induces the electric field observed by the comoving frame of the plasma. In the scale larger than several Debye lengths, the free charge density should tend to vanish because of the same reason for the charge quasineutrality of the unmagnetized plasma. This can be regarded as the generalization of the concept of charge quasineutrality to the magnetized, relativistic plasma. We call this concept "free charge quasineutrality". When the magnetic field is so strong and the thin plasma rotates so fast as assumed in a pulsar magnetosphere that the number density of the plasma particles is smaller than the Goldreich-Julian density divided by the elementary electric charge e, the free charge density becomes significant and the electric field observed by the comoving frame of the plasma remains. The parallel component of this electric field to the magnetic field accelerates plasma particles directly. In the region of the remaining one-direction electric field component along the magnetic field, plasma is swept by the electric field and the vacuum called "outer gap" appears [4,5]. Even in such vacuum, when plasma enters into it, the plasma is separated into positively and negatively charged fluids and the two fluids move to the opposite directions along the magnetic field to decrease the electric field component. Thus, in a strongly magnetized, relativistic plasma, the free charge tends to be canceled to keep the free charge neutrality.

According to the above consideration, in a scale larger than several Debye lengths, it has been assumed that the charged components of plasma move so that the electric field accelerating the charged components decreases so as to restore the free charge quasineutrality. Here, we report a charge separation instability in an unmagnetized plasma rotating around a black hole, which will induce the free charge density and electric field exponentially. To investigate the charge separation of plasmas around the black holes, we use generalized GRMHD equations derived by Koide [6]. We present the linear analysis of the charge separation of plasmas near Kerr black holes. We found the well-known plasma oscillation in the stable disk region outside of an innermost stable circular orbit (ISCO) around a black hole. On the other hand, in a circularly rotating plasma inside of the ISCO, we found an instability of the charge separation. The charge separation instability does not happen in the stable disk region outside of the ISCO. Furthermore, even in the unstable disk inside of the ISCO, the growth rate of the charge separation is smaller than that of the disk instability. However, when the plasma density is much lower than the critical density (the very low plasma density case), the growth rates of the two instabilities become comparable and the charge separation instability

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becomes apparent. That is, due to the charge separation instability, the unstable disk falling into the black hole can be charged.

In Sec. II, we present a linear analysis of charge separation in a stationary rotating disk plasma around a Kerr black hole with a brief summary of the generalized GRMHD equations. The analysis shows an instability of charge separation in the plasma inside of the ISCO around a black hole. In Sec. III, we summarize the results and discuss briefly the astrophysical meanings of the charge separation instability.

II. CHARGE SEPARATION IN PLASMA DISK AROUND BLACK HOLE

We investigate the simplest process of charge separation in an unmagnetized disk rotating around a Kerr black hole.

A. Brief summary of generalized GRMHD equations

In this subsection, we briefly summarize the generalized GRMHD equations; see Koide [6] in more detail. We investigate a charge separation using the generalized GRMHD equations of plasmas in the space-time, $x^\mu = (t, x^1, x^2, x^3)$ around a black hole where a line element ds is given by $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$ (Equations (18), (24), and (59) with Equations (25) and (58) of Koide [6]). Throughout this paper, except for a paragraph in Sec. III, we use the unit system where light speed is unity and the energy densities of electric field $\boldsymbol{E}$ and magnetic field $\boldsymbol{B}$ are given by $E^2/2$ and $B^2/2$ in the Minkowski space-time, respectively.

First of all, we summarize the generalized GRMHD equations for a general case briefly as follows:

$$\nabla_\nu(\rho U^\nu) = 0, \tag{1}$$

$$\nabla_\nu T^{\mu\nu} = 0, \tag{2}$$

$$\begin{aligned}\frac{1}{ne}\nabla_\nu\left(\frac{\mu h^\dagger}{ne}Q^{\mu\nu}\right) = {}& U^\nu F^\mu{}_\nu - \eta(J^\mu - \rho_e' U^\mu) \\ &+ \frac{1}{2ne}\nabla^\mu(\Delta\mu p - \Delta p) - \frac{\Delta\mu}{ne}J^\nu F^\mu{}_\nu \\ &+ \eta\rho_e'\Theta U^\mu,\end{aligned} \tag{3}$$

and Maxwell equations

$$\nabla_\nu {}^*F^{\mu\nu} = 0, \tag{4}$$

$$\nabla_\nu F^{\mu\nu} = J^\mu, \tag{5}$$

where the energy-momentum tensor $T^{\mu\nu}$ and "charge-current density tensor" $Q^{\mu\nu}$ are given by

$$\begin{aligned}T^{\mu\nu} \equiv {}& pg^{\mu\nu} + h^\dagger U^\mu U^\nu + \frac{\mu h^\ddagger}{(ne)^2}J^\mu J^\nu \\ &+ \frac{2\mu\Delta h^\dagger}{ne}(U^\mu J^\nu + J^\mu U^\nu) \\ &+ F^\mu{}_\sigma F^{\nu\sigma} - \frac{1}{4}g^{\mu\nu}F^{\kappa\lambda}F_{\kappa\lambda},\end{aligned} \tag{6}$$

$$\begin{aligned}Q^{\mu\nu} \equiv \frac{en}{\mu h^\dagger}K^{\mu\nu} = \frac{en}{h^\dagger}\bigg[&\frac{h^\ddagger}{ne}(U^\mu J^\nu + J^\mu U^\nu) \\ &+ 2\Delta h^\dagger U^\mu U^\nu - \frac{\Delta h^\sharp}{(ne)^2}J^\mu J^\nu\bigg].\end{aligned} \tag{7}$$

Equation (3) presents the general relativistic generalized Ohm's law. In Eq. (3), the left-hand side expresses the inertia effect and transport of kinetic energy and momentum of the current, the first two terms of the right-hand side correspond to all terms of the "standard" Ohm's law with resistivity η, the third term represents the thermo-electromotive force, the fourth term expresses the Hall effect, and the last term comes from the equipartition of the thermalized energy due to the friction force between the two fluids. Here, $\rho_e' = -J^\nu J_\nu$ is the charge density observed by the local rest frame of the plasma and Θ is the rate of equipartition with respect to the thermalized energy due to friction (for details, see Appendix A of Koide [7]). We follow the notations used by Koide [6] with respect to physical variables except that we use $Q^{\mu\nu}$ instead of $K^{\mu\nu}$. Here, we used the two-fluid model, where we assumed the plasma consists of positively charged particles with charge e and mass m_+ and negatively charged particles with charge $-e$ and mass m_- (Appendix A). We used the typical mass of a plasma particle $m \equiv m_+ + m_-$, normalized reduced mass $\mu \equiv m_+ m_-/m^2$, and normalized mass difference $\Delta\mu \equiv (m_+ - m_-)/m$. The variables ρ, $h^\dagger$, p, $n \equiv \rho/m$, Δp, and $\Delta h^\dagger$ are the mass density, enthalpy density, pressure, number density, pressure difference of two fluids, and difference of two-fluid enthalpy density. Furthermore, ∇_μ, U^μ, and J^μ are the covariant derivative, 4-velocity, and 4-current densities, respectively, and $F_{\mu\nu}$ is the electromagnetic strength tensor and ${}^*F_{\mu\nu}$ is the dual tensor of $F_{\mu\nu}$. Here, the electric field is given by $E_i = F_{i0}$ and the magnetic field is $F_{ij} = \sum_k \epsilon_{ijk}B_k$ (ϵ_{ijk} is the Levi-Civita symbol), where the alphabetic index (i, j, k) runs from 1 to 3. We also use the variables related to the enthalpy density,

$$\begin{aligned}h^\ddagger &\equiv h^\dagger - \Delta\mu\Delta h^\dagger, \\ \Delta h^\sharp &\equiv \Delta\mu h^\dagger - \frac{1-3\mu}{2\mu}\Delta h^\dagger.\end{aligned} \tag{8}$$

It is noted that Eq. (5) yields the equation of continuity with respect to the current,

$$\nabla_\nu J^\nu = 0. \tag{9}$$

We assume that off-diagonal spatial elements of the metric $g_{\mu\nu}$ vanish: $g_{ij} = 0 (i \neq j)$. Writing the nonzero components by $g_{00} = -h_0^2$, $g_{ii} = h_i^2$, $g_{i0} = g_{0i} = -h_i^2 \omega_i$, we have $ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -h_0^2 dt^2 + \sum_{i=1}^3 [h_i^2 (dx^i)^2 - 2h_i^2 \omega_i dt dx^i]$. When we define the lapse function α and shift vector β^i by $\alpha = [h_0^2 + \sum_{i=1}^3 (h_i \omega_i)^2]^{1/2}$, $\beta^i = \frac{h_i \omega_i}{\alpha}$, the line element ds is written by $ds^2 = -\alpha^2 dt^2 + \sum_{i=1}^3 (h_i dx^i - \alpha \beta^i dt)^2$. We also have $g = -(\alpha h_1 h_2 h_3)^2$. Using the "zero-angular-momentum observer (ZAMO) frame" $\hat{x}^\mu$, where the line element ds is given by $ds^2 = -d\hat{t}^2 + \sum_i (\hat{x}^i)^2 = \eta_{\mu\nu} d\hat{x}^\mu d\hat{x}^\nu$, we have the 3 + 1 formalism of the generalized GRMHD and the Maxwell equations. As for equations including only derivatives of contravariant vectors A^μ or antisymmetric 2nd rank tensors $A^{\mu\nu}$, we obtain their 3 + 1 formalism easily using Equations $\nabla_\nu A^\nu = \frac{1}{\sqrt{-g}} \partial (\sqrt{-g} A^\nu)$ or $\nabla_\nu A^{\mu\nu} = \frac{1}{\sqrt{-g}} \partial (\sqrt{-g} A^{\mu\nu})$. With respect to any equation including a term of derivative of the symmetric 2nd rank tensor,

$$\nabla_\nu S^{\mu\nu} = H^\mu, \tag{10}$$

the 3 + 1 formalism is given by

$$\begin{aligned} &\frac{\partial}{\partial t} \hat{S}^{00} + \frac{1}{h_1 h_2 h_3} \sum_j \frac{\partial}{\partial x^j} \left[\frac{\alpha h_1 h_2 h_3}{h_j} \left(\hat{S}^{0j} + \beta^j \hat{S}^{00} \right) \right] \\ &+ \sum_j \frac{1}{h_j} \frac{\partial \alpha}{\partial x^j} \hat{S}^{j0} + \sum_{j,k} \alpha \beta^k (G_{kj} \hat{S}^{kj} - G_{jk} \hat{S}^{jj}) \\ &+ \sum_{j,k} \sigma_{jk} \hat{S}^{jk} = \alpha \hat{H}^0, \end{aligned} \tag{11}$$

$$\begin{aligned} &\frac{\partial}{\partial t} \hat{S}^{i0} + \frac{1}{h_1 h_2 h_3} \sum_j \frac{\partial}{\partial x^j} \left[\frac{\alpha h_1 h_2 h_3}{h_j} \left(\hat{S}^{ij} + \beta^j \hat{S}^{i0} \right) \right] \\ &+ \frac{1}{h_i} \frac{\partial \alpha}{\partial x^i} \hat{S}^{00} - \sum_j \alpha \left[G_{ij} \hat{S}^{ij} - G_{ji} \hat{S}^{jj} + \beta^j (G_{ij} \hat{S}^{0i} - G_{ji} \hat{S}^{0j}) \right] \\ &+ \sum_j \sigma_{ji} \hat{S}^{0j} = \alpha \hat{H}^i, \end{aligned} \tag{12}$$

where $G_{ij} \equiv -\frac{1}{h_i h_j} \frac{\partial h_i}{\partial x^j}$ and $\sigma_{ij} \equiv \frac{1}{h_j} \frac{\partial}{\partial x^j} (\alpha \beta^i)$.

B. Linear analysis of charge separation in stationary disk

For simplicity, we consider a plasma of a stationary thin disk rotating around a Kerr black hole with zero pressure ($p = \Delta p = 0$). The space-time $x^\mu = (t, r, \theta, \phi)$ around the Kerr black hole with a mass M and rotation parameter a is given by the metrics, $h_0 = (1 - 2r_g r/\Sigma)^{1/2}$, $h_1 = \sqrt{\Sigma/\Delta}$, $h_2 = \sqrt{\Sigma}$, $h_3 = \sqrt{A/\Sigma} \sin\theta$, $\omega_3 = 2r_g^2 ar/A$, and $\omega_i = 0$ ($i = 1, 2$). Here, $r_g = GM$ is the gravitational radius (G is the gravitational constant), $\Delta = r^2 - 2r_g r + (ar_g)^2$, $\Sigma = r^2 + (ar_g)^2 \cos^2\theta$, and $A = \{r^2 + (ar_g)^2\}^2 - \Delta (ar_g)^2 \sin^2\theta$. In this metric, the lapse function is $\alpha = \sqrt{\Delta\Sigma/A}$. The Schwarzschild radius of the black hole is given by $r_S = 2r_g$. The 3-velocity of the circularly rotating disk observed by the ZAMO frame, called the Kepler velocity, V_K, is given by the quadratic equation

$$\alpha G_{31} V_K^2 + (\alpha \beta^3 G_{31} + \sigma_{31}) V_K + \frac{1}{h_1} \frac{\partial \alpha}{\partial r} = 0. \tag{13}$$

In investigating linear behavior of charge separation in the stationary disk, we assume charge separation is weak, $|\rho'_e| = |-J^\nu J_\nu| \ll en$. Then, we can use an approximation of the enthalpy density and enthalpy difference density as

$$h^\dagger \approx mn, \tag{14}$$

$$\Delta h^\dagger \approx \frac{-m}{2e} \rho'_e, \tag{15}$$

$$h^\ddagger \approx mn + \frac{2\mu m \Delta\mu}{e} \rho'_e, \tag{16}$$

$$\Delta h^\sharp \approx mn\Delta\mu + \frac{1 - 3\mu}{e} m\rho'_e, \tag{17}$$

because of $n_\pm \approx n \pm \frac{m_\mp}{em} \rho'_e$. The generalized GRMHD equations reduce to

$$\nabla_\nu (\rho U^\nu) = 0, \tag{18}$$

$$\begin{aligned} mn\nabla_\nu \Bigg[&U^\mu U^\nu + \frac{\mu}{(ne)^2} \left(1 + \frac{2\mu\Delta\mu\rho'_e}{en} \right) J^\mu J^\nu \\ &- \frac{\mu\rho'_e}{(ne)^2} (U^\mu J^\nu + J^\mu U^\nu) \Bigg] = J^\nu F^\mu{}_\nu, \end{aligned} \tag{19}$$

$$\frac{1}{\omega_p^2} \nabla_\nu Q^{\mu\nu} = \left(U^\nu - \frac{\Delta\mu}{ne} J^\nu \right) F^\mu{}_\nu - \eta [J^\mu - \rho'_e (1 + \Theta) U^\mu], \tag{20}$$

where $\omega_p \equiv \sqrt{(ne)^2/(\mu\rho)} = \sqrt{ne^2/(\mu m)}$ is the plasma frequency, and $Q^{\mu\nu}$ is approximated by

$$Q^{\mu\nu} \approx U^\mu J^\nu + J^\mu U^\nu - \rho'_e U^\mu U^\nu. \tag{21}$$

To perform the linear analysis of the charge separation in the hydrostatic equilibrium plasma rotating around the Kerr black hole, we consider only the perturbation with respect to the static electric field,

$$\hat{J}^\mu = \tilde{J}^\mu, \qquad \hat{\rho}_e = \tilde{\rho}_e \tag{22}$$

$$\hat{F}_{i0} = \tilde{E}_i, \qquad \hat{F}_{ij} = 0, \tag{23}$$

where the tildes indicate the infinitesimally small variables, and we do not consider perturbation to the hydrostatic equilibrium,

$$\rho = \bar{\rho}, \qquad n = \bar{n}, \qquad \hat{U}^\mu = \bar{U}^\mu. \tag{24}$$

In the ZAMO frame, the 4-velocity is given by $\bar{U}^0 = \gamma_K = (1 - V_K^2)^{-1/2}$, $\bar{U}^1 = \bar{U}^2 = 0$, $\hat{U}^3 = \gamma_K V_K$. The linear analysis requires the Ohm's law (Eq. (3)), the equation of continuity about current (Eq. (9)), and the Gauss law of electrostatics (temporal component of Eq. (5)),

$$\frac{1}{\omega_P^2}\nabla_\nu \tilde{Q}^{\mu\nu} = \bar{U}^\nu \tilde{F}_\nu^\mu - \eta[\tilde{J}^\mu - (\rho_e' + \rho_e'\Theta)\bar{U}^\mu], \tag{25}$$

$$\nabla_\nu \tilde{J}^\nu = 0, \tag{26}$$

$$\nabla_\nu \tilde{F}^{0\nu} = \tilde{J}^0. \tag{27}$$

Using Eqs. (25)–(27) and (12), we obtain the 3 + 1 formalism of the Ohm's law, equation of continuity about current, and Gauss's law for the electric field (see also Equations (63) and (67) of Koide [6]),

$$\begin{aligned}\frac{\partial}{\partial t}\tilde{Q}^{i0} = &-\Big[\frac{1}{h_1h_2h_3}\sum_j \frac{\partial}{\partial x^j}\Big(\frac{\alpha h_1h_2h_3}{h_j}(\tilde{Q}^{ij} + \beta^j\tilde{Q}^{i0})\Big)\\ &+ \frac{1}{h_i}\frac{\partial\alpha}{\partial x^i}\tilde{Q}^{00} - \sum_j \alpha\{G_{ij}\tilde{Q}^{ij} - G_{ji}\tilde{Q}^{jj}\\ &+ \beta^j(G_{ij}\tilde{Q}^{0i} - G_{ji}\tilde{Q}^{j0})\}\Big]\\ &+ \alpha\omega_P^2\Big[\bar{U}^\nu \tilde{F}^i{}_\nu - \eta[\tilde{J}^i - (\rho_e' + \rho_e'\Theta)\bar{U}^i]\Big],\end{aligned} \tag{28}$$

$$\frac{\partial}{\partial t}\tilde{\rho}_e = -\frac{1}{h_1h_2h_3}\sum_j \frac{\partial}{\partial x^j}\Big(\frac{h_1h_2h_3}{h_j}\tilde{J}^j\Big), \tag{29}$$

$$\tilde{\rho}_e = \sum_j \frac{1}{h_1h_2h_3}\frac{\partial}{\partial x^j}\Big(\frac{h_1h_2h_3}{h_j}\tilde{E}_j\Big). \tag{30}$$

Here, we used the following approximation:

$$\tilde{Q}^{00} \approx \gamma_K(2\tilde{\rho}_e - \gamma_K\tilde{\rho}_e'), \tag{31}$$

$$\tilde{Q}^{i0} = \tilde{Q}^{0i} \approx \gamma_K\tilde{J}^i + (\tilde{\rho}_e - \gamma_K\tilde{\rho}_e')\bar{U}^i, \tag{32}$$

$$\tilde{Q}^{ij} \approx U_K[\delta^{3i}\tilde{J}^j + \delta^{3j}\tilde{J}^i - U_K\delta^{i3}\delta^{j3}(\gamma_K\tilde{\rho}_e - U_K\tilde{J}^3)], \tag{33}$$

where $\tilde{Q}^{00}$ can be regarded as modified charge density and $\tilde{Q}^{i0}$ corresponds to the modified current density. When we use the relation

$$\rho_e' = J^\nu U_\nu = \gamma\rho_e - \boldsymbol{J}\cdot\boldsymbol{U} \approx \gamma_K\tilde{\rho}_e - \bar{\boldsymbol{U}}\cdot\tilde{\boldsymbol{J}},$$

where $\boldsymbol{J} \equiv (J^1, J^2, J^3)$ and $\boldsymbol{U} \equiv (U^1, U^2, U^3)$ are the 3-current density and 3-velocity, respectively, we have

$$\hat{Q}^{00} \approx \gamma_K[(1 - U_K^2)\tilde{\rho}_e + \gamma_K U_K\tilde{J}^3], \tag{34}$$

$$\hat{Q}^{i0} \approx \gamma_K[\tilde{J}^i + U_K^2\delta^{i3}\tilde{J}^3 - U_K^3\delta^{i3}\tilde{\rho}_e]. \tag{35}$$

Because in the present linear analysis, we can assume the quasicharge neutrality, thus we have an approximation of $\rho_e'\Theta$ as,

$$\rho_e'\Theta \approx \frac{\Delta\mu}{2ne}|\boldsymbol{J}'|^2, \tag{36}$$

where $|\boldsymbol{J}'|^2 = (\gamma - 1)\rho_e^2 - 2\gamma\rho_e\boldsymbol{U}\cdot\boldsymbol{J} + |\boldsymbol{J}|^2 + (\boldsymbol{U}\cdot\boldsymbol{J})^2$ [6]. Then, when J' is infinitesimally small, we have $\rho_e'\Theta \approx 0$.

Here, we assume the perturbation is symmetric with respect to the polar axis and the equatorial plane, $\partial/\partial x^i = \delta^{1i}\partial/\partial x^1$. Then, the equations with respect to the perturbation of the charge separation at the equatorial plane are as follows:

$$\begin{aligned}\frac{\partial}{\partial t}\tilde{J}^1 &= [\alpha G_{31}(2V_K + \beta^3) + \sigma_{31}](V_K\tilde{\rho}_e - \tilde{J}^3)\\ &+ \frac{\alpha\omega_P^2}{\gamma_K}(\gamma_K\tilde{E}_1 - \eta\tilde{J}^1),\end{aligned} \tag{37}$$

$$\frac{\partial}{\partial t}(\tilde{J}^3 - \hat{V}_K^3\tilde{\rho}_e) = -\frac{1}{h_1h_2h_3^2\gamma_K^3}\frac{\partial}{\partial r}(\alpha h_2h_3^2\hat{U}_K\tilde{J}^1) \tag{38}$$

$$+ \omega_p^2\frac{\alpha}{\gamma_K^2}\Big[\tilde{E}_3 - \eta\Big(\gamma_K\tilde{J}^3 - U_K\tilde{\rho}_e\Big)\Big], \tag{39}$$

$$\frac{\partial\tilde{\rho}_e}{\partial t} = -\frac{1}{h_1h_2h_3}\frac{\partial}{\partial r}(\alpha h_2h_3\tilde{J}^1), \tag{40}$$

$$\tilde{\rho}_e = \frac{1}{h_1h_2h_3}\frac{\partial}{\partial r}(h_2h_3\tilde{E}^1). \tag{41}$$

To derive Eq. (37), we used Eq. (13). For simplicity, we assume the wave length of the perturbation is much smaller than the characteristic scale length of the metrics and the Keplerian rotation around the black hole, $\sim r_S$, and we put the perturbation is proportional to $\exp(ikr - i\omega t)$. Here, we note that we have to consider the relation between the derivatives

$$\begin{aligned}&\frac{1}{\alpha U_K h_1h_2h_3^2}\frac{\partial}{\partial r}(\alpha h_2h_3^2U_K\tilde{J}^1) - \frac{1}{\alpha h_1h_2h_3}\frac{\partial}{\partial r}(\alpha h_2h_3^2\tilde{J}^1)\\ &= \frac{\tilde{J}^1}{L_K}\frac{\partial L_K}{\partial r},\end{aligned} \tag{42}$$

where $L_K = h_3 U_K$ is the specific angular momentum of the Keplerian disk. Finally, we obtain the dispersion relation of the charge separation in the plasma disk rotating circularly around the Kerr black hole,

$$\left(\frac{\omega}{\alpha}\right)^3 + 2i\eta'\left(\frac{\omega}{\alpha}\right)^2 - [\eta'^2 + \omega_p^2 - 2g_K\Lambda_K]\frac{\omega}{\alpha} - i\eta'\omega_p^2 = 0, \tag{43}$$

where $\eta' = \omega_p^2\eta/\gamma_K$,

$$g_K = \frac{V_K}{2\alpha}[\alpha G_{31}(2V_K + \beta^3) + \sigma_{31}] = -\frac{1}{h_1}\frac{\partial}{\partial r}\log\alpha - \frac{V_K}{2\alpha}(\alpha G_{31}\beta^3 + \sigma_{31}), \tag{44}$$

$$\Lambda_K = \frac{1}{h_1\gamma_K^2 L_K}\frac{\partial L_K}{\partial r}. \tag{45}$$

Here, it is noted that the stability condition of the accretion disk is given by $\partial L_K/\partial r > 0$ ($\Lambda_K > 0$), and the condition $\partial L_K/\partial r = 0$ ($\Lambda_K = 0$) yields the radial coordinate of the ISCO, $r = r_{ISCO}$.

In the case of zero resistivity ($\eta = 0$), Eq. (43) yields the dispersion relation of the charge separation in the plasma disk as

$$\omega^2 = \alpha^2(\omega_p^2 - 2g_K\Lambda_K) \equiv \omega_0^2. \tag{46}$$

In the region inside of the ISCO, when

$$\omega_p < \sqrt{2g_K\Lambda_K}, \tag{47}$$

the charge separation becomes unstable. However, it is noted that the disk is unstable with the growth rate $\gamma_{disk} = \alpha\sqrt{2g_K\Lambda_K}$, which is larger than the growth rate of the charge separation instability. In the very low plasma density case ($\omega_p \ll \gamma_{disk}$), the growth rate of the disk instability and the charge separation instability become comparable. Then, in this situation, the charge separation instability may appear in the disk falling into the black hole and may make the black hole charged. On the other hand, in the other usual plasma density case, the charge separation instability is forbidden or inhibited behind the disk instability.

To investigate the effect of resistivity, we consider a solution for a very weak resistivity limit, $\alpha\eta' \ll \omega_0$. In this approach, we treat the difference $\Delta\omega = \omega - \omega_0$ is an infinitesimal variable which is comparable to $(\alpha\eta'/\omega_0^2)\omega_0$. The dispersion relation (43) yields

$$2\omega_0^2\Delta\omega + i\eta'[2\omega_0^2 - \alpha^2\omega_p^2] = 0, \tag{48}$$

and we have the solution

$$\omega = \omega_0 - i\frac{\alpha^2\eta'}{2\omega_0^2}(\omega_p^2 - 4g_K\Lambda_K) = \alpha(\omega_p^2 - 2g_K\Lambda_K)^{1/2} - i\frac{\alpha^2\eta'(\omega_p^2 - 4g_K\Lambda_K)}{\omega_p^2 - 2g_K\Lambda_K}. \tag{49}$$

It is noted that even in the case of $\omega_0^2 > 0$ (stable state in the zero resistivity case), $\omega_p^2 - 4g_K\Lambda_K$ can become negative in the unstable region ($\Lambda_K < 0$), which means the resistivity induces the charge separation instability, while in the outside of the ISCO, $r > r_{ISCO}$, the instability is forbidden. On the other hand, in the case of $\omega_0^2 < 0$, the resistivity stabilizes the charge separation instability as shown by the last term of the right-hand side of Eq. (49).

III. DISCUSSION

We have shown the charge separation instability of the circularly rotating plasma inside the ISCO ($r \le r_{ISCO}$) around the Kerr black hole. This instability is forbidden in the stable disk outside of the ISCO. Furthermore, even in the unstable disk region inside of the ISCO, the growth rate of the charge separation instability is smaller than that of the disk instability. However, when the plasma density is much lower than the critical density, growth rates of the charge separation and disk instabilities become comparable and the charge separation instability becomes apparent. Then, the charge separation instability makes the disk plasma falling into the black hole charged and may eventually charge the hole. The critical plasma density is expressed as

$$n_{crit} \equiv \frac{2\mu m}{e^2}g_K\Lambda_K, \tag{50}$$

where the critical density n_{crit} is given by Eq. (47) with $\omega_p^2 = n_{crit}e^2/(\mu m)$ in the zero resistivity case. With respect to the resistive case, Eq. (49) indicates that the weak resistivity makes the charge separation instability happen easier in the stable region of the charge separation instability inside of the ISCO, while in the unstable region of the ideal MHD situation, the resistivity stabilizes the instability. In a case with resistivity, the growth rate of the charge separation instability is also smaller than that of the disk instability inside of the ISCO. Then, unless the plasma density n is much less than the critical density n_{crit} so that the growth rate of the charge separation instability is comparable to that of the disk instability, the charge separation instability is hidden behind the disk instability inside of the ISCO and is also forbidden in the stable disk around the astrophysical black holes. However, we emphasize again that when the plasma density is much less than the critical density, $n \ll n_{crit}$, the charge separation instability may be apparent as the charged disk falling into the black hole.

The mechanism of the charge separation instability is explained by the following schematic picture. Consider that the accretion disk is composed of two disks of positively charged particles (+ disk) and of negatively charged particles (− disk) (Fig. 1). Both of the purely charged disks (± disks) are unstable in the region $r < r_{\rm ISCO}$. However, even in the unstable region, when one disk (± disk) falls and the other disk (∓ disk) shifts outward, the electric field is induced and tends to suppress the charge separation. When the electric field is strong enough, it causes plasma oscillation. On the other hand, when the electric field can not become so strong (in the case of $n \ll n_{\rm crit}$), the suppression is not strong (effective) enough and the charged disks separate each other increasingly so that the charge separation is induced exponentially. In the outer region of the ISCO, $r > r_{\rm ISCO}$, the plasma oscillation is always induced by the charge separation because of the stability of the ± disks. Using this intuitive picture of the charge separation instability, we suggest that the charge separation instability with small wave number k is also possible in the unstable disk around the rotating black hole, when the plasma oscillation frequency $\omega_{\rm p}$ is small enough and the plasma density is sufficiently small. This picture also suggests that in a magnetized plasma disk, the similar instability of charge separation would be caused to break the free charge quasineutrality when the purely charged disk is gravitationally unstable and the plasma density is low enough. This charge separation grows exponentially until the nonlinear effects begin to suppress the instability. For example, the disk charged by the charge separation instability falls into the black hole and charges the black hole. The electric field of the charged black hole will suppress the fall of the charged disk and induce the fall of the oppositely charged plasma around the black hole.

The charge separation instability may also happen in the pulsar magnetosphere when the radius of the ISCO is larger than the central star radius. However, when the magnetic

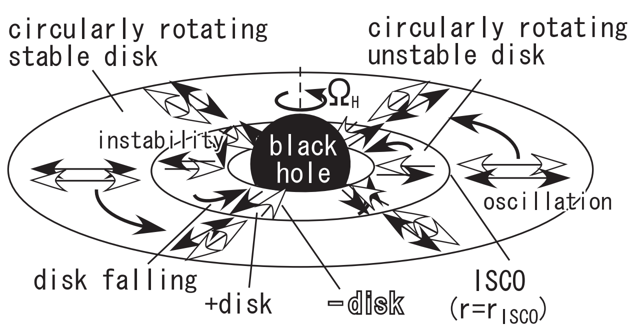

FIG. 1. Schematic picture of the charge separation instability of the plasma in an unstable region around a black hole ($r < r_{\rm ISCO}$). The neutral disk consists of positively/negatively charged disks (+ / − disks); when the electric field due to the charge separation is not strong enough to suppress the charge separation, for example, the positive disk shifts outward and the negative disk falls into the black hole exponentially in the unstable disk region. The reverse is also true. This shift and falling cause the exponential charge separation.

field is extremely strong, the free charge due to the charge separation instability would not be induced because the magnetic field suppresses the gravitational instability of the disk. Plasma dynamics of the pulsar magnetosphere outside of the relativistic star were investigated with the Schwarzschild metric by Henriksen and Rayburn [8]. They discussed the net charge separation and the instability, while they did not perform the linear analysis of the charge separation in bulk plasma. They considered the large-scale net charge separation within the free charge quasineutrality and the charge separation instability produced by the current driven plasma turbulence which leads charge fluctuation on small scales less than the Debye length.

Here, we estimate the critical density of the charge separation instability (Eq. (50)) for an individual astrophysical object. For a rough estimation of the instability condition, we consider the cases of the Schwarzschild black holes with the mass $M_{\rm BH}$ ($a = 0$). The critical density of the instability is expressed in the MKSA system of units (SI unit) by

$$n_{\rm crit} = \frac{\mu m r_{\rm S}}{2\mu_0 e^2} \frac{3r_{\rm S} - r}{\sqrt{r^5 (r - r_{\rm S})^3}}, \tag{51}$$

where μ_0 is the magnetic permeability in vacuum. We estimate it at $r = 2r_{\rm S}$ to get

$$\begin{aligned} n^*_{\rm crit} &= \frac{\mu m}{m_{\rm e}} \frac{m_{\rm e}}{2^{7/2} \mu_0 e^2 r_{\rm S}^2} \\ &= 2.5 \times 10^{12} (r_{\rm S}[{\rm m}])^{-2} \frac{\mu m}{m_{\rm e}}\ [{\rm m}^{-3}] \\ &= 2.8 \times 10^5 \left(\frac{M_{\rm BH}}{M_\odot}\right)^{-2} \frac{\mu m}{m_{\rm e}}, \end{aligned} \tag{52}$$

where $m_{\rm e}$ is the electron mass. If we assume the electron-proton and electron-positron plasmas, we have $\mu m/m_{\rm e} = 1$ and $\mu m/m_{\rm e} = 1/2$, respectively. In this paragraph, we set $\mu m/m_{\rm e} = 1$. We use values of the typical density n of accretion disks around black holes of individual objects listed by Koide [6]. When we consider the active galactic nucleus (AGN) of M87 whose central black hole mass is $M_{\rm BH} = 3 \times 10^9 M_\odot$ ($r_{\rm S} = 9 \times 10^{12}$ m) [9], the critical density of the charge separation instability is estimated as $n^*_{\rm crit} = 3 \times 10^{-14}\ [{\rm m}^{-3}]$. This value is extremely small compared not only to the estimated value at the accretion disk around the black hole of $n = 8 \times 10^{21}\ [{\rm m}^{-3}]$, but also to the averaged particle number density in the extragalactic region, $n_{\rm U} \sim 10\ [{\rm m}^{-3}]$ [10]. In the case of Sgr A*, a supermassive black hole in the Galaxy, whose central black hole mass is $M_{\rm BH} = 4.4 \times 10^6 M_\odot$ ($r_{\rm S} = 1.3 \times 10^{10}$ m) [11], the critical density is $n^*_{\rm crit} = 1.4 \times 10^{-8}\ [{\rm m}^{-3}]$. This is also extremely small compared to the value at the accretion disk, $n = 9 \times 10^{23}\ [{\rm m}^{-3}]$. As we estimated above, the charge separation instability hardly happens around the supermassive black holes. In the case of black hole X-ray binaries, for example, the micro-quasar, GRS1915 + 105, whose central black hole mass is $M_{\rm BH} = 14 M_\odot$ ($r_{\rm S} = 4.2 \times 10^4$ m) [12], the critical density is $n^*_{\rm crit} = 1.4 \times 10^3$ [m^{-3}]. This density is much smaller than the density at the accretion disk around the black hole, $n = 4 \times 10^{27}$ [m^{-3}]. However, a density smaller than the critical density may be realized in a low density disk around a single stellar-mass black hole, for example. In the case of a very low density disk, the charge separation instability may be caused, while it would be difficult to observe because of its low activity of the thin disk.

The charge instability causes the falling of the charged plasma into the black hole and will charge the black hole. Here, we estimate the influence of the electric field of the charged black hole to the ambient plasma in the following case. We assume that the one-component charged fluid with the height $H \sim r_{\rm S}$, the inner radius $r_{\rm inner} \sim 2r_{\rm S}$, the outer radius $r_{\rm outer} \sim 2r_{\rm S} + L$, and the density $n \sim n^*_{\rm crit}$ falls due to the charge separation instability and is swallowed by the black hole. The charge contained in the one-component fluid is $Q \sim 8\pi r_{\rm S}^2 L n^*_{\rm crit} e$. The electric and gravitational forces which act on a charged particle with the mass m_- and the charge $-e$ located at $r = r$ are $F_{\rm E} \sim \frac{1}{4\pi\epsilon_0}\frac{eQ}{r^2}$ and $F_{\rm grav} \sim G\frac{mM_{\rm BH}}{r^2}$, respectively. We estimate the disk scale L where the electric and gravitational forces become comparable: $F_{\rm E} \sim F_{\rm grav}$. It yields $L \sim \frac{\epsilon_0 m_- c^2}{4e^2 n^*_{\rm crit} r_{\rm S}}$. Using Eq. (52), we have the simple expression

$$\frac{L}{r_{\rm S}} \sim 2^{3/2}\frac{m}{m_+} \sim 2.8, \tag{53}$$

where the ratio does not depend on the black hole mass, $M_{\rm BH}$. This suggests that the small disk with the scale of the Schwarzschild radius supplies so much charge to the black hole through the charge separation instability within short time scale of the instability that the charged black hole influences the plasma dynamics around it. This electric field will suppress the further charging of the black hole.

It is true that the charge separation instability is caused only in the unstable disk region. However, the current induced by the instability may reach the stable disk region because of the inertia of the current. The current supplies the net charge in the stable disk around the black hole. Thus, the charge separation due to the instability may induce a distinctive drastic phenomena in the plasma of the stable disk regions around black holes, while the net charge may be canceled by the charge supply from the outer disk. For example, the gravitational magnetic reconnection can be induced by the charge separation [6]. If the charge separation is significant, it also causes a strong electric field. This strong electric field may accelerate particles which can explain high energy cosmic rays as well as in the ''outer gap'' of the pulsar magnetosphere [4,5]. The cause of these distinctive phenomena of plasma around black holes will be clarified with more detailed analysis of the generalized GRMHD equations and numerical simulations. The numerical simulations of the generalized GRMHD beyond the ideal GRMHD [e.g., [13,14]] and the resistive relativistic MHD with the acausal relativistic Ohm's law [15] would provide a useful and essential tool for such analysis. This is our next subject.

In the last paragraph of this paper, we suggest a radiation process associated with the charge separation instability. The charge separation instability does not depend on the wave number k in the zero resistivity case as shown by Eq. (43). Thus, a very strong electric field with a very large wave number can be caused through the instability. When a high energy charged particle passes through the strong electric field, the particle is accelerated and decelerated reciprocally, and emits strong radiation. This radiation should be detected around the inner edge of disks of black holes.

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APPENDIX: BASIS OF GENERALIZED GRMHD EQUATIONS

The generalized GRMHD equations are derived from the general relativistic two-fluid equations of plasma, which is composed of positively charged particles with charge e and mass m_+ and negatively charged particles with charge e and mass m_- [6]. The variables of the generalized GRMHD equations are defined by the average and difference of the variables of the two-fluid equations. We note a variable of fluid composed by the positively charged particles with a subscript ''+'' and that of negative fluid with ''−''. We write the variables of two fluids as: $n_\pm$ is the number density, $\gamma'_\pm$ is the Lorentz factor observed by the center of mass frame of the charged fluids, $p_\pm$ is pressure, $h_\pm$ is the enthalpy density, $U^\mu_\pm$ is 4-velocity. The variables of the generalized GRMHD equations are based as follows:

$$\rho = m_+ n_+ \gamma'_+ + m_- n_- \gamma'_-, \tag{A1}$$

$$n = \frac{\rho}{m}, \tag{A2}$$

$$p = p_+ + p_-, \tag{A3}$$

$$\Delta p = p_+ - p_-, \tag{A4}$$

$$U^\mu = \frac{1}{\rho}(m_+ n_+ U^\mu_+ + m_- n_- U^\mu_-), \tag{A5}$$

$$J^\mu = e(n_+ U_+^\mu - n_- U_-^\mu), \tag{A6}$$

$$h^\dagger = n^2\left(\frac{h_+}{n_+^2} + \frac{h_-}{n_-^2}\right), \tag{A7}$$

$$\Delta h^\dagger = \frac{mn^2}{2}\left(\frac{h_+}{m_+ n_+^2} - \frac{h_-}{m_- n_-^2}\right). \tag{A8}$$

We also use the variables with respect to the enthalpy density:

$$h^\ddagger = \frac{n^2}{4\mu}\left[\frac{h_+}{n_+^2}\left(\frac{2m_-}{m}\right)^2 + \frac{h_-}{n_-^2}\left(\frac{2m_+}{m}\right)^2\right] = h^\dagger - \Delta\mu\Delta h^\dagger, \tag{A9}$$

$$\Delta h^\sharp = -\frac{n^2}{8\mu}\left[\frac{h_+}{n_+^2}\left(\frac{2m_-}{m}\right)^3 - \frac{h_-}{n_-^2}\left(\frac{2m_+}{m}\right)^3\right] = \Delta\mu h^\dagger - \frac{1-3\mu}{2\mu}\Delta h^\dagger. \tag{A10}$$

Incidentally, we sometimes assume the plasma consists of two perfect fluids with the equal specific heat ratio, Γ. The equations of states are

$$h^\dagger = n^2\left[\frac{m_+}{n_+} + \frac{m_-}{n_-} + \frac{\Gamma}{2(\Gamma-1)}\left\{\left(\frac{1}{n_+^2} + \frac{1}{n_-^2}\right)p + \left(\frac{1}{n_+^2} - \frac{1}{n_-^2}\right)\Delta p\right\}\right], \tag{A11}$$

$$\Delta h^\dagger = 2\mu m n^2\left[\frac{1}{n_+} - \frac{1}{n_-} + \frac{\Gamma}{2(\Gamma-1)}\left\{\left(\frac{1}{m_+ n_+^2} - \frac{1}{m_- n_-^2}\right)p + \left(\frac{1}{m_+ n_+^2} + \frac{1}{m_- n_-^2}\right)\Delta p\right\}\right], \tag{A12}$$

where

$$n_\pm \equiv \left[n^2 \mp \frac{2m_\mp n}{em} U^\nu J_\nu - \left(\frac{m_\mp}{em}\right)^2 J^\nu J_\nu\right]^{1/2}, \tag{A13}$$

corresponds to the particle number density of each charged fluid (see Equations (74)–(78) of Koide [6]).[1]

The energy-momentum tensors $T^{\mu\nu}$ and the charge-current density tensor $Q^{\mu\nu}$ are given by

$$T^{\mu\nu} = T_+^{\mu\nu} + T_-^{\mu\nu} + T_{\rm EM}^{\mu\nu}, \tag{A14}$$

$$Q^{\mu\nu} = \frac{en}{h^\dagger}\left(\frac{1}{m_+}T_+^{\mu\nu} - \frac{1}{m_-}T_-^{\mu\nu}\right), \tag{A15}$$

where $T_\pm^{\mu\nu} = g^{\mu\nu}p_\pm + h_\pm U_\pm^\mu U_\pm^\nu$ are the energy-momentum tensor of the two fluids and $T_{\rm EM}^{\mu\nu} = F^\mu{}_\sigma F^{\nu\sigma} - \frac{1}{4}g^{\mu\nu}F^{\kappa\lambda}F_{\kappa\lambda}$ is the Maxwell stress tensor.

The relativistic two-fluid equations come from the continuity equations of particle number and conservation law of energy and momentum:

$$\nabla_\nu(n_\pm U_\pm^\nu) = 0, \tag{A16}$$

$$\nabla_\nu(h_\pm U_\pm^\mu U_\pm^\nu) = -\nabla^\mu p_\pm \pm e n_\pm U_{\pm\nu}^\mu \pm R^\mu, \tag{A17}$$

where R^μ is the frictional 4-force density between the two fluids. It connected with the resistivity and current as

$$R^\mu = -\eta n e[J^\mu - \rho_{\rm e}'(1+\Theta)U^\mu]. \tag{A18}$$

[1]Equations (76) of Koide [6] contain several typographical errors. Here, we correct them in Eq. (A13). The variable $\rho_\pm$ in Equation (76) of Koide [6] is given by $\rho_\pm = mn_\pm$.

[1] P.M. Bellan, *Fundamentals of Plasma Physics* (Cambridge University Press, Cambridge, 2006).
[2] J. Frank, A. King, and D. Raine, *Accretion Power in Astrophysics* (Cambridge University Press, Cambridge, 1985).
[3] P.G. Goldreich and W.H. Julian, Astrophys. J. **157**, 869 (1969).
[4] N.J. Holloway, Nature (London) **246**, 6 (1973).
[5] N.J. Holloway and M.H.L. Pryce, Mon. Not. R. Astron. Soc. **194**, 95 (1981).
[6] S. Koide, Astrophys. J. **708**, 1459 (2010).
[7] S. Koide, Astrophys. J. **696**, 2220 (2009).
[8] H.N. Henriksen and D.R. Rayburn, Mon. Not. R. Astron. Soc. **166**, 409 (1974).
[9] F. Macchetto, A. Marconi, D.J. Axon, A. Cappeti, W. Sparks, and P. Crane, Astrophys. J. **489**, 579 (1997).
[10] S. Weinberg, *Gravitation and Cosmology* (John Wiley & Sons, New York, 1972).
[11] R. Genzel, F. Eisenhauer, and S. Gillessen, Rev. Mod. Phys. **82**, 3121 (2010).
[12] J. Greiner, J.G. Cuby, and M.J. McCaughrean, Nature (London) **414**, 522 (2001).
[13] S. Koide, T. Kudoh, and K. Shibata, Phys. Rev. D **74**, 044005 (2006).
[14] J.C. McKinney, Mon. Not. R. Astron. Soc. **368**, 1561 (2006).
[15] N. Watanabe and T. Yokoyama, Astrophys. J. **647**, L123 (2006).